Benefit-Cost Ratios: A Word of Caution

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•IN THE EVALUATION of capital expenditure proposals for public projects, especially in highway facility planning, a method of comparing design alternatives called the benefit-cost ratio is widely used. The author offers a word of caution in the application of this method and makes a suggestion for those who compare the economy of design alternatives. The study of capital expenditure evaluation is generally referred to as engineering economy (1 through 10).

The benefit-cost ratio is a method of comparing economic alternatives. It is used to determine (a) which alternative, if any, is worthwhile, and (b) which alternative offers the greatest economy. Specifically, it is the ratio of annual benefits (such as reduced cost to users of the facility) to annual costs (such as maintenance, operation, and the average annual share of capital costs). This method is used in comparing alternatives for many types of public projects (e.g., water treatment services, recreational facilities, flood control, and public parks); the examples which follow, however, will be in the language of highway alternatives. The following notations are used:

S = investment costs on an annual basis;
M = maintenance costs on an annual basis;
$S + M$ = highway costs on an annual basis;
R = road user costs on an annual basis;
I = investment;
n = estimated life of facility, in years; and
i = interest rate.

The subscripts 0 and 1 identify data pertaining to the original and proposed road facilities, respectively.

CRITERIA FOR ECONOMY

Because the benefit-cost ratio is the ratio of annual benefits to annual costs (1, p. 27), the ratio for the notation given can be shown as

$$\text{Benefit-cost} = \frac{R_0 - R_1}{S_1 + M_1 - S_0 - M_0} \tag{1}$$

If the resulting ratio is higher than the prescribed minimum ratio the proposal "passes" the benefit-cost ratio test.

Rate of return can be computed by simply equating annual savings with annual costs of obtaining such savings:

$$R_0 - R_1 + M_0 - M_1 = (I_1 - I_0) \text{ (capital recovery factor in which n is given, and i is unknown)} \tag{2}$$

Eq. 2 is then solved for i. If the result is higher than the prescribed minimum rate of return, the proposal "passes" the rate of return test.

Example 1

Five alternative proposals for a highway facility are being considered. Each requires an investment of \$20,000 and each has a life of 10 years. A 5 percent interest rate is

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used in computing capital recovery. Other estimates pertaining to the alternatives are given in Table 1. For each alternative in Table 1, find (a) the benefit-cost ratio, and (b) the rate of return on the investment.

Solution of Example 1

$$\text{Benefit-cost ratio} = \frac{R_0 - R_1}{S_1 + M_1 - S_0 - M_0} = \frac{R_0 - R_1}{(S_1 - S_0) - (M_0 - M_1)} \quad (3)$$

Because

$S_1 - S_0 = (I_1 - I_0)$ (capital recovery factor in which i = 5%, n = 10)
= (20,000) (0.1295)
= $2,590

Then for alternative A,

$$\text{Benefit-cost ratio} = \frac{6,000}{2,590 + 410} = 2.0$$

and similarly for alternatives B, C, D, and E. Rate of return can be computed by

$R_0 - R_1 + M_0 - M_1 = (I_1 - I_0)$ (capital recovery factor in which i = ?, n = 10)

for alternative A (using CRF for capital recovery factor):

6,000 - 410 = $20,000 (CRF - i - 10)
(CRF - i - 10) = 0.2795

by use of tables (4, pp. 538-557) and interpolation:

$i \simeq 25$ percent

and similarly for alternatives B, C, D, and E. Results of the calculations are given in Table 2.

Example 2

Six alternative proposals for a highway facility are being considered. An interest rate of 5 percent is to be used. Estimates pertaining to competing alternatives are given in Table 3.

For each alternative in the table, find (a) the benefit-cost ratio, (b) the rate of return on the investment, and (c) the savings-cost ratio, $(R_0 - R_1 + M_0 - M_1) / (S_1 - S_0)$.

Solution of Example 2

Results in Table 4 are obtained by calculations similar to those of example 1.

TABLE 1

ESTIMATES PERTAINING TO CERTAIN HIGHWAY ALTERNATIVES

Alternative	Decrease in Road-User Costs, R_0-R_1 ($)	Decrease in Maintenance Costs, M_0-M_1 ($)	Gross Savings R_0-R_1 + M_0-M_1 ($)
A	6,000	-410	5,590
B	3,000	1,090	4,090
C	1,000	2,090	3,090
D	200	2,490	2,690
E	-1,000	3,090	2,090

TABLE 2

COMPARATIVE ECONOMY OF CERTAIN HIGHWAY ALTERNATIVES BY TWO METHODS

Alternative	Decrease in Road-User Costs, R_0-R_1 ($)	Decrease in Maintenance Costs, M_0 - M_1 ($)	Gross Savings R_0-R_1 + M_0-M_1 ($)	Benefit-Cost Ratio[1]	Rate of Return (%)[2]
A	6,000	-410	5,590	2.0	25
B	3,000	1,090	4,090	2.0	16
C	1,000	2,090	3,090	2.0	9
D	200	2,490	2,690	2.0	6
E	-1,000	3,090	2,090	2.0[3]	1

[1]Rounded to nearest tenth.
[2]Rounded to nearest whole percent.
[3]This is a rather facetious alternative and ratio; although negative numerator and denominator cancel each other, ratio only indicates that every dollar decrease in cost to agency that provides highway facility is accompanied by two-dollar increase in cost to road user.

