Importance of Inlet Design on Culvert Capacity

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THE design of a culvert inlet has a significant bearing upon the relationship of the head to the discharge of a culvert. Its relative importance is contingent upon the type of flow occurring in the culvert, which in turn is governed by the location of the control section. For part-full flow the control may be either at the inlet or the outlet depending on whether the slope is hydraulically steep or mild. In the case of short culverts, control may be at the inlet even for horizontal or mild slopes.

The head-discharge curves of culverts having square-edged inlets have been compared with those for culverts having rounded inlets to illustrate the conditions for which a head-advantage may be obtained by using a rounded inlet. These comparisons have been made for three categories of culvert flow: long culverts on steep slopes, long culverts on mild slopes, and short culverts. Dimensionless head-discharge curves have been plotted for culvert flow in each category. For culverts on steep slopes, experimental data have been compared with the computed values and, since the agreement was reasonably good, serve as a basis for the analysis of flow in culverts operating under conditions other than those for which the tests were made.

The greatest head advantage for a particular discharge of the rounded inlet over that of a square-edged inlet was found for those cases in which the control section was located at the inlet. These were long culverts on steep slopes or short culverts where the length was negligible as regards barrel frictional resistance to flow. For long culverts on mild slopes, the head-advantage was far less pronounced.

● FROM a practical point of view, probably the most serious deficiency in the planning of simple culverts used in highways is in the culvert inlet. All too frequently the culvert is assumed to have much greater capacity than, in fact, it has; this reduction in capacity is frequently attributable to inadequacy of the culvert inlet.

Quite generally, the deficiencies of the inlet are thought of only in terms of their effect upon the head loss with the culvert flowing full; in reality this effect is of relatively minor importance in differentiating between good inlets and the poorest inlets customarily used. The important consideration is the overall hydraulics of the culvert in conveying runoff from one side of an embankment to the other, without impairing the roadway by overflow during high rates of runoff.

In general, the objective in designing a culvert is to provide a structure which will, under the conditions imposed, discharge a given flow with the least head; if the head and discharge are specified, the objective is to provide the most-economical culvert which, normally, is one with the least cross-sectional area.

The factors which combine to determine the character of flow in a culvert include all the design variables: slope, size, shape, length, and roughness of the culvert, the headwater and tailwater elevations, and inlet and outlet geometry. A convenient hydraulic classification of culverts is based on the location of the culvert control which is, in turn, determined by the relative magnitudes of the design variables. The nature of a control section is such that flow conditions downstream of the section do not affect the flow upstream of the section within a specified range of discharges. The principal flow characteristics are determined by location of the culvert control which for part-full flow may be either at the inlet or the outlet.

Control at the inlet usually occurs when the culvert has a steep slope and a free outlet; it may also occur with the culvert on a mild slope, provided the culvert is relatively short and the outlet is free. In one case of control at the inlet, the flow passes through critical depth at or near the inlet and supercritical flow exists through the barrel of the culvert. As disturbances cannot be propagated upstream in supercritical flow, it is apparent that the headwater elevation is dependent only on the geometry of the inlet and the discharge. This condition exists within a specific range of discharges; if this range is exceeded, the culvert may flow full and the control section will change.

For long culverts on a mild slope, flowing partly full, the control is usually at the outlet; with a free outlet the flow will pass through critical at the outlet. As a result, the headwater elevation is dependent on the discharge, wall friction, and inlet characteristics. If the tailwater is high enough to create a depth greater than critical at the outlet, the control is the tailwater elevation at the outlet, and the headwater is a function of the tailwater elevation as well as the other variables.

The preceding discussion of various control sections is included only to illustrate types which may exist. These will be discussed in more detail in subsequent sections.

The importance of inlet design as related to culvert capacity is contingent to a large extent upon the position of the control section. For inlet control, the geometry of the inlet has a significant influence upon the head required for a given discharge. A square-edged inlet causes separation to occur at the entrance and inhibits full flow in the culvert. A properly rounded inlet, on the other hand, avoids the separation and promotes full utilization of the barrel for flow. As a result of the availability of additional head in the culvert, the required water-surface elevation in the headwater pool is reduced — frequently very significantly reduced. When the control is at the outlet or when the culvert flows full, the geometry of the inlet becomes far less significant.

A comprehensive discussion of culvert entrances would necessarily be rather lengthy because of the many types involved. For example, the culvert may have a rounded, beveled, square, or bell-mouthed inlet. It may be in a defined or an undefined channel. It may be installed with the inlet flush or protruding (re-entrant) through a vertical or sloping headwall. Wing walls or warped transitions may be utilized. In most instances these variations will have a bearing on the culvert capacity. The square-edged inlet and the rounded inlet represent, in a sense, two extremes of inlet geometry. It appears that most culverts would possess inlets that fall somewhere between the two limits. The curves presented in this paper represent (for the case of circular culverts with a flush headwall) these two extremes of head-discharge curves, with the curves for other types falling between. However, a sharp-edged protruding inlet might be even worse hydraulically than the square-edged inlet.

Experimental and analytical investigations have for several years been undertaken at the St. Anthony Falls Hydraulic Laboratory, University of Minnesota, for the purpose of studying specific hydraulic characteristics on both full-scale culverts of various roughnesses (1) and dimensions (up to 3 ft. in diameter) and on smaller scale models. Tests with specific regard to entrance conditions of culverts were conducted in part under the sponsorship of the Minnesota State Highway Department and the United States Bureau of Public Roads.[1] These have been supplemented by student thesis research and other studies at the St. Anthony Falls Laboratory.

GLOSSARY

A	Cross-sectional area of the flow stream
A_o	Cross-sectional area of the culvert
α	Kinetic energy factor defined by Equation 5
b	Width of the stream at the water surface
C_c	Coefficient of contraction
C_v	Coefficient of velocity
d	Depth of flow in the culvert
d_c	Critical depth of flow
D	Diameter of the culvert
f	Friction factor in Darcy's formula

[1] Unpublished except for project reports. There will also be issued for limited distribution through the sponsorship of the Minnesota Highway Department Project Report No. 37 of the St. Anthony Falls Hydraulic Laboratory, "Effect of Inlet Design on Capacity of Culverts on Steep Slopes," giving results of specific culvert inlet experiments in more detail.

F Fall of culvert in length L so $S = F/L$

g Acceleration due to gravity

H Depth above culvert invert of headwater

H_o Specific energy with respect to culvert in-invert

K_e Entrance loss coefficient for full flow

L Length of culvert

n The Manning roughness coefficient

Q Discharge

Q_c Critical discharge

R Hydraulic radius of the flow stream

R_o Hydraulic radius of the culvert

S Slope of culvert

S_c Critical slope

θ Angle of inclination of the culvert from the horizontal

V Mean velocity of the flow stream

V_c Critical velocity of the flow stream

V_i Velocity at particular point in cross section

Salient experimental investigations were conducted in an apparatus constructed primarily for studies of this type. It consists of a channel 12 in. deep, 30 in. wide, and 50 ft. long in which culvert models of various sizes can be installed. The upstream 10-ft. section is separated from the remainder of the channel by a transverse bulkhead which normally forms the headwall of the culvert. This section has walls 28 in. high, as compared with 12 in. in the remainder of the channel, to permit variation of the head pool elevation. A second bulkhead is installed in the channel at the outlet end of the culvert model. The slope of the complete unit can be varied from 0 to 10 percent. Figures 1 and 2 illustrate the basic equipment. The model used in the studies was constructed of 4-in. diameter Lucite pipe and had an overall length of 35 ft. The ends of the pipe were flush with the bulkheads which formed the end walls of the culvert. The inlet section was removable so that square-edged and rounded inlets (Fig. 2) could be interchanged. The rounded inlet used in these tests had a radius of rounding equal to 15 percent of the pipe diameter.[2] Piezometers were located at frequent intervals along the culvert for pressure measurements.

In an earlier series of tests, the culvert was tested with both the inlet and outlet submerged in order to obtain data on frictional losses and entrance-loss coefficients for full flow. However, in the series of tests here concerned, the outlet was completely free, i.e., the jet was unsupported and discharged into the atmosphere. In the practical or applied case the tailwater may be raised considerably before causing any essential modification of the flow in the culvert.

