Repeated Stresses in Highway Bridges

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This report presents an approach to certain problems associated with repeated stresses in highway bridges. This approach includes a proposed method for predicting the frequencies of various levels of stress to which various members of highway bridges may be subjected as a result of individual or vehicle group loads encountered in the various compositions and volumes of traffic for given periods of time or throughout the life of a given structure. For the investigation of varying numbers of repetitions of various intensities of stress and how they may be related to present design criteria for fatigue, it is highly desirable that a reliable method be available for predicting the frequencies of such stresses. It is believed that the method presented accomplishes this objective by providing the means for predicting the frequencies of various levels of stress produced by heavy vehicle loads in any particular part or member of a given bridge corresponding with given traffic conditions. The proposed method takes into account (a) the frequency distribution of heavy truck loads measured in terms of their H-equivalencies on various spans, (b) the lateral placement of vehicles in highway traffic, and (c) the relative frequencies of the stress-producing effects of these loadings at any selected point in a given bridge.

• **THIS PAPER** presents a suggested procedure for estimating the number of repeated stresses of varying intensities produced by heavy truck loads in highway bridges. The proposed method is made possible by the following facts:

1. Vehicles by type (automobiles, buses, light trucks, heavy trucks, etc.) have been found to occur at random in ordinary highway traffic. This provides the means for estimating the frequencies of specified vehicle groups at a given location, such as at a bridge, mathematically based on the elementary laws of chance.
2. The time and/or distance spacings of vehicles have been found to occur at random in ordinary highway traffic. This randomness makes it possible for traffic engineers and others to apply statistical methods and the theory of probability to traffic problems that could not be solved satisfactorily on the basis of judgment alone (5).
3. The sizes and weights of heavy trucks and their H-equivalencies on various span lengths also have been found to occur at random in ordinary highway traffic. This randomness provides the means for describing the frequency distributions of gross vehicle weights and H-equivalencies for various span lengths on a mathematical basis (1, 5, 6, 9). Once the H-equivalency of a given truck has been determined for a given span, its stress-producing effects may be evaluated mathematically or from charts (see Figs. 7-9).
4. For any simple-span beam bridge of given length, design designation, and type of construction, it has been found that the percent of total design moment per interior beam caused by dead load remains about the same, irrespective of the lateral spacing of beams or stringers (3, 6, 10). Similarly, for a given span the percent of total design moment (or stress) per interior beam caused by live load plus impact also remains

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about the same, irrespective of the beam spacings (3, 6, 10). These findings permit the live- and dead-load moments per interior beam or per 10-ft lane to be generalized for the interior beams of simple-span bridges of given construction type and design designation (Fig. 5).

In order to simplify the presentation of this proposed method, the discussion and illustrative examples are based on maximum bending moments in simple-span beam bridges of H15-44 design consisting of a concrete deck of minimum thickness supported by unencased steel beams. It is also assumed that the steel beams in these bridges are so spaced and the deck is of such rigidity that the maximum live-load bending stress produced in an interior stringer by a single vehicle in one lane only will amount to C = 75 percent of that produced by identical vehicles in each lane simultaneously. A more detailed discussion of this concept is given in Appendix A. The ratio of dead-load stresses to total design stresses for such steel beam bridges is smaller than for any of the heavier types of construction, such as reinforced concrete deck girder bridges. This light type of construction is used as a basis of discussion because conclusions concerning the stress-producing effects of a given truck or trucks will be on the conservative side.

EQUIVALENT H TRUCK LOADINGS

If a given heavy truck produced a maximum live-load moment of 445.6 kip-ft, with no impact, on a 50-ft span, it would be the same as that produced by an H20 truck on that span. On a 50-ft span, therefore, this truck would be converted into or rated as an equivalent H truck load weighing 20 tons, or simply an equivalent H20 truck loading. Similarly, if a given truck would produce as much live-load moment on a given span as an H26.4 truck it would have an H-equivalency of 26.4 tons, or 52.8 kips on that span. A more detailed discussion of equivalent H truck loadings is given in Appendix A, and the relative frequencies of equivalent H truck loadings for various spans, as reported by the national truck weight (loadometer) study of 1954, are given in Table 1.

