Vertical and Horizontal Bridging with the Kelsh Stereoscopic Plotter

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• SECOND ORDER stereoscopic plotting instruments are capable of obtaining a theoretically correct solution of double point resection under certain conditions. Hypothetically, it is possible to adapt the "base-in, base-out" method of first order optical train aerial triangulation instruments to a Kelsh double projection, stereoscopic plotter and achieve results for limited photogrammetric operations (1).

This technique for transferring the external orientation of a controlled stereoscopic model to uncontrolled stereoscopic models is based on the principle of spatial resection. Each succeeding photograph printed as a transparency on optically flat glass and called a diapositive in this paper is oriented to the preceding one, which contains a correct internal and external solution. By carrying the orientation forward in this manner, a series of highly predictable errors are incurred which can be adjusted to yield accurate information.

The equipment used in this test was a standard two projector Kelsh photogrammetric instrument and a desk calculator. The highway department's electronic computer was not used in any of the calculations, although the adjustment of errors could be accomplished by strictly analytical means with the aid of a computer.

No claims are made for originality of the basic principles involved, but a presentation of the procedures developed and a summary of the results obtained may be of interest to those involved in compilation of maps by photogrammetric methods.

PREPARING THE DIAPOSITIVE

Each diapositive plate is marked with 9 evenly spaced crosses near its perimeter and center (Fig. 1). This can be accomplished by making fine cuts in the emulsion by use of a razor blade along a straightedge. These marks are not critical as to exact location with the exception of the center cross referred to hereafter as the radial center, which should be accurately positioned at the intersection of undrawn line extensions between each pair of fiducial marks. The crosses near the perimeter should be about 2 in. from the edge of the photographic plate to prevent them from being projected off of the table top.

DOUBLE PROJECTION INSTRUMENT OPERATION

The first and last stereoscopic model of the aerial photography strip to be bridged should be fully controlled. The control points required are the same as in accepted practice for bridging by use of first order, optical train, instruments, two horizontal three vertical control points on every fifth stereoscopic model, andfull control for every sixteenth stereoscopic model of no less than three horizontal and five vertical control points. A base sheet, long enough to encompass a five-model section, is prepared on a scale stable base drafting film and all horizontal control points are accurately plotted thereon to the scale at which the stereoscopic models will be given absolute orientation.

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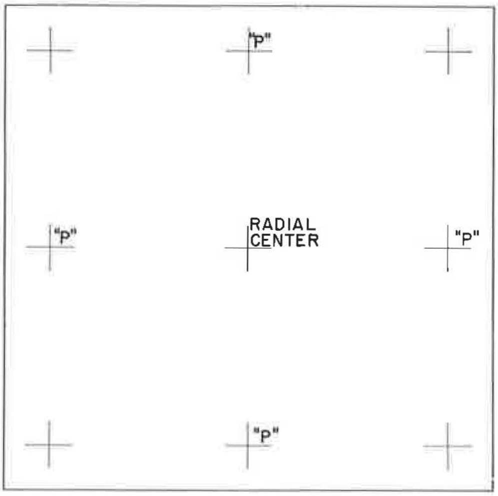

Figure 1. Marked diapositive plate.

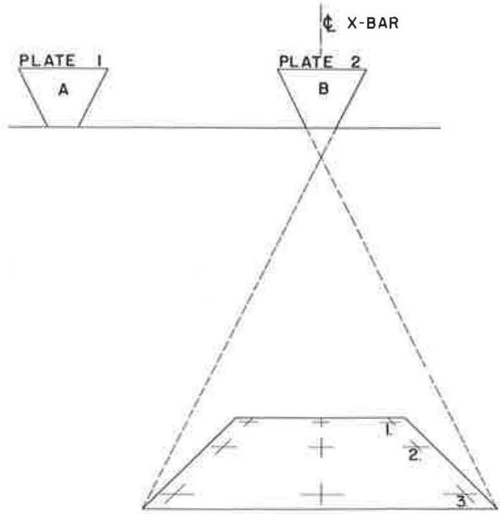

Figure 2. Starting position of projectors.