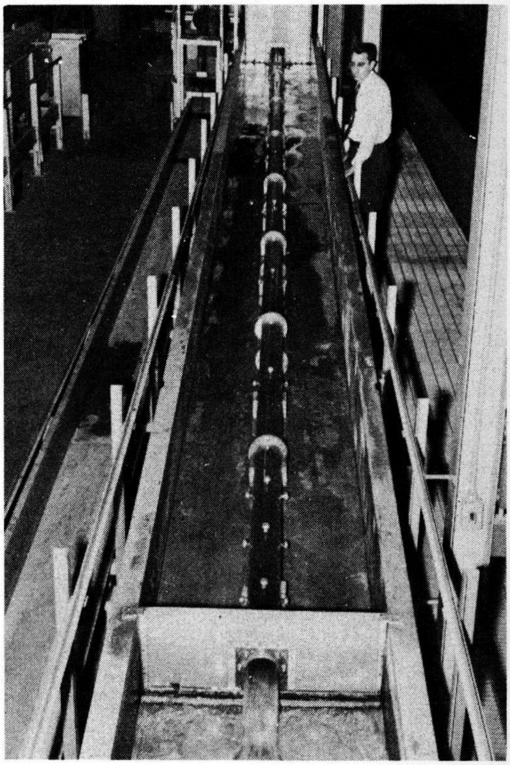

Figure 1. Variable-slope culvert model.

Data were obtained on the height of the head pool above the inlet invert for variations in inlet type, discharge, and culvert slope. When the culvert flowed full for at least a portion of the length, such as sometimes occurred when using a rounded inlet, data on the hydraulic gradient and the magnitude of pressure fluctuations were obtained.

[2] A theoretical explanation for the use of 15 percent of the pipe diameter as the radius of rounding is based upon recognition that for a sharp-edged orifice the coefficient of contraction is nearly 0.61; thus the entrance area must be 1/0.61 times the area at the vena contracta so that $(D_e/D_c)^2 = 1/0.61$ or $D_e = 1.28\,D_c$. Thus a 15 percent D enlargement of the entrance satisfies the criterion. Actually also this has been established experimentally and reported in "Suppression of Pipe Intake Losses by Various Degrees of Rounding" by J. B. Hamilton (Bulletin 51, Engineering Experiment Station, University of Washington, November, 1929), which corresponds exactly to the theoretical explanation of the authors.

Results of these experimental studies are summarized herein; there is also given an analysis of the flow conditions based upon fundamental hydraulics. Figures 4 and 5 illustrate some of the flow types which may occur in culverts with free outlets. The discussion has been restricted to culverts with free outlets because the case of culverts flowing with submerged outlets has been treated rather fully in other publications, and because of space limitations.

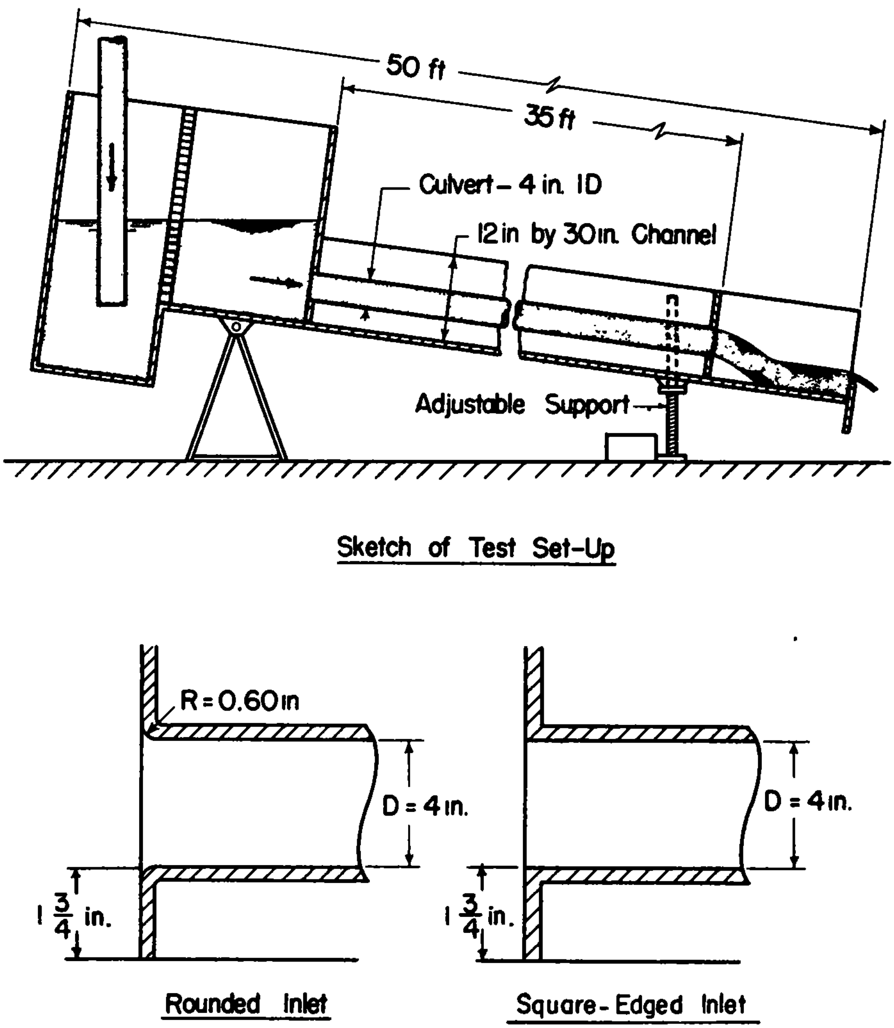

Figure 2. Equipment and inlets used in model tests.

Figure 6 illustrates some of the experimental data obtained for square-edged and rounded inlets. It may be noted that for culverts on steep slopes the head required for a specified discharge is much less for a rounded inlet than for a square-edged inlet. This is especially pronouned for values of $Q/D^{5/2}$ in excess of four. The head advantage of the rounded inlet is dependent on the culvert slope and on the culvert length. An explanation of the flow conditions with the model culvert on a 4 percent slope may be of interest as a typical test. With a square-edged inlet the culvert flowed part-full for the complete test range which included values of $Q/D^{5/2}$ up to 9.0. Larger discharges were not used because the required head would have exceeded the height of the head tank walls. With a rounded inlet the culvert flowed part-full for values of $Q/D^{5/2}$ less than 4.0 ($H/D < 1.3$). For $4.0 < Q/D^{5/2} < 8.5$, the culvert either alternated between full and part full (slug flow or mixed flow); this caused the headwater elevation to fluctuate between H/D values of about 1.2 to 1.5. For values of $Q/D^{5/2}$ in excess of 8.5, the culvert flowed full. The head-discharge curve is illustrated in Figure 6.

In some instances the culvert behavior and the head-discharge curves are dependent on the culvert length as well as the slope and other variables. An analysis of flow conditions for (1) long culverts on steep slopes, (2) long culverts on mild slopes, and (3) short culverts (where barrel wall friction has negligible influence on flow pattern) is presented following a discussion of some basic principles. Typical problems are solved as examples of each type.

as a closed conduit or pipe. The pressure gradient then no longer necessarily coincides with the water surface. When a straight culvert flows full, the headwater level is, of course, above the crown of the culvert; however, the culvert does not necessarily flow full when the headwater is above the crown, even though this height may be several times the diameter of the culvert. The complete range of hydraulic relationships between discharge and head on the culvert includes both part-full and full-flow conditions, and the different types of flow follow different algebraic relationships. These relationships can now be quite adequately defined.

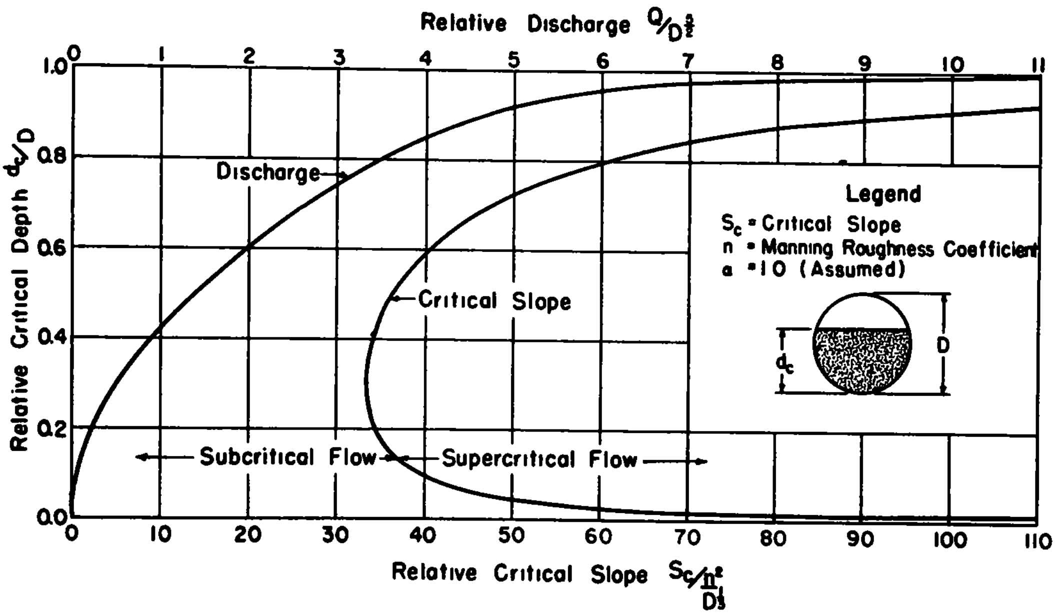

Figure 3. Critical culvert slope as a function of depth.

