Thickness of Flexible Pavements by the California Formula Compared to AASHO Road Test Data

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For the past several years, the State of California has been using a pavement structural design method based on test road data and on observed performance of pavement structures. The original formula, containing factors for traffic, supporting power of the soil, and slab strength of the pavement and base layers, has been modified at times as better information became available.

This paper describes not only the design formula but also modifications suggested from a study of the AASHO Test Road data. Correlation with test track data is shown.

•SOILS and granular materials have been used in building construction, for walls, floors, and pavements, for many thousands of years. Obviously, the ancients must have had a great amount of practical knowledge about the use of such materials. When the designers and builders of ballistae, catapults, and similar engines of war turned their attention to other forms of construction, precise methods for estimating the potential behavior of materials began to emerge. The need to design stable earthworks was probably most pressing on the military engineers and one of these, Charles Augustin Coulomb (1736-1806), was among the first to propose a formula by means of which the stability of earthwork embankments might be computed. Nevertheless, in spite of the long history of engineering works involving earthy materials, formulas for calculating the bearing capacity of soils have not been as reliable or perhaps not as well understood as are formulas for bridge members and other structures.

Engineering is a profession that requires an understanding of several sciences and disciplines but which depends primarily on a knowledge of materials and how the materials will perform or "stand up" under given conditions. The typical engineer has a working knowledge of physics, mechanics, mathematics, and is acquainted with a collection of somewhat inexact numbers and values optimistically referred to as "the strength of materials." The strength concept seem to be reasonable, sound, and "common sense." However, it is deceptively simple and can be misleading. A layman knows that a 12- by 12-in. timber beam will sustain a greater load than will be 2- by 4-in., and can also grasp the idea that a steel beam will support a greater load than a wooden beam of the same dimensions. Carpenters, millwrights, masons, and even architects have designed and constructed some fairly elaborate structures without very much in the way of recognizable engineering training. However, though the strength properties of wood, stone, or iron may be reasonably well appraised by experience or intuition, this approach has been less successful in estimating the ability of soils and foundations to sustain loads.

A great deal of the difficulty may be ascribed to the lack of means for identifying and measuring the important properties of the materials involved. Although the "strength idea" is accepted almost spontaneously and instinctively and presents no serious difficulties when applied to such things as steel, timber, and reinforced concrete, it does become a little blurred and the image rather fuzzy when one tries to apply this term to the properties of soils. It becomes even more elusive when applied

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to cohesionless sand and fails completely to describe the properties of liquids such as water.

Webster's dictionary defines "strength":

> Power to resist force; solidity or toughness; the quality of bodies by which they endure the application of force without breaking or yielding; a measure of the cohesion of material; firmness; coherence; as the strength of bone, beam, wall, rope, et cetera.

The word "strength" obviously has many meanings and shadings, and it does not mean the same thing when applied to different materials and circumstances. One may speak of a strong wind or a strong current of water but what is meant is that when either a gas or a liquid is in motion it can exert considerable force. A "strong man" may also be able to exert considerable force but he cannot necessarily withstand as much as a "weak woman." At least women have shown that they often have great powers of resistance! One speaks of a strong steel cable or a nylon rope, and such strands are strong in the sense of the dictionary definition meaning "cohesion." For most engineering materials, the word strength actually denotes only tensile strength, but materials such as soils can "endure the application of force" and yet possess little or no tensile strength. It, therefore, appears that a more precise general term for these properties is "resistance." This term is explicit and may be applied without confusion to a variety of materials. Thus, a strong steel wire or a cable requires a considerable force to overcome its resistance to breaking. A column of stone blocks or a dry rubble wall exerts considerable resistance to compressive forces. Even more pertinent to this discussion, the common materials of the earth's crust (rock, sand, gravel, soil, or mud) can all be shown to offer measurable degrees of resistance to applied forces. But these materials have little cohesion and hence little or no "strength" unless combined with an artificial binder such as asphalt or portland cement, and even the tensile strength of concrete is not very great compared to steel, for example.

THE PAVEMENT PROBLEM

All pavements, regardless of type, rest upon the materials of the earth's surface, and though there are a few examples of relatively solid rock subgrades, the vast majority of highway pavements are supported by soils or related granular materials having low cohesive strength. Nevertheless, a wide variety of soils have "what it takes" to support pavements if the pavement structure is "properly designed." This means that soils possess some pertinent property other than cohesive strength and this property is easily identified as interparticle friction. The importance of both friction and cohesion was recognized by Coulomb, and values for each appear in his formulas.

To apply the principles of engineering to the structural design of a pavement, the engineer must know what properties of materials are involved. Lack of reliable tests has been on of the greatest stumbling blocks. Many of the tests that have been applied to soils and paving materials do not provide measures of fundamental properties. For example, if one wishes to measure the tensile strength of steel, a carefully prepared specimen is attached to the jaws of a testing machine and the force required to pull the specimen apart is measured. This is a direct measurement of an important property. If the strength of concrete is involved, a carefully prepared test cylinder or cube is subjected to a direct compression loading. However, even though steel and concrete are often combined to produce reinforced concrete structures, one rarely attempts to measure the properties in combination. The individual strength properties are evaluated by separate tests. Unfortunately, in the case of soils and other granular materials, a number of test methods are affected by the two distinct properties acting simultaneously.

Many tests provide no means for differentiating between such radically different attributes as friction and the cohesive resistance. Though the resistance to deformation or displacement due to friction is fairly well defined (if not well measured), the cohesive "strength" or resistance is generally defined as "that portion of the resistance to sliding that is not affected by the pressure." This is a negative definition and differs from the dictionary definition of cohesive strength. In effect then, the soil mechanics definition of cohesion does not define what cohesion is, it merely says what it is not. The other element of confusion arises from the use of such devices as the Mohr circle analysis in which the intercept of the Mohr envelope on the vertical scale is defined as "cohesion." Tests on certain obviously cohesionless materials have shown a definite value for the intercept which would therefore be defined as "cohesion." Finally, a great many have been "thrown off the track" by the substitution of such terms as "shear strength" which by itself is not a property of materials; the total resistance to shear being again composed of variable portions of frictional and cohesive resistance. The resistance due to each of these dissimilar properties combines to produce the total resistance in an endless variety of combinations. The use of tests such as the CBR test, several varieties of direct shear tests, or unconfined compression tests, all tend to reflect or summarize some arbitrary combination of friction and cohesion. The relative proportions depend on the geometry of the test specimen and speed of loading which usually differ considerably from the conditions on an actual roadway.

Both geologists and agronomists have studied fragmentary stone and the finer decomposition products called "soil" and each group has developed classification schemes and names for the numerous varieties of rock, gravel, sand, and soil types. These classifications have their uses and have proved helpful to the engineer but none are directly fitted to the engineer's problem. As stated by Feld, "an adequate soil classification scheme for engineers should be based upon engineering properties." All this leads up to the point that soil, sand, gravel, and other naturally occurring mineral materials possess a number of properties and characteristics and can be variously described according to geologic origin, petrographic classification, grain size, soil texture, mineralogical composition, or even the chemical compounds involved. These classifications may or may not indicate the suitability of the material or the best means of treatment for engineering purposes.

As with all the other sciences concerned with soils, the engineer needs to know what properties are important to him and what determines the ability of the soil to support loads, and having identified these properties he must then know what test methods to use to measure them. This is a step that must be made first as no reliable or valid mathematical formula for structural design can be developed unless it includes numerical values to express real and essential properties of the materials involved.

In 1948 a design formula for calculating the thickness of pavements (1) was reported which includes an expression for the measured resistance value of various soil or granular layers and for the tensile strength or cohesive resistance of all elements composing the pavement structure. The basic data for the relationships developed were derived from a small but full-scale project known as the Brighton test track constructed by the California Division of Highways in 1940. For an expenditure of less than $100,000, it was possible to construct and operate a test track that included eight different types of base material varying in thickness from 3 to 18 in. resting on the same saturated silty clay soil having a CBR value of about 3 or an R-value of approximately 17. The track was subjected to a loaded truck and at the end of the operation it was evident that the thickness required for the various types of base did not show any consistent relationship to the CBR value or the resistance value for the base material itself, but there was an orderly and consistent trend with the tensile strength of the materials as measured by the cohesiometer. This test track made it possible to assign tentative values to some of the variables such as the effects of wheel load and repetition. Though the underlying soil on the test track was uniform throughout and gave no range of value, some additional check points were obtainable from observations on the State highway system. A few scattered examples where the pavement thickness had been varied over different types of soils made it possible to establish a relationship. The establishment of a scale of values for soil support was greatly simplified by the fact that the thickness of pavement structure required bears a linear relationship to the resistance value of the soil as measured by the stabilometer. There was no opportunity to introduce a variation in tire pressure so the effects of this variable were not established. The equation developed at the time was

$$T = \frac{(K\,P\sqrt{a}\,\log r)\,(P_h/P_v - 0.10)}{\sqrt[5]{C}} \quad (1)$$

in which

T = thickness of cover (base and pavement) (in.);
K = 0.0175 for best correlation but without any factor of safety (for design purposes, it is suggested that $K = 0.02$);
P_h = transmitted horizontal pressure in the stabilometer test (psi);
P_v = applied vertical pressure in the stabilometer test (typically 160 psi);
P = effective tire pressure (psi);
a = effective tire area (sq in.);
r = number of load repetitions; and
C = tensile strength of the cover material as measured by the cohesiometer in grams per square inch (approximately equals modulus of rupture × 45.4).