1. The first step in this process is recording the exterior orientation which is accomplished as follows:
   a. The initial stereoscopic model is oriented relatively and absolutely to conventional control surveyed on the ground for mapping purposes. Projector B (Fig. 2) containing diapositive No. 2 should occupy the center of the X-bar so that the cone of projection is visible on the table top. The tracing table index should be locked and not changed during triangulation.
   b. Record three pass points on the glass plate which will be on the leading edge of the stereoscopic model. One of the points must be near the flight azimuth and its use is more critical as to elevation than horizontal definition. The other two pass points, one near each corner of the stereoscopic model, are important as horizontal control bridging points and should be sharply defined.
   c. Disconnect the telescoping guide rod of the illuminating light of projector B from the tracing table and guide this table to each of the nine marked areas projecting them on the table top. Carefully mark each of their positions on the scale stable base sheet (Fig. 3).
2. The exterior orientation of this diapositive is now recorded. The only case in which these marked points will again correspond to their projected positions is when the photographic plate is in its original relationship to the working surface of the instrument table plane.
3. Recovery of the exterior orientation may now be made in the following manner:
   a. Rotate both projectors 180° (Fig. 4). Remove diapositive No. 1. Diapositive No. 3 may now be placed in projector A (Fig. 5) without disturbing the interior orientation of diapositive No. 2. Perform a normal relative orientation.
   b. Rotate the manuscript 180°. Disconnect the guide rod of projector B and observe once more the pattern of crosses as recorded after the initial absolute orientation.
   c. Adjust the X-bar, using the three suspension screws until the projected marks again fit their plotted positions (Fig. 6). The base sheet should be oriented to the radial center cross and a double correction applied to each of the 4 points, marked P (Fig. 1), and this is accomplished by reorienting on the radial center each time. When these 5 points agree, check the 4 corner points and apply minor corrections. If all 9 points do not fit, go back and check for residual Y-parallax.

Figure 3. Recording the exterior orientation.