BASIC CONSIDERATIONS, CRITICAL DEPTH AND SLOPE IN PART-FULL FLOW

A culvert may flow either full or partly full, depending upon the specific hydraulic conditions. In part-full flow, the culvert behaves as an open channel with a free surface, the depth of flow being less than the vertical diameter or height of the culvert. In fullflow, the culvert behaves as a closed conduit or pipe.

For part-full flow, the total energy per unit weight of water referred to the culvert invert is called the specific energy H_o and may be written as

$$H_o = \frac{\alpha V^2}{2g} + d \qquad (1)$$

where V is the mean velocity, d is the depth, g is the acceleration due to gravity, and α is a kinetic energy correction factor, the numerical value of which depends upon the velocity distribution over the cross section. (For uniform velocity distribution, α is unity.)

The minimum value of the specific energy corresponds to the critical flow conditions, for which it can be shown analytically

$$\left(\frac{Q_c}{D^{5/2}}\right) = \left(\frac{\pi}{4}\right)^3 \frac{g}{\alpha} \left(\frac{A/A_o}{b/D}\right)^3 \qquad (2)$$

where Q_c is the critical discharge, A and A_o are respectively the cross-sectional area of the flow and of the culvert, b is the surface width, and D is the diameter of the culvert.

In order to eliminate Q, if Equation 2 is combined with the Manning formula

$$\left[Q = (1.486/n)\ AR^{2/3}\ S_c^{\ 1/2}\right]$$

an expression results for the critical slope and may be written as

$$\frac{S_c D^{1/3}}{n^2} = \frac{2.26g}{\alpha} \frac{(A/A_o)}{(b/D)\ (R/R_o)^{4/3}} \qquad (3)$$

In this equation S_c is the critical slope of the culvert, n is the Manning roughness coefficient, and R and R_o are respectively the hydraulic radii of the flow and the culvert section. In Figure 3,

$$S_c/(n^2/D^{1/3})$$

and $Q_c/D^{5/2}$ have been plotted as functions of d/D. For very small depths and for depths approaching the magnitude of the culvert diameter, the critical slope be-

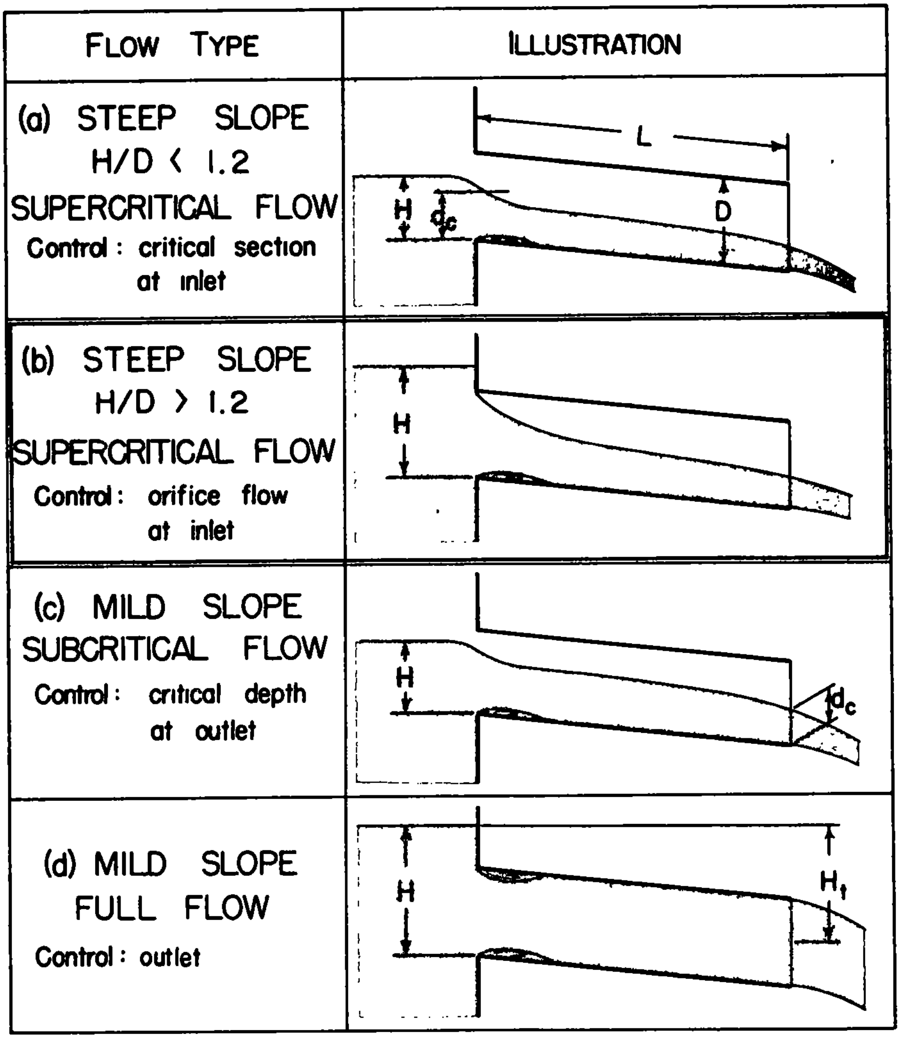

Figure 4. Typical flow conditions for square-edged inlet.

comes quite large, but over the wide intermediate normal range of part-full flow conditions through the culvert the critical slope varies within narrower limits. If the actual slope is greater than S_c (see Equation 3) for a given discharge, normal flow in the culvert will be superciritcal and the depth less than critical. If the slope is greater than critical for this discharge, the culvert will flow part-full for its entire length. For a slope less than critical the culvert will flow part-full if it is short enough that retardation of flow by barrel wall friction is insufficient to induce critical flow, or full if it is sufficiently long.

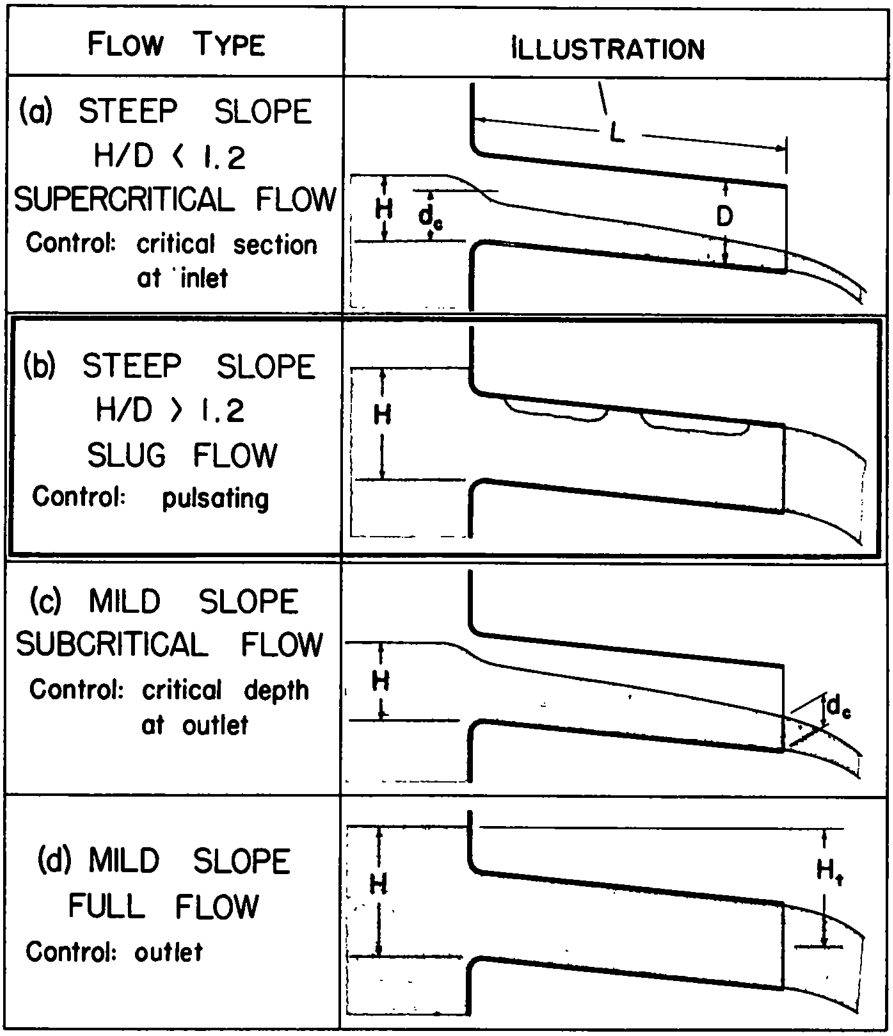

Figure 5. Typical flow conditions for rounded inlet.

slope is less than S_c, the normal flow will be subcritical and the depth greater than critical.