Eq. 1 was simplified by reducing the effects of load and repetition to an expression termed the traffic index and by reducing the stabilometer data to a resistance value R. Eq. 1 then becomes

$$T = 0.095\,\frac{(\text{traffic index})\,(90-R)}{\sqrt[5]{\text{cohesion value}}} \quad (2)$$

in which

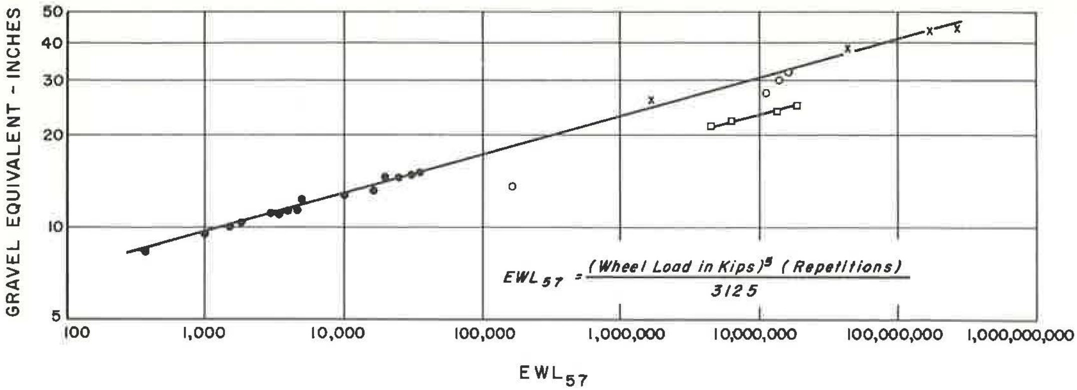
T = required thickness of cover; and
R = resistance value by stabilometer.

This equation was used for the design of pavements, and any discrepancies that became apparent between prediction and performance were noted and modifications in the testing and design procedure were introduced as seemed to be warranted.

On the completion of the WASHO test road in Idaho, attempts were made to check the California formula by comparison with the performance on the WASHO test road. Unfortunately, the design of this project was such that only a very few definite points could be established. Although the usable data from the WASHO road agreed with the predictions of the formula, they were insufficient to confirm its validity over any substantial range (Fig. 1).

The tremendously larger AASHO test road in Illinois furnishes a great deal more data and gives a much wider range of values for checking a previously established structural design formula. To make a comparison between calculated values and test road data, the various materials, basement soils, granular base, subbase, and asphaltic pavement were tested and evaluated according to the California procedures. The wheel loads and number of trips were converted through the equivalent wheel load calculation to the traffic index number. With values derived by laboratory tests of the Illinois materials and calculations for the traffic, it is possible to arrive at a design thickness based on the California formula (1957 Model). The calculated thicknesses may then be compared with the actual thickness reported to be necessary on the test road. The correlation is shown later in Figure 5. The statistical values showing a standard error of estimate of ± 2.7 in. and a coefficient of correlation of 0.87 (Appendix B) seem to confirm the ability of the California design formula to predict the thickness of pavement required for a wide variety of traffic loads and materials.

The test road data, however, neither prove nor disprove the applicability of the California formula to other types of soil or granular base materials. The test road pavement structures were supported by only one type of basement soil. Because of this lack of variables on the AASHO project, it is not possible to develop a design formula by using the test road data alone. Also, the statistical-type formulas developed by the road test staff have no terms or identities that permit application to soils differing in properties and ability to support loads from those used on the test road. The test road formula does not identify or indicate means for measuring the properties or physical



TEST TRACK DESIGNATION	WHEEL LOAD	TYPE OF SOIL SUPPORT
▫ WASHO		17 Avg. CBR Soil
• Brighton	6 KIP	4 Avg. CBR Soil
○ Stockton	25 KIP	5 Avg. CBR Soil
x Stockton	40 KIP	

Figure 1. Correlation of EWI_{57} with gravel equivalent.

conditions that account for the performance of the subbases, bases, and asphalt pavement types.

FACTORS TO BE CONSIDERED IN A DESIGN FORMULA

A design formula for the structural elements of a pavement should embody all the important factors that affect the ability of the pavement structure to sustain vehicle loads over a substantial period of years. There have been many formulas proposed. M. S. Kersten (2) has listed 22 different ones. Some of these were based on theoretical concepts, others were completely empirical, and some represented a mixture of the two approaches. The factors that influence the over-all performance of a pavement are so numerous and the desirable attributes of a pavement are so diverse that it seems impossible or highly improbable that all of these variables can ever be included in a single formula, or if such a formula were constructed, only a highly sophisticated electronic calculator could hope to reach a solution. Even then, a certain allowance would be needed for the inability to do a perfect job.

Figure 2 is included to show the variables that can affect the performance of an asphalt pavement. At least 30 items have been identified. However, design formulas rarely need to cover every factor, and many of the variables shown in the figure can be ignored or combined into a single element in the formula.

As an example of the simplification that is possible and quite practicable, an adequate structural design might be described as one that produces an economical or efficient pavement that will neither crack nor deform under the assumed traffic during the design life of the pavement. (Guarding against disintegration types of failure is primarily a question of mixture design and quality of materials rather than a structural design problem.) Column 3 of Figure 2 shows that there are three primary factors; namely, the effects of traffic, the strength of the pavement, and the ability of the foundation to support the load. The primary factors have the following relationship:

$$T = \frac{KD\,(90-R)}{S} \tag{3}$$

in which T = thickness; K = constant; D = destructive effect of traffic; R = resistance value of support; and S = strength of pavement structure.

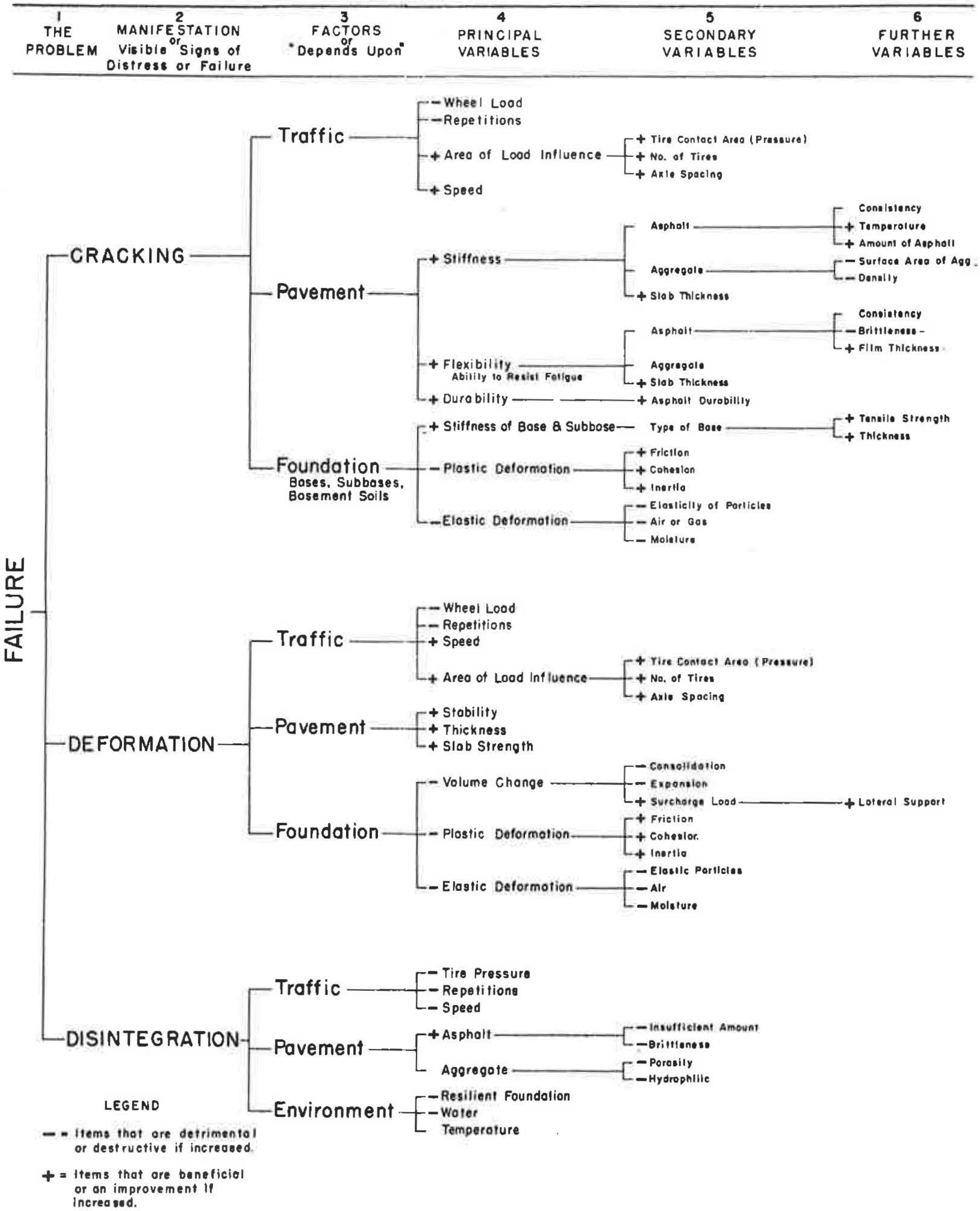


Figure 2. Analytical chart showing variables that must be evaluated for structural design of asphalt pavements.

To derive a number to express the effect of traffic, it is necessary to consider Columns 4 and 5 which list some of the subdivisions that make up the traffic load effect or "the destructive effect of traffic." The principal variables are the total wheel load in contact with the pavement and the number of times this load passes over the pavement. The area of load influence is a factor but the problem has thus far been simplified for highway traffic as the maximum tire pressure on most motor trucks is in the order of 70 or 80 psi for the heavier vehicles. The axle spacing or "the proximity factor" is confined to only two typical configurations; namely, single axles some 15 ft apart or tandem axles (2 axles within 4 ft). Although the comparative effects of tandem axles vs single axles differ markedly as those between flexible pavements and rigid pavements, nevertheless, it is possible to convert these two types of axle spacings to a common denominator for each type of pavement.

Examining all the available data which include the Brighton test track, the Stockton track (constructed by the Corps of Engineers), the WASHO and the AASHO projects, it appears that the relative effects of traffic may be expressed as follows for flexible pavement design:

$$TI = 1.30\left(\frac{W}{5}\right)^{0.050} r^{0.119} \tag{4}$$

in which

TI = traffic index;
W = wheel load in kips for tandem axles (W = 1.10 individual wheel load); and
r = number of load applications.