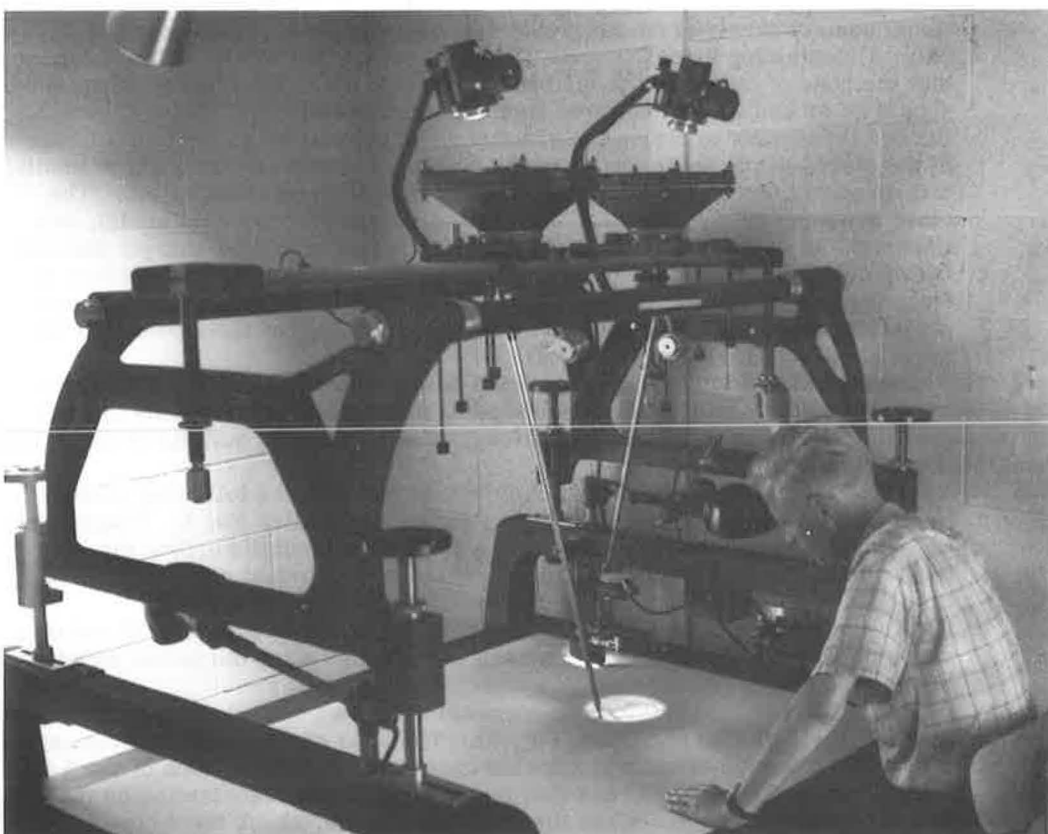

Figure 4. Rotating the projectors.

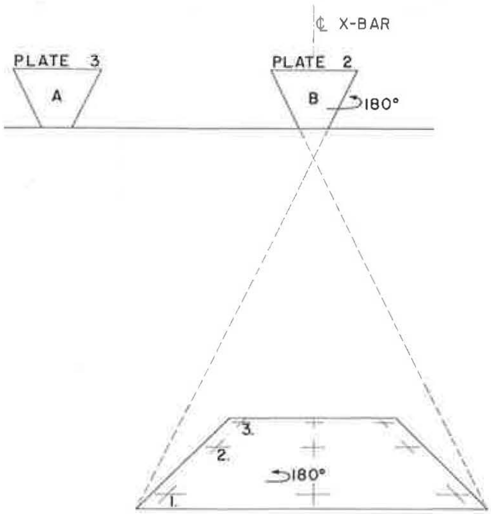

Figure 5. Rotation of projectors.

d. Reconnect the guide rods of the illumination lamps and observe the center pass point stereoscopically. Adjust the air base with the scaling (X-motion) knob until the previously marked elevation is read. If this correction can be applied without moving the center projector, absolute orientation is achieved. If the center projector has been moved, however, the sheet must be reoriented.

4. The next step is minor adjustments.

a. Measure the elevation of the pass point in each corner of the stereoscopic model and make orientation adjustments to bring them to their proper elevation at the scale of the stereoscopic model. To do this, use the leveling screws and scale adjustment, but do not change the tracing table index.

b. Recheck the projected marks and horizontal pass points. When all of these factors agree, absolute orientation is completed.

5. After securing the scale stable sheet to the working surface of the table, the instrument operator selects a series of suitable photographic images to serve as vertical and horizontal pass points to serve as supplemental control points to each of which he