Once the H-equivalency of a given truck has been determined for a given span, its stress-producing effects can be found by Eq. 3, as explained in Appendix A. The same procedure may be used for each heavy truck reported by a loadometer survey; that is, determine the H-equivalency for each truck for each span length (Table 1).

DESIGN-STRESS RATIOS

Design-stress ratios, Q, define the ratios of total actual stresses to total design stresses at any point in a bridge. For example, if design calculations for a 50-ft steel beam bridge of H15 design show a maximum dead-load stress of 8.28 ksi and a maximum live-load plus impact stress of 9.72 ksi in one of the interior stringers, the total design stress would be 8.28 + 9.72 = 18.00 ksi. If a given heavy truck would produce a maximum live-load plus impact stress of 14.56 ksi, the total actual stress in this stringer would be 8.28 + 14.56 = 22.84 ksi. In this case, the design-stress ratio would be Q = 22.84/18.00 = 1.27. The truck under consideration would produce a total stress of 1.27 times the basic design stress of 18.00 ksi, or an overstress of 27 percent. Design-stress ratios resulting from various H-equivalences on 30-, 50-, and 100-ft spans for one truck in one lane only and one truck in each lane simultaneously with varying allowances for impact are shown in Figures 7-9.

Thus, with any given volume of traffic containing a known percentage of heavy trucks (13 tons or more), whose H-equivalencies had been determined (or estimated) such as those given by Table 1, it would be a simple matter to determine the numbers of stress repetitions of various levels that would result from such traffic — taking the vehicles one at a time and assuming that each one would be so positioned laterally to produce maximum live-load stress. For a traffic volume of 500 vph for 50 years with 5 percent heavy trucks and H-equivalencies as given by Table 1, there would be a total of 11 million heavy trucks equally divided between the two directions. The numbers of design-stress ratio that would result, taking the trucks one at a time, with and without impact on a 50-ft and a 100-ft span, would be as given by Tables 2 and 3, respectively.

TABLE 1

RELATIVE FREQUENCIES OF EQUIVALENT H TRUCK LOADINGS[a] FOR 16,888 HEAVY TRUCKS OF THE SIX MAJOR TYPES (9)

