# An Anti-Rear-End Collision System 

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$\bullet$ IN THE STUDY of electronic aids to highway safety, a particularly alluring subject has been the application of electronics to longitudinal control of individual cars. Two desired gains from this application are a reduction in the number of rear-end collisions and an increase in safe traffic density. In simplified form, this problem is one of controlling a following car of velocity $\mathrm{v}_{2}(\mathrm{t})$ in response to a sensed lead car of velocity $\mathrm{v}_{1}(\mathrm{t})$, so that a finite distance is maintained between the cars. Of course, this assumes that the following car is overtaking the lead car and may not change lanes. If both the lead car's and the following car's velocities, $\mathrm{v}_{1}(\mathrm{t})$ and $\mathrm{v}_{2}(\mathrm{t})$, respectively, were known functions of time, the synthesis of the longitudinal control system would be a rather straightforward problem. Naturally, such is not the case. The velocity $v_{1}(t)$ may be one of a multitude of forms. The problem is then to obtain the best response of the following car to all forms of $v_{1}(t)$.

The simplest type of control would be one that would maintain a constant headway. This type of control is basically unstable and cannot be maintained. One may then try to maintain the relative velocity, $\mathrm{v}(\mathrm{t})=\mathrm{v}_{1}(\mathrm{t})-\mathrm{v}_{2}(\mathrm{t})$, equal to zero. This is now stable, but provides no consistent headway. Steady-state headway is now a function of the initial headway and initial relative velocity when the following car comes under the influence of the lead car. One may now combine the two previous forms of control, using relative velocity to stabilize the constant headway control systems. This control is characterized by large overshoots in headway in correcting for sudden changes in $v_{1}(t)$. If any car in a line of cars is disturbed from a constant velocity, the resulting disturbance in headway will build up with distance behind the originally disturbed car until a collision occurs in the line of cars. This is referred to as asymptotic instability.

The shortcomings of straightforward control techniques caused attention to be turned to the human driver. A mathematical relation which describes the motion of his car in response to that of a lead car is termed a driving criterion. A number of these criteria have been proposed as a result of measurements of traffic flow, and are generally of functional form.

$$
\begin{equation*}
\mathrm{a}_{2}(\mathrm{t})=\mathrm{f}\left[\mathrm{v}(\mathrm{t}), \mathrm{h}(\mathrm{t}), \mathrm{v}_{2}(\mathrm{t})\right] \tag{1}
\end{equation*}
$$

The functions in general are continuous functions of time. Here, $a_{2}(t)$ is acceleration of the driver's car, and $h(t)$ is the time integral of the relative velocity equal to the distance between the cars. A number of these relations have been examined (1), and allof them have been found inadequate to describe what the driver is actually doing. Incidentally, because the criteria variables are those of absolute and relative motions of the cars, the response of the controlled car will be the same whether it is achieved by a human driver or by a fully automatic acceleration control system.

The inadequacy of the continuous criteria functions led to a more detailed examination of existing human driver characteristics obtained from an analog computer simulator in which the driver was following a lead car of constant velocity. This is referred to as the small signal case, because only small perturbations from zero relative velocity and from the desired constant headway are encountered. Test data indicated that a driver changes his acceleration according to a change in sense (from positive to negative or vice versa) of the relative velocity. However, the form of the acceleration (rather constant discrete levels) was independent of the form of the $v(t)$ function

[^0](continuously varying). This led Barbosa to propose the decision point model of the human driver. This model as developed by Todosiev helps to explain several characteristics of the human driver in the car-following situation. Mainly, though, it suggests that the driver instead of continuously tracking a continuous time variable, actually selects a constant acceleration and holds it until the variable exceeds some arbitrary threshold, at which time he changes to another level of acceleration and holds that, etc.

The development of the automatic longitudinal control technique has paralleled this break from continuous control. The results predicted by preliminary anaysis of these systems indicate a significantly improved performance by the threshold system, as shown later.

