Simulation of Traffic Flow to Obtain Volume Warrants for Intersection Control

RUSSELL M. LEWIS, Assistant Professor of Civil Engineering, Rensselaer Polytechnic Institute, and
HAROLD L. MICHAEL, Associate Director, Joint Highway Research Project, Purdue University

The paper reports the results of a research project in which a digital simulation model was developed to determine volume warrants at street intersections. The particular type of intersection studied was the four-legged, right-angled intersection of a high-volume major arterial street with a lower-volume minor arterial street. The major arterial had four travel lanes with parking prohibited, whereas the minor arterial had two travel lanes with parking permitted on both sides. Both arterials were operated as two-way streets.

Two types of intersection control were studied, the semi-traffic-actuated signal and the two-way stop sign. The delays encountered at the intersection were measured and used as criteria for the establishment of warrants.

• THE CONTROL of vehicular traffic at street intersections has been one of the most studied items in the traffic engineering field, yet much remains unknown. Intersections are the critical element of streets in that their characteristics determine the efficiency and capacity of the entire street system. Here, one common area must accommodate the vehicular flow of two streets and the conflicting maneuvers of their several approaches.

Several methods of traffic control have been developed for intersections. These include the basic right-of-way rule, stop signs, and various types of traffic signals. General warrants have been proposed for these methods of control based on vehicular volume, pedestrian traffic, accident records, and other factors. These warrants were developed in part on empirical data, but in some cases are little more than "rules of thumb." Although significant effort has been devoted to the determination of warrants for fixed-time traffic signals, specific warrants for actuated signals are lacking (30).

One of the foremost problems in the development of warrants is the difficulty in determining the specific behavior of a general class of intersections. Computer simulation, however, offers tremendous possibilities in this area. Digital simulation possesses some unique properties when compared with more conventional methods, such as the important advantage of bringing the traffic facility into the laboratory for study under practically limitless conditions. Precise control of the dynamic traffic process can be maintained and many unwanted variables eliminated. Parameters are varied at the discretion of the programer, rather than by chance alone.

Simulation on digital computers is not new, but comparatively little work has been done in this area. Production-model, general-purpose computers were not readily available until about 1954. In 1956, three digital computer simulations were reported in the traffic engineering literature. Gerlough (16) discussed the simulation of traffic flow on a freeway. Wong (46) described the simulation of a portion of a multilane boulevard. Goode, Pollmar, and Wright (18) constructed a model of a signalized intersection. Two separate studies of intersection simulation performed by Benhard (5)

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and Lewis (27) dealt with the intersection of 2 two-lane streets with actuated signal control. The models were greatly simplified in that turning and passing were prohibited. These and other early simulations of an intersection permitted only a limited and somewhat arbitrary action of vehicles. Later investigations incorporated several refinements. The simulation of freeway interchange traffic was presented by Perchonok and Levy (33) and Glickstein, Findley, and Levy (17). Wohl (45) developed a model depicting the traffic behavior in a freeway merging area. A recent paper by Stark (38) described the simulation of nine blocks of a city street, and research performed by Kell (25) involves the simulation of the intersection of 2 two-lane urban streets under various types of traffic control.

PURPOSE AND SCOPE

The purpose of this research was twofold. The first phase of the study was the development of a model, whereby a traffic intersection could be simulated on a digital electronic computer. The particular intersection chosen for study was a four-legged, right-angled intersection of a high-volume major arterial street with a lower-volume minor arterial street. The major arterial had four lanes with parking prohibited, and the minor arterial had two travel lanes with parking permitted on both sides. Both arterials were operated as two-way streets. The intersection is typical of many intersections located in intermediate urban areas and in suburban areas.

The second phase of the study was the operation of the simulated intersection under two appropriate types of traffic control—the two-way stop sign and the semi-traffic-actuated signal. The purpose was to establish a realistic set of volume warrants for the given class of intersection. Such warrants were to indicate when, from the standpoint of delay, it would be advantageous to go from stop sign control to actuated signal control. The major variables used were the traffic volumes carried by the two streets.

Delay was a most important factor in the determination of volume warrants. From an economic viewpoint, the type of traffic control device preferred is one that results in the minimum delay to motorists. Total or overall delay is the type of delay that has the greatest significance when comparing two types of intersection control (6). Total delay encompasses any delay as caused by the existence of traffic control devices and interaction with other vehicles. An undelayed straight-through vehicle will pass through the intersection area at its desired speed. An undelayed turning vehicle will decelerate to a safe turning speed and then regain its desired velocity. Any travel time in addition to these requirements is considered a delay.

To relate delays observed at various levels of volume, the figure of merit used was average delay per vehicle. It was realized, however, that the intersection may be operated so that the average delay per vehicle is small, but the average delay per side-street vehicle is excessive. To picture this situation, the average delays per vehicle for each street were also considered. To permit comparison with field studies and utilization in economic studies, stopped delay was included as an additional output of the simulation model.

DEVELOPMENT OF SIMULATION MODEL

Mode of Representation

Various methods can be used to represent the flow of traffic within the computer. The first traffic simulations employed a physical notation (16, 18). Binary 1's were used to represent vehicles and 0's were used to indicate the spaces between vehicles. Groups of memory cells were figuratively placed end to end to represent the roadway. Algebraic manipulations caused the 1's to change position, thereby simulating the flow of traffic. With this mode of representation, the vehicles must occupy only certain specified locations (bit positions) along the roadway and individual vehicles have no identity as such.

The memorandum notation uses an entire word to represent a vehicle. Various parts of the word are used for such individual characteristics as its time of entry into the system and its desired velocity. These parts may be extracted and interpreted as desired. This method is more versatile in that each vehicle's characteristics are identifiable as it moves through the system, making it possible to compute the delays associated with an individual vehicle.

Most simulation programs using the memorandum notation have considered the roadway as a series of unit blocks, which represent the various positions a vehicle can occupy (46). Each block is one lane wide and has a length equivalent to some fraction or multiple of the unit vehicle length. Thus, a vehicle may occupy only a limited number of discrete positions. Velocity and acceleration are step functions of the unit block and the time increment of scanning. This procedure is adequate for some models, but offers severe restrictions when realistic total delays are desired.

A third method of representation has been called a mathematical notation (16). This form of representation is similar to the memorandum notation, except that, in addition to its other characteristics, each vehicle is associated with its own position indicator. Its position is therefore continuous within the accuracy of the computer. At any time a vehicle's new position can be computed simply by adding its velocity (in units related to the time increment) to its previous position coordinate. Spacings between vehicles are available from their respective coordinates and the vehicle length.

A fully mathematical notation has generally been avoided, as it requires a more complicated logic. Maneuvers, such as turns, which must be accomplished at a specified location are more difficult when the vehicle may occupy any position at the start of the maneuver. Furthermore, the mathematical processing of vehicles is more complex, thereby increasing the computer time required. On the other hand, the elimination of limitations on the position increment will allow some increase in the size of the time increment for the same model accuracy.

The mode of representation employed is a variation of the mathematical notation. Because an algebraic compiler was selected for the coding of the program, the entire representation had to be in an algebraic format. Moreover, because bit manipulation could not be performed within the scope of the compiler, the various vehicle characteristics could not be coded within the same word. One word had to be used for each characteristic. Inasmuch as a vehicle was then composed of several computer words, it became cumbersome to shift the vehicle for its relative position changes in the system. Movement was accomplished, in effect, by making the roadway "flow" past the vehicle.

The entire roadway system was represented by a three-dimensional mathematical array (Fig. 1). The length dimension corresponded to relative position along the roadway; that is, vehicle data were stored in adjoining array elements in the same order as the vehicles occupied a particular lane. The vertical dimension of the array accommodated all the information or characteristics of each particular vehicle, and the width dimension represented the several traffic lanes.

Because the vehicles did not move within the array, a very long array would have been needed to handle all the traffic within a study period. Thus, the memory capacity of the computer would soon have been exceeded. To circumvent this problem, the concept of a "circular array" was employed. The ends of the array were mathematically connected to provide a roadway that was sufficient in length to handle all the traffic within the study section. Two items of information were kept in special registers for each traffic lane—the index position of the lead vehicle and the number of vehicles in the lane. Knowing this information enabled the extraction of the characteristics for any vehicle by stating its relative position with respect to the lead vehicle. Each vehicle maintained its own record of its absolute position, or X-coordinate. When a vehicle left a lane, the lead index was shifted to the vehicle immediately behind, and the lane count was decreased by one.

In actuality, all drivers of vehicles within the roadway system are continually and simultaneously making decisions and modifying their behavior. The computer, however, can make only one simple logical choice at a time. To control all the occurrences at any given instant, it must process all decisions sequentially. In other words, it must process each decision for every vehicle, for each vehicle in every lane, and for each lane within the system. It must do this in accordance with a prescribed sequence for each instant of time to be considered.

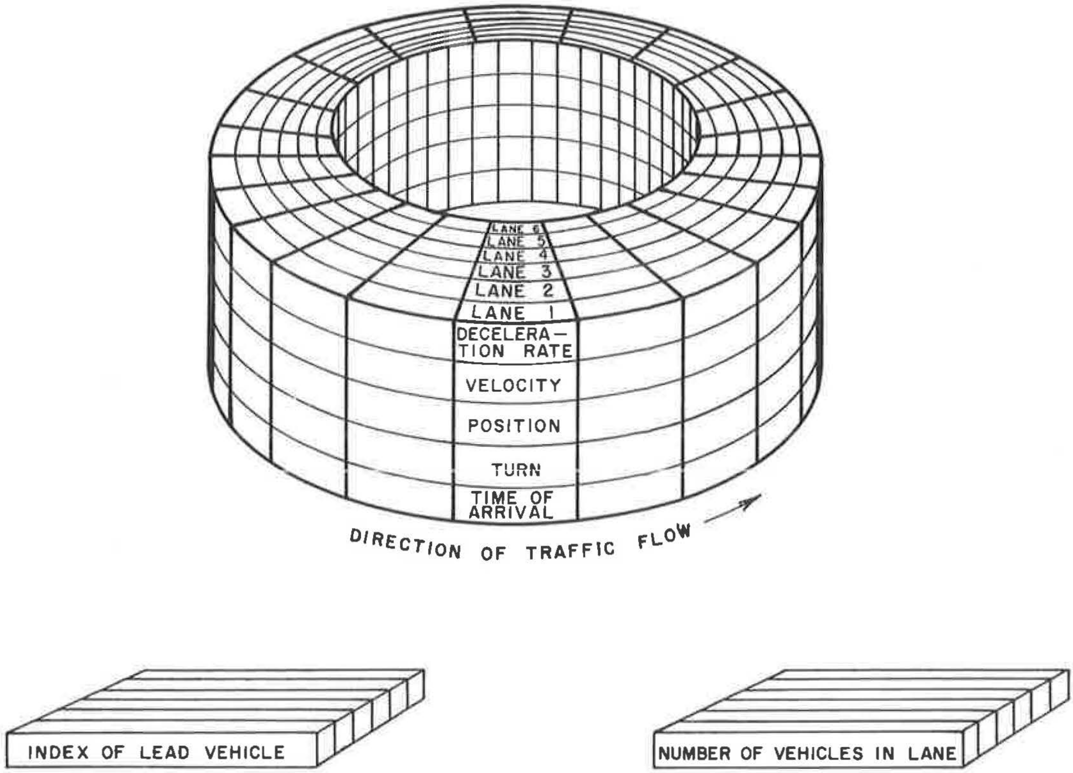


Figure 1. Computer representation of traffic.

The selection of a suitable time increment is most important. If the time increment chosen is too large, it will not be possible to simulate all the events that may occur. If it is too small, many additional computations will be required for each event. This will result in additional computer time, thereby increasing the cost of running the problem on the computer.

The increment selected must be no smaller than the smallest event to be simulated. The time for all other events must be some multiple of the time increment. This requirement will apply to such items as the traffic controller settings, acceptable gaps for crossing, and reaction times.

In most simulations, a critical factor is the minimum headway for vehicles; that is, because vehicles may enter the system only at each time increment, their minimum time spacing will be equal to, or some multiple of, this increment. A method was developed for this study which isolated vehicle generation from the time increment. The only requirement was that the minimum intervehicular headway and the time increment be some multiple of each other. A time increment of 1 sec between successive scans of the system was selected as adequately meeting this criterion.

Mathematics of Vehicle Behavior

It was postulated that vehicles are operated so as to minimize their delays. All vehicles attempted to travel at an average velocity $\overline{V}$ of 30 mph or 44 ft/sec. Units of feet and seconds were used throughout this study for the sake of simplicity.

A uniform rate of speed change was assumed under free-flowing conditions. Although observed rates of acceleration are not quite uniform, the uniform case supplies an adequate approximation of the real case. It was assumed that all vehicles would attempt to employ an acceleration rate $\overline{A}$ or 3 ft/sec^2 (20, 37, 42). It was recognized, however, that higher rates of acceleration are used in crossing maneuvers when vehicles are under the pressure of opposing traffic flows. This behavior pattern was accommodated for vehicles accelerating from a stopped or near stopped condition at stop signs and for left-turn maneuvers. In such cases, vehicles must accelerate rapidly to take advantage of available gaps in the traffic stream. For this case, accelerations of 6, 5, and 4 ft/sec^2 were used for the first 3 seconds and an acceleration of 3 ft/sec^2 was used thereafter.

Studies have shown that deceleration rates of 8 to 9 ft/sec^2 are comfortable, and rates of up to 16 ft/sec^2 can be used without severe discomfort (22, 44). Typical deceleration rates are approximately twice the value of typical acceleration rates (3). Under free-flowing conditions, the average deceleration rate $\overline{D}$ which all vehicles attempted to use was assumed to be 6 ft/sec^2. However, when drivers are presented with the amber signal, much higher rates may be expected. For this situation, a deceleration rate of up to 12 ft/sec^2 was used (11, 32).

In the simulation model, velocity and rates of acceleration and deceleration were sometimes affected by the presence of other vehicles within the system. In no case, however, was $\overline{V}$ or $\overline{A}$ exceeded.

Car-Following Procedure.—In recent years, there has been much interest in the development of car-following models. Newell (31) and Greenberg (19) have used physical analogies based on the kinetic theory of gases and on fluid dynamics. Research involving actual field studies and theoretical investigations has been reported by Pipes (34), Chandler, Herman, and Montroll (8), Herman et al. (23), Gazis, Herman, and Potts (13), and Kometani and Sasaki (26). Most of these studies are concerned with the capacity or near capacity situation, in which cars are following each other as closely as possible. They attempted to relate the spacing between successive vehicles to such factors as the velocity and acceleration of the lead vehicle and velocity and reaction time of the following vehicle.

For this research problem, a car-following model was needed which would be applicable for a wide range of traffic volumes including well-below capacity conditions. Such a model was developed along practical lines which yielded relatively realistic results insofar as delays were concerned. This car-following relationship was based on the premise that vehicles do not collide, and that they are operated in a safe manner. The safety margin, however, may be extremely small. This premise is justified by the fact that the number of accidents is infinitesimal as compared with the number of opportunities for their occurrence.

Vehicles stopped in a queue are at some average minimum spacing which includes the vehicle length and a clear space. This average minimum spacing P is measured from the front bumper of the lead vehicle to the front bumper of the following vehicle. Field studies have shown that P is approximately 22 ft (4, 21, 39).

When vehicles are moving at the same speed, the minimum desired spacing S (measured from front to front of adjacent vehicles) has been shown to be linearly related to velocity V (21). The relationship chosen for the uniform velocity case was

$$S = P + K_1V \tag{1}$$

in which K_1 is a constant with the dimension of time. When all units are given in feet and seconds, $K_1 = 1$ sec. This equation is substantiated by the practical consideration of braking behavior. For example, two vehicles are traveling at equal velocities and at minimum spacing; both have similar braking capabilities. If the preceding vehicle stops and if a brake reaction time of 1 sec is assumed for the following vehicle, both vehicles will come to rest with S equal to P.

For a following vehicle traveling at a higher speed than the preceding vehicle, the spacing relationship selected was

$$S = P + K_1V + \frac{K_2}{2\overline{D}} (V - V')^2 \tag{2}$$

in which K_2 is a constant with the dimension of velocity. When all units are in feet and seconds, K_2 = 1 ft/sec. V' is the velocity of the preceding vehicle. When a preceding vehicle is traveling at a uniform velocity (or stopped), Eq. 2 provides for the deceleration of a following vehicle to the velocity of the preceding vehicle at an average rate of approximately $\overline{D}$. If a preceding vehicle is also decelerating, higher decelerations of the following vehicle result. The maximum rate of deceleration for this case can be shown to occur when a following vehicle is traveling at $\overline{V}$ and approaching a preceding vehicle that has a velocity of $\overline{V}/2$, and at the instant when the following vehicle starts to slow down to maintain its required spacing, the preceding vehicle also starts decelerating. For this unusual case, the deceleration rate of the following vehicle is 11 ft/sec^2 when $\overline{V}$ = 44 ft/sec. This maximum rate is still within the reasonable range of deceleration rates.