Equivalent H Truck Loadings	Span (ft) 10	20	30	40	50	60	80	100	Infinite G.V.W.
5	0.04	—	—	—	—	—	—	—	—
6	0.21	—	—	—	—	—	—	—	—
7	0.75	0.12	0.01	—	—	—	—	—	—
8	2.01	0.15	0.15	0.07	0.01	—	—	—	—
9	2.87	1.55	0.49	0.26	0.16	0.05	—	—	—
10	7.16	4.14	3.32	1.87	0.67	0.14	0.01	0.01	0.01
11	15.02	9.74	5.51	5.19	2.75	1.40	0.82	0.76	0.75
12	27.30	10.92	11.43	8.36	4.90	3.85	2.04	1.74	1.53
13	26.83	11.55	10.51	8.94	7.14	4.24	2.58	1.90	1.19
14	11.03	10.57	9.27	7.40	6.90	6.07	3.22	2.05	1.12
15	3.61	12.59	10.56	9.56	7.59	5.94	4.99	3.36	0.77
16	1.84	19.87	10.51	10.18	8.59	7.52	5.42	4.29	0.60
17	0.69	9.28	18.41	15.81	11.17	6.89	5.90	5.91	1.15
18	0.31	5.90	11.95	14.19	12.99	9.53	6.96	6.09	2.90
19	0.12	2.08	3.60	9.06	14.08	11.23	6.85	7.10	4.00
20	0.06	0.69	2.19	5.03	11.85	11.62	7.08	5.91	5.48
21	0.09	0.52	1.01	1.73	4.97	17.31	10.04	7.40	5.55
22	0.02	0.15	0.55	1.04	3.39	6.45	11.20	8.98	4.51
23	0.02	0.07	0.25	0.63	1.13	3.41	14.16	13.21	4.52
24	0.01	0.02	0.09	0.31	0.72	2.14	8.69	12.01	4.58
25	0.01	0.01	0.04	0.15	0.52	0.92	4.45	9.54	6.18
26	—	0.02	0.04	0.05	0.20	0.60	2.59	3.15	8.72
27	—	0.02	0.03	0.05	0.08	0.30	1.48	2.80	9.77
28	—	0.01	0.01	0.02	0.05	0.16	0.69	1.65	8.71
29	—	—	0.04	0.02	0.03	0.06	0.38	0.92	5.65
30	—	0.01	—	0.02	0.01	0.04	0.20	0.57	4.08
31	—	0.01	—	0.02	0.03	0.02	0.08	0.27	3.10
32	—	0.01	—	0.01	0.02	0.04	0.05	0.18	2.59
33	—	—	0.01	—	0.01	0.01	0.02	0.05	2.91
34	—	—	0.01	—	—	0.01	0.01	0.05	2.79
35	—	—	0.01	0.01	—	0.01	0.04	0.01	2.61
36	—	—	—	0.01	0.02	0.01	0.01	0.03	2.00
37	—	—	—	0.01	0.01	0.01	0.01	0.02	1.05
38	—	—	—	—	—	—	0.01	0.02	0.59
39	—	—	—	—	0.01	0.01	—	—	0.29
40	—	—	—	—	—	0.01	—	—	0.13
41	—	—	—	—	—	—	0.01	0.01	0.07
42	—	—	—	—	—	—	0.01	—	0.02
43	—	—	—	—	—	—	—	—	0.02
44	—	—	—	—	—	—	—	0.01	0.03
45	—	—	—	—	—	—	—	—	0.01
50	—	—	—	—	—	—	—	—	0.01
51	—	—	—	—	—	—	—	—	0.01
Total	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Max. H Truck	25	32	35	37	39	40	42	44	51
Avg. H Truck	12.24	14.37	15.16	15.93	17.15	18.41	20.25	21.17	25.60
Min. H Truck	5	7	7	8	8	9	10	10	10
Poisson's Coeff., Z	7.2	7.4	8.2	7.9	9.2	9.4	10.3	11.2	15.6

[a]Equivalent H truck loadings based on moments produced by gross vehicle weights on simple spans.

TABLE 2

NUMBER OF STRESS REPETITIONS[a] IN MOST HIGHLY STRESSED STRINGER IN EACH DIRECTION OF A 50-FT SIMPLE-SPAN BRIDGE OF H15-44 DESIGN[b]

Equivalent H Truck Loading	Poisson Distribution for Z = 9.2	Number of Vehicles	Design-Stress Ratio[c]	
			Full Allowance for Impact[d]	No Allowance for Impact[e]
8	0.000101	556	0.676	0.628
9	0.000930	5,115	0.703	0.649
10	0.004276	23,518	0.730	0.670
11	0.013113	72,122	0.757	0.691
12	0.030160	165,880	0.784	0.712
13	0.055494	305,217	0.811	0.733
14	0.085091	468,001	0.838	0.754
15	0.111834	615,087	0.865	0.775
16	0.128609	707,349	0.892	0.796
17	0.131467	723,069	0.919	0.817
18	0.120950	665,225	0.946	0.838
19	0.100158	556,369	0.973	0.859
20	0.077555	426,553	1.000	0.880
21	0.054885	301,868	1.027	0.901
22	0.036067	198,369	1.054	0.922
23	0.022121	121,666	1.081	0.943
24	0.012720	69,960	1.108	0.964
25	0.006884	37,862	1.135	0.985
26	0.003518	19,349	1.162	1.006
27	0.001704	9,372	1.189	1.027
28	0.000784	4,312	1.216	1.048
29	0.000343	1,887	1.243	1.069
30	0.000144	792	1.270	1.090
31	0.000057	314	1.297	1.111
32	0.000022	121	1.324	1.132
33	0.000008	44	1.351	1.153
34	0.000003	17	1.378	1.174
35	0.000001	6	1.405	1.195
Total	1.000000	5,500,000		

[a]Stress effects based on continuous traffic volume of 500 vph (12,000 per day) containing 5% heavy vehicles (in excess of 13 tons gross weight).
[b]Assumed useful life of 50 years; a total of 11 million heavy vehicles occur on this span one at a time, divided equally between the two directions.
[c]See Figure 8 for design-stress ratio equations.
[d]$Q = 0.0270\ H + 0.460$.
[e]$Q = 0.0210\ H + 0.460$.