This equation assumes a tire pressure in the range of 50 to 100 psi but does not provide for effects of extreme variation in tire pressure as there are insufficient data available to indicate how variation in tire pressure may affect the performance of a road structure.

Figure 2 also shows there are a number of factors that compose the over-all properties of the pavement. Primarily, there is a question of stiffness or the resistance to bending. The term "stiffness" has been borrowed from a report by L. W. Nijboer and C. van der Poel (3). Nijboer computes stiffness from

$$S = \frac{F_p}{X_p}\ (12) \tag{5}$$

in which

F_p = force acting on pavement in newtons [limits of F_p between 10^4 newtons (1 ton) and 2×10^4 newtons (2 tons), respectively]; and
X_p = deflection of the pavement in microns.

Therefore, the term "stiffness" bears a simple mathematical relationship to the deflection of the pavement, and as used by Nijboer, "stiffness" implies the resistance of all components including the pavement, bases, subbases, and the underlying soil. For design purposes it seems preferable to associate the concept of stiffness with the pavement and base structures alone, in which case there will not be a consistent relationship between "stiffness" and "deflection" as the character of the supporting soil will then represent a variable: resilience.

Stiffness of a "flexible" pavement is influenced by the thickness, the type and amount of asphalt, and the temperature. This means that an asphalt pavement has a high degree of stiffness during cold weather and it also means that the lower courses provide greater stiffness in warm weather than the same mixture in the surface layer exposed directly to the sun. The stiffness of all materials can be expected to increase with the thickness of the layer but in the case of asphalt pavements the effect is enhanced by the lower temperatures in the bottom courses, especially where the pavement is of substantial depth. Flexibility is more or less the opposite number, or complement, of stiffness. This is a property not easily measured but it may enable a pavement to survive the flexing over resilient or springy foundations. It is a difficult value to include in a simple design formula.

The word "stiffness" is also not entirely applicable or adequate to express the manner in which a pavement structure functions. The concept of "stiffness" is readily visualized in the case of a thick asphalt pavement. It is even more descriptive of a portland cement concrete slab, but a substantial layer of crushed stone or gravel will have the same effect, within the limits of its own resilience, in reducing deflections. Precisely speaking, the term stiffness hardly seems appropriate for a bed of cohesionless material. Nevertheless, in the absence of a better term, a thick layer of sand or gravel may be said to have "stiffness." The question of pavement stability and resistance to water action are properties that fall into the area of mix design and need not ordinarily be considered in a structural design formula.

The process of assigning strength or resistance values to foundation materials must resolve a great many variables due to the wide variety of materials that may be involved. The treated bases and subbases may possess properties similar to that of the pavement layer, whereas granular bases and underlying soils are generally low or completely lacking in tensile strength or cohesive properties. As inferred in the preceding, a great deal of the so-called fundamental or theoretical approach to the design problem has focused attention on the elastic properties but for the most part it is the plastic properties of soils, subbases, and granular bases that have caused the most trouble. Again, one must recognize the very dissimilar response of friction and cohesion to most tests or loads.

The stabilometer furnishes a means for measuring the internal friction or granular materials under load. When solid particles such as stone or sand grains are coated with asphalt or wet clay, a lubrication effect is introduced as soon as a sufficient quantity of the lubricant has been added. Obviously, the amount needed and effects produced may vary considerably. Rough crushed stone particles are difficult to lubricate, whereas smooth polished gravel and sands will tolerate only small amounts of asphalt or wet clay additions. The problem of stability of asphalt pavements or the ability of granular bases and subbases to support a pavement depends very largely on the friction or the degree to which the friction has been reduced or lost by lubrication. Thus, the designer of bituminous mixtures or clay-bound stone bases is confronted with the fact that the very materials added to increase the cohesion (strength) will also reduce the friction through lubrication whenever sufficient amounts have been added.

When the cohesive effect is provided by a viscous liquid such as asphalt it becomes impossible to summarize the two unlike properties except under some specific condition of load area and speed of loading. Furthermore, the two properties are individually important and each is most effective in certain regions or zones of the pavement structure. A bed of cohesionless crushed stone, gravel, or sand will support traffic provided the surface is covered with an adequate thickness of material that does possess some cohesion. A surface treatment or seal coat on a gravel road is an example, but to be successful, a certain depth of the gravel must have some coherence or cementing action furnished by a soil binder. In contrast, a thin seal coat would be completely ineffective on a bed of clean beach sand. There is ample evidence therefore to show that an adequate pavement structure must provide an upper layer of material having some coherence or tensile strength, and the thickness of this layer must increase with increasing wheel loads. Beyond this critical depth, a completely cohesionless gravel or sand will serve quite well and will often prove to be less critical and give more lasting service than will base and subbase layers cemented with natural materials. Natural materials may consist of soil including clay or fines produced by degradation of the aggregate. Figure 3 shows the regions in the pavement structure where cohesion and friction are most influential or important. Figure 4 is an alignment chart suggesting the depths of pavement and/or cohesive base layer that is required over a completely cohesionless material.

For various magnitudes of wheel loading, the AASHO test road furnishes examples that supplement observations on the performance of actual highways. On Loop 2, the thin bituminous surface treatment resting directly on the soil gave a better performance and sustained a greater number of trips before failure than did the same thickness of surface resting on the gravel, yet the soil had a lower CBR and a lower R-value, and would be considered to be far less adequate by most methods of evaluation thus far

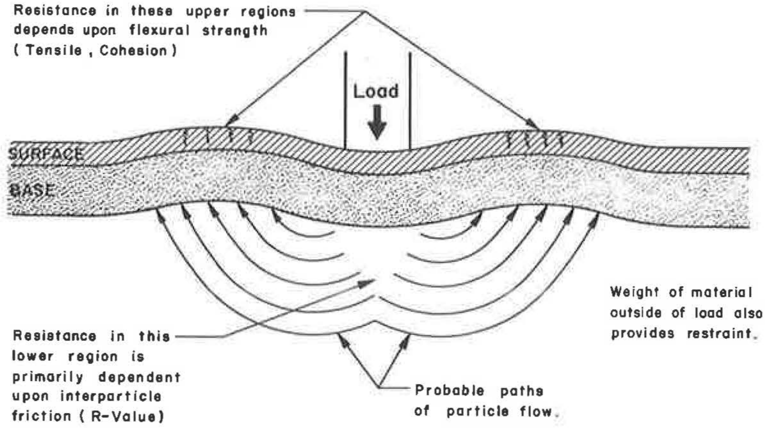


Figure 3. Plastic flow phenomena in soils supporting a pavement.

developed. Referring to Loop 5, the wedge sections 457, 458, 467, and 468 also demonstrate that the failure of the pavement was due to the gravel base as it failed as readily with 15 in. of base depth as with 5 in.

In the California formula, one of the factors that reflects the effect of pavement thickness is the (90 - R) factor which in effect states that a material of 90 R-value would be of sufficient strength to support any highway traffic load. This expression was developed from early data when the formula was devised and was based on extrapolations from rather light traffic. Furthermore, the factor appeared to correlate with California experience.

The data from the AASHO Road Test would indicate that a more rational approach to determining thickness of pavement for heavy traffic would be to use a factor of (100 - R). This would provide adequate thickness over most of the sections that appeared to fail because of inadequate base cover. This adjustment in the (90 - R) factor is made possible through more accurate information on the effect of traffic and also by adjustment of the cohesion factor in the formula. Again, in the gravel base wedge sections in Loop 5 of the Road Test, 4 $\frac{1}{2}$ in. of asphalt concrete, in lieu of the 3 in. provided would have been required over the base material if any of the base thicknesses were to have survived the Test Road traffic. Likewise, in Loop 4, for the same wedge of gravel base, it would appear that $3\frac{3}{4}$ in. in lieu of 3 in. would be required for the traffic of Loop 4 to have been satisfactorily carried over the wedge for the duration of the project. These increased thicknesses of surfacing over these cohesionless gravel materials would have allowed the effect of gravel thickness to have been measured in a uniform and consistent manner, with the principal variable being thickness of base.

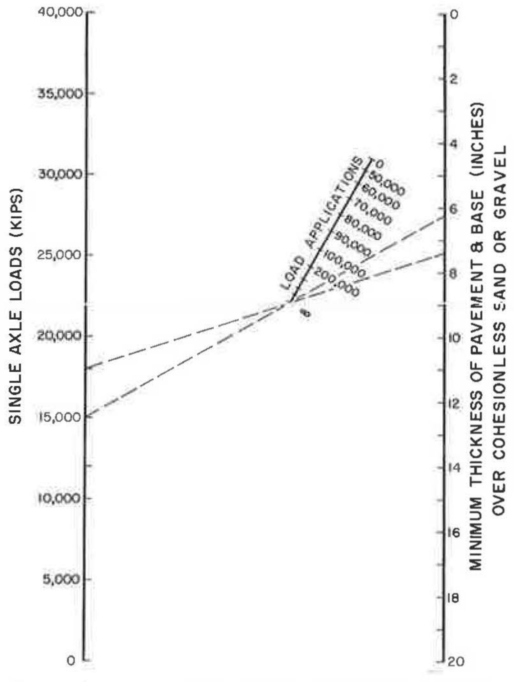


Figure 4. Alignment chart indicating thickness of cohesive layer required over cohesionless sand or gravel.

Figure 5 shows the correlation between the thickness computed by the California method (1957 revision) (4) and the actual minimum thickness found to be adequate on the AASHO test road. It appears that the greatest discrepancy between the predictions of the California formula and the actual performance is in the bituminous base sections, and it is therefore evident that the assumed cohesive strength value that has been used for California asphalt pavements is not adequate to account for the performance of the thick asphalt section on the test road.