TABLE 3

ESTIMATES PERTAINING TO CERTAIN HIGHWAY ALTERNATIVES

Alternative	Decrease in Road-User Costs, R_0 - R_1 ($)	Decrease in Maintenance Costs M_0 - M_1 ($)	Gross Savings R_0-R_1 + M_0-M_1 ($)	Investment I_1 - I_0 ($)	Estimated Life (yr)
F	300	1,000	1,300	8,000	10
G	628	1,000	1,628	10,000	10
H	15,275	1,000	16,275	100,000	10
I	5,501	-4,000	1,501	10,000	50
J	4,188	-2,195	1,993	10,000	10
K	4,038	-1,055	2,983	10,000	5

Analysis

The preceding calculations show that for any given benefit-cost ratio the rate of return on investment is not fixed. In example 1, five alternatives having benefit-cost ratio of 2.0 exhibit returns that vary from 1 to 25 percent. The decision indicated by the benefit-cost ratio method does not agree with the decision indicated by rate of return method.

If public funds should be allocated to their various purposes so as to maximize the long-run gains (such as reduced cost to users of the facility and decreased maintenance cost to operators of the facility) of such investments, it follows that a criterion that satisfactorily measures the desirability of alternatives is mandatory.

In example 1, the benefit-cost ratio fails to reveal the investment alternative that maximizes the return on public funds invested. In example 2, the benefit-cost ratio makes three equivalent alternatives (F, G, and H) appear to be not equivalent. Worse yet, the alternatives that would maximize the return on public funds (alternatives I, J, or K) appear by the benefit-cost ratio method to be least desirable of the six alternatives. The examples show three crucial defects in the benefit-cost ratio method:

1. It sometimes fails to discriminate so as to point out the alternative that maximizes the return on public funds.
2. It sometimes discriminates among alternatives that provide equivalent returns on public funds.
3. It sometimes yields results that point to the selection of alternatives that do not maximize the return on public funds.

These defects in the benefit-cost ratio method are not corrected by the savings-cost ratio method. As can be seen in the comparison of alternatives I, J, and K, the savings-cost ratio method is responsive to differences in the lives of alternatives; it will as a matter of fact, generally bias the results to favor the longer-lived alternative. Interestingly enough, the savings-cost ratio method is similar to an inverted payoff period; still more interesting is that the bias introduced is just the opposite. Payoff period as a criterion tends to favor short-lived alternatives; savings-cost ratio as a criterion tends to favor long-lived alternatives.

If the benefit-cost ratio method fails to discriminate properly in the instances shown, then it can hardly be expected to determine satisfactorily the sequence of investment proposals that should be followed by a public body.

The preceding examples suggest that the rate-of-return method should be used at least as a check in the evaluation of proposed capital expenditures for public facilities. When there is more than a single capital expenditure and several life expectancies are involved, or when a deferred expenditure is involved, the rate of return is computed by successive trial values. Although the rate-of-return method can be more complex computationally, proper evaluation of the usually large capital expenditures for proposed public facilities compensates many times over for the extra effort. Example 3 demonstrates that even the more complex problems require only added computational time.

Example 3

It has been proposed that a certain highway be replaced by a relocated route. Estimates of lives and costs of the relocated route are 20 years and $100,000 for the paving, 40 years and $200,000 for the grading and drainage, 60 years and $50,000 for the right-of-way.

It is expected that the proposed route will require $40,000 every ten years for major roadway rehabilitation. Road user costs are expected to decrease $92,000 per year with the proposed route, whereas maintenance costs are expected to increase by $20,000 per year. Find the rate of return on the investment.

Solution of Example 3

Equivalent annual costs of capital expenditures are obtained by multiplying each expenditure by the appropriate capital recovery factor (CRF). For recurring deferred

TABLE 4

COMPARATIVE ECONOMY OF CERTAIN HIGHWAY ALTERNATIVES BY THREE METHODS

Alternative	Decrease in Road-User Costs, $R_0 - R_1$ ($)	Decrease in Maintenance Costs, $M_0 - M_1$ ($)	Gross Savings, $R_0-R_1 + M_0-M_1$ ($)	Investment, $I_1 - I_0$ ($)	Estimated Life (yr)	Benefit-Cost Ratio[1]	Rate of Return (%)	Savings Cost Ratio[1]
F	300	1,000	1,300	8,000	10	8.3	10	1.4
G	628	1,000	1,628	10,000	10	2.1	10	1.3
H	15,275	1,000	16,275	100,000	10	1.3	10	1.3
I	5,501	-4,000	1,501	10,000	50	1.2	15	2.7
J	4,188	-2,195	1,993	10,000	10	1.2	15	1.5
K	4,038	-1,055	2,983	10,000	5	1.2	15	1.3

[1]Rounded to nearest tenth..
[2]Rounded to nearest whole percent.