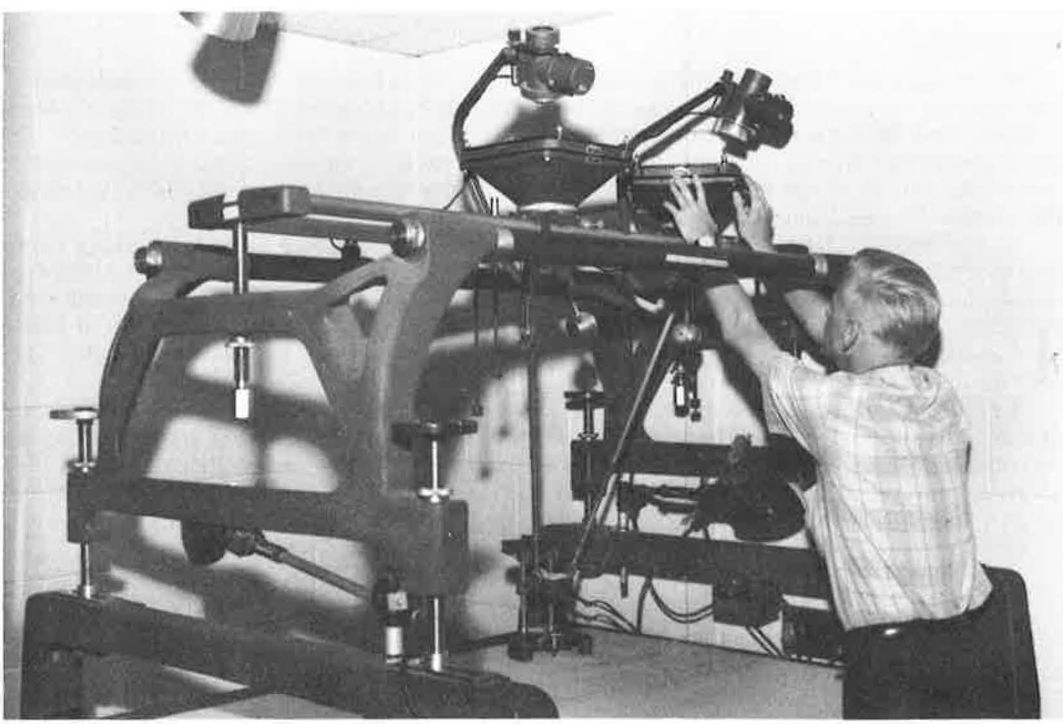

Figure 6. Recovering the exterior orientation.

applies an identification mark on the photograph, and also marks its position on the base sheet along with the radial center of each diapositive. It is desirable, for adjustment purposes, to select points for vertical control where all points will be nearly the distance from the azimuth of the photographic flight line.

6. Before transcribing the succeeding orientation from plate No. 3, both projectors of the instrument must be moved so projector A occupies the center of the X-bar. Absolute orientation is repeated using the control points marked on the base sheet before the cross marks on the next diapositive being used are recorded on the base sheet. This step is essential so all cross marks on this diapositive will be projected on the working surface of the table top at one time.

The instrument operation procedure outlined is repeated for each succeeding diapositive to the match edge of the terminal or controlled stereoscopic model.

The stereoscopic models should not be adjusted to conform to scattered field survey control points encountered in the photographic strip before all work is completed to the end of the bridge. Similarly, no attempt should be made to correct for errors observed in the terminal model. It is from these errors, which are measured and noted in horizontal position and in elevation, that adjustment begins.

This method is based on the assumption error has not been eliminated but controlled, insomuch as it has been cumulated in a systematic manner. The only errors which can be corrected are systematic, therefore the ability and concentration of the stereoscopic instrument operator is of primary importance. Major accidental errors will disrupt the bridged control in an easily identifiable way, eliminating danger of endeavoring to compile maps using extremely erroneous supplemental control data.

The vertical error is caused by a multitude of factors, including lens characteristics, flatness of the working surface of the table top, and human perception. Photogrammetric instrument operators should not be changed during any phase of photogrammetrically bridging to establish needed supplemental control, as resolution of Y-parallax has the greatest effect and is subject to individual interpretation.

ADJUSTMENTS FOR REDUCING ERRORS

Horizontal

The horizontal error is a direct result of the vertical error. The points selected for vertical bridging of control rise as the bridging is extended along the strip of photographs, thus the error is increasingly positive. This tends to displace horizontal positions back towards the beginning of the strip, causing an increasing shortness in horizontal length of the bridge. Consequently, reduction of the raw data is begun with the horizontal positions rather than the vertical error.

As previously stated, the horizontal error cumulates directly at an increasing rate, therefore it is necessary to correctly apportion the ultimate error among the stereoscopic models of the photography strip. This correction is more accurately measured vertically than horizontally, thus a standard procedure used in bridging by use of Multiplex aeroprojectors was adapted to this problem of bridging by use of Kelsh double projection instruments (2).

A straight line should be drawn on the base sheet (Fig. 7) through the radial centers of the first and last diapositives used in the photogrammetric bridging strip. Select a coordinated horizontal point on the initial stereoscopic model and on the terminal stereoscopic model, referred to for convenience as points A and B. Points A′, B′, and C′ are located on the central axis at the perpendicular of points A, B and C.