To evaluate the Test Road performance of thicker asphalt concrete sections properly, it was necessary to revise the scale of cohesion values used in the California formula. The original formula assumed there was a cohesion of 100 for gravel and no materials would be less than 100. However, in trying to evaluate the AASHO test road, it became evident that the gravel base, for example, had far less than 100 cohesion. Actual tests performed on this material indicated a cohesion of only 20 g per lineal inch. Cohesions on the crushed rock base material had a value of only 30. To obtain more accurate definition with the design formula, it appeared expedient to change the basic cohesion for cohesionless materials (such as the AASHO subbase) from 100 to 20 and to use a value of 30 for crushed rock bases. The use of a more cohesive material (such as asphalt) or a cementitious material (such as portland cement) has a greater effect in the reduction of thickness of section. An evaluation of the effect that bituminous bases have on performance of the wedge sections of the Test Road provides some information. Table 15 of AASHO Road Test Report 5 gives information showing the equivalencies in terms of inches of gravel for both the bituminous-treated and the cement-treated bases. From the AASHO information, the equivalencies in Table 1 were developed.

TABLE 1

EQUIVALENCIES OF TREATED BASES

App. (1,000)	Base Type[a]	Equivalency (in. of stone base per inch of treated base)							
		Loop 3		Loop 4		Loop 5[b]		Loop 6[c]	
		12K-S	24K-T	18K-S	32K-T	22.4K-S	40K-T	30K-S	48K-T
100	CTB			1.8	2.2			1.9	2.0
	BTB	2.9	3.2			2.4	2.3	3.0	2.9
300	CTB			1.7	1.8			1.6	1.6
	BTB	2.8	2.9			2.2	2.3	1.9	2.3
500	CTB			1.7	1.6			1.5	1.5
	BTB	2.7	3.0			2.3	2.4	1.7	2.1
700	CTB			1.6	1.6			1.5	1.5
	BTB	3.1	3.1			2.2	2.4	1.7	1.8
900	CTB			1.6	1.6			1.5	1.5
	BTB	3.3	3.1			2.2	2.4	1.7	1.7
1,114	CTB			1.6	1.6			1.5	1.5
	BTB	3.4	3.1			2.2	2.4	1.7	1.7
Average	CTB			1.7	1.7	(1.65)	(1.65)	1.6	1.6
	BTB	3.0	3.1			2.3	2.4	2.0	2.1

[a]CTB = cement-treated base; BTB = bituminous-treated base.
[b]Because there was no stone base wedge section in Loop 5, the average equivalency for CTB (1.65) from Loops 4 and 6 was assumed to be correct for Loop 5 also, and this value was used for comparison with the BTB sections. Data for Loop 5 are, therefore, interpolations.
[c]For Loop 6, 4 in. of subbase was replaced by 3.5 in. of stone base for comparison purposes.

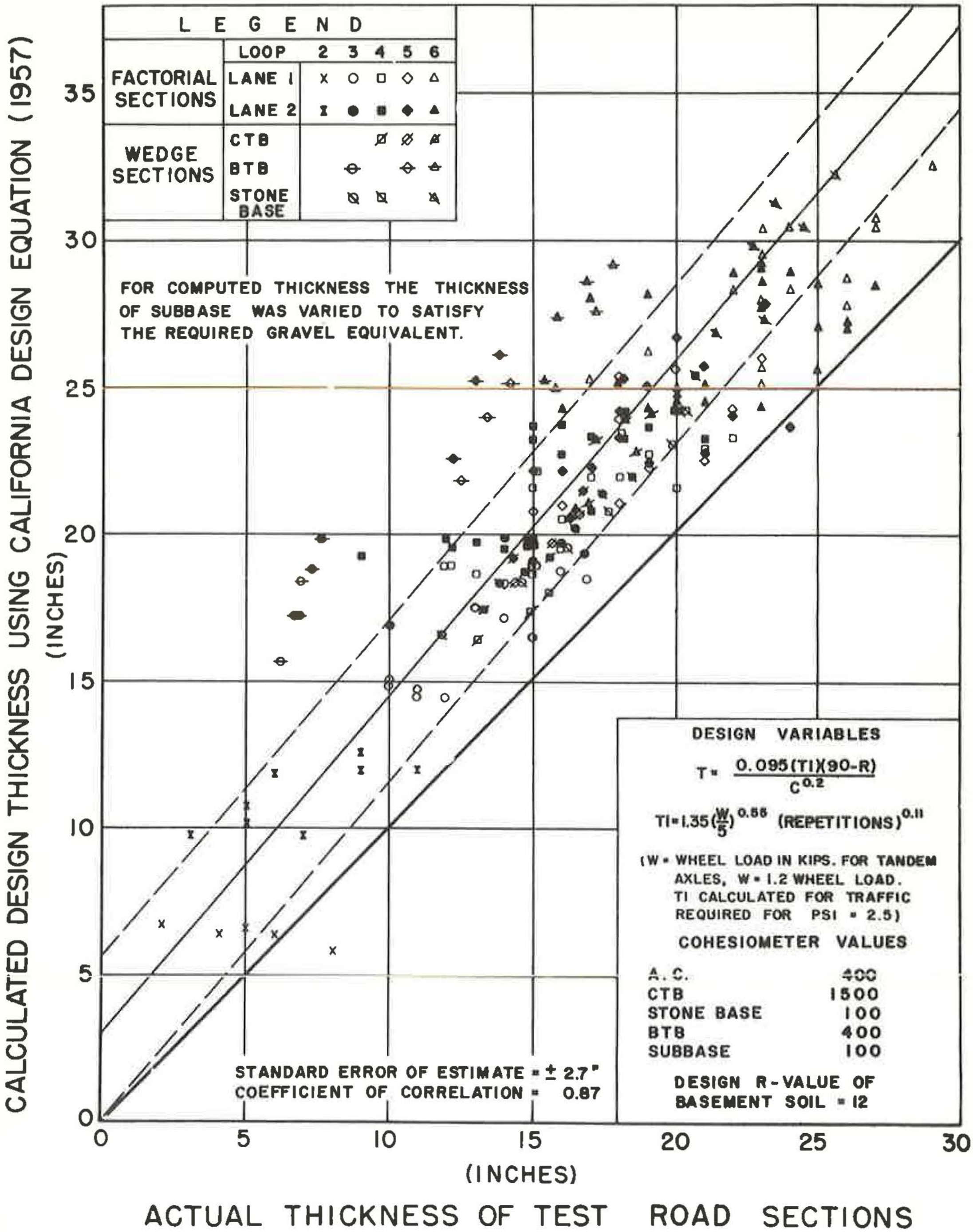


Figure 5. Thickness of test sections at AASHO Test Road vs calculated design thickness using California design equation (1957).

Table 1 would indicate that cement-treated bases have an equivalency of 1.65 in. of gravel to 1 in. of base. This agrees quite favorably with California experience in which a factor of 1.75 to 1 is currently being used.

From the information on bituminous bases, on the other hand, it is apparent that the magnitude of load has a marked effect on the equivalency of bituminous bases. It is suspected that there is also an effect due to depth of layer and the number of repetitions. However, in these latter two cases, it was not possible to isolate the variables by means of the information available. It is possible that one effect offsets the other.

A study of air temperature data at the AASHO test road and corresponding pavement temperature data indicated that an approximate average pavement temperature of about 72° would represent the over-all condition of the test road pavement. Cohesion (tensile strength) tests were made on AASHO pavement cores tested at various temperatures. The results are shown in Figure 6. At 72°, the cohesion value of the AASHO mix is 5,000 g per lineal inch. The recovered penetration of the asphalt in these cores was 37. (Cohesion test is performed by breaking a 2½- by 4-in. diameter test specimen by bending. Cohesion value equals the grams per lineal inch to break specimen when the load is applied on a 30-in. lever arm (6).)

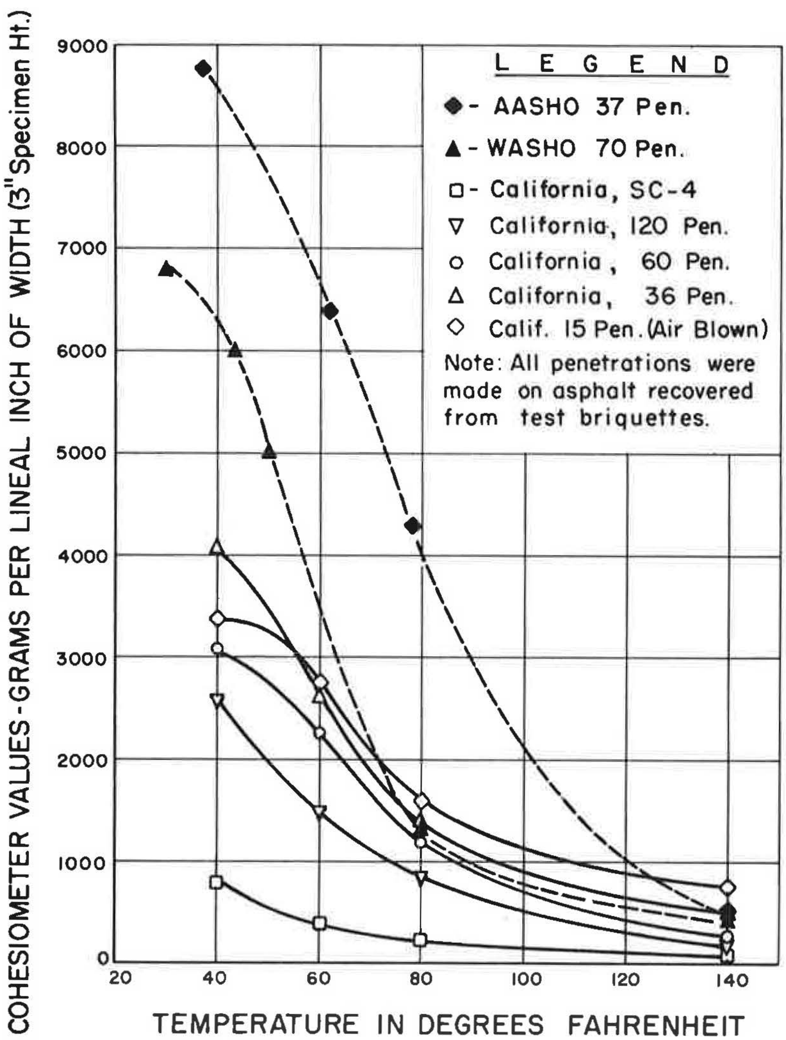


Figure 6. Cohesiometer values of bituminous pavement at various temperatures.