If the head is above the culvert crown, the depth within the culvert at the inlet is governed by the contraction and the character of the flow in the barrel is dependent upon the length and slope. If the actual

LONG CULVERTS WITH FREE OUTLETS ON STEEP SLOPES

In the case of culverts with steep slopes, that is [$S > S_c$ (Fig. 3)], the transition from subcritical flow in the approach channel to the super-critical flow in the culvert takes place at the culvert inlet (Figs. 4a and 5a) and corresponds to the condition under which Equation 2 applies. If we assume that the energy loss from the head pool to the critical section is negligible, we may write

$$\frac{H}{D} = \frac{\alpha V_c^2}{2gD} + \frac{d_c}{D} \qquad (4)$$

where from Equation 2

$$\frac{\alpha V_c^2}{2gD} = \frac{A}{2bD} = \frac{\pi}{8}\,\frac{(A/A_0)}{(b/D)}$$

If α is defined as the ratio of the average of the velocity heads of the individual flow filaments to the velocity head based upon the average velocity through the gross cross section, it may be written (2) as

$$\alpha = \frac{1}{A}\int_A \left(\frac{V_i}{V}\right)^2 dA \qquad (5)$$

If it is further assumed that the velocity between the vena contracta and the culvert wall is zero, then the value of α at the vena contracta is

$$\alpha = \frac{1}{C_c} \qquad (6)$$

Since the contraction coefficient depends upon the geometry of the inlet, the value of α will also depend upon the geometry and, of course, the depth of the inlet.

For the square-edged inlet and approach conditions used in these experiments, computed values of α varied from 1.25 to 1.42 as the depth changed from 0.1 D to 0.9 D at the inlet (3). The head-discharge curve computed from Equations 2 and 4 using these computed values of α agreed well with the measured head-discharge curve for the square-edged inlet (Fig. 6).

For the fully-rounded inlet where no separation occurs, it is assumed that $\alpha = 1.0$, that is, uniform velocity distribution just inside the culvert inlet. The head-discharge curve for the rounded inlet was also computed from Equations 2 and 4 with $\alpha = 1.0$ and compared with the measured curve for the rounded inlet in Figure 6.

Agreement with the measured values for the head-discharge curves were obtained up to values of H/D of about 1.2 in each case. This appears to be the limit of H/D for which a free surface can be maintained through the inlet; that is, the flow is not in contact with the inlet crown. Two separate curves are obtained, one for the square-edged inlet and one for the rounded inlet.

As the discharge increases, so that H/D is greater than about 1.3, the flow will normally be in contact with the wall entirely around the periphery of the entrance. With the square-edged inlet, separation at the corner will cause a contraction of the jet (Fig. 4b). If, in addition, the culvert is on a steep slope or is not too long, the culvert will not flow full. Hence, it may be assumed that the inlet operates in the same manner as an orifice. The equation for the discharge through an orifice under low heads may be written (4) as

$$\frac{Q}{D^{5/2}} = C_c \frac{\pi}{4}\sqrt{2g}\left(\frac{H}{D} - \frac{1}{2}\right)^{1/2}\left[1 - \frac{1}{128\ (H/D - 1/2)^2}\right] \qquad (7)$$

The term in brackets represents the effect of head on the velocity distribution in the orifice, particularly for low heads, and may be considered as a coefficient of velocity such that

$$C_v = \left[1 - \frac{1}{128\ (H/D - 1/2)^2}\right] \qquad (8)$$

which value rapidly approaches unity with increase in head.

With the square-edged inlet some experiments were made with the barrel removed from the inlet so that the inlet was a true orifice. The head-discharge curve so measured coincided with that measured with the barrel in place, indicating that for this particular arrangement of inlet and approach channel in which the bed of the approach channel was below the inlet invert, the inlet is similar to an orifice. The coefficient of contraction also varies somewhat with the head and may be approximated by a consideration of the geometry of the inlet and the head pool. The computed head-discharge curve for the square-edged inlet agreed with experimental data for values of H/D> 1.4 when the inlet acts as an orifice is shown in Figure 6 as a continuation of the curve for part-full flow at the inlet. A transition occurs for H/D between 1.2 and 1.4 from part-full flow at the inlet to orifice flow.

The measured head-discharge curve for the square-edged inlet corresponds very closely to that presented by Mavis (5) also for a square-edged inlet.

On the other hand, when the inlet is well rounded, separation at the inlet does not occur (Fig. 5b); consequently, the culvert begins immediately to flow full in the neighborhood of the inlet. The zone of full flow rapidly extends down the culvert toward the outlet. In the process of moving toward the outlet, an added head due to the slope of the culvert becomes effective. This added head tends to increase the discharge in the culvert above that of the inflow to the approach channel. The increased discharge causes a lowering of the water surface just upstream of the inlet. As the water surface is lowered it reaches a point where vortices form at the inlet and air is sucked into the culvert and increases the pressure to atmospheric. This breaks the seal and with the loss of the added velocity head, the discharge decreases below that of the inflow, and the water surface rises until the inlet is again sealed and the culvert again starts to flow full. The cycle then repeats itself; pulsating flow develops through the culvert. The relationship of head to discharge in the region of pulsating flow has not been determined analytically so that dashed curves have been drawn through the experimental points in Figure 6. In this range the data for all slopes from 2 to 8 percent fall on the same curve. The two lines represent the range of fluctuation of the head in pulsating flow.

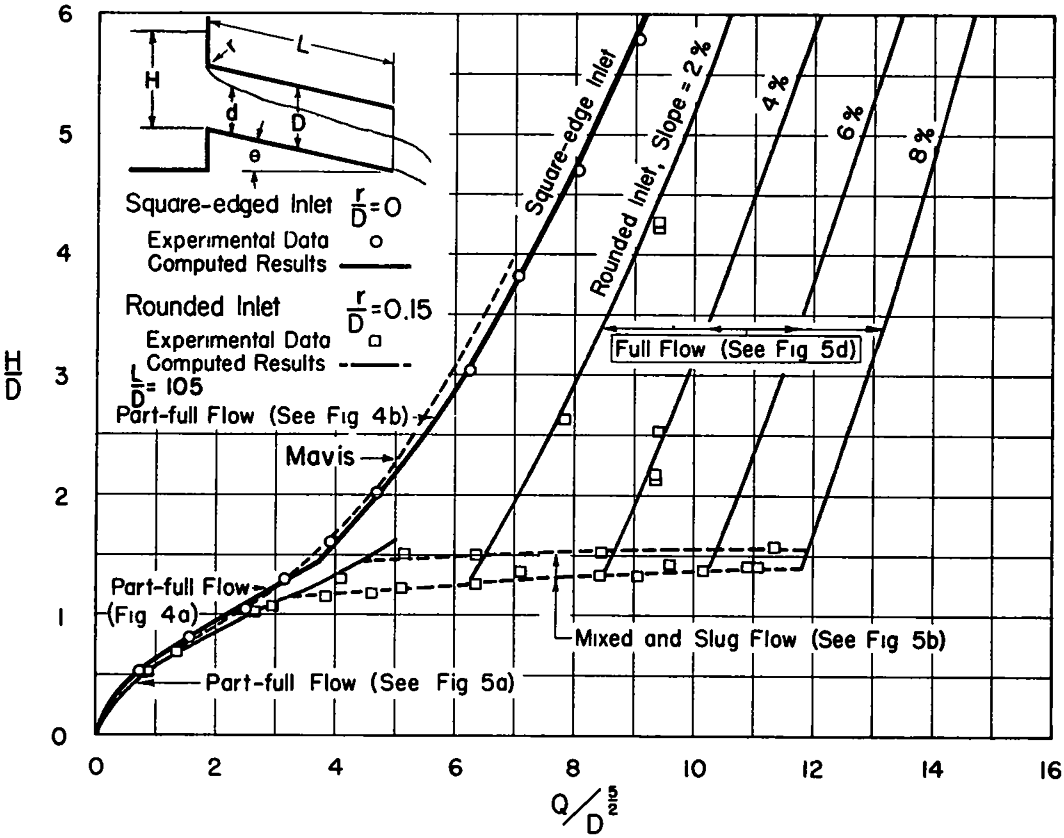

Figure 6. Comparison of head-discharge curves for square-edged and rounded inlets for long culvert on steep slope (from experiments on model culvert).

When the inflow is large enough for a particular slope to permit the "slug" to extend the entire length of the culvert before the headwater is drawn down sufficiently to permit the intake of air, the "slug" or "mixed" flow phenomena ceases and the culvert flows full continuously.

