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Research Report 15-B

Culvert Hydraulics

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Culvert Hydraulics

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Foreword

The Committee on Surface Drainage of Highways herewith presents two papers on culvert hydraulics. The first paper, "Importance of Inlet Design on Culvert Capacity," is a report on tests made at the St. Anthony Falls Hydraulic Laboratory on a 4-in., round, lucite culvert model for the Minnesota State Highway Department and the Bureau of Public Roads. The second paper, "Model Studies for Tapered Inlets for Box Culverts," reports tests on a 4-in., square, lucite culvert model made at Oregon State College for the Oregon State Highway Commission and the Bureau of Public Roads.

The committee believes these two papers are the most significant contribution to knowledge of culvert hydraulics since Yarnell, Woodward, and Nagler published Iowa Bulletin No. 1, "The Flow of Water Through Culverts," in 1926.

Both papers demonstrate clearly that most culverts as now constructed may be very inefficient under certain conditions of common occurrence, because the full capacity of the barrel cannot be utilized after the entrance is substantially submerged. It is actually possible for water to be ponded enough to overflow the roadway where the cover over the culvert is more than the culvert height at the same time that the barrel is flowing partly full. This condition can be remedied simply by improving the inlet so as to eliminate the contraction of flow forced by the usual square-edged entrance. Then the barrel will flow full, and in addition, the head available in the fall of the culvert, which is not utilized when the control is at the entrance, will become fully effective. The result, for culverts with appreciable fall in relation to height, is that the headwater pool can be lowered by amounts as great or greater than the culvert height. In other words, by improving the entrance of a 5-ft. culvert it may be possible to lower the headwater as much as 5 ft.

There are situations, however, where relatively little lowering of headwater is obtained by improving the entrance. The conclusion of Carl Izzard's discussion of the Oregon paper develops the limitations and shows how the results of the tests on both pipe and box culverts can be applied to practical design problems.

Both papers make reference to the need for further investigation along certain lines. The committee was aware of some of these deficiencies and at meetings held in the fall of 1952 drew up suggestions which were passed on to the Iowa State Highway Commission when it was learned that a new cooperative research project on culvert hydraulics was being formulated at the Iowa Institute of Hydraulic Research. That work is under way and initially will consist of model tests exploring the characteristics of other modifications of box and pipe culvert entrances with the aim of finding the most practical designs for typical situations.

Contents

Model Studies of Tapered Inlets for Box Culverts

ROY H. SHOEMAKER, JR., Project Research Engineer and Instructor of Civil Engineering, and LESLIE A. CLAYTON, Assistant Professor of Civil Engineering, Oregon State College

Model studies of box culverts on steep grades were conducted in cooperation with and sponsored by the Oregon State Highway Commission and the Bureau of Public Roads. Objectives of the experiments were: (1) clarification of the theory of operation of box culverts and (2) modification of the design of the Oregon State Highway standard inlet in order to increase the over-all effectiveness of the culvert as a drainage structure.

Test data were taken from a 1:12 scale model of a 4- by 4-ft. box culvert 82 ft. long. This model was provided with a trapezoidal approach channel and a section of embankment slope with means for installing different types of inlets without major changes in the model.

The program consisted of testing three basic types of inlets: the Oregon State Highway Commission standard inlet with no flare or taper, an inlet with a taper in the sides and top of 1 to 10, and an inlet designed primarily for operation under entrance control. All three of these inlets were provided with wing walls at an 8 to 12 angle with the axis of the culvert. Essentially, the initial testing consisted of three general types of comparisons: (1) analysis of the operation of all three inlets in nonsubmerged states, (2) operation of the standard inlet as a sluice gate, and (3) full-flow operation of the standard and tapered inlets.

The nonsubmerged operation of the inlets followed the theory of entrance control with critical depth, and it was indicated that designs for entrance control could be based upon critical depth theory with little or no modification. In the submerged state the standard inlet operated normally as a sluice gate, while the 1:10 tapered inlet showed no sluice contraction and flowed full automatically as it became submerged, with a resultant increase in flow as compared to the standard inlet. It was discovered that the increase in capacity was the result of: (1) the shift in the lower energy reference from inlet to outlet of the culvert and (2) the increase in the effective area of flow due to elimination of the contraction at entrance which occurred with the standard inlet. Test results indicate that the standard inlet flowing as a sluice can be treated with existing orifice theory and that existing theory regarding pipe flow can be used for the full-flow conditions of the tapered inlet.

It was discovered that the culvert with standard inlet could be made to flow full artificially by temporary elimination of the sluice contraction and would remain full so long as air was prevented from entering the inlet section. Under ordinary full operation, however, air admitted through the action of vortices in the upstream pool caused the culvert to revert to sluice operation.

A practical approach to assurance that the culvert barrel would flow full and remain full upon submergence of its inlet was to modify the inlet to eliminate the contraction at that location. The procedure followed was to form a taper in the entrance by extension of the top slab of the culvert upstream from the parapet wall over the wing walls and the extension of a portion of the wing walls to meet the

top slab; thus the tapered section was formed by the wing walls and the top slab. The shortest practical length of extension was determined experimentally to be that required to produce an area ratio of entrance to culvert barrel of 2 to 1. The resulting inlet design showed a substantial increase in capacity, upon inlet submergence, over any other inlet tested. Effectively, this inlet allowed no change in headwater level from the discharge required to just submerge the inlet to that required by the culvert flowing full. With the culvert on a 4-percent grade and operating at the head of submergence, the modified inlet allowed an increase of approximately 100 percent over that obtainable from the culvert equipped with the standard inlet. Experiments with flat, 4-, and 8-percent grades confirmed a hypothesis that, within limits, the ratio of areas required for the foregoing type of operation is the same, regardless of the slope or angle of wing walls.

Conclusions derived from the experiments were: (1) a significant saving of materials could result from designing culverts on steep grades to flow full, (2) the formation of a tapered inlet by the extension of the top slab and wing walls would be a practical solution to the problem of assuring full flow, and (3) by proper application existing theory is adequate for the design of culverts on steep grades.

● COSTS of drainage structures are known to be high percentages of highway construction costs, and of these structures box culverts comprise a significant portion. For the biennium from July 1950 to June 1952, the State of Oregon alone spent \$676,000 on box culverts as compared to \$8,500,000 for all highway structures and a total of \$38,000,000 for actual highway construction. Thus, any improvement in the design of box culverts which would allow reductions in size for given installations could result in major savings in construction costs.

With these savings in mind, engineers of the Bureau of Public Roads in Division 8 employed an improved inlet design for box culverts installed in forest highway projects as early as 1948. The improvement, which was in the form of an enlargement of the entrance in the form of a taper, was prompted by field observations that many box culverts on steep grades flowed less than half full, even when operating at maximum discharge with deeply submerged entrances. Subsequently, several culverts with enlarged entrances installed along the Pacific Highway near Canyonville, Oregon, were subjected to severe floods, and the operation of one of these was much better than had been expected on the basis of theoretical analysis.

In the meantime, the Oregon State Highway Commission became interested in using this type of inlet design on some of their box culverts. The unexpectedly good operation of the tapered inlet culverts on the Canyonville project (constructed by the Bureau of Public Roads) clearly emphasized the need for a through understanding of the hydraulics of this type of structure. As a result, the Oregon State Highway Commission and the Bureau of Public Roads agreed to jointly sponsor laboratory tests to investigate the problem by means of scale models. An agreement was entered into with Oregon State College to conduct the investigation, and work was started in June 1951 as an engineering experiment station project. The laboratory experiments were completed in November 1952.

The model studies reported here were made on the basis of two general objectives. The first of these was to investigate the theory of operation of the Oregon State Highway Commission standard box culvert on steep grades with a free overfall at the discharge end. This investigation was intended to include studies of both nonsubmerged and submerged inlet operation. The second objective was to determine means for improving the effectiveness of operation of this culvert barrel by means of tapered inlets. In the progress of the experiment, this objective was modified to apply to determination of an economical means of causing the culvert barrel to flow full upon submergence of the inlet. A corollary objective was to investigate somewhat the effect of the geometry of the inlet section (with wing walls) upon the operation of the culvert.

The model studies reported here were conducted with box culverts on steep and flat grades and having free overfalls, and the results of these experiments are necessarily applicable only to structures in this category.

THEORETICAL CONSIDERATIONS

The operation of a culvert through its overall range of discharge can be subdivided into two phases, each of which is dependent upon a different head-discharge relationship. The first of these applies through the range of discharges during which the inlet is not submerged. If the culvert is on a steep grade and the flow in the upstream pool is subcritical (the ordinary case), critical depth will occur in the region of the entrance (1) with an accompanying acceleration of the flow. The location of this depth will be near the entrance to the uniform barrel in the case of a continuous flow line and near the break in slope in case there is a drop in the flow line.

Since the flow in the culvert barrel must be supercritical, the effects of roughness and slope of the culvert barrel cannot be reflected upstream to the entrance; consequently, the geometry of the entrance alone (specifically the width at which critical depth occurs) determines the quantity of flow carried by the culvert for a given upstream pool elevation. Thus, when the inlet of a culvert on a steep grade is not submerged, the structure can be said to operate under critical depth control at the entrance. In most cases it should be possible to compute the discharge through such a culvert by knowledge of the width at which critical depth should theoretically occur. The following equation, based upon the relation of critical depth with discharge for a rectangular cross section, makes it possible to compute the upstream pond level, above the flow line at the control section, for a given discharge per foot of width of barrel (1):

$$H = 1.5(q^2/g)^{1/3} \quad \text{.............} \quad (1)$$

If the entrance of a culvert is square edged, its submerged operation may be considered analogous to that of a sharp-edged orifice, discharging horizontally, on the premise that the momentum of the fluid approaching the entrance nonaxially will cause a contraction in the area of flow downstream from the opening. It has been shown that, in the case of an orifice, the contracted area (or "vena contracta") is the controlling area with respect to discharge computations (2). The energy available for producing flow is, in this instance, a function of the head measured between the center line of the orifice (the location of the pressure line in the case of a nonsupported jet) and the upstream energy grade line,

$$Q = A_j C_v \sqrt{2gH} \quad \text{.............} \quad (2)$$

where A_j represents the area of the jet at the vena contracta, C_v the coefficient of velocity of the orifice.

For a culvert with a square-edged entrance, if the flow downstream of the entrance is unobstructed, it is reasonable to assume that approximately the same relationship will hold. In the ordinary case, the flow line is more or less a continuation of the upstream channel flow line, and

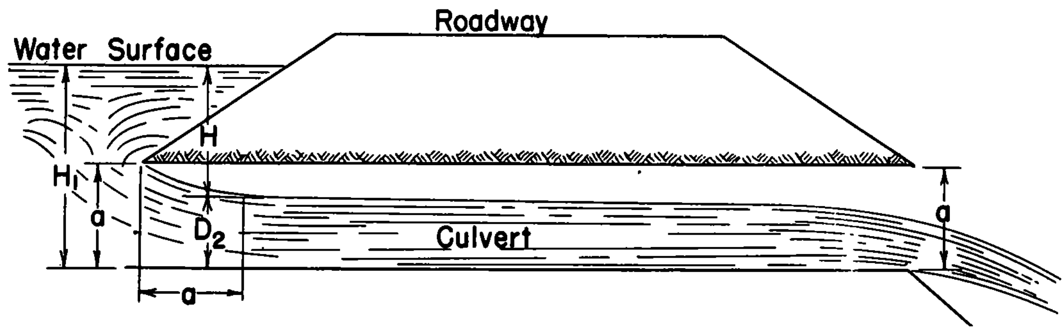


Figure 1. Definitive sketch for sluice-type operation of a culvert.

wing walls are provided at the sides of the entrance so that only a top contraction should occur, as in a sluice gate. The flow producing energy (see Fig. 1) would then be measured between the upstream energy grade line and the water surface at the contracted area, with the realization that the hydraulic grade line (or pressure line) is in the water surface in the case of a supported jet; thus

$$Q = W D_2 C_v \sqrt{2g H_2} \quad \ldots\ldots\ldots\ldots (3)$$

in which W is the width at the vena contracta, D_2 the depth at the vena contracta, C_v the coefficient of velocity, and H_2 the available head. Both Equations 2 and 3 can be derived upon the basis of the energy equation and the continuity equation, taking into consideration all velocity and pressure heads. A more rational concept of Equation 3 is that the head H_2 is the energy available for conversion into velocity head, measured from a datum through the water surface at the vena contracta, and that if there were no losses in the jet, the coefficient of discharge of the sluice would be the decimal fraction of the area of the opening that was available at the contraction. In all cases, however, there is some loss of head in the jet, so in terms of Equation 3, the coefficient of discharge would be:

$$C_d = \frac{D_2}{a} C_v \quad \ldots\ldots\ldots\ldots (4)$$

where D_2 is the depth at the vena contracta and a is the height of the opening.

In contrast to the sluice operation of the submerged inlet, operation of a full culvert barrel utilizes not only the energy available with respect to its entrance elevation, but any additional head provided by the fall in the length of the barrel. Furthermore, the area of flow is the total area of the culvert, which gives an opening considerably greater than the contracted area of the sluice. As with a short tube, however, if the entrance of the culvert is square edged, the top contraction will occur even with the barrel full, with a consequent low pressure area in the vicinity of this contraction (3). Continuing the analogy with a short tube, if air is admitted to the contraction, the opening will revert to sluice operation, providing there is no obstruction to the flow downstream from the entrance.

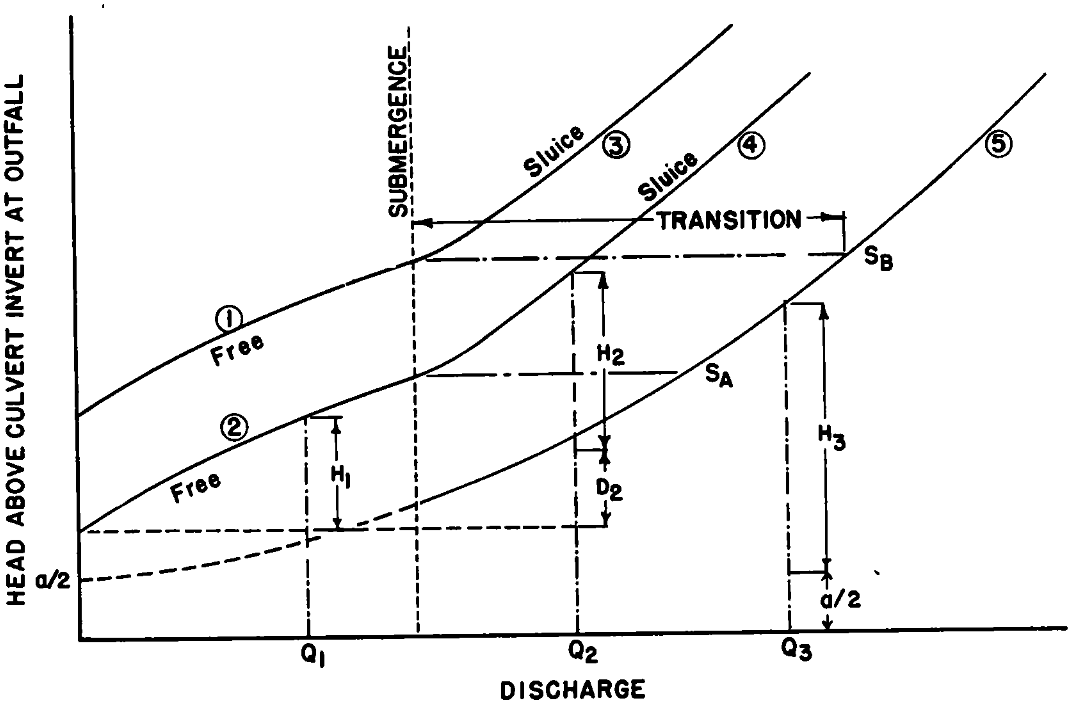


Figure 2. Rating curves for typical culvert installations.

The energy available to produce flow when the culvert barrel is flowing full should be measured from the upstream energy grade line to the pressure line at the discharge end of the culvert, which line will be located near the center of the jet in the case of free overfall, provided that the velocity head is not less than 0.8 times the height of the culvert (6), or in the water surface in the case of a supported jet. The discharge, then, will be a function not only of the entrance loss and barrel friction loss, but of the slope and length of the culvert barrel.

With this information it is possible to make an analysis of the operation of a culvert with its inlet submerged, on the basis of (1) operation analogous to a sluice gate and (2) operation analogous to a pipe flowing full. Rating curves for typical culvert installations are given in Figure 2, showing the discharge characteristics for two culverts of the same length on grades S_A and S_B.

Three types of curves are shown in this figure; one covering the range of discharges during which the inlet is not submerged, another showing the operation as a sluice gate, and the third a single curve covering the possible total range of full flow. Curves 1 and 2 originate at the invert elevation of the culvert entrance and assume that critical depth control occurs at that point. Curve 5 is not influenced by the grade of the culvert because the full flow discharge is determined by the difference in elevation of the water surface at the inlet and the position of the pressure grade line at the outlet, which is taken as a height of a/2 above the outfall invert. The effective head for producing discharge is shown for each case, and the formulas for discharge are as follows:

Curve 2 (critical depth control):

$$Q_1 = W(g)^{1/2}\left[\frac{H_1}{1.5}\right]^{3/2} \text{ ....(5)}$$

Curve 4 (sluice):

$$Q_2 = C_d A\ (2g\ H_2)^{1/2} \text{ ....(6)}$$

Curve 5 (full):

$$Q_3 = A\left[2g(H_3 - \text{losses})\right]^{1/2} \text{ ....(7)}$$

Equation 4 and Figure 2 ignore the drop in the flow line between the entrance of the culvert and the vena contracta due to the grade of the culvert. In very steep grades this would have to be considered.

The ordinary box culvert with the Oregon standard inlet and with free outfall should follow Curves 1 and 3 or 2 and 4, depending upon the grade of the barrel. If the culvert can be made to flow full upon submergence of its inlet, the discharge under a given head should be increased considerably, as shown on the chart; the amount of increase being dependent upon the grade of the culvert and the length of the culvert barrel.

In the transition range between entrance control and full flow, the quantity of flow required by a full culvert barrel would, in the cases shown, be in excess of that supplied by the approach channel, and the result would be an intermittent free-full condition, during which the headwater level would be restricted to a height equal to or less than the height of the culvert entrance. Thus the portion of the full-flow curve available for use at a given grade would be that part above a horizontal line drawn from the head of submergence of the culvert.

Discharge calculations for culverts operating in any of the three manners previously discussed can be made from Equations 5, 6, and 7, provided that proper choice is made of coefficients and that allowance is made for the grade and length of the culvert.

The model-to-prototype scale ratio chosen for these experiments was 1 to 12, which is quite conservative for this type of study. In most cases the water changed elevation rapidly, indicating that the forces of gravity and inertia are the predominant forces acting. Since the relationship between these forces is defined by the Froude number, scale ratios for amplifying various quantities such as depth, velocity, and discharge to prototype scale can be derived by reference to the equality of Froude numbers. There may be some question as to the validity of the discharge scale ratio in the case of the barrel flowing full, because of the increased importance of viscous forces due to fluid friction. However, a sufficiently practical evaluation of full-flow discharge through a full-scale culvert can be made on the basis of existing data on pipe friction factors and entrance loss coefficients, and there is no particular need for consideration of scale errors in model roughness which would affect similarity relationships with respect to full-flow conditions.

There can be little doubt as to the similarity of operation of the model inlets to the operation of geometrically similar full-scale culvert inlets. Therefore, the results from entrance control conditions in the model experiments should be representative of the operation of full-scale culverts of similar construction, and these results should be of value in future design.

THE CULVERT MODEL

The hydraulic model upon which these experiments were made comprised an intake box, approach channel, culvert inlet and barrel, and a triangular weir, as shown in Figure 3.

The intake box was supplied through a 4-in. line from the laboratory pumping system, and was 4 by 5 by 3 ft. deep, the 5-ft. side being connected to the approach channel, the bottom of which was approximately 1½ ft. above the bottom of the box.

The approach channel was 10 ft. long and 26 in. wide at the bottom, having sides 16 in. high with slopes of 2 to 1 at the bottom and the remaining 8 in. vertical. The channel terminated in a simulated section of highway embankment with a 2 to 1 slope for the first 8 in. of height and a vertical end wall. The embankment slope was cut to receive flanged sections, forming flush joints at the wall and floor. In the construction of the channel, use was made of ½-in. exterior plywood (painted) on wood and steel framing, and Plexiglas for the end section containing the embankment slope. Channel slope adjustments were made possible be means of blocks and four leveling screws.

The inlet sections, which will be described later, were connected by flanges to a 4- by 4-in. Plexiglas culvert barrel 6 ft. long. The barrel was supported by a steel I beam, which was provided with leveling screws at the ends for slope adjustments. Free overfall from the culvert barrel was directed into a box equipped with a triangular weir which discharged into a sump. The sump used was a tank calibrated for volumetric measurements and was employed at the start of the experiments for calibration of the weir.

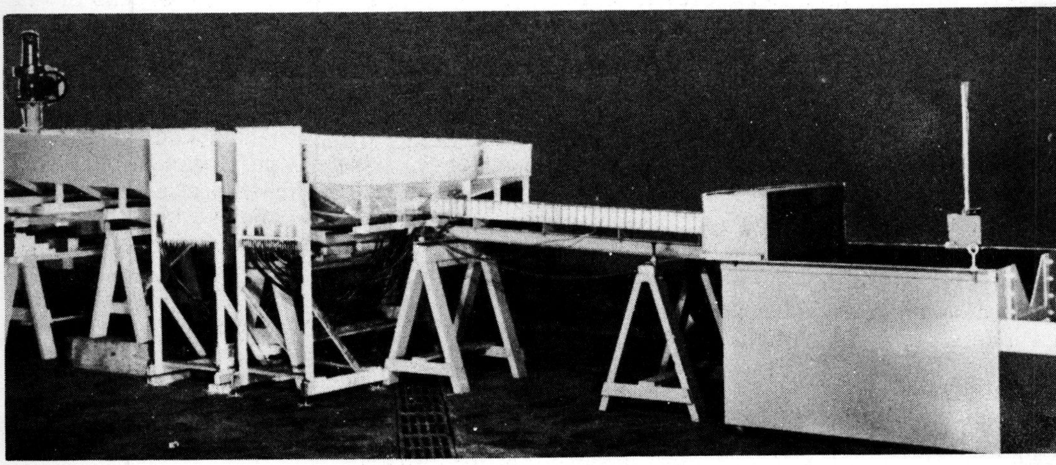


Figure 3. General view of model.

Inlet sections tested in these experiments can be compared by reference to Table 1. The only major deviation from the Oregon State Highway Commission plans was that for all inlets the wing walls were extended to meet the toe of the embankment, which is not the usual case in practice.

Adjustments in slope were made with an engineer's level and a sharp-pointed rod divided into tenths of an inch. All readings were taken to an estimated accuracy of 0.01 in.

Hydraulic Measurements

Discharge. A 90-deg. triangular, sharp-crested weir with a capacity of approximately 0.9 cu. ft. per sec. was used for the measurement of discharge, in conjunction with a hook gage readable

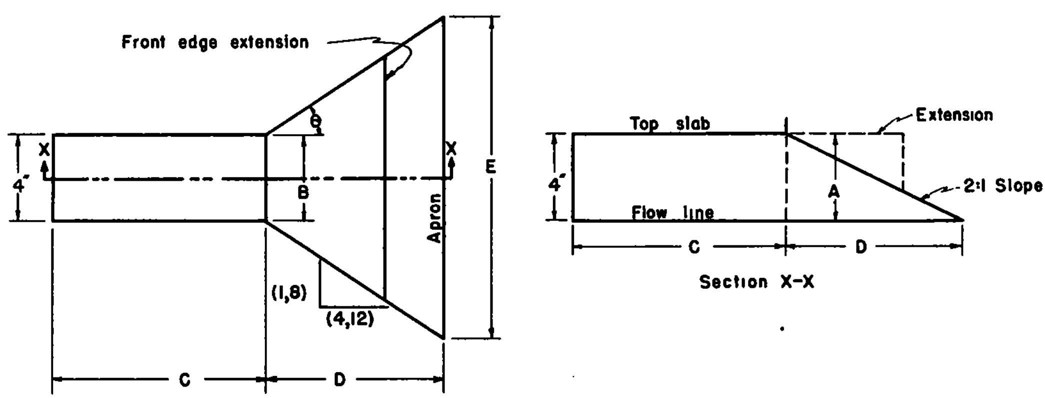
TABLE 1

Inlet Models

All dimensions in inches

No.	Entrance A	Entrance B	Entrance Area	C	θ	Apron D	Apron E	Remarks
1	4	4	16	10	8: 12	8. 25	15	Oregon State Highway Comm. standard (Dwng 9656). Control inlet for experiments
2	5	6	30	10	8: 12	10. 25	20	Standard tapered inlet (Dwng 9656). 1: 10 side and top tapers
3	4. 63	4. 63	21. 4	5. 27 See Note 6	8: 12	9. 21	17. 3	Developed by Hydraulics Branch, Bur. of Pub. Roads (Hydraulic Information Circ. No. 2)
4	4	8. 63	34. 6	13. 5	8: 12	4. 75	15	Modification of No. 1. 3. 5-in. top slab extension. See Note 5
5	4	4	16	10	1: 4	8. 25	8. 12	Modification of No. 1
6	4	6	24	14	1: 4	4. 25	8. 12	Modification of No. 5. 4-in. top slab extension.
7	4	6. 75	27	15. 5	1: 4	2. 75	8. 12	Modification of No. 5. 5½ in. top slab extension

1. Actual dimensions taken from inlet models.
2. All models constructed of ⅛-in. Flexiglas.
3. Flanges provided for joining inlets to 4- by 4-in. culvert barrel.
4. Parapet wall the same height for all inlet models.
5. Top slab extended parallel to flow line between wing walls, and wing walls built up to meet extension.
6. Drop of 1. 76 in. in flow line within inlet section (dimension C, below).



to 0. 001 ft. installed in a stilling well. The weir was calibrated in place by use of a volumetric tank in which a rise of 1 ft. represented an increase of 170. 7 cu. ft. of water. The rate of rise of the water in the tank was measured by an electrical-contact point gage and a stop watch, with readings taken to provide an accuracy of three significant figures. A sufficient number of weir-hook gage readings were taken for each run in the model tests to provide a reliable average observation, and the discharge values were read from a rating curve prepared from the calibration.

Pressure and Water Level. Pressure and water level indications were taken from piezometer tubes connected to the bottom of the channel and culvert. The piezometer holes were made by a No. 40 drill, the connecting tubing was $\frac{1}{4}$-in. I.D. transparent "Tygon" and the piezometer tubes were $\frac{1}{2}$-in. I.D. glass. Special precautions were taken to remove burrs from the edges of the holes, and where possible, the holes were drilled from the inside out with a drill press before assembly of the model. Air was eliminated from the tubing before each test by agitating the water in each tube with a syringe until visual inspection revealed the absence of bubbles.

proach channel at that location. The remaining connections were made to the apron, inlet, and culvert barrel; their locations for the various test runs are given in the appendix.

The piezometer readings were taken photographically with a tripod-mounted Kodak "Tourist 800" camera provided with an f/4.5 Anastar lens, using Plus-X 620 roll film. It was found that roll film was most satisfactory for the purpose (for most cases), because of the speed usually required to ready the camera for the next picture. In location of the camera, par-

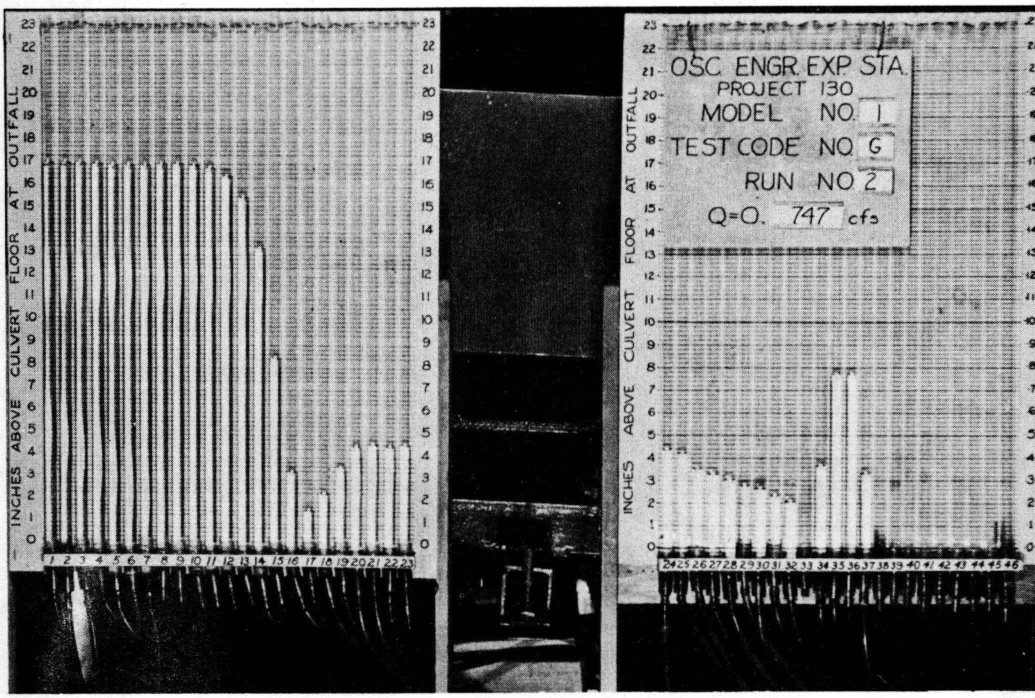


Figure 4. Typical data picture for Inlet 1 flowing full, showing piezometer boards and test run designations. Tubes 2 and 10 to 32 inclusive are along centerline of channel and culvert.

The glass tubes were mounted in two banks on boards provided with 0.2-in. divisions which were ruled with india ink on white acetate. To provide protection against water damage, the boards were painted with clear synthetic enamel. The piezometer boards were mounted on standards provided with leveling screws.

Three piezometer connections located in the approach channel were used for indications of the upstream water level. These connections were 18 in. upstream of the toe of the embankment slope and were at the center and sides of the invert of the ap-

allax was minimized by setting the camera at least 6 ft. from the boards and level with the centers of the boards. Light was provided by four reflector flood lamps mounted two to a board, one board above and one below the piezometer boards, to eliminate shadows.

Because of the requirements for enlargement (in many cases a single reading of the two boards was made on one quarter of a $2\frac{1}{4}$- by $3\frac{1}{4}$-in. negative and then enlarged to 5 by 7 in.), correct exposure and uniform lighting were necessary for elimination of graininess in the negatives; consequently, the exposures were measured by a meter and a neutral-gray test card. No coloring of the water in the tubes was required, because the lighting delineated the meniscus at the top of each water column. Satisfactory readings were subsequently taken from 5- by 7-in. glossy prints made on average contrast paper. Figure 4 shows a sample photograph to demonstrate the method of identification of runs.

Approximate water level readings in the culvert barrel were provided by ½-in. divisions in black paint parallel to the axis of the culvert and 2-in. divisions perpendicular to the axis. When backed by white paper, the lines were quite useful for interpretation of photographs taken to record the general flow patterns.

Accuracy of Measurements

Before discussing the accuracy of the measurements it may be well to describe the flow conditions in the approach channel for the three culvert grades studied.

For all grades, a pool of some variety was formed at the entrance of the culvert. With the culvert on a flat grade, a pool formed for the length of the channel, so that no great turbulence occurred. Although the velocity distributions were notably nonuniform in the channel, the kinetic energy in the pool was negligible.

When the model was on a 4-percent grade, the flow conditions were much more widely varied. For the small discharges, a hydraulic jump occurred in the approach channel, its distance from the culvert inlet varying with the discharge. However, for all except the smallest flows, the water surface a short distance downstream of the jump was reasonably calm, and the velocities were small enough to be neglected in energy calculations. Conditions were such that the hydraulic jump disappeared shortly after the inlet submerged, since the top of the inlet was approximately at the same level as the upstream end of the channel.

Flow conditions in the approach channel for the 8-percent grade were much-less satisfactory than those for the flatter grades. In this case, the location of the hydraulic jump was, for all but a few cases, in the approach channel, with the result that the surface of the water was quite turbulent. The velocities encountered when the culvert was flowing full were so great that there was considerable turbulence in the channel at the culvert inlet even at the highest heads. Due to the nature of the flow, it was impossible to obtain satisfactory measurements in most cases.

In the calm pools that existed at the various grades, formation of free vortices was common. These vortices had a considerable effect upon the flow conditions in some of the inlet models by the tendency to admit air to low-pressure areas.

It is believed that the discharge measurements were the most accurate of the quantities measured by reason of the calibration of the weir in place, for the experiments. This accuracy was set at three significant figures.

The piezometric readings were, in themselves, accurate to 0.05 in. but, depending upon the flow conditions at the culvert connections, were accurate to different degrees as indications of depth or pressure. For depth measurements, it is assumed that the damping provided by small holes and connecting tubing allowed representative readings in locations where the flow was stable or, at least, parallel to the channel or culvert invert. These locations were (1) in the approach channel when the depth was great enough to provide a reasonably calm pool and (2) in the culvert where the depth was either constant or changing very gradually. Regions of curvilinear flow occurred at the entrance of the culvert due to the acceleration of the water and in the vicinity of the outlet; at such locations piezometer indications are not reliable as depth measurements. In pressure measurement for the full culvert, the readings were assumed accurate only in locations where the velocities could be deduced to have normal distribution. There was considerable evidence of impact on the connections in certain regions, especially those on the apron at the entrance of the culvert, where the water velocities had definite downward components.