TABLE 3

NUMBER OF STRESS REPETITIONS[a] IN MOST HIGHLY STRESSED STRINGER IN EACH DIRECTION OF A 100-FT SIMPLE-SPAN BRIDGE OF H15-44 DESIGN[b]

Equivalent H Truck Loading	Poisson Distribution for Z = 11.2	Number of Vehicles	Design Stress Ratio[c]	
			Full Allowance for Impact[d]	No Allowance for Impact[e]
10	0.000013	71	0.755	0.728
11	0.000153	842	0.770	0.740
12	0.000858	4,719	0.785	0.752
13	0.003202	17,611	0.800	0.765
14	0.008965	49,308	0.815	0.777
15	0.020082	110,452	0.830	0.789
16	0.037487	206,178	0.844	0.801
17	0.059979	329,884	0.859	0.813
18	0.083970	461,835	0.874	0.826
19	0.104496	574,728	0.889	0.838
20	0.117035	643,692	0.904	0.850
21	0.119163	655,396	0.919	0.862
22	0.111220	611,710	0.934	0.874
23	0.095820	527,010	0.949	0.887
24	0.076656	421,608	0.964	0.899
25	0.057236	314,798	0.979	0.911
26	0.040065	223,058	0.993	0.923
27	0.026396	145,178	1.008	0.935
28	0.016424	90,332	1.023	0.948
29	0.009682	53,251	1.038	0.960
30	0.005422	29,821	1.053	0.972
31	0.002892	15,906	1.068	0.984
32	0.001472	8,096	1.083	0.996
33	0.000717	3,944	1.098	1.009
34	0.000335	1,842	1.113	1.021
35	0.000150	825	1.128	1.033
36	0.000065	358	1.142	1.045
37	0.000027	148	1.157	1.057
38	0.000011	60	1.172	1.070
39	0.000004	22	1.187	1.082
40	0.000002	11	1.202	1.094
41	0.000001	6	1.217	1.106
Total	1.000000	5,500,000		

[a]Stress effects based on continuous traffic volume of 500 vph (12,000 per day) containing 5% heavy vehicles (in excess of 13 tons gross weight)

[b]Assumed life of 50 years; a total of 11 million heavy vehicles occur on this span one at a time, divided equally between the two directions.

[c]See Figure 9 for design-stress ratio equations.

[d]$Q = 0.0149\ H + 0.606$.

[e]$Q = 0.0122\ H + 0.606$.

FREQUENCIES OF SPECIFIED VEHICLE GROUPS

In a previous report (5) it was shown that Poisson's frequency distributions (12) could be used to determine how often two or more specified vehicles, such as two or more heavy trucks, could be expected to occur within a given length of bridge or a specified distance near the midspan of a bridge. The results of this study are summarized for 250 and 500 vph containing 5 percent heavy trucks in Figures 1 and 2, respectively.

It has also been found that Poisson's Law provided a good estimate for the frequency distribution of gross vehicle weights and H-equivalencies on various spans (Table 1). Here the Poisson equation is