When the culvert is flowing full, the head-discharge relationship may be determined by the application of Bernoulli's theorem to the flow so that

$$\frac{H}{D}-\frac{1}{2}+\frac{L}{D}\sin\theta=\frac{8}{\pi^2\cdot g}(1+K_e+f\frac{L}{D})\left(\frac{Q}{D^{5/2}}\right)^2 \quad (9)$$

where L is the length of the culvert; θ is the angle of inclination of the culvert from the horizontal so that $S = \sin\theta$, or $L/D \sin\theta = F/D$ where F is the fall in length L; f is the friction factor which for smooth culverts is a function of the Reynolds number, and K_e is the entrance loss coefficient for full flow.

In Equation 9 it is assumed that the pressure line is at the center of the culvert at the outlet. The location depends upon the value of $Q/D^{5/2}$, being above the center for small values of $Q/D^{5/2}$ and approaching the center of the culvert as $Q/D^{5/2}$ increases (3).

Inspection of Equation 9 indicates that for full flow the head-discharge curve depends upon slope, length, and roughness of the culvert as well as the entrance loss; therefore the dimensionless curve will be different for each culvert as well as for each slope of the culvert.

For the culvert model tested with the rounded inlet, the head discharge follows the critical depth curve for rounded inlets up to $H/D \cong 1.2$, at which point slug or mixed flow starts. As the discharge increases the slug or mixed flow continues and the head follows the slug-or mixed-flow curves to the point where the full-flow curve for the particular culvert intersects the mixed-flow curves. When this point is reached, the curve continues up the full-flow curve and the culvert flows full. For the model with the rounded inlet, full flow occurred at the point where the head-discharge curve for full flow meets the curves for mixed or slug flow. This point, of course, varied with the culvert slope, since a different full flow curve applies to each slope. In Figure 6, the experimental points on the full flow curve indicate that the culvert was flowing full. In these computations the factors corresponding to the experiments were used in order that a comparison with the experimental results might be made. Here $L/D = 105$, $K_e = 0.08$, and the value of f as a function of Reynolds number, were obtained from previous experiments on the same culvert.

In Figure 6 a comparison may be made of the effect on the head-discharge curve of rounding the inlet corners. It is apparent that for headwater elevation above the crown of the culvert ($H/D>1.5$), a very strong advantage in the head which is required to pass a given discharge, accrues to the culvert with the rounded inlet. In the region where the flow passes through critical at the inlet (that is, $H/D<1.2$), the head advantage in a rounded inlet is less.

The experimental results presented in Figure 6 were obtained from experiments on the model culvert (4 in. in diameter). In general, good agreement was obtained with curves computed on the basis of hydraulic principles with the exception of the curves for mixed and slug flows. This phenomenon forms the transition between part-full critical flow at the inlet and full flow for culverts with rounded inlets and was based entirely upon the model experiments. An analytical solution for this phase is desirable before extension of the results to prototype culverts is undertaken.

Example of Culvert Flow on Steep Slopes

In order to illustrate the foregoing principles, assume that a prefabricated concrete culvert 3 ft. in diameter and 300 ft. long is to be laid on a 2 percent slope to discharge 140 cfs. Assume further that the outlet is free and that a headwall at the entrance provides a flush inlet. For the concrete pipe the following factors apply:

$$\frac{L}{D} = 100 \qquad \frac{Q}{D^{5/2}} = 9.0$$

n (partly full flow)[3] = 0.011
f (full flow)[3] = 0.015; (n=0.010 to 0.011) approximately

The factor

$$\frac{S}{n^2/D^{1/3}} = \frac{0.02}{(0.011)^2/(3.0)^{1/3}} = 239$$

is considerably greater than the values given in Figure 3 for the critical slope throughout the greater portion of the depth. Consequently, the culvert lies on a steep slope. If it is assumed that a square-edged inlet has been provided (note here

[3] Based on full-scale experiments (1) Customarily in the past higher n values have been used for concrete pipe and such higher values might be proper for inferior or deteriorated pipe

that a socket end is not as severe as a square-edged inlet), the head required for a discharge of 140 cfs. can be obtained directly from Figure 6 since the head-discharge curve for culverts on steep slopes with square-edged inlets is independent of the characteristics of the barrel. From the figure it appears that for

$$\frac{Q}{D^{5/2}} = 9.0, \ \frac{H}{D} = 5.80$$

Consequently, to discharge 140 cfs. through the culvert will require a head of 17.4 ft. above the invert or 14.4 ft. over the culvert crown.

If the inlet were rounded so that no separation at the inlet occurred, the culvert would flow full when the upstream water surface became high enough to seal the entrance. If it is assumed that for a discharge of 140 cfs. the culvert will flow full, Equation 9 will describe the flow or, in addition to the factors given above, we have

$$\frac{L}{D} \sin\theta = 100 \times 0.02 = 2.0$$

$$K_e = 0.08$$

Then

$$\frac{H}{D} - \frac{1}{2} + 2.0 = 0.0252\,(1+0.08+1.50)\,9^2$$

$$\frac{H}{D} = 3.77$$

Since H/D as computed is greater than 1.5, the assumption that the culvert flows full for the prescribed discharge is satisfied. For a rounded inlet then the head required to discharge 140 cfs. is 11.3 ft. above the invert or only 8.3 ft. above the crown of the culvert as compared to 14.4 ft. above the crown if the inlet had been square-edged. The difference is attributable entirely to the entrance condition.

If the culvert had been placed on a 4 percent slope the head above the crown would have been about 2.3 ft. (Fig. 6). In this case, since H/D for $Q/D^{5/2} = 9.0$ is only slightly above the zone of pulsating flow, the flow in the culvert could conceivably be pulsating. If the slope had been 5 percent, certainly pulsating flow would occur in the culvert with the rounded inlet. However, even in this case with a culvert on a 2 percent slope, the head required would be considerably less than that required if the inlet had been square-edged.

LONG CULVERTS WITH FREE OUTLETS ON HORIZONTAL OR MILD SLOPES

The distinction between a long and short culvert in the hydraulic sense is significant when for a particular discharge the culvert is on a mild slope. A long culvert may be qualitatively defined as one which is sufficiently long to flow full on a mild slope when the head is above the culvert crown. If the culvert is short, the supercritical flow caused by the inlet contraction passes through the culvert without making contact with the culvert crown and the inlet assumes the control. If the culvert is long enough, the water surface profile would rise to the crown or the flow would pass through a hydraulic jump to reach the crown. When this happens, the jump or mixed flow would pump air from the space upstream, reducing the pressure thus causing the jump to move upstream. Either it would reach the inlet and the culvert would flow full, or the headwater elevation would be reduced enough to permit vortices to form and air to be sucked into the culvert. In this case, a slug or pulsating flow would develop.

When the culvert is horizontal, or at least the slope is less than S_c as defined by Equation 3, the flow in the culvert at depths less than D must be subcritical and the control section moves to the outlet end of the culvert. For larger discharges the culvert will flow full. For those discharges where the culvert flows partly full, the water surface assumes the profile of a drawdown curve passing through critical depth at the outlet and acquiring a relative depth at the inlet end of the culvert that depends on the slope, length, and roughness of the culvert (Figs. 4c and 5c). This relative depth is independent of the geometry of the inlet, and hence is the same whether the inlet is square-edged or rounded. If Bernoulli's equation is written between a point upstream of the inlet and a point within the culvert just downstream of the inlet, there is obtained for the head upstream the expression

$$\frac{H}{D} = \frac{d}{D} + \frac{8}{\pi^2 \cdot g}\,(1 + K_e)\left(\frac{Q}{D^{5/2}}\right)\frac{1}{(A/A_o)^2} \qquad (10)$$

where H/D is the relative head acting on the culvert and d/D is the relative depth within the inlet. Equation 10 applies both to the square-edged and rounded inlets; the difference is in the magnitude of the entrance loss coefficient K_e. Experiments on the 4-in. Lucite culvert indicated that for the square-edged inlet $K_e = 0.43$ and for a well-rounded inlet ($r/D \geq 0.15$) $K_e = 0.08$. Experiments on full-scale prefabricated concrete culverts (1) with socket-end inlets showed that for reentrant inlets $K_e = 0.15$, and for flush inlets $K_e = 0.10$. For given discharge through a particular culvert will depend on the magnitude of K_e corresponding to whether the inlet is square-edged or rounded. The same factors applicable to part-full flow may also be applied to full flow. The head-discharge curves for culverts on a zero slope may be compared in Figure 7 to show the effect of