There was no practical means for estimating error caused by the foregoing effects, and as a general rule, all peizometric readings should be regarded with reservation due to this circumstance. However, since only qualitative information was required in many cases, the piezometers were considered satisfactory for procurement of the data. Furthermore, it should be mentioned that, for similar flow con-

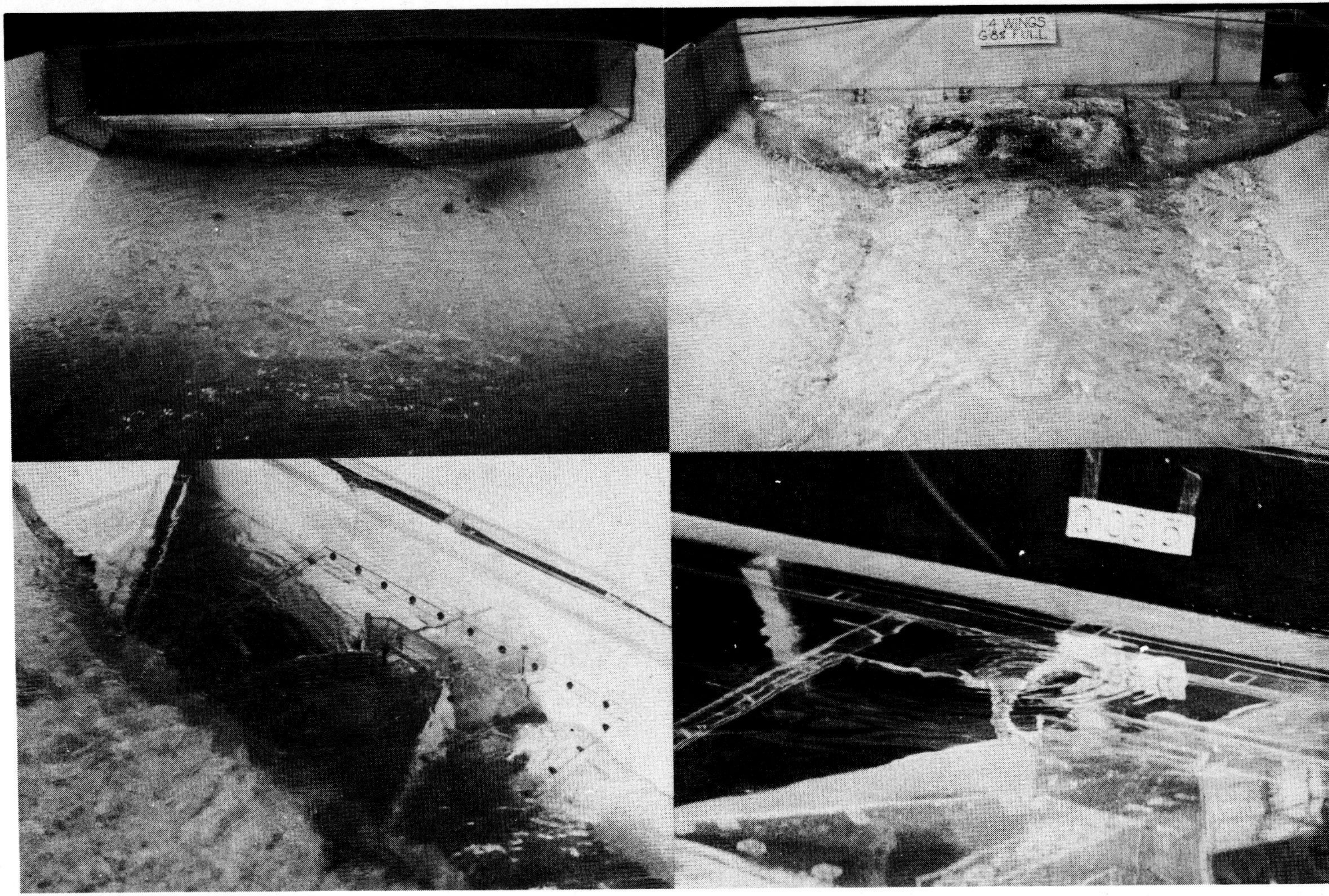


Figure 5. Flow conditions in approach channel: Upper-left, 4-percent grade, Inlet 1 installed; upper-right, 8-percent grade, Inlet 5 installed; lower-left, 4-percent grade, flow into Inlet 1; and lower-right, 4-percent grade, typical vortex in pool.

ditions in two arrangements, the piezometer readings should also be similar, so that they should be satisfactory for use in obtaining comparison data. There was remarkable consistency in the piezometric data throughout the experiments, especially in the measurements taken for the purpose of rating the culvert. Results were even uniform for the conditions when the model was set on an 8-percent grade and the water in the approach channel was extremely turbulent.

Experimental Procedure

As a general rule, for discharge rating experiments, the inlet to be tested was run through its total range of discharge with an experimental plot carried for the purpose of obtaining the required intervals between observations. For a given run, the discharge was set at approximately the value to produce the desired upstream pool elevation, after which visual pool elevation readings were taken until equilibrium was reached by the model. Successive hook-gage readings at the weir were then taken until the average of three readings was constant, after which a series of three to five readings were averaged. The discharge reading determined from the weir-rating curve was then inserted into the placard on the piezometer boards, together with data for identification of the run, and a picture was taken of the boards. If there was much variation of the column heights, several photographs were taken.

As a means of providing a constant check on the data during the experiments, a discharge-rating curve was plotted concurrently from visual observations of headwater level, and if the run followed the proper trend, the discharge was adjusted to a new value.

The specific order of testing the various inlets, along with the identification of the test runs, is given in the appendix. Generally, the tests were run in the following order: First, Inlets 1, 2, and 3 were tested on a 4 percent grade, after which Inlet 1 was reinstalled to obtain data not taken in the first tests. Subsequently, modifications were made to Inlet 1 to improve its operation, and at the completion of these tests the grade was reduced to zero. After testing Inlets 1 and 4 on the flat grade, Inlets 5 and 6 were developed. The grade was then adjusted to 4 percent for tests of Inlets 5, 6, and 7 and then was increased to 8 percent for comparison of Inlet 1 and its final modifications, Inlets 4, 5, and 7.

RESULTS

The presentation of the results of these experiments will be separated into two sections, one dealing with the operation of Inlets 1, 2, and 3, and the other dealing with modifications of Inlet 1.

Inlets 1, 2, and 3, General (4 Percent Grade)

The operation of Inlets 1, 2, and 3 can be best described by reference to Figure 6, which contains rating curves for the inlets. These curves, which were obtained from the model with the barrel set to a 4-percent grade, describe the discharge characteristics of the inlets for their total range of operation on the given grade. Hydraulic grade lines (or pressure lines) obtained from piezometer readings are included in the appendix as a means of presentation of the original data from the experiments.

Inlet 1 normally followed the free-flow discharge curve with critical depth control at entrance occurring for the nonsubmerged condition and with operation closely resembling that of a sluice gate for the submerged condition, as shown in Figure 10. For all but the highest discharges, the culvert grade was sufficient to produce accelarating flow in the culvert barrel, and for all discharges the flow was controlled at the entrance. As shown in Figure 10, the major part of the sluice contraction occurred within the first 4 in. of the culvert barrel, after which the depth either decreased gradually or remained reasonably constant.

Full-flow conditions for this inlet could be attained only by placing a temporary obstruction to flow at the outfall or by inducing turbulence in the stream at the culvert entrance. Under full-flow condition, the discharge for a given headwater elevation was increased considerably, substantiating the general theory of the inlet operation.

When the culvert was flowing full, the contraction of streamlines at the inlet section persisted and was accompanied by a local drop in pressure a short distance beyond the entrance. The minimum pressure occurred approximately 4 in. downstream of the entrance. Whenever sufficient air was admitted to the inlet section, the water broke away from the top of the culvert barrel at the entrance and the typical sluice contraction formed. Subsequently, the culvert reverted to free-flow operation with a consequent rise of the headwater pool. If the quantity of air was limited, the contraction would occur at the entrance, but a hydraulic jump would form downstream in the culvert barrel, with the result that only part of the barrel would flow free. As the quantity of air supply was increased, the jump would move downstream until eventually the water surface in the culvert would be completely free.

After the culvert barrel had been artificially filled, it would remain full, so long as no air was admitted to the entrance. In the ordinary case, however, a free vortex would form in the pool above the entrance and admit air to the inlet in sufficient quantity to allow the culvert to revert to sluice operation. It was, therefore, decided that with the culvert on a 4-percent grade the full-flow operation of the culvert equipped with Inlet I was unstable and, therefore, unreliable as a possibility for field application.

The operation of Inlet I was also studied with the culvert on a flat grade in order to determine whether or not the sluice contraction would cause the culvert to flow free upon inlet submergence. On this grade, as the discharge was increased, the control was critical depth near the outlet until the water surface in the upstream portion of the culvert barrel met the top of the barrel. This occurred at the upper end of the barrel, because there was always a drawdown of the water surface along the length of the culvert barrel to critical depth near the outlet. Subsequently, the culvert barrel flowed full until vortices forming at the corners of the parapet wall vented the sluice contraction. The length of the vented portion increased with discharge until the water surface was completely free for the length of the barrel. The head at which the foregoing vortices

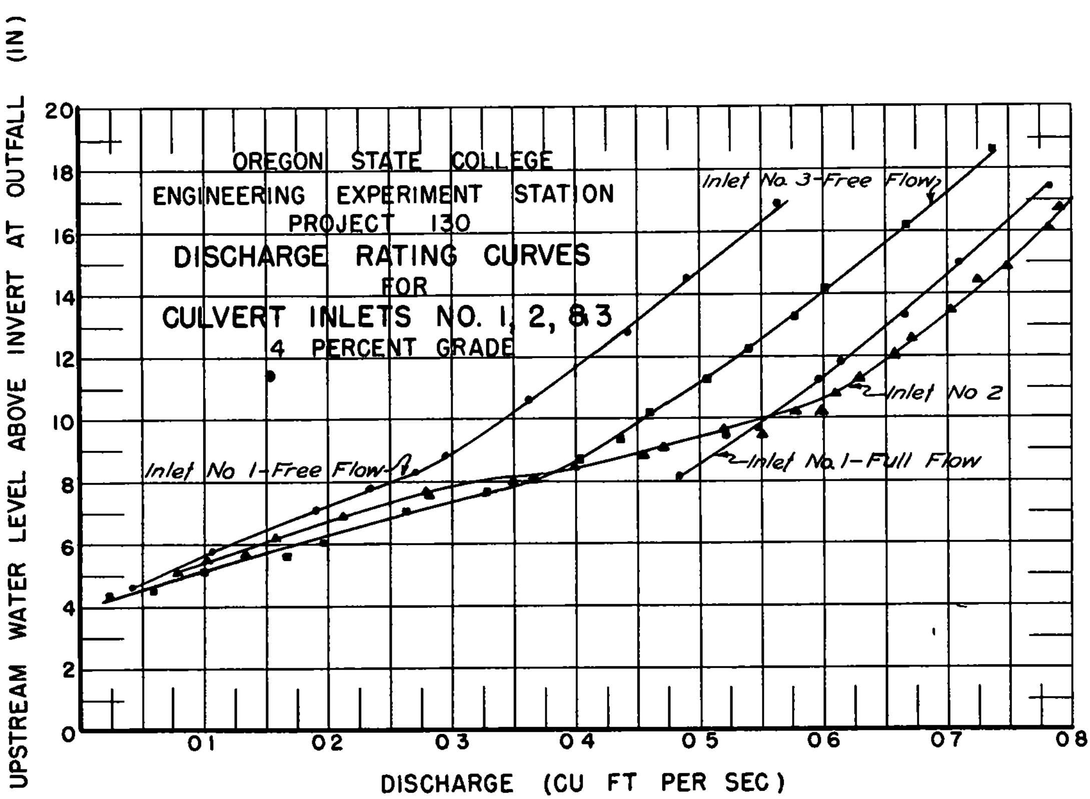


Figure 6. Discharge rating curves for culvert Inlets 1, 2, and 3.

formed was very low, and the range of discharges during which the culvert barrel flowed full was insignificant with respect to the total capacity of the culvert. When the culvert barrel was flowing free with its inlet submerged, the depth of the water increased downstream of the contraction because of the lack of slope of the barrel. Since the velocity of water leaving the contraction was greater at higher heads, the increased momentum of the jet allowed less downstream rise as the head was increased. By reference to the foregoing description, it can be seen that the length of a culvert on a flat grade, as well as the headwater level, are major factors in establishing the type of flow within the culvert barrel. It is conceivable that some long culverts on flat grades may never be capable of flowing free for their entire lengths, because of the resistance losses in the barrel.

The discharge characteristics of Inlet 2 differed radically from those of Inlet 1. As the discharge through the culvert was gradually increased, the culvert flowed under entrance control until the headwater pool intersected the top of the tapered section, after which the tapered section and then the culvert barrel flowed full. The rating curve for Inlet 2 (see Fig. 6) shows a large increase in flow for a small increase in head after the inlet was submerged. This range of operation on the model was characterized by an intermittent free-full discharge in the culvert barrel, a condition caused by the lack of sufficient flow in the approach channel to supply a full culvert barrel at the head of submergence. The rating curve followed very closely the trend of the full-flow curve of Inlet 1 after the culvert barrel was completely full.

When the culvert barrel was flowing full, there was no local pressure drop at the entrance to the culvert, implying that the contraction experienced when Inlet 1 was flowing full was not present in this case. The absence of this contraction allowed the culvert barrel to remain full, even upon the admission of air to the entrance; thus, vortex action in the pool above the entrance had no appreciable effect upon the full-flow capacity of the culvert. It was therefore concluded that full flow was the normal submerged condition of the culvert with Inlet 2 installed and that the principle of operation of the inlet was worthy of further consideration.

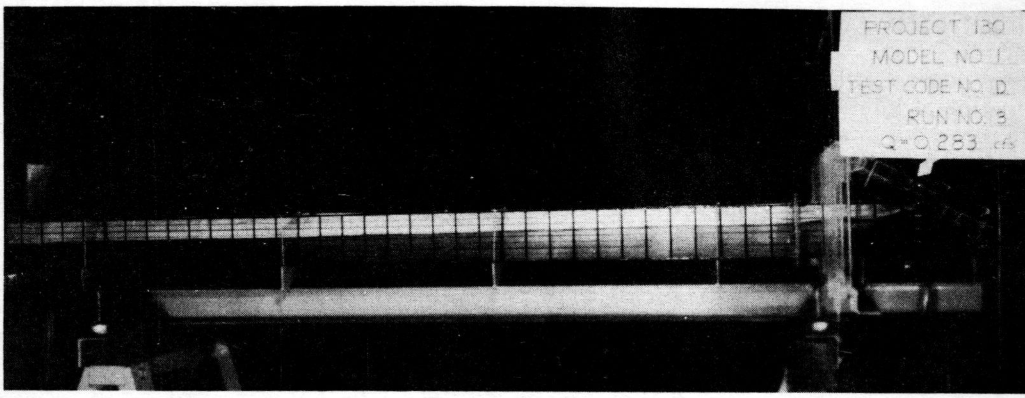


Figure 7. Model on 4 percent grade with Inlet 1 installed. The inlet is not submerged. Note depth at outfall.

Inlet 3 was designed primarily for operation in a nonsubmerged condition with critical depth control at the entrance. In the design, a drop in the flow line was provided within a tapered entrance section in order to accelerate the water to the velocity required for uniform flow in the culvert barrel. The design provided for an overall grade, including the inlet section, of 4 percent, in which case the slope of the culvert barrel was 1.85 percent.

With Inlet 3 installed the model operated as designed in the nonsubmerged range of discharges and proved to be more satisfactory in this range than the others tested (see Fig. 6). However, the combination of the drop in the flow line and abrupt angle between the tops of the taper and culvert barrel caused a contraction of the water surface at the entrance to the culvert barrel, so that the normal submerged operation of the model under these circumstances was free flow.

Since the over-all operation of this inlet was less satisfactory than that of Inlet 2, further analysis of this design was abandoned in favor of full-flow studies of other inlet types. Had the angle between the tops of the taper and culvert barrel been made less abrupt by streamlining the boundary, it is probable that the culvert barrel could have been made to flow full automatically, but limitations of time did not permit investigation of this possibility.

Critical Depth Control (4 Percent Grade)

The degree with which the nonsubmerged operation of Inlets 1, 2, and 3 conformed with the theory can best be described by use of the equation

$$H = 1.5\left[\frac{q^2}{g}\right]^{1/3}$$

where H is the total head above the culvert inlet at the location of critical depth (the control point) and represents the upstream pool elevation above that point when velocity head in the pool is neglected. Figure 8 is based upon the foregoing equation, with the dotted lines plotted for the widths shown. The locations of critical depths computed from given discharges were determined by reference to hydraulic grade lines plotted from the piezometer tube readings. These locations are shown in Figure 9 with sketches describing the configurations of the inlets in the vicinity of the control points. For Inlet 3, the location of critical depth was upstream of the break in slope and between the wing walls; since this location varied with discharge, the average width of 5.15 in. was used for the plot in Figure 8 simply as a reference for the plotted points.

The points plotted in Figure 8 are the actual upstream pool elevations above the control points as determined from piezometer readings. For the 4-in. width, the experimental data match the theoretical heads (dotted lines) very closely, and for the 5.15-in. width there is exceptional correspondence, considering that the width used for the dotted-line plot was an average and open to question.

Figures 8 and 9 demonstrate the extent to which critical depth theory can be used to compute discharge through a culvert. If it is possible to estimate the location of the control section, the discharge can be accurately determined from the elevation and width at that section. With Inlets 1 and 2, in cases where the grade of the culvert is relatively flat (but still supercritical for the given discharge), a reasonably accurate computation could be made upon the assumption that critical depth occurred at the entrance to the uniform culvert barrel, but on steeper grades the error caused by the difference between the

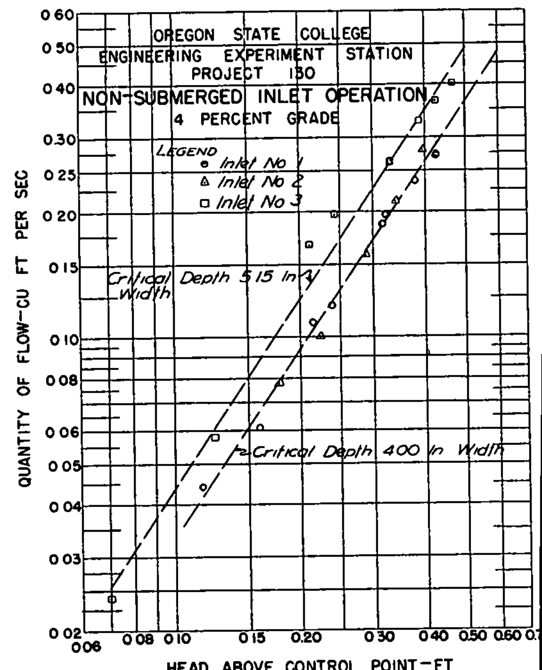
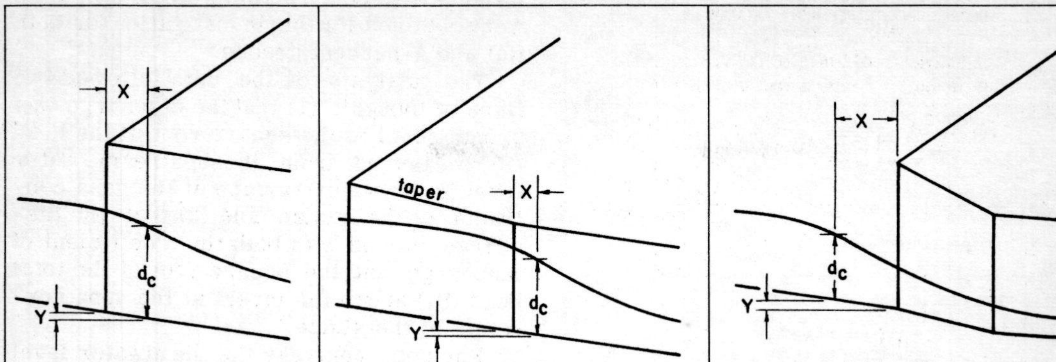


Figure 8. Nonsubmerged operation of culvert Inlets 1, 2, and 3. Dashed lines are plotted from theoretical calculations.

elevation of the control section and that of the upstream end of the culvert could be significant. From the results of these experiments, the maximum discharge error resulting from the foregoing assumption was found to be 7 percent for Inlet 1 and 6 percent for Inlet 2, with the model on a 4-percent grade. For Inlet 3 the variation of width at which the control is located makes necessary a more-precise determination of the location of the control for discharge computations.

The conformance of the experimental data with the theoretical curves in Figure



Inlet No. 1

Q cfs	X in.	Y in.
0.044	1.05	0.04
0.061	2.30	0.09
0.108	2.60	0.10
0.119	3.90	0.16
0.148	3.05	0.12
0.188	3.40	0.14
0.192	2.80	0.11
0.236	2.45	0.10
0.271	2.15	0.09

Inlet No. 2

Q cfs	X in.	Y in.	W in.
0.150	2.0	0.08	4.00
0.200	3.2	0.13	4.00
0.250	3.9	0.16	4.00
0.300	1.8	0.07	4.00
0.350	-1.3	0.05	4.26
0.387	-2.3	0.09	4.46

Inlet No. 3

Q cfs	X in.	Y in.	W in.
0.024	0.0	0.00	4.63
0.058	0.2	0.01	4.90
0.166	0.7	0.03	5.56
0.197	0.5	0.02	5.30
0.262	0.5	0.02	5.30
0.328	0.5	0.02	5.30
0.366	0.4	0.01	5.16
0.403	0.3	0.01	5.04

Figure 9. Measured location of critical depth.

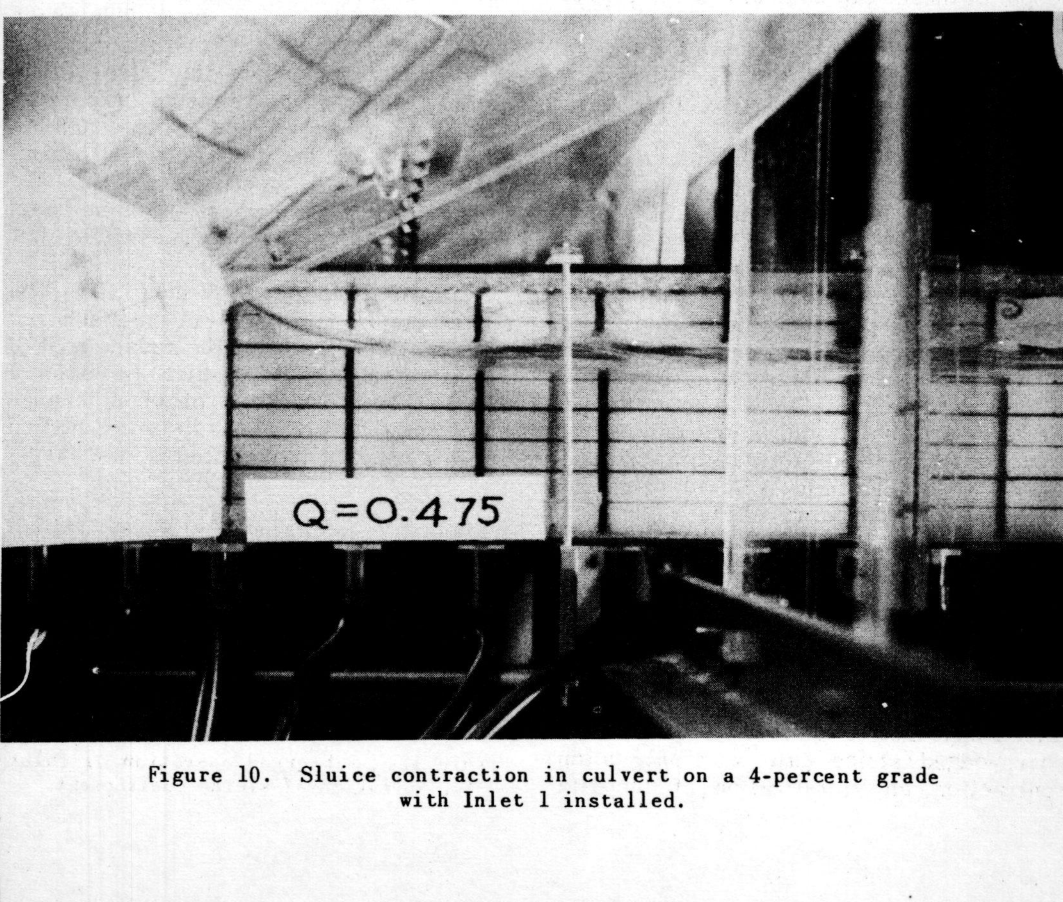


Figure 10. Sluice contraction in culvert on a 4-percent grade with Inlet 1 installed.

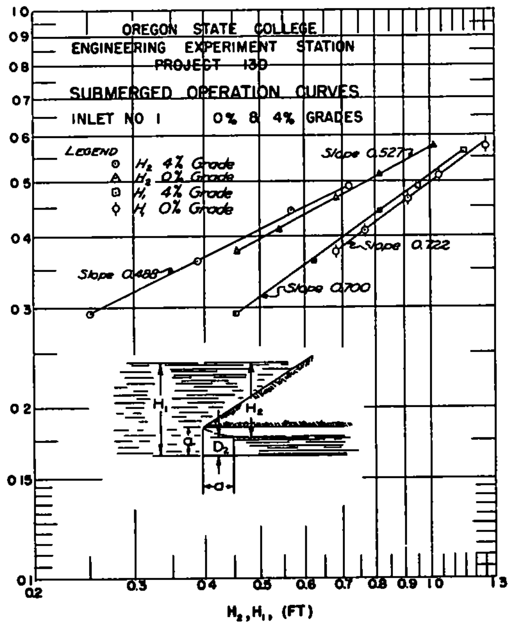


Figure 11. Submerged operation curves for Inlet 1.

8, indicates that the location of the control sections in these experiments was sufficiently accurate. However, since the scope of these experiments with respect to location of critical depth was limited, no attempt will be made here to generalize the results to apply to the experimental inlets on different grades or to other types of inlets.

There may be some question as to the accuracy of the use of piezometric measurements for the determination of the location of critical depth in these experiments, since the measurements were taken in a region of curvilinear flow. It is known that, in the case of downward curvature of the water surface, the centrifugal action of the water can cause a piezometric depth indication to be less than the actual depth; conversely, upward curvature can cause piezometric depths to be greater than actual depths. At the location of critical depth, however, the curvature of the water surface is zero, in which case the surface effects should cause little error in piezometric indications.

Sluice-Gate Operation

Because the submerged operation of Inlet 1 appeared to be similar to that of a sharp-edged sluice gate with only a top contraction, an investigation of the data was made to confirm this similarity. Data were obtained for this investigation for both flat and 4-percent grades.

The analysis of the data followed two lines of thought: (1) that the discharge was proportional to the square root of the head (H_2) measured from the upstream water level to the water surface at the vena contracta of the sluice and (2) that the discharge varied with both the coefficient of discharge and the square root of the total head (H_1) above the invert at the vena contracta of the sluice.

For both analyses the headwater level was determined by piezometer readings, and the water level at the vena contracta was obtained by direct observation through the side of the culvert at a distance of 4 in. downstream of the entrance. The computed head was then plotted against discharge on log-log graph paper and the results are shown in Figure 11 for both analyses. In both cases the plotted points defined straight lines, giving the indication that, within the range of these experiments, the discharge was proportional to some constant power of the head for either type of analysis. Using the sluice head, H_2, the slopes of the lines for the two grades were close to 0.5, which shows that the theoretical assumptions were nearly correct in this case. Since the head in these experiments was small with respect to the height of the opening, some variation from the proportionality of discharge to the square root of the head might be expected, much on the same basis as with orifices discharging under low heads.

With reference to Figure 11, it will be noted that in no case was the discharge directly proportional to the square root of the head, a condition which requires a variation of the coefficient of discharge with some function of the head if Equation 6 is to be used for computation of discharge.

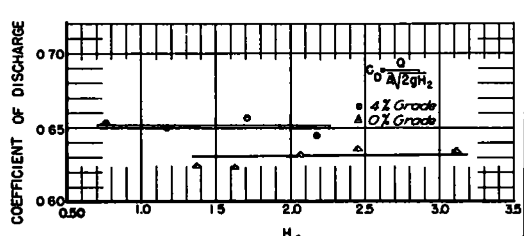


Figure 12. Submerged operation of Inlet 1, variation of sluice coefficient.

Plots of C_d against H/a are shown in Figures 12 and 13, demonstrating the manner of variation of C_d for the two analyses. In view of the fact that in no case could the coefficient of discharge be assumed constant, it is apparent that the analysis based on H_1 would be the most useful in the application of the results, since only one variable need be chosen in order to make a discharge calculation from a given H_1. In contrast, in order to utilize the results of the analysis involving H_2, the depth at the vena contracta in addition to the coefficient of discharge must be determined for a given head. This depth was constant when the culvert was on the flat grade but increased slightly with discharge when the grade was 4 percent.

A point of interest arising from the results of both investigations was the variation of the discharge coefficients with culvert slope. The increase of the coefficients with slope is expected for two reasons: (1) for a given upstream pool elevation the effective head above the entrance area is greater for the horizontal culvert than it is for the inclined culvert and (2) the vertical angle with which the water approaches the top of the opening is less when the culvert is inclined, with the result that the contraction should be less. Insufficient data were obtained from these experiments to make possible an analysis of the foregoing variation, and the necessity of further experiments for this purpose is indicated.

Full-Flow Operation

With reference to the full-flow rating curves in Figure 6, it will be noted that with the same barrel slope and length the discharge capacity of Inlet 2 was slightly greater than that of Inlet 1. The reason for the increased capacity of Inlet 2 can be explained on the basis of the pipe-flow theory. The total head causing flow in the culvert at a given slope is the sum of the loss of head caused by the conditions at entrance, the loss caused by the fluid friction in the culvert barrel, and the velocity head in the culvert. The head loss caused by the entrance conditions is a function of the shape of the entrance and is usually expressed in terms of the velocity head in the culvert. Since the downstream turbulence caused by a tapered-inlet section will be less than that for a uniform-inlet section with a square top edge due to the lack of an entrance contraction, the head loss at a given discharge will also be less, with the result that the capacity of the culvert is increased by use of a tapered inlet.

The analysis of the friction and entrance losses in the full culvert barrels requires a culvert long enough so that the turbulence caused by the entrance conditions is not present in the lower reaches of the barrel, since a definite trend in friction loss must be established. Although the pressure lines (see appendix) appear to establish reasonable trends, it is not believed that the barrel length (approximately 20 diameters in most cases) was sufficient for exact determinations of entrance and friction loss coefficients in these studies.

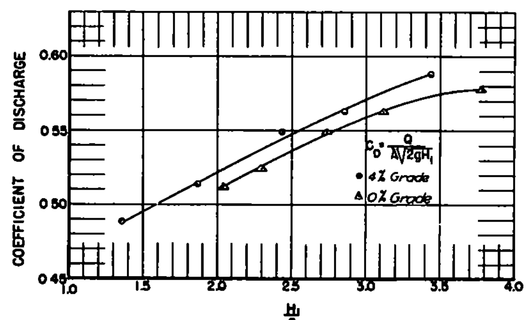


Figure 13. Submerged operation of Inlet 1, variation of sluice coefficient.

Modification Experiments

Tests on Inlets 1 and 2 indicated that a desirable objective of these experiments was the development of a culvert inlet which would allow the culvert barrel to flow full upon inlet submergence. This objective was based upon the increased discharge capacity of the culvert model when equipped with Inlet 2 over that of the model with Inlet 1 operating as a sluice. Since it appeared possible, as a result of preliminary experiments, to make simple modifications to Inlet 1 to accomplish the foregoing objective, it was decided to explore the possibilities of modifications in lieu of attempting major changes of inlet design.

The submerged operation of Inlets 1 and 2 indicated that the elimination of the entrance contraction was necessary for stable full-flow operation of a culvert barrel. Since the approach conditions to Inlet 2 were almost identical with those of Inlet 1, there was some reason to believe that

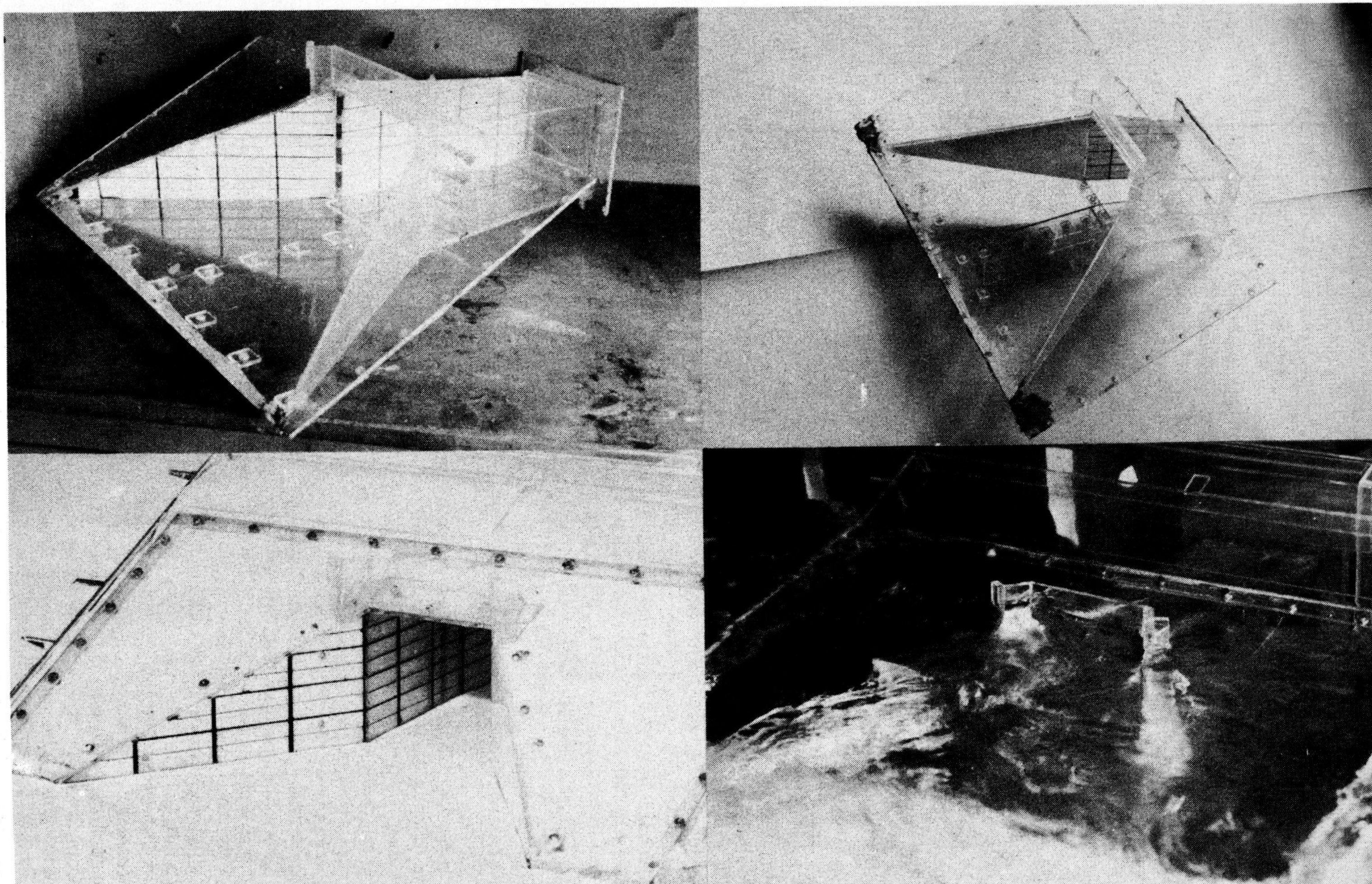


Figure 14. Inlet models: Upper-left, Inlet 2, top and side taper on 1:10 slopes to 4- by 4-in. flanged end; upper-right, Inlet 3; lower-left, Inlet 1 installed in model; and lower-right, flow into Inlet 1 with 4-in. top slab extension (the culvert barrel is flowing full in this case).

if a contraction occurred at the entrance of Inlet 2 it would bear the same relationship to entrance area as did the contraction of Inlet 1. The sluice experiments with Inlet 1 indicated a contraction of approximately two thirds of the entrance area was to be expected at the entrance of Inlet 2, and since the area of the entrance of the latter inlet was nearly twice that of the culvert barrel, it was possible that the contraction which should have been present in the inlet was suppressed by the enlargement of the entrance.