To compare a normal California mix using a good crushed California aggregate and asphalt manufactured on the Pacific Coast the remaining series of tests shown in Figure 6 were performed. For these California mixes, the cohesion at 72° would be only 2,000 g per lineal inch.

Most observers would agree that the equivalency of a rigid layer of material, in terms of inches of gravel, should be directly related to its tensile strength and its depth of section. A somewhat different situation exists in the case of bituminous layers, for in this case, strength is related not only to composition but also to temperature (Fig. 6). A bituminous mix varies in temperature from top to bottom, consequently there is a variation in that portion of its strength that is dependent on the viscosity of the asphalt binder.

To evaluate the property of cohesion, an empirical formula was developed to fit AASHO conditions:

$$C = \text{cohesion at } 72^{0} \left(\frac{8}{W+2}\right)^{2.5} \tag{6}$$

in which

C = equivalent cohesion; and
W = applied wheel load in kips (≥ 6).

Also, for gravel equivalency (GE),

$$GE = \left(\frac{C}{\text{cohesion of gravel}}\right)^{0.2} \tag{7}$$

Figure 6 indicated that mixes in themselves have widely divergent tensile strength characteristics; in ordinary highway design problems, an equivalency correction for wheel load would not be a simple matter because mixed traffic is involved and the weight of individual axles is rarely known, except on a statistical basis. However, assuming that lightly traveled roads will generally be designed for light loads, and heavy industrial roads will be subjected to heavy loads, a general relationship between equivalency and traffic index can be established.

Figure 7 is an empirical development from AASHO test road data which provides a means of adjusting equivalency for mixes that do not have the tensile strength characteristics of the AASHO asphalt concrete. These reductions in equivalency are necessary and need to be considered if flexible pavements are to be designed with the assurance of an adequate life. In California, therefore, it is proposed that a series of equivalencies be used that are based on the predicted traffic.

The proposed equivalencies taken from Figure 7 are given in Table 2. It covers a complete range of traffic currently using California streets and highways.

The coefficients in Table 2 would appear to challenge the validity of the coefficients D_1, D_2, and D_3 which were developed in the formula explaining the performance of the AASHO Road Test. These coefficients were obtained by statistical analysis of the factorial sections and most surely expressed what happened at the AASHO Road Test, yet there are the wedge sections and they, by this analysis at least, do not necessarily agree with the factorial sections. If the evidence reported by the British Road Test (8) that 6 in. of bituminous base is equivalent to 10 in. of gravel is added to this, as well as Nichols' report from Virginia (9) concerning distress of a number of asphalt base projects in which the total base and surfacing equaled 9 in., it would appear that there are other factors to consider before a single standardized ratio of equivalencies can be established for use under all conditions and all geographical areas. In Table 2 there is an attempt to indicate the ranges of equivalencies that might be encountered due to varying traffic conditions or varying quality of the asphalt concrete layer itself.

In 1957, the method for calculating traffic index in the California formula (4) was revised. The formula, based on test road data and experience available at that time, was

$$TI = 1.35 \left[\left(\frac{W}{5}\right)^{5} \text{repetitions} \right]^{0.113} \tag{8}$$

in which

TI = traffic index, a number directly proportional to the required thickness of structural section.

The AASHO Road Test data were reviewed to determine the validity of the exponents in the formula. The number of applications at present serviceability index (PSI) = 2.5 was plotted vs the gravel equivalent of the individual sections. The plots on log log paper yielded the slopes given in Table 3.

Table 3 shows that the use of different base materials results in different deterioration rates due to applications of a given load. However, the estimating of future traffic for purposes of design is, at best, only an approximation. Therefore, refinements in the exponent due to base type are not justified until methods of traffic prediction are greatly improved. To encompass all reasonable possibilities, it appears that the exponent of 0.119 would provide a reasonably satisfactory value.

Using the same procedure as the preceding, a tabulation was made for the same test sections in which curves of wheel load vs gravel equivalent were plotted for the indicated number of applications. The slopes are determined for the wheel load exponent. The tabulation is given in Table 4 and typical curves are shown in Figure 8.

In Table 4 the factorial sections were omitted because sufficient data were not available to interpolate exact thicknesses for given numbers of repetitions.

The average value of 0.48 is sufficiently close to a theoretical value of 0.50 to justify the use of the latter figure. Using the value of 0.50 the formula for thickness becomes

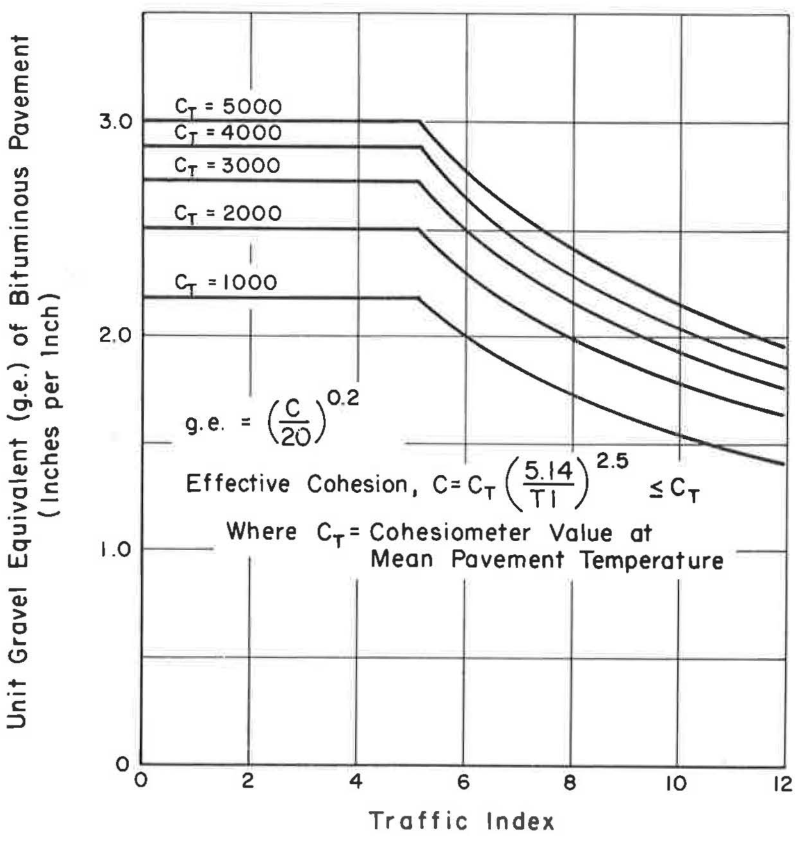


Figure 7. Gravel equivalent of bituminous pavement based on AASHO Test Road analysis.

TABLE 2

PROPOSED EQUIVALENCIES FOR BITUMINOUS MATERIALS SHOWING THICKNESS OF GRAVEL LAYER REQUIRED TO EQUAL 1 IN. OF ASPHALT CONCRETE PAVEMENT

Class of Road	Traffic Index Range	Gravel Equivalency (in.) AASHO Mat'l.	Calif. Mat'l.
Heavy industrial	12	2.0	1.6
	11	2.1	1.7
Heavy truck traffic	10	2.2	1.8
	9	2.3	1.9
Medium truck traffic	8	2.4	2.0
	7	2.6	2.1
Light truck traffic	6	2.8	2.3
Residential streets	5	3.0	2.5
	4	3.0	2.5

TABLE 3

SLOPE VALUES OF APPLICATION VS GRAVEL EQUIVALENT CURVES

Loop	Lane	Slope of Application vs Gravel Equivalent Curve: All Factorial Sections	BTB Wedge	CTB Wedge	Stone Wedge
3	1	0.118	0.088		0.137
	2		0.099		0.111
4	1	0.146		0.100	0.067
	2	0.141		0.127	0.064
5	1	0.093	0.100	0.082	
	2	0.103	0.080	0.103	
6	1	0.097	0.162	0.086	0.046
	2	0.090	0.161	0.099	0.044
Avg.		0.112	0.115	0.099	0.078

$$T = \text{constant } W^{0.50} r^{0.119} \tag{9}$$

in which

T = thickness;
W = wheel load; and
r = repetitions.

From Eq. 9, wheel load constants may be calculated which may be applied to mixed traffic:

$$\frac{T_1}{T_2} = \left(\frac{W_1}{W_2}\right)^{0.50} \left(\frac{r_1}{r_2}\right)^{0.119} \tag{10}$$

If $T_1 = T_2$; $W_1 = 5{,}000$ lb; and r_2 = one repetition of load W_2, then

$$r_1 = \left(\frac{W_2}{5}\right)^{4.2} \text{ equivalent 5-kip wheel loads (EWL)} \tag{11}$$

TABLE 4

SLOPE VALUES OF WHEEL LOAD VS GRAVEL EQUIVALENT CURVES

Applications of Load	Slope of Wheel Load vs Gravel Equivalent Curve: BTB Wedge	CTB Wedge	Stone Wedge	All Wedges
100,000	0.504	0.411	0.563	
300,000	0.535	0.476	0.488	
500,000	0.595	0.431	0.455	
700,000	0.636	0.394	0.380	
900,000	0.653	0.349	0.349	
1,114,000	0.668	0.359	0.347	
Avg.	0.599	0.403	0.430	0.48

The constants are called EWL_{62} to differentiate from previous EWL calculations made by the California Division of Highways.