## THRESHOLD MODEL

In studies of human driver characteristics on a driving simulator, Barbosa (1) observed typical phase plane trajectories (plots of relative velocity as a function of headway) of the human driving a car that is overtaking a lead car in a two-car situation of the form shown in Figure 1. These trajectories are typified by two basically different regions (́or modes) of operation. In the first region, for $v=v_{1}-v_{2} \ll 0$ and the leaud car velocity $\mathrm{v}_{1}$ equal to a constant, the driver is found to maintain velocity $\mathrm{v}_{2}$ of the trailing car equal to a constant until the headway diminishes to a value below a certain reference level, or safety threshold. At this point the driver takes action by decelerating, thus reducing the relative velocity v to the vicinity of zero. As might be anticipated, with increasing values of initial relative velocity, the values of the reference level of headway at which the driver initiates action become greater. In the second region, for $\mathrm{v}_{2} \cong \mathrm{v}_{1}$ and $\mathrm{v}_{1}=$ constant, it is found that the headway oscillates in a semirandom fashion about some mean value of steady-state headway. This latter region is presently being investigated on a statistical basis by Todosiev (2). The model of human driver performance in this region has been termed variously as the "decision point

model" or, more recently, as the "action point model." The statistical studies to date have disclosed that the periodicity and amplitude of oscillations are semi-random. As of this time, a deterministic mathematical expression of human driver performance has not yet been achieved. However, data of these statistical tests suggest that the human driver is, on the average, performing as a threshold device.

An automatic control system, based on the decision point model, that has been termed a multi-mode relay control system is presently under study.

The principle of this system was derived from human driver performance in the following manner. The human driver apparently observes a changing condition of $v$ and $h$ as he overtakes a lead car. When the danger level becomes too high, he decelerates. In the synthesis of the automatic system the supposition is made that the "danger level" is directly proportional to the magnitude of a constant deceleration needed to bring the car to the same velocity as that of the lead car at a fixed distance behind the lead car. For a given rate of closure, the danger level increases as the headway decreases, because in stopping within smaller distances higher decelerations are needed. To be quite specific, it is assumed both that the driver always wants to use the same constant deceleration $A$ and that he desires that the minimum headway of the maneuver be one that he would maintain in the steady state. This minimum headway is denoted K for now. When the headway decreases below that for which the deceleration A will alleviate the condition (return v to zero) with a minimum headway K, the situation becomes too dangerous, and the driver will decelerate at A. An automatic system can be made to do this.

Assuming that the relative velocity and headway can be measured by the equipment in the following car, this same equipment can calculate the headway H needed to reduce v to zero with final headway K .

$$
\begin{equation*}
\mathrm{H}=\mathrm{g}(\mathrm{v})=\mathrm{K}+\frac{\mathrm{v}^{2}}{2 \mathrm{~A}} \tag{2}
\end{equation*}
$$

in which
$\mathrm{K}=\mathrm{h}_{\mathrm{SS}}=$ steady-state headway;
$\mathrm{v}=$ relative velocity; and
$\mathrm{A}=$ acceleration constant (or level).
As the measured headway $h$ becomes less than $H$, a relay is switched which activates a brake actuator to give a deceleration $A$.

The preceding concepts are shown in Figure 2 for the case of one switching function (or level of danger) with $A=A_{1} g$. From this figure the headway $h$ diminishes at constant relative velocity until the headway error signal $\epsilon_{h}$ becomes slightly negative. At this instant, the trailing car decelerates with a constant deceleration of $\mathrm{A}_{1} \mathrm{~g}$ along the $\mathrm{A}_{1} \mathrm{~g}$ relay switching function (path $A$ ) until $v$ becomes zero at $K=h_{S S}$. However, it is possible because of physical circumstances that the trailing car might not decelerate at exactly $\mathrm{A}_{1} \mathrm{~g}$. If the actual value of deceleration were slightly greater (such as $\mathrm{A}_{2} \mathrm{~g}$ ), the actual trailing car trajectory would be along path $B$, or if slightly less (such as $\mathrm{A}_{3} \mathrm{~g}$ ), the actualtrajectory would be along path C.

This latter situation suggests then that a practical implementation of this system should have at least two or more different relay switching functions (or levels of danger). The lowest level of danger is represented by the smallest value of constant deceleration, whereas the greatest level of danger necessary to avoid a rearend collision is represented by the largest value of constant deceleration. This



Figure 3. Case of two relay switching functions, $A=A_{1} g$, and $A=A_{2} g\left(A_{2} \cong 5 A_{1}\right)$.
situation is shown in Figure 3 for the case of two relay switching functions with decelerations of $A_{1} g$ and $A_{2} g$. If in the $A_{1} g$ mode, the actual deceleration is less than $\mathrm{A}_{1} \mathrm{~g}$, the actual trajectory taken will be along path B until intersection occurs with $\mathrm{A}_{2} \mathrm{~g}$ level of greatest danger. When this occurs, the deceleration of the trailing car steps to $\mathrm{A}_{2}$ (or approximately thereof) along path C, thus averting a potential collision course along path $B$ in the absence of the $\mathrm{A}_{2} \mathrm{~g}$ switching function.