In addition, the tendency for the formation of a contraction at the entrance of Inlet 2 should have been less because of the reduction of the approach velocities in the vicinity of the entrance.

On the basis of the foregoing analysis, one practical possibility for modification of Inlet 1 was to enlarge the entrance by extending a portion of the wing walls to meet an extension of the top slab upstream of the parapet wall (see Fig. 14). The taper thus formed was only in the sides of the inlet, with no change in culvert height within the inlet.

Since the contraction experienced with Inlet 1 was approximately two thirds of the entrance area, the first trial in entrance enlargement was extension of the top slab to provide an entrance area of 1.5 times the area of the culvert barrel. Trial runs with this modification showed elimination of the top contraction, with the result that the culvert barrel would flow full upon submergence of its inlet. It was discovered, however, that side contractions occurred, whenever air was admitted to the entrance of the culvert, indicating that either the side taper was too abrupt or that the tapered section was too short. The principal difference between the operation of this inlet and that of Inlet 1 was that, if the supply of air was discontinued, the culvert barrel would automatically fill.

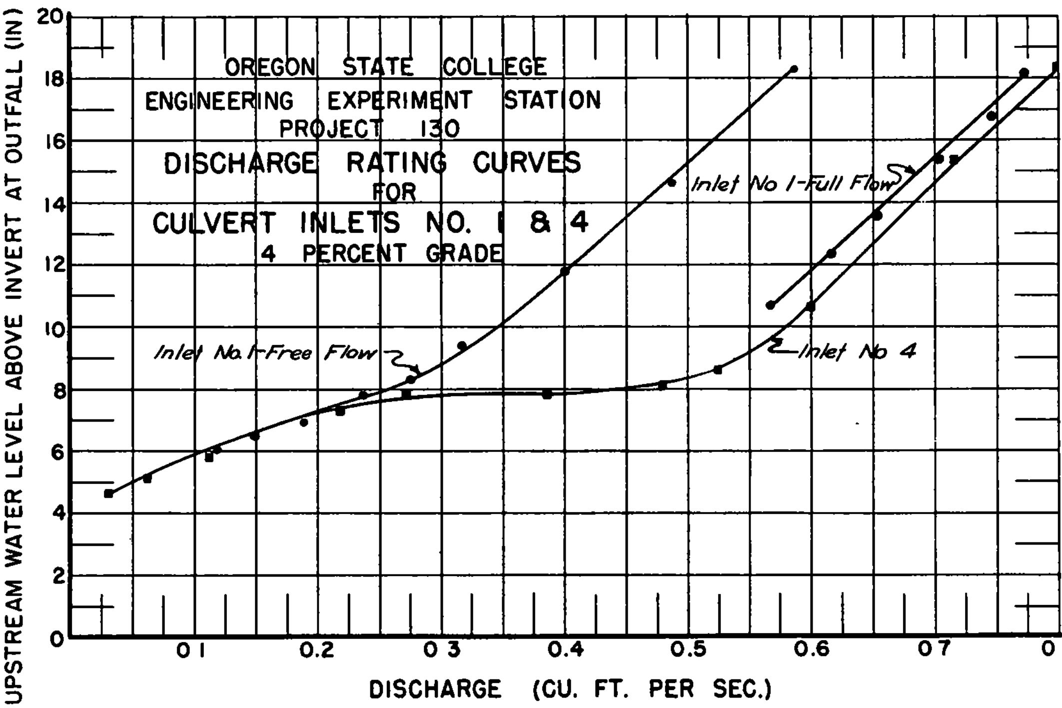


Figure 15. Discharge rating curves for culvert Inlets 1 and 4, 4-percent grade.

Considering the possibility that a greater enlargement of area than 1.5 to 1 would be necessary, a sheet aluminum extension of 4 in., giving an area ratio of 2.33 to 1, was installed in the model. Trial runs indicated that the extension provided was adequate for the elimination of all contractions and that the culvert barrel would flow full for all values of H_1/a greater than 1.13. Subsequently, the extension was shortened progressively by ¼-in. steps until the side contractions occurred. At a top slab extension of 2½ in. (area ratio of 1.83 to 1) it was decided that the full-flow operation of the culvert was unstable, and the shortening tests were discontinued. Complete tests were then run on the inlet equipped with a more precise extension of Plexiglas, 1 in. longer than the minimum length previously determined. This inlet, hereafter designated as Inlet 4, thus had a top slab extension of 3.5 in., providing an area enlargement of 2.16 to 1 (see Fig. 14).

section during the total range of full-flow discharges.

While the model was on the flat grade for investigation of the sluice-gate operation of Inlet 1, a series of tests was run on Inlet 4 to determine the advantage of full flow on this grade. The resulting rating curve in Figure 16 shows that there is an increase in the full capacity of the culvert equipped with Inlet 4 over that of the culvert operating as a sluice. This advantage

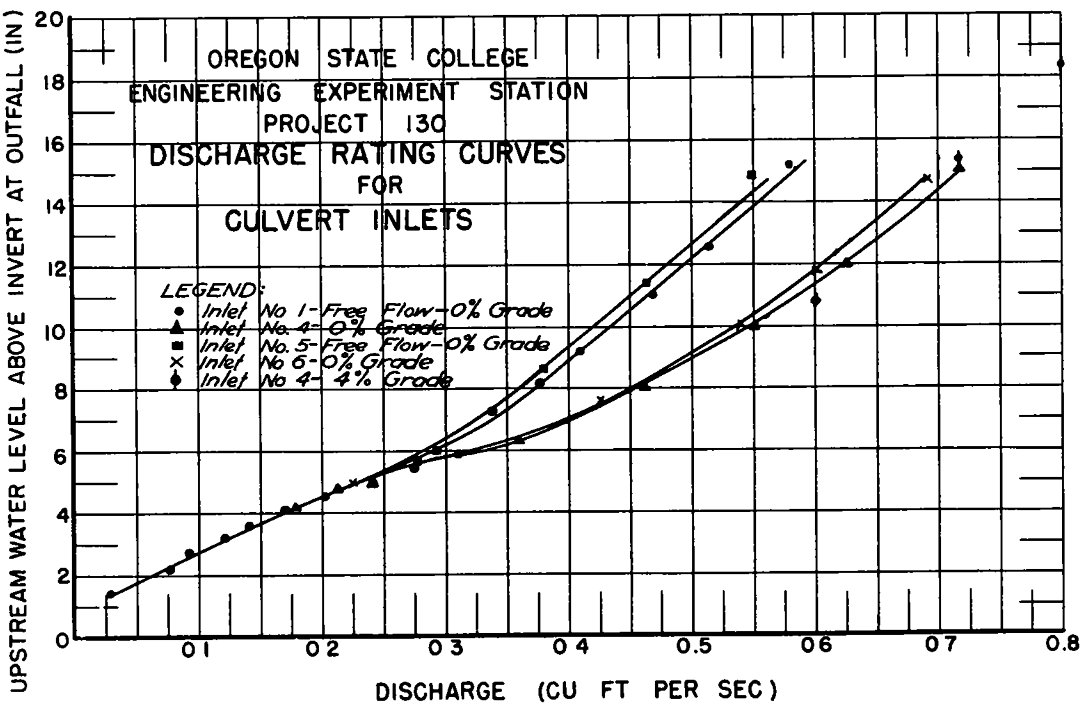


Figure 16. Discharge rating curves for culvert inlets.

The operation of Inlet 4 is compared with that of Inlet 1 in rating curves in Figure 15 for the 4-percent grade. In this figure it is seen that upon submergence of Inlet 4 the discharge increased to a value nearly double that required for submergence before the upstream water level increased perceptibly. During this increase of discharge the culvert barrel flowed intermittently free and full, with very little change in upstream water level. Figure 14 shows the flow conditions at the entrance of Inlet 1 with a 4-in. top-slab extension and operating in this range of discharges. The hydraulic grade lines obtained from these tests (see appendix) show that there was no contraction within the entrance

is presumed to be due to an increase in the flow area by the elimination of the entrance contraction, in addition to the fact that the culvert barrel was sufficiently short that friction losses in the barrel were not excessive. It is conceivable that the advantage due to the increase in area of flow could be lessened by friction losses in a longer culvert barrel.

In order to add to the general value of the experiments, it was decided to determine to some extent the effect of variation of wing-wall angle upon the necessary length of the top-slab extension. Inlet 1 was modified by reduction of the wing-wall angle to 1 to 4, an angle chosen as the minimum for which any significant savings could be made in materials required for construction of the apron between the wings. The resulting inlet was designated as Inlet 5.

It was determined that, when the culvert was on the flat grade, a top slab extension providing an entrance enlargement of 1.5 to 1 (Inlet 6) was sufficient for elimination of the entrance contractions, and the results are shown in Figure 16. On the 4-percent grade, however, it was found that the operation of Inlet 6 was unstable, because of strong vortex action at low heads. This operation was considered undesirable, since there was such variation of piezometer readings that it was impossible to obtain satisfactory measurements. Furthermore, the presence of strong vortex action in a culvert installation should be undesirable, because of the tendency of the vortices to suck floating drift into the inlet at times when the capacity of the culvert would be the most critical.

On the 4-percent grade the same procedure used for the development of Inlet 4 was then applied to Inlet 5, and it was determined that a minimum entrance enlargement of 2 to 1 was required for stable operation of the inlet. As the top slab extension was shortened from an original length of 8 in., it was noted that the tendency for the formation of vortices above the entrance appeared to be a function of the length of the tapered section, with apparently greater tendencies with shorter sections. Since the velocity at the entrance is a function of the area of the entrance, it was concluded that the tendency for the formation of vortices above the entrance was a definite function of the entrance velocity. This indication was also observed during the development of Inlet 4; however, to a lesser degree. The enlargement of the entrance area in a culvert, then, should not only eliminate the undesirable effects of vortices but also should discourage their formation.

Observation of the hydraulic grade lines for the flat and 4-percent grades when Inlet 4 was installed raised some question as to the possible operation of the inlet with the culvert on steeper grades. The principal reason for this uncertainty was the probability that the pressure gradients for full discharges at low heads would have sufficiently flat slopes to fall below the invert of the culvert at the entrance. Under these conditions subatmospheric pressures would exist at the entrance with the possibility for unstable full-flow operation, as was the case when Inlet 1 was flowing full on a 4-percent grade. Furthermore, the unsatisfactory operation of Inlet 6 on the 4-percent grade indicated that the necessary inlet enlargement could be a function of the culvert grade.

Accordingly, the grade of the model was increased to 8 percent, and the operation of Inlets 1, 4, 5, and 7 (the minimum top slab extension of Inlet 5 on the 4-percent grade) were compared for identical discharges under similar flow conditions. On this grade the operation of Inlets 4 and 7 was entirely satisfactory with stable conditions occurring in the full culvert for all discharges. The full operation of Inlets 1 and 5 was unstable, as was the case on the flatter grades.

Pressure measurements (see appendix) showed that subatmospheric pressures occurred at the entrance when Inlet 4 flowed full under a low head, but no undesirable effects resulted from this condition. It was interesting to note that the increased slope of the pressure gradients resulting from greater losses caused the pressure at the entrance to rise as the discharge was increased.

CONCLUSIONS AND RECOMMENDATIONS

Nonsubmerged Inlet Operation

For box culverts on steep grades where critical depth control must be at the entrance, these experiments indicated that the application of existing theory can produce satisfactory results in discharge calculations, if variation of the location of the control section can be ignored. It was determined that error resulting from the assumption of a fixed control would be small in cases where only the variation of the elevation of the control section is involved but would be great when the width of the section is also variable.

Submerged Inlet Operation

The normal submerged operation of a culvert provided with an inlet similar to Inlet 1 should follow the sluice theory as presented in this paper, and the experimental results provide a satisfactory means for calculation of the discharge through such a culvert. Since in these tests the maximum headwater depth above the culvert invert at the entrance was limited to four times the height of the culvert, the experimental results are necessarily applicable only to cases within that limit. However, it is believed that the experimental range of operation is typical of the majority of culvert installations, so that the limits of the experiments do not seriously affect the usefulness of the results.

The general operation of a full-scale culvert provided with Inlet 4 should be similar to that of the model, and the range of operation between critical depth control and complete full flow can be analyzed by application of the laws of similitude to the results of these experiments. However, for reasons previously discussed, the results from the full-flow experiments cannot be accurately applied to full-scale culverts for the purpose of discharge calculations, and resort must be made to the use of available data from full-scale culvert studies for this purpose.

The submerged operation of Inlets 2 and 4 leaves little doubt as to the possible economy that can be gained by the design of a culvert on a steep grade to flow full, since for a given discharge and upstream-pool elevation a smaller cross section of culvert barrel is required for a full culvert than for one discharging as a sluice gate. This advantage is not necessarily limited to cases where it is possible to operate culverts under considerable inlet submergence, because of the extreme increase in capacity of culverts equipped with Inlets 2 and 4 at the head of submergence. For example, with reference to the rating curves in Figure 15 for the models on a 4-percent grade, a culvert equipped with an inlet similar to Inlet 1 and with a height equal to that of Inlet 4 would require a width of approximately twice that of Inlet 4, in order to discharge an equal quantity of flow under the same low head. It should be noted that the example cited is applicable only to culverts on the 4-percent grade. The advantage of full flow would, of course, vary with the grade and length of the culvert, being less marked on a flatter grade.

The results have indicated that a culvert inlet similar to Inlet 4 is a practical and economical design for the purpose of causing a culvert barrel to flow full upon inlet submergence. The simplicity of construction of this inlet over that of Inlet 2 is the factor determining its choice, since the two inlets are nearly comparable in discharge capacity (within the range from 0.57 to 0.80 cu. ft. per sec., Inlet 2 has the slight advantage). In addition, the inlets of existing culverts having insufficient capacity by reason of an inability to flow full at inlet submergence can be modified to be similar to Inlet 4.

Recommendations for Further Experiments

1. The study of the nonsubmerged operation of the culvert inlets suggested the necessity for investigations of the variation of the location of the critical depth control point with discharge and culvert grade.

2. Since it was determined that the sluice coefficient of a culvert varied with both grade and head, it is recommended that further investigations be made under widely varied conditions in order to obtain some generally applicable results.

3. The inlet design resulting from these experiments was undoubtedly but one of many possibilities for causing a box culvert to flow full, and further experimentation could possibly provide a more simple and economical design for the purpose. As an example, curved sections were not investigated in these experiments.

4. Tests of pipe culvert and arches on steep grades, made on the same basis as these experiments, could possibly produce more economical designs than those in use at the present time.

ACKNOWLEDGMENTS

The authors wish to express appreciation to Carl F. Izzard and R.E. Tarbet, Bureau of Public Roads, and G. S. Paxson and R. C. Edgerton, Oregon State Highway Commission, for assistance and guidance in the preparation of this paper. Assistance rendered in conducting the experimental work by L. A. Herr, Bureau of Public Roads, and M. H. Karr and S. C. Ditsworth, students at Oregon State College, is also greatly appreciated.

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Appendix

Identification of Experiments

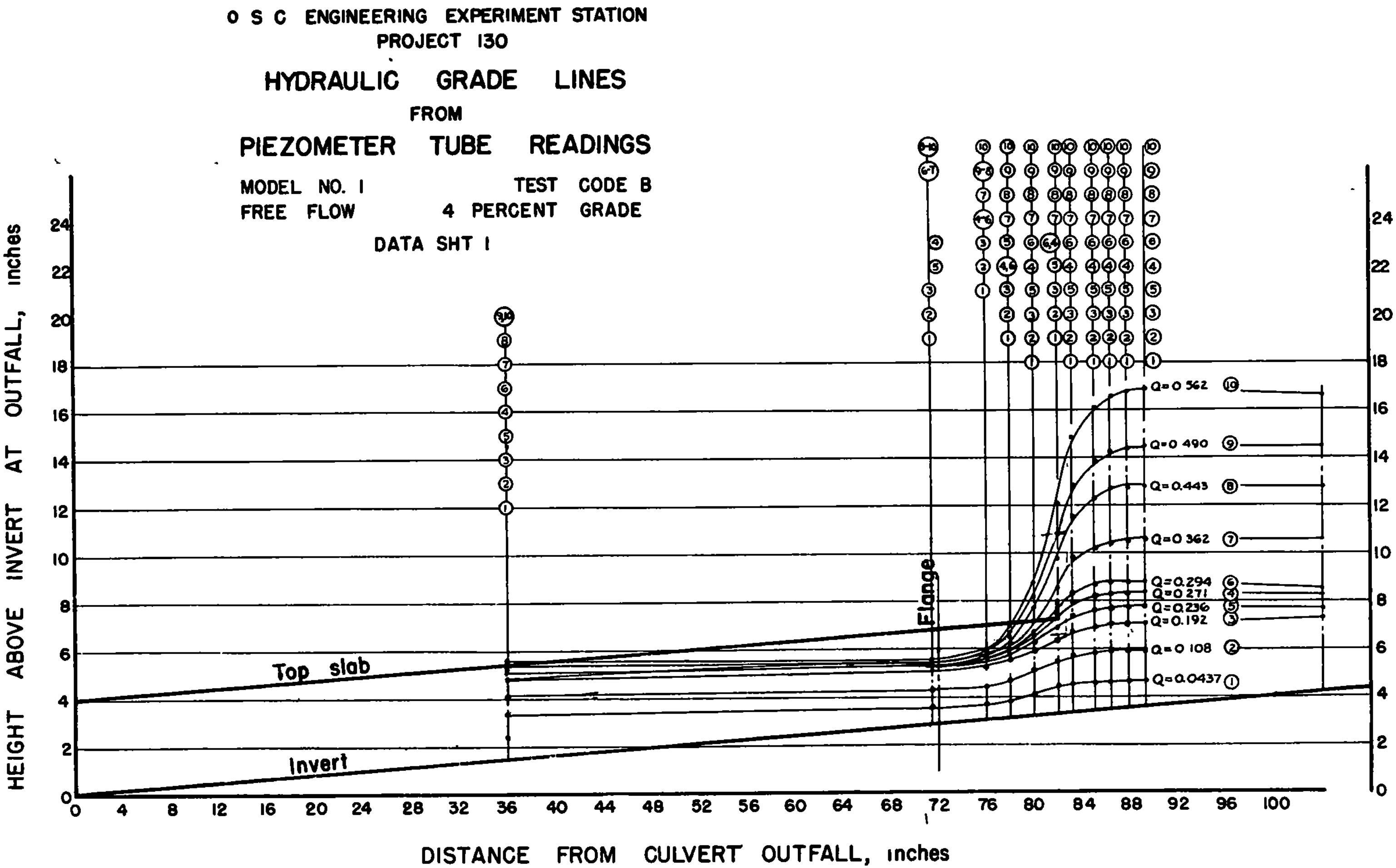
The organization of the experimental data was made in the following manner: (1) a run was designated as a set of readings taken at a given discharge through the model and (2) a series of runs taken for a specific purpose were grouped into a set and coded with a letter.

The following tests were made on the inlet models. For all tests reported a complete series of piezometer readings were taken unless otherwise indicated.

Inlet Number	Test Code	Grade Percent	Purpose	Data Sheet Number
1	A	4	Preliminary experiments (not reported)	-
	B	4	Free flow, total range of heads	1
	C	4	Full flow, total range of heads	2
	D, E	4	Descriptive photographs	-
	F	4	Free flow, total range of heads (re-run)	3
	G	4	Full flow, total range of heads (re-run)	4
	J	0	Free flow, total range of heads	6, 7, 8
2	A, A-1	4	Total range of heads	9, 10, 11
3	A	4	Free flow, total range of heads	12
4	H	4	Modification development experiments	5
	I	4	Total range of heads	13, 14
	M	0	Total range of heads	15
5	K	0	Free flow, total range of heads	16
	O	4	Modification experiments	17
6	L	0	Full flow, total range of heads	18

Test code P included comparison experiments of Inlets 1, 4, 5, and 7 on the 8-percent grade as given below:

Inlets	Condition	Data Sheet Number
1, 5	Critical depth control, identical discharges	19
1, 5	Submerged free, identical discharges	20
1, 4, 5, 7	Submerged full, two sets of identical discharges	21, 22


O S C ENGINEERING EXPERIMENT STATION
PROJECT 130
HYDRAULIC GRADE LINES
FROM
PIEZOMETER TUBE READINGS
MODEL NO. 1
TEST CODE B
FREE FLOW
4 PERCENT GRADE
DATA SHT 1
HEIGHT ABOVE INVERT AT OUTFALL, inches
DISTANCE FROM CULVERT OUTFALL, inches
Top slab
Invert
Flange
Q=0.562 ⑩
Q=0.490 ⑨
Q=0.443 ⑧
Q=0.362 ⑦
Q=0.294 ⑥
Q=0.271 ④
Q=0.236 ⑤
Q=0.192 ③
Q=0.108 ②
Q=0.0437 ①

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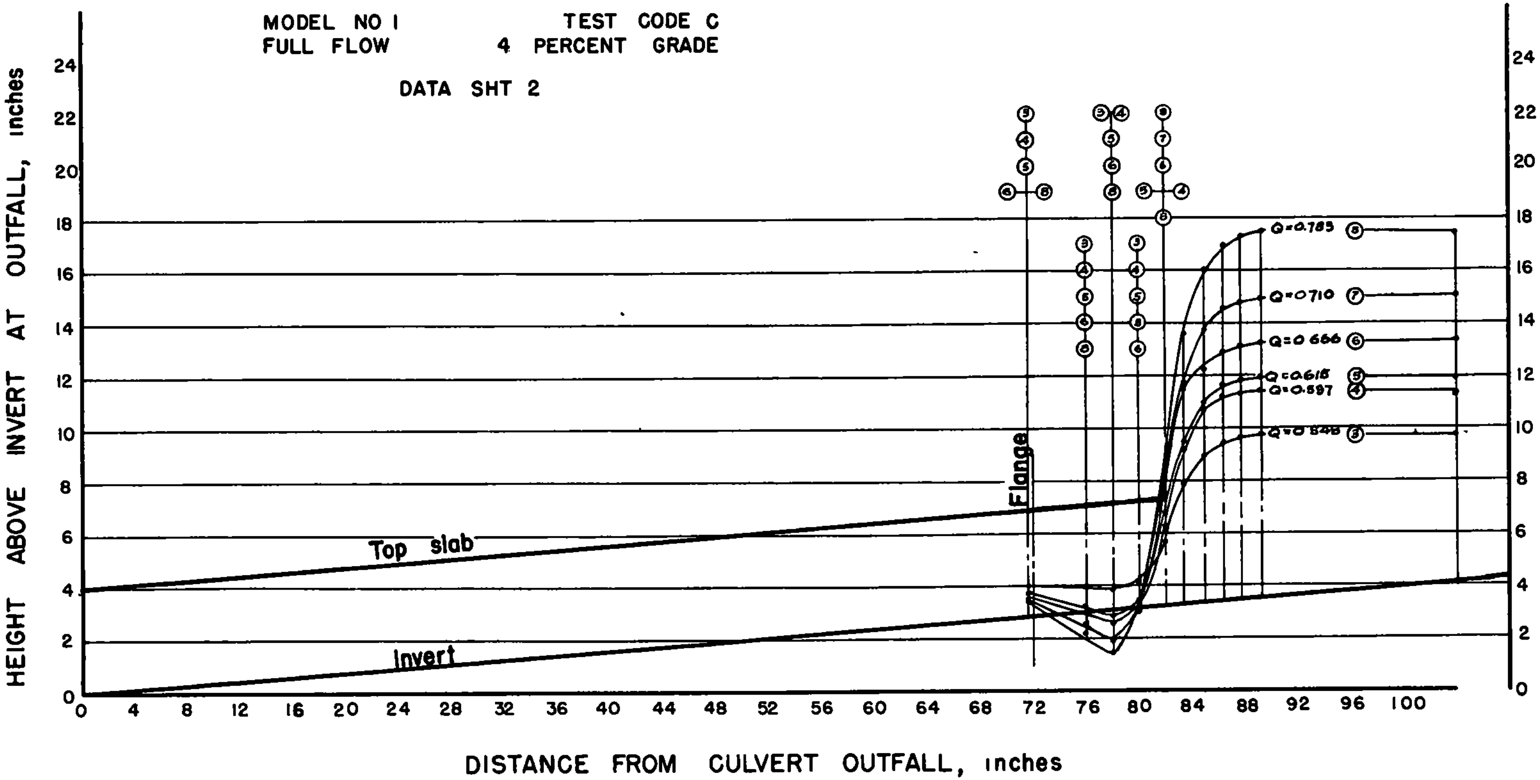
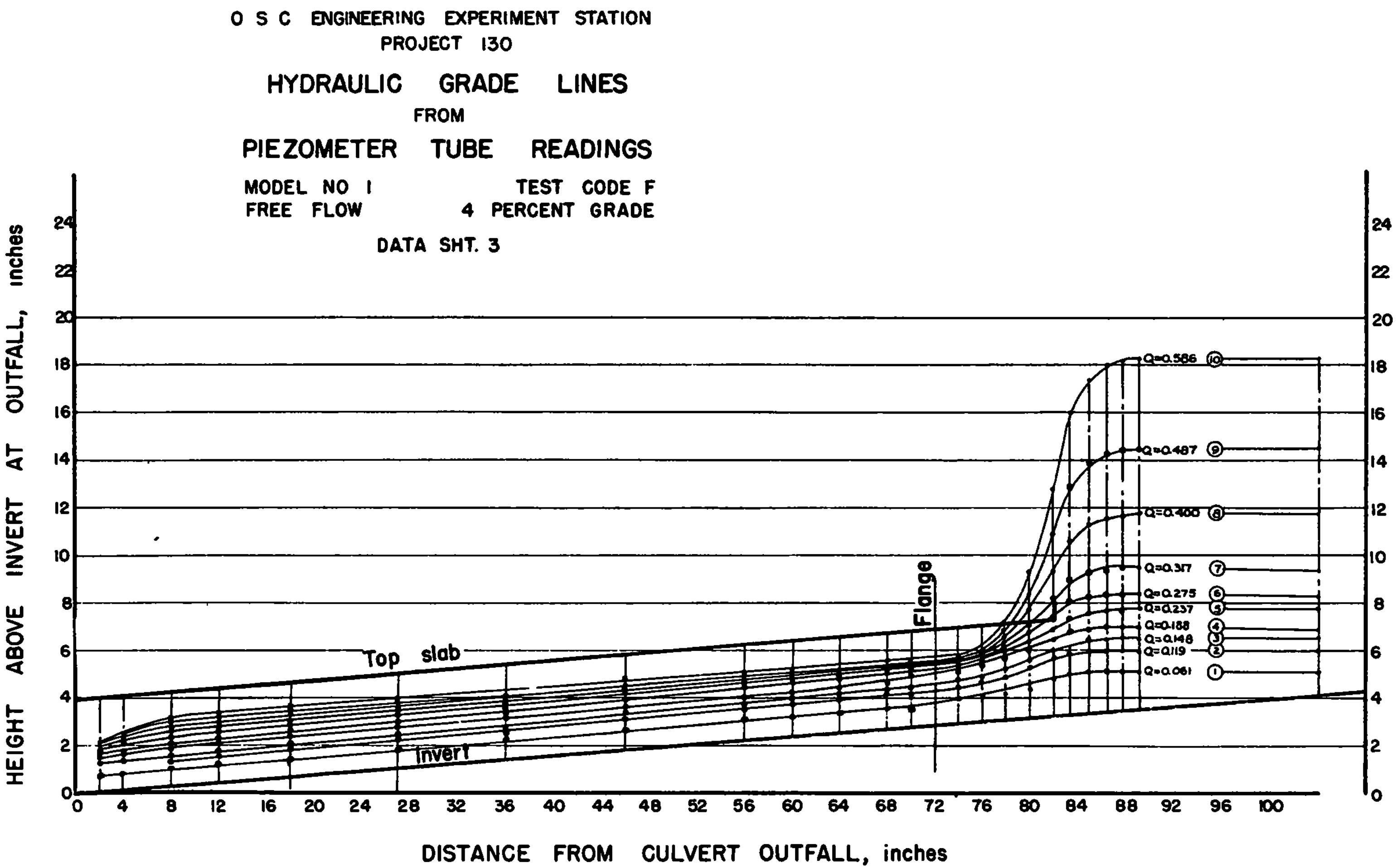
PROJECT 130