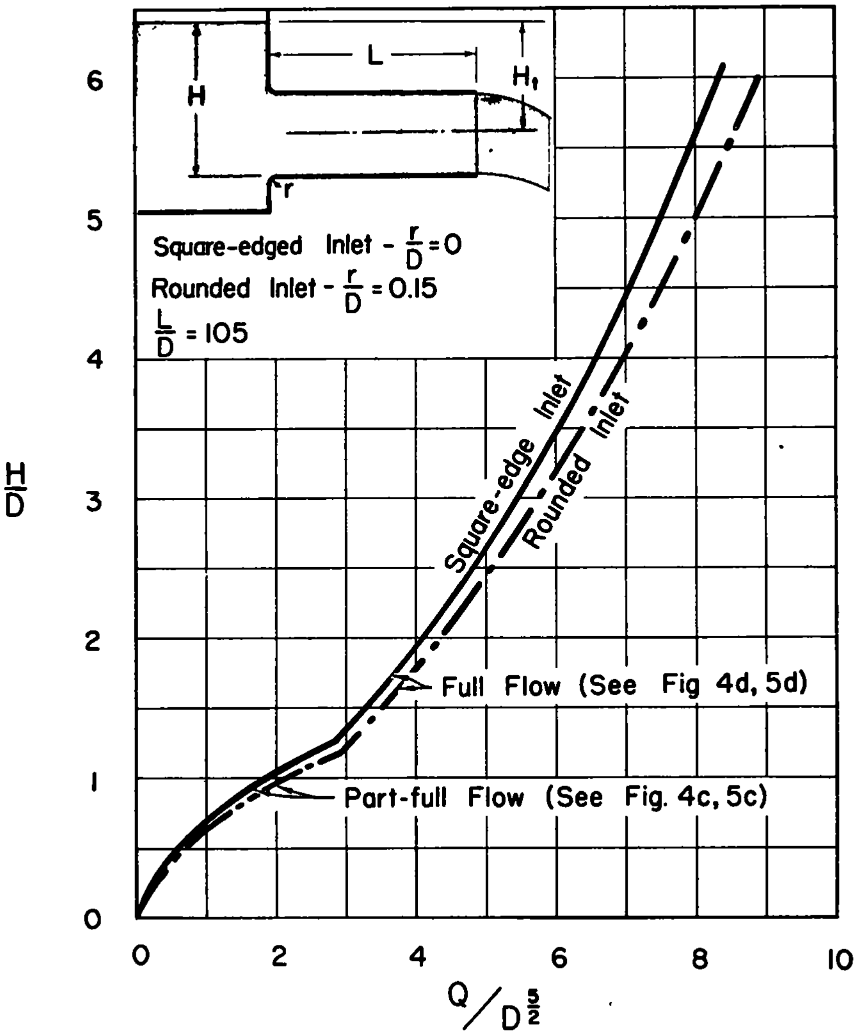

Figure 7. Comparison of computed head-discharge curves for square-edged and rounded inlets for long culvert on zero slope.

culverts fabricated from corrugated metal pipes, the corresponding entrance losses were as follows: projecting (re-entrant) inlet $K_e = 0.85$, flush inlet $K_e = 0.50$. For larger relative discharges, a point will be reached when the culvert will flow full throughout its length (Figs. 4d and 5d). When this occurs, Equation 9 applies. Here again the difference in head required for a inlet rounding on the required head. The curves in Figure 7 were computed to indicate the influence of rounding the inlet and are not based on experimental data.

Example of Flow in Horizontal Culvert (Zero Slope)

If it is assumed that the culvert described in the previous hypothetical example had been laid horizontally rather than on a 2 percent slope, the influence of inlet geometry on the flow in culverts on mild slopes may be illustrated. Again the factors which apply, assuming a square-edged or rounded inlet are as follows:

$\frac{L}{D} = 100$ K_e (square-edged) = 0.43

$\frac{Q}{D^{5/2}} = 9.0$ K_e (rounded) = 0.08

$\sin\theta = 0$ f = 0.015 (or about 0.011 for Manning n)

If it is assumed as before that the culvert flows full, then

$$\frac{H}{D} - \frac{1}{2} + \frac{L}{D}\sin\theta = \frac{8}{\pi^2 \cdot g}\left(1 + K_e + f\frac{L}{D}\right)\left(\frac{Q}{D^{5/2}}\right)^2$$

For the square-edged inlet

$$\frac{H}{D} = 0.0252\,(1 + 0.43 + 1.50)\;9^2 + 0.50 - 0 = 6.47$$

H = 6.47 x 3.0 = 19.41 ft. above invert

For the rounded inlet

$$\frac{H}{D} = 0.0252\,(1 + 0.8 + 1.50)\;9^2 + 0.50 = 5.77$$

or

H = 5.77 x 3.0 = 17.31 ft. above invert

The computed value of H/D indicates that the assumption that the culvert flows full is valid.

In this case the advantage of using a rounded inlet is approximately 2.1 ft. of head.

SHORT CULVERTS

When a culvert is short, the flow characteristics become relatively independent of the slope, and the factors that involve the length become comparatively unimportant. (In this connection the barrel-wall roughness comes into consideration: a smooth-walled culvert can be considerably longer than a rough-walled culvert and still be classified as "short.") Consequently, the control section is essentially at the inlet for all conditions. Therefore the head-discharge relationship for part-full flow should be much the same as for culverts on a steep slope in the case of both the square-edged and rounded inlets. The head-discharge curve for the square-edged inlet when the headwater elevation is above the top of the pipe is the same as that for a similar culvert on a steep slope. In the case of the short culvert with the rounded inlet flowing full, Equation 9 with $L \rightarrow 0$ or becoming very small as regards wall friction would describe the flow, the magnitude of $L/D \sin\theta$ and $f(L/D)$ both being negligible. Between the part-full phase and the full-flow phase there exists a transition zone of pulsating flow in which the culvert is alternately full and partly full.

The head-discharge curves for short culverts of any slope have been computed on the above basis and plotted in Figure 8 for comparison. In these computations it was assumed that L could be considered equal to zero, and the entrance loss coefficient K_e for the rounded inlet, as before, was assumed equal to 0.08.

It is apparent from the plot that a considerable advantage in head is gained for the larger discharges by the simple expedient of rounding the inlet to reduce the degree of contraction of the jet.

Example of Flow in Short Culverts

Consider the hypothetical culvert previously described again modified by reducing its length to the point where pipe friction is a negligible amount; the culvert will be taken as horizontal. Then, using the same discharge as before ($Q/D^{5/2}=9.0$), we may take the value of H/D directly from the curve for the square-edged inlet in Figure 8, since H/D is a function of inlet geometry only. Therefore

$$\frac{H}{D} = 5.80$$

or

H = 5.80D = 17.4 ft. above the invert

On the other hand, if the inlet is rounded, the value of H/D may also be taken from Figure 8 since in this case too the head-discharge relationship depends only on the inlet geometry. Here

$$\frac{H}{D} = 2.55$$

and

$H = 2.55D = 7.65$ ft. above the invert

In this case the advantage in head of the rounded inlet over the square-edged inlet amounts to 9.75 ft., a quite significant amount.

able part of the analysis. As part of a thesis project Madhav Manohar performed a rather extensive series of experiments to study the flow in culverts on steep slopes using both a square-edged and a rounded inlet. His experiments covered the range