$$P(n) = Z^n e^{-Z}/n! \qquad (1)$$

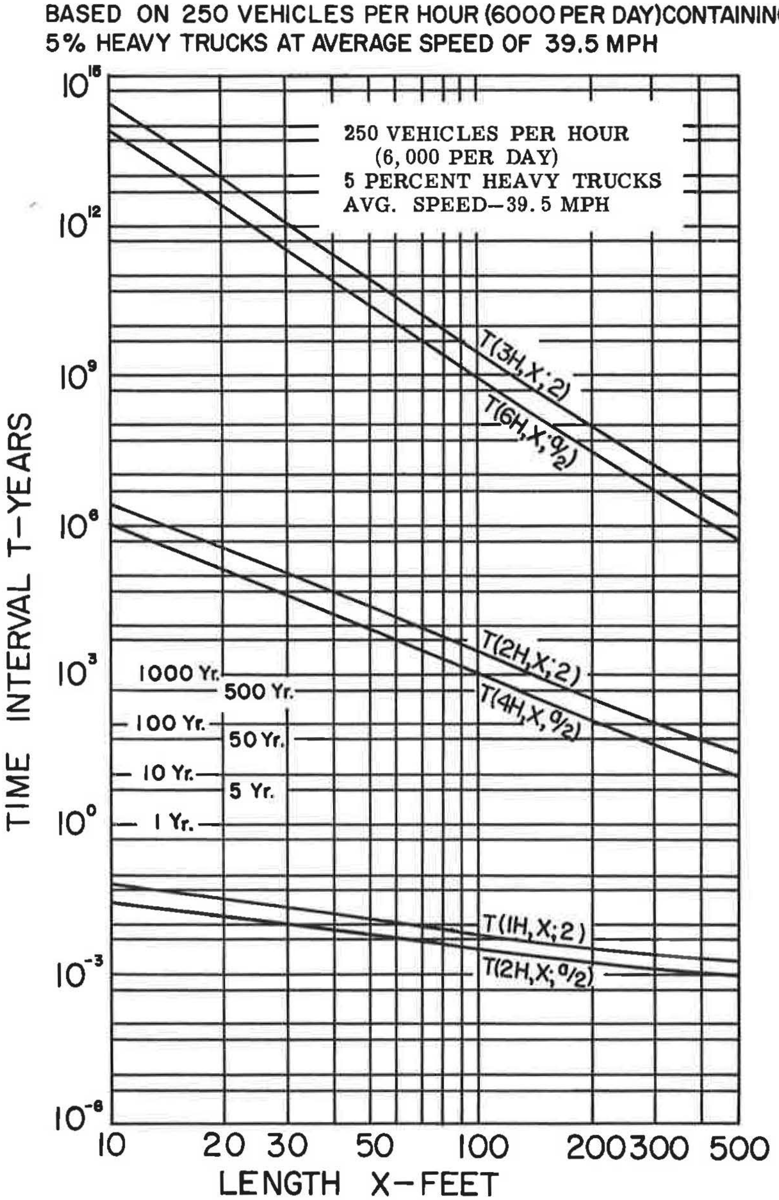


Figure 1. Time interval for typical specified vehicle groups occurring within specified lengths.