expenditures, the equivalent annual cost is obtained by multiplying the deferred expenditure by the sinking fund factor (SFF). For nonrecurring deferred expenditures, the equivalent annual cost is obtained by multiplying the deferred expenditure by the present worth factor for a single sum and then by the capital recovery factor. In example 3, a solution is obtained as follows:

$$\text{Benefits} = \text{Cost of obtaining benefits} \tag{4}$$

$$\begin{aligned}\$92,000 - \$20,000 = {} & \$100,000\ (\text{CRF} - i - 20) + \$200,000\ (\text{CRF} - i - 40) + \\ & \$\ 50,000\ (\text{CRF} - i - 60) + \$\ 40,000\ (\text{SFF} - i - 10)\end{aligned}$$

and the solution is obtained by successive trials:

At i = 15 percent,

$$\begin{aligned}\$72,000 & \neq \$15,976 + \$30,112 + \$7,500 + \$1,970 \\ & \neq \$55,568\end{aligned}$$

At i = 20 percent,

$$\begin{aligned}\$72,000 & = \$20,536 + \$40,028 + \$10,000 + \$1,541 \\ & = \$72,105\end{aligned}$$

Therefore, the rate of return is about 20 percent.

SUMMARY

The suggested use of the rate of return method for public projects (as a check or as an independent method) is not as drastic as it appears. The comprehensive data prepared by AASHO (1) on road user costs would be used exactly as before; the changes in maintenance costs and proposed expenditures and lives would be estimated as at present. The benefit-to-cost concept need not be lost; finding of the rate of return requires computation of the ratio of gross annual savings to users and operators of the facility (benefits) to investment (cost). The author is not alone, nor even first (7, p. 8) to suggest that AASHO consider changing to the rate-of-return method of evaluating economic alternatives. The danger involved in continued use of only the benefit-cost ratio method is in the possibility that some high-yield projects will be delayed or denied because funds have been exhausted in some low-yield projects.

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Discussion

GERALD A. FLEISCHER, Assistant Professor of Industrial Engineering, University of Michigan*—Professor Smith offers two examples of application of the benefit-cost ratio method which, he claims, show the three crucial defects of (a) sometimes failing to discriminate so as to point out the alternative which maximizes the return on public funds; (b) sometimes discriminating among alternatives which provide equivalent returns on public funds; and (c) sometimes yielding results which point to the selection of alternatives which do not maximize the return on public funds.

In view of these criticisms, Professor Smith suggests that "the rate of return method should be used at least as a check in the evaluation of proposed expenditures for public facilities," and offers a third example demonstrating the use of this preferred method.

The writer objects to the conclusion that the benefit-cost method is conceptually invalid. Although it is agreed that the rate of return method is preferable, this is due to a number of reasons other than inherent verity.

The purpose, then, of this discussion is to demonstrate that the benefit-cost ratio method, when properly applied, is a valid technique for choosing among alternatives competing for limited resources. This is done by using the same examples offered by the author in his attempt to demonstrate the opposite. That is, it will be shown that the rate of return and benefit-cost ratio methods are equivalent.

Notation

To maintain consistency with the basic paper, the author's notation has been retained with only minor changes.

To convert an initial investment I to an equivalent uniform series S, it is necessary to use the appropriate capital recovery factor for a given interest rate i and a given number of interest periods n. This factor is indicated by (crf - i% - n).

Basic Equations

The benefit-cost ratio is commonly defined as the ratio of annual benefits to annual costs, although there exists some question as to which consequences of an investment are benefits and which are costs. Clearly, a benefit is a negative cost, and vice versa. Table 5 gives the various combinations of notation elements.

TABLE 5

COMBINATIONS IN BENEFIT-COST ANALYSES

Costs	Symbol			
	Old	New	Benefits	Costs
Road user	R_0	R_1	$R_0 - R_1$	$R_1 - R_0$
Maintenance	M_0	M_1	$M_0 - M_1$	$M_1 - M_0$
Investment	S_0	S_1	$S_0 - S_1$	$S_1 - S_0$

It is generally agreed that effects on road user costs should be included in the numerator of the benefit-cost ratio, and it is likewise agreed that changes in investment costs should be shown in the denominator. However, there are several ways of handling the increase (or decrease) in maintenance costs, as follows:

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$$B/C = \frac{(R_0 - R_1)}{(S_1 - S_0) + (M_1 - M_0)} \tag{1}$$

$$B/C = \frac{(R_0 - R_1)}{(S_1 - S_0) - (M_0 - M_1)} \tag{3}$$

$$B/C = \frac{(R_0 - R_1) + (M_0 - M_1)}{(S_1 - S_0)} \tag{5}$$

The minor differences between Eqs. 1 and 3 are obvious; they are essentially the same. Eq. 5 differs from the other two in defining a reduction in maintenance costs as a benefit rather than a negative cost. (The choice of numerator or denominator for maintenance costs is irrelevant, although this question has been the source of considerable controversy in recent years. The acceptance criterion is whether or not the benefit-cost ratio exceeds unity, thus the absolute value of the ratio is unimportant. When the same number is added or subtracted to both the numerator and denominator of a fraction, the fraction cannot change from greater than one to less than one, or vice versa. One must only insure that the definition of the ratio is applied consistently in any given problem.)