HYDRAULIC GRADE LINES FROM PIEZOMETER TUBE READINGS

MODEL NO I — TEST CODE C
FULL FLOW — 4 PERCENT GRADE

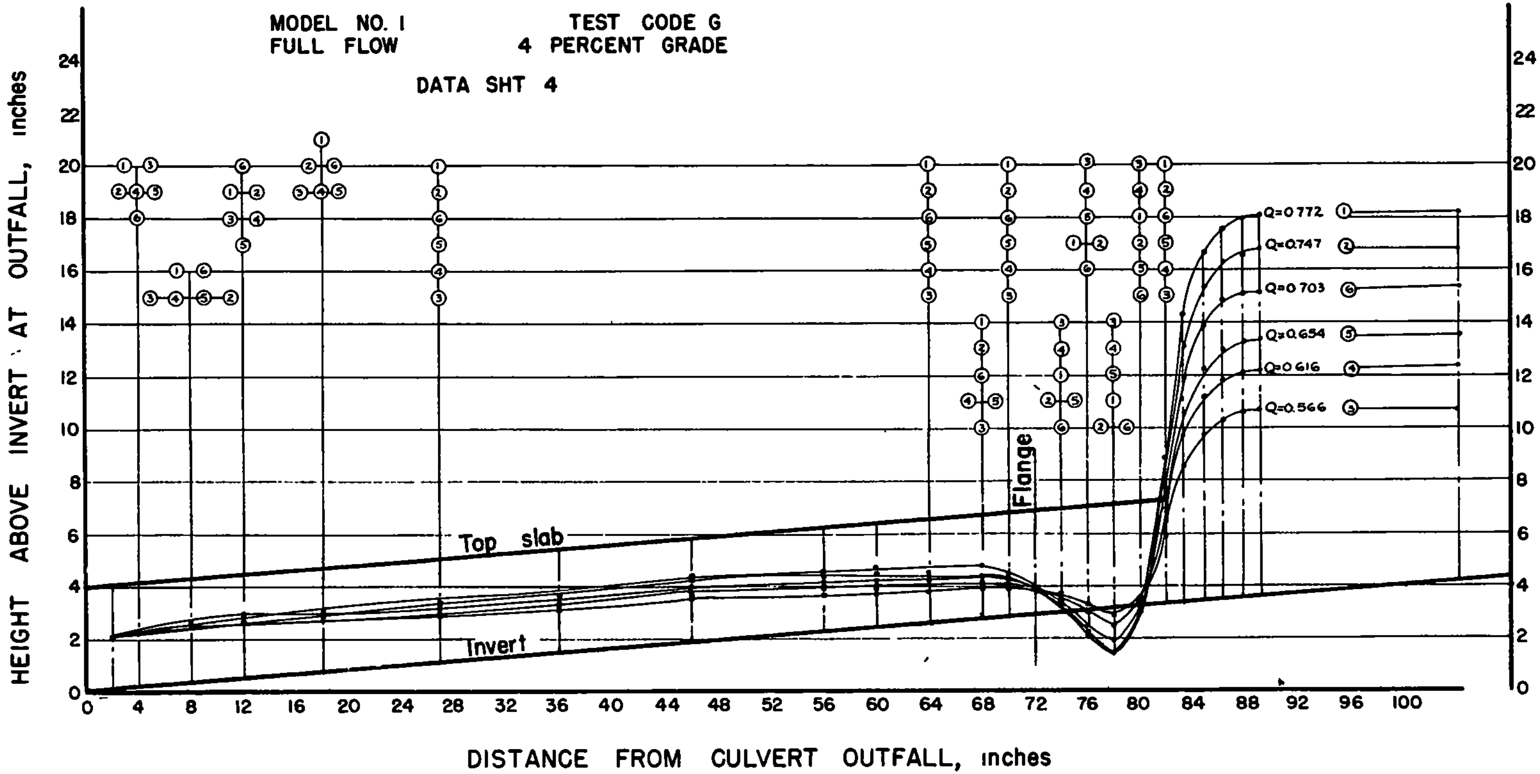
DATA SHT 2

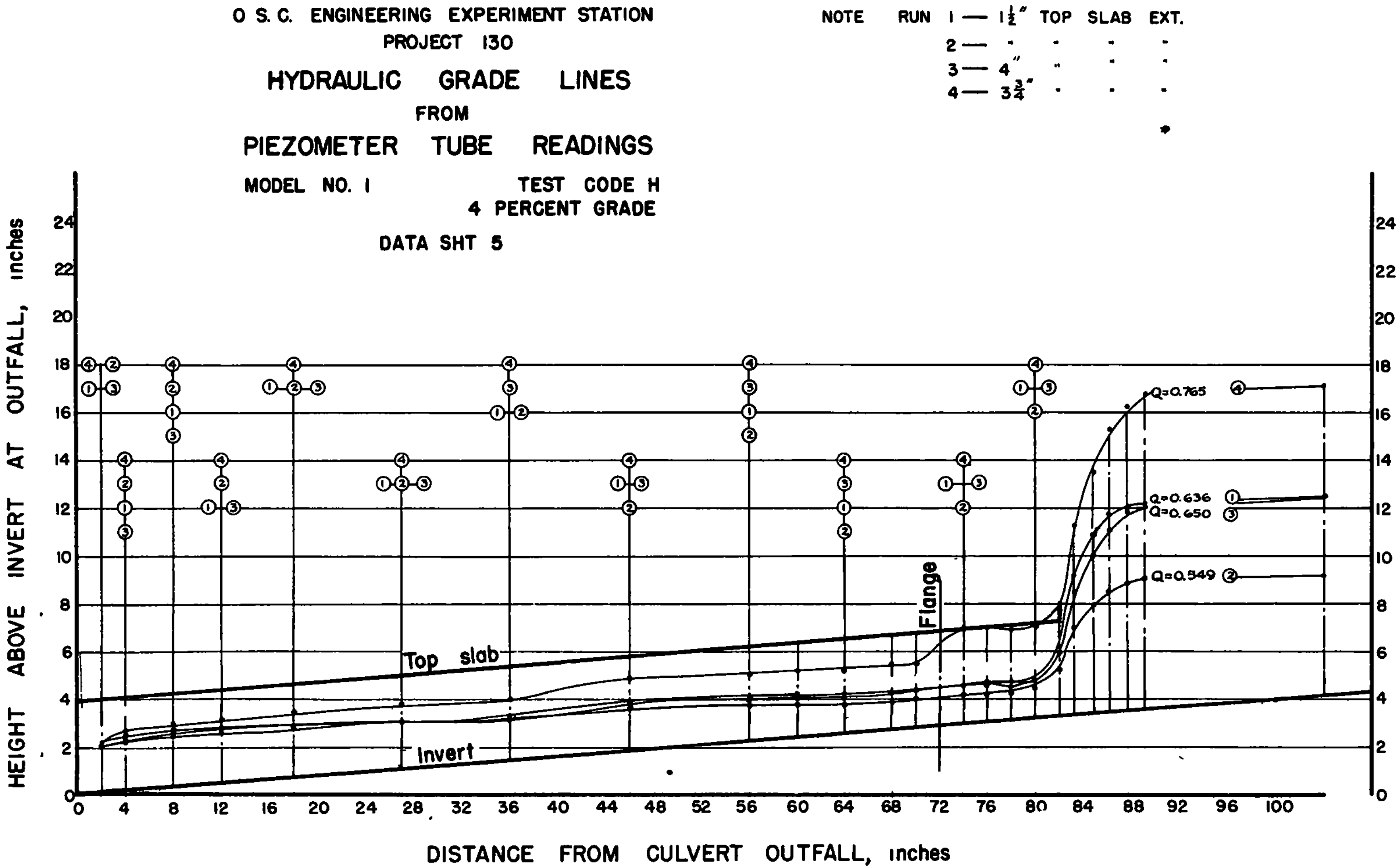


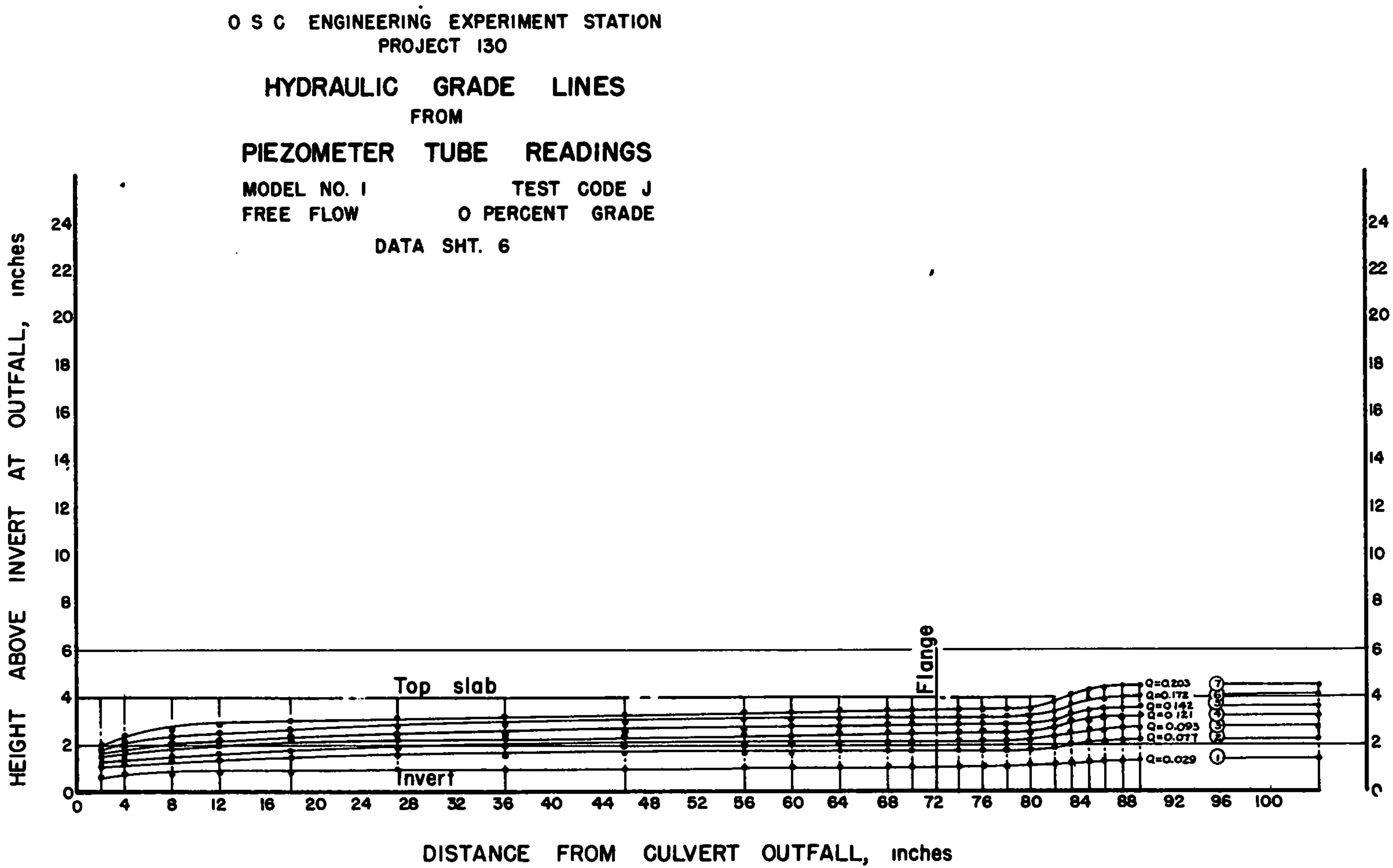

O S C ENGINEERING EXPERIMENT STATION
PROJECT 130
HYDRAULIC GRADE LINES
FROM
PIEZOMETER TUBE READINGS
MODEL NO 1
TEST CODE F
FREE FLOW
4 PERCENT GRADE
DATA SHT. 3
HEIGHT ABOVE INVERT AT OUTFALL, inches
DISTANCE FROM CULVERT OUTFALL, inches
0 4 8 12 16 20 24 28 32 36 40 44 48 52 56 60 64 68 72 76 80 84 88 92 96 100
0 2 4 6 8 10 12 14 16 18 20 22 24
Top slab
Invert
Flange
Q=0.586 (10)
Q=0.487 (9)
Q=0.400 (8)
Q=0.317 (7)
Q=0.275 (6)
Q=0.237 (5)
Q=0.188 (4)
Q=0.148 (3)
Q=0.119 (2)
Q=0.061 (1)

O S. C ENGINEERING EXPERIMENT STATION
PROJECT 130

HYDRAULIC GRADE LINES
FROM
PIEZOMETER TUBE READINGS

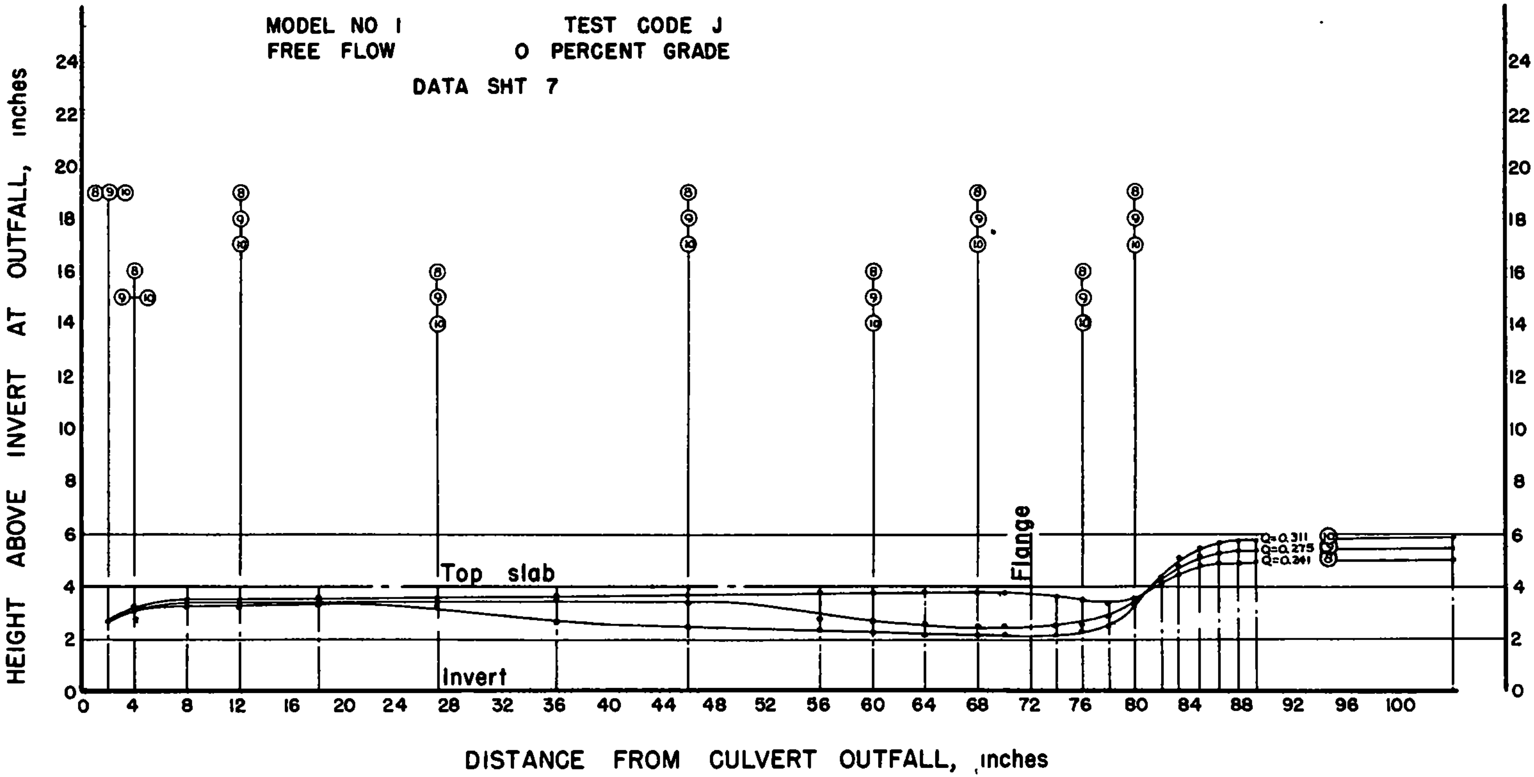



O S. C. ENGINEERING EXPERIMENT STATION
PROJECT 130
HYDRAULIC GRADE LINES
FROM
PIEZOMETER TUBE READINGS
MODEL NO. 1
TEST CODE H
4 PERCENT GRADE
DATA SHT 5
NOTE RUN 1 — 1½" TOP SLAB EXT.
2 — " " " "
3 — 4" " " "
4 — 3¾" " " "
Q=0.765
Q=0.636
Q=0.650
Q=0.549
Top slab
Invert
Flange
HEIGHT ABOVE INVERT AT OUTFALL, inches
DISTANCE FROM CULVERT OUTFALL, inches


O S C ENGINEERING EXPERIMENT STATION
PROJECT 130
HYDRAULIC GRADE LINES
FROM
PIEZOMETER TUBE READINGS
MODEL NO. 1
TEST CODE J
FREE FLOW
0 PERCENT GRADE
DATA SHT. 6
HEIGHT ABOVE INVERT AT OUTFALL, inches
DISTANCE FROM CULVERT OUTFALL, inches
Top slab
Invert
Flange
Q=0.203 (7)
Q=0.172 (6)
Q=0.142 (5)
Q=0.121 (4)
Q=0.093 (3)
Q=0.077 (2)
Q=0.029 (1)

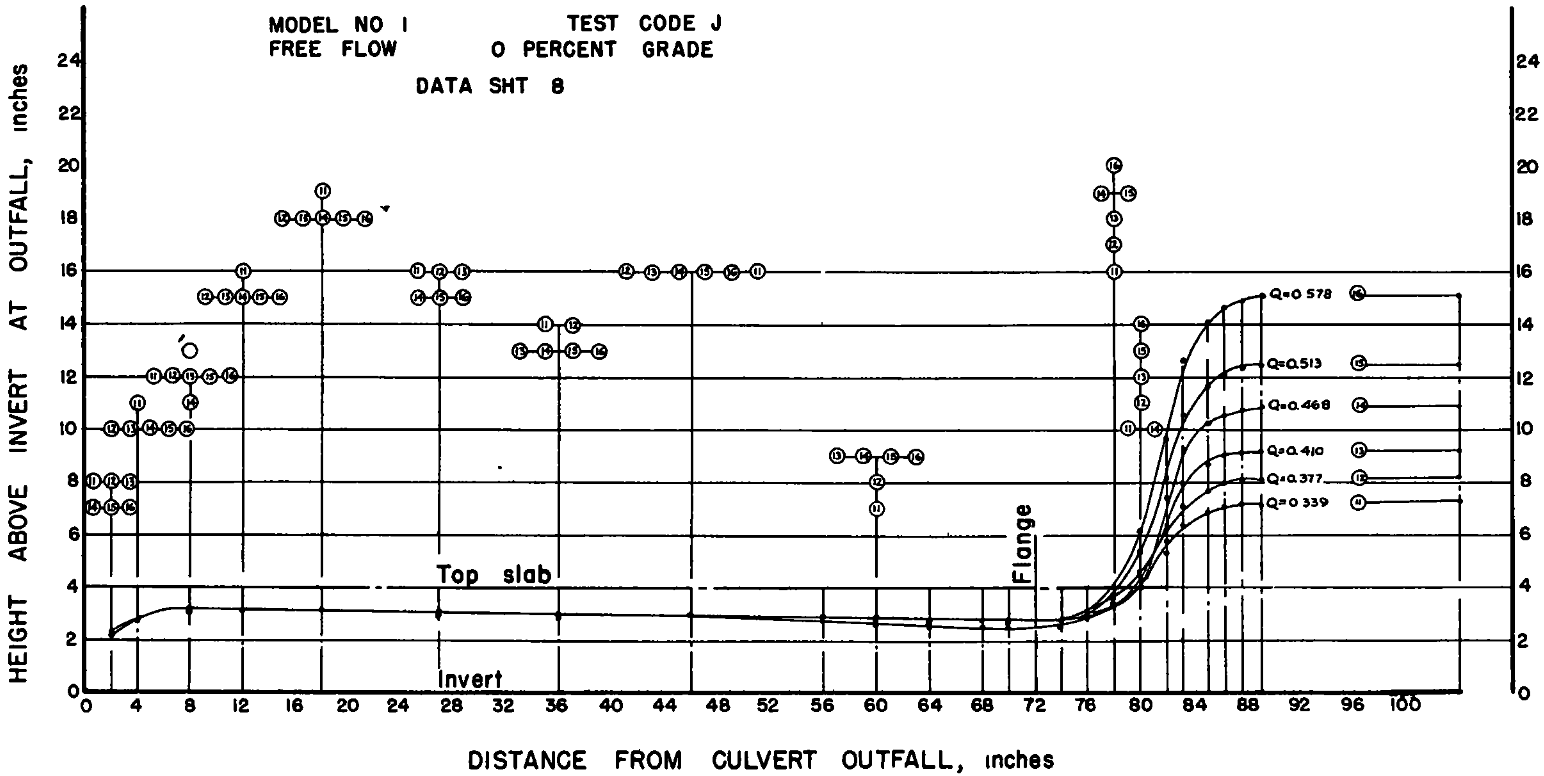
O S C. ENGINEERING EXPERIMENT STATION
PROJECT 130

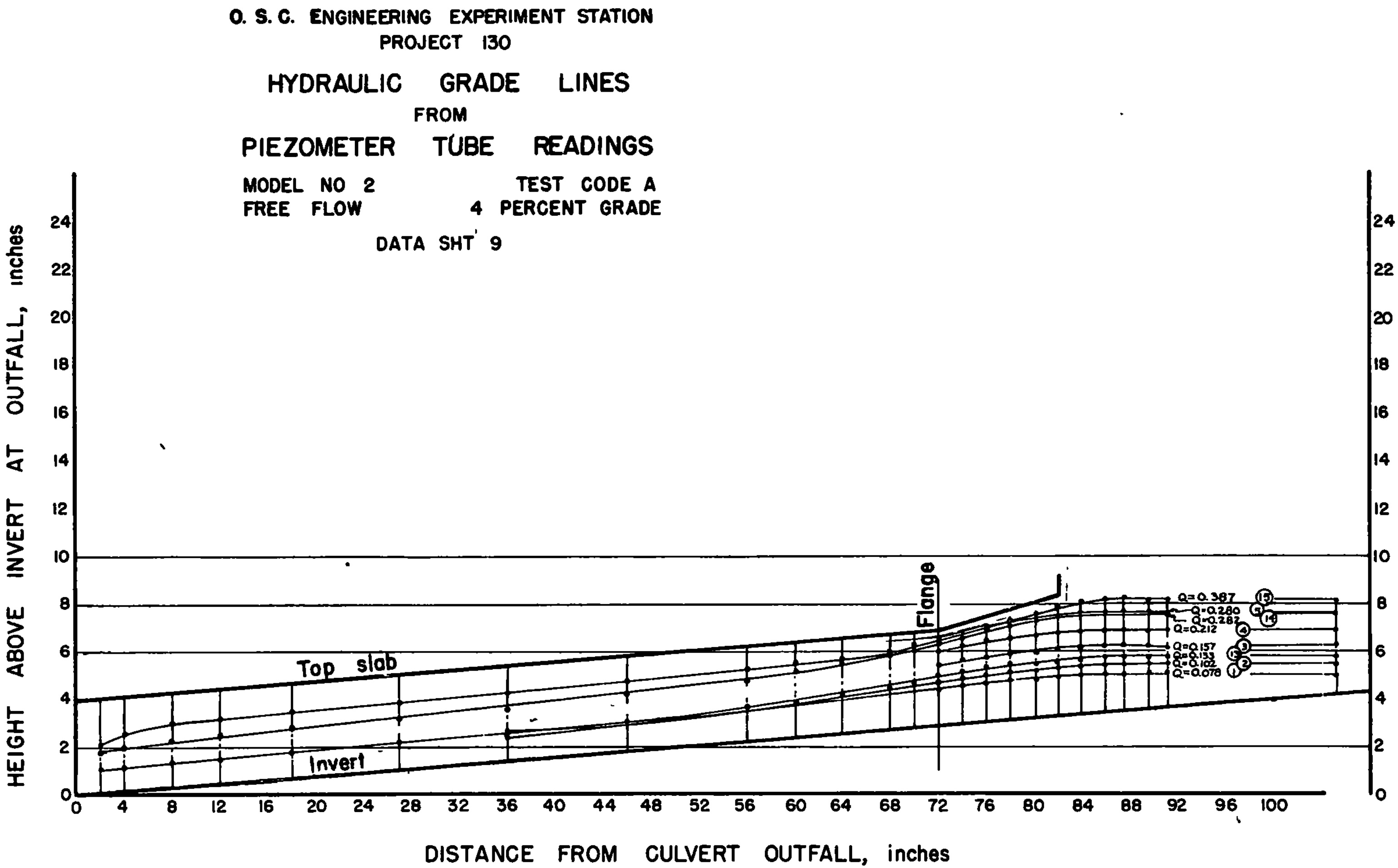
HYDRAULIC GRADE LINES FROM PIEZOMETER TUBE READINGS

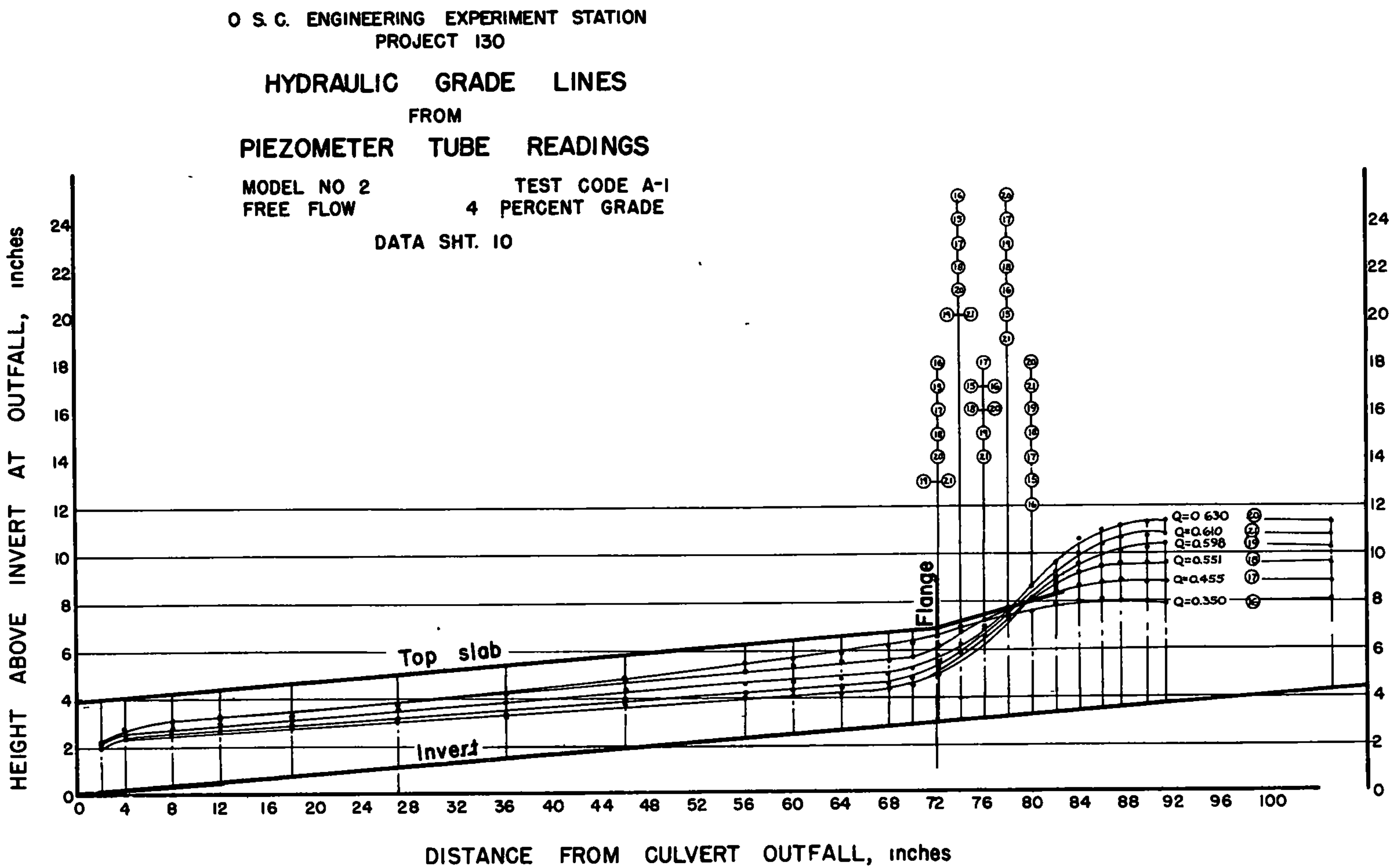


O S C ENGINEERING EXPERIMENT STATION
PROJECT 130

HYDRAULIC GRADE LINES FROM PIEZOMETER TUBE READINGS

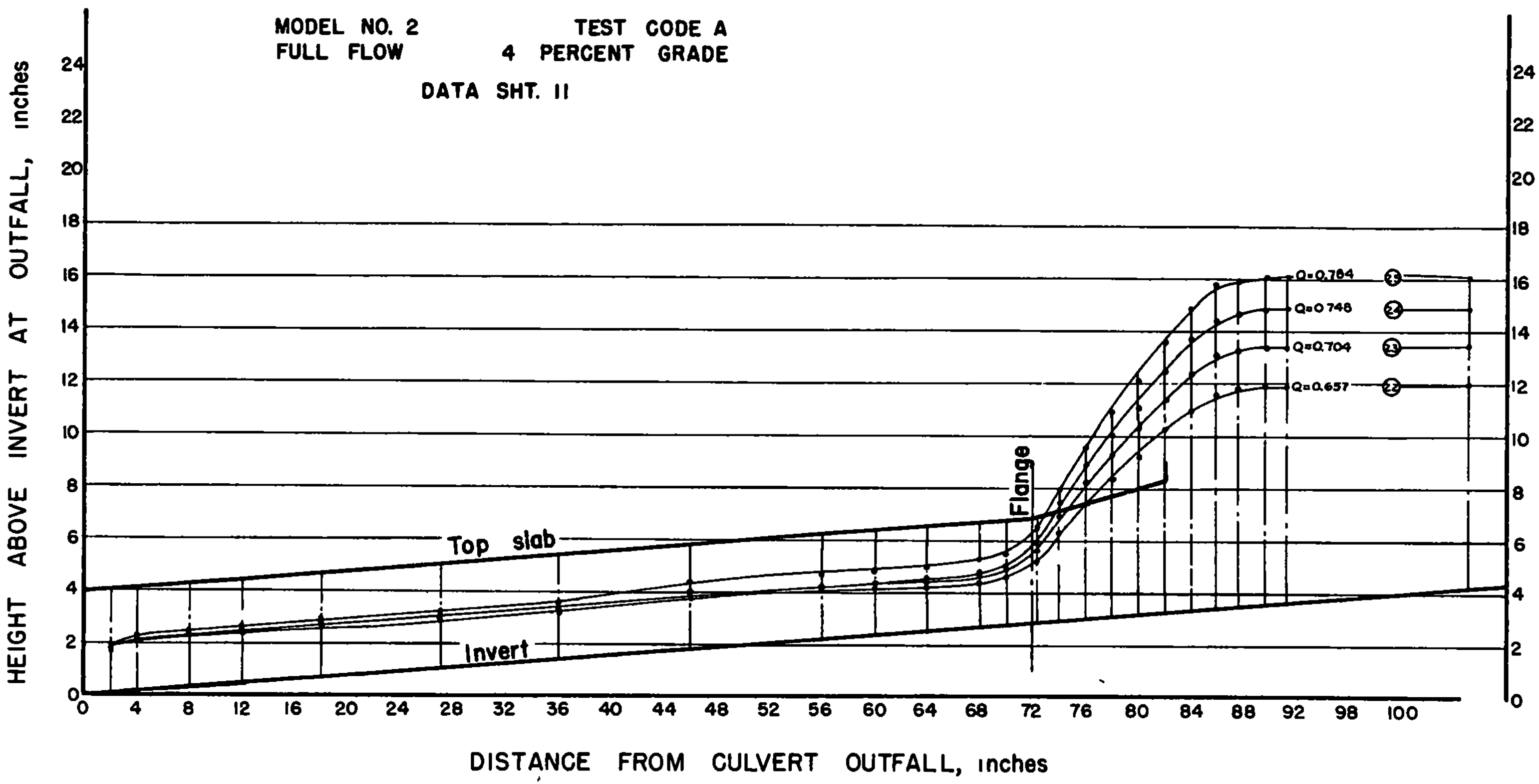


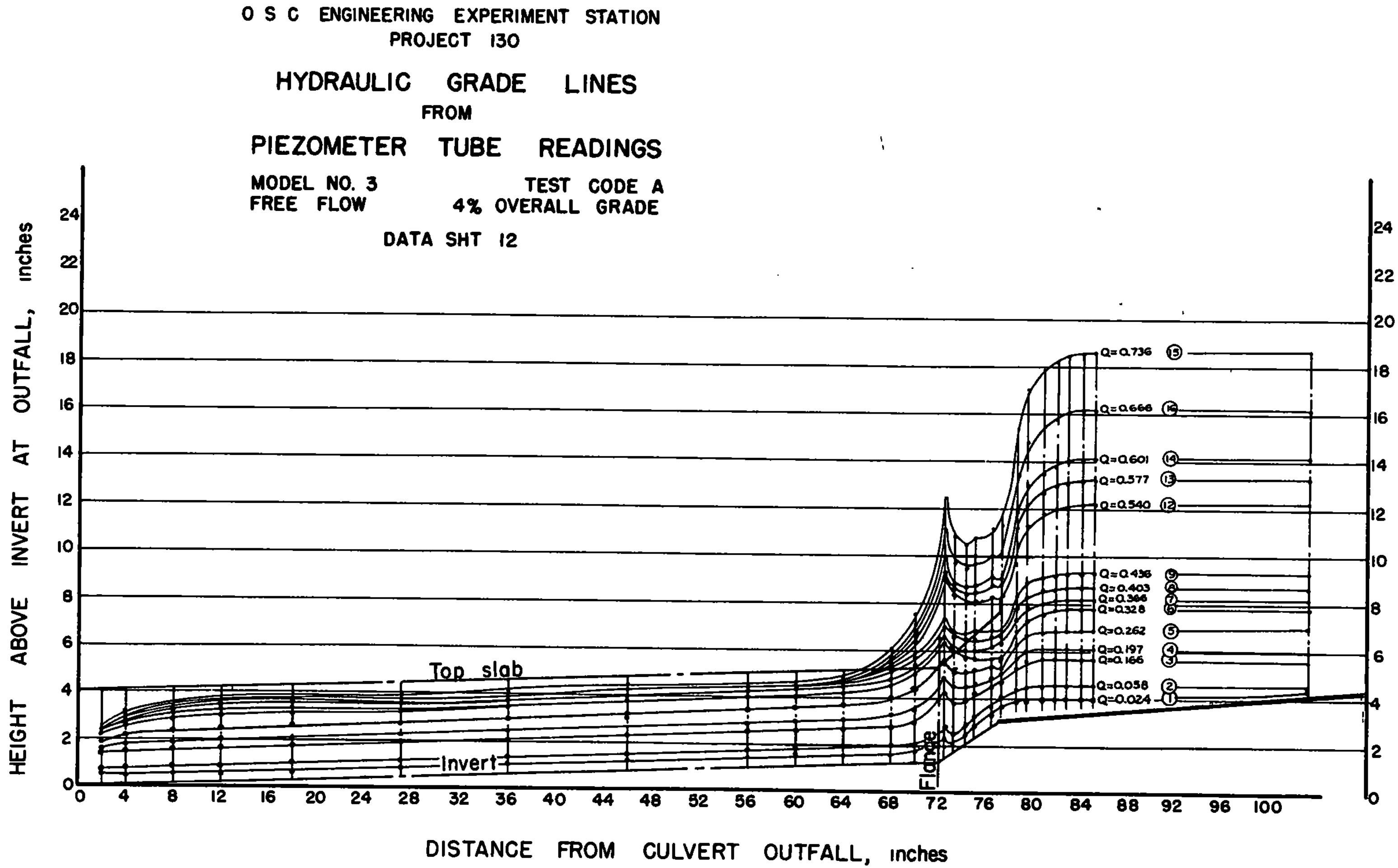

O. S. C. ENGINEERING EXPERIMENT STATION
PROJECT 130
HYDRAULIC GRADE LINES
FROM
PIEZOMETER TUBE READINGS
MODEL NO 2
TEST CODE A
FREE FLOW
4 PERCENT GRADE
DATA SHT 9
HEIGHT ABOVE INVERT AT OUTFALL, inches
DISTANCE FROM CULVERT OUTFALL, inches
Top slab
Invert
Flange
Q=0.387
Q=0.280
Q=0.282
Q=0.212
Q=0.157
Q=0.133
Q=0.102
Q=0.078


O S. C. ENGINEERING EXPERIMENT STATION
PROJECT 130
HYDRAULIC GRADE LINES
FROM
PIEZOMETER TUBE READINGS
MODEL NO 2
TEST CODE A-1
FREE FLOW
4 PERCENT GRADE
DATA SHT. 10
HEIGHT ABOVE INVERT AT OUTFALL, inches
DISTANCE FROM CULVERT OUTFALL, inches
Top slab
Invert
Flange
Q=0.630
Q=0.610
Q=0.598
Q=0.551
Q=0.455
Q=0.350

O S C ENGINEERING EXPERIMENT STATION
PROJECT 130

HYDRAULIC GRADE LINES FROM PIEZOMETER TUBE READINGS




O S C ENGINEERING EXPERIMENT STATION
PROJECT 130
HYDRAULIC GRADE LINES
FROM
PIEZOMETER TUBE READINGS
MODEL NO. 3
TEST CODE A
FREE FLOW
4% OVERALL GRADE
DATA SHT 12
HEIGHT ABOVE INVERT AT OUTFALL, inches
DISTANCE FROM CULVERT OUTFALL, inches
Top slab
Invert
Flange
Q=0.736 (15)
Q=0.666 (16)
Q=0.601 (14)
Q=0.577 (13)
Q=0.540 (12)
Q=0.436 (9)
Q=0.403 (8)
Q=0.366 (7)
Q=0.328 (6)
Q=0.262 (5)
Q=0.197 (4)
Q=0.166 (3)
Q=0.058 (2)
Q=0.024 (1)