Obtain the correction factor, ΔZ, by use of the following:

$$\Delta Z = \frac{H \times E}{L} \qquad (1)$$

in which

$E = A'B' - A'C'$;
$H = A'B'$ / number of stereoscopic models; and
$L = AB$.

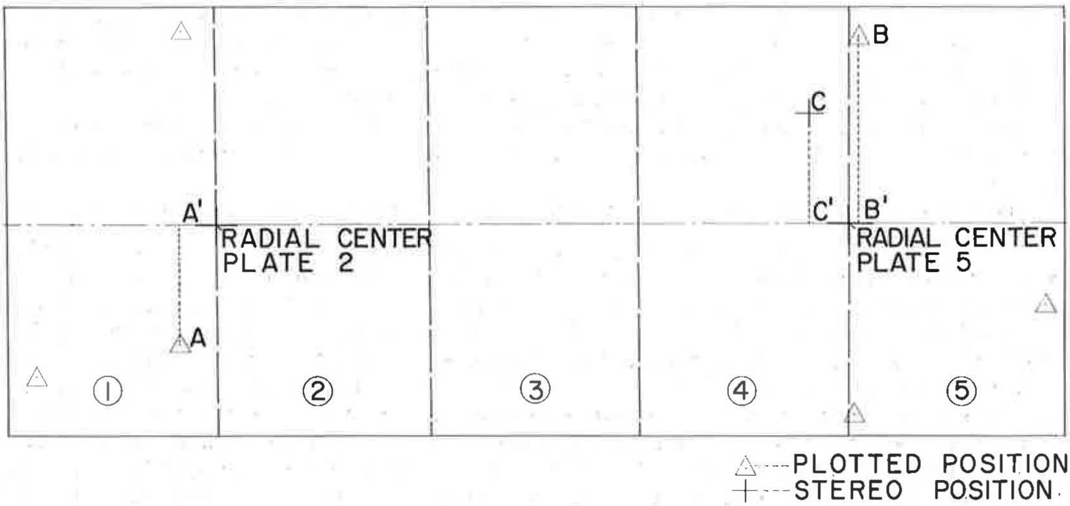

Figure 7. Horizontal plan.

Reset the first uncontrolled stereoscopic model using the unadjusted, bridged control. Subtract ΔZ from the elevation of the center pass point and, by changing the air base, reduce the apparent elevation of the point. Control the swing of the base sheet with the marked position thereon of the radial center and remark from the stereoscopic model onto the base sheet all control points and pass points. Now reset all the bridged stereoscopic models of the photography strip, using the new position of the pass point near each corner of each of the successive stereoscopic models to adjust the base and radial centers to control their swing. As this is done, level each model on the unadjusted vertical control. Prepare an overlay showing plotted point A (Fig. 7) and all stereoscopic pass points. Holding point A, swing point C, making it coincidental with point B. Prick each stereoscopic pass point in its adjusted position on the base sheet. If a significant error in the X direction remains at B, the entire procedure may be repeated until this disparity is removed.

Vertical

Due to the inherent limitations of the Kelsh stereoscopic plotter as an instrument for bridging supplemental vertical control, a graphic solution produced results which were, for all practical purposes, as good as could be obtained. Furthermore, the ease and efficiency of this method recommends it from an economic standpoint.

It might be useful to first describe some of the characteristics of the error which will be adjusted. If the deviation of each bridged point from the correct datum is plotted in its proper horizontal relationship to each preceding point, the successive points lie very nearly on two smooth curves, one for pass points lying on the far side and one for pass points lying on the near side of the trace of the flight line. The two curves are due to error on the Y-axis of the photographic strip which, compounded by the much greater error in the X direction, usually makes it necessary to consider each side of the line of flight separately. The slope of these curves on the graph cumulatively increases because the elements which induce the initial inaccuracy remain constant throughout the operation. Therefore, the final discrepancy is the product of a standard error and the preceding total plus a percentage of the previous error incorporated every time an orientation is recorded.