The author defines his benefit-cost ratio as including the maintenance costs in the denominator (as in Eqs. 1 and 3), and uses the term "savings-cost ratio" when shifting the effect of changes in maintenance costs to the numerator (as in Eq. 5).

To find the unknown rate of return in certain special cases (such as Example 1), the following equation may be used with n given:

$$(\text{crf} - i\% - n) = \frac{(R_0 - R_1) + (M_0 - M_1)}{(I_1 - I_0)} \tag{6}$$

Given the capital recovery factor for a certain n, it is only necessary to consult the appropriate tables for various values of i and interpolate if necessary. (Eq. 6 may be derived quite easily by using Equation 1, 3 or 5, and setting the benefit-cost ratio equal to unity.)

Example 1

Benefit-Cost Ratio Method.—The basic data provided by the author are given in Cols. 1 through 4, Table 6. It is assumed that each alternative has a life of 10 years and that an interest rate of 5 percent per annum is used.

Two observations should be made about the benefit-cost ratios shown in Col. 7. The first, pointed out by Professor Smith, is that the benefit-cost ratio shown for alternative E is spurious. Col. 2 indicates that there is an increase in road user costs accompanied by a smaller decrease in costs to the highway agency, as shown in Col. 6. Or, as the author puts it, "...although the negative numerator and denominator cancel each other, the ratio indicates only that every dollar decrease in cost to the agency that provides the highway facility is accompanied by a two-dollar increase in cost to the road user."

The second point is that only an analysis of incremental benefit-cost ratios will yield the proper solution. Thus, the numbers in Col. 7, Table 6, have no other value than to indicate that alternatives A through D are each acceptable. The incremental analysis is still needed to determine which one should be chosen.

Unfortunately, the author ranked his alternatives in descending, rather than ascending, order of costs. Thus, to simplify the arithmetic, the writer has started with alternative E and worked backward to alternative A. (This procedure simply eases computational effort; the final result is the same regardless of sequence.)

First, alternative E is compared with the possibility of doing nothing; i.e., employing the resources elsewhere rather than investing in the project. Inasmuch as E is not economically feasible, the next move is to D. The result is that annual benefits will increase by \$200 while costs will increase by only \$100. Thus the incremental benefit-cost ratio ($\Delta B/C$) is 2.0.

TABLE 6

EXAMPLE 1:
BENEFIT-COST RATIO SOLUTION

Alt. (1)	$(R_0 - R_1)$ (\$) (2)	$(M_0 - M_1)$ (\$) (3)	$(I_1 - I_0)$ (\$) (4)	$(S_1 - S_0)$[a] (\$) (5)	Col. 5 - 3 (\$) (6)[a]	B/C[b] (7)	B - C[c] (\$) (8)	Compare[d] (9)	ΔB (10)	ΔC (11)	ΔB/C (12)
A	6,000	-410	20,000	2,590	3,000	2.0	3,000	|A/B|	3,000	1,500	2.0
B	3,000	1,090	20,000	2,590	1,500	2.0	1,500	|B/C|	2,000	1,000	2.0
C	1,000	2,090	20,000	2,590	500	2.0	500	|C/D|	800	400	2.0
D	200	2,490	20,000	2,590	100	2.0	100	|D/ϕ|	200	100	2.0
E	-1,000	3,090	20,000	2,590	- 500	2.0	- 500	|E/ϕ|	-	-	-

[a]Col. 4 × (crf - 5% - 10) = 0.1295 Col. 4.
[b]By Eq. 3.
[c]$B - C = (R_0 - R_1) + (M_0 - M_1) - (S_1 - S_0)$.
[d]ϕ denotes the alternative "Do nothing, employ resources elsewhere."

The next question is whether or not alternative C is economically superior to D. The difference between these two is that an \$800 increase in benefits will be accompanied by a \$400 increase in costs. Because the incremental benefit-cost ratio exceeds unity (i.e., 2.0), C is accepted and D is disregarded.

Continuing this process results in the selection of alternative A inasmuch as B is superior to C and A is superior to B. This is the correct answer, as is shown in Col. 8, where the "excess of benefits over costs" values have been computed in a straightforward manner. (The reader may wonder why—if a simple evaluative method such as that described up to Col. 8 is available—there is discussion of a method which requires a rather cumbersome iterative technique. Why indeed? But rather than digress at this point to discuss the philosophy of choice of method, this question is left with the reader. The purpose here is only to demonstrate that the benefit-cost ratio method is valid, regardless of its computational intricacies.)

Rate of Return Method.—Table 7 is a summary of calculations necessary to select the most economic alternative by use of the rate of return method. As before, the basic data for each of the five alternatives are given in Cols. 1 through 4.

Col. 5 is simply a calculation of the numerators appropriate to Eq. 6. The denominator values are given in Col. 4, and Col. 6 represents the capital recovery factors for unknown interest rates i and n = 10 years. The actual (solving) interest rates have not been shown in Col. 7 because they are irrelevant; it is only necessary to know whether or not i is greater than the minimum attractive rate of return, 5 percent. That is, will the highway agency be able to invest its dollars in one of these alternatives at a rate of return greater than 5 percent? Inasmuch as the capital recovery factor for i = 5% and n = 10 is 0.1295, it is only necessary that the values in Col. 6 be greater than this number. Only alternative E fails this test and hence should be omitted from further consideration.