A preliminary analysis of a multi-mode control system with three levels of deceleration and one level of acceleration has been performed. The three levels of constant deceleration chosen were 0.02 g (neutral gear), 0.1 g (brake actuation), and 0.5 g (brake actuation), and the one level of constant acceleration was set at 0.1 g . These values are compatible with the vast majority of motor car capabilities under normal circumstances. A block diagram of this control system, along with means of calculating the relay switching functions from $v$ and $h$, is shown in Figure 4. Some of the results of this analysis are shown in Figures 5 and 6. From Figure 5, it is found that the multimode control system is capable of averting a rear-end collision starting from the largesignal (or transient) situation; in addition, a stable limit cycle is established in the vicinity of zero relative velocity. The latter is accomplished by having the control system drop out of the deceleration mode and into the 0.1 g constant acceleration mode at a threshold relative velocity of $v=+2.5 \mathrm{mph}$. The maximum headway amplitude of the stable limit cycle was found to be about 15 ft , with a period of oscillation slightly less than 15 sec .

In the region of small values of relative velocity v and headway close to the desired steady-state value, it may be desirable to switch into a linear velocity tracking mode rather than go into the nonlinear stable limit cycle. Large disturbances outside the bounds of this linear region (of the phase plane) would then return the system to its nonlinear threshold modes. The ability of this system to handle a large disturbance from the steady state is shown in Figure 6. In this instance, it is assumed that, while in the stable limit cycle ( $\mathrm{v}_{1}=$ constant), the lead car suddenly undergoes a large deceleration of 0.5 g . When this occurs, the trailing car is thrown out of the limit cycle, and its resultant trajectory is as shown in Figure 6. Along path A, the sense of the relative acceleration a is such that the trailing car is accelerating toward the lead car at 0.48 g . The same is true along path B except the acceleration is equal to 0.4 g . Along path C , the relative acceleration is zero $\left[a=a_{1}-a_{2}=-0.5-(-0.5)=0\right]$ and the relative velocity $v$ is constant. At the end of path $C$, the lead car velocity $v_{1}$ finally becomes zero, which means that the trailing car decelerates toward zero relative velocity at a safe value of headway. The headway amplitude required for these maneuvers was found to be about seven car lengths (approximately 125 ft ).

An essential consideration in performing longitudinal control in the two-car situation is the accuracy and time lag involved in processing relative velocity and headway information to the trailing car. This matter is of importance whether the headway error signal $\epsilon_{h}$ be used to trigger a display presented to the human driver, or whether $\epsilon_{h}$ be used to actuate an automatic control system. The detection and subsequent calculation of $\mathbf{v}$ and $h$ as presently conceived is based on the concept of propagating zones of influence behind the lead car. If the trailing car is at a headway less than some critical value, then the trailing car will be within the influence of the lead car. As a result, a sequence of voltage pulses of fixed amplitude and varying time base are transmitted to the trailing car by means of a communication link. From these voltage pulses, the relative velocity and headway are calculated in the trailing car. Figure 7 shows that time $t_{b}$ at which the zones are reset are inversely proportional to the lead car velocity $\mathrm{v}_{1}$, whereas the times $\mathrm{T}_{\mathrm{o}}$ are directly proportional to headway h .


Figure 4. Multi-mode relay control system.

One means of performing the necessary calculations which has been studied in some detail is shown by a simplified block diagram (Fig. 8). This computing system, although comprised of analog and digital computing components, is basically analog in character. On the basis of the preliminary analysis, the steady-state response ( $\mathrm{v}_{1}=$ constant, $\mathrm{h}=\mathrm{h}_{\mathrm{SS}}=$ constant) is good; that is, small amplitude errors and little time lag. However, for the large signal transient cases of either (a) $\mathrm{v}_{2}=$ constant, and $\mathrm{v}_{1}$ suddenly changes rapidly, or (b) $\mathrm{v}_{1}=$ constant, and trailing car just enters zone of influence of lead car, the amplitude errors and time lags become rather large.


Figure 5. Phase-plane trajectories of multi-mode control system with three levels of deceleration and one level of acceleration.