The details of using this method to obtain constants applicable to mixed traffic are outlined by Sherman (4). Briefly, the method consists of a statistical sample of traffic as weighed at various loadometer stations throughout the State. The development of the method is given in Table 5 where axle weights have been grouped together to show variations within classes of trucks (such as 2-, 3-, 4-, 5-, or 6-axle trucks. In the table, wheel load factors for the 3-, 4-, 5-, and 6-axle trucks show a variation within a given wheel load group. This is due to allowance for tandem effect. Based on road test data, a 10 percent effect was allowed for each pair of tandems included. The number of tandem vehicles for each class of truck is estimated, using tables published in House Document 91, 1st Session, 86th Congress. This document contains a large sample of truck combinations and loadings for various geographical areas of the United States. It contains sufficient information to establish the percentage of single- and tandem-axle combinations for each load group. These percentages were applied to the loadometer tables of the California Division of Highways to determine the average wheel load factor for each class of truck and for each loading.

Table 6 gives the totals arrived at in Table 5 and develops the EWL_{62} constants for computing average daily traffic.

Because California traffic counts are reported as the total vehicles in two directions, the truck constants developed in the last column of Table 6 are for these bidirectional counts. Further the constants in Table 6 are based on 1959 traffic, and any increase in allowable load limits will result in higher constants. These constants multiplied by the estimated number of trucks of each axle grouping will total to the design equivalent 5,000-lb wheel loads (EWL). Constants could also be determined quite readily for equivalent 9,000-lb wheel loads.

TABLE 5

CALCULATIONS TO DETERMINE YEARLY ADT CONSTANTS FOR TRUCK GROUPS BASED ON 1959 STATEWIDE LOADOMETER SURVEY[a]

Axle Group (kips)	Wheel Load (kips)	2-Axle Trucks			3-Axle Trucks			4-Axle Trucks			5-Axle Trucks			6-Axle Trucks		
		EWL Per Axle	No. Axles	EWL	EWL[b] Per Axle	No. Axles	EWL	EWL[b] Per Axle	No. Axles	EWL	EWL[b] Per Axle	No. Axles	EWL	EWL[b] Per Axle	No. Axles	EWL
2- 8	2	0.02	1,939	39	0.02	931	14	0.02	1,104	21	0.02	3,313	55	0.01	153	3
8- 9	4¼	0.51	115	59	0.45	241	108	0.48	108	51	0.39	859	331	0.44	31	13
9-10	4¾	0.81	77	63	0.72	212	153	0.73	90	65	0.62	492	302	0.73	36	26
10-11	5¼	1.23	64	79	1.08	157	170	1.10	53	58	0.93	253	235	1.10	27	29
11-12	5¾	1.80	49	88	1.58	105	165	1.51	54	82	1.37	261	357	1.55	19	29
12-13	6¼	2.54	45	114	2.25	76	212	2.16	50	108	1.99	290	578	2.22	11	25
13-14	6¾	3.52	34	120	3.16	71	224	3.00	56	168	2.93	409	1,198	3.07	23	70
14-15	7¼	4.75	28	134	4.24	105	444	4.05	64	259	3.97	515	2,041	4.15	20	83
15-16	7¾	6.3	28	177	5.6	114	641	5.3	55	290	5.2	667	3,494	5.5	12	66
16-17	8¼	8.2	15	123	7.3	66	483	6.9	53	365	6.8	675	4,611	7.2	8	57
17-18	8¾	10.5	29	305	9.4	38	358	8.8	39	342	8.7	615	5,362	9.1	6	55
18-19	9¼	13.2	15	198	11.9	16	190	11.2	26	290	11.0	276	3,039	11.5	1	12
19-20	9¾	16.5	3	50	14.8	1	15	13.9	9	126	13.8	40	551	—	—	—
20-22	10½	22.6	4	90	20.1	—	—	18.8	4	76	18.6	11	205	—	—	—
22-24	11½	33	1	33	32	—	—	28	1	28	27	9	245	—	—	—
24-26	12½	47	—	—	45	—	—	39	—	—	39	3	117	—	—	—
Total No. Axles			2,446			2,133			1,763			8,688			347	
Total EWL				1,672			3,177			2,329			22,721			468

[a]Percentage of single and tandem axles extracted from House Document No. 91, 86th Congress, 1st Session, March 2, 1959.
[b]Based on tandem effect (i.e., one tandem = one single 10 percent heavier than tandem wheel load).

TABLE 6

TABLE OF AVERAGE DAILY TRUCK CONSTANTS FOR VARIOUS CLASSES OF TRUCKS

No. of Axles Per Truck	Total No. Axles	Total EWL	EWL per Axle	EWL per Truck	EWL for 365 Days[a]	EWL/Year for One Truck in One Direction[b]
2	2,446	1,672	0.684	1.368	499	250
3	2,133	3,177	1.489	4.467	1,630	815
4	1,763	2,329	1.321	5.284	1,929	965
5	8,688	22,721	2.615	13.075	4,772	2,385
6	347	468	1.349	8.094	2,954	1,475

[a]Constants when traffic counts cover traffic in one direction only.
[b]Constants when traffic counts include bidirectional traffic.

The EWL may be converted to traffic index by

$$TI = 1.30\ (EWL_{62})^{0.119} \tag{12}$$

A typical traffic index calculation is shown in Appendix A.

Those who are familiar with and have used the California method previously will note a substantial reduction in the EWL constants. However, the relation between constants (i.e., the ratio of 2-axle to 5-axle or 3-axle to 6-axle vehicles) has not greatly changed. Also, for a given traffic situation the new EWL constants will result in virtually the same traffic index. For example, in Appendix A, the EWL_{57} would have a traffic index of 10.7, whereas the new 1962 constant will yield a traffic index of 10.9.

Having re-evaluated the various factors of the design formula in light of the AASHO data, it would appear the formula should be changed to read

$$\text{Thickness} = \frac{0.070\ (\text{traffic index})\ (100 - \text{resistance value})}{(\text{cohesion})^{0.2}} \tag{13}$$

Also in Appendix A is a typical example showing the pavement thickness calculation using the nomograph (Fig. 11) that solves the suggested new formula. This calculation illustrates how each layer may be evaluated, one on top of the other, to give the most economical thickness of cover material. Naturally, when applying this formula on a broad-scale highway system, some additional factors of safety may be allowed, especially when the traffic factor cannot be accurately estimated. It is, of course, uneconomical to change structural sections too often on a single project so that some "rounding off" in sections is needed. For these reasons, may States provide design standards for minimum thicknesses of pavement and base for certain traffic conditions and allow only the subbase layer to be varied. In the example shown in Appendix A, however, the thickness determined by formula is shown.

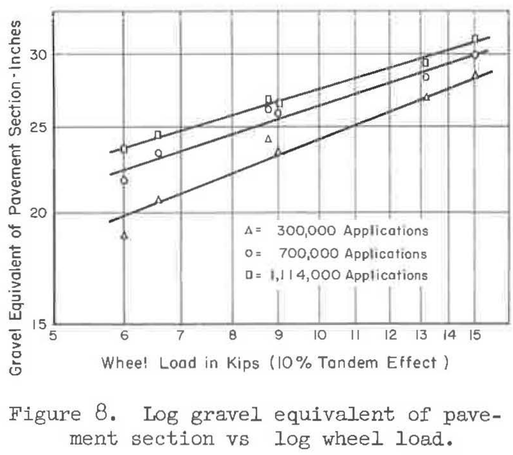


Figure 8. Log gravel equivalent of pavement section vs log wheel load.

By introducing an expression for an increased tensile strength allowance,

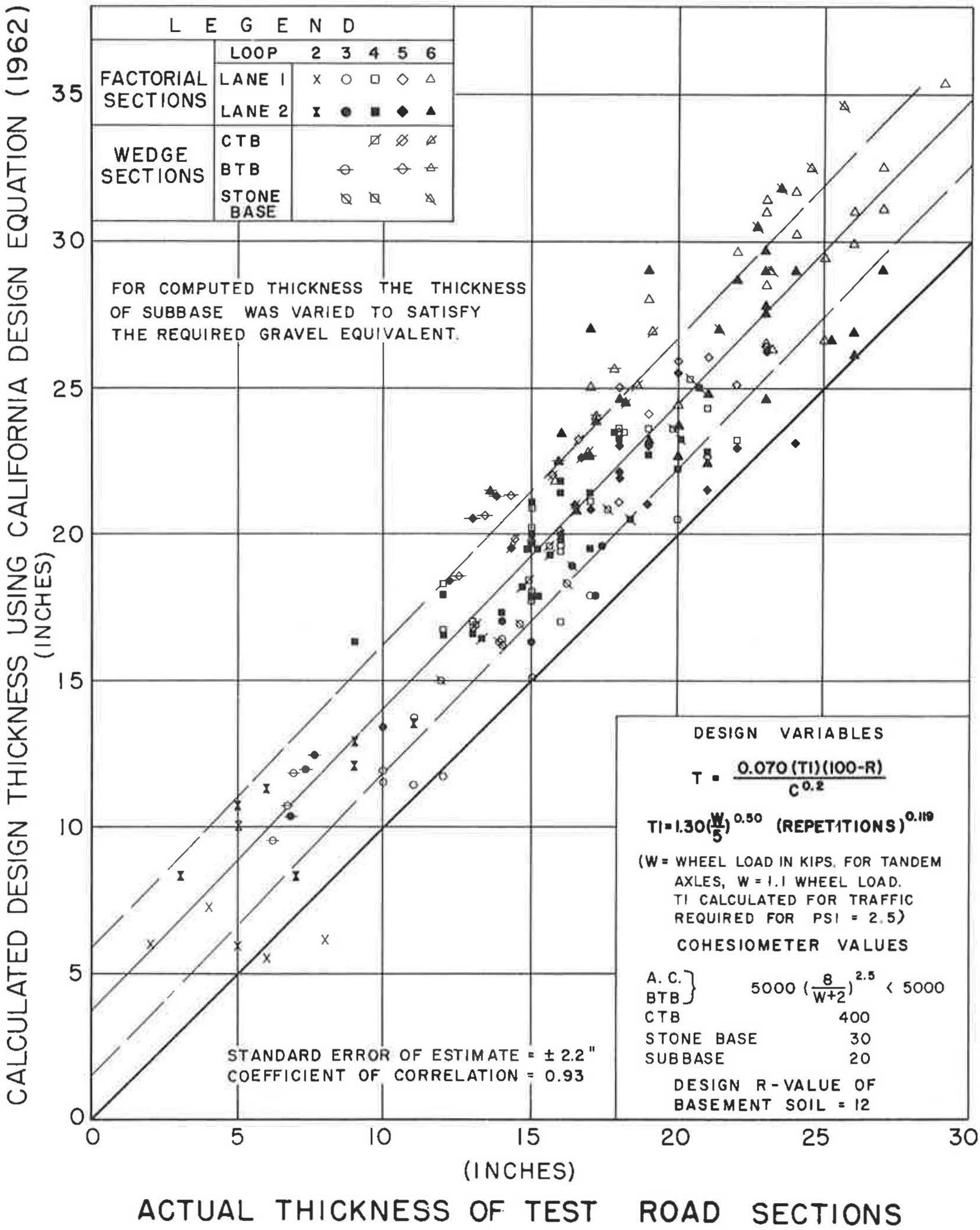


Figure 9. Thickness of test sections at AASHO Test Road vs calculated design thickness using California design equation (1962).