As in the case of the benefit-cost ratio method, it is now necessary to look at the prospective rates of return yielded by increments of investment. This analysis represented by Cols. 8 through 12, follows a pattern similar to the incremental analysis

TABLE 7

EXAMPLE 1: RATE OF RETURN SOLUTION

Alt. (1)	$(R_0 - R_1)$ (\$) (2)	$(M_0 - M_1)$ (\$) (3)	$(I_1 - I_0)$ (\$) (4)	Col. 2 + 3 (\$) (5)	(crf - i% - 10)[a] (6)	i% (7)	Compare (8)	Δ Num.[b] (\$) (9)	Δ Denom.[c] (\$) (10)	Δ CRF[d] (11)	Δ i% (12)
A	6,000	- 410	20,000	5,590	0.2795	> 5	|A/B|	1,500	0	∞	> 5
B	3,000	1,090	20,000	4,090	0.2045	> 5	|B/C|	1,000	0	∞	> 5
C	1,000	2,090	20,000	3,090	0.1545	> 5	|C/D|	400	0	∞	> 5
D	200	2,490	20,000	2,690	0.1345	> 5	|D/ϕ|	2,690	20,000	0.1345	> 5
E	-1,000	3,090	20,000	2,090	0.1045	< 5	|E/ϕ|	-	-	-	-

[a]$(\text{crf - i\% - 10}) = \dfrac{\text{Col. 2 Col. 3}}{\text{Col. 4}} = \dfrac{(R_0 - R_1) + (M_0 - M_1)}{(I_1 - I_0)}$.

[b]Δ Num. = increment in Col. 5 between alternatives.
[c]Δ Denom. = increment in Col. 4 between alternatives.

[d]$\Delta\,\text{CRF} = \dfrac{\Delta\ \text{Num.}}{\Delta\ \text{Denom.}} = \dfrac{\text{Col. 9}}{\text{Col. 10}}$.

employed in the benefit-cost ratio method. (Again, the writer has elected to start with alternative D and work backward to A in order to simplify calculations.) For example, the consequence of selecting alternative D rather than "doing nothing" is to increase initial investment by $20,000 in order to receive a benefit of $2,690 each year for 10 years. Because the computed capital recovery factor (0.1345) is greater than that represented by the minimum attractive rate of return (0.1295), alternative D is acceptable.

The incremental effect of selecting alternative C rather than D is now examined. Although there will be no increase in initial costs, annual benefits will increase by $400 each year for 10 years. The incremental rate of return is therefore infinite. Continuing this pair-wise process until the last alternative has been considered, it is found that alternative A is the most economically feasible. (The actual rate of return on total investment for alternative A, found by reference to compound interest tables, is approximately 25%.) Of course, this is the same solution obtained by both the "excess of benefits over costs" and the benefit-cost ratio methods.

Example 2

Input Data.—The second example deals with six alternatives for a proposed highway facility. Data concerning road user costs, maintenance costs, initial investments, and estimated lives are given in Table 8. An interest rate of 5 percent is used. It is demonstrated in the following that the identical, correct solution may be obtained by using both the benefit-cost ratio and rate of return methods, and that the so-called "savings-cost ratio method" also yields a valid solution.

Benefit-Cost Ratio Method.—The results of the application of the benefit-cost ratio method are given in Table 9. It should be noted that the B/C values in Col. 6 erroneously indicate that alternative F is superior to the others. It is erroneous because the incremental analysis must be completed before being able to determine the most economical alternative.

The procedure outlined in Table 9 is identical with that used in Example 1 with one exception. In the preceding example it was fairly obvious that calculations could be minimized by starting with alternative D and working backward to A. Here it is not so readily evident; therefore, one begins with F and works down to K. By following the arithmetic, the procedure should be clear.

Another similarity to the preceding example is the generation of specious benefit-cost ratios (see Col. 11 for alternatives I, J, and K). For example, the choice of I rather than H results in a reduction in benefits of $9,774, but the associated cost reduction is only $7,402. Although the negative signs algebraically cancel each other, the resulting benefit-cost ratio should clearly be negative. One must be wary of blindly following rules of algebra without reference to common sense.

The benefit-cost ratio method indicates that alternative H is best. Referring again to the results of the "excess of benefits over costs" method—here given in Col. 7—H is

TABLE 8

EXAMPLE 2: INPUT DATA

Alt. (1)	$(R_0 - R_1)$ ($) (2)	$(M_0 - M_1)$ ($) (3)	$(I_1 - I_0)$ ($) (4)	n (yr) (5)
F	300	1,000	8,000	10
G	628	1,000	10,000	10
H	15,275	1,000	100,000	10
I	5,501	-4,000	10,000	50
J	4,188	-2,195	10,000	10
K	4,038	-1,055	10,000	5