Because the computer program processed vehicles sequentially by proceeding down the lane backwards, the preceding vehicle had already been relocated at the time the following vehicle was processed. The decision as to which variation of the spacing equation was applicable was therefore based on the current velocity of the preceding vehicle. Thus, the generalized spacing equation used was

$$S = P + K_1 V_t + \frac{K_2}{2\overline{D}} (V_t - V_t')^2 C \qquad (3)$$

in which C = 1, when $V_{t-1} > V_t'$, and C = 0, when $V_{t-1} \leq V_t'$. The subscripts refer to increments of time. For the case in which the preceding vehicle is at a higher velocity, acceleration limitations become significant and spacing is seldom critical.

Spacing Restriction. —Spacing is merely one of several restrictions that limit vehicle movement. When spacing is critical, speed is adjusted so that at any time a vehicle's position with respect to the preceding vehicle is no closer than desired. By using this restriction, the rate of deceleration for the model was not specified directly but was permitted to vary over some range. Ordinarily the deceleration rate did not exceed the value $\overline{D}$ used in the spacing equation.

To derive the spacing restriction, Z_s is set equal to the distance that a following vehicle travels during one time increment. Using subscripts to refer to time and primes for the preceding vehicle, the movement during one increment of time is as shown in Figure 2.

Assuming a uniform rate of acceleration during each time increment and using a time increment equal to unity, the basic equation for movement during each time increment is

$$Z = \tfrac{1}{2} (V_{t-1} - V_t) \qquad (4)$$

Figure 2 shows that

$$Z_s = X_t' - X_{t-1} - S \qquad (5)$$

First, in the case in which $V_{t-1} > V_t'$, the appropriate spacing condition is selected

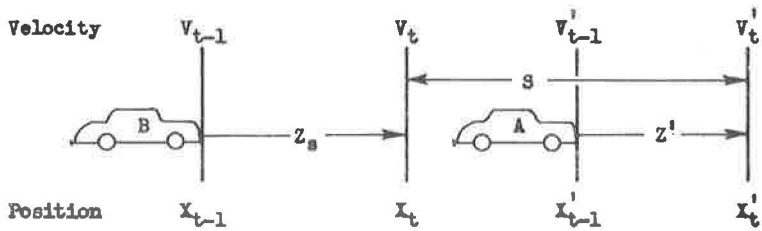


Figure 2. Diagram of spacing restriction.

from Eq. 3. Substituting this value for S in Eq. 5,

$$Z_s = X_t' - X_{t-1} - P - V_t - \frac{1}{2\overline{D}} (V_t - V_t')^2 \tag{6}$$

Replacing V_t by the value from Eq. 4,

$$Z_s = X_t' - X_{t-1} - P - 2Z_s + V_{t-1} - \frac{1}{2\overline{D}} (2Z_s - V_{t-1} - V_t')^2 \tag{7}$$

Expanding by the quadratic formula, and selecting the significant root,

$$Z_s = \tfrac{1}{2} V_{t-1} + \tfrac{1}{2} V_t' - \frac{3\overline{D}}{4} + \left[\frac{9\overline{D}^2}{16} - \frac{\overline{D}}{4} V_{t-1} - \frac{3\overline{D}}{4} V_t' + \frac{\overline{D}}{2} (X_t' - X_{t-1} - P)\right]^{1/2} \tag{8}$$

when $V_{t-1} > V_t'$.

Next, in the case in which $V_{t-1} \leq V_t'$, using Eqs. 3 and 5,

$$Z_s = X_t' - X_{t-1} - P - V_t \tag{9}$$

Substituting for V_t the value obtained from Eq. 4, and solving for Z_s,

$$Z_s = \frac{1}{3}\left[X_t' - X_{t-1} - P + V_{t-1}\right] \tag{10}$$

when $V_{t-1} \leq V_t'$.

Acceleration Restriction.—Another restriction used in the model and involved with vehicle behavior is based on acceleration. Stated simply, this restriction assumes that when free to do so, a vehicle will continue to accelerate at $\overline{A}$ until the maximum permissible velocity $\overline{V}$ is attained. If Z_a is the distance that a vehicle travels in one time increment based on the acceleration restriction, then, considering the time increment as unity and using the relationship in Eq. 4,

$$Z_a = \tfrac{1}{2}\left[V_{t-1} + (V_{t-1} + \overline{A})\right] \tag{11}$$

in which $(V_{t-1} + \overline{A})$ must be $\leq \overline{V}$.

Stopping Restriction.—The model also permits a vehicle decelerating for a traffic control device, such as a traffic signal or stop sign, to adjust its speed for each time increment. If Z_d is the distance traveled during a time increment based on the stopping restriction and x is the distance between the vehicle and the stopping point at time t-1, then, using the basic motion equations based on uniform acceleration,

$$V_t^2 = 2D\,(x - Z_d) \tag{12}$$

By solving Eq. 12 for V_t and substituting this value in Eq. 4,

$$Z_d = \tfrac{1}{2} V_{t-1} + \tfrac{1}{2}\left[2D(x - Z_d)\right]^{1/2} \tag{13}$$

By using the quadratic formula to solve for Z_d and selecting the significant root,

$$Z_d = \tfrac{1}{2} V_{t-1} - \frac{D}{4} + \left(\frac{D^2}{16} - \frac{D}{4} V_{t-1} + \frac{D}{2} x\right)^{1/2} \tag{14}$$

Turning Restriction.—During a turning maneuver, it was assumed in the development of the model that a free-flowing vehicle will decelerate uniformally up to a point during the turn called the "turn point," and that once past the turn point the vehicle will accelerate normally. If v is the maximum velocity permitted at the turn point, and x is the distance from the turn point at time t-1, Z_t is the distance the vehicle will travel in one time increment in accordance with the turning restriction. The basic law of motion for uniform acceleration applicable to this situation is

$$V_t^2 - 2\overline{D}\,(x - Z_t) = v^2 \qquad (15)$$

Substituting in Eq. 4 the value for V_t obtained from Eq. 15 gives

$$Z_t = \tfrac{1}{2}\, V_{t-1} + \tfrac{1}{2}\left[v^2 + 2\overline{D}\,(x - Z_t)\right]^{1/2} \qquad (16)$$

Solving by the quadratic formula and selecting the significant root,

$$Z_t = \tfrac{1}{2}\, V_{t-1} - \frac{\overline{D}}{4} + \left(\frac{\overline{D}^2}{16} + \frac{v^2}{4} - \frac{\overline{D}}{4}\, V_{t-1} + \frac{\overline{D}}{2}x\right)^{1/2} \qquad (17)$$

This equation is only applicable when the turning vehicle does not proceed beyond the turn point during the given time increment. When $Z_t > x$, a different solution obtains. It is convenient for this solution to consider first whether the maximum velocity permitted at the turn point can be exceeded. Based on acceleration capabilities, the maximum velocity possible at the turn point is given by

$$V_{max} = \left(V_{t-1}^2 + 2\overline{A}x\right)^{1/2} \qquad (18)$$

If $V_{max} \leq v$, the turning restriction is not applicable. When $V_{max} > v$, the alternate solution for Z_t is required.

In the latter situation, if (1-T) is the time required for the vehicle to reach the turn point, the velocity at the turn point will be equal to v, and the time required is given by the distance divided by the mean velocity:

$$1 - T = \frac{2x}{V_{t-1} + v} \qquad (19)$$

The time available for acceleration after passing the turn point is then given by T, and

$$Z_t = x + v\,T + \frac{\overline{A}\,T^2}{2} \qquad (20)$$

For this one instance, a special computation of V_t is required, as the change in velocity is not at a uniform rate during the previous time increment. The applicable equation is

$$V_t = v + \overline{A}\,T \qquad (21)$$

The turn point was located at some point approximately midway through the turning maneuver. Turning vehicles with a relatively high initial velocity start decelerating at some point before turning and start to regain speed at some point during the turn (12). Vehicles with a low initial speed, however, may accelerate throughout a major portion of the turning maneuver or even during the entire movement. A midturn location for the turn point would have this effect. From the standpoint of delay, its location is not critical.

Turning speeds depend to some extent on the direction of turn. There is a tendency to use a slower turning speed for right turns than for left turns due to the shorter turning radius available. The relative lack of interference for right turns, however, may have the opposite effect. Equal turning speeds were therefore assumed for both the left- and right-turn maneuvers. The maximum velocity at the turn point has been observed to be about 15 ft/sec (12, 15, 36); therefore, this value was used.

Vehicle Processing.—During every time increment, each vehicle was processed by proceeding sequentially down each lane in a direction opposite to traffic flow. The following procedure was used for any particular vehicle.

1. The distance traveled during the time increment was computed in accordance with each of the relevant restrictions to movement. These restrictions may have been due to spacing (Eqs. 8 or 10), acceleration (Eq. 11), stopping (Eq. 14) or turning requirements (Eqs. 17 or 20).

2. The critical Z was selected as the smallest of the ones computed in step 1. If the critical Z was negative, it was replaced by zero.

3. The new X-coordinate was computed as

$$X_t = X_{t-1} + Z \tag{22}$$

4. The new velocity was computed in accordance with Eq. 4 as

$$V_t = 2Z - V_{t-1} \tag{23}$$

If, however, the turning restriction was critical and when the vehicle had passed the turn point during the time increment, the velocity was determined by Eq. 21. When V_t was negative, it was replaced by zero.

Vehicle Generation

Vehicle generation was accomplished by using a theoretical probability distribution. The headways, or time spacings between vehicles, were determined by a modified binomial distribution which incorporated the contagious or platooning effect of vehicular traffic and provided for the existence of a minimum headway (29).

A pseudo-random number series was generated according to the multiplicative congruential scheme investigated by Taussky and Todd (40). The random number series could be reset at will to two different initial values, thereby providing two independent reproducible series. The ability to reproduce the series was essential to assure that identical traffic occurred when the intersection was operated under each of the two different types of traffic control.

When a vehicle was generated, it was considered to have arrived. The time of arrival, T_a, of a vehicle was defined as the time at which it would have reached a given point in the roadway had it experienced no delay. It can be seen that arrival was independent of intersection conflicts and the effect of traffic control devices. The point at which arrival occurred was termed the beginning of the lane and was designated X_0. Ordinarily, a long approach lane would be needed if the effect of the backup of traffic was not to be felt at the beginning of the lane. Such a long approach lane would have resulted in an added computational load because many additional vehicles would have been included in the system.

To eliminate the necessity for a long approach lane, a backlog list was used. When a vehicle was generated, it was placed directly in the backlog and designated by its time of arrival. Its turning maneuver, if any, was determined binomially using a pseudo-random number. Its time of arrival and turn data were stored in a circular array similar to the lane array previously described. This backlog file provided the additional function of separating the generation time increment from the scanning time increment.

Entering was defined as the process of leaving the backlog and starting down the approach lane. Vehicles were entered so as to minimize their potential delay. Because the acceleration rate was less than the deceleration rate, vehicles were entered at the maximum velocity.

The beginning of the lane was located sufficiently far back from the stop line so that the turning and stopping restrictions were not applicable. Thus, there were only two factors affecting the entering movement. The location of an entering vehicle in a lane was determined as follows. The distance traveled by a vehicle during one scan time increment while entering, based on the spacing relationship of Eq. 3, is

$$Z_{e_1} = X_t' - X_0 - P - \overline{V} - \frac{1}{2\overline{D}} (\overline{V} - V_t')^2 \tag{24}$$

The second consideration is how far the vehicle could have traveled since its time of arrival; that is,

$$Z_{e_2} = \overline{V} (t - T_a) \tag{25}$$

in which t refers to the clock time at this instant. The critical Z_e is the smaller of the two, as obtained by Eqs. 24 and 25. Negative values of Z_e merely mean that the vehicle remains in the backlog.

The backlog was inspected at each scanning time increment to determine whether the first vehicle listed could enter. It can be seen that vehicles were entered in the same position that they would have occupied had the scanning time increment been equal to the generation increment. Thus, a vehicle was entered with an X-coordinate of $(X_0 + Z_e)$ and a velocity of $\overline{V}$.

Description of Intersection

Physical Characteristics. — The intersection studied was a four-legged, right-angled intersection of a high-volume major arterial with a lower-volume minor arterial street. Hereafter, these streets are called the main street and the side street, respectively. The main street had four traveled lanes 11 ft wide, with parking prohibited. The side street had four 10-ft lanes with parking permitted, thereby providing but two travel lanes. This same configuration is applicable for side streets with low volume and with parking prohibited, for at low traffic volumes a multilane side-street approach is used as if it has but one travel lane. This layout also approximates a rural intersection where the side street is but 20 ft wide and a larger curb radius is employed.

Figure 3 shows the intersection. The stop line was located 12 ft behind the extensions of the curb lines. The near stop line for all approaches was designated as station 2,000 ft. The beginning of the lane was a program variable with a minimum of 0 ft. The end of the lane was located 350 ft beyond the far stop line for each maneuver. The possible maneuvers were left turn (LT), straight through (ST) and right turn (RT).

Release points were established where the scanning of vehicles was no longer required. These points occurred at locations where a vehicle no longer blocked either following vehicles making different maneuvers or vehicles from the opposing approach. For example, when a side-street RT vehicle reached its release point (33 ft beyond the stop line), a following ST or LT vehicle was free to proceed. Likewise, it did not conflict with an opposing side-street LT vehicle. In Figure 3, the arrowheads show the release points for two approaches of the intersection. The complete lane stationing is given in Table 1.

Rules of Operations. — Because no device yet conceived can duplicate all the characteristics of man, the vehicle operator, certain simplifications had to be imposed in the simulation. Such simplifications should be of such an order that the problem can be solved efficiently, yet not so overly simplified that the results would be meaningless. The intent, therefore, was to rule out both the unusual and insignificant behavior patterns, and maintain the behavior typical of the vast majority of vehicles and their operators.

To this end, certain general rules were established for the formulation of the model with vehicle behavior postulated as follows:

1. Vehicles enter the system in accordance with a prescribed random distribution.
2. Turning maneuvers are made in accordance with the desires of each vehicle as determined randomly at the time it enters the system.
3. Vehicles travel so as to minimize their delays.
4. The maximum velocity is fixed at 44 ft/sec.
5. Free-flowing acceleration is at a uniform rate of 3 ft/sec^2, except in special instances where a higher initial rate may be used.

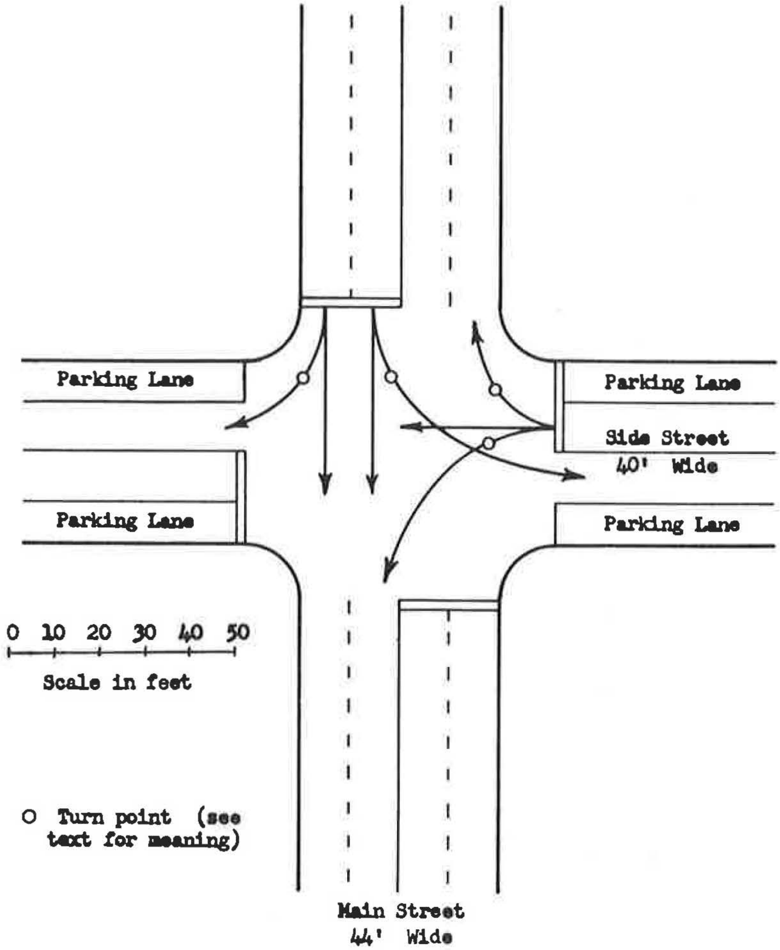


Figure 3. Diagram of intersection.