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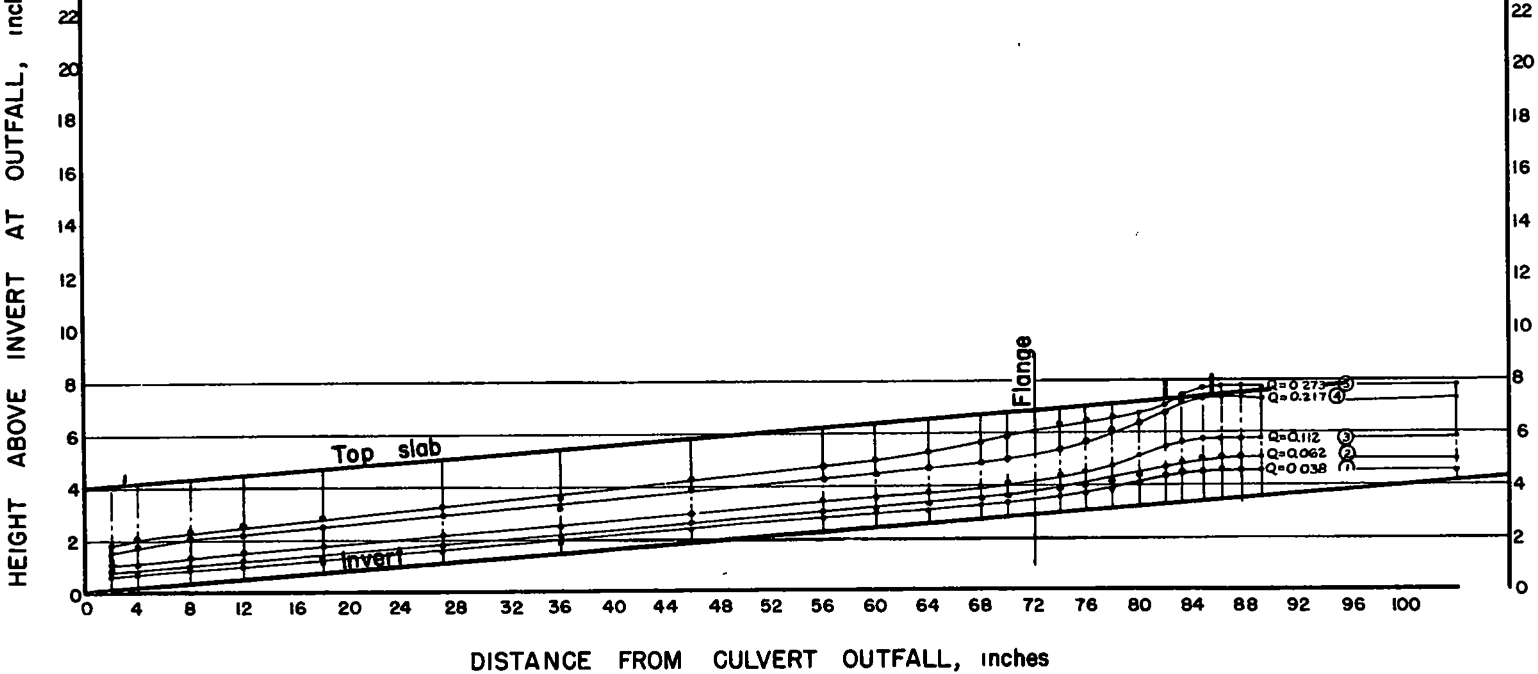
PROJECT 130

HYDRAULIC GRADE LINES FROM PIEZOMETER TUBE READINGS

MODEL NO 4 TEST CODE 1
4 PERCENT GRADE

DATA SHT 13

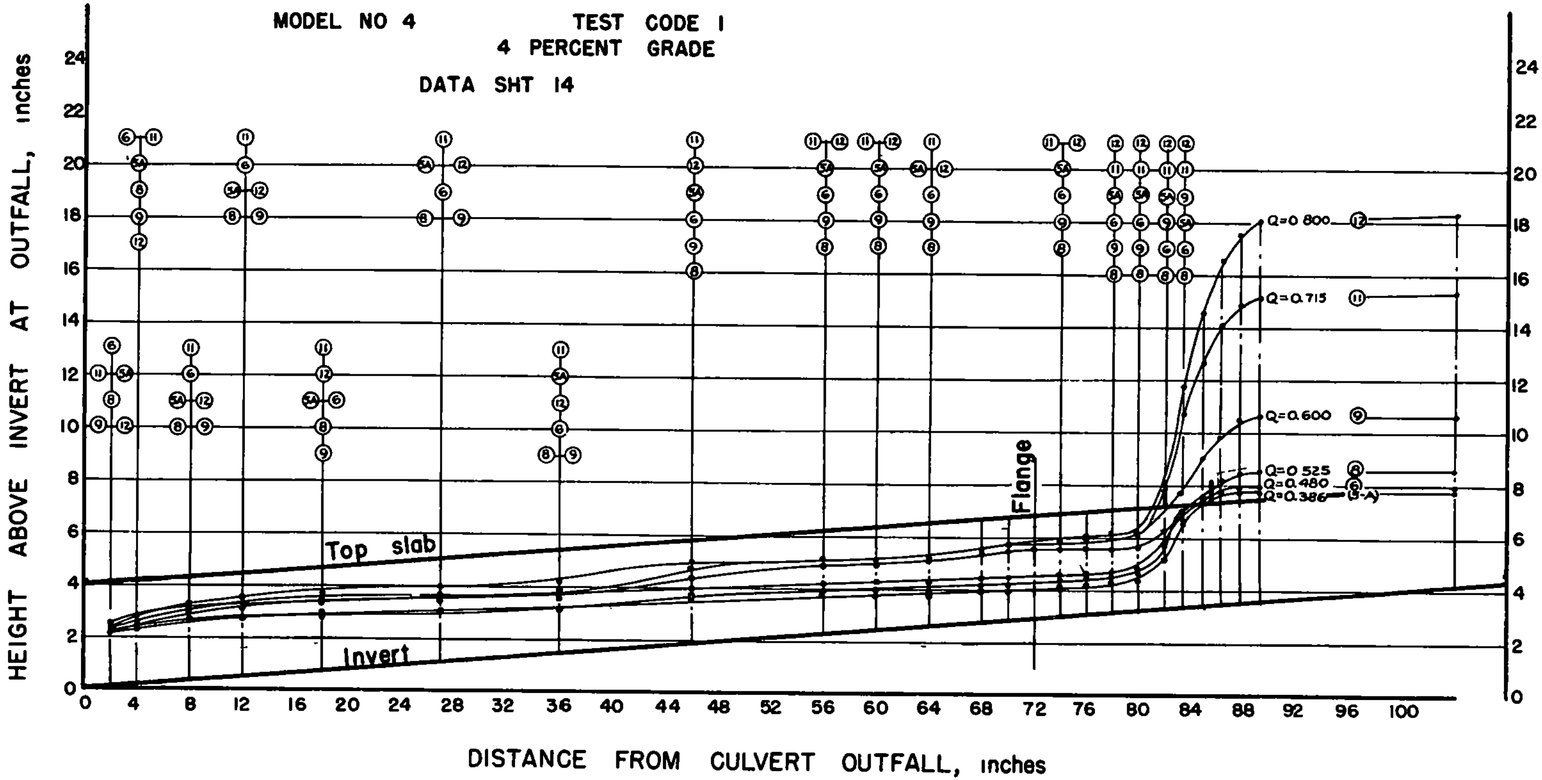


O S C ENGINEERING EXPERIMENT STATION
PROJECT 130

HYDRAULIC GRADE LINES FROM PIEZOMETER TUBE READINGS

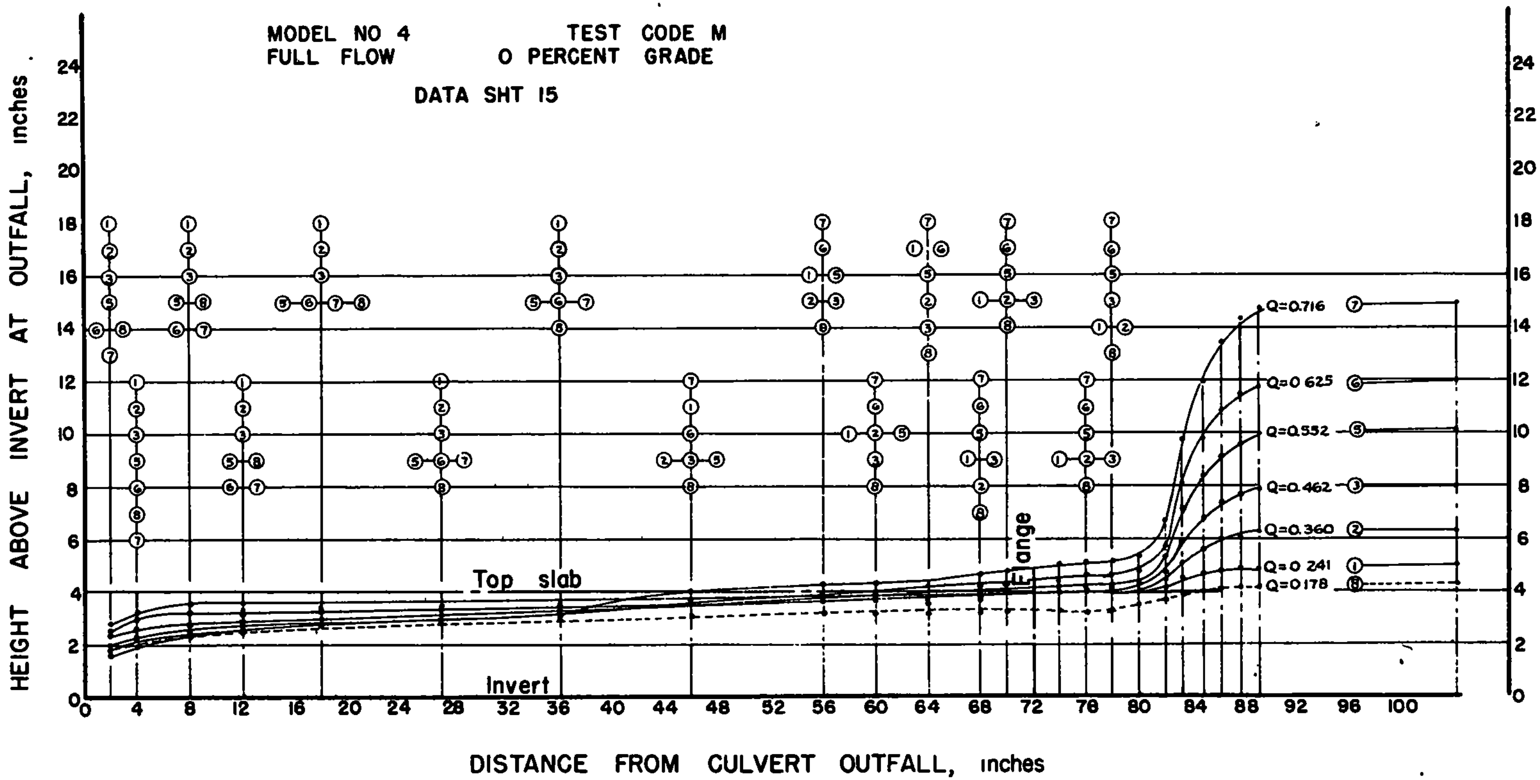
MODEL NO 4 TEST CODE I
4 PERCENT GRADE

DATA SHT 14



O. S. C ENGINEERING EXPERIMENT STATION
PROJECT 130

HYDRAULIC GRADE LINES FROM PIEZOMETER TUBE READINGS

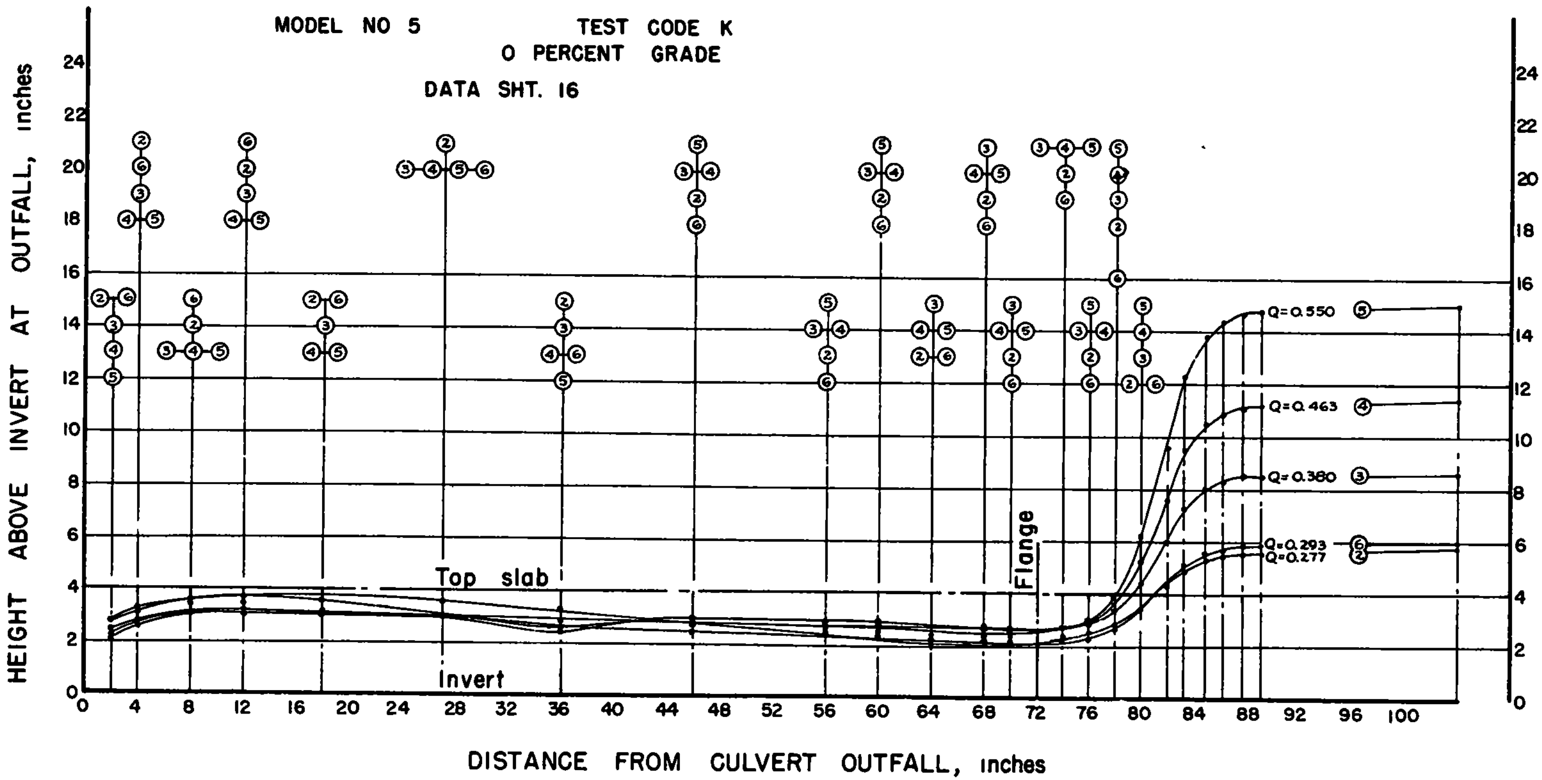


O S C ENGINEERING EXPERIMENT STATION
PROJECT 130

HYDRAULIC GRADE LINES FROM PIEZOMETER TUBE READINGS

MODEL NO 5 TEST CODE K
0 PERCENT GRADE

DATA SHT. 16

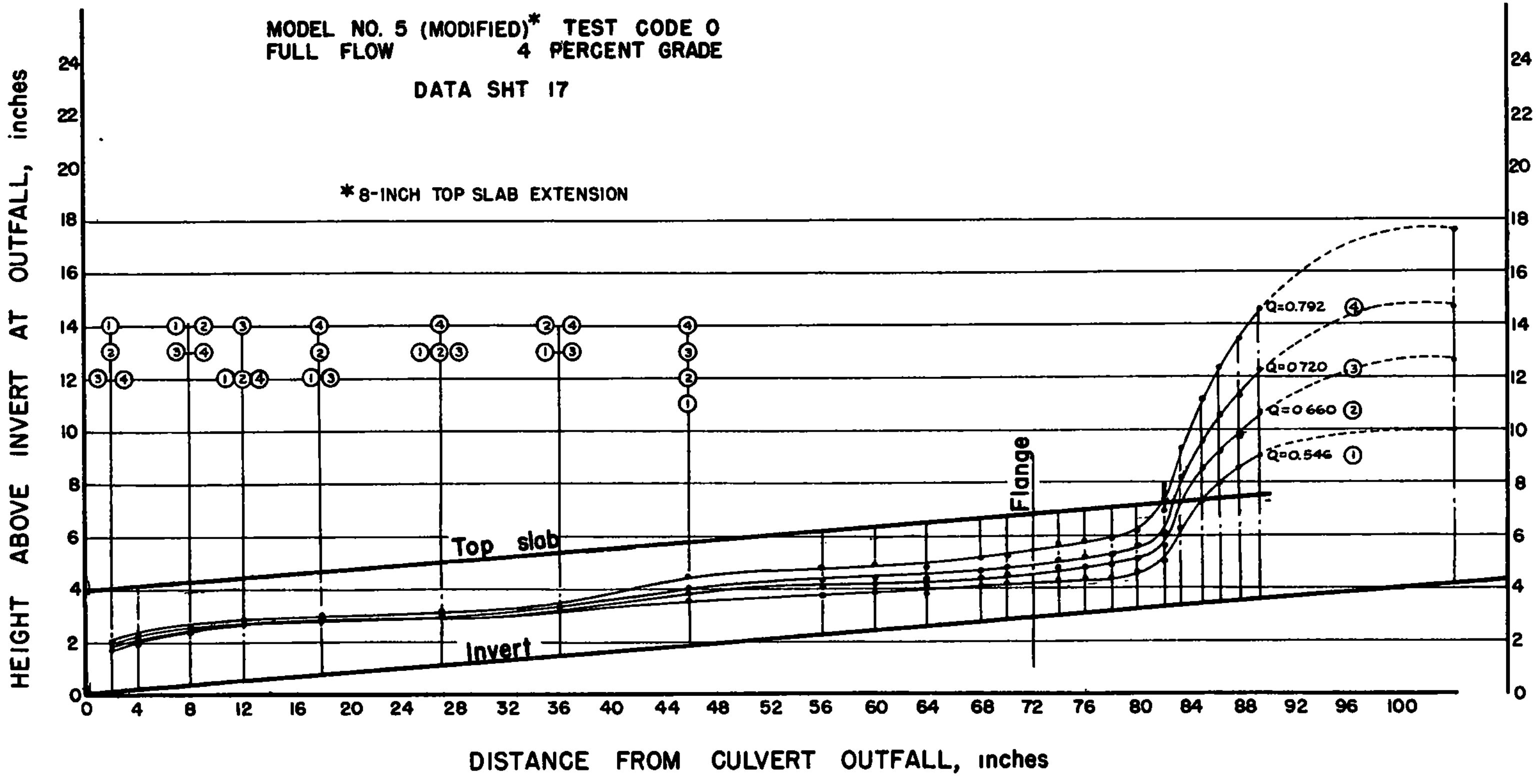


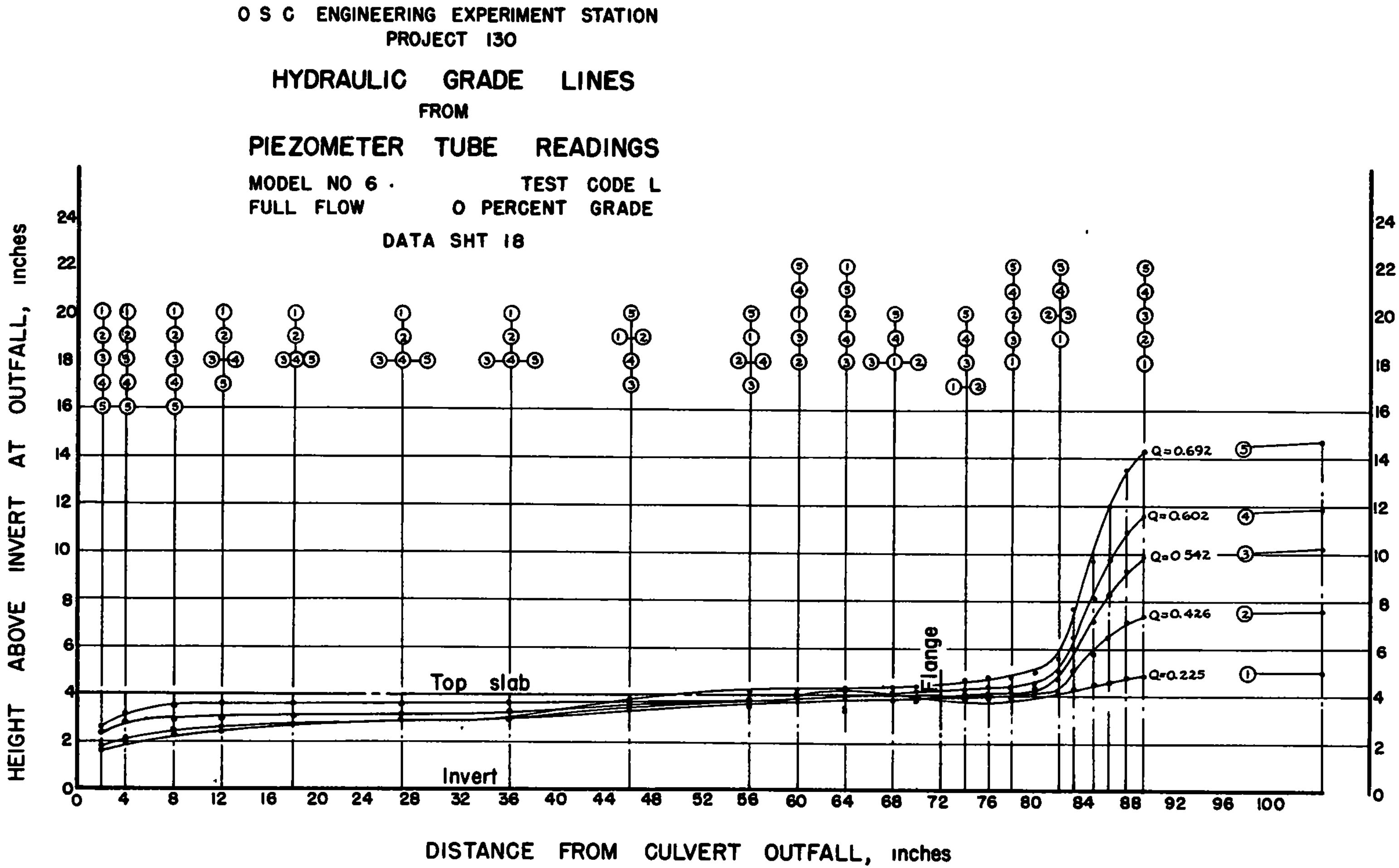
O S C ENGINEERING EXPERIMENT STATION
PROJECT 130

HYDRAULIC GRADE LINES FROM PIEZOMETER TUBE READINGS

MODEL NO. 5 (MODIFIED)* TEST CODE O
FULL FLOW 4 PERCENT GRADE

DATA SHT 17

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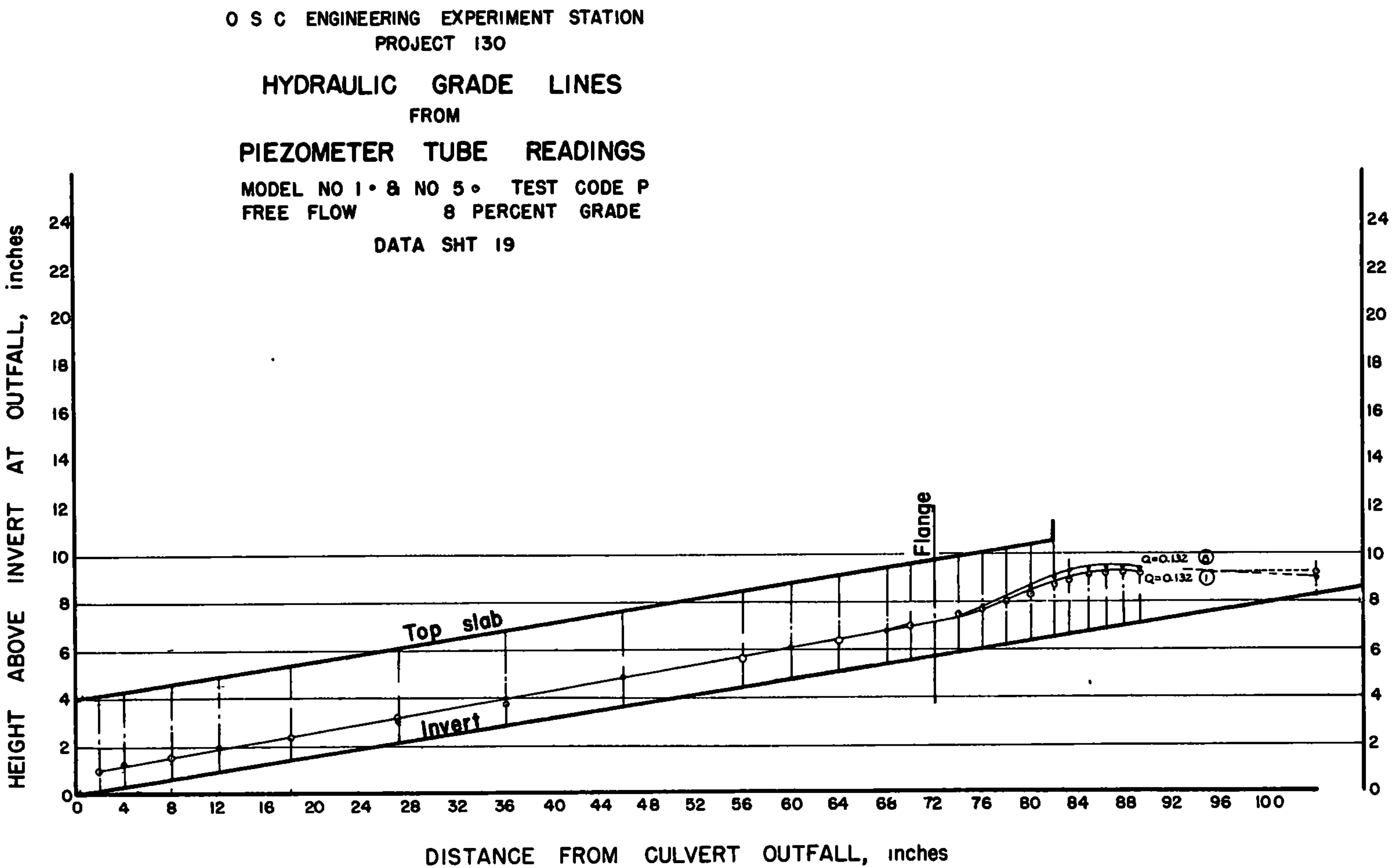
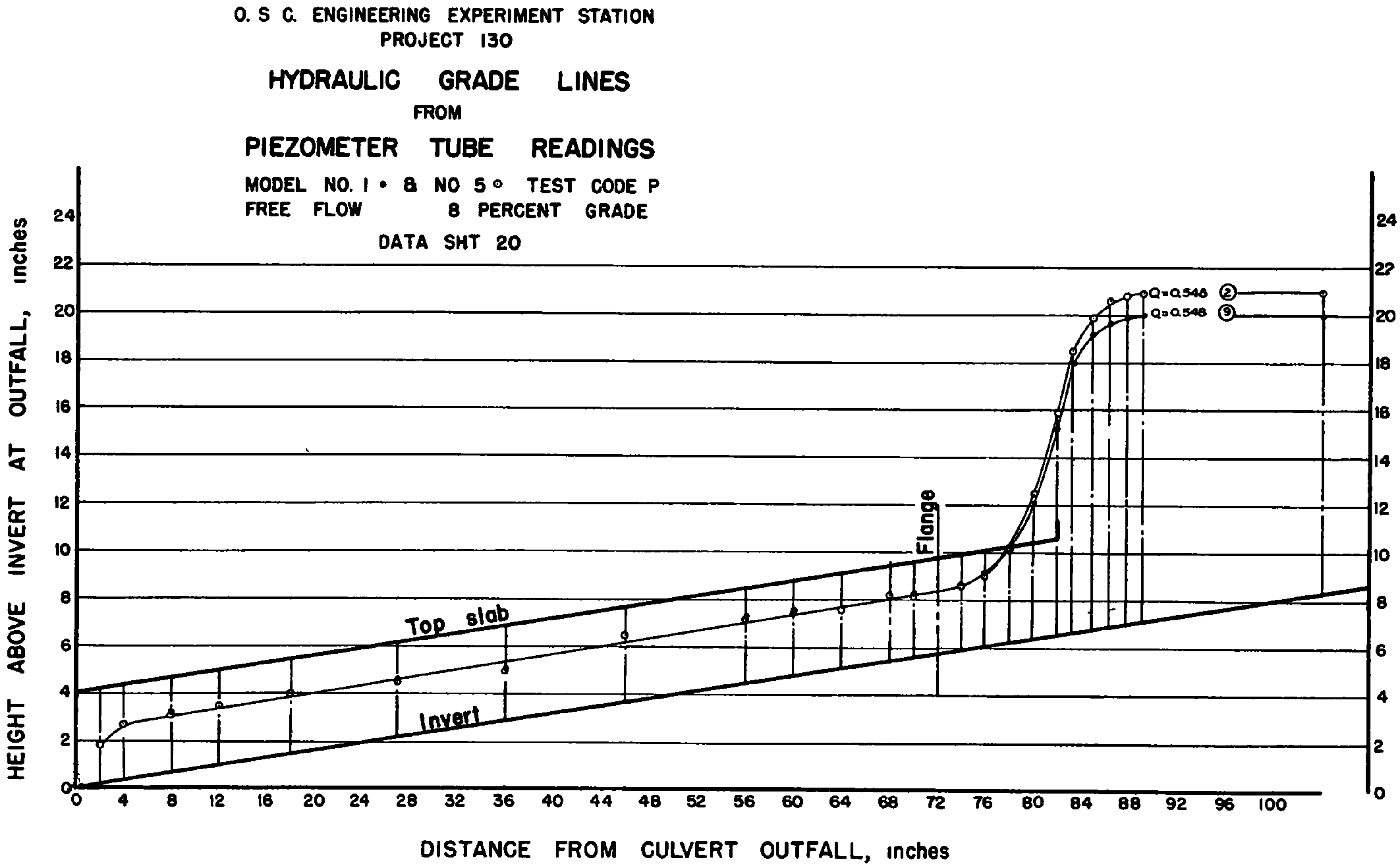
PROJECT 130
HYDRAULIC GRADE LINES
FROM
PIEZOMETER TUBE READINGS
MODEL NO 6
TEST CODE L
FULL FLOW
0 PERCENT GRADE
DATA SHT 18
HEIGHT ABOVE INVERT AT OUTFALL, inches
DISTANCE FROM CULVERT OUTFALL, inches
Top slab
Invert
Flange
Q=0.692
Q=0.602
Q=0.542
Q=0.426
Q=0.225

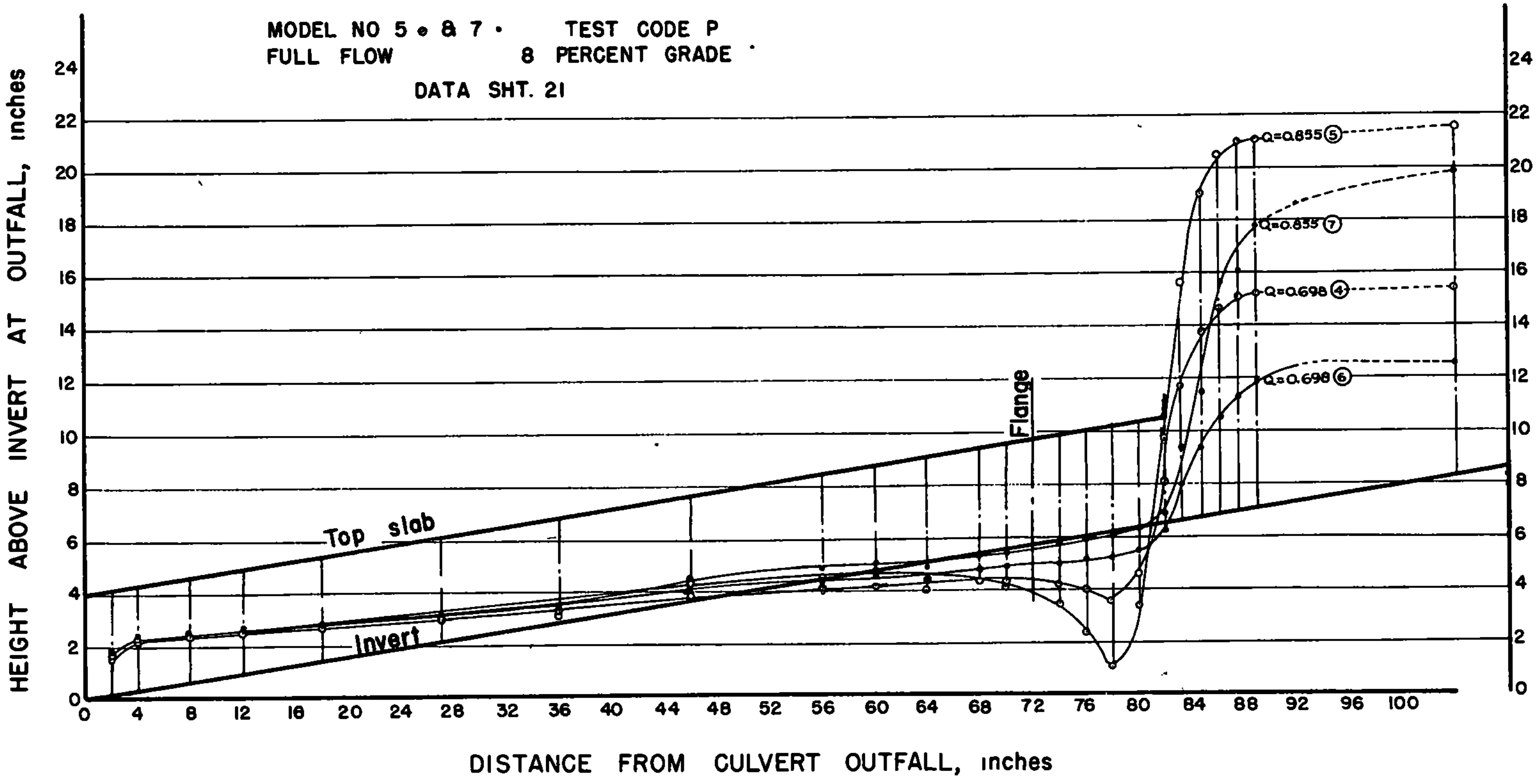

O S C ENGINEERING EXPERIMENT STATION
PROJECT 130
HYDRAULIC GRADE LINES
FROM
PIEZOMETER TUBE READINGS
MODEL NO 1 • & NO 5 ∘ TEST CODE P
FREE FLOW 8 PERCENT GRADE
DATA SHT 19
HEIGHT ABOVE INVERT AT OUTFALL, inches
0 2 4 6 8 10 12 14 16 18 20 22 24
DISTANCE FROM CULVERT OUTFALL, inches
0 4 8 12 16 20 24 28 32 36 40 44 48 52 56 60 64 68 72 76 80 84 88 92 96 100
Top slab
Invert
Flange
Q=0.132 5
Q=0.132 1