TABLE 9

EXAMPLE 2: BENEFIT-COST RATIO SOLUTION

Alt. (1)	Benefit[a] ($) (2)	(crf - 5% - n) (3)	$(S_1 - S_0)$[b] ($) (4)	Cost[c] ($) (5)	B/C[d] ($) (6)	B-C[e] ($) (7)	Compare[f] (8)	Δ B ($) (9)	Δ C ($) (10)	Δ B:C (11)
F	300	0.1295	1,036	36	8.3	264	|F/φ|	300	36	8.3
G	628	0.1295	1,295	295	2.1	333	|G/F|	328	259	1.3
H	15,275	0.1295	12,950	11,950	1.3	3,325	|H/G|	14,647	11,650	1.3
I	5,501	0.0548	548	4,548	1.2	953	|I/H|	-9,774	-7,402	1.3[g]
J	4,188	0.1295	1,295	3,490	1.2	698	|J/H|	-11,087	-8,460	1.3[g]
K	4,038	0.2310	2,310	3,365	1.2	673	↓K/H↓	-11,237	-8,585	1.3[g]

[a] $(R_0 - R_1)$ = Col. 2, Table 8.
[b] $(S_1 - S_0) = (I_1 - I_0)$ (crf - 5% - n).
[c] Col. 4 - Col. 3, Table 8.
[d] Col. 2/Col. 5.
[e] Col. 2 - Col. 5.
[f] φ denotes the alternative "Do nothing, employ resources elsewhere."
[g] Actually, < 1.

seen to be superior to the other alternatives. This was predictable, of course, because the methods are equivalent.

Savings-Cost Ratio Method.—The author makes a distinction between the benefit-cost ratio and savings-cost ratio methods, although the writer pointed out in an earlier section that the only difference between these two methods is the location of maintenance costs in either the numerator or the denominator of the ratio, and the two methods will lead to identical solutions.

The writer's results using the savings-cost ratio solution are given in Table 10. (Again, the specious ratios shown for alternatives I, J, and K in Col. 8 should be noted.) The method used is identical to that shown in Table 9, except that the absolute values of the ratios are slightly different due to the location of maintenance costs in the numerator rather than in the denominator. The incremental analysis—beginning with F and working down through K—indicates that alternative H is the most economical. This checks with the results of the preceding section.

It is notable in passing that the absolute values of the benefit-cost ratios (Col. 6, Table 9) are neither equal to, nor provide the same ranking as, the computed savings-cost ratios (Col. 4, Table 10). This is not surprising, as the location of the maintenance costs has been shifted. However, the comparison is irrelevant inasmuch as (a) absolute values have no meaning when selecting among alternatives, and (b) only the incremental

TABLE 10

EXAMPLE 2: SAVINGS-COST RATIO SOLUTION

Alt. (1)	Cost[a] ($) (2)	Savings[b] ($) (3)	S/C[c] (4)	Compare (5)	Δ S ($) (6)	Δ C ($) (7)	Δ S/C (8)
F	1,036	1,300	1.3	|F/φ|	1,300	1,036	1.3
G	1,295	1,628	1.3	|G/F|	328	259	1.3
H	12,950	16,275	1.3	|H/G|	14,647	11,655	1.3
I	548	1,501	2.7	|I/H|	-14,774	-12,402	1.2[d]
J	1,295	1,993	1.5	|J/H|	-14,282	-11,655	1.2[d]
K	2,310	2,983	1.3	↓K/H↓	-13,292	-10,640	1.2[d]

[a] Col. 4, Table 9.
[b] Col. 2, Table 8 + Col. 3, Table 8.
[c] Col. 3/Col. 2.
[d] Actually, < 1.

analysis will yield the proper solution. (The statement about absolute values must be qualified. One needs only to determine if the ratio is greater than or less than unity. How much greater or how much less is irrelevant, all other factors being considered.)

Rate of Return Method.—Most of the calculations necessary for the rate of return solution are given in Table 11. As in the preceding benefit-cost and savings-cost solutions, the incremental analysis begins with F and works down through K. However, due to the difference in service lives of some of the alternatives, the method of calculating the incremental rates of return differs slightly from the procedure used in the first example.

The incremental rates of return for the pair-wise comparisons of F with "doing nothing," G with F, and H with G have been determined as in Example 1. This is possible because alternatives F, G, and H have equal lives (10 years); thus there is only one capital recovery factor in the solution equation, and it may be determined directly. Alternative I, however, has a life of 50 years, hence the solution equation for the differences between I and H is 0 = [100,000 (crf - i% - 10) - 16,275] - [10,000 (crf - i% - 50) - 1,501]. The unknown interest rate i may be determined by testing the equation using various values of i until the equality is satisfied. But, since the interest here is only in determining if the solving value for i is greater or less than 5%, one can simply substitute the appropriate capital recovery factors for i = 5% in this equation. Thus, [100,000 (crf - 5% - 10) - 16,275] - [10,000 (crf - 5% - 50) - 1,501] = [100,000 (0.1295) - 16,275] - [10,000 (0.0548) - 1,501] = [12,950 - 16,275] - [548 - 1,501] = -3,325 + 953 = -2,372. Because this value is negative, the solving rate of return must be less than 5%. Thus, alternative I is economically inferior to H and may be disregarded.