TABLE 1

STATIONING OF INTERSECTION[1]

Street	Movement	Release Point (ft)	Distance Between Stop Lines (ft)	End of Lane (ft)
Main	LT	2,070	61	2,411
	ST	2,041	64	2,414
	RT	2,041	33	2,383
Side	LT	2,057	61	2,411
	ST	2,034	68	2,418
	RT	2,034	33	2,383

[1] Beginning of lane = Variable ($\geq$ 0 ft)
Near stop line = 2,000 ft
Turn point = 2,016 ft
LT wait point = 2,016 ft
(2,032 ft for left-turn-hold position)
End of lane = 350 ft from far stop line

6. Free-flowing deceleration is at a uniform deceleration rate of 6 ft/sec^2. For stopping at an amber traffic signal, rates of 6 to 12 ft/sec^2 may be used.
7. All vehicles are approximately 17 ft long and, when stopped, have a fixed minimum spacing of 22 ft.
8. Pedestrian interference is negligible and therefore neglected.
9. For the main street all right turns are made from the outside lanes, and all left turns are made from the inside lanes.
10. Passing is permitted for second- and third-in-line ST main-street vehicles when the lead vehicle is decelerating to perform a turning maneuver.
11. The turning maneuvers of approaching vehicles are not indicated to opposing traffic until they reach the extension of the near curblines.
12. In situations of equal advantage, vehicles from the south or west approaches give way to vehicles on the north and east approaches, respectively.
13. Turning vehicles must not exceed a velocity of 15 ft/sec at the turn point (a point located 16 ft beyond the near stop line.)
14. Vehicles react to preceding vehicles and to traffic controls in accordance to the behavior equations previously derived.
15. Merging and crossing maneuvers are made in accordance with fixed gap-acceptance criteria.
16. Vehicles follow the preceding vehicle only until such time as the preceding vehicle reaches the release point.
17. Vehicles are released as soon as their movement is independent of the intersection. The time for the vehicle to reach the end of its lane is then computed and added to its travel time.
18. When the intersection is operating under signal control, there is a location called the "left-turn-hold position." One vehicle from a side-street approach can wait for an acceptable turning gap at this location without obstructing any side-street maneuvers other than a following left turn.
19. LT vehicles will not proceed past a point 16 ft beyond the near stop line (32 ft for vehicles in the left-turn-hold position) until they can be released through an acceptable gap in the opposing traffic stream.
20. When the intersection is operating under two-way stop control, all side-street vehicles give way to all main-street vehicles. Furthermore, no delays are incurred by main-street vehicles due to the presence of side-street traffic.
21. No vehicles travel backwards, collide, or break down.

Description of Traffic Controller

Semi-traffic-actuated control is applicable to intersections of a heavy-volume or high-speed road with a lightly-traveled minor road. Traffic actuation of the signal is by means of detectors placed on the side street only. The signal is normally green on the main street, changing to the side street only as a result of detector actuation. In the type of controller used in this study, the side-street green is proportioned to the side-street volume of traffic with some maximum limit. On expiration of the required or maximum side-street interval, the green signal automatically reverts to the main street where it remains for a predetermined minimum interval. This type of control provides for a minimum of disturbance to main-street traffic at the intersection.

The adjustable time intervals used in semi-traffic-actuated control are as follows:

1. Main-street minimum green interval,
2. Main-street amber interval,
3. Side-street initial green interval,
4. Side-street extension green interval,
5. Side-street maximum green interval, and
6. Side-street amber interval.

Performance Characteristics

The effect of the behavior equations is to fix the relationships that must exist with respect to position and velocity. Each equation did not establish a specific behavior pattern but merely placed boundaries on behavior. Thus, the turning restriction did not force all vehicles within a certain zone to decelerate at a specified rate. It simply stated that for each position there was a velocity that could not be exceeded. In application, a vehicle could actually be accelerating in conformance with this restriction.

Free-flowing acceleration and deceleration were essentially uniform and at specified rates. Other speed changes were not fixed directly. They could vary over a specified range, and could be non-uniform from one time increment to the next.

Starting Performance. —The behavior equations were based on the performance characteristics of individual vehicles. To assure their adequacy, the behavior of a traffic stream in the model should be compared with field observations of traffic flow.

For example, a line of vehicles has stopped at minimum spacing at a red traffic signal. When the light turns green, the lead vehicle must react to the signal changes and then start to move. Likewise, each successive vehicle must react in turn to the preceding vehicle before getting under way. This reaction time has been observed to be approximately 1 sec per vehicle when pedestrian interference is negligible (21). The initial acceleration rate is in the range of 5 ft/sec^2, but this rate decreases materially after the first few seconds (3, 21).

In the model, however, a different situation existed. When the first-in-line vehicle was free to move, it accelerated uniformly at 3 ft/sec^2. Once the lead vehicle had moved, all other vehicles were free to move sequentially in accordance with the spacing restriction. Each vehicle, therefore, experienced an instantaneous creeping, which decreased in magnitude for positions farther away from the stop line. This creeping effect was such that it compensated for the high initial acceleration of the real vehicle. If actual starting in the model is defined as occurring when a vehicle attains a velocity of a few feet per second, then the equivalent reaction times for successive vehicles were very nearly 1 sec per vehicle. Despite initial deviations, the model starting performance gave similar eventual results insofar as delay was concerned.

Extensive field studies have been conducted to determine queue starting headways. Greenshield's (21) well-publicized values for passenger cars at urban intersections are 3.8 sec for the first-in-line car with subsequent values ranging down to 2.2 sec for the fifth car, and a constant of 2.1 sec per car thereafter. These values are the time intervals after the green signal for each subsequent car in the queue to enter the intersection (pass beyond the extension of the near curbline). Bartle, Skoro, and Gerlough (2) conducted similar tests at signalized intersections and obtained a mean value of 3.83 sec for the first car to enter the intersection. Other research studies (10) have yielded similar results.

Figure 4 shows the queue starting headways for the intersection model. Using a 1-sec reaction time for the first-in-line vehicle, it can be seen that it required 3.8 sec for it to enter the intersection. Subsequent headways decreased to about 2.1 sec for the fourth vehicle. By the time the twentieth vehicle entered the intersections, it was traveling at a velocity close to $\overline{V}$ and the minimum intervehicular headway of 1.5 sec existed.

Reaction Time. —Perception-reaction time requirements were not included as a program variable. This characteristic of behavior is recognized, however, and was indirectly included in several applicable situations.

In the model, vehicles instantaneously reacted to certain events, such as the changes in traffic signal aspects. The model traffic signal, however, was set so that each aspect was displayed after a delay which was equal to the reaction time required for the real signal. Assuming that the reaction time to all signal aspects is the same, the signal timing would be unaffected. The model signal phasing, therefore, was considered to have a 1-sec lag as compared to the real signal.

As derived, the car-following equations neglect reaction time. Some reaction delay obviously exists in the real situation, and research has been performed to determine its magnitude (8). It has also been observed that the reaction time may be zero in some cases. Second-in-line vehicles often react directly to a traffic signal change, and following vehicles may react directly to the speed changes of the vehicle in front of the preceding vehicle (21). The inclusion of a reaction lag in the car-following equations would have added realism, but would not have had an appreciable effect on delay. It

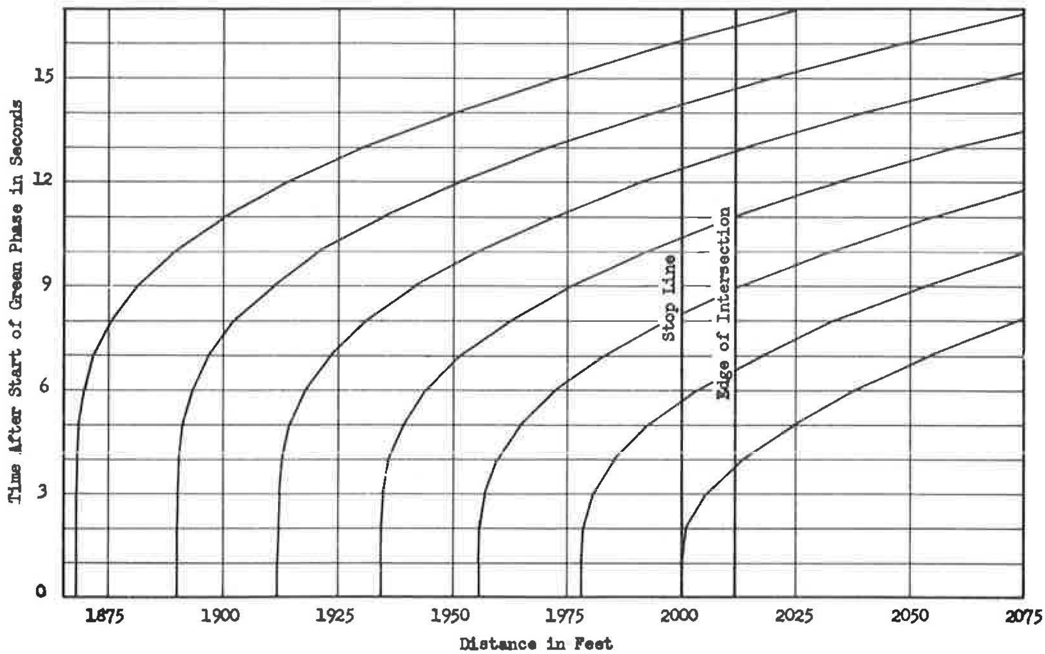


Figure 4. Queue starting headways.

has already been shown that the starting performance for following cars provided delays that are essentially equivalent to observed behavior.

Theoretical studies (8) have demonstrated that a reaction lag can cause instability. The result of this instability would be an amplification of speed changes by following vehicles that may reach a resonant condition. Such behavior does occasionally occur in nature, as is evidenced by some chain-type rear-end collisions. This type of behavior was undesirable in the model because it is an uncommon occurrence. For the car-following equations used in the model, velocity oscillations were damped for following vehicles.

PROGRAMING AND RUNNING MODEL

Flow Charting

Figure 5 is the flow chart for the simulation program. This chart points out the relationships between the four routines included in the program.

Input and Initialization Routine. —For each new problem, the input and initialization routine first reads in the specifications for the run. The input data are summarized in Table 2. This routine next initializes the program by computing constants for the problem, zeroing counters, and setting switches.

Because the intersection is initially devoid of traffic, some time is necessary to load the system with vehicles and reach a statistical steady-state condition. This time is called the transient time and is an input variable. Data collected during the transient time are not statistically significant and must be disregarded. After the transient time has expired, a partial reset takes place. This resets all values that are used in the computation of delays.

Traffic Controller Routine. —The traffic controller routine is shown in Figure 6. Signal phases are adjusted in accordance with the demands of side-street traffic as determined by the detector switch. The detector switch is used at two places in the program. Once the main-street minimum green interval is timed out, switch C is used

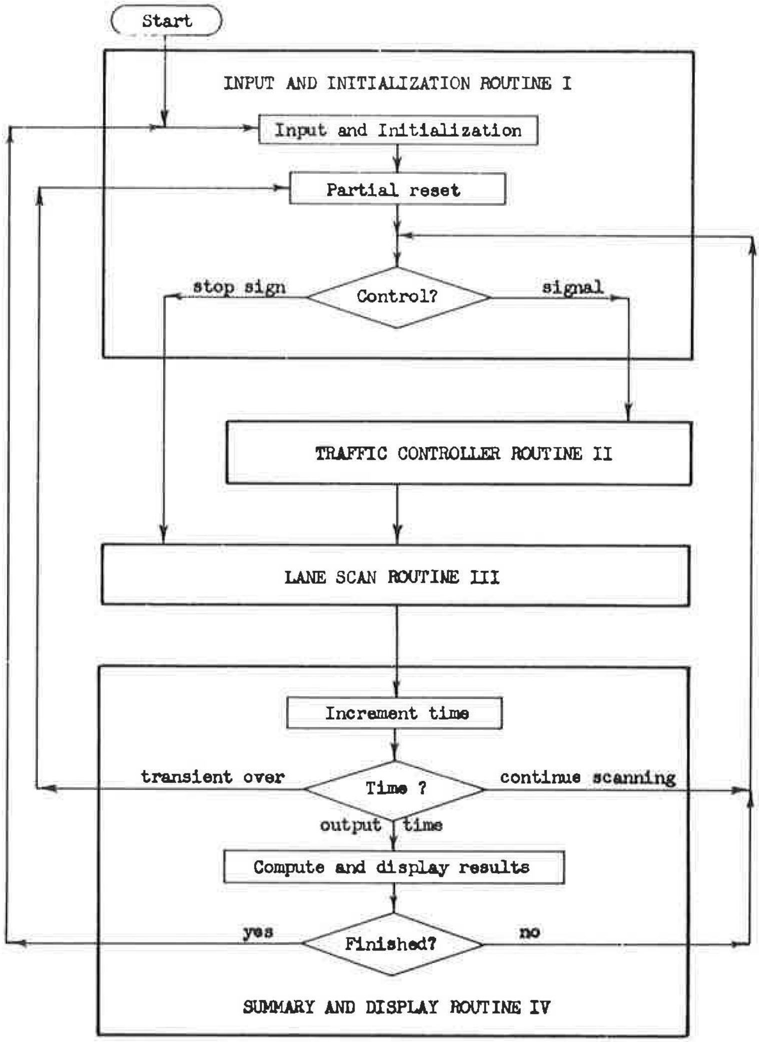


Figure 5. Master flow chart.

to initiate a new side-street green phase. Detector switch M is used to reset a new extension interval during the side-street green phase. In addition to the action of the lane scan routine, the detector switch may also be set by the controller itself in subroutine R. This accomplishes the memory feature whereby a new side-street green phase will be initiated if the side-street maximum green interval is timed out before the completion of an extension green interval. Once the detector switch is actuated, it will remain in that position until action is taken by the controller. Actuation is canceled when subroutine P is reached.

Lane Scan Routine.—It was originally planned to handle the simulation of the intersection under stop-sign control and signal control as two separate projects. It soon became obvious that many portions of the programs were common to both types of control, and they were incorporated into a single program. Further study indicated that programing economy could also be achieved by making the same program elements handle all six approach lanes. Figure 7 is a flow chart for the lane scan routine. Most of the logic shown in this flow chart is the switching necessary to permit this single routine to handle all six approach lanes under either of two types of traffic control.

TABLE 2

INPUT INFORMATION

Run identification number
Control mode (signal or stop sign)
Production mode (output format)
Reset mode (selects random number series)
Backlog limit
Transient time
Sample time
Number of samples
Beginning of lanes for each street
Position of detectors on each side-street approach
Critical lag for each side-street approach
Traffic volume for each lane
Headway distribution parameters
Fraction of right and left turns for each lane
Traffic signal controller intervals

The lane setup subroutine A1 initializes the program for the scanning of each particular lane. Lanes are scanned in the following order: north outside, north inside, south outside, south inside, east, and west. Next, the settings for the M switches are selected. These switches are all set identically to any of four positions, depending on which street is to be scanned and whether the control mode is signal or stop sign.

The vehicle setup subroutine A2 establishes the procedure whereby each vehicle within the lane is processed. For each time increment, scanning starts with the lead vehicle and proceeds sequentially to the last vehicle in the lane. On entering this routine initially, the register containing the number of vehicles in the lane is examined. If the lane is empty, the scanning process is bypassed and control is transferred to the vehicle generation subroutine W.

The spacing bypass subroutine C1 provides for a special case applicable only to side-street vehicles when operating under signal control. In this one instance, an ST or RT second-in-line vehicle is not required to follow a lead vehicle that is in the LT hold position. All other non-lead vehicles must behave in accordance with the spacing restriction.