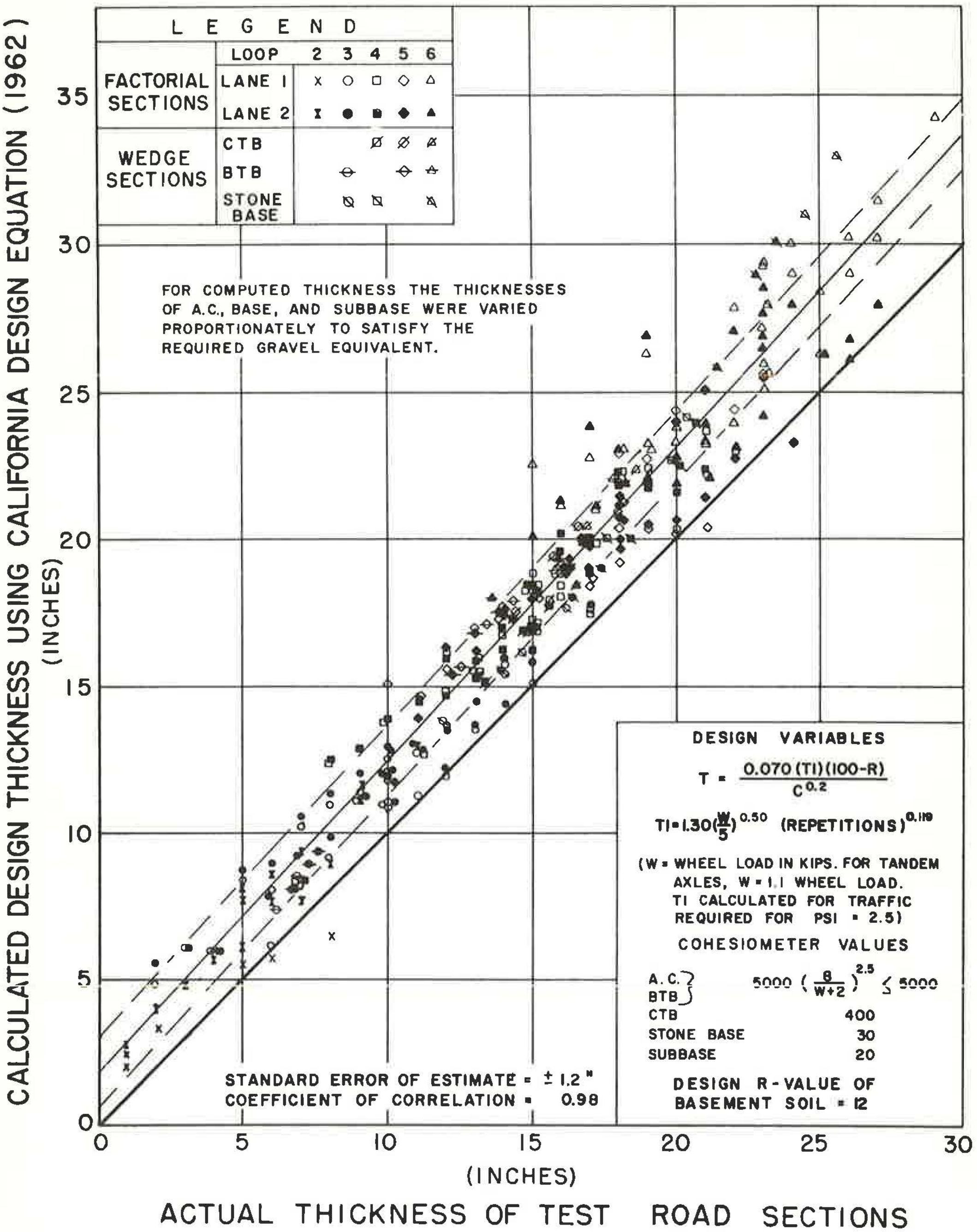


Figure 10. Thickness of test sections at AASHO Test Road vs calculated design thickness using California design equation (1962) (equivalent sections).

coupled with a readjustment of the load and repetition exponents, a better correlation with the test road data is obtained (Fig. 9). The improved correlation is measured numerically by the reduction in the standard error of estimate from ± 2.7 in. shown in Figure 5 to ± 2.2 in. and the increase in coefficient of correlation from 0.87 to 0.93.

Figures 5 and 9 contain the statement that the sections that failed during the first spring thaw are omitted. The reason for doing this was lack of time to study all of the sections on the flexible pavement portions of the Test Road, and because most of the highway mileage in California is in frost-free areas, an analysis was made first on those sections that survived the first spring thaw. These two charts (Fig. 5 and 9) report the results of this study.

Also, in Figure 9 all the error is being placed in the subbase layer. This gives a maximum error of estimate and a minimum coefficient of correlation when such things are evaluated in terms of thickness of section. The reason for this is obvious in that the error between actual and calculated thickness must be determined first in terms of gravel equivalent thickness, then converted to inches of surface, base, and subbase. Subbase, having the lowest equivalency, gives the greatest error. Surface material, having the highest equivalency, will give the lowest error.

An example of how the correlation factors might be changed is shown by Figure 10 which shows the data for all sections on the road test. This represents the same plot as that in Figure 9 except that the difference in gravel equivalent was prorated by thickness of layer to surface, base, and subbase. When this is done, the error of estimate ± 2.2 in. in Figure 9 becomes ± 1.2 in. and the coefficient of correlation raises to 0.98.

SUMMARY

Figures 9 and 10 serve to illustrate the influence of the method used to judge the efficiency of a design formula. These figures also show that the thicknesses computed by means of the California formula (based on measured properties of the basement soil, the subbase, base, and surface, also the effects of traffic expressed by the traffic index) are in nearly all cases equal to or greater than the thickness indicated in the serviceability index of 2.5 on the test road. A similar relationship could be shown for 2.0 or 1.5 serviceability index. This is the only relationship that can be justified, as a design formula should provide a structure stronger than any section known to fail. In other words, no portions are expected to show failure within the design life of the project. It may be argued that this provides too great a factor of safety and that the theoretical thickness, in many cases, would be excessive compared to the depths reported as just adequate on the test road. In judging the validity of a pavement design formula by comparing the calculated thickness with test road data, the following facts must be considered:

1. Every effort was made to secure a high degree of uniformity on the test road, and no such uniformity of performance can be expected on a highway constructed by ordinary methods.
2. Traffic was continued on the test road for a period of only two years. This means that the test road did not undergo the large number of cycles ranging from high to low temperature and from wet to dry which affects the performance of a highway over a period of many years.
3. The asphaltic pavements and bases on the test road were only two years old at the end of the test. Virtually all asphalts harden to some degree and become brittle with age. One could not assume an equally good performance over a long period of time on the average highway.

Taking these considerations into account, any design formula should be on the conservative side and provide some factor of safety over the thickness and strength of pavement which appeared to be barely adequate on the test road. The following are primary and important advantages of the California formula:

1. The California procedure utilizes numerical values derived from physical tests of the basement soil, the subbase, base and pavement.

2. The California method provides a logical means for converting miscellaneous traffic wheel loads to a single number—the traffic index. This number bears a direct linear relationship to the thickness of pavement structure required.

3. The California method has been in use for approximately 13 years and has demonstrated that it can accommodate wide variations in the type of soil, type of base, and type of pavement as well as variations in wheel loads and in the number of load repetitions.

ACKNOWLEDGMENTS

In addition to the many persons who have worked on the California pavement design formula for previous papers, the authors wish to acknowledge the help of several whose combined efforts made this paper possible. The review of the AASHO Road Test data was carried out by Parks M. Adams, David W. Eckhoff, Carl R. Sundquist, and Robert O. Watkins. Correlation tests were performed under the general direction of Ernest Zube. Clyde Gates, Daniel Howe, and Merle Nelson all contributed to the testing program and analysis of results.