Alternative J may be compared to H as before, because each has a service life of 10 years. However, the resulting capital recovery factor is shown in parentheses in Col. 10, Table 11 because it is somewhat misleading. A $90,000 reduction in initial cost results in a reduction of $14,282 in operating and maintenance savings each year for 10 years. Thus the computed capital recovery factor (0.1587) is applicable to choosing H rather than J. (That is, if H is chosen rather than J the initial cost will be increased by $90,000, but the annual operating and maintenance savings will be increased by $14,282.) Because the capital recovery factor for i = 5% and n = 10 is 0.1295, the rate of return for choosing H over J is greater than 5%. It follows that the rate of return for J over H is less than 5%.

Finally, alternative K must be compared with H, but the estimated service life for K is only 5 years. Thus i must be chosen so that the following equation is satisfied: 0 = [100,000 (crf - i% - 10) - 16,275] - [10,000 (crf - i% - 5) - 2,983]. Substituting i = 5% gives [100,000 (crf - 5% - 10) - 16,275] - [10,000 (crf - 5% - 5) - 2,983] = [100,000 (0.1295) - 16,275] - [10,000 (0.2310) - 2,983] = [12,950 - 16,275] - [2,310 - 2,983] = -3,325 + 673 = -2,652. Again, because the solution is a negative value, it is

TABLE 11

EXAMPLE 2: RATE OF RETURN SOLUTION

Alt. (1)	$(I_1 - I_0)$ ($) (2)	Ann. Sav.[a] ($) (3)	n (yr) (4)	CRF[b] (5)	i% (6)	Compare (7)	Δ Col. 2 ($) (8)	Δ Col. 3 ($) (9)	Δ CRF[c] (10)	Δ i% (11)
F	8,000	1,300	10	0.1628	10	|F/ϕ|	8,000	1,300	0.1625	10
G	10,000	1,628	10	0.1628	10	|G/F|	2,000	328	0.1640	10
H	100,000	16,275	10	0.1628	10	|H/G|	90,000	14,647	0.1627	10
I	10,000	1,501	50	0.1501	15	|I/H|	-d	-d	-d	<5
J	10,000	1,993	10	0.1993	15	|J/H|	-90,000	-14,282	(0.1587)	<5
K	10,000	2,983	5	0.2983	15	↓K/H↓	-d	-d	-d	<5

[a]Annual savings resulting from investment (Col. 3, Table 10).

[b]$CRF = (crf - i\% - n) = \frac{(R_0 - R_1) + (M_0 - M_1)}{(I_1 - I_0)}$.

[c]Δ CRF = Col. 9/Col. 8.

[d]See text for discussion of special form of analysis.

concluded that the true rate of return of the increment is less than 5%. Hence alternative H is superior to all others being considered in the problem. (This is the same solution, of course, which was obtained by the other methods.)

Example 3

Problem Statement. —The author presented a third example using only the rate of return method. He said: "Although the rate of return method can be more complex computationally, proper evaluation of the usually large capital expenditures for proposed public facilities compensates many times over for the extra effort. Example 3 demonstrates that even the more complex problems require only added computational time." It is demonstrated in the following that the benefit-cost ratio method will also lead to a "proper evaluation."

This problem deals with a proposed relocation of an existing highway. The basic data are as follows:

Item	First Cost ($)	Service Life (yr)
Right-of-way	50,000	60
Grading and drainage	200,000	40
Paving	100,000	20
Roadway rehabilitation	40,000	10

Further, adoption of the new location is expected to decrease road user costs by $92,000 per year and increase maintenance costs by $20,000 per year. Two alternatives are involved—do nothing or relocate the highway.

Rate of Return Method. —The rate of return on the proposed investment is that value of i which satisfies the equation: 50,000 (crf - i% - 60) + 200,000 (crf - i% - 40) + 100,000 (crf - i% - 20) + 40,000 (sff - i% - 10) - 92,000 + 20,000 = 0. Wherein (sff - i% - 10) is the mnemonic form of the "sinking fund factor" for i = 10% and n = 10.

One would normally begin a trial-and-error procedure until the appropriate i is found (in this case about 20%). This is verified as follows, using i = 20%: 50,000 (crf - 20% - 60) + 200,000 (crf - 20% - 40) + 100,000 (crf - 20% - 20) + 40,000 (sff - 20% - 10) - 92,000 + 20,000 = 50,000 (0.20001) + 200,000 (0.20014) + 100,000 (0.20536) + 40,000 (0.03852) - 92,000 + 20,000 = 105 (or almost 0).

The analysis is not complete, however. It is still not known whether or not the proposal should be accepted. To do so, the project rate of return must be compared with that available by investing elsewhere, usually stated as the "minimum attractive rate of return." In Examples 1 and 2 this value was given as 5%. Assuming that the same value applies to this problem, it is now possible to state that relocation of the highway is preferable to doing nothing; that is, the funds invested in the new facility will yield 20%, which is greater than the expected return from other potential (but unknown) investments.