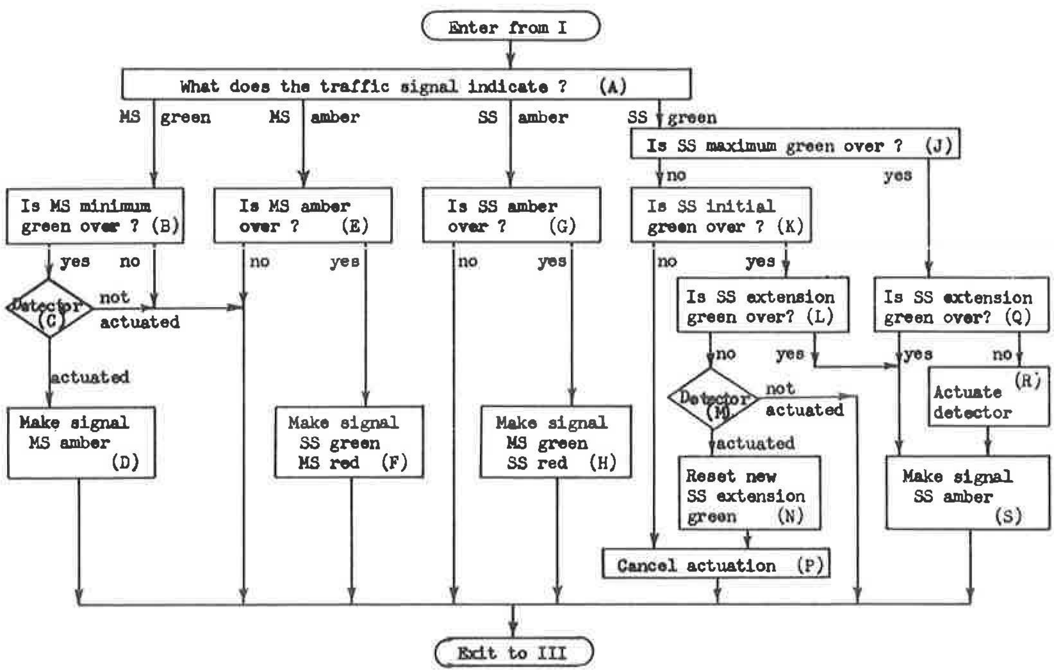
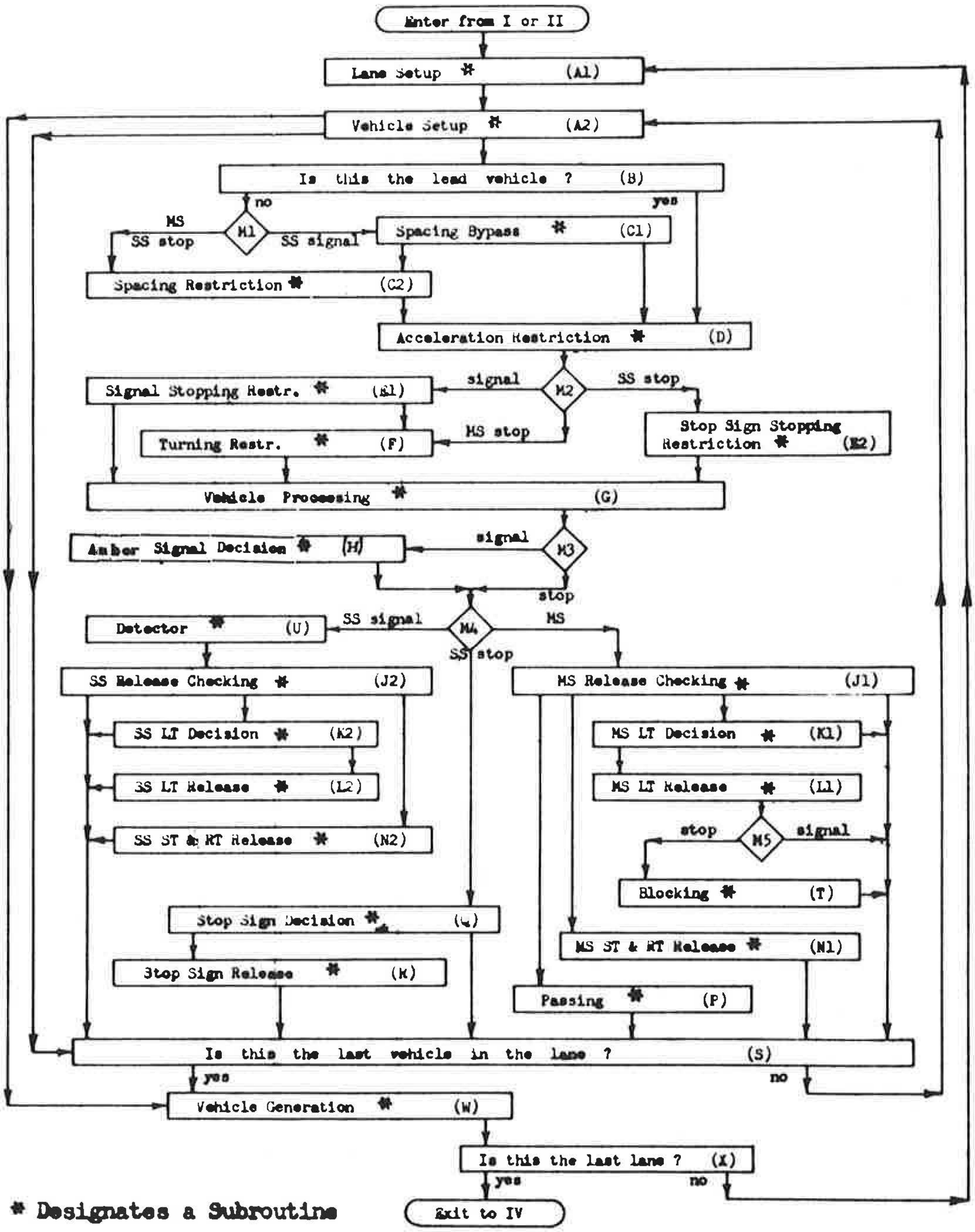


Figure 6. Traffic controller routine II.



*** Designates a Subroutine**

Figure 7. Lane scan routine III.

When the traffic signal is employed, the stopping restriction is handled by subroutine E1. The first vehicle to stop at an amber signal has been "tagged" in the amber signal decision subroutine. Only tagged vehicles are processed by this subroutine; subsequent vehicles that stop at the amber or red signal do so in accordance with the car-following criteria.

The stopping restriction is handled by subroutine E2 for stop-sign control. A deceleration rate of 6 ft/sec^2 is always used in the computation of Z_d at a stop sign.

Vehicle processing takes place in subroutine G. The new X-coordinate and velocity for the current clock time are computed in accordance with the relevant behavior restrictions. If the velocity is less than 4.5 ft/sec, the pertinent stopped delay counter is incremented. A different counter is used for each lane and each turning movement.

Vehicle behavior when confronted by an amber traffic signal is taken care of in the amber signal decision subroutine H. When the traffic signal is green, no action is taken. When the signal changes to amber, the routine checks each vehicle to determine which one will be the first to stop. The criterion used is that the vehicle must be able to stop at the stop line with a uniform rate of deceleration which does not exceed 12 ft/sec^2. If the required deceleration rate is less than 6 ft/sec^2, a value of 6 is substituted for the computed deceleration rate. When a vehicle meets this criterion, it is tagged, and its applicable rate of deceleration is recorded for use by subroutine E1. If no vehicle is found which can stop within the acceptable deceleration range or if the lane is empty, a procedure is established whereby the next vehicle that enters the lane will be tagged. Tagging attempts are terminated once a suitable vehicle is tagged or when the signal turns green.

After the vehicle has been repositioned for the current time increment, it is necessary to determine whether it is able to be released. The release checking subroutines are J1 and J2. The prerequisite for release is that the vehicle has reached or passed the release point. If an ST or RT vehicle can be released, control is then transferred to the appropriate release routine. If an LT vehicle is in a position to intercept the opposing traffic stream, control is transferred to a decision routine.

Special considerations are involved when a vehicle must cross an opposing stream of traffic. First, the crossing vehicle examines the position of other vehicles within the intersection area, as defined by the extensions of the curblines. The presence of any vehicle within this area may block the desired movement. It is assumed that once a vehicle enters the intersection area, its path through the intersection becomes obvious. On the other hand, a vehicle that has not reached the intersection area is as yet uncommitted. The crossing vehicle must assume that the approaching vehicle can make any acceptable turning maneuver.

The logic sequence that the crossing vehicle performs is now established. The following questions are asked for each opposing approach:

1. Are there any vehicles in the opposing approach lane? (Vehicles that have passed the release point are automatically excluded.)
2. What is the effect of any vehicles that are now within the intersection area?
3. Is there sufficient time to cross before a vehicle from the opposing approach lane will enter the intersection, assuming that it will make the critical maneuver? (Vehicles stopping at an amber traffic signal do not conflict with an LT vehicle.)

To determine whether an acceptable gap exists in the opposing traffic stream, the time for the opposing vehicle to reach the intersection area is computed. This computation is based on the assumption that the opposing vehicle continues moving at its present velocity. The distance between the edge of the intersection area and the point of physical contact provides a factor of safety, which will permit some acceleration of the opposing vehicle. The available crossing time is computed as determined by each conflicting vehicle that opposes the crossing maneuver. The shortest time found is selected as the critical one.

For LT vehicles, the time to reach its release point is first computed. This clearance time is based on the turning vehicle's accelerating in accordance with the applicable behavior equations. It is recognized that the turning vehicle may undergo a high initial acceleration from a stopped or near stopped condition. This latter clearance time is computed by assuming that it uses this higher initial acceleration from a stopped condition at the LT wait point. When the smaller of these two clearance times is equal to or less than the time available, the LT vehicle proceeds. If an acceptable gap does not exist, and if the vehicle has passed the LT wait point, it is moved back to this point and stopped.

In the real situation, decisions are not made at a single point. The LT vehicle continually examines the opposing traffic stream as it approaches the intersection and adjusts its velocity accordingly. Such behavior is complex and difficult to simulate. The procedure employed in the model is not realistic, but it yields similar results. The loss of advantage due to a complete stop is offset by the higher initial starting acceleration. High decelerations will be experienced by following vehicles. In actuality, lesser rates of deceleration would be required over a longer distance from the intersection. This difference has a minor effect on the delays to following vehicles, because they are blocked by the LT vehicle. If, as is often the case, the LT vehicle must stop in the real situation, the simulation results are equivalent.

The release subroutines are L1, N1, L2, and N2. The side-street subroutines are used only when the intersection is operating under signal control. The procedure employed in all these subroutines is similar (Fig. 8). For the ST and RT subroutines, the vehicle will have already reached the release point. The time to reach the end of the lane is computed by using the vehicle's present position and velocity.

The time to reach the end of the lane is then added to the existing travel time as determined by the difference between the clock time and time of arrival. To obtain the delay, the travel time as required for an unimpeded free-flowing vehicle is subtracted from the actual travel time. This delay is then added to the appropriate delay counter; a different one being used for each turning movement and for each lane. Finally, various housekeeping functions are performed. These consist of adjusting the counter for the number of vehicles in the lane and the register which holds the index of the lead vehicles.

The stop-sign decision subroutine Q is used when the side-street vehicle is less than 3 ft from the stop line. Thus, a vehicle may be released with a velocity as high as 6 ft/sec. This provision accommodates the fact that some vehicles at a stop sign will proceed without making a full stop. The procedure for the determination of an available gap in main-street traffic is similar to that previously described for the LT decision subroutines. Certain vehicles, such as main-street LT vehicles, may still occupy the intersection area even though they have been removed from the lane arrays. That these vehicles may have the proper effect on vehicles stopped at a stop sign, blocking registers are employed. A separate register is used for each side-street approach and for each turning movement. The registers contain the earliest clock time at which the various side-street vehicles may proceed.

The critical lag is the smallest time lag that a crossing vehicle will accept. This quantity is an input variable. If the available time for crossing is equal to or greater than the critical lag, the side-street vehicle is released. The critical lag required for ST and LT vehicles has been found to be similar. RT vehicles will accept a shorter time lag due to the merging nature of this maneuver. Raff (35) found that the critical lag for RT vehicles is about 80 percent of that for the other maneuvers. Greenshields (21) observed that this value was approximately 68.4 percent. A compromise value of 75 percent of the critical lag was used for RT vehicles in the model.

Vehicles released from a stop sign are assumed to use a high rate of initial acceleration as previously described. Delay computations are performed in a manner similar to that employed by the other release routines. The values of the delay for each released vehicle are stored in a special file which is later examined to select the 85th percentile delay.

Until it clears the intersection area, a side-street vehicle released from the stop line may block vehicles from the opposing approach. The blocking time is dependent on the turning maneuvers of the blocking and the blocked vehicles. As each vehicle is released, the blocking registers for the opposite approach are set. The appropriate blocking time is added to the clock time. If the resultant time is later than the time presently contained in the blocking register, the register is reset to the new value.

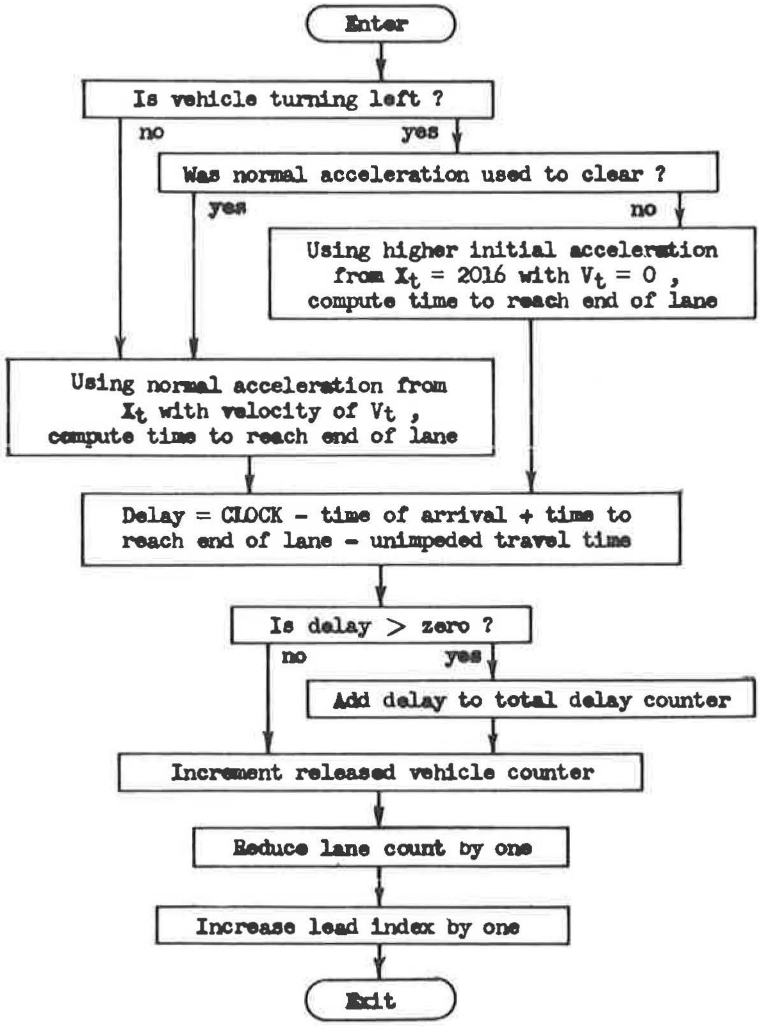


Figure 8. Flow chart for typical release subroutine.

The earliest time of release of the subsequent vehicle at the stop sign is controlled by the car-following equations. The following vehicle may not be released until it has reached a position which is less than 3 ft from the stop line. Various delays will be experienced by the following vehicle depending on the position-velocity combinations that exist for both vehicles. If both vehicles are stopped at a minimum spacing at the time that the first vehicle is released, 4 sec are required for the following vehicle to move into the release position. Likewise, if the following vehicle is in the process of decelerating when the first vehicle is released, at least 4 sec are required before the following vehicle can be released.

The blocking subroutine T performs the blocking functions necessitated by the release of main-street LT vehicles. This subroutine is bypassed when the intersection is operating under signal control. Side-street vehicles located to the left of the main-street approach are delayed for the actual clearance time required for the main-street vehicle. Side-street vehicles located to the right of the main-street approach are delayed for 1 sec less than this required clearance time.

The generation of vehicles is accomplished by subroutine W (Fig. 9). A pair of switches operates so that generation is ordinarily attempted twice each time increment, thereby providing for headways in ½-sec steps. The action of these switches, however, is such that once a vehicle is generated, a subsequent generation will not be attempted for 1½ sec.

When a vehicle is generated, its time of arrival is recorded as the clock time, or as the clock time plus ½ sec, whichever is applicable. The newly arrived vehicle is placed immediately in the backlog file and its turning maneuver is determined randomly. Entering is attempted once each time increment for the earliest vehicle in the backlog.

Summary and Display Routine.—Once the six lanes have been scanned, the summary and display routine is entered and the simulated clock is incremented. When each designated sample time has been reached, the data collected during that sample are displayed. After the last sample, additional data are summarized and displayed. Output information is given in Table 3.

Coding

The developed program is coded in the IBM 709/7090 FORTRAN language (24). The FORTRAN (formula translation) system accepts a source program written in a language

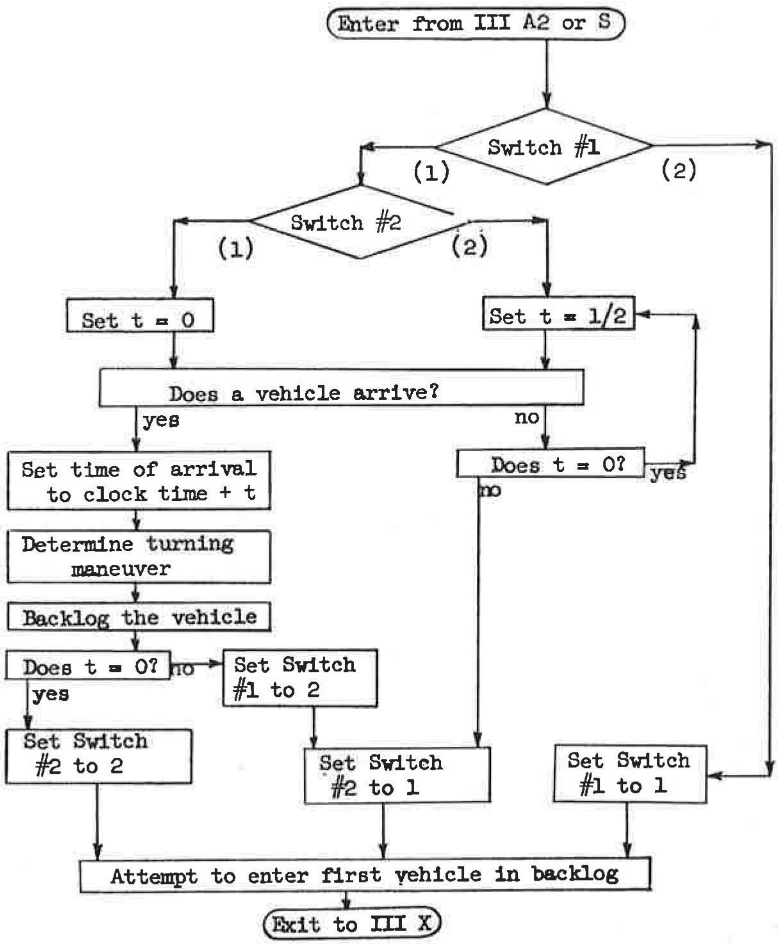


Figure 9. Vehicle generation subroutine.