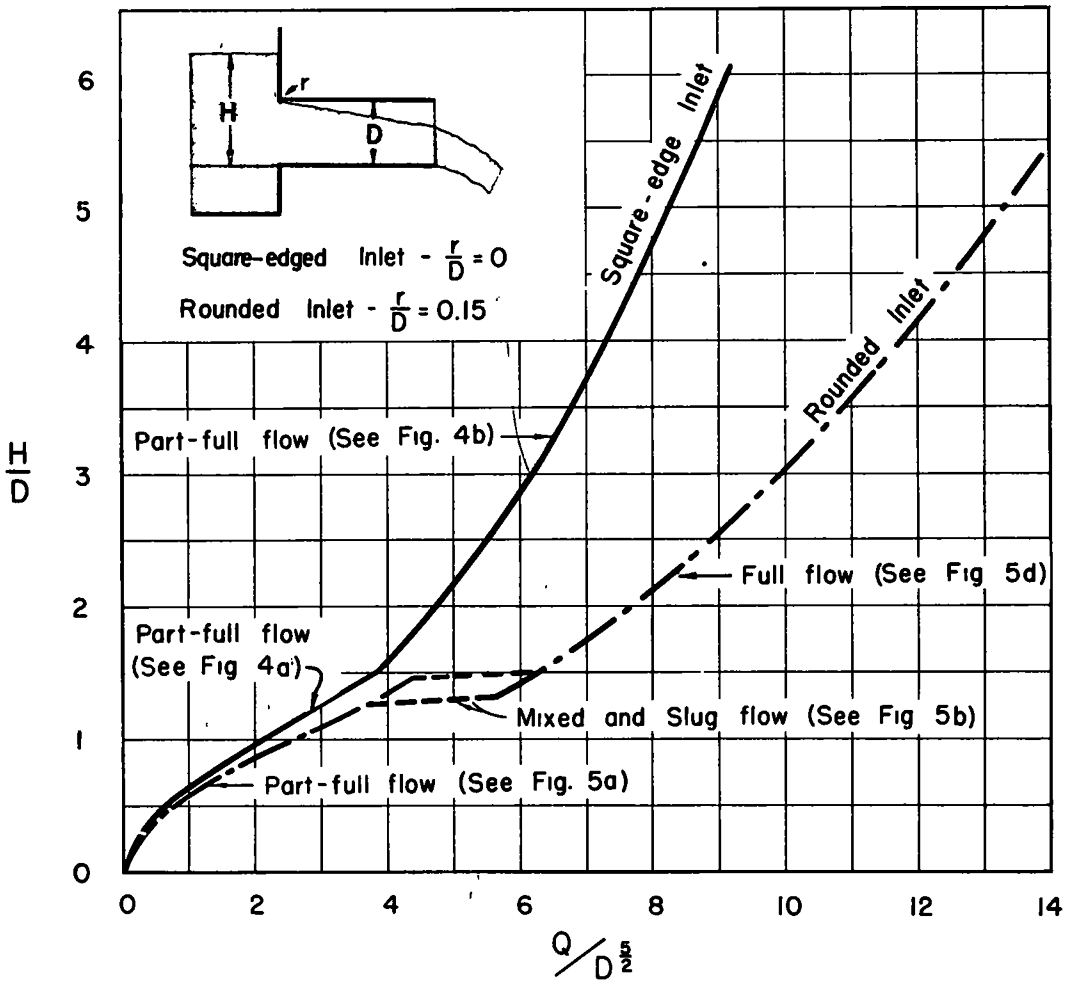

Figure 8. Comparison of computed head-discharge curves for square-edged and rounded inlets for short culvert, control at inlet (culvert horizontal and pipe friction negligible).

of orifice flow through the square-edged inlet, and the slug-flow and mixed-flow phases for rounded inlets, supplementing earlier tests.

ACKNOWLEDGMENT

The experiments described here and used in the discussion of the influence of inlet geometry on the capacity of culverts were performed at the St. Anthony Falls Hydraulic Laboratory under the general supervision of Lorenz G. Straub, director. The project leader of those under the sponsorship of the Minnesota State Highway Department and the Bureau of Public Roads was Henry M. Morris who did a considerable part of the analysis. As part of a thesis project Madhav Manohar performed a rather extensive series of experiments to study the flow in culverts on steep slopes using both a square-edged and a rounded inlet. His experiments covered the range of orifice flow through the square-edged inlet, and the slug-flow and mixed-flow phases for rounded inlets, supplementing earlier tests.

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Discussion

F. T. MAVIS and T. E. STELSON, Department of Civil Engineering, Carnegie Institute of Technology, Pittsburgh, Pennsylvania — In this paper there are many points of similarity, and even identity, with studies (1) of the hydraulics of culverts published by The Pennsylvania State College in 1942 as Engineering Experiment Station Bulletin 56. Abstracts of eleven studies which had been conducted earlier at the State University of Iowa, beginning with the pioneer work of David L. Yarnell, Floyd A. Nagler, and Sherman M. Woodward (2) were reproduced there by permission (3). Further work has been done at Carnegie Institute of Technology by civil engineering staff and students (4).

Straub, Anderson, and Bowers have verified the types of flow and the head-discharge curves for conduits with square-edged entrances which should by this time be generally known. They have added information concerning entrance-loss factors for rounded entrances. These contributions should be reassuring and helpful to designing engineerings.

However, we would call attention to several points in the paper and raise some questions that may be interpreted as cautions:

1. Consider the example (following Equation 8) of a culvert 3 ft. in diameter and 300 ft. long discharging 140 cu. ft. per sec. (at an average velocity of 20 ft. per sec.) Is this typical of good practice? How would this fill-ripping velocity be handled at the outlet? Wouldn't the designer want to buy a bigger pipe in this case — and wouldn't the pipe salesman be willing to sell it to him?

2. If the culvert in this example is to be full, the discharge necessary to keep the pipe flowing full must first pass the inlet section as a control before the control point can move down to the outlet. Unless the culvert is first submerged by backwater, the cycle of operation during a storm would be either to flow part full from beginning to end; or to flow part full, then full (steadily or slug-wise), and finally part full (or empty). Computations and sketches of all types of flow are detailed in Bulletin 56 and in Concrete Pipe Lines (5).

3. Rounding the inlet of a culvert may increase the discharge for a given headwater depth; because (1) the rounded inlet reduces the contraction of the flow when the culvert flows part-full, or reduces the entrance-loss coefficient when the culvert flows full and (2) the rounded inlet may cause the culvert to flow full instead of part-full. If the culvert flows full and the slope is steep enough, negative pressures may increase the effective head.

Rounding inlets for Reason 3(a) is sound and may be easily analyzed by methods previously developed for weir-orifices. Rounding inlets for Reason 3(b) is more likely to be questionable practice than clever design. To illustrate, look closely at the example which follows Equation 8. The velocity head and entrance loss is 6.1 ft. for the culvert with a rounded inlet. Yet at the crown the water is only 2.3 ft. deep. Can a negative pressure of 3.8 ft. be maintained at a point that is only 2.3 ft. below the free surface of the headwater pool? Undoubtedly a vortex would form and relieve the negative pressure unless the flow were well baffled. When the negative pressure is relieved, the headwater level may rise to 7.6 ft. above the invert at entrance. Note that if the slope of the culvert in this example were 5 percent (instead of 4 percent) the headwater level would have been figured to be below the crown, requiring a negative pressure in the atmosphere above the entrance — and this is clearly impossible!

To extrapolate model data to prototypes is tricky at best — and the caution that is necessary if subatmospheric pressures are involved is well illustrated by extending this example.

Laboratory studies and analyses such as those reported in the paper and discussion help engineers understand how a culvert behaves under a given set of field conditions. They can help even more in unscrambling hydrologic data when a culvert is used as a flow-measuring device. Incidentally, a culvert is a practical tool for measuring discharge and one that is perhaps too seldom used. This paper will lend additional confidence to engineers to use culverts to measure stream flow (6).

The engineer who designs and builds culverts that are to serve as adequate drainage structures in the uncertain future may be inclined to consider some such suggestions as these:

1. Don't overlook outlet velocities. What will happen if they are too high?

2. Don't expect pipes with rounded entrances to work miracles. They may discharge more water for a given total head than pipes with square-edged entrances, but sometimes it may be better to lose head under control in the pipe rather than below the outlet.

3. Don't make a "long" culvert "small" and "steep" merely to gain hydraulic advantage, forgetting that it may sniff air and need to be cleaned.

4. Keep designs and design-computations simple and checkable. The uncertainties of stormy weather are much greater than the uncertainties of culvert-hydraulics; and the answer to "What's worth figuring?" will stem as much from the hardheadedness of engineering judgment as from the niceties of hydraulic science.

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CARL F. IZZARD, Chief, Hydraulic Research Branch, Bureau of Public Roads — This paper demonstrates conclusively the fact that under certain conditions culverts with square-edged entrances cause excessive headwater because the barrel does not flow full. The common assumption that any culvert will flow full if laid on a slope equal to or less than the friction slope may be wrong, particularly in the case of relatively short, smooth culverts. The paper demonstrates that a rounded edge will cause the barrel to flow full; as will be pointed out later, this does not always mean that the headwater depth will be reduced from that for the square-edged entrance.

The types of flow illustrated in Figures 4 and 5 deserve careful study. As recognized by the authors all the possible cases are not covered. One common case is contracted flow as in Figure 4(b) but on a subcritical slope. This can occur when the length of the culvert is such that the momentum of the flow carries it out of the culvert before the water surface can rise to the top of the barrel.

Contracted flow is fully developed for $H/D = 1.5$ which corresponds to $Q/D^{5/2} = 3.9$ approximately. At this relative discharge critical depth as indicated by the discharge curve in Figure 3 is about 0.84D. Since the contracted depth is about 0.6D the flow in the contracted section must always be at supercritical velocity. Thus it is possible with the square-edged entrance to have supercritical flow on a mild or even level slope. The main point is that contracted flow with headwater as shown for the square-edged inlet in Figure 6 is not confined to culverts on steep slopes.

The curve for critical slope in Figure 3 is useful for distinguishing in Figure 4 between critical depth control at the inlet (a) and at the outlet (c) but, for the reason stated in the previous paragraph, does not govern for type (b). The form of the profile beyond the contraction in type (b) depends on the friction slope at the contracted depth, the water surface dropping if slope exceeds this friction slope or rising if it does not.