As in the horizontal adjustment (Fig. 7), it is convenient to locate the points on an instrument coordinate system with the ground trace of the aircraft line of flight as the X-axis. Distances along this line are measured at bridging scale from the first controlled radial center. The line formed by using each pass point near each corner of each stereoscopic model lying each side of the flight line is considered as a unit, usually designated positive and negative, but to avoid ambiguity, referred to herein as red and blue. With direction of the photogrammetric bridge extending from left to the right, the upper half is red, the center line (flight line) is green and the lower half is blue. Points are numbered VR-1, vertical red number one, VB-5 vertical blue number five, and so forth.

The error-curve may now be predicted from the final error in the following manner:

Locate perpendiculars to the bridged vertical control pass points along the central axis. Record the distance to each point and selecting a convenient scale, plot this information on cross-section paper (Fig. 8). Choose a useful vertical scale and plot the observed elevation error of each vertical control pass point from its match edge in the terminal stereoscopic model plotted above its horizontal position on the graph.

Divide, for each vertical control pass point, the graph height of this line into as many equal segments as there are uncontrolled stereoscopic models in the photogrammetric bridge (Fig. 8). With one end of a straightedge at the origin of the X-axis, draw radiating lines through each division on the vertical axis. Mark, either red or blue, the juncture of the appropriate radial center with these radiating lines. A smooth curve through each set of colored points will give the best approximation of the error-curve for the vertical control pass points each side of the flight axis. Center points are corrected midway between the two curves.

In actual practice this method was refined to a "point to point" solution as follows:

The horizontal (X-axis) position and the vertical (Y-axis) position of each point was plotted at the same scale. The origin used for the Y-axis is the furthermost control point (Fig. 9). Lines were radiated from the beginning point of the X-axis through each point on the vertical Y-axis. The appropriate color was marked at the intersection of each radial line with a line drawn vertically at a right angle to the X-axis from the plotted horizontal position of each applicable point on the X-axis. This yields one curve at three vertical scales. To obtain the correction factors, divide the number of small "squares" between origin point of X-axis and each control point into its elevation error. This gives the value, in feet of elevation, for each small "square." The factor for the center points is an average of the red and blue figures. Multiply the height of each point intersection by the proper correction factor to obtain the error in elevation of each bridged vertical control point.

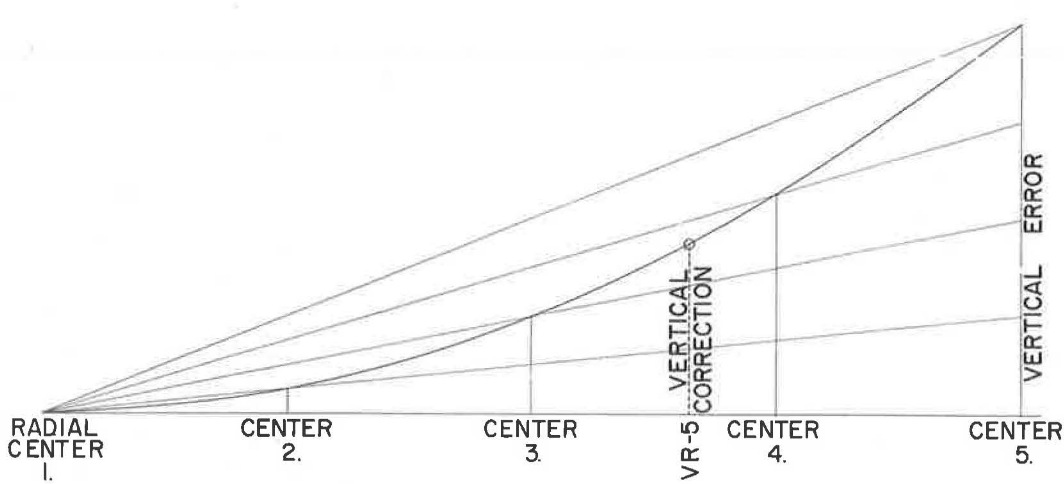

Figure 8. Vertical error curve.