TABLE 3

OUTPUT INFORMATION

Description	Number of Items Involved
Problem specifications	
(All items included in Table 2)	--
Results for each sample:	
Cumulative number of vehicles generated	14[a]
Cumulative number of vehicles released	14[a]
Number of vehicles currently in lanes	6
Number of vehicles currently in backlogs	6
Cumulative total delay	14[a]
Cumulative stopped delay	14[a]
Average total delay per vehicle in sample	3[b]
Average stopped delay per vehicle in sample	3[b]
Run summary:	
Mean of average total delay per vehicle for samples	3[b]
Mean of average stopped delay per vehicle for samples	3[b]
Variability of sample averages of total delay	3[b]
Variability of sample averages of stopped delay	3[b]
Overall average total delay per vehicle for run	3[b]
Overall average stopped delay per vehicle for run	3[b]
Actual volume of traffic in vehicles per hour	3[c]
Actual percent of directional distribution	2[c]
Actual percent of right turns	2[c]
Actual percent of left turns	2[c]
85th percentile total delay on side street for stop sign control	1

[a]By lane and by turning movement.
[b]For main street, side street, and both streets.
[c]For main street and side street.

that closely resembles the ordinary language of mathematics. The system uses the computer to convert this mathematical language into a machine language, which is actually used in running the problem. There are several points of interest concerning the manner by which the simulation is converted into an algebraic format.

The approach lanes are represented by a three-dimensioned circular array (Figure 1) with dimensions of 6, 100, and 5. The first dimension denotes the particular lane. The second dimension refers to the relative position within the lane. Vehicles are stored in order, starting with the lead vehicle, and they do not ordinarily change position. (Vehicles shift positions within lane arrays only for passing maneuvers.) Two separate arrays are used to keep track of the index of the lead vehicle and the number of vehicles in each lane. This information is updated each time a vehicle enters or is released from a lane. The 100 index positions are sufficient to store a solid line of stopped vehicles when the beginning of the lane is designated as X = 0.0 ft. A special technique is employed whereby, when index position 100 is reached, the next position behind it is given as index position 1; thereby providing the continuous circular feature of the lane array.

The third dimension for the lane array refers to the vehicle characteristics. The values stored represent the time of arrival, turning movement, X-coordinate, velocity, and deceleration rate. The turn indicator is set to a negative, zero, or positive value to signify left turn, straight through, or right turn, respectively. The deceleration rate stored is that computed by the amber signal decision subroutine as the rate required to stop at the traffic signal. If the deceleration rate is zero, the car is not tagged. A vehicle's deceleration register is also used for two special purposes which do not occur concurrently with stopping for the amber signal. A negative deceleration rate for a main-street vehicle signifies that the vehicle has passed from an outside lane to an inside lane during the current time increment. For a side-street vehicle, a negative deceleration rate indicates that the vehicle occupies the LT hold position.

The backlog file is likewise represented by a three-dimensioned circular array, but with dimensions of 6, 100, and 2. The dimensions have the same meaning as in the lane array, except that only the first two vehicle characteristics are stored. The 100 vehicle positions should accommodate the most severe traffic jam. If the backlog is filled to capacity, however, a special indication is included in the output. In addition to this feature, a backlog limit may be specified which will delete the remainder of the problem when the limit is exceeded.

The entire input for a particular problem can be contained on two IBM cards. There are many variables that are fixed in the program, such as the maximum acceptable rate of deceleration at an amber traffic signal. The FORTRAN language facilitates the location of these variables in the code. Even though the program would have to be recompiled, it is a simple matter to alter the values of such variables.

The simulation was programed for an IBM 7090 computer with 32,000 words of core storage. The program, as written, requires 15,271 words of core storage of which 668 words are used by special debugging routines. The lane and backlog arrays use 4,200 words and the file for side-street delays with stop-sign control uses 4,000 words of storage. By reducing the length of the approach lanes and the backlog lists and by reducing and/or eliminating the delay file and debugging routines, the program could be run on a computer with but 8,000 words of core storage.

The model may be classified as the periodic scanning type, in which each vehicle is processed during each time increment. Previously, models of the event-scanning type have been used. In this latter type, processing is bypassed except when an event occurs that necessitates some action of the vehicle. The reduction in scanning time thus achieved, however, is in good part offset by the additional logic employed. The behavior equations used in this model, moreover, are such that continual processing is required. With the recent tremendous advances in computer technology, the efficiency of the simulation model is rapidly becoming unimportant. A favorable real time to computer time ratio is achieved by the program. The ratio is 45 to 1; this means that 1-hr traffic can be simulated in about 1⅓ min on the computer. A complete description of the computer program is found elsewhere (28).

Selection of Intersection Parameters

It is difficult to make a general comparison of two types of traffic control at intersections. Even for a particular intersection, traffic patterns vary throughout the day. Furthermore, there is often a wide operational latitude possible for a specific control type. The variables involved may be classified by three categories of factors: geometric, traffic, and control.

The geometric design of the intersection was fixed by using typical dimensions and characteristics for the class of intersection studied. It was then necessary to determine the values of the remaining variables in order to compare directly the effect of the two traffic control types.

Traffic Factors. —Some traffic factors, such as velocities, rates of acceleration, and vehicle size, were incorporated in the behavior equations. The magnitude of these quantities was dependent in part on the composition of traffic. A single vehicle type was used to approximate the mixed vehicles in the traffic stream. This average vehicle had properties that were essentially similar to those of passenger cars, except for a slight reduction in acceleration capability to account for the presence of trucks. Other traffic factors include such items as directional distribution, lane distribution, and the frequency of turns. Although it would have been desirable to investigate the individual effect of each of these items, the computer time required to simulate the intersection under the complete range of possible conditions was well beyond the scope of this project.

For urban streets in intermediate areas, the typical proportion of traffic flowing in the major direction has been found to be 60 percent of the total street volume (1). Therefore, the directional distribution was fixed at a 60:40 percent value.

For rural highways, lane distribution is a function of traffic volume. At low volumes, the major portion of traffic will use the outside lane. On four-lane rural highways, the proportion of traffic in the outside lane has been found to vary from 88 percent at very low volumes to 40 percent as capacity was approached (9). Wagner and May presented data for the lane distribution on a heavily-traveled four-lane urban expressway. Their value for the proportion of traffic in the outside lane was approximately 57 percent throughout the range of volumes observed (43). Equivalent information for urban streets is lacking. The proportion of turns and the proximity of intersections undoubtedly have a significant effect on lane selection.

Field studies were conducted by the authors on two urban arterial streets in West Lafayette, Ind. Both were four-lane streets with parking prohibited. Two 15-min recording traffic counters were placed side by side; one with the road tube covering both lanes going in one direction, and the other tube covering only the outside lane. The hoses were cut off $2\frac{1}{2}$ ft short of the lane lines so that vehicles straddling two lanes were distributed equally to both counts. The data obtained indicated that, at traffic volumes below capacity, lane distribution remained relatively constant. Even at extremely low volumes a significant number of vehicles selected the inside lane. A lane distribution with 60 percent of the vehicles using the outside lane was inconsistent with these observations. This value was therefore used throughout the range of volumes employed in the simulation. Because passing was accommodated for main-street vehicles, delay was not sensitive to lane distribution.

The percentage of turns commonly used for urban intersections is 10 percent for each turning direction (9). This value is typical for the intersection of two similar streets. The simulated intersection was composed of two streets of different character. The proportion of vehicles turning into the side street would usually be less than that of vehicles turning into the main street. The percentage of turns may differ for each approach and may even be a function of traffic volume. The percentage of turns was fixed in the simulation, however, and typical values were chosen as 7 percent for the main street and 14 percent for the side street. That is, 7 percent of the traffic entering the intersection from each main-street approach turned left, and another 7 percent turned right. For the main street, all RT vehicles used the outside lane, and all LT vehicles used the inside lane. Due to the lane distribution factor, a higher percentage of vehicles in the inside lane turned as compared with the outside lane.

The remaining traffic factors were the traffic volumes on the two intersecting streets. These volumes could not be fixed, as they are the fundamental variables with which delay is associated.

Stop Sign Factors.—In his study of vehicle performance at urban two-way stop signs, Raff (35) developed the concept of a critical lag. Critical lag was defined as the lag having the property that the number of accepted lags shorter than it is the same as the number of rejected lags longer than it. A lag is in turn defined as the time interval between the arrival of a side-street vehicle at the intersection and the arrival of the next main-street vehicle. A main-street vehicle is considered to have arrived when it enters the area bounded by the extension of the curblines. A side-street vehicle arrives when it reaches its lowest speed or, if it is following behind another side-street vehicle, it arrives when the preceding vehicle enters the intersection area.

The critical lag is the single value used to represent the pattern of acceptance and rejection of lags. The four intersections studied by Raff yielded values for the critical lag of 4.6, 4.7, 5.9, and 6.0 sec. The higher values were observed at intersections that correspond more closely to the intersection under study. Of all the factors affecting the critical lag, the most significant was found to be sight distance; that is, shorter lags are associated with poorer sight distances. The sight distances for the two intersections with the shorter lags were typical for downtown areas, whereas the sight distances at the other two intersections were more typical of intermediate areas.

In Greenshields' study (21), a different quantity was used to evaluate performance at a stop sign. Greenshields' "minimum acceptable time gap" is defined as the gap that will be accepted by more than 50 percent of the drivers. This time gap is measured as the time required for the main-street vehicle approaching from the left to reach the point of conflict. The point of conflict is in turn described as the intersection of the centerlines of the two vehicle paths. Because the distance to this point is greater than the distance to the intersection area, a slightly larger value would be expected for Greenshields' gap as compared with Raff's lag. Greenshields' quantity was observed to be 6.1 sec and is 0.2 sec longer on the average than the critical lag for similar intersections.

A recent study by Bissel (7) resulted in a probability distribution for gap acceptance at stop signs. The median value for ST vehicles was 5.8 sec with the 15 and 85 percentiles at 3.9 and 8.5 sec, respectively. The median value for LT vehicles was about 0.4 sec greater. Neither Raff nor Greenshields segregated left turns because the differences between LT and ST vehicles were found to be very small.

Raff's terminology and definitions were employed in this study as they are more rigorous. A value of 5.8 sec was used as the typical critical lag for stop signs. A second value of 4.8 sec was used to indicate the effect of changing this quantity. The single values were considered representative of the actual distributions of acceptable lags. In most field studies, the lag is measured as it occurs after the fact. Both in the model and in reality, the lag can only be estimated by the driver before the maneuver takes place.

Traffic Signal Factors.—There are seven basic variables involved in semi-traffic-actuated signal control. These are the six adjustable intervals employed by the controller and the location of the side-street detectors. The two amber interval settings should be based on geometric and traffic factors. The most widely-used amber interval is 3 sec long. It has been shown both theoretically and in field studies that this short clearance time may result in a "dilemma zone" of considerable length (14, 32). In other words, there is a portion of the approach lane in which a vehicle can neither safely stop nor clear the intersection before the expiration of the amber interval. Corresponding behavior took place in the model. In such cases where the model vehicle could not stop within the acceptable limits of deceleration, the vehicle automatically continued through the intersection. Even though it may not have cleared before the start of the opposing green interval, it cleared in sufficient time to avoid physical contact. Inasmuch as the 3-sec clearance interval is prevalent, both amber intervals were fixed at this value.

Of the five remaining variables, three are interdependent. The side-street initial green interval plus one extension interval combines to provide the minimum side-street green time. This minimum green time must be of sufficient duration to clear a queue of vehicles occupying the space between the stop line and the detector. Pedestrian considerations may also bear on the minimum green, as this time should accommodate pedestrians crossing the main street. Using a walking speed of $3\frac{1}{2}$ ft/sec and allowing a 5-sec leeway, a desirable minimum green time is 18 sec.

Optimal controller settings, with respect to delay, are dependent on traffic volumes. If delay on the main street is to be minimized, the detector should be placed near the stop line and short side-street initial and extension intervals used. If delay on the side street is to be minimized, the detector should be placed at some distance from the stop line. Then an approaching side-street vehicle may clear without even decelerating. Ordinarily, settings cannot be changed when volumes vary, and compromise values must be used. Because traffic volume on the main street is most always greater, the delay to main-street vehicles is usually critical.

Two sets of signal variables were used; one with the detectors placed at 150 ft, and one with the detectors 21 ft from the stop lines. These settings provide for the cases where pedestrians must be considered and where pedestrian movements are negligible. They also correspond to attempts to minimize side-street delay and minimize main-street delay. The effect of intermediate detector locations can be estimated by interpolation of the resultant delays.

When the detector is placed approximately 150 ft from the stop line, the side-street initial and extension green intervals should be set at 13 and 5 sec, respectively. About 7 vehicles may be stopped between the stop line and the detector. As determined by the behavior equation, 17½ sec are needed to move a queue of 7 vehicles so that the front of the seventh vehicle is 17 ft beyond the extension of the far curbline. This behavior is shown as the time to reach a position where X = 2,073 ft (Fig. 3). An eighth vehicle may also clear by using one-half the amber interval. A ninth vehicle in the queue would cross the detector during the fourteenth second of green, thereby gaining an additional 5-sec extension interval. All subsequent vehicles would similarly be cleared up to the time at which the side-street maximum green interval has expired.

The side-street extension interval should be long enough to clear a vehicle approaching a green signal once it has actuated the detector. An interval of 5 sec is adequate using only occasionally a portion of the amber interval.

Once the main-street minimum green interval has expired, these settings require an approaching side-street vehicle to slow to approximately 24 ft/sec before receiving the green aspect. The settings closely correspond to the values recommended in the "Manual on Uniform Traffic Control Devices" (30).

When the detector is placed 21 ft from the stop line, the side-street initial and extension green intervals should be 2 and 4 sec, respectively. A second-in-line vehicle would thus cross the detector 3 sec after the start of the green aspect, thereby resetting the extension interval. All subsequent vehicles in a queue may similarly be cleared. A side-street vehicle approaching a red aspect would normally reach a complete stop and then wait 1 sec before receiving the right-of-way.

Minimum delays have occurred when the main-street minimum green interval is relatively short, as more flexibility is provided by the controller's capability to react quickly to side-street actuation. On the other hand, the side-street maximum green had only a minor effect on delay (5). The nature of this interval is such that it is rarely timed out. The interval will be used fully only when it is actually required. Practical values for these two intervals are in the range of 30 sec. These values would provide for reasonable cycling of the right-of-way as capacity conditions are approached.

The two sets of traffic signal variables used are given in Table 4. The model detector is actuated by the front bumper of a vehicle, whereas the real detector is more often actuated by the front tires. The model detector was therefore placed 3 ft closer to the stop line than the corresponding nominal position.

TABLE 4

TRAFFIC SIGNAL VARIABLES USED

Variable	Time (sec) for Distance Between Detector and Stop Line of	
	150 Ft[a]	21 Ft[b]
Main-street minimum green	30	30
Main-street amber	3	3
Side-street initial green	13	2
Side-street extension green	5	4
Side-street maximum green	30	30
Side-street amber	3	3

[a]Station of model detector, 1,853 ft.
[b]Station of model detector, 1,982 ft.

Checking the Program

Once the program was debugged, several runs were made using special output features which provided, in a readable format, detailed information on the behavior of each vehicle during each time increment. The use of this information resulted in several refinements in the program. Extensive testing of the intersection in this manner assured that the program was properly written and that the action of the vehicles was reasonable and realistic.

Field validation of the model was well beyond the scope of this study. Such validation is a most difficult undertaking even when unlimited resources are available. It was hoped that the model could be checked against some of the delay studies reported in the traffic engineering literature. In each case, however, certain necessary information was lacking in these studies. In some cases, the delay data were measured over a period of several hours, and the variations in traffic volume throughout the study period were not recorded. In other cases, such items as traffic distribution and turning movements were not observed. The basic problem in validation is simply that information of the type readily obtained from the model is extremely difficult to measure in the field.

The merit of the model, therefore, must be judged by the manner in which it was constructed. The traffic characteristics on which it was based are those that have been extensively studied and found similar at different locations. These characteristics included velocity, acceleration, spacing, and gap acceptance.

Insofar as possible, identical models were used to represent the intersection as operating under the two types of traffic control. The effects of certain possible inaccuracies in delays as determined for the two control types were thus significantly reduced; that is, differences in delay could be realistic even though the absolute values of delay may have been somewhat distorted. The use of model comparison also permitted such variables as parking interference, pedestrian movements, and intersection geometry to be eliminated as direct considerations.

Selection of Approach Length and Running Time

Runs were also made to test the effect of changing the length of approach lanes. These lanes had to be long enough for an entering vehicle to stabilize its behavior before reaching any of the critical points in the lane. These critical locations were the point where vehicles began decelerating for a stop, the location of the detectors, and the farthest point investigated by a vehicle crossing the traffic stream as it searched for an acceptable gap. A beginning of lane coordinate X_0 of 1, 650 ft adequately met these requirements. This provided an approach lane of 350 ft before the location of the stop lines.