O. S. C. ENGINEERING EXPERIMENT STATION
PROJECT 130
HYDRAULIC GRADE LINES
FROM
PIEZOMETER TUBE READINGS
MODEL NO. 1 • & NO 5 ○ TEST CODE P
FREE FLOW 8 PERCENT GRADE
DATA SHT 20
Q=0.548 ②
Q=0.548 ⑨
Flange
Top slab
Invert
HEIGHT ABOVE INVERT AT OUTFALL, inches
DISTANCE FROM CULVERT OUTFALL, inches
0
2
4
6
8
10
12
14
16
18
20
22
24
0
4
8
12
16
20
24
28
32
36
40
44
48
52
56
60
64
68
72
76
80
84
88
92
96
100

O S C ENGINEERING EXPERIMENT STATION
PROJECT 130

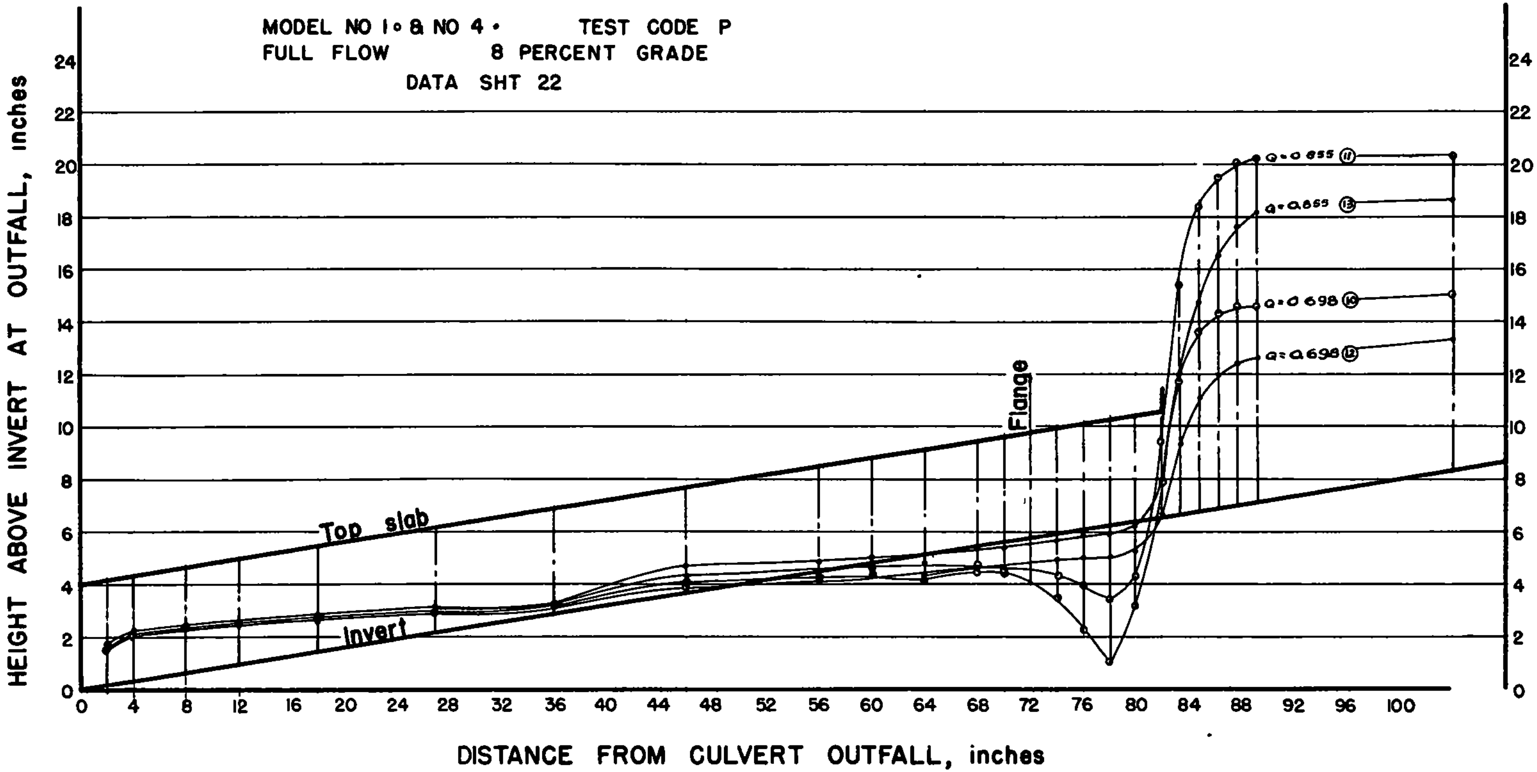
HYDRAULIC GRADE LINES
FROM
PIEZOMETER TUBE READINGS

MODEL NO 5 ∘ & 7 • TEST CODE P
FULL FLOW 8 PERCENT GRADE
DATA SHT. 21



O S C ENGINEERING EXPERIMENT STATION
PROJECT 130

HYDRAULIC GRADE LINES
FROM
PIEZOMETER TUBE READINGS



Discussion

W. O. REE, Project Supervisor, Soil Conservation Service Research, Stillwater Outdoor Hydraulic Laboratory, Stillwater, Oklahoma — The writer has recently completed model studies of some box culverts in order to determine their suitability as runoff measuring devices. In the course of these experiments the sluice-gate-type flow was observed. It was noted that nearly 50 percent of the culvert cross-section was not occupied by water at the time of peak flow. This seemed to be a waste of space and that something could be done. Therefore it was not exactly surprising to learn that at the very same time Shoemaker and Clayton were performing experiments with this in mind.

In the short time available it is impossible to give this fine paper the detailed study it deserves. However, a few comments which occur to the writer will be made. The writer agrees with the authors in their choice of formula to describe sluice gate flow. It is the most practical and further it agrees with the analytically derived expression:

$$Q = \frac{C_c}{\sqrt{1 + \frac{C_c a}{H_1}}} a\sqrt{2gH_1}$$

where

$$C_d = \frac{C_c}{\sqrt{1 + \frac{C_c a}{H_1}}}$$

These formulas are taken from "Elementary Mechanics of Fluids" by Hunter Rouse. The notation is the same as the authors. The additional term C_c, is the contraction coefficient and is the ratio of the depth of the jet to the height of the opening.

The writer found in his experiments that critical depth theory gave a satisfactory estimate of the head-discharge relationship for nonsubmerged flows through steep culverts. The expression derived from the experiments was:

$$\frac{Q}{W^{5/2}} = 3.06\left(\frac{h}{W}\right)^{3/2}$$

This compares well with the theoretical relationship for flow at entrance at critical depth, since the theoretical coefficient is 3.09 instead of 3.06. In the foregoing expression W is the culvert width, and h is head referred to culvert floor at entrance.

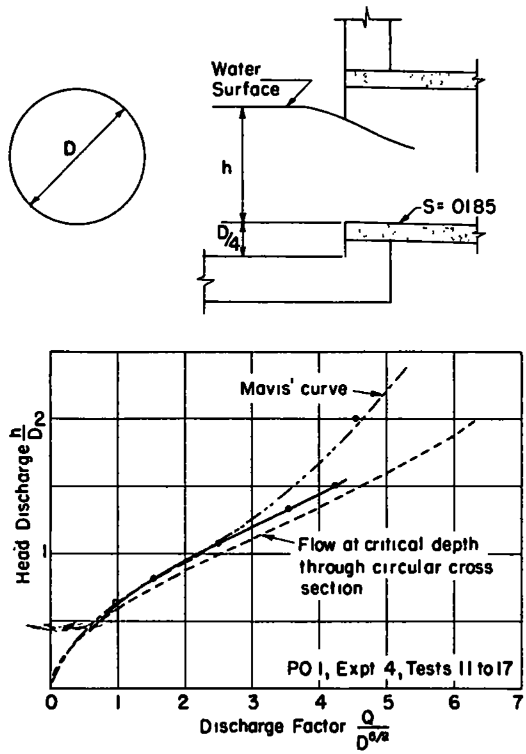


Figure A. Head-discharge relationship square-edged circular entrance for control at entrance, pipe outlet spillway.

Other recent experiments at the Stillwater Outdoor Hydraulic laboratory may be of interest here. Tests were made to determine the hydraulic characteristics of a pipe outlet spillway such as used on detention reservoirs in agricultural flood control works. In these tests the loss coefficients of the component parts of the spillway were evaluated. The first section of the spillway is a 24-in. reinforced-concrete culvert pipe 108 ft. long and laid on a 0.0185 slope. At the entrance to this pipe is a straight 4-ft. -wide wall with the wing walls perpendicular to it. The invert of the pipe is 6 in. above the apron of the inlet structure. Three entrance forms for

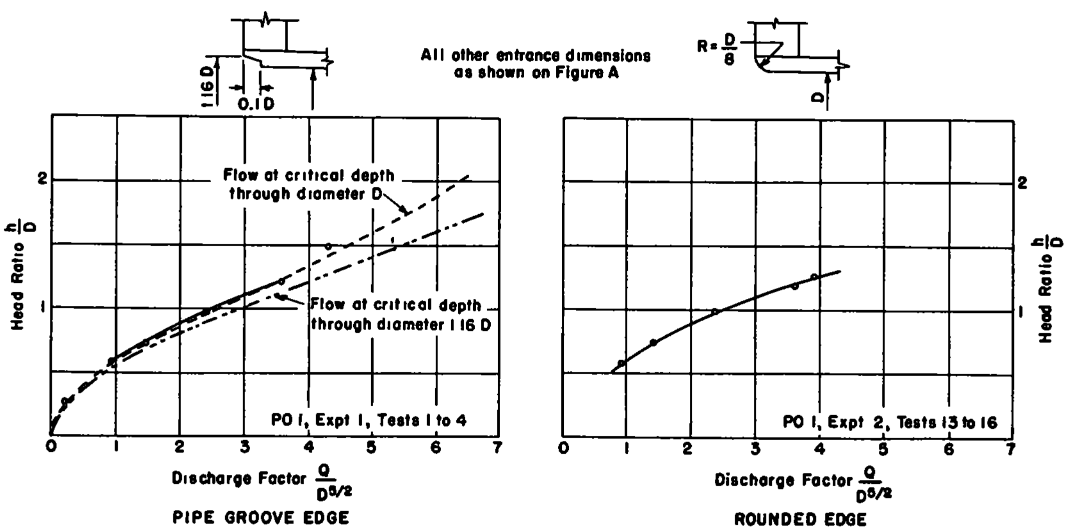


Figure B. Head-discharge relationships for circular entrances for control at entrance, pipe-outlet spillway.

the pipe were tested. For full pipe flow the entrance coefficients obtained were:

Standard pipe groove entrance $K_e = 0.33$

Rounded entrance, 3-in. radius $K_e = .27$

Square-edged entrance $K_e = .70$

The values of K_e are to be applied to the velocity head to determine the loss in feet.

Tests also were made on these same entrances for flows less than full. Since the pipe below was on a steep grade the entrance controlled the discharge. The results of these tests are shown Figures A and B. There are dimensionless rating curves with a discharge factor, $\frac{Q}{D^{5/2}}$, plotted against the corresponding head ratio, h/D. In these ratios h is the head above the invert at pipe entrance, D is the pipe diameter, and Q the volumetric discharge rate. Some additional curves are shown for comparison purposes. On the square-edged diagram are the curve from Mavis' experiments and a curve showing the head-discharge relationship for flow at critical depth through a circular cross-section. Critical depth curves for both the pipe diameter are shown on the pipe groove data plot to determine which diameter controlled the flow. Evidently the smaller diameter controls. Since these data are limited no further explanatory remarks will be made. However, the data will be of some value in the form presented.

CARL F. IZZARD, Chief, Hydraulic Research Branch, Bureau of Public Roads—The investigation described in this paper had its beginning in the conviction of engineers in the Portland Division Office of the Bureau of Public Roads that culvert barrels on steep grades were larger than necessary and that the main problem was to get the water through the entrance. They developed the design designated as Inlet 2 in this report. The tests by Shoemaker and Clayton amply confirm the value of this type of inlet and show why it works. By careful observation of the model in operation they developed even simpler modifications of a standard culvert, as in Inlet 4, which would accomplish the same purpose. The latter type has the advantage that it can easily be built onto an existing culvert with flared wingwalls by simply extending the top slab out from the headwall and building up the wingwalls to meet this extension.

The highway engineer should take note, however, that the degree of reduction in headwater depth obtainable by tapering the entrance depends on the difference between the headwater resulting from what the authors call "sluice" action and that for the same barrel flowing full. This difference can be readily determined by first computing the headwater depth for entrance control and comparing it with the headwater depth which would occur if the barrel flowed full. The latter depends on the length, roughness, and slope of the barrel, as well as the entrance loss coefficient.

Fortunately, charts to facilitate these computations are already available to highway engineers. Public Roads Hydraulic Chart No. 1043 gives the headwater depth with entrance control and agrees closely with the curve for Inlet 1 in Figure 6. Chart No. 1041 gives the head loss in a culvert flowing full. The entrance loss coefficient for a tapered entrance can be assumed as 0.1 (based on velocity head in the uniform barrel). The length of the culvert should include the length of the tapered entrance. An example of the use of these charts is given at the conclusion of this discussion.

Operation of Inlets 1, 2, and 3

Figure 8 indicates that nonsubmerged-inlet operation on a steep grade is controlled by critical depth, and Figure 9 shows variations in the location of critical depth. An analysis of the data made by plotting the drop in pressure against the distance from each piezometer to the plane of the entrance, both in terms of critical velocity head, shows that for a standard culvert, Inlet 1, critical depth occurs within the barrel a distance of about 1.4 times critical depth. The water surface, as sketched in Figure 9, will strike the top of the entrance when critical depth becomes about equal to 0.85 times the culvert height.

For Inlet 2, tapered entrance, critical depth occurs about the same (or somewhat shorter) relative distance within the uniform barrel, the position being less well defined than for Inlet 1. For both inlets, the barrel slope was 4 percent which was definitely supercritical. Obviously critical depth control near the entrance requires that the barrel slope be supercritical.

A study of data for Inlet 3 shows that critical depth occurs at or very close to the entrance, i.e., where the barrel steepens abruptly. Critical depth in a channel of uniform width would be expected to occur upstream from the break but in this case the converging walls force the critical section to occur about at the break. The total head is actually slightly less than the minimum total head for critical depth probably because of negative pressure due to lack of aeration of the underside of the nappe.

Inlet 3 is designed in accordance with the criteria set forth in Hydraulic Information Circular No. 2 (Public Roads), page 19. The model tests indicate that the culvert operates as expected provided the inlet is not submerged. Once the barrel flows full there is no advantage to the break in grade; in fact, as shown in Figure 6, the head-discharge curve is higher than for Inlet 2. The primary advantage of Inlet 3 over Inlet 2 would be in discharging water more rapidly on the rising hydrograph, thus leaving more storage area available to knock the peak off the hydrograph. Also, where either Inlet 2 or Inlet 3 is expected always to flow partly full, the headwater with Inlet 3 will always be less than that for Inlet 2, because the control section for the latter is based on the width of the uniform barrel whereas Inlet 3 has the control section at the widened entrance.

The head H_2, as plotted in Figure 11, was determined as the difference between the elevation of the water surface at a point 4 in. from entrance as observed through the side of the flume and the elevation of the pool as determined by piezometric readings. The difference in discharge coefficients shown in Figure 12 for the 0- and 4-percent slopes is not explained, but it is not surprising considering the fact that the actual mean depth at the vena contracta is difficult to measure. It may be noted, however, that a plot of observed head H_1 against $V^2/2g$ for the full barrel, both of which are quite accurately determined, yields an equation

$$H_1 + 3aS_o = 0.267 + 2.33 \frac{V^2}{2g}$$

which fits the data for both slopes. The left side of this equation represents the head H_1 plus the fall to a point 3a or 12 in. from the entrance. A study of the piezometric profiles indicates that the latter point more nearly indicates the position of the vena contracta, which conceivably could be different on the center line from what it appears to be as viewed through the side wall. From the sketch in Figure 11, since H_2 is the drop in water surface from H_1 to D_2 the equation can be written

$$H_1 + 3a\,S_o = D_2 + H_2.$$

By comparison of the two equations above it follows that D_2 = 0.267 ft. and H_2 = 2.33

$\frac{V^2}{2g}$ by this indirect method. From Equation 6, solved for H_2, it follows that $C_d = (1/2.33)^{1/2} = 0.655$ which agrees fairly well with the value of C_d shown in Figure 12 for the 4-percent slope. Substituting $D_2 = 0.267$ and $C_d = 0.655$ in Equation 4 gives

$C_v = \frac{0.655 \times 0.333}{0.267} = 0.82$ indicating a loss of energy between the two sections.

A comparison of curves for Inlet 1 on a zero slope in Figure 16 with curve for same inlet on 4-percent slope in Figure 15 indicates that adding the fall of $(3a\ S_o) = 3 \times 4 \times 0.04 = 0.48$ in. to the latter curve will cause it to coincide very nearly with the curve for Inlet 1 on zero slope. [Note: Figure 15 based on re-run of Inlet 1, as shown in data Sheets 3 and 4, is more accurate than similar curve in Figure 6.]

Modification Experiments

The authors are to be commended for the careful observation which led to the modifications of Inlet 1 and development of Inlet 4 which for practical purposes is the equivalent of Inlet 2. Further tests with wingwalls on a 1-to-4 angle demonstrated that the area of the entrance should be about twice the barrel area in order to eliminate excessive vortex action and to enable the barrel to flow full.

Unfortunately time did not permit testing modifications of Inlet 2 in which the rate of taper of the barrel (1 to 10 for Inlet 2) was varied. However, it is reasonable to assume that rates of 1 to 1.5 and 1 to 4 would be satisfactory provided the area of entrance was twice the barrel area, since these angles were satisfactory on Inlets 4 and 7, respectively. It also appears that the enlargement may be made on the sides only or on the sides and top. Keeping the wall height constant would have a construction advantage and would also be advantageous hydraulically because it would cause the barrel to begin flowing full at a lower upstream water level. Submergence of the entrance by about 20 percent of the entrance height, as shown in Figure 6 for Inlet 2, and in Figure 15 for Inlet 4, appears to be necessary before the barrel can begin to flow full.

Comparison with Results on Model Pipe Culvert

There is very good agreement in the conclusions which can be drawn from the Oregon tests on square box culverts and the Minnesota tests on round pipe culverts, both with free outlets:

1. Both operate with critical depth control on supercritical slopes, this relation being affected in only a minor degree by the rounding on the pipe or the wingwalls on the box. Critical depth control will cease to exist when discharge in the pipe exceeds $4D^{5/2}$ or discharge in the box exceeds $4BD^{3/2}$.
   (D = diameter, or height;
   B = width of box; all in feet).
2. Both operate with contracted flow when entrance is square-edged, the contraction becoming substantially constant for headwater greater than 1.5 times the entrance height.
3. Elimination of the contraction in either type causes the barrel to flow full for part of the length or for the full length as the discharge rate increases to the point where utilization of the entire fall available in the barrel is required.
4. The headwater depth for both pipe and box culverts can be reduced by improving the inlet and causing the barrel to flow full, provided the head losses in the full barrel when added to the elevation of the pressure line at the outlet give a headwater elevation lower than that for entrance control with contracted flow in the partly-full barrel. Since the losses in the full barrel depend on length, size, and roughness of the barrel as well as entrance loss coefficient, all these variables must be considered but the fall in the barrel generally determines when the improved entrance will be advantageous. A guide covering most, but not all, situations is that the fall in the barrel should be at least 0.4 of the culvert height and that the head loss (H) for the full barrel must be less than 2.5 times the velocity head in the barrel.
5. The minimum head above the crown of a culvert expected to flow full with improved inlet is about 0.2 of the entrance height. If the head loss for the full barrel when added to the elevation of the pressure line at the outlet plots below this minimum elevation, the barrel will flow full for only part of its length and the pool elevation will

not drop below this minimum elevation. (Discharge must, of course, be above the limitation for critical depth control cited in 1. above)

Discussion of Culvert Head-Discharge Relations

The foregoing conclusions indicate the many variables involved in the head-discharge relation of culverts with free outlets. The best way of comparing alternate culverts which might be used at a given site is to make a diagram similar to Figure A which shows the headwater depth as a function of $(Q/100)^2$. The abscissa could be in terms of Q to the first power, but since the head for either entrance control (above 1.5D) or full flow with outlet control varies as Q^2, the curves plot as straight lines in this form. A scale is added for Q for convenience.

Two curves are drawn for each barrel size, one for a square-edged inlet, and one for an improved inlet meeting the criteria set forth in the previous summary discussion. In the case of the boxes the enlargement of the entrance was assumed to be entirely in the width, the height being the same as the barrel height. For the pipe an adequate rounding to eliminate the contraction is assumed. The curves are plotted only for headwater greater than culvert height.

The curves were computed from Public Roads Hydraulic Charts 1041, 1042 and 1043, minor deviations of computed points from a straight line being ignored. The curve for entrance control is dotted above HW = 1.5D, and the curve for outlet control is not drawn below HW = 1.5D, since the culvert cannot be depended on to flow full unless the inlet is submerged by at least 0.4D, a slightly higher limit being set to be conservative. The horizontal line connecting the two curves indicates this minimum headwater for the full barrel.

The curves for entrance control are plotted directly from the values of headwater computed from Chart 1042 for pipe or 1043 for boxes. With full flow, however, the head H read from Chart 1041 (with C_e = 0.1 and n = 0.015) is plotted above the elevation of the outlet crown in relation to the inlet invert, which assumes the pressure line to be at the outlet crown. The culvert

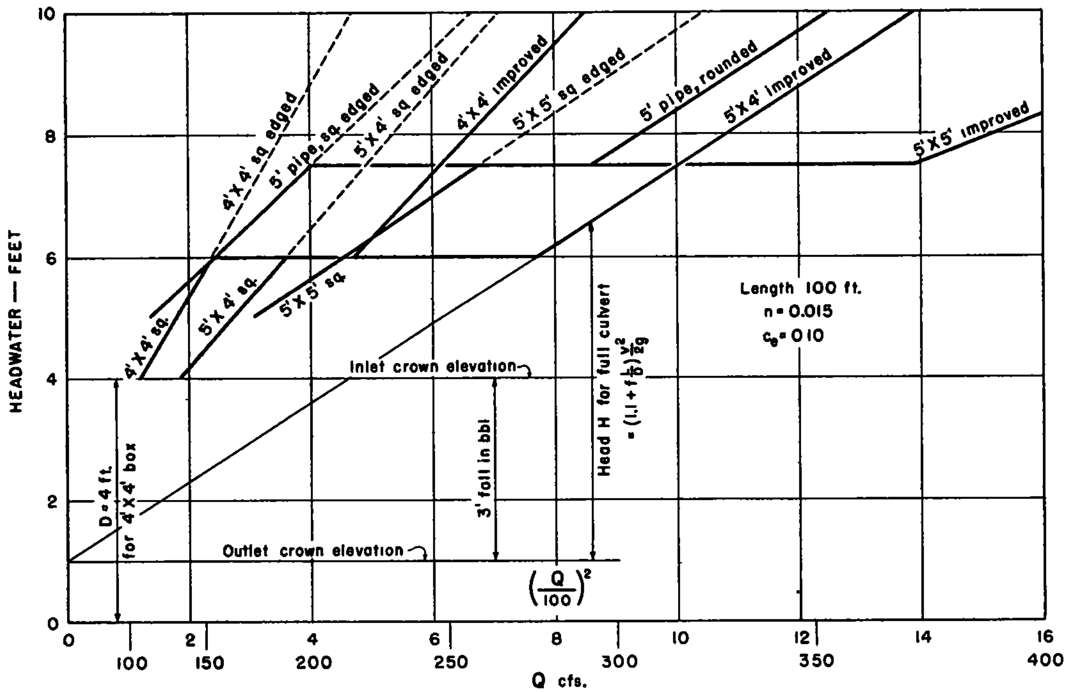


Figure A. Typical head-discharge curves for headwater greater than culvert height for culverts with and without improved entrances.

length is uniformly 100 ft., disregarding slight changes which would normally occur due to difference in culvert height, as these differences would not change the head loss significantly. The fall is 3 ft., making the slope 3 percent.

In plotting curves for entrance control, points should be computed over the entire range above 1.5D. Only two points need be plotted for a full culvert; this line extended must pass through the outlet crown elevation at zero discharge.

The half hour which might be required to plot a diagram such as Figure A should be worthwhile as in similar cases (with appreciable fall in the barrel), it is usually possible with an improved entrance either to use a smaller barrel or to gain reserve discharge capacity with the same barrel size. In other cases it may be shown that the extra cost of an improved entrance, if any, may result in no appreciable saving in headwater beyond the difference in entrance loss represented by 0.4 of the velocity head. The latter does become appreciable for velocities in excess of 10 or 12 ft. per sec.

To illustrate, we may note in Figure A that if the limiting HW = 6 ft. and Q = 210 cfs., a 4 by 4 box with improved inlet is the equivalent of either a 5 by 5 box with square-edged inlet, or a 5 by 4 box with improved inlet. If the head is increased to 8 ft., the latter then has a capacity of about 330 cfs. as compared to 260 cfs. for the 4 by 4 improved inlet. This could mean that if 210 cfs. is the 10-yr. peak runoff the 25-yr. flood, being about 25 percent greater, could be handled by the 4 by 4 improved inlet with 2 ft. increased headwater. On the other hand, the 5 by 4 improved inlet could handle a 50 percent greater flood at the 8-ft. stage, which would be about a 50-yr. flood. The designer must then decide whether or not the increase in cost for the larger structure is justified by the increased protection afforded, taking into consideration the conditions at the site.

Attention is called to the fact that the 5-ft. pipe with improved inlet operates with about 1 ft. more headwater than that for the 5 by 4 improved inlet box for any discharge greater than 300 cfs. This difference is due primarily to the elevation of the outlet crown. The lower height of the 5 by 4 improved inlet also accounts for the fact that it operates at a lower head than the 5 by 5 improved inlet for discharges from 210 to about 320 cfs. In this case, however, the lower head is due to the fact that more depth is needed to submerge the 5-ft. high entrance. Above 320 cfs. the increased area of the 5 by 5 becomes effective so that it will carry 18 percent more discharge with headwater at 7.5 ft. and an increasing percentage for higher heads.

The following table shows the relative increase in discharge for these particular culverts resulting from improving the inlet when headwater is 2D.

Increase in discharge resulting from improving entrance if HW = 2D

Size	Discharge: Square edged	Discharge: Improved	Percent increase
4 by 4	187 cfs.	256 cfs.	37
5 by 4	232	328	41
5 by 5	325	450	39
5-ft. pipe	256	353	38

Another comparison is the amount of lowering of the headwater which is possible at the discharge where the barrel with improved inlet begins to flow full for entire length, as compared to the same culvert with square-edged inlet.

Decrease in headwater at discharge where improved culvert begins to flow full for entire length

Size	Discharge cfs.	Headwater, ft.: Square Edged	Headwater, ft.: Improved	Decrease in headwater, ft.
4 by 4	217	10.2	6.0	4.2
5 by 4	277	10.7	6.0	4.7
5 by 5	373	12.2	7.5	4.7
5-ft. pipe	293	12.0	7.5	4.5

The form of diagram in Figure A lends itself especially well to studying the effect of fall in the barrel upon the comparative headwater elevations for a given barrel size with and without an improved inlet. The line for the full culvert is fixed in slope but the position depends on the elevation of the pressure line at the outlet. Consequently by drawing a parallel line through the outlet crown elevation plotted at Q = 0, the operating curve for any fall in the culvert of the same length is immediately determined, bearing in mind that it is fully effective only for headwater greater than 1.5D. In the case of the 4 by 4 box a line parallel to the 4 by 4 improved inlet line drawn through the headwater of 6 ft. where it intersects the line for the 4 by 4 square-edged inlet, indicates that a fall of 0.5 ft. is necessary to equalize capacities. Consequently on this slope there is no advantage to using an improved inlet at this headwater although there is a slowly increasing advantage for higher pool elevations. The following table gives equivalent values for the other culvert sizes.

Minimum fall for equivalent discharge with square-edged and improved inlets at H = 1.5D

Size	Fall	Headwater	Discharge
4 by 4	0.5	6.0	152
5 by 4	0.4	6.0	190
5 by 5	0.2	7.5	260
5-ft. pipe	0.1	7.5	200

The general characteristics of the head-discharge curves plotted in the form of Figure A are as follows:

1. The entrance control curve depends only on the dimensions of the entrance and is fixed in position for a given culvert. Length, roughness and slope of culvert are immaterial, except as a rising water surface may force the barrel to flow full.

2. The curve for full flow has a slope which depends on the entrance loss coefficient, the roughness, length, hydraulic radius and area of the barrel. The position of the curve depends only on the elevation of the pressure line at the outlet.

3. The horizontal line for minimum submergence is drawn at 1.5D which is probably conservative; culverts may actually begin to flow full at inlet for somewhat lower heads.

4. For headwater less than 1.2D critical depth controls and the head-discharge curve would not plot as a straight line in Figure A because head is not directly proportional to the discharge squared. For such low heads the barrel will usually be flowing with a free water surface and the form of the inlet has a relatively small effect on headwater (except in the special case of an enlarged inlet with a steep drop in the tapered portion of the barrel, (Inlet 3 in Oregon report).

The Bureau of Public Roads is preparing a series of charts with head-discharge curves for any size and length of culvert with square-edged entrance operating under low head. This is possible because the variation in headwater with length is either zero as on supercritical slopes, or of small magnitude on mild slopes. These curves will enable direct comparison of headwater for various sizes with only a minor correction for longer culverts on relatively flat slopes and will obviate use of Charts 1042 and 1043 in this range.

Importance of Inlet Design on Culvert Capacity

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THE design of a culvert inlet has a significant bearing upon the relationship of the head to the discharge of a culvert. Its relative importance is contingent upon the type of flow occurring in the culvert, which in turn is governed by the location of the control section. For part-full flow the control may be either at the inlet or the outlet depending on whether the slope is hydraulically steep or mild. In the case of short culverts, control may be at the inlet even for horizontal or mild slopes.

The head-discharge curves of culverts having square-edged inlets have been compared with those for culverts having rounded inlets to illustrate the conditions for which a head-advantage may be obtained by using a rounded inlet. These comparisons have been made for three categories of culvert flow: long culverts on steep slopes, long culverts on mild slopes, and short culverts. Dimensionless head-discharge curves have been plotted for culvert flow in each category. For culverts on steep slopes, experimental data have been compared with the computed values and, since the agreement was reasonably good, serve as a basis for the analysis of flow in culverts operating under conditions other than those for which the tests were made.

The greatest head advantage for a particular discharge of the rounded inlet over that of a square-edged inlet was found for those cases in which the control section was located at the inlet. These were long culverts on steep slopes or short culverts where the length was negligible as regards barrel frictional resistance to flow. For long culverts on mild slopes, the head-advantage was far less pronounced.

● FROM a practical point of view, probably the most serious deficiency in the planning of simple culverts used in highways is in the culvert inlet. All too frequently the culvert is assumed to have much greater capacity than, in fact, it has; this reduction in capacity is frequently attributable to inadequacy of the culvert inlet.

Quite generally, the deficiencies of the inlet are thought of only in terms of their effect upon the head loss with the culvert flowing full; in reality this effect is of relatively minor importance in differentiating between good inlets and the poorest inlets customarily used. The important consideration is the overall hydraulics of the culvert in conveying runoff from one side of an embankment to the other, without impairing the roadway by overflow during high rates of runoff.

In general, the objective in designing a culvert is to provide a structure which will, under the conditions imposed, discharge a given flow with the least head; if the head and discharge are specified, the objective is to provide the most-economical culvert which, normally, is one with the least cross-sectional area.

The factors which combine to determine the character of flow in a culvert include all the design variables: slope, size, shape, length, and roughness of the culvert, the headwater and tailwater elevations, and inlet and outlet geometry. A convenient hydraulic classification of culverts is based on the location of the culvert control which is, in turn, determined by the relative magnitudes of the design variables. The nature of a control section is such that flow conditions downstream of the section do not affect the flow upstream of the section within a specified range of discharges. The principal flow characteristics are determined by location of the culvert control which for part-full flow may be either at the inlet or the outlet.

Control at the inlet usually occurs when the culvert has a steep slope and a free outlet; it may also occur with the culvert on a mild slope, provided the culvert is relatively short and the outlet is free. In one case of control at the inlet, the flow passes through critical depth at or near the inlet and supercritical flow exists through the barrel of the culvert. As disturbances cannot be propagated upstream in supercritical flow, it is apparent that the headwater elevation is dependent only on the geometry of the inlet and the discharge. This condition exists within a specific range of discharges; if this range is exceeded, the culvert may flow full and the control section will change.

For long culverts on a mild slope, flowing partly full, the control is usually at the outlet; with a free outlet the flow will pass through critical at the outlet. As a result, the headwater elevation is dependent on the discharge, wall friction, and inlet characteristics. If the tailwater is high enough to create a depth greater than critical at the outlet, the control is the tailwater elevation at the outlet, and the headwater is a function of the tailwater elevation as well as the other variables.