Earth Curvature Correction

If the photogrammetric bridge extends for a long distance, the earth's curvature in feet should be computed by the standard expression:

$$V = 0.167 L^2 \qquad (2)$$

where L is the length of the photogrammetric bridge in miles.

V can be plotted on an expanded scale in the center of the graph. An arc through V from end to end of the strip provides a simple method of determining this correction for each bridged vertical.

If the photography used is not the "distortion-free" type, an additional correction is required to minimize the effects of lens distortions. The photography used for this test project was taken with a Wild R-C 8 aerial camera, which eliminated the need for considering lens distortions.

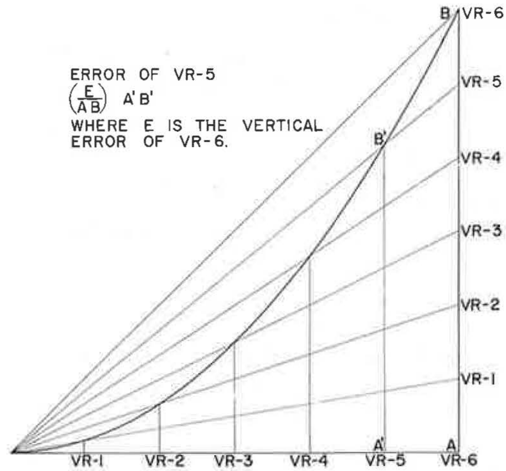

Figure 9. Refined error curve.

RESULTS

Selection of Projects and Scales

The survey projects selected for this investigation were chosen to obtain a variety of topography and photography scales. Every "photogrammetric bridge" was plotted and computed at least twice and the length of several of the bridges was varied a number of times. Each flight strip of photographs selected for the photogrammetric bridging work had been used previously in photogrammetric compilation of maps which had been field checked. After bridging, the elevation ascertained thereby for each vertical control pass point was compared with the elevation measured for it by field surveys on the ground. It may have been possible to achieve better precision with ideally placed ground survey control at a standard distance from the ground trace of the flight line, but position for such control was not selected with bridging in mind. Positioning of the points varied considerably in Y-distance from the flight line.

Horizontal Accuracy

The accuracy achieved in establishing the position of horizontal supplemental control was much better than accomplished for vertical control. At present there is no coordinatograph in Georgia. Consequently, measurement of the errors gave rise to conjecture. Perhaps some of the positional errors were due to slight variations in the plane coordinate grid lines and in plotting the surveyed horizontal control. Aerial photography scale did not have an effect on the real horizontal errors. Thus, large scale photography yields quite accurate results at map compilation scale. In observation of numerous bridged horizontal points throughout the entire test project, the greatest discrepancy noted was less than 0.05 in. at the stereoscopic model scale at which the photogrammetric bridging was done. The error was so minute that it could not be reliably measured with present equipment.

For such reasons, it was not deemed useful to present any more than a representative tabulation of results attained (Table 1). More work should be done with a coordinatograph, to better determine the magnitude of error. In addition, virtually no difference in horizontal accuracy was noted between bridges of 5 and 7 stereoscopic models. How many stereoscopic models can be used between surveyed control points in photogrammetric bridging with double projection instruments and maintain acceptable accuracy has not as yet been determined.

Vertical Accuracy

There are three major factors affecting the quality of the vertical accuracy:

1. Ability of the stereoscopic instrument operator.
2. Length of the photogrammetric bridge in number of stereoscopic models.
3. Calibration of the stereoscopic plotter.

Much more precision is demanded of the photogrammetric instrument operator in bridging vertical control than in photogrammetrically compiling conventional topographic maps. In the compilation of topographic maps there is usually an allowable error of plus or minus one-half contour interval. Moreover, the photogrammetric instrument operator engaged in bridging cannot eliminate all error while obtaining supplemental control, thus requiring the subsequent mapping work to be virtually errorless. Therefore, at present, this method of establishing supplemental control does not acceptably fulfill all requirements for much of the large-scale topographic mapping needed for highway design and preparation of detailed construction plans.