The beginning of the lane also affects vehicle behavior in that it fixes the relative time within a time increment that a free-flowing vehicle reaches the various critical points in the lane. To assure a comparison in which the only variable was the length of lane, two X_0's were selected which differed by a multiple of 44 ft (the distance traversed by a free-flowing vehicle during a 1-sec increment). Two runs were made with X_0's of 22 and 1, 650 ft. Care was taken to assure that the identical traffic was used for each run. No significant difference in total delay was observed for these runs. However, stopped delay is recorded only for vehicles within the lane. Stopped delay would be underestimated in a situation where a long line of stopped vehicles filled the approach lanes and the backlog contained additional vehicles. An X_0 of 1, 650 ft was used for all production runs.

Additional preliminary runs were made to investigate the variability of the delay data. A 90-min run was made for each type of control, using 30 samples of 3 min. Various groupings of the sample data were tried, and the standard deviation of the sample means was used as an index of the variability. Traffic signal control resulted in considerably less variability than stop sign control. For either case, the additional data obtained beyond 1-hr running time had little effect on the average delay. A 1-hr run of eight 7. 5-min samples was selected for all production runs. This plan provided a reasonable compromise between sample size and number of samples. A 5-min transient time was used for all production runs.

Procedure for Production Runs

Most production runs were made using the regular random number series option of the program. This assured that the identical traffic was generated when the two different control devices were tested at the same volume levels. Furthermore, because a separate series of random numbers was used to generate vehicles for each street, the volume level of one street could be varied without affecting the traffic pattern on the other. It was desirable that the volume levels for each street remain fixed to locate the points of equal delay accurately.

As both the generation of vehicles and the selection of turning maneuvers were done randomly, the actual traffic characteristics for samples of short duration deviated from the ones specified. Minor variations also occurred when identical traffic patterns were generated. This latter variation was caused by slight differences in the pattern of vehicle release at the beginning and end of a 1-hr run. As these differences were small, the characteristics were averaged for each street and volume level. The average traffic characteristics are given in Table 5.

The backlog limit was arbitrarily set at 20 vehicles. When this backlog was exceeded, there were usually 8 to 10 vehicles occupying the distance between the beginning of the

TABLE 5

VALUES OF ACTUAL TRAFFIC CHARACTERISTICS

Random Number Series	Street	Traffic Volume (veh./hr)		Actual Traffic Characteristics (%)		
		Specified	Actual	Directional Distribution	Right Turn	Left Turn
Regular	Side	42	43	51.8	24.0	11.5
		84	84	58.1	17.7	9.6
		125	125	54.8	14.2	10.4
		250	251	58.8	13.2	14.0
		375	363	60.8	13.0	13.2
		500	522	60.7	13.0	12.8
	Main	125	113	60.2	7.1	4.4
		250	252	58.8	7.5	8.3
		500	531	61.0	6.0	5.8
		750	789	59.9	8.1	8.9
		1,000	1,018	59.3	7.5	7.0
		1,250	1,283	59.8	7.0	7.4
		1,500	1,489	58.9	7.4	6.7
Alternate	Side	42	44	55.0	16.6	11.4
		84	72	56.5	15.3	9.6
		125	116	65.1	16.5	12.5
		250	249	56.0	14.9	12.7
		375	363	62.2	15.9	15.3
		500	--	58.8	15.6	13.5
	Main	125	136	62.5	9.6	3.7
		250	257	62.5	8.2	6.6
		500	513	57.9	7.4	7.6
		750	766	60.1	8.2	6.8
		1,000	1,035	60.8	7.1	6.6
		1,250	1,296	61.0	6.9	7.3
		1,500	1,519	60.4	7.1	7.4

lane and the stop line. In each case when a run was terminated due to the exceeding of the backlog limit, it was obvious that the possible capacity of the approach was exceeded.

Computer runs were made in several shifts. The results of one run made possible a more intelligent selection of specifications for subsequent runs. In some cases, this procedure reduced the number of volume combinations required for subsequent runs, because it was known which combinations were likely to be critical.

Two average delays were computed by the program. The first was the average of the sample means and the second was the overall average delay. Because the number of vehicles released during each sample varied, the values differed for the two quantities. The differences were minor, but the overall average delay was the one used in the analysis and is the one shown in the figures.

The possibility existed of underestimating average total delay as capacity was approached. Because total delay was computed and recorded only at the time of a vehicle's release, the delays to vehicles still in the system were not measured. Such a situation could be detected, however, by examining the number of vehicles in the lanes and the size of the backlogs, and noting a drop in the rate at which vehicles were released.

RESULTS AND ANALYSIS

Two-Way Stop Control

In analyzing the average total delay that resulted when the intersection was operated under stop sign control, it was first advantageous to consider the two streets separately. The delay to main-street traffic was due only to the interaction among main-street vehicles and was completely independent of the traffic on the side street. The major factor contributing to this delay was the LT vehicle. Delays also occurred when ST vehicles were forced to slow down behind turning vehicles. In particular, when two turning vehicles are traveling at minimum headway, the second vehicle will be delayed an additional amount. Due to the spacing restriction, the minimum headway increases as velocity is reduced. The average total delay per main-street vehicle is a function of main-street volume (Fig. 10). Even though the values of these delays are small,

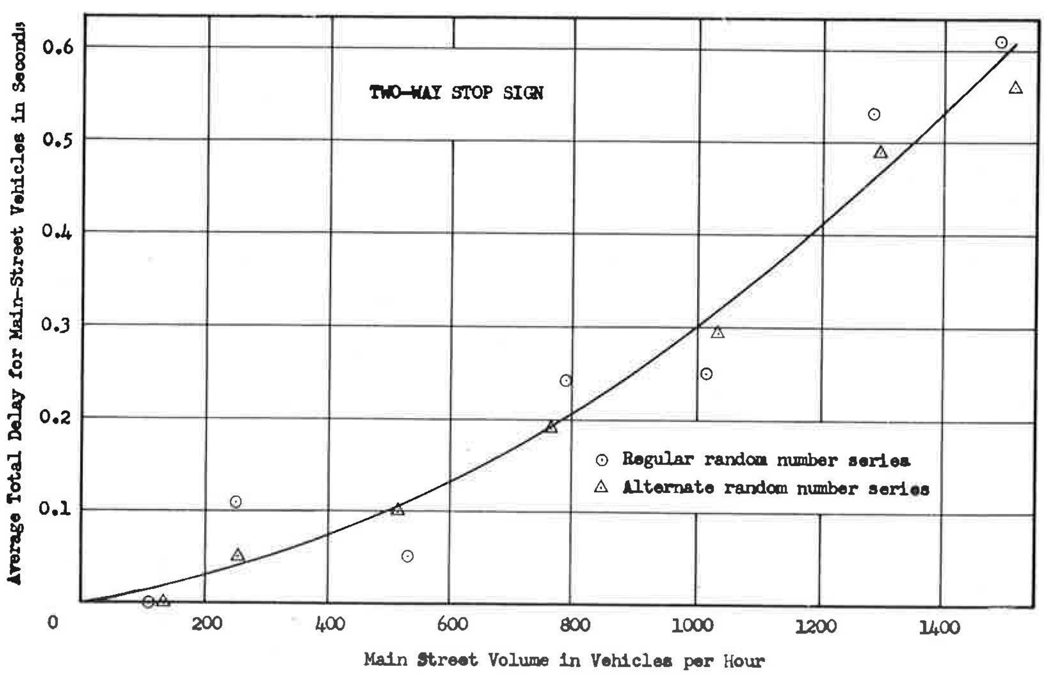


Figure 10. Average total delay per main-street vehicle, two-way stop sign.

they become a significant portion of the total delay when the main-street volume is considerably greater than side-street volume.

The delay data for side-street vehicles when they were controlled by stop signs contained a fair amount of scatter, but trends were clearly evident. When the data were plotted, an exponential relationship was indicated. By fixing the side-street volume and plotting the natural logarithm of delay vs main-street volume, an interesting relationship was obtained. Subtracting a constant amount from each delay resulted in relatively straight lines. Similar results were obtained when main-street volume was fixed and side-street volume was varied.

The constant amount of delay which existed for each side-street vehicle was the time lost in deceleration and acceleration. In the model the magnitude of this portion of delay is known for free-flowing vehicles. It amounts to 8.67 sec for vehicles generated at an even second and 9.17 sec for vehicles generated on one-half a second. As the probabilities are equal for the two cases, a mean value of 8.9 sec was used. The concept of a "wait" was then defined as the total delay per side-street vehicle minus 8.9 sec. It is the wait and not the total delay which is more identifiable by the vehicle operator.

Stopped delay might have been used in place of wait, but it neglects some delays that actually occur. For example, when the lead vehicle is released from a queue at the stop line, following vehicles may exceed a 4.5-ft/sec velocity (stopped delay was defined as any velocity 4.5 ft/sec or less) as they change position in the queue. Stopped delay thus tends to underestimate waiting time.

To place the origin corresponding to zero waiting time on the graphs, the quantity "wait plus one second" was used. Figure 11 shows the relationship between wait plus one second and traffic volume. The lines are drawn directly through the data points, and the linear trend is clearly shown. Other sets of data indicated a similar relationship.

The major portion of the variability in the delays observed under stop sign control occurred on the side street. The standard deviation of the sample means increased as the value of the delay increased. The standard deviation for the average total delay per side-street vehicle generally varied between 7 and 50 sec. Because the variability in side-street delays was greater than desired, it was decided to check the results against an independent set of data. The alternate random number series option in the program was used to obtain data based on different main-street and side-street traffic. These results likewise contained a fair amount of scatter, yet comparison of the two sets of data revealed that the results were quite similar. In spite of the variation between the short time samples, the overall delay characteristics of the 1-hr runs were essentially reproducible.

Figure 11 shows the actual data points for a critical lag of 5.8 sec when the alternate random number series was used. Figure 12 shows the corresponding data points for the regular random number series, as well as a visual fit to both sets of data. Straight lines were assumed in constructing this fit. To a minor extent, the data points were weighted in constructing the fit by using additional information available from the computer output. Such items as actual traffic characteristics, size of backlogs, and variability were considered. Figure 13 shows a similar fit for the average side-street wait when the critical lag was 4.8 sec. One should not extrapolate these curves for higher main-street volumes. The relationships shown hold true only when the capacity of the side-street approaches is not exceeded. When capacity is exceeded, delay is associated with the additional variable of time, and the given curves will underestimate the average wait.

The average total delay per side-street vehicle for the known traffic volumes was then recomputed by adding 8.9 sec to the values obtained from the smoothed curves for the average side-street wait. A new average total delay per main-street vehicle for the same volumes was also found by using the smoothed curve shown in Figure 10. A new value for the average total delay per vehicle for all vehicles was then computed based on this information and the known traffic volumes for each street. This computation was performed by using the weighted mean concept.

A significant advantage of this smoothing process was that it tended to eliminate

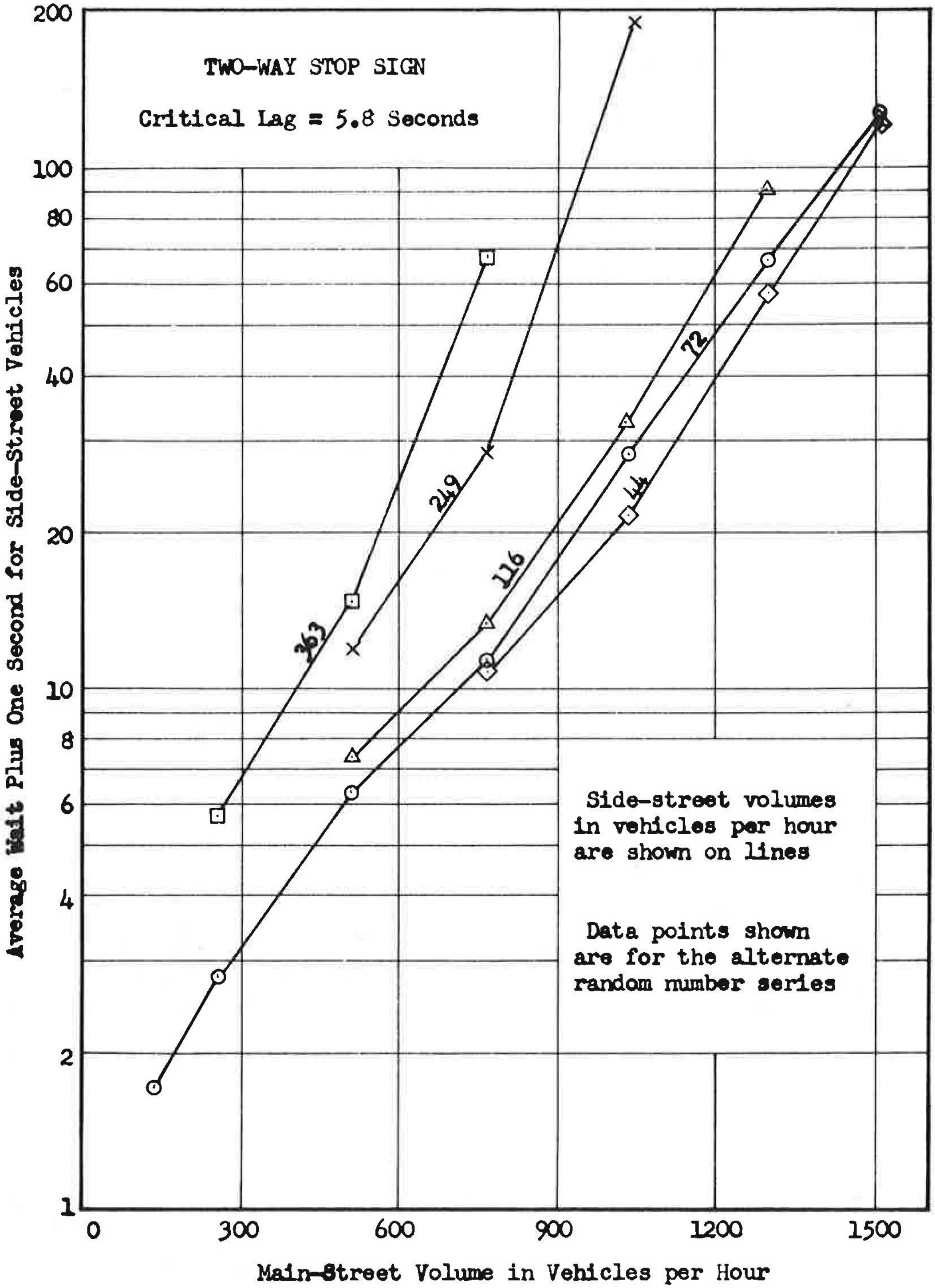


Figure 11. Relationship between average wait per side-street vehicle and traffic volume, two-way stop sign.

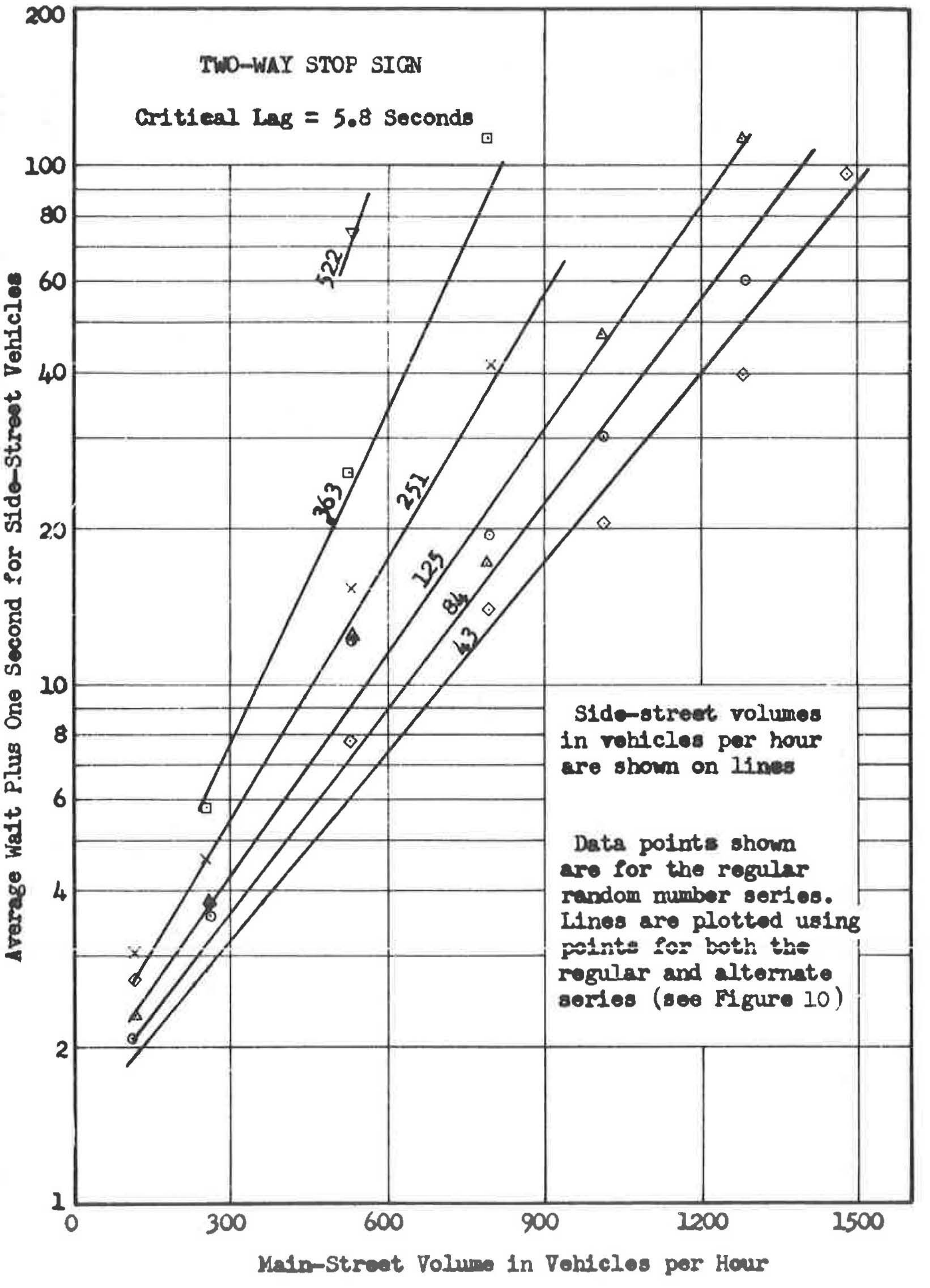


Figure 12. Average wait per side-street vehicle, two-way stop sign with 5.8-sec critical lag.