Benefit-Cost Ratio Method. —Using an interest rate of 5%, the first step is to convert all consequences of the proposal to uniform annual series. The consequences are then assigned to "benefits" or "costs" and the benefit-cost ratio is computed. (The computations are fairly simple in this example because only two alternatives are being considered. The complex incremental technique is necessary only when there are three or more alternatives.)

$$(R_0 - R_1) = \$92{,}000$$

$$
\begin{aligned}
\$\ 50{,}000\ (\text{crf} - 5\% - 60) &= 50{,}000\ (0.05283) = 2{,}642 \\
200{,}000\ (\text{crf} - 5\% - 40) &= 200{,}000\ (0.05828) = 11{,}656 \\
100{,}000\ (\text{crf} - 5\% - 20) &= 100{,}000\ (0.08024) = 8{,}024 \\
40{,}000\ (\text{sff} - 5\% - 10) &= 40{,}000\ (0.07950) = \underline{3{,}180} \\
(S_1 - S_0) &= \$25{,}502
\end{aligned}
$$

$$(M_1 - M_0) = \$20{,}000$$

$$B/C = \frac{(R_0 - R_1)}{(S_1 - S_0) + (M_1 - M_0)} = \frac{\$92,000}{\$25,502 + \$20,000} = 2.0$$

Because the resulting benefit-cost ratio is greater than unity, the new proposal should be accepted. Moreover, the savings-cost ratio method yields the same solution:

$$S/C = \frac{(R_0 - R_1) - (M_1 - M_0)}{(S_1 - S_0)} = \frac{\$92,000 - \$20,000}{\$25,502} = 2.8$$

It is emphasized again that although the absolute values of the two ratios may differ the same course of action is indicated because each ratio is greater than unity.

Summary

In addition to demonstrating the techniques of incremental analysis, the objective of this discussion is to show the equivalence of various analytical methods. The examples used by Professor Smith to show "three crucial defects in the benefit-cost ratio method" have been re-analyzed here to illustrate that these so-called defects are matters of procedural error rather than inherent invalidity. Moreover, it has been shown that the differences between the benefit-cost ratio and savings-cost ratio methods are effectively inconsequential.

Although the "excess of benefits over costs," benefit-cost ratio, savings-cost ratio, and rate of return methods are equivalent insofar as they lead to the correct choice among alternative investment proposals, it is not meant to imply that each of them is equally effective as a practical analytical tool. Certainly the author and the writer agree on this point. In fact, the writer views the benefit-cost ratio method (and other such methods based on a ratio) with considerable disfavor. However, since the purpose here is simply to discuss validity and not relative efficacy, the question of choice of method is left to another time and place.

GERALD W. SMITH, Closure—The paper is concerned with exceptions to a general (benefit-cost ratio) approach to problems. Its object is to illustrate that sometimes the general rule is imperfect. To argue such an exception is more difficult, for the argument must show that all of the supposed exceptions are untrue (otherwise, exceptions still exist).

As in most questions, the opinion differences between author and discussers arise from differences in assumptions. The paper treats sets of alternatives (a) without restriction as to whether alternatives are mutually exclusive or non-mutually exclusive, and (b) without restriction as to the "cut-off" or "minimum acceptable" benefit-cost ratio used by the analyst.

The comments are appropriate only when all of three conditions are met: (1) the alternatives are mutually exclusive, (2) the analyst using the benefit-cost ratio technique applies a cut-off of minimum acceptable benefit-cost ratio of 1.0, and (3) when it is reasonable for the analyst using a minimum acceptable ratio of, say 2.0, coupled with an interest rate of, say 5%, to change his method so that he now applies a minimum acceptable ratio of 1.0 and an interest rate of 5%.

A question is raised here: is there a variety of minimum acceptable benefit-cost ratios, other than 1.0, in use? The widely used AASHO report (1) does not suggest use of the 1.0 minimum acceptable ratio, even in the "second benefit" incremental approach presented on page 151. It is on this basis that the paper leaves unrestricted the question of what minimum acceptable benefit-cost ratio will be used by the analyst.

The conclusions which may be drawn from the paper and comments as a total seem to be:

1. If alternatives are non-mutually exclusive and if a minimum acceptable ratio other than 1.0 is used, the benefit-cost ratio can lead to erroneous conclusions.
2. If alternatives are non-mutually exclusive and if a minimum acceptable ratio of 1.0 is used, the benefit-cost ratio can lead to erroneous conclusions.

3. If alternatives are mutually exclusive and if a minimum acceptable ratio other than 1.0 is used, the benefit-cost ratio can lead to erroneous conclusions.
4. If alternatives are mutually exclusive, and if a minimum acceptable ratio of 1.0 is used, and if it is reasonable to use a ratio of 1.0 coupled with an interest rate of 5%, instead of some other ratio, perhaps 2.0, coupled with an interest rate of 5%, the incremental benefit-cost approach illustrated in the comments can be applied as shown to yield correct conclusions.

The divergence of approach is not as great as it might appear. This may be illustrated by Example 6, pages 40-44 of the AASHO report (1). The conclusion (p. 44) that "Plan 1 is more desirable than Plan 2" is questionable. Both parties agree that the question is one of interpretation, that a benefit-cost ratio for Plan 1 of 5.16 and a benefit-cost ratio for Plan 2 of 4.75 does not necessarily mean that Plan 1 is better than Plan 2 if the plans are mutually exclusive. Both parties agree that supplementary rate of return analyses (incremental if appropriate) would help the analyst avoid erroneous conclusions.