The preceding discussion of various control sections is included only to illustrate types which may exist. These will be discussed in more detail in subsequent sections.

The importance of inlet design as related to culvert capacity is contingent to a large extent upon the position of the control section. For inlet control, the geometry of the inlet has a significant influence upon the head required for a given discharge. A square-edged inlet causes separation to occur at the entrance and inhibits full flow in the culvert. A properly rounded inlet, on the other hand, avoids the separation and promotes full utilization of the barrel for flow. As a result of the availability of additional head in the culvert, the required water-surface elevation in the headwater pool is reduced — frequently very significantly reduced. When the control is at the outlet or when the culvert flows full, the geometry of the inlet becomes far less significant.

A comprehensive discussion of culvert entrances would necessarily be rather lengthy because of the many types involved. For example, the culvert may have a rounded, beveled, square, or bell-mouthed inlet. It may be in a defined or an undefined channel. It may be installed with the inlet flush or protruding (re-entrant) through a vertical or sloping headwall. Wing walls or warped transitions may be utilized. In most instances these variations will have a bearing on the culvert capacity. The square-edged inlet and the rounded inlet represent, in a sense, two extremes of inlet geometry. It appears that most culverts would possess inlets that fall somewhere between the two limits. The curves presented in this paper represent (for the case of circular culverts with a flush headwall) these two extremes of head-discharge curves, with the curves for other types falling between. However, a sharp-edged protruding inlet might be even worse hydraulically than the square-edged inlet.

Experimental and analytical investigations have for several years been undertaken at the St. Anthony Falls Hydraulic Laboratory, University of Minnesota, for the purpose of studying specific hydraulic characteristics on both full-scale culverts of various roughnesses (1) and dimensions (up to 3 ft. in diameter) and on smaller scale models. Tests with specific regard to entrance conditions of culverts were conducted in part under the sponsorship of the Minnesota State Highway Department and the United States Bureau of Public Roads.[1] These have been supplemented by student thesis research and other studies at the St. Anthony Falls Laboratory.

GLOSSARY

A	Cross-sectional area of the flow stream
A_o	Cross-sectional area of the culvert
α	Kinetic energy factor defined by Equation 5
b	Width of the stream at the water surface
C_c	Coefficient of contraction
C_v	Coefficient of velocity
d	Depth of flow in the culvert
d_c	Critical depth of flow
D	Diameter of the culvert
f	Friction factor in Darcy's formula

[1] Unpublished except for project reports. There will also be issued for limited distribution through the sponsorship of the Minnesota Highway Department Project Report No. 37 of the St. Anthony Falls Hydraulic Laboratory, "Effect of Inlet Design on Capacity of Culverts on Steep Slopes," giving results of specific culvert inlet experiments in more detail.

F Fall of culvert in length L so $S = F/L$

g Acceleration due to gravity

H Depth above culvert invert of headwater

H_o Specific energy with respect to culvert invert

K_e Entrance loss coefficient for full flow

L Length of culvert

n The Manning roughness coefficient

Q Discharge

Q_c Critical discharge

R Hydraulic radius of the flow stream

R_o Hydraulic radius of the culvert

S Slope of culvert

S_c Critical slope

θ Angle of inclination of the culvert from the horizontal

V Mean velocity of the flow stream

V_c Critical velocity of the flow stream

V_i Velocity at particular point in cross section

Salient experimental investigations were conducted in an apparatus constructed primarily for studies of this type. It consists of a channel 12 in. deep, 30 in. wide, and 50 ft. long in which culvert models of various sizes can be installed. The upstream 10-ft. section is separated from the remainder of the channel by a transverse bulkhead which normally forms the headwall of the culvert. This section has walls 28 in. high, as compared with 12 in. in the remainder of the channel, to permit variation of the head pool elevation. A second bulkhead is installed in the channel at the outlet end of the culvert model. The slope of the complete unit can be varied from 0 to 10 percent. Figures 1 and 2 illustrate the basic equipment. The model used in the studies was constructed of 4-in. diameter Lucite pipe and had an overall length of 35 ft. The ends of the pipe were flush with the bulkheads which formed the end walls of the culvert. The inlet section was removable so that square-edged and rounded inlets (Fig. 2) could be interchanged. The rounded inlet used in these tests had a radius of rounding equal to 15 percent of the pipe diameter.[2] Piezometers were located at frequent intervals along the culvert for pressure measurements.

In an earlier series of tests, the culvert was tested with both the inlet and outlet submerged in order to obtain data on frictional losses and entrance-loss coefficients for full flow. However, in the series of tests here concerned, the outlet was completely free, i.e., the jet was unsupported and discharged into the atmosphere. In the practical or applied case the tailwater may be raised considerably before causing any essential modification of the flow in the culvert.

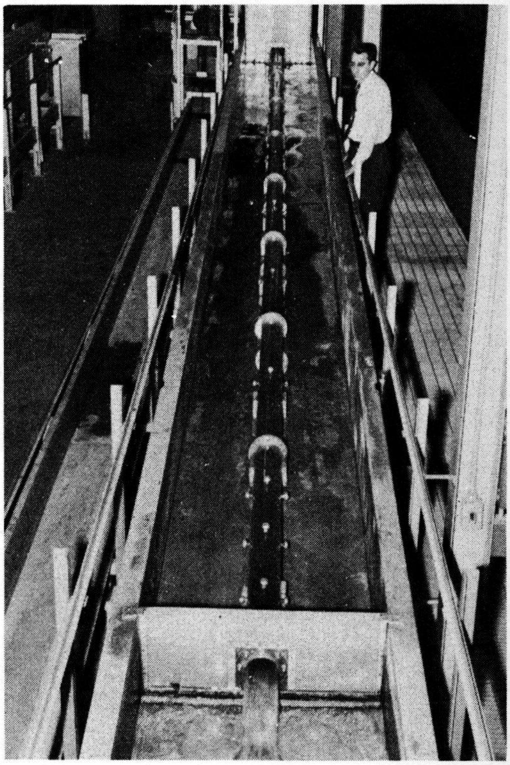

Figure 1. Variable-slope culvert model.

Data were obtained on the height of the head pool above the inlet invert for variations in inlet type, discharge, and culvert slope. When the culvert flowed full for at least a portion of the length, such as sometimes occurred when using a rounded inlet, data on the hydraulic gradient and the magnitude of pressure fluctuations were obtained.

[2] A theoretical explanation for the use of 15 percent of the pipe diameter as the radius of rounding is based upon recognition that for a sharp-edged orifice the coefficient of contraction is nearly 0.61; thus the entrance area must be 1/0.61 times the area at the vena contracta so that $(D_e/D_c)^2 = 1/0.61$ or $D_e = 1.28\,D_c$. Thus a 15 percent D enlargement of the entrance satisfies the criterion. Actually also this has been established experimentally and reported in "Suppression of Pipe Intake Losses by Various Degrees of Rounding" by J. B. Hamilton (Bulletin 51, Engineering Experiment Station, University of Washington, November, 1929), which corresponds exactly to the theoretical explanation of the authors.

Results of these experimental studies are summarized herein; there is also given an analysis of the flow conditions based upon fundamental hydraulics. Figures 4 and 5 illustrate some of the flow types which may occur in culverts with free outlets. The discussion has been restricted to culverts with free outlets because the case of culverts flowing with submerged outlets has been treated rather fully in other publications, and because of space limitations.

Figure 6 illustrates some of the experimental data obtained for square-edged and rounded inlets. It may be noted that for culverts on steep slopes the head required for a specified discharge is much less for a rounded inlet than for a square-edged inlet. This is especially pronouned for values of $Q/D^{5/2}$ in excess of four. The head advantage of the rounded inlet is dependent on the culvert slope and on the culvert length. An explanation of the flow conditions with the model culvert on a 4 percent slope may be of interest as a typical test. With a square-edged inlet the culvert flowed part-full for the complete test range which included values of $Q/D^{5/2}$ up to 9.0. Larger discharges were not used because the required head would have exceeded the height of the head tank walls. With a rounded inlet the culvert flowed part-full for values of $Q/D^{5/2}$ less than 4.0 ($H/D < 1.3$). For $4.0 < Q/D^{5/2} < 8.5$,

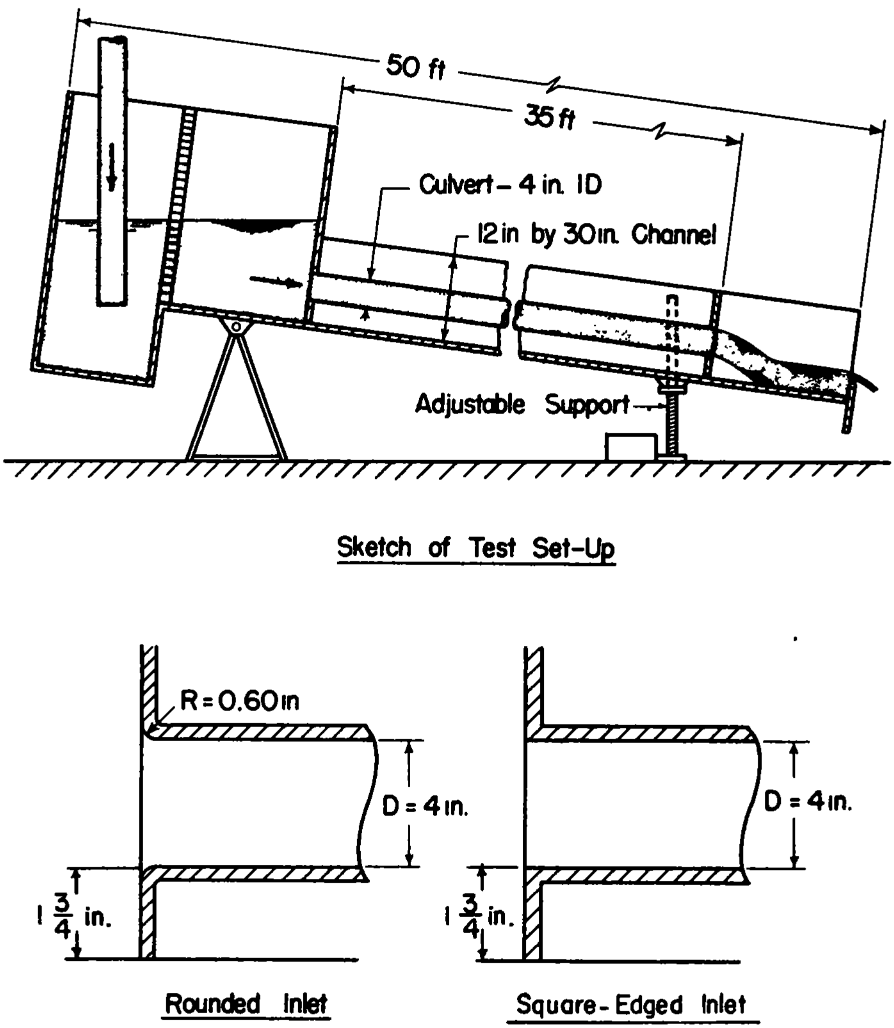

Figure 2. Equipment and inlets used in model tests.

the culvert either alternated between full and part full (slug flow or mixed flow); this caused the headwater elevation to fluctuate between H/D values of about 1. 2 to 1. 5. For values of $Q/D^{5/2}$ in excess of 8. 5, the culvert flowed full. The head-discharge curve is illustrated in Figure 6.

In some instances the culvert behavior and the head-discharge curves are dependent on the culvert length as well as the slope and other variables. An analysis of flow conditions for (1) long culverts on steep slopes, (2) long culverts on mild slopes, and (3) short culverts (where barrel wall friction has negligible influence on flow pattern) is presented following a discussion of some basic principles. Typical problems are solved as examples of each type.

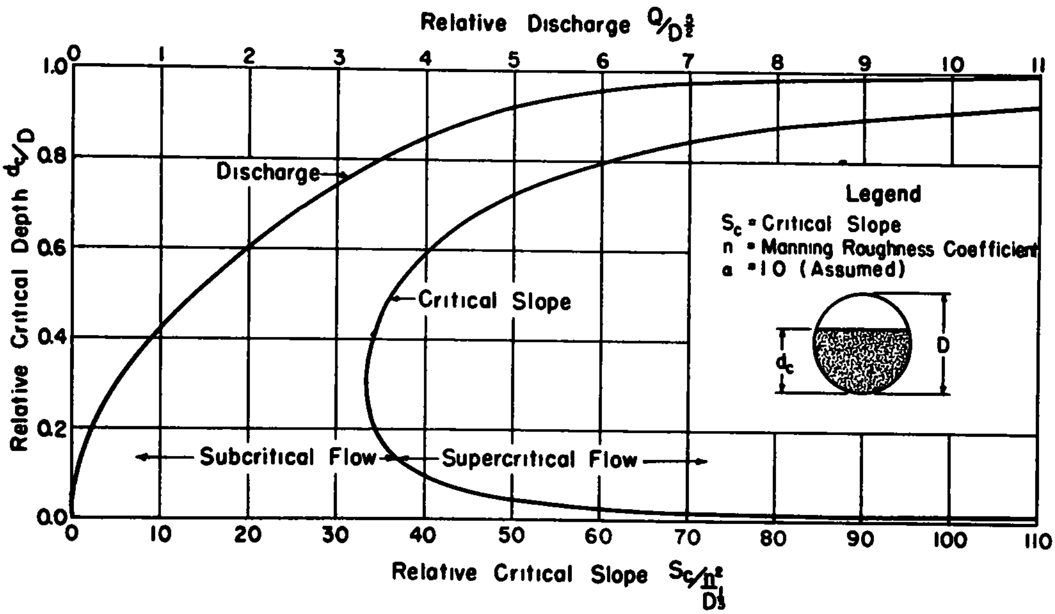

Figure 3. Critical culvert slope as a function of depth.

BASIC CONSIDERATIONS, CRITICAL DEPTH AND SLOPE IN PART-FULL FLOW

A culvert may flow either full or partly full, depending upon the specific hydraulic conditions. In part-full flow, the culvert behaves as an open channel with a free surface, the depth of flow being less than the vertical diameter or height of the culvert. In fullflow, the culvert behaves as a closed conduit or pipe. The pressure gradient then no longer necessarily coincides with the water surface. When a straight culvert flows full, the headwater level is, of course, above the crown of the culvert; however, the culvert does not necessarily flow full when the headwater is above the crown, even though this height may be several times the diameter of the culvert. The complete range of hydraulic relationships between discharge and head on the culvert includes both part-full and full-flow conditions, and the different types of flow follow different algebraic relationships. These relationships can now be quite adequately defined.

For part-full flow, the total energy per unit weight of water referred to the culvert invert is called the specific energy H_o and may be written as

$$H_o = \frac{\alpha V^2}{2g} + d \qquad (1)$$

where V is the mean velocity, d is the depth, g is the acceleration due to gravity, and α is a kinetic energy correction factor, the numerical value of which depends upon the velocity distribution over the cross section. (For uniform velocity distribution, α is unity.)

The minimum value of the specific energy corresponds to the critical flow conditions, for which it can be shown analytically

$$\left(\frac{Q_c}{D^{5/2}}\right)=\left(\frac{\pi}{4}\right)^3\frac{g}{\alpha}\left(\frac{A/A_o}{b/D}\right)^3 \qquad (2)$$

where Q_c is the critical discharge, A and A_o are respectively the cross-sectional area of the flow and of the culvert, b is the surface width, and D is the diameter of the culvert.

In order to eliminate Q, if Equation 2 is combined with the Manning formula

$$\left[Q = (1.486/n)\ AR^{2/3}\ S_c^{\ 1/2}\right]$$

an expression results for the critical slope and may be written as

$$\frac{S_c D^{1/3}}{n^2} = \frac{2.26g}{\alpha}\ \frac{(A/A_o)}{(b/D)\ (R/R_o)^{4/3}} \qquad (3)$$

In this equation S_c is the critical slope of the culvert, n is the Manning roughness coefficient, and R and R_o are respectively the hydraulic radii of the flow and the culvert section. In Figure 3,

$$S_c/(n^2/D^{1/3})$$

and $Q_c/D^{5/2}$ have been plotted as functions of d/D. For very small depths and for depths approaching the magnitude of the culvert diameter, the critical slope be-

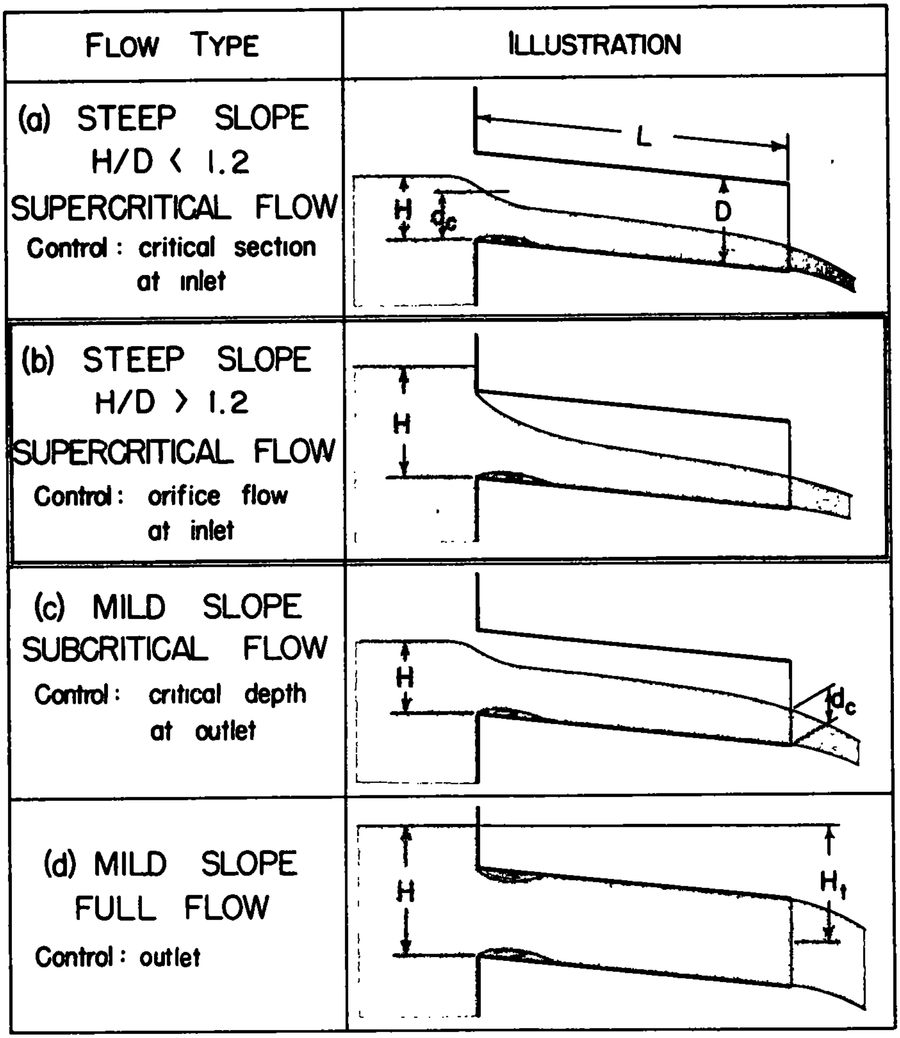

Figure 4. Typical flow conditions for square-edged inlet.

comes quite large, but over the wide intermediate normal range of part-full flow conditions through the culvert the critical slope varies within narrower limits. If the actual slope is greater than S_c (see Equation 3) for a given discharge, normal flow in the culvert will be supercriritcal and the depth less than critical. If the slope is greater than critical for this discharge, the culvert will flow part-full for its entire length. For a slope less than critical the culvert will flow part-full if it is short enough that retardation of flow by barrel wall friction is insufficient to induce critical flow, or full if it is sufficiently long.

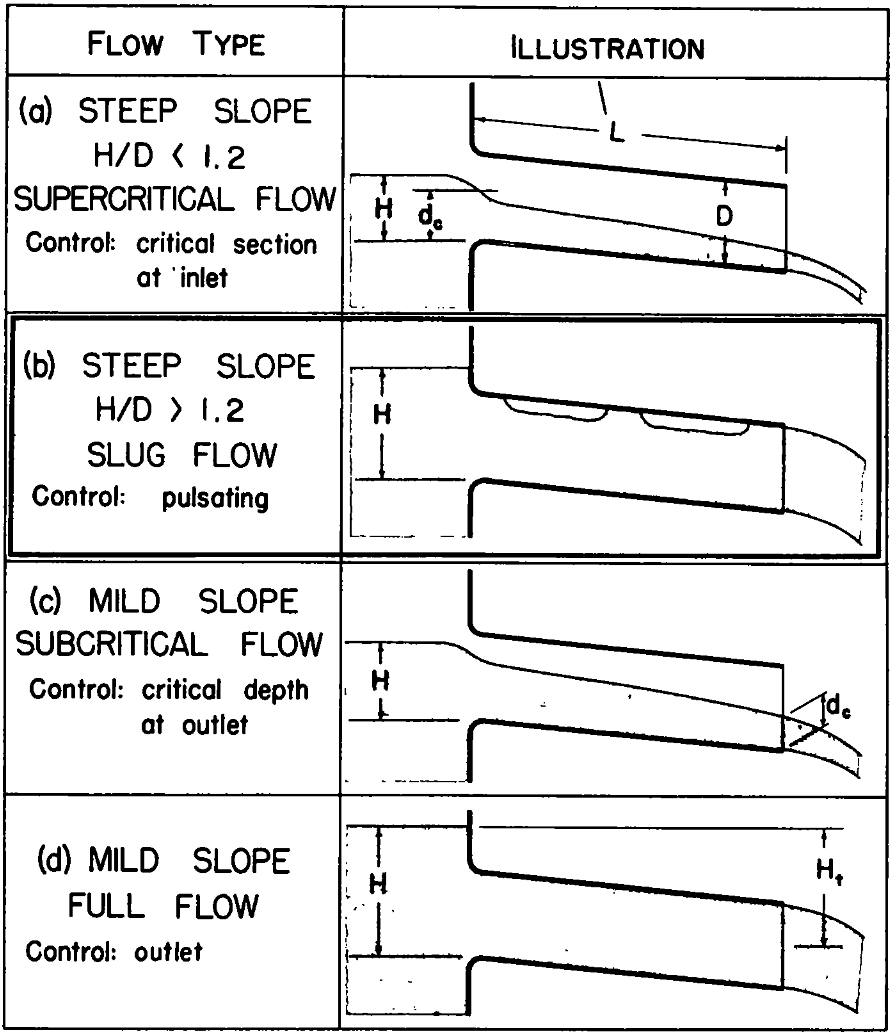

Figure 5. Typical flow conditions for rounded inlet.

slope is less than S_c, the normal flow will be subcritical and the depth greater than critical.

If the head is above the culvert crown, the depth within the culvert at the inlet is governed by the contraction and the character of the flow in the barrel is dependent upon the length and slope. If the actual

LONG CULVERTS WITH FREE OUTLETS ON STEEP SLOPES

In the case of culverts with steep slopes, that is [$S > S_c$ (Fig. 3)], the transition from subcritical flow in the approach channel to the super-critical flow in the culvert takes place at the culvert inlet (Figs. 4a and 5a) and corresponds to the condition under which Equation 2 applies. If we assume that the energy loss from the head pool to the critical section is negligible, we may write

$$\frac{H}{D} = \frac{\alpha V_c^2}{2gD} + \frac{d_c}{D} \qquad (4)$$

where from Equation 2

$$\frac{\alpha V_c^2}{2gD} = \frac{A}{2bD} = \frac{\pi}{8}\,\frac{(A/A_o)}{(b/D)}$$

If α is defined as the ratio of the average of the velocity heads of the individual flow filaments to the velocity head based upon the average velocity through the gross cross section, it may be written (2) as

$$\alpha = \frac{1}{A}\int_A \left(\frac{V_i}{V}\right)^2 dA \qquad (5)$$

If it is further assumed that the velocity between the vena contracta and the culvert wall is zero, then the value of α at the vena contracta is

$$\alpha = \frac{1}{C_c} \qquad (6)$$

Since the contraction coefficient depends upon the geometry of the inlet, the value of α will also depend upon the geometry and, of course, the depth of the inlet.

For the square-edged inlet and approach conditions used in these experiments, computed values of α varied from 1.25 to 1.42 as the depth changed from 0.1 D to 0.9 D at the inlet (3). The head-discharge curve computed from Equations 2 and 4 using these computed values of α agreed well with the measured head-discharge curve for the square-edged inlet (Fig. 6).

For the fully-rounded inlet where no separation occurs, it is assumed that $\alpha = 1.0$, that is, uniform velocity distribution just inside the culvert inlet. The head-discharge curve for the rounded inlet was also computed from Equations 2 and 4 with $\alpha = 1.0$ and compared with the measured curve for the rounded inlet in Figure 6.

Agreement with the measured values for the head-discharge curves were obtained up to values of H/D of about 1.2 in each case. This appears to be the limit of H/D for which a free surface can be maintained through the inlet; that is, the flow is not in contact with the inlet crown. Two separate curves are obtained, one for the square-edged inlet and one for the rounded inlet.

As the discharge increases, so that H/D is greater than about 1.3, the flow will normally be in contact with the wall entirely around the periphery of the entrance. With the square-edged inlet, separation at the corner will cause a contraction of the jet (Fig. 4b). If, in addition, the culvert is on a steep slope or is not too long, the culvert will not flow full. Hence, it may be assumed that the inlet operates in the same manner as an orifice. The equation for the discharge through an orifice under low heads may be written (4) as

$$\frac{Q}{D^{5/2}} = C_c\,\frac{\pi}{4}\sqrt{2g}\left(\frac{H}{D} - \frac{1}{2}\right)^{1/2}\left[1 - \frac{1}{128\,(H/D - 1/2)^2}\right] \qquad (7)$$

The term in brackets represents the effect of head on the velocity distribution in the orifice, particularly for low heads, and may be considered as a coefficient of velocity such that

$$C_v = \left[1 - \frac{1}{128\,(H/D - 1/2)^2}\right] \qquad (8)$$

which value rapidly approaches unity with increase in head.

With the square-edged inlet some experiments were made with the barrel removed from the inlet so that the inlet was a true orifice. The head-discharge curve so measured coincided with that measured with the barrel in place, indicating that for this particular arrangement of inlet and approach channel in which the bed of the approach channel was below the inlet invert, the inlet is similar to an orifice. The coefficient of contraction also varies somewhat with the head and may be approximated by a consideration of the geometry of the inlet and the head pool. The computed head-discharge curve for the square-edged inlet agreed with experimental data for values of H/D> 1.4 when the inlet acts as an orifice is shown in Figure 6 as a continuation of the curve for part-full flow at the inlet. A transition occurs for H/D between 1.2 and 1.4 from part-full flow at the inlet to orifice flow.

The measured head-discharge curve for the square-edged inlet corresponds very closely to that presented by Mavis (5) also for a square-edged inlet.

On the other hand, when the inlet is well rounded, separation at the inlet does not occur (Fig. 5b); consequently, the culvert begins immediately to flow full in the neighborhood of the inlet. The zone of full flow rapidly extends down the culvert toward the to atmospheric. This breaks the seal and with the loss of the added velocity head, the discharge decreases below that of the inflow, and the water surface rises until the inlet is again sealed and the culvert again starts to flow full. The cycle then repeats itself; pulsating flow develops through the culvert. The relationship of head to discharge in the region of pulsating flow has not been determined analytically so that dashed curves have been drawn through the experimental points in Figure 6. In this

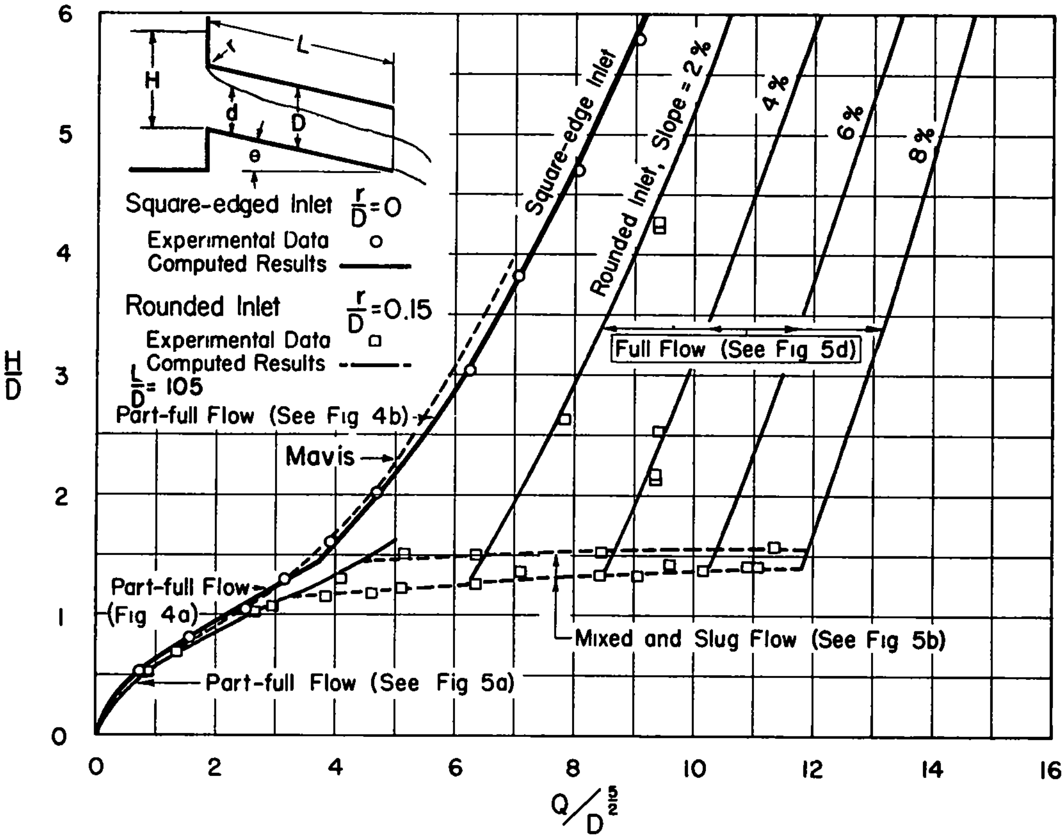

Figure 6. Comparison of head-discharge curves for square-edged and rounded inlets for long culvert on steep slope (from experiments on model culvert).

outlet. In the process of moving toward the outlet, an added head due to the slope of the culvert becomes effective. This added head tends to increase the discharge in the culvert above that of the inflow to the approach channel. The increased discharge causes a lowering of the water surface just upstream of the inlet. As the water surface is lowered it reaches a point where vortices form at the inlet and air is sucked into the culvert and increases the pressure range the data for all slopes from 2 to 8 percent fall on the same curve. The two lines represent the range of fluctuation of the head in pulsating flow.

When the inflow is large enough for a particular slope to permit the "slug" to extend the entire length of the culvert before the headwater is drawn down sufficiently to permit the intake of air, the "slug" or "mixed" flow phenomena ceases and the culvert flows full continuously.

When the culvert is flowing full, the head-discharge relationship may be determined by the application of Bernoulli's theorem to the flow so that

$$\frac{H}{D} - \frac{1}{2} + \frac{L}{D}\sin\theta = \frac{8}{\pi^2 \cdot g}\left(1+K_e+f\frac{L}{D}\right)\left(\frac{Q}{D^{5/2}}\right)^2 \quad (9)$$

where L is the length of the culvert; θ is the angle of inclination of the culvert from the horizontal so that $S = \sin\theta$, or $L/D \sin\theta = F/D$ where F is the fall in length L; f is the friction factor which for smooth culverts is a function of the Reynolds number, and K_e is the entrance loss coefficient for full flow.

In Equation 9 it is assumed that the pressure line is at the center of the culvert at the outlet. The location depends upon the value of $Q/D^{5/2}$, being above the center for small values of $Q/D^{5/2}$ and approaching the center of the culvert as $Q/D^{5/2}$ increases (5).

Inspection of Equation 9 indicates that for full flow the head-discharge curve depends upon slope, length, and roughness of the culvert as well as the entrance loss; therefore the dimensionless curve will be different for each culvert as well as for each slope of the culvert.

For the culvert model tested with the rounded inlet, the head discharge follows the critical depth curve for rounded inlets up to $H/D \cong 1.2$, at which point slug or mixed flow starts. As the discharge increases the slug or mixed flow continues and the head follows the slug-or mixed-flow curves to the point where the full-flow curve for the particular culvert intersects the mixed-flow curves. When this point is reached, the curve continues up the full-flow curve and the culvert flows full. For the model with the rounded inlet, full flow occurred at the point where the head-discharge curve for full flow meets the curves for mixed or slug flow. This point, of course, varied with the culvert slope, since a different full flow curve applies to each slope. In Figure 6, the experimental points on the full flow curve indicate that the culvert was flowing full. In these computations the factors corresponding to the experiments were used in order that a comparison with the experimental results might be made. Here $L/D = 105$, $K_e = 0.08$, and the value of f as a function of Reynolds number, were obtained from previous experiments on the same culvert.

In Figure 6 a comparison may be made of the effect on the head-discharge curve of rounding the inlet corners. It is apparent that for headwater elevation above the crown of the culvert ($H/D > 1.5$), a very strong advantage in the head which is required to pass a given discharge, accrues to the culvert with the rounded inlet. In the region where the flow passes through critical at the inlet (that is, $H/D < 1.2$), the head advantage in a rounded inlet is less.

The experimental results presented in Figure 6 were obtained from experiments on the model culvert (4 in. in diameter). In general, good agreement was obtained with curves computed on the basis of hydraulic principles with the exception of the curves for mixed and slug flows. This phenomenon forms the transition between part-full critical flow at the inlet and full flow for culverts with rounded inlets and was based entirely upon the model experiments. An analytical solution for this phase is desirable before extension of the results to prototype culverts is undertaken.