Figure 6. Phase-plane trajectories of a large disturbance from steady state.


Figure 7. Signals processed to trailing vehicle.


An example of the first situation is shown in Figure 9, in which it is assumed that before $t=0$ the lead car velocity $\mathrm{v}_{1}$ was constant at 100 ft per sec. At $\mathrm{t}=0$, the lead car is subjected to a constant deceleration of 0.4 g for a period of about 5 sec . The actual instantaneous value of $\mathrm{v}_{1}$ and the value of $\mathrm{v}_{1}$ as calculated in the trailing car are also shown in the figure.

## COMPARISON OF THRESHOLD AND CONVENTIONAL LONGITUDINAL CONTROL SYSTEMS

The simplest automatic longitudinal control system examined is one in which zones of influence are set up in the highway as in the threshold system described in the preceding section. However, the signal used by the controlled car is simply the "on-off" signal received from passing over actuated and nonactuated highway blocks. The criteria differential equation of control (2) shows the influence of the lead car on the following car:

$$
\begin{equation*}
\tau_{\mathrm{a}} \mathrm{a}_{2}(\mathrm{t})=-\mathrm{v}_{2}(\mathrm{t})+\mathrm{v}_{2}\left(1-\left[1-\mathrm{v}_{1}(\mathrm{t}) \mathrm{h}(\mathrm{t})\right]\right) \tag{3}
\end{equation*}
$$

in which
$\tau_{\mathrm{a}}=$ effective time constant of controlled car; and
$\mathrm{V}_{2}=$ desired steady-state velocity in absence of lead car.
This equation means that as the lead car velocity and/or the headway is reduced, the following car's velocity (taking into account its inertia and friction) is reduced from the steady-state, open-road velocity $\mathrm{V}_{2}$.

Although the system is unstable so that the controlled car may not remain in the influence of the lead car in the steady state. Figure 10 shows the collision avoidance curve for the case of the controlled car (originally at 60 mph ) overtaking a lead car at 30 mph . Figure 1 shows that the constant deceleration curve avoids the collision better at a lower level of deceleration than does the control system. The reason for this is that the control system at first decelerates too little, and later in the maneuver must make up for it with increased deceleration.

The performance for the same case with the threshold system is shown in Figure 11. It is adequate to avoid collisions with stability in the steady state. The figure actually


Figure 9. Actual lead vehicle velocity and lead vehicle velocity as calculated in trailing vehicle.


Figure 10. Simple automatic system performance.


Figure 11. Performance of threshold system in avoiding rear-end collision.
shows a worse case, where the controlled automobile's brakes are not producing the required nominal 0.1 g of deceleration. Because of this, the car undergoes 0.5 g of deceleration for a short period. If the car's brakes were properly adjusted to give at least the nominal 0.1 g , the collision would be avoided with a final headway of 125 ft and the deceleration would never exceed 0.1 g . This is similar to the constant deceleration cases in Figure 10.

The analog computing system of Figure 8, although possessing large time lags in the transient situations, probably performs as well as the human does under similar circumstances. However, these large time lags are of such a nature that the complete capabilities and versatility of the automatic control system could not be realized in surpassing the performance of the human driver.

Moreover; the actual circuits that would be used to physically implement the system of Figure 8 are somewhat complex, and from the standpoint of reliability and cost, it is desirable to reduce the amount of equipment required in the car to a minimum. To these ends, a system for calculating $v$ and $h$ has been proposed recently in the form of a sampled-data system. Analysis of this latter system completed thus far indicates that the accuracy and time response are quite good for both the steady-state and the large signal transient regions.

## CONCLUSIONS

Significant improvements were made in both the human driver description and the automatic acceleration control system when the transfer was made from continuous function driving criteria to threshold criteria. Also, study of the human driver has pointed the way to a feasible automatic system. It is hoped that as the decision point model of the human driver becomes more accurate, it will also describe the driver in the transient state, such as rapidly overtaking a lead car. This description combined with the results of a study of the best possible motion of the controlled car in the transient state should yield the optimum driving system, characterized by a reduction in rear-end collisions and higher safe traffic density.

## REFERENCES

1. Barbosa, L. de C., "Studies of Traffic Flow Models." Master's thesis, Ohio State University (1961).
2. Todosiev, E., Ohio State University Engineering Experiment Station Report EES 202-1 (July 1962).

[^0]:    Paper sponsored by Committee on Electronic Research in the Highway Field.