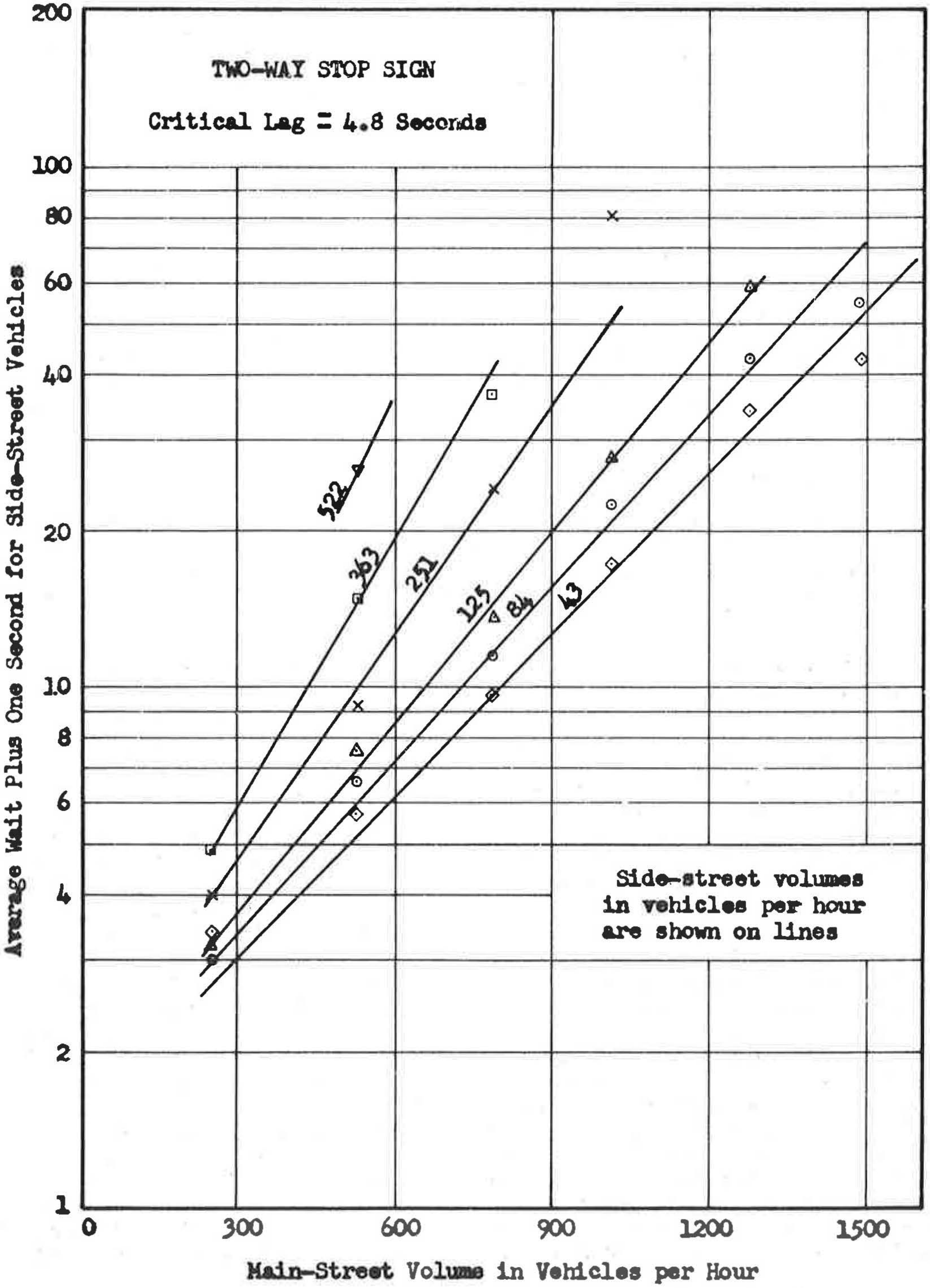


Figure 13. Average wait per side-street vehicle, two-way stop sign with 4.8-sec critical lag.

the variations due to the individual traffic patterns and the deviation of the traffic characteristics from the specified mean values. The adjusted values of average total delay per vehicle are shown in Figures 14 and 15 for critical lags of 5. 8 and 4. 8 sec, respectively. As curves are drawn directly through the adjusted data points (not shown), the uniform shape of the curves demonstrates the efficiency of the smoothing process. The adjusted curves are nevertheless a reasonable fit to the plotted original data points.

Semi-Traffic-Actuated Signal Control

When the intersection was operated under actuated signal control, the standard deviation of the sample means rarely exceeded a few seconds for the average delay per vehicle. Inasmuch as the variability of the data was small, the data were used directly. One property of semi-traffic-actuated control is that the average delay per side-street vehicle is independent of the volume of traffic on the main street, which is clearly shown in the data.

Curves for the average total delay per vehicle for the two detector locations are shown in Figures 16 and 17. The individual computed data points are also shown. Smooth curves were then drawn through these points so as to reduce the variability in the data caused by the individual traffic patterns and the deviation of the traffic characteristics from the specified values.

Development of Volume Warrants

The first consideration in establishing possible volume warrants was minimizing the average total delay for all vehicles. By superimposing the delay curves for one traffic control type on the curves for the other type, the points of equal delay were determined. These points were then plotted as a function of the traffic volumes on the two streets, and the line of equal delay was drawn. The various combinations of the two critical lags and two detector locations (Figs. 14 through 17) yielded four lines of equal delay (Fig. 18). Lines of equal delay for other critical lags and detector locations may be estimated by interpolation.

By entering Figure 18 with known main-street and side-street volumes, the intercept of the two volume lines is found. The point of intersection may then be related to a line of equal delay. If the point lies above the appropriate line of equal delay, the average total delay per vehicle would be less for actuated signal control at that intersection. Conversely, if the point lies below the line, delay would be less for two-way stop control. When the point falls on or even close to the line, local conditions and factors other than delay may prevail. When the point falls some distance from the line, one of the control devices would be clearly superior from the standpoint of minimizing overall delay. The advantage of stop sign control over actuated signal control varies for different low-volume combinations, with the maximum reduction in average total delay per vehicle being 6 to 7 sec per vehicle.

A second, yet equally important, consideration is that delays should not be excessive for either street. Both control types studied usually operate so that the delays to side-street traffic are greater than the delays to main-street traffic, and the delay per side-street vehicle under stop sign control is the critical factor. Figure 19 is the second warrant diagram, showing for stop-sign control the average wait per side-street vehicle as a function of the traffic volume on the two streets.

For a critical lag of 5. 8 sec curves for average waits of 30, 60, and 90 sec are shown. For a critical lag of 4. 8 sec, curves for average waits of 30 and 60 sec are drawn. The average wait acceptable on the side street may be a function of side-street volume. With smaller side-street volumes, greater waits may be considered reasonable. The curves shown were obtained directly from Figures 12 and 13. The values of the 85th percentile wait remained relatively constant for each average wait (Fig. 19).

Application of Volume Warrants

The procedure for using the warrant diagrams is as follows. The first warrant diagram (Fig. 18) is entered with the traffic volumes for the two streets. The appropriate

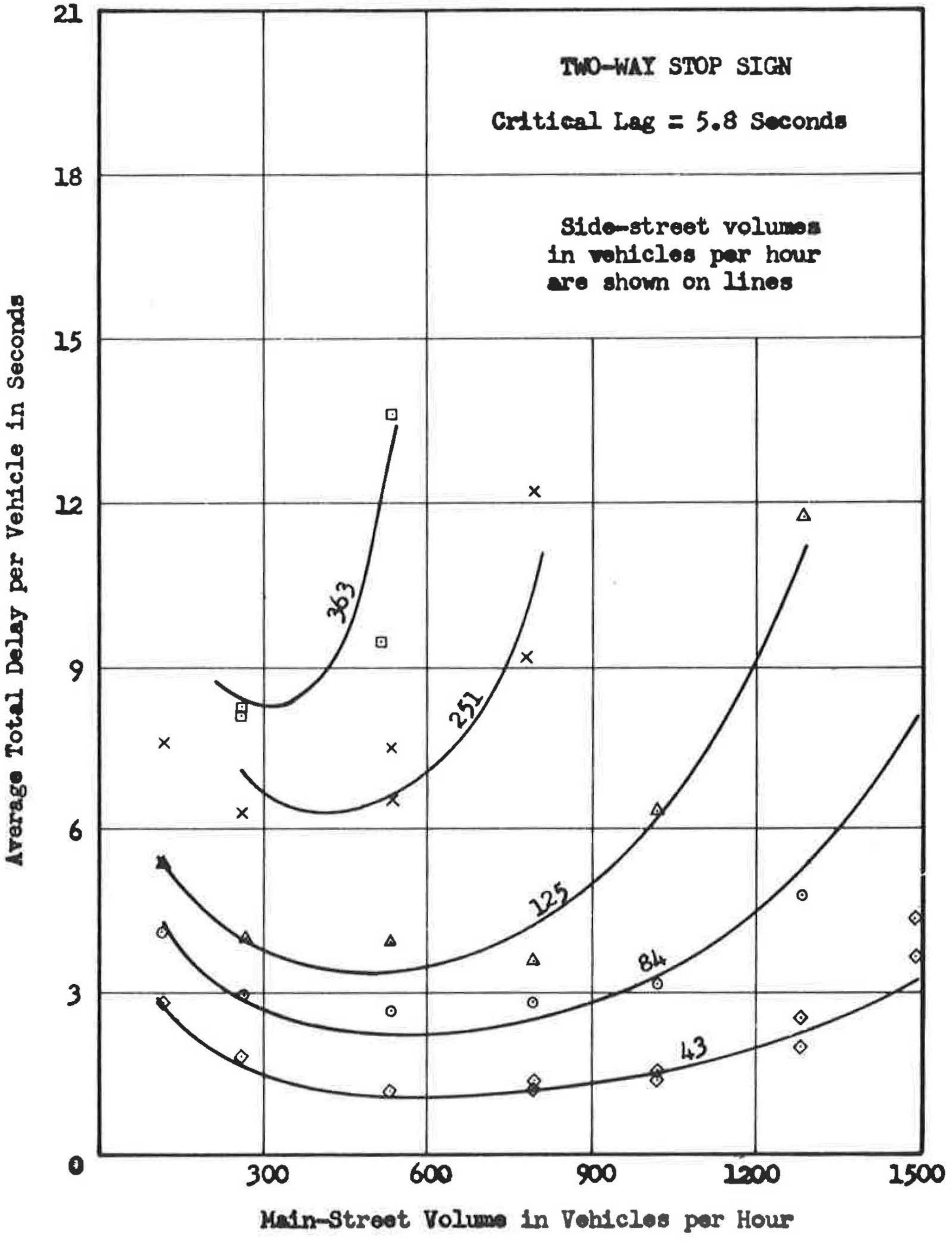


Figure 14. Average total delay per vehicle for all vehicles, two-way stop sign with 5.8-sec critical lag.

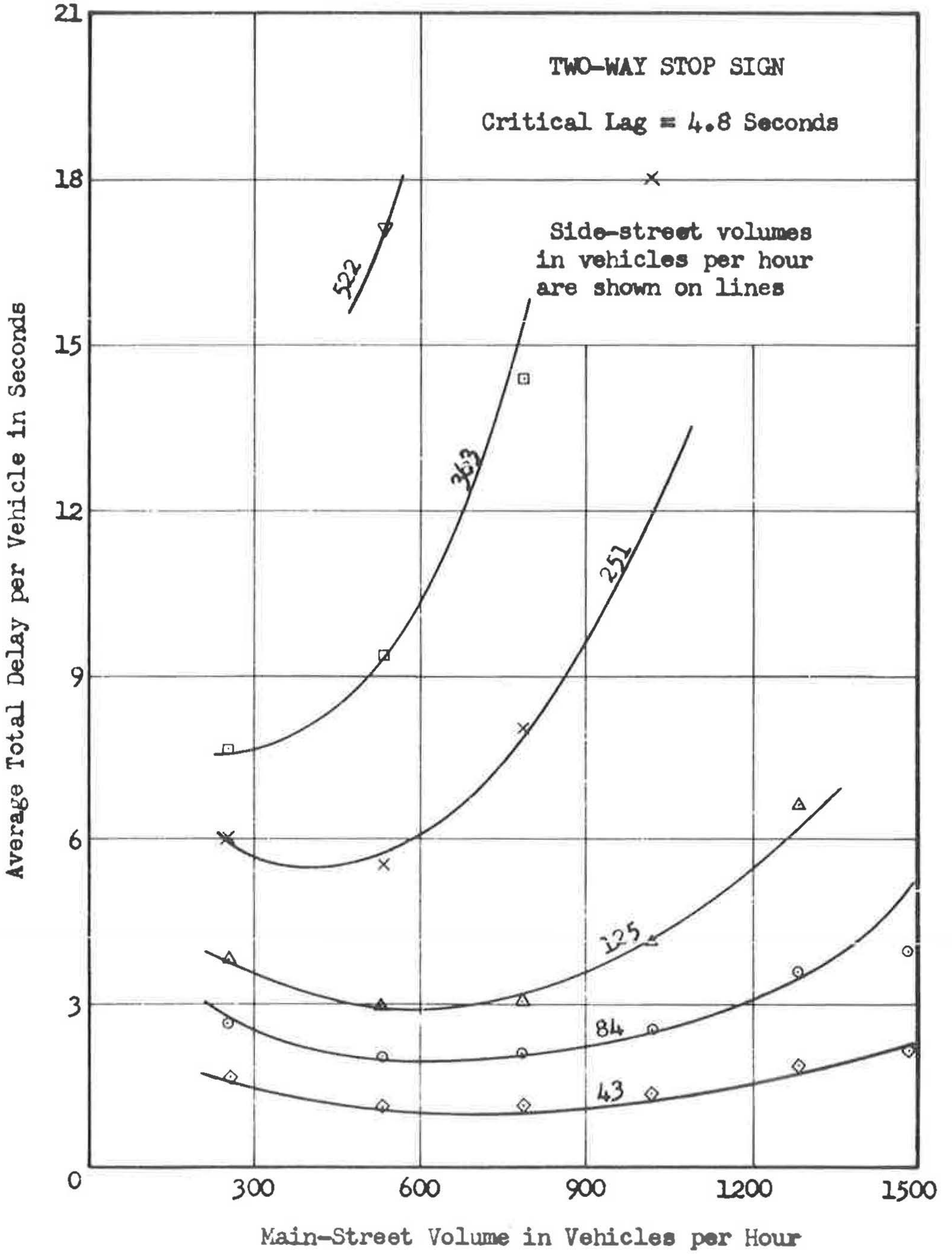


Figure 15. Average total delay per vehicle for all vehicles, two-way stop sign with 4.8-sec critical lag.