Example of Culvert Flow on Steep Slopes

In order to illustrate the foregoing principles, assume that a prefabricated concrete culvert 3 ft. in diameter and 300 ft. long is to be laid on a 2 percent slope to discharge 140 cfs. Assume further that the outlet is free and that a headwall at the entrance provides a flush inlet. For the concrete pipe the following factors apply:

$$\frac{L}{D} = 100 \qquad \frac{Q}{D^{5/2}} = 9.0$$

n (partly full flow)[3] = 0.011
f (full flow)[3] = 0.015; (n = 0.010 to 0.011) approximately

The factor

$$\frac{S}{n^2/D^{1/3}} = \frac{0.02}{(0.011)^2/(3.0)^{1/3}} = 239$$

is considerably greater than the values given in Figure 3 for the critical slope throughout the greater portion of the depth. Consequently, the culvert lies on a steep slope. If it is assumed that a square-edged inlet has been provided (note here

[3] Based on full-scale experiments (1) Customarily in the past higher n values have been used for concrete pipe and such higher values might be proper for inferior or deteriorated pipe

that a socket end is not as severe as a square-edged inlet), the head required for a discharge of 140 cfs. can be obtained directly from Figure 6 since the head-discharge curve for culverts on steep slopes with square-edged inlets is independent of the characteristics of the barrel. From the figure it appears that for

$$\frac{Q}{D^{5/2}} = 9.0, \ \frac{H}{D} = 5.80$$

Consequently, to discharge 140 cfs. through the culvert will require a head of 17.4 ft. above the invert or 14.4 ft. over the culvert crown.

If the inlet were rounded so that no separation at the inlet occurred, the culvert would flow full when the upstream water surface became high enough to seal the entrance. If it is assumed that for a discharge of 140 cfs. the culvert will flow full, Equation 9 will describe the flow or, in addition to the factors given above, we have

$$\frac{L}{D} \sin\theta = 100 \times 0.02 = 2.0$$

$$K_e = 0.08$$

Then

$$\frac{H}{D} - \frac{1}{2} + 2.0 = 0.0252\,(1+0.08+1.50)\,9^2$$

$$\frac{H}{D} = 3.77$$

Since H/D as computed is greater than 1.5, the assumption that the culvert flows full for the prescribed discharge is satisfied. For a rounded inlet then the head required to discharge 140 cfs. is 11.3 ft. above the invert or only 8.3 ft. above the crown of the culvert as compared to 14.4 ft. above the crown if the inlet had been square-edged. The difference is attributable entirely to the entrance condition.

If the culvert had been placed on a 4 percent slope the head above the crown would have been about 2.3 ft. (Fig. 6). In this case, since H/D for $Q/D^{5/2} = 9.0$ is only slightly above the zone of pulsating flow, the flow in the culvert could conceivably be pulsating. If the slope had been 5 percent, certainly pulsating flow would occur in the culvert with the rounded inlet. However, even in this case with a culvert on a 2 percent slope, the head required would be considerably less than that required if the inlet had been square-edged.

LONG CULVERTS WITH FREE OUTLETS ON HORIZONTAL OR MILD SLOPES

The distinction between a long and short culvert in the hydraulic sense is significant when for a particular discharge the culvert is on a mild slope. A long culvert may be qualitatively defined as one which is sufficiently long to flow full on a mild slope when the head is above the culvert crown. If the culvert is short, the supercritical flow caused by the inlet contraction passes through the culvert without making contact with the culvert crown and the inlet assumes the control. If the culvert is long enough, the water surface profile would rise to the crown or the flow would pass through a hydraulic jump to reach the crown. When this happens, the jump or mixed flow would pump air from the space upstream, reducing the pressure thus causing the jump to move upstream. Either it would reach the inlet and the culvert would flow full, or the headwater elevation would be reduced enough to permit vortices to form and air to be sucked into the culvert. In this case, a slug or pulsating flow would develop.

When the culvert is horizontal, or at least the slope is less than S_c as defined by Equation 3, the flow in the culvert at depths less than D must be subcritical and the control section moves to the outlet end of the culvert. For larger discharges the culvert will flow full. For those discharges where the culvert flows partly full, the water surface assumes the profile of a drawdown curve passing through critical depth at the outlet and acquiring a relative depth at the inlet end of the culvert that depends on the slope, length, and roughness of the culvert (Figs. 4c and 5c). This relative depth is independent of the geometry of the inlet, and hence is the same whether the inlet is square-edged or rounded. If Bernoulli's equation is written between a point upstream of the inlet and a point within the culvert just downstream of the inlet, there is obtained for the head upstream the expression

$$\frac{H}{D} = \frac{d}{D} + \frac{8}{\pi^2 \cdot g}\,(1 + K_e)\left(\frac{Q}{D^{5/2}}\right)^2 \frac{1}{(A/A_o)^2} \qquad (10)$$

where H/D is the relative head acting on the culvert and d/D is the relative depth within the inlet. Equation 10 applies both to the square-edged and rounded inlets; the difference is in the magnitude of the entrance loss coefficient K_e. Experiments on the 4-in. Lucite culvert indicated that for the square-edged inlet $K_e = 0.43$ and for a well-rounded inlet $(r/D \geq 0.15)$ $K_e = 0.08$. Experiments on full-scale prefabricated concrete culverts (1) with socket-end inlets showed that for reentrant inlets $K_e = 0.15$, and for flush inlets $K_e = 0.10$. For culverts fabricated from corrugated metal pipes, the corresponding entrance losses were as follows: projecting (re-entrant) inlet $K_e = 0.85$, flush inlet $K_e = 0.50$. For larger relative discharges, a point will be reached when the culvert will flow full throughout its length (Figs. 4d and 5d). When this occurs, Equation 9 applies. Here again the difference in head required for a given discharge through a particular culvert will depend on the magnitude of K_e corresponding to whether the inlet is square-edged or rounded. The same factors applicable to part-full flow may also be applied to full flow. The head-discharge curves for culverts on a zero slope may be compared in Figure 7 to show the effect of inlet rounding on the required head. The curves in Figure 7 were computed to indicate the influence of rounding the inlet and are not based on experimental data.

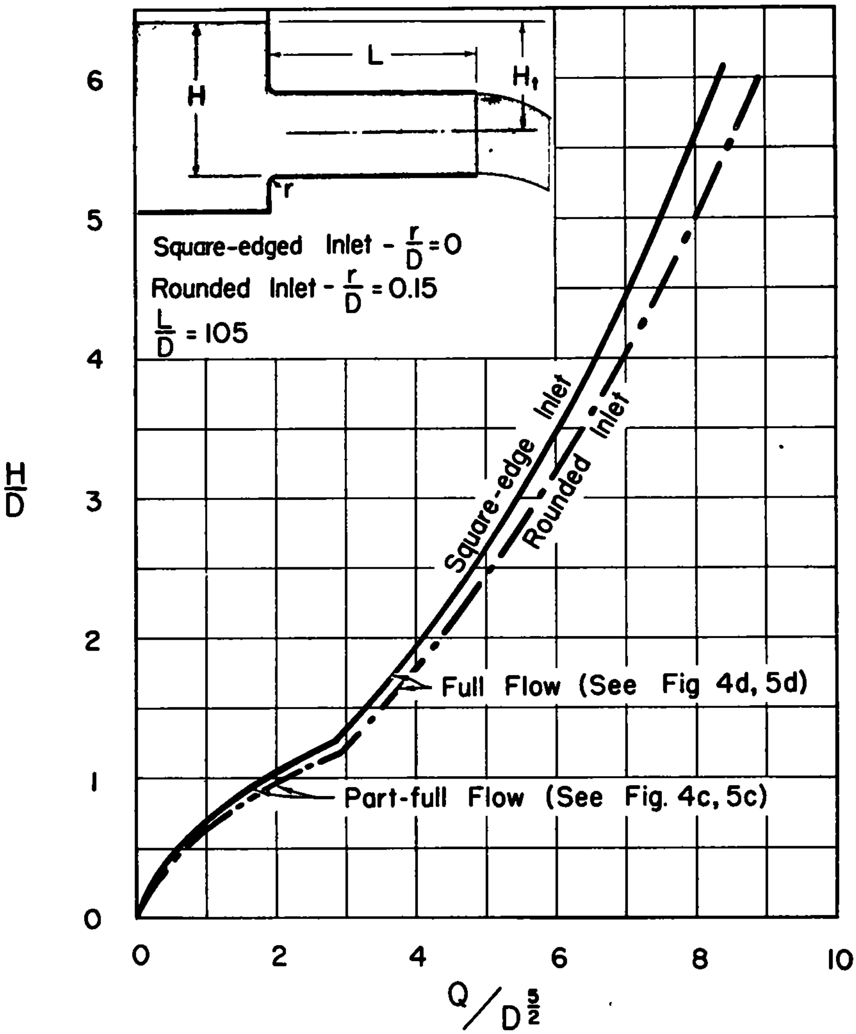

Figure 7. Comparison of computed head-discharge curves for square-edged and rounded inlets for long culvert on zero slope.

<u>Example of Flow in Horizontal Culvert (Zero Slope)</u>

If it is assumed that the culvert described in the previous hypothetical example had been laid horizontally rather than on a 2 percent slope, the influence of inlet geometry on the flow in culverts on mild slopes may be illustrated. Again the factors which apply, assuming a square-edged or rounded inlet are as follows:

$\frac{L}{D} = 100$ K_e (square-edged) = 0.43

$\frac{Q}{D^{5/2}} = 9.0$ K_e (rounded) = 0.08

$\sin\theta = 0$ f = 0.015 (or about 0.011 for Manning n)

If it is assumed as before that the culvert flows full, then

$$\frac{H}{D} - \frac{1}{2} + \frac{L}{D}\sin\theta = \frac{8}{\pi^2 \cdot g}\left(1 + K_e + f\frac{L}{D}\right)\left(\frac{Q}{D^{5/2}}\right)^2$$

For the square-edged inlet

$$\frac{H}{D} = 0.0252\,(1 + 0.43 + 1.50)\,9^2 + 0.50 - 0 = 6.47$$

H = 6.47 x 3.0 = 19.41 ft. above invert

For the rounded inlet

$$\frac{H}{D} = 0.0252\,(1 + 0.8 + 1.50)\,9^2 + 0.50 = 5.77$$

or

H = 5.77 x 3.0 = 17.31 ft. above invert

The computed value of H/D indicates that the assumption that the culvert flows full is valid.

In this case the advantage of using a rounded inlet is approximately 2.1 ft. of head.

SHORT CULVERTS

When a culvert is short, the flow characteristics become relatively independent of the slope, and the factors that involve the length become comparatively unimportant. (In this connection the barrel-wall roughness comes into consideration: a smooth-walled culvert can be considerably longer than a rough-walled culvert and still be classified as "short.") Consequently, the control section is essentially at the inlet for all conditions. Therefore the head-discharge relationship for part-full flow should be much the same as for culverts on a steep slope in the case of both the square-edged and rounded inlets. The head-discharge curve for the square-edged inlet when the headwater elevation is above the top of the pipe is the same as that for a similar culvert on a steep slope. In the case of the short culvert with the rounded inlet flowing full, Equation 9 with $L \rightarrow 0$ or becoming very small as regards wall friction would describe the flow, the magnitude of $L/D \sin\theta$ and $f(L/D)$ both being negligible. Between the part-full phase and the full-flow phase there exists a transition zone of pulsating flow in which the culvert is alternately full and partly full.

The head-discharge curves for short culverts of any slope have been computed on the above basis and plotted in Figure 8 for comparison. In these computations it was assumed that L could be considered equal to zero, and the entrance loss coefficient K_e for the rounded inlet, as before, was assumed equal to 0.08.

It is apparent from the plot that a considerable advantage in head is gained for the larger discharges by the simple expedient of rounding the inlet to reduce the degree of contraction of the jet.

Example of Flow in Short Culverts

Consider the hypothetical culvert previously described again modified by reducing its length to the point where pipe friction is a negligible amount; the culvert will be taken as horizontal. Then, using the same discharge as before ($Q/D^{5/2}=9.0$), we may take the value of H/D directly from the curve for the square-edged inlet in Figure 8, since H/D is a function of inlet geometry only. Therefore

$$\frac{H}{D} = 5.80$$

or

H = 5.80D = 17.4 ft. above the invert

On the other hand, if the inlet is rounded, the value of H/D may also be taken from Figure 8 since in this case too the head-discharge relationship depends only on the inlet geometry. Here

$$\frac{H}{D} = 2.55$$

and

H = 2.55D = 7.65 ft. above the invert

In this case the advantage in head of the rounded inlet over the square-edged inlet amounts to 9.75 ft., a quite significant amount.

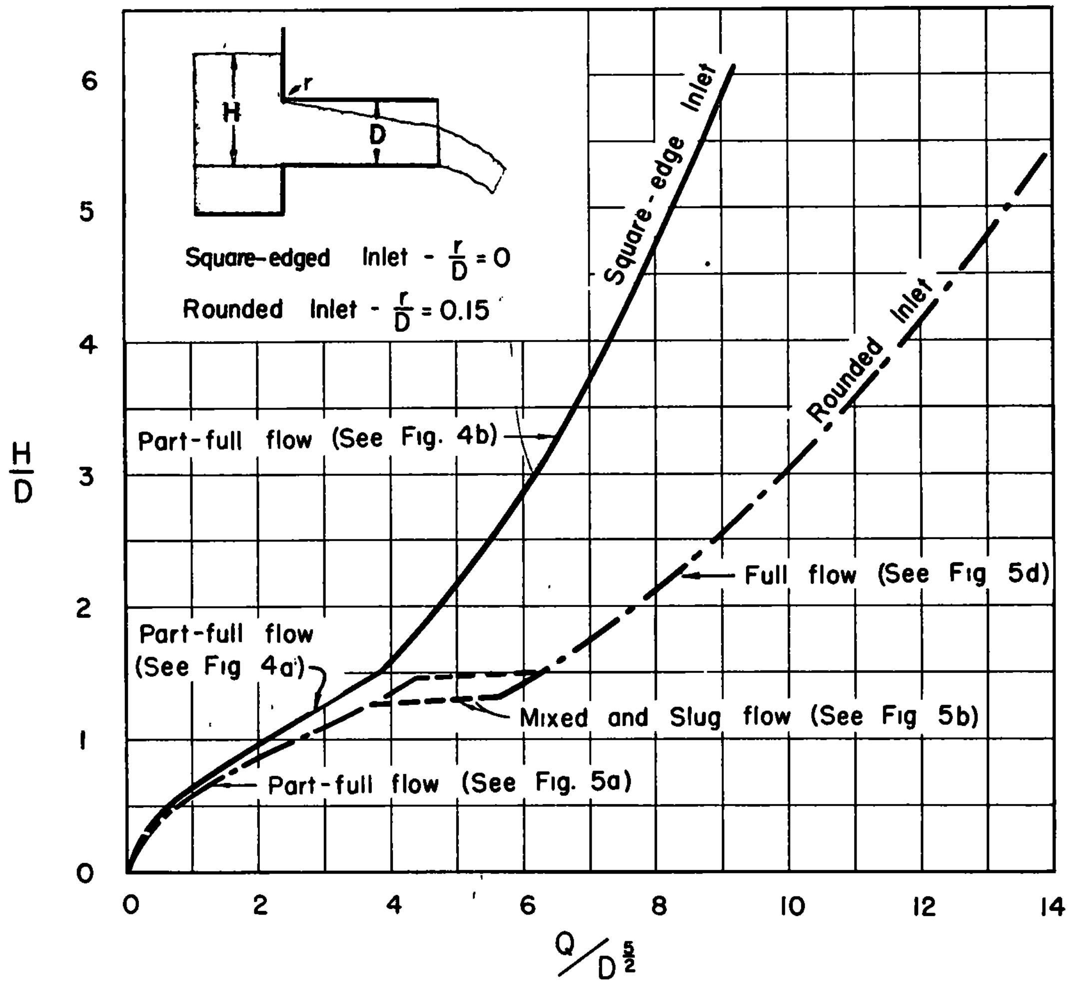

Figure 8. Comparison of computed head-discharge curves for square-edged and rounded inlets for short culvert, control at inlet (culvert horizontal and pipe friction negligible).

ACKNOWLEDGMENT

The experiments described here and used in the discussion of the influence of inlet geometry on the capacity of culverts were performed at the St. Anthony Falls Hydraulic Laboratory under the general supervision of Lorenz G. Straub, director. The project leader of those under the sponsorship of the Minnesota State Highway Department and the Bureau of Public Roads was Henry M. Morris who did a considerable part of the analysis. As part of a thesis project Madhav Manohar performed a rather extensive series of experiments to study the flow in culverts on steep slopes using both a square-edged and a rounded inlet. His experiments covered the range of orifice flow through the square-edged inlet, and the slug-flow and mixed-flow phases for rounded inlets, supplementing earlier tests.

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Discussion

F. T. MAVIS and T. E. STELSON, Department of Civil Engineering, Carnegie Institute of Technology, Pittsburgh, Pennsylvania — In this paper there are many points of similarity, and even identity, with studies (1) of the hydraulics of culverts published by The Pennsylvania State College in 1942 as Engineering Experiment Station Bulletin 56. Abstracts of eleven studies which had been conducted earlier at the State University of Iowa, beginning with the pioneer work of David L. Yarnell, Floyd A. Nagler, and Sherman M. Woodward (2) were reproduced there by permission (3). Further work has been done at Carnegie Institute of Technology by civil engineering staff and students (4).

Straub, Anderson, and Bowers have verified the types of flow and the head-discharge curves for conduits with square-edged entrances which should by this time be generally known. They have added information concerning entrance-loss factors for rounded entrances. These contributions should be reassuring and helpful to designing engineerings.

However, we would call attention to several points in the paper and raise some questions that may be interpreted as cautions:

1. Consider the example (following Equation 8) of a culvert 3 ft. in diameter and 300 ft. long discharging 140 cu. ft. per sec. (at an average velocity of 20 ft. per sec.) Is this typical of good practice? How would this fill-ripping velocity be handled at the outlet? Wouldn't the designer want to buy a bigger pipe in this case — and wouldn't the pipe salesman be willing to sell it to him?

2. If the culvert in this example is to be full, the discharge necessary to keep the pipe flowing full must first pass the inlet section as a control before the control point can move down to the outlet. Unless the culvert is first submerged by backwater, the cycle of operation during a storm would be either to flow part full from beginning to end; or to flow part full, then full (steadily or slug-wise), and finally part full (or empty). Computations and sketches of all types of flow are detailed in Bulletin 56 and in Concrete Pipe Lines (5).

3. Rounding the inlet of a culvert may increase the discharge for a given headwater depth; because (1) the rounded inlet reduces the contraction of the flow when the culvert flows part-full, or reduces the entrance-loss coefficient when the culvert flows full and (2) the rounded inlet may cause the culvert to flow full instead of part-full. If the culvert flows full and the slope is steep enough, negative pressures may increase the effective head.

Rounding inlets for Reason 3(a) is sound and may be easily analyzed by methods previously developed for weir-orifices. Rounding inlets for Reason 3(b) is more likely to be questionable practice than clever design. To illustrate, look closely at the example which follows Equation 8. The velocity head and entrance loss is 6.1 ft. for the culvert with a rounded inlet. Yet at the crown the water is only 2.3 ft. deep. Can a negative pressure of 3.8 ft. be maintained at a point that is only 2.3 ft. below the free surface of the headwater pool? Undoubtedly a vortex would form and relieve the negative pressure unless the flow were well baffled. When the negative pressure is relieved, the headwater level may rise to 7.6 ft. above the invert at entrance. Note that if the slope of the culvert in this example were 5 percent (instead of 4 percent) the headwater level would have been figured to be below the crown, requiring a negative pressure in the atmosphere above the entrance — and this is clearly impossible!

To extrapolate model data to prototypes is tricky at best — and the caution that is necessary if subatmospheric pressures are involved is well illustrated by extending this example.

Laboratory studies and analyses such as those reported in the paper and discussion help engineers understand how a culvert behaves under a given set of field conditions. They can help even more in unscrambling hydrologic data when a culvert is used as a flow-measuring device. Incidentally, a culvert is a practical tool for measuring discharge and one that is perhaps too seldom used. This paper will lend additional confidence to engineers to use culverts to measure stream flow (6).

The engineer who designs and builds culverts that are to serve as adequate drainage structures in the uncertain future may be inclined to consider some such suggestions as these:

1. Don't overlook outlet velocities. What will happen if they are too high?

2. Don't expect pipes with rounded entrances to work miracles. They may discharge more water for a given total head than pipes with square-edged entrances, but sometimes it may be better to lose head under control in the pipe rather than below the outlet.

3. Don't make a "long" culvert "small" and "steep" merely to gain hydraulic advantage, forgetting that it may sniff air and need to be cleaned.

4. Keep designs and design-computations simple and checkable. The uncertainties of stormy weather are much greater than the uncertainties of culvert-hydraulics; and the answer to "What's worth figuring?" will stem as much from the hardheadedness of engineering judgment as from the niceties of hydraulic science.

Discussion References

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CARL F. IZZARD, Chief, Hydraulic Research Branch, Bureau of Public Roads — This paper demonstrates conclusively the fact that under certain conditions culverts with square-edged entrances cause excessive headwater because the barrel does not flow full. The common assumption that any culvert will flow full if laid on a slope equal to or less than the friction slope may be wrong, particularly in the case of relatively short, smooth culverts. The paper demonstrates that a rounded edge will cause the barrel to flow full; as will be pointed out later, this does not always mean that the headwater depth will be reduced from that for the square-edged entrance.

The types of flow illustrated in Figures 4 and 5 deserve careful study. As recognized by the authors all the possible cases are not covered. One common case is contracted flow as in Figure 4(b) but on a subcritical slope. This can occur when the length of the culvert is such that the momentum of the flow carries it out of the culvert before the water surface can rise to the top of the barrel.

Contracted flow is fully developed for $H/D = 1.5$ which corresponds to $Q/D^{5/2} = 3.9$ approximately. At this relative discharge critical depth as indicated by the discharge curve in Figure 3 is about 0.84D. Since the contracted depth is about 0.6D the flow in the contracted section must always be at supercritical velocity. Thus it is possible with the square-edged entrance to have supercritical flow on a mild or even level slope. The main point is that contracted flow with headwater as shown for the square-edged inlet in Figure 6 is not confined to culverts on steep slopes.

The curve for critical slope in Figure 3 is useful for distinguishing in Figure 4 between critical depth control at the inlet (a) and at the outlet (c) but, for the reason stated in the previous paragraph, does not govern for type (b). The form of the profile beyond the contraction in type (b) depends on the friction slope at the contracted depth, the water surface dropping if slope exceeds this friction slope or rising if it does not.

The example of culvert flow on a steep slope curiously enough does not meet the assumed condition that the slope is steep, although as stated in the previous paragraph, critical slope does not govern. At $Q/D^{5/2} = 9.0$ the relative critical depth is 0.99 and the corresponding critical slope, mathematically, would be 0.0240 as computed from King's Handbook of Hydraulics, Table 116. Actually critical depth in this range can have no real significance. (The friction slope for the full culvert would be slightly more.) Since the barrel slope is only $s = 0.02$ the depth of flow in this culvert would increase from the contracted depth and might even fill the barrel before the full length was attained.

Practically the assumption of $n = 0.010$ (or 0.011) is not realistic as culverts installed in the field cannot be expected to be as smooth at the joints as the culvert tested by Dr. Straub. With a higher value of n the culvert in this example would almost certainly flow full, even with the square-edged entrance.

For the culvert on the 2-percent slope the outlet velocity will be nearly 20 ft. per sec. and would require some type of energy dissipator if an erodible soil were present. Enlarging the size of the culvert would not reduce the outlet velocity appreciably as a 3.5-ft.-diameter culvert would not flow full on this slope, unless the roughness was at least $n = 0.013$.

As noted by the authors the limits for slug flow in the model as shown by dotted lines in Figure 6 may not be entirely correct for the prototype. This follows from the fact that model tests involving entrained air are qualitatively indicative of prototype performance but may not give true quantitative results. The error is not likely to be large, however.

The involved equation for orifice flow (Equation 7) fortunately does not need to be used if a graph such as Figure 6 is available. Analysis of the data indicates that the following equation fits the data very well in the range above $Q/D^{5/2} = 4$.

$$\frac{H}{D} = 0.59 + 0.067\left(\frac{Q}{D^{5/2}}\right)^2 \qquad (10)$$

The orifice theory strictly is not applicable to this case since the jet is not free; it happens to fit the data closely, probably because the pressure line at the free water surface in the contracted section is close to the center of the entrance.

It is unfortunate that both Mavis and the authors of this paper chose to set the model culvert above the flow line of the approach channel. Additional tests are needed to determine to what extent, if any, the bottom contraction affects the vena contracta in the culvert which governs the headwater-discharge relation. Furthermore, the tests would have been more representative of normal field conditions if the outlet jet had been supported on an apron at the invert elevation. This would affect the elevation of the pressure line and can be allowed for in computations involving culverts flowing full.

While the rounding of 0.15D causes the model of length 105D to flow full, there is no positive evidence that a very short culvert would also flow full.

There appears to be no good reason for expressing the head loss in a full culvert as a function of $(Q/D^{5/2})$ (Equation 9). Highway engineers generally have nomographs available for determining the head loss in a full culvert. The headwater depth is then determined by subtracting the fall and adding the height of the pressure line at the outlet above the invert. Equation 9 assumes the pressure line to be at the center of the outlet which is true only for a free jet when $V^2/2g > 0.8D$ (see Mavis (5) page 28). For most culvert installations the jet is supported and the pressure line is probably at or close to the crown of the culvert and conservatively may be assumed at the crown.

The explanation given by the authors of the operation of a culvert with rounded entrance is illuminating and by far the most valuable information contained in the paper. Lest the unwary should be led astray it should be emphasized that in Figure 6 curves for the culvert with rounded inlet on various slopes apply only to a very smooth pipe having a length of 105 diameters. Attention is also called to the fact that for discharge in excess of $Q/D^{5/2} = 7$ the pressure line at the entrance will be below the invert. In actual practice such negative pressures probably should be avoided. There is some doubt that the prototype could actually be depended on to operate on the lower line for slug flow in Figure 6 at $Q/D^{5/2} > 7$. Further investigation of this pressure problem is needed.

A simple test for indicating the advantage to be obtained by using a rounded entrance on a given culvert is covered in the discussion of the Oregon paper.

LORENZ G. STRAUB, ALVIN G. ANDERSON, and CHARLES E. BOWERS, Closure-The authors are pleased with the interest that has variously been expressed in the paper and hope that the end result will be some improvement in modern culvert design. In responding to the written discussions re-emphasis is here made that the purpose of the presentation was to demonstrate the "importance of inlet design on culvert capacity." The many other aspects of culvert design and practice are not considered as a part of the treatment covered. The primary objective of the paper has been to emphasize that a greater culvert capacity and hydraulic efficiency can be obtained, particularly for short culverts and for long culverts on steep slopes, by proper attention to characteristics of the inlet. Quite frequently in the past improved design of the inlet has been associated too strongly with the local head loss rather than with the more important aspect of the influence of the inlet in overall behavior of the entire culvert.

The terms "long" and "short," "steep slope," and "mild slope" when applied to a given culvert are relative and their applicability depends among other things upon the roughness characteristics of the culvert itself, also upon the discharge. They are qualitative expressions which can be defined when such factors as roughness, discharge, and the like are given. Thus, Izzard mentions that it is possible to have supercritical or contracted flow on a level or mild slope. In a qualifying sense this is true, provided the culvert is not too long.

The authors disagree with Izzard that an n value for concrete culvert pipe of 0.011 is not realistic. On the contrary, there is positive evidence that many of the customary values which have been taken for granted in practice are really quite unrealistic and misleading, both for concrete and corrugated metal pipes. In regard to the conducting of experiments with the culvert invert at the elevation of the approach channel, such experiments are the logical next step to the more-idealized studies of culvert entrances free from the approach channel as reported in this paper. However, the authors wish to point out that the orifice-flow philosophy basically would lead one to surmise that the entrance conditions would be similar with the approach channel at the invert elevation as with the approach channel at the lower elevation of the reported tests. Suppression of contraction on the invert side distorts the vena contracta but the contraction coefficient does not change materially. Exploratory tests at the St. Anthony Falls Laboratory bear out this fact.

Izzard calls attention to an empirical equation (Equation 10) which he presents in preference to the authors' Equation 7. The suggested Equation 10 is probably quite adequate for normal use, but it is restricted to a square-edged inlet. For other types of inlets it would be necessary to set up a new empirical equation if this method were to be used. The important significance of Equation 7 is that it is generally applicable and not limited to one specific type of culvert entrance, because it is based upon the degree of contraction by the factor involving the contraction coefficient. A method of estimating the contraction coefficient was developed which considers the shape of the inlet (Reference 3). The applicability of the basic orifice equation to the square-edged inlet used in the experiments was demonstrated by tests giving the same head-discharge curve when the culvert was removed and the inlet became an orifice.

Equation 9 was written in terms of the discharge, or $Q/D^{5/2}$, only for convenience in order to plot head discharge curves for culverts flowing full. There are, of course, other ways in which the head loss can be expressed, possibly in more-convenient forms for particular uses. The assumption that the pressure line is at the center of the outlet is probably a good approximation for full flow with a free outlet for the cases considered.

In regard to the rounding of 0.15D, contrary to the question raised, it is quite positive that a short culvert would flow full with this type of entrance. The 0.15D rounding insures the vena contracta being the full size of the pipe at the entrance; pipe friction produces further resistance to flow in the full pipe. Tests with model culverts as short as 10 diameters with rounded entrance invariably flowed full with slopes from 0 to as steep as 10 percent, provided the discharge was sufficient to develop a head of the order of ½ D above the crown of the entrance; for lower discharges pulsating slug flow develops in accordance with the chart shown in Figure 6.

In regard to the part of the discussion of Mavis and Stelson, which seems to question the desirability and advantages of rounding the culvert entrance, some supplementary comments are desirable. First, relative to high discharge velocities, this is a matter entirely separate from hydraulic efficiency of the culvert and should be so treated. Here one is concerned particularly with proper exit design, which requires further analytical and experimental treatment not within the scope of the paper. Relative to the implication of the development of an absurd negative pressure just beyond the entrance of a rounded culvert, this is of course recognized as not possible. Actually, under such conditions slug flow develops; air is drawn in and full head is temporarily interrupted. There is thus not a steady state flow for this condition but a pulsating flow of varying velocity, the pattern of which changes as the average rate of discharge changes while the head over the entrance to the culvert varies but slightly as shown experimentally in Figure 6. The authors agree with the cautions outlined by Mavis and Stelson, although probably not completely with the implications that a casual reader obtains. Regardless of uncertainties of stormy weather and other unpredictable conditions, every economical advantage should be taken in producing hydraulically the most-efficient design; this must also take cognizance of energy dissipation at the discharge end of the culvert which should be the subject of further treatment. Thus if a designer chooses to make the culvert "adequately large" he should still take advantage of getting highest practical hydraulic efficiency for handling the unpredictable storm runoff.

The authors appreciate the discussions and questions raised by Izzard and by Mavis and Stelson in providing further clarification of the problems of culvert design.

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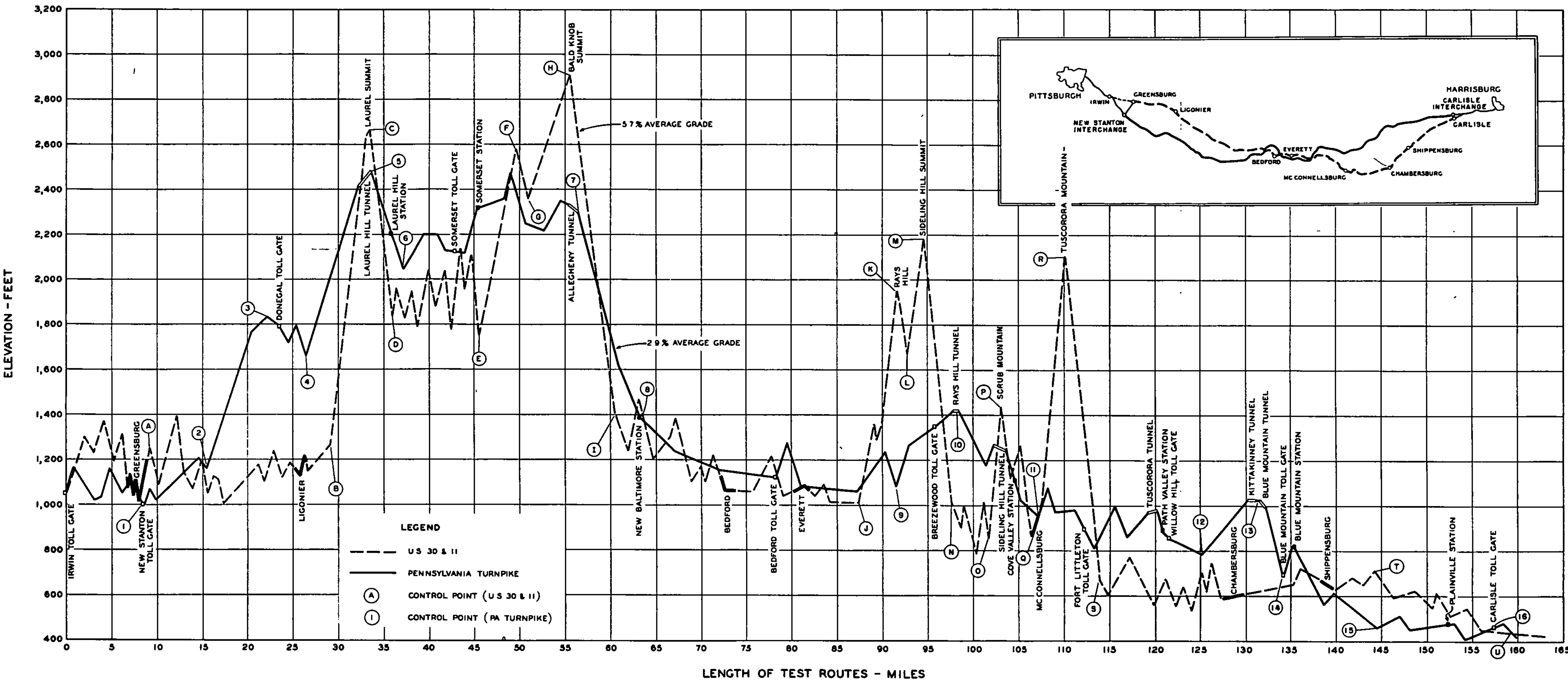
FIGURE 1 - TEST ROUTES

PROFILE AND SKETCH OF PENNSYLVANIA TURNPIKE, U.S. 30 AND U.S. 11



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