TABLE 1

REPRESENTATIVE TABULATION

Point No.	Error (in.)
1	0.00
2	0.00
3	0.01
4	0.03
5	0.02
6	0.01
7	0.01
8	0.00
9	0.00

The results from bridging supplemental vertical control as subsequently explained and as noted in Table 2 are based on the following aerial photography scales and contour invervals:

Mapping Scale (ft to 1 in.)	Contour Interval (ft)
250	2
500	5
1,000	10

Two lengths were considered for the photogrammetric bridges; control on the first and seventh stereoscopic models and control on the first and fifth. On the longer bridges it was found that 74 percent of the bridged vertical control points were not in error more than plus or minus one-half contour interval and 40 percent not more than plus or minus one-quarter contour interval. For the bridges controlled on stereoscopic models one and five, 98 percent were not in error more than one-half contour and 75 percent more than one-fourth contour interval. It seems likely from these results that control on the first and fourth stereoscopic models would yield dramatically better results.

Extremely small changes in instrument calibration can produce fairly large changes in the unadjusted vertical error. This causes no particular problem if the photogrammetric bridging is completed with little interruption using a well-calibrated instrument. The final error of a single strip bridged twice may double on one of the operations, but the residual error will remain virtually the same, barring accidental error which usually affects single points.

More work needs to be done to prove the reliability of this process in all cases. As previously mentioned, information is lacking on both shorter and longer photogrammetric bridges than reported herein. Consequently, more of such bridging tests should be undertaken using scattered ground control, and smaller and larger numbers of stereoscopic models in each bridge.

TABLE 2

VERTICAL ACCURACY

Photography Scale (ft/in.)	Control Error Less Than (ft)	Control in Stereo Models (%)	
		1 and 7	1 and 5
250	1.5	100	100
	1.25	87.1	100
	1.0	67.7	93.8
	0.75	58.1	93.8
	0.50	41.9	68.8
	0.25	25.8	31.3
500	3.5	87.5	100
	2.0	75.0	100
	1.50	50.0	100
	1.0	25.0	85.7
	0.5	12.5	80.0
1,000	4	--	100
	3	--	83.3
	2.5	--	66.7
	2.0	--	50.0
	1.5	--	33.3
	1.0	--	16.7

CONCLUSIONS

Truly reliable information on cost is unavailable at this stage, but some speculation can be made regarding these experimental projects.

A photogrammetric bridge of five stereoscopic models should require about 30 man-hours of work, including all adjustments to completion. Comparative costs are difficult to compile as costs will vary from place to place. But in Georgia photogrammetric bridging work represents roughly only one-fifth of the normal cost of establishing supplemental control by surveys on the ground.

The most efficient use of photogrammetric bridging of supplemental control would be for block area mapping. The field survey parties must measure a traverse along a highway route comprising a strip area for which control must be established regardless of the number of stereoscopic models, whether or not they are to be bridged. The control traverse measuring comprising a series of parallel strips of aerial photographs, for mapping a broad area, could proceed at a right angle to the direction of the aerial photography strips and cross mapping the project area at the interval of the number of stereoscopic models to be included in each photogrammetric bridge.

The most immediate benefit of this process of photogrammetric bridging has been determination of erroneous field surveyed control. Large vertical or horizontal discrepancies will generally be apparent without adjusting the raw data and smaller errors will be discovered after adjustment.

For stereoscopic models not containing reliable control on which to end the photogrammetric bridge, a form of cantilever extension may be employed. Set up the stereoscopic model preceding the last reliably controlled model. Now bridge the controlled model and observe the amount of vertical error apparent at the extreme edge. Correct this error and bridge the questionable models, applying the same correction after each bridging operation. In some situations, determining which of several control points should not be used can eliminate sending a field survey party back to an area to make essential checks.

Other areas of application might be:

1. Small-scale topographic mapping.
2. Horizontal bridging using field surveyed vertical control.
3. Planimetric mapping.

Results achieved thus far suggest continuation of this test project. Many answers obtained point the way to further variations in technique. Better results are expected as the photogrammetric investigations using double projection instruments are continued.

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