The example of culvert flow on a steep slope curiously enough does not meet the assumed condition that the slope is steep, although as stated in the previous paragraph, critical slope does not govern. At $Q/D^{5/2} = 9.0$ the relative critical depth is 0.99 and the corresponding critical slope, mathematically, would be 0.0240 as computed from King's Handbook of Hydraulics, Table 116. Actually critical depth in this range can have no real significance. (The friction slope for the full culvert would be slightly more.) Since the barrel slope is only $s = 0.02$ the depth of flow in this culvert would increase from the contracted depth and might even fill the barrel before the full length was attained.

Practically the assumption of $n = 0.010$ (or 0.011) is not realistic as culverts installed in the field cannot be expected to be as smooth at the joints as the culvert tested by Dr. Straub. With a higher value of n the culvert in this example would almost certainly flow full, even with the square-edged entrance.

For the culvert on the 2-percent slope the outlet velocity will be nearly 20 ft. per sec. and would require some type of energy dissipator if an erodible soil were present. Enlarging the size of the culvert would not reduce the outlet velocity appreciably as a 3.5-ft.-diameter culvert would not flow full on this slope, unless the roughness was at least $n = 0.013$.

As noted by the authors the limits for slug flow in the model as shown by dotted lines in Figure 6 may not be entirely correct for the prototype. This follows from the fact that model tests involving entrained air are qualitatively indicative of prototype performance but may not give true quantitative results. The error is not likely to be large, however.

The involved equation for orifice flow (Equation 7) fortunately does not need to be used if a graph such as Figure 6 is available. Analysis of the data indicates that the following equation fits the data very well in the range above $Q/D^{5/2} = 4$.

$$\frac{H}{D} = 0.59 + 0.067\left(\frac{Q}{D^{5/2}}\right)^2 \qquad (10)$$

The orifice theory strictly is not applicable to this case since the jet is not free; it happens to fit the data closely, probably because the pressure line at the free water surface in the contracted section is close to the center of the entrance.

It is unfortunate that both Mavis and the authors of this paper chose to set the model culvert above the flow line of the approach channel. Additional tests are needed to determine to what extent, if any, the bottom contraction affects the vena contracta in the culvert which governs the headwater-discharge relation. Furthermore, the tests would have been more representative of normal field conditions if the outlet jet had been supported on an apron at the invert elevation. This would affect the elevation of the pressure line and can be allowed for in computations involving culverts flowing full.

While the rounding of 0.15D causes the model of length 105D to flow full, there is no positive evidence that a very short culvert would also flow full.

There appears to be no good reason for expressing the head loss in a full culvert as a function of $(Q/D^{5/2})$ (Equation 9). Highway engineers generally have nomographs available for determining the head loss in a full culvert. The headwater depth is then determined by subtracting the fall and adding the height of the pressure line at the outlet above the invert. Equation 9 assumes the pressure line to be at the center of the outlet which is true only for a free jet when $V^2/2g > 0.8D$ (see Mavis (5) page 28). For most culvert installations the jet is supported and the pressure line is probably at or close to the crown of the culvert and conservatively may be assumed at the crown.

The explanation given by the authors of the operation of a culvert with rounded entrance is illuminating and by far the most valuable information contained in the paper. Lest the unwary should be led astray it should be emphasized that in Figure 6 curves for the culvert with rounded inlet on various slopes apply only to a very smooth pipe having a length of 105 diameters. Attention is also called to the fact that for discharge in excess of $Q/D^{5/2}$ = 7 the pressure line at the entrance will be below the invert. In actual practice such negative pressures probably should be avoided. There is some doubt that the prototype could actually be depended on to operate on the lower line for slug flow in Figure 6 at $Q/D^{5/2} > 7$. Further investigation of this pressure problem is needed.

A simple test for indicating the advantage to be obtained by using a rounded entrance on a given culvert is covered in the discussion of the Oregon paper.

LORENZ G. STRAUB, ALVIN G. ANDERSON, and CHARLES E. BOWERS, Closure-The authors are pleased with the interest that has variously been expressed in the paper and hope that the end result will be some improvement in modern culvert design. In responding to the written discussions re-emphasis is here made that the purpose of the presentation was to demonstrate the "importance of inlet design on culvert capacity." The many other aspects of culvert design and practice are not considered as a part of the treatment covered. The primary objective of the paper has been to emphasize that a greater culvert capacity and hydraulic efficiency can be obtained, particularly for short culverts and for long culverts on steep slopes, by proper attention to characteristics of the inlet. Quite frequently in the past improved design of the inlet has been associated too strongly with the local head loss rather than with the more important aspect of the influence of the inlet in overall behavior of the entire culvert.

The terms "long" and "short," "steep slope," and "mild slope" when applied to a given culvert are relative and their applicability depends among other things upon the roughness characteristics of the culvert itself, also upon the discharge. They are qualitative expressions which can be defined when such factors as roughness, discharge, and the like are given. Thus, Izzard mentions that it is possible to have supercritical or contracted flow on a level or mild slope. In a qualifying sense this is true, provided the culvert is not too long.

The authors disagree with Izzard that an n value for concrete culvert pipe of 0.011 is not realistic. On the contrary, there is positive evidence that many of the customary values which have been taken for granted in practice are really quite unrealistic and misleading, both for concrete and corrugated metal pipes. In regard to the conducting of experiments with the culvert invert at the elevation of the approach channel, such experiments are the logical next step to the more-idealized studies of culvert entrances free from the approach channel as reported in this paper. However, the authors wish to point out that the orifice-flow philosophy basically would lead one to surmise that the entrance conditions would be similar with the approach channel at the invert elevation as with the approach channel at the lower elevation of the reported tests. Suppression of contraction on the invert side distorts the vena contracta but the contraction coefficient does not change materially. Exploratory tests at the St. Anthony Falls Laboratory bear out this fact.

Izzard calls attention to an empirical equation (Equation 10) which he presents in preference to the authors' Equation 7. The suggested Equation 10 is probably quite adequate for normal use, but it is restricted to a square-edged inlet. For other types of inlets it would be necessary to set up a new empirical equation if this method were to be used. The important significance of Equation 7 is that it is generally applicable and not limited to one specific type of culvert entrance, because it is based upon the degree of contraction by the factor involving the contraction coefficient. A method of estimating the contraction coefficient was developed which considers the shape of the inlet (Reference 3). The applicability of the basic orifice equation to the square-edged inlet used in the experiments was demonstrated by tests giving the same head-discharge curve when the culvert was removed and the inlet became an orifice.

Equation 9 was written in terms of the discharge, or $Q/D^{5/2}$, only for convenience in order to plot head discharge curves for culverts flowing full. There are, of course, other ways in which the head loss can be expressed, possibly in more-convenient forms for particular uses. The assumption that the pressure line is at the center of the outlet is probably a good approximation for full flow with a free outlet for the cases considered.

In regard to the rounding of 0.15D, contrary to the question raised, it is quite positive that a short culvert would flow full with this type of entrance. The 0.15D rounding insures the vena contracta being the full size of the pipe at the entrance; pipe friction produces further resistance to flow in the full pipe. Tests with model culverts as short as 10 diameters with rounded entrance invariably flowed full with slopes from 0 to as steep as 10 percent, provided the discharge was sufficient to develop a head of the order of ½ D above the crown of the entrance; for lower discharges pulsating slug flow develops in accordance with the chart shown in Figure 6.

In regard to the part of the discussion of Mavis and Stelson, which seems to question the desirability and advantages of rounding the culvert entrance, some supplementary comments are desirable. First, relative to high discharge velocities, this is a matter entirely separate from hydraulic efficiency of the culvert and should be so treated. Here one is concerned particularly with proper exit design, which requires further analytical and experimental treatment not within the scope of the paper. Relative to the implication of the development of an absurd negative pressure just beyond the entrance of a rounded culvert, this is of course recognized as not possible. Actually, under such conditions slug flow develops; air is drawn in and full head is temporarily interrupted. There is thus not a steady state flow for this condition but a pulsating flow of varying velocity, the pattern of which changes as the average rate of discharge changes while the head over the entrance to the culvert varies but slightly as shown experimentally in Figure 6. The authors agree with the cautions outlined by Mavis and Stelson, although probably not completely with the implications that a casual reader obtains. Regardless of uncertainties of stormy weather and other unpredictable conditions, every economical advantage should be taken in producing hydraulically the most-efficient design; this must also take cognizance of energy dissipation at the discharge end of the culvert which should be the subject of further treatment. Thus if a designer chooses to make the culvert "adequately large" he should still take advantage of getting highest practical hydraulic efficiency for handling the unpredictable storm runoff.

The authors appreciate the discussions and questions raised by Izzard and by Mavis and Stelson in providing further clarification of the problems of culvert design.