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REFERENCES

1. Hveem, F. N., and Carmany, R. M., "The Factors Underlying the Rational Design of Pavements." HRB Proc., 28:101-136 (1948).
2. Kersten, M. S., Cosgrove, F. H., Goldbeck, A. T., Haines, R. M., and Olmstead, F. R., "Progress Report of Subcommittee on Methods of Measuring Strength of Subgrade Soil—Review of Methods of Design of Flexible Pavements." HRB Proc., 25:8-18 (1945).
3. Nijboer, L. W., and van der Poel, C., "A Study of Vibration Phenomena in Asphaltic Road Constructions." Proc., AAPT, vol. 22 (1953).
4. Sherman, G. B., "Recent Changes in the California Design Method for Structural Sections of Flexible Pavement." Proc., 1st Annual Highway Conference (1958).
5. Throughout the text references are made to data from the AASHO Road Test. These data are obtained primarily from "The AASHO Road Test, Report 5, Pavement Research." HRB Special Report 61 E (1962).
6. California Division of Highways Materials Manual of Testing and Control Procedures, vol. I.
7. "Monroe Culculating Machine Methods—General Statistics." Monroe Calculating Machine Company, Inc. (1960).
8. Lee, A. R., and Croney, D., "British Full-Scale Pavement Design Experiments." Internat. Conf. on the Structural Design of Asphalt Pavements (1962).
9. Nichols, F. P., Jr., "Effects of Compaction and Subgrade Stabilization on Deflection and Performance of Virginia Pavements." Internat. Conf. on the Structural Design of Asphalt Pavements (1962).

Appendix A

TYPICAL EXAMPLE OF PAVEMENT THICKNESS DESIGN

Given the resistance value of a basement soil = 20, as measured by the Hveem stabilometer, cohesion of gravel = 20, cohesion of crushed stone base = 30, cohesion of asphalt concrete = 2,000, and the average daily truck traffic shown in Table 7, the number of trucks counted in each class is multiplied by the yearly EWL constants to determine the annual EWL.

TABLE 7

Truck Class by Axle	Average Daily No. of Trucks[a]	EWL (5,000-lb) Yearly Constants	Yearly EWL
2	679	250	169,750
3	344	815	280,360
4	295	965	284,675
5	1,539	2,385	3,670,515
6	113	1,475	166,675
Total			4,571,975

[a]Two-directional count.

Assuming that in 10 years the traffic will have increased 50 percent, the average annual design EWL is $\frac{1.0 + 1.5}{2}$ (4,571,975) = 5,715,000 EWL. The total design EWL for 10 years is 10 (5,715,000) = 57,150,000 EWL.

Traffic index (TI) is calculated from the EWL by Eq. 12:

$$TI = 1.30\ (EWL)^{0.119}$$

For the preceding example TI = 10.9; therefore, 11.0 should be used.

Pavement Thickness Calculation

The required gravel equivalent GE is determined by

$$GE = \frac{0.070\ (\text{traffic index})\ (100 - \text{resistance value})}{(\text{cohesiometer value of gravel})^{0.2}} \qquad (14)$$

For the example, GE = 33.8 in.

Surface Thickness

To determine the thickness of asphalt concrete required, the nomograph in Figure 11 is used. The California specifications require a crushed aggregate base to have an 80 R-value minimum. With a straightedge, Scale E is intersected at 80 R-value and Scale F at 11.0 traffic index. The intersection of this line with Scale G is the thickness of gravel equivalent required. Using this value of 8.5 in. gravel equivalent as a turning point, Scale H is intersected at the appropriate value of cohesion for the AC. This cohesion value is found from

$$C = C_T \left(\frac{5.14}{TI}\right)^{2.5} \leq C_T \qquad (15)$$

in which C_T = 2,000 and TI = 11.0; therefore, C = 300. The intersection of this line with Scale I gives 4.9 in. of asphalt concrete required. In design, 5 in. should be used.

Base Thickness

Using California Standard Specifications of 60 R-value minimum for subbase materials (this value can be and is modified in the Special Provisions to fit local aggregate conditions), Figure 11 shows a gravel equivalent of 16.9 in. needed over the subbase materials. Because the 5-in. AC is equivalent to 8.6 gravel equivalent inches, 8.3 in. remains to be satisfied by the base material. A cohesion of 30 for a good crushed rock product would indicate 7.5 in. to be satisfactory. Therefore, 8.0 in. of base material should be used.

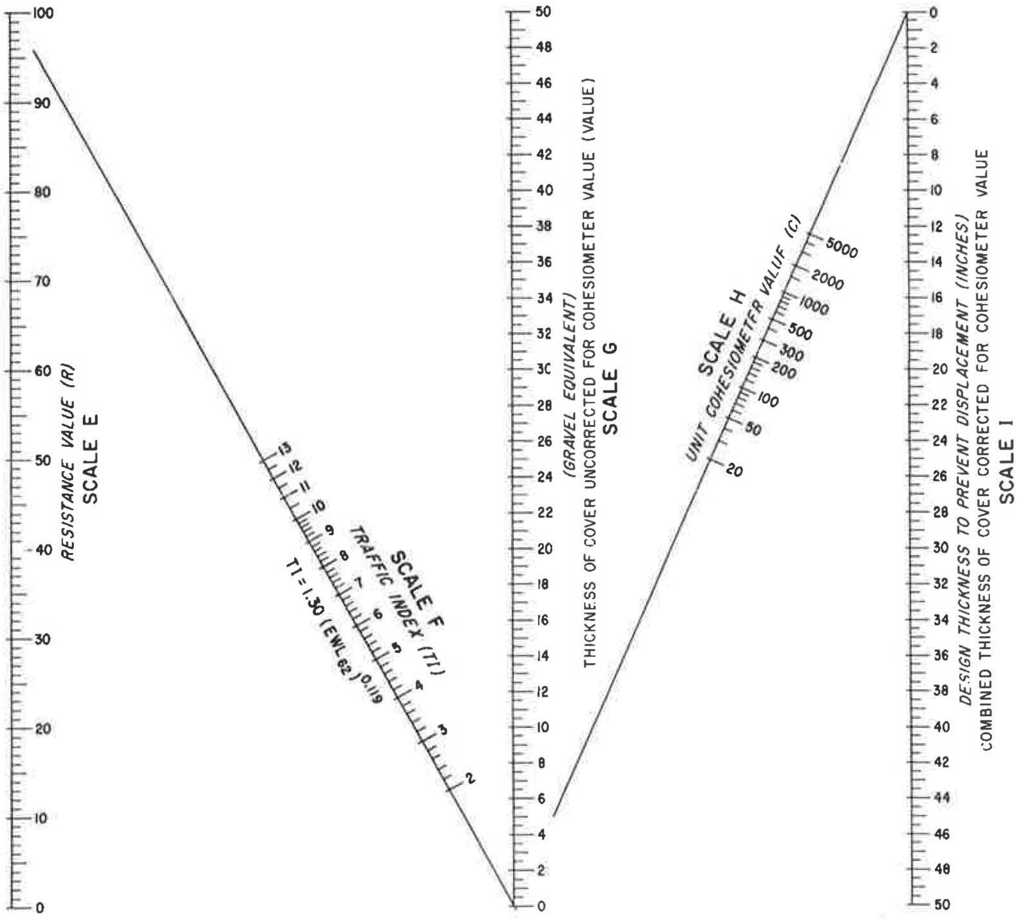

Figure 11. Nomograph for solving $T = \frac{0.070\ (TI)\ (100 - R)}{C^{0.2}}$

Subbase Thickness Design

Figure 11 also shows that a 20 R-value basement soil with TI = 11.0 requires a gravel equivalent of 33.8 in. The GE of surface and base is 5-in. surface = 8.6-in. GE, and 8-in. base = 8.8-in. GE; and the total GE = 17.4 in.

Required thickness of subbase is, therefore, 33.8 - 17.4 = 16.4 in. Thus, 17.0-in. subbase should be used.

The minimum allowable structural section over 20 R-value basement soil for very heavy truck traffic is 5-in. AC, 8-in. Class I aggregate base, and 17-in. 60 R-value subbase, for a total thickness of 30 in.

Various other structural sections that might also be found satisfactory for the preceding traffic and soil conditions would be 5-in. asphalt concrete, 8-in. cement-treated base, 11-in. subbase, for a total of 24 in.; and 5-in. asphalt concrete, 8-in. bituminous-treated base, 12-in. subbase for a total of 25 in.

Appendix B

DEFINITION OF STATISTICAL TERMS

Coefficient of Correlation

Linear correlation is used to determine whether a relationship exists between two variates. There may be a direct, an inverse, or no relationship between variates.

Pearson's coefficient of correlation for ungrouped data has theoretical limits of ± 1. A value of r approaching +1 indicates a direct relationship between the variates, whereas a value approaching -1 indicates an inverse relationship. A value of r tending toward 0 indicates that no relationship exists between the variates.

$$r = \frac{\dfrac{\Sigma xy}{N} - \dfrac{\Sigma x \Sigma y}{N^2}}{\sigma_x \sigma_y} \tag{16}$$

in which

$$\sigma_x = \sqrt{\frac{\Sigma x^2}{N} - \frac{(\Sigma x)^2}{N^2}};$$

$$\sigma_y = \sqrt{\frac{\Sigma y^2}{N} - \frac{(\Sigma y)^2}{N^2}};$$

x = actual thickness (inches);
y = computed thickness (inches);
N = number of points;
σ = standard deviation; and
r = coefficient of correlation.

Line of Regression

If the plotted data indicate a linear relationship between the variates, then a straight line that best fits the data is called a line of regression. The general equation is expressed as $y = mx + b$ and the values of m and b are found by using the method of least squares.

$$y = mx + b \tag{17}$$

in which

$$m = \frac{N\Sigma xy - \Sigma x \Sigma y}{N\Sigma x^2 - (\Sigma x)^2};$$

$$b = \frac{\Sigma y \Sigma x^2 - \Sigma x \Sigma xy}{N\Sigma x^2 - (\Sigma x)^2};$$

N = number of points;
y = computed thickness (inches);
x = actual thickness (inches);
m = slope; and
b = y-intercept.

Standard Error of Estimate

Standard error of estimate (7) measures the concentration of the points clustered about the line of regression. A zone drawn parallel to the line of regression on either side at a vertical distance S_y will include approximately 67 percent of the points. A vertical distance $2S_y$ will include approximately 95 percent of the points.

$$S_y = \sigma_y \sqrt{1-r^2} \tag{18}$$

in which

σ = standard deviation;
r = coefficient or correlation; and
S_y = standard error of estimate (inches).