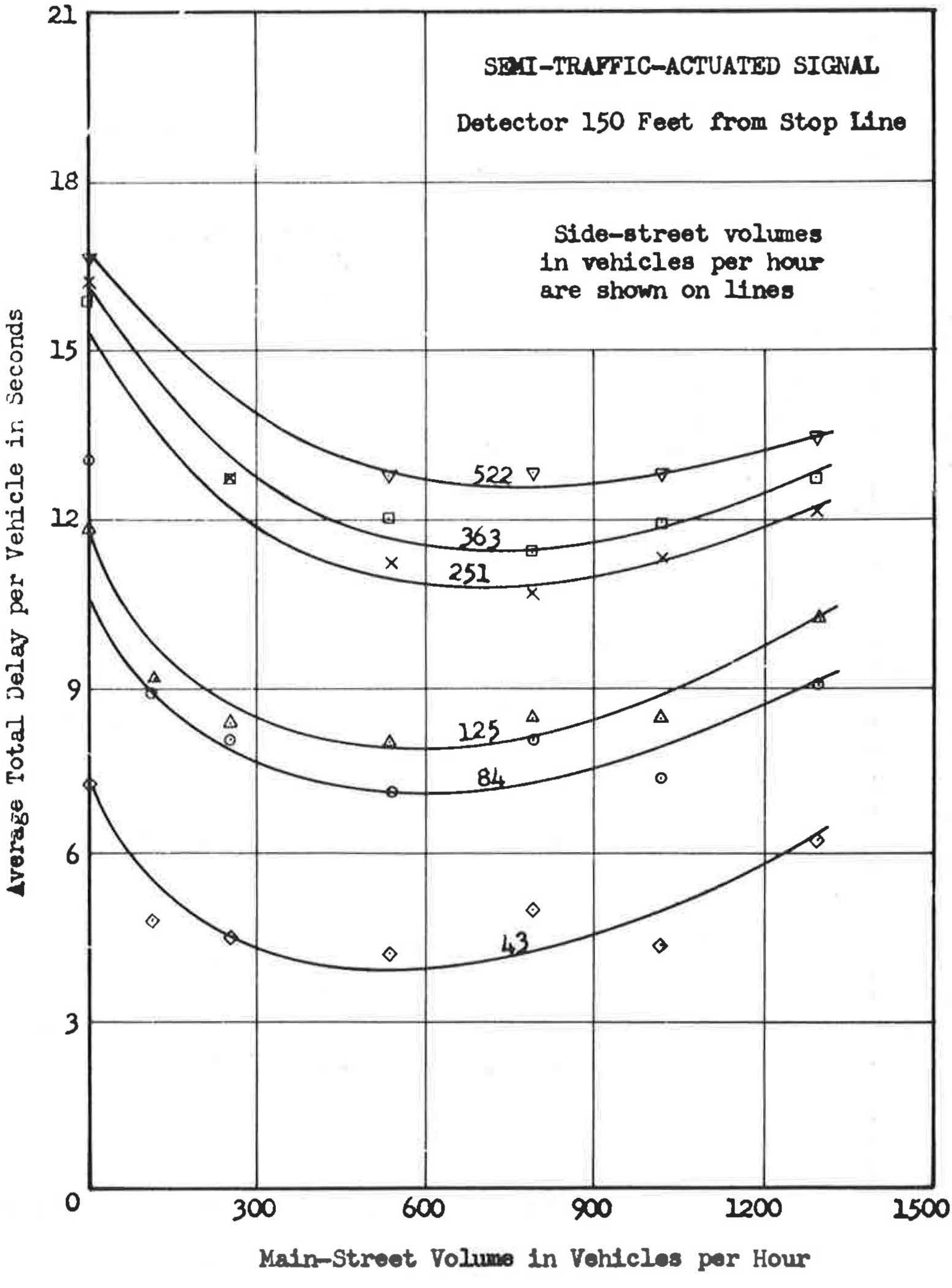


Figure 16. Average total delay per vehicle for all vehicles, actuated signal with detector 150 ft from stop line.

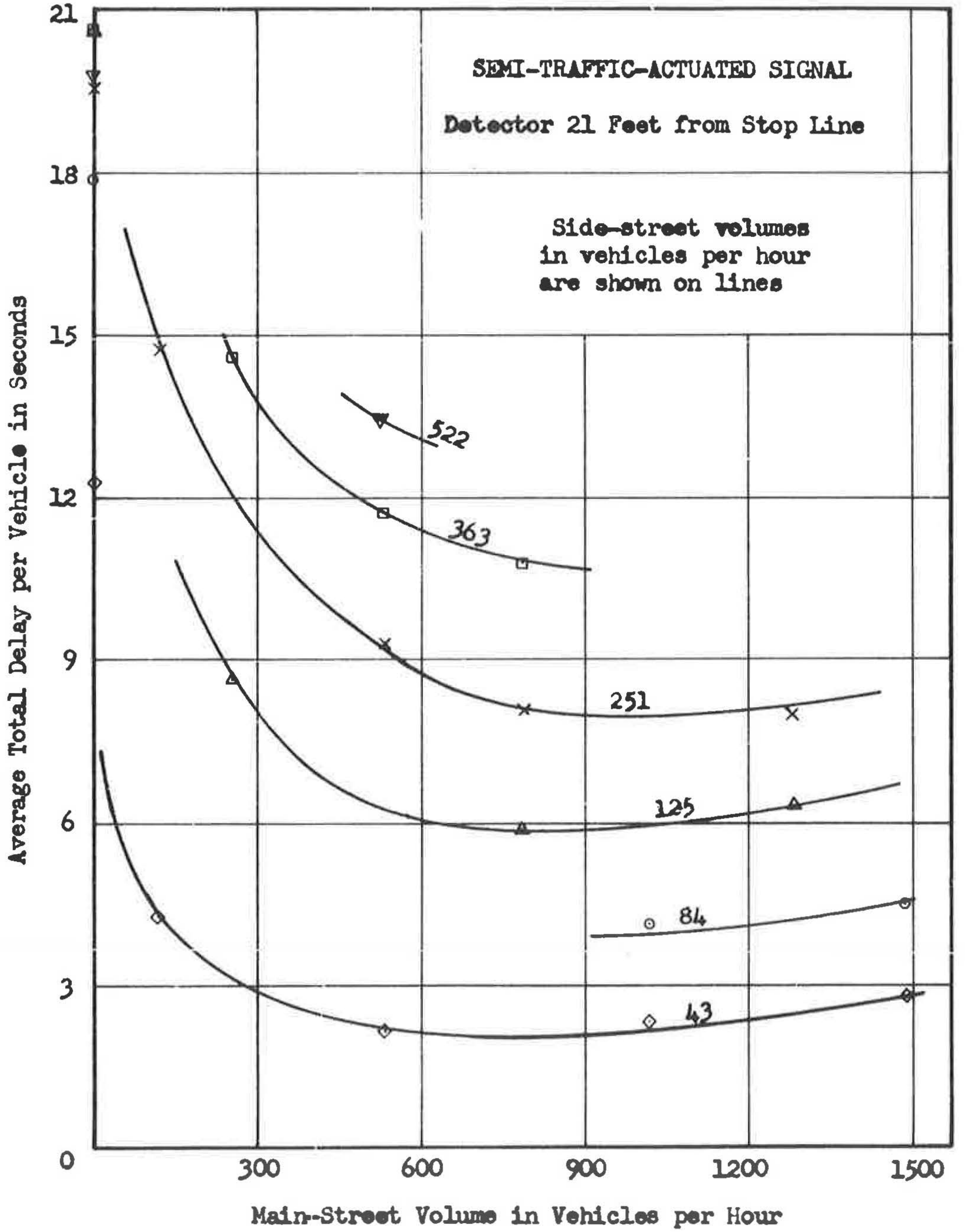


Figure 17. Average total delay per vehicle for all vehicles, actuated signal with detector 21 ft from stop line.

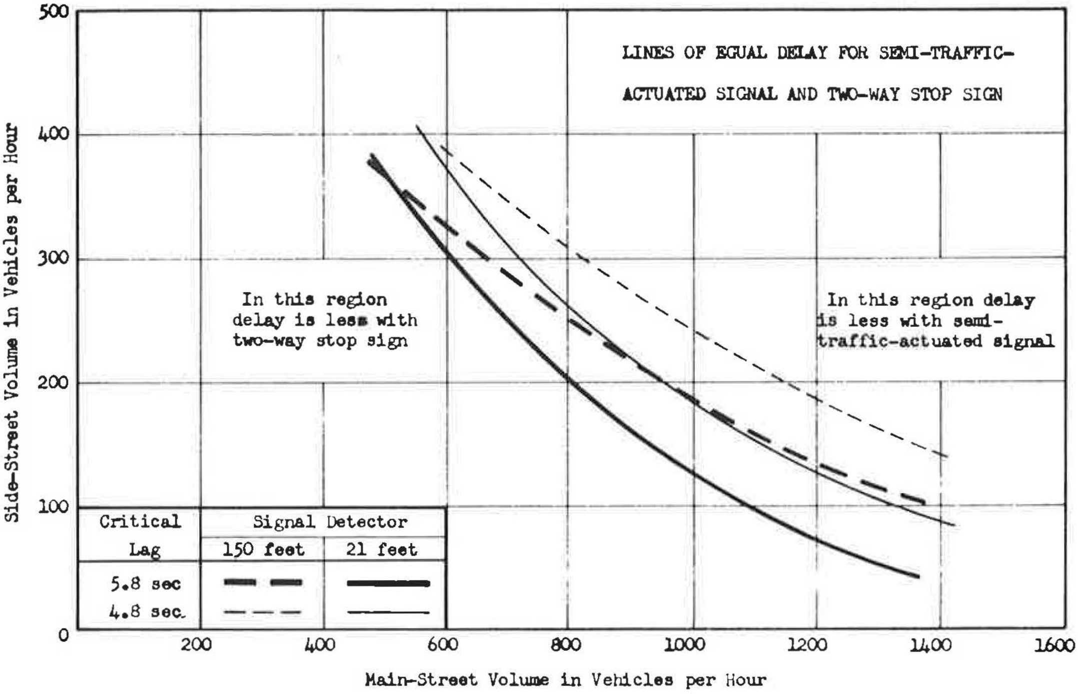


Figure 18. Warrant diagram based on minimizing average total delay per vehicle for all vehicles.

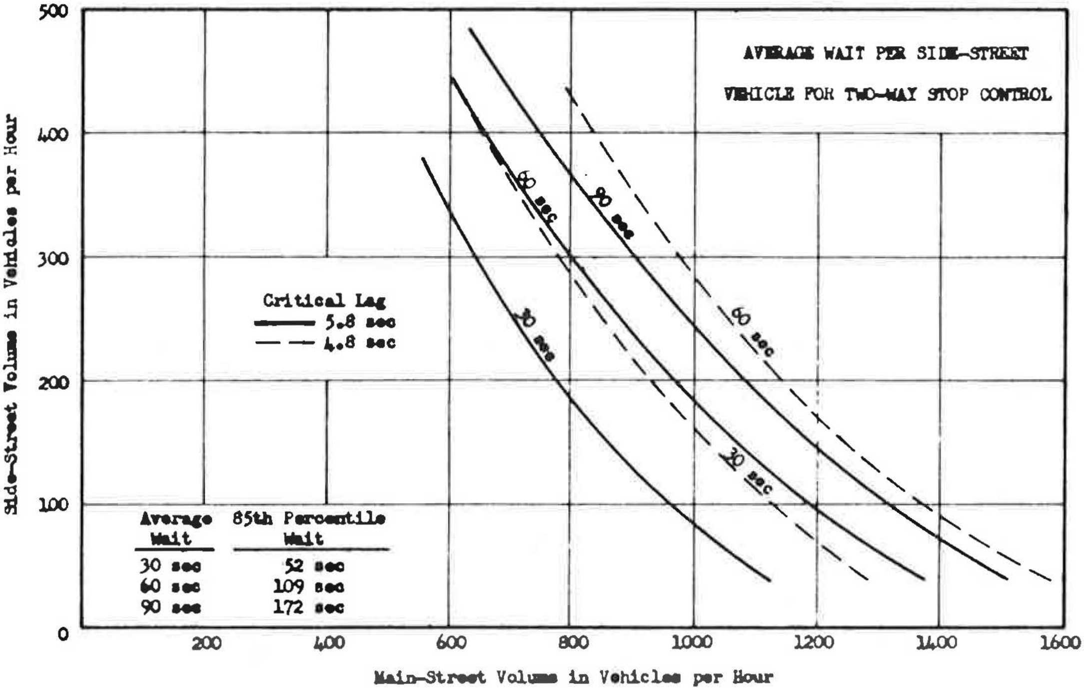


Figure 19. Warrant diagram based on delay to side-street vehicles for two-way stop sign.

line of equal delay is used to determine which control type will minimize delay. Judgment must be exercised if the decision is not clearly indicated. If two-way-stop control is indicated, the second warrant diagram should be consulted. The average wait that will occur on the side street is found from Figure 19. If this wait is considered by the majority of "reasonable men" to be excessive, then stop sign control should probably not be used. The magnitude of the difference in delay for the two control types should also be obtained from Figures 14 through 17.

These warrant curves were developed using specific traffic characteristics. The following are the actual characteristics that occurred with the percent of turns given as the percentage of the approach volume: directional distribution, 59 percent, right turns on side street, 14 percent; left turns on side street, 12 percent; right turns on main street, 7 percent; and left turns on main street, 7 percent. Other traffic factors such as the approach speed were also fixed. Moreover, the geometric factors were specified, and certain of the traffic controller settings were arbitrarily selected. If the warrants are used with care, however, the results should be indicative of the behavior of the general class of intersection.

As traffic volumes vary throughout the day, the problem naturally arises concerning which traffic volumes to use. If the critical factor is the wait on the side-street under stop sign control, then the highest volumes anticipated should be considered. This procedure will assure that the two-way stop will remain operational, and that the capacity of the side-street approaches will not be exceeded.

A rigorous approach to minimizing total delay would entail use of the hourly variation in traffic volumes throughout a typical day. The day may then be divided into several periods for which the volume characteristics remain relatively constant. Figures 14 through 17 may be used to determine the average delay per vehicle for each period. The average delay per vehicle for the entire day may then be computed by weighing each period delay by the number of vehicles using the intersection during that period. By performing this computation for each control type, the difference in average delay for the typical day is readily found.

Although the investigation of delays at pretimed traffic signals was not included in this study, the results obtained are applicable in part to pretimed signals. Except in unusual circumstances, the delays due to pretimed signals are greater than the delays due to actuated signals. Actuation reduces the allocation of the right-of-way to approaches where it is not required. Therefore, if two-way stop control can be shown to be preferable from the standpoint of delay to semi-traffic-actuated control, in all likelihood it will be preferred to pretimed signal control.

An additional application of the delay information is concerned with the flashing operation of traffic signals. When a traffic signal is set to flash a red light on the side street and an amber light on the main street, the signal is operationally equivalent to a two-way stop. Although it is generally believed that the delays caused by actuated control are small enough that flashing operation is not warranted, the data obtained in this study indicate otherwise for some volume combinations that are likely to occur at signalized intersections during some hours of the day. Again using the assumption that delays at pretimed signals are greater than at actuated signals, the delay data may also be used to indicate when flashing operation of pretimed signals would be advantageous.

Analysis

The volume warrants developed in this study are not complete within themselves. They are based solely on the considerations of delay. Although delay is perhaps the major factor, in the final analysis many other factors should be considered. These factors include the differences in accident potential, the types of traffic control used at adjacent intersections, pedestrian movements, and local conditions.

The delay data are directly applicable to the particular type of intersection studied when the geometric, traffic, and control factors are similar to those used in the study. Extrapolation of these data should be done only with caution.

Even though two-way stop control may result in lower average delay to all vehicles at even high-volume combinations, hazardous conditions may result which make such control unwise. The impatience of drivers may cause side-street vehicles to accept dangerously small gaps in the main-street traffic stream. Furthermore, the motorists acceptability of delay should be considered. It has been stated that motorists may be more willing to accept longer delays at a signal than shorter delays at a stop sign (41). This willingness may stem from the fact that the signal provides a certainty of right-of-way, whereas the stop sign does not.

It is of interest to compare the warrants developed in this study to the warrants presented in the "Manual on Uniform Traffic Control Devices" (30). No specific volume warrants are given for actuated signal control, but such warrants are given for pretimed signals. Two types of warrants are given, and for each the minimum volume warrant is satisfied when

> ...for each of any 8 hours of an average day the traffic volumes given (in tables) exist on the major street and on the higher-volume minor-street approach to the intersection. The major-street and the minor-street volumes are for the same 8 hours. During those 8 hours the direction of higher volume on the minor street may be on one approach during some hours and on the opposite approach during other hours.

By applying a 60:40 percent directional distribution to the side street, the "minimum vehicular warrant" becomes 600 and 250 vehicles per hour on the main street and the side street, respectively. Likewise, the "interruption of continuous traffic warrant" is 900 and 125 vehicles per hour for the two streets. Because for each of 8 hr these volumes must be equaled or exceeded, the average volume during this period will be higher than the minimum. These volume figures, therefore, are not directly comparable to the warrant diagrams in this study.

CONCLUSIONS

1. The digital simulation model performed in the desired manner and provided comprehensive delay information that would be most difficult if not impossible to obtain by more conventional methods.

2. The volume warrants developed in this study for type of intersection control are directly applicable to intersections of the class studied when they are operating within the range of conditions considered. The trends in the delay data are of general interest, moreover, and should contribute to understanding the effect of traffic control type on delay at all intersections.

3. When the intersection was operated under two-way stop-sign control, the following conclusions were drawn from the results: (a) Unless an average wait in excess of 30 sec per vehicle is acceptable on the side street, the critical factor in determining the adequacy of stop-sign control will generally be the delay to side-street vehicles. The interruption of continuous traffic will then be the primary justification for abandoning the two-way stop in favor of a higher type of control. (b) The average wait per side-street vehicle is quite sensitive to the gap acceptance criteria employed by the motorists.

4. The following conclusions were reached in regards to semi-traffic-actuated signal control: (a) For many volume combinations which occur during portions of the day, the overall delay to all vehicles would be materially reduced by placing traffic signals on flashing operation. (b) For normal volume distributions (the majority of traffic on the main street) the average delay per vehicle for all vehicles is lowest when the detectors are placed close to the side-street stop lines.

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