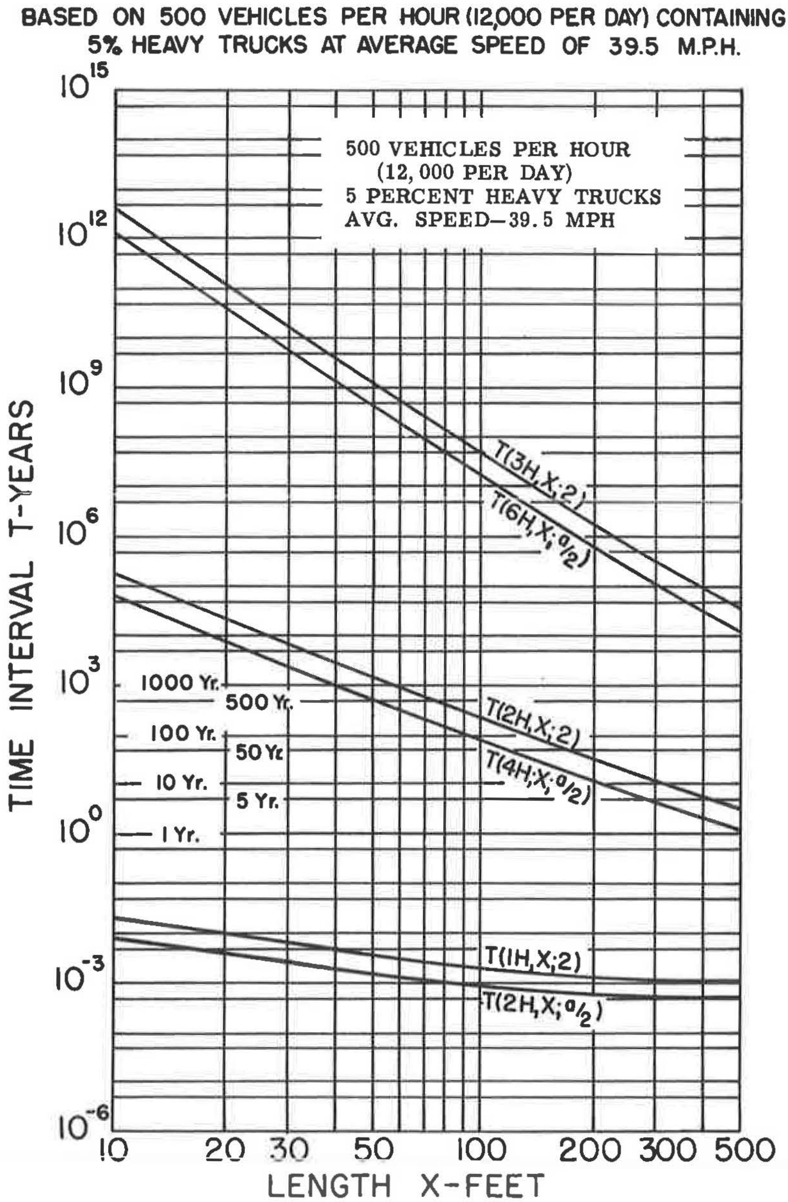


Figure 2. Time interval for typical specified vehicle groups occurring within specified lengths.

Each vehicle constitutes a sample whose H-equivalency is measured in tons. On a 60-ft span, for example, if the H-equivalency fell between 18.60 and 19.49 tons, it would be classified as an equivalent H19-ton truck loading. Table 1 gives the observed frequencies of equivalent H truck loadings, on various spans, for 16,888 heavy trucks reported by the 1954 truck weight survey. For the 50-ft span, the minimum H-equivalency is 8 tons and the average is 17.15 (or 17.2), and the Poisson coefficient is 9.2. This means that the average is 9.2 tons removed from the minimum, or the expected average is 9.2 cells removed from the zero cell. Eq. 1, therefore, would be read: the probability that the H-equivalency of a given vehicle will be n-cells (or in this case n-tons) larger than the smallest or zero cell, when the average is Z cells greater than the zero cell, is equal to $Z^n e^{-Z}/n!$

TABLE 4

NUMBER OF STRESS REPETITIONS[a] IN MOST HIGHLY STRESSED STRINGER IN EACH DIRECTION OF A 50-FT SIMPLE-SPAN BRIDGE OF H15-44 DESIGN[b]

Equivalent H Truck Loadings	Poisson Distribution for Z = 18.0	Number of Occurrences	Design-Stress Ratio[c]	
			Full Allowance for Impact[d]	No Allowance for Impact
8	0.000	—	—	—
9	0.000	—	—	—
10	0.001	2	0.820	0.740
11	0.003	6	0.856	0.768
12	0.012	24	0.892	0.796
13	0.039	78	0.928	0.824
14	0.088	176	0.964	0.852
15	0.144	288	1.000	0.880
16	0.182	364	1.036	0.908
17	0.183	366	1.072	0.936
18	0.148	296	1.108	0.964
19	0.099	198	1.144	0.992
20	0.056	112	1.180	1.020
21	0.027	54	1.216	1.048
22	0.011	22	1.252	1.076
23	0.004	8	1.288	1.104
24	0.002	4	1.324	1.132
25	0.001	2	1.360	1.160
Total	1.000	2,000		

[a]Stress effects based on continuous traffic volume of 500 vph (12,000 per day) containing 5% heavy vehicles (in excess of 13 tons gross weight).
[b]Assumed life of 50 years, resulting from 2,000 occurrences in each direction of one heavy truck in each of two adjacent lanes simultaneously.
[c]See Figure 8 for design-stress ratio equations.
[d]Q = 0.0360 H + 0.460.
[e]Q = 0.0280 H + 0.460.

TABLE 5

NUMBER OF STRESS REPETITIONS[a] IN MOST HIGHLY STRESSED STRINGER IN EACH DIRECTION OF A 100-FT SIMPLE-SPAN BRIDGE OF H15-44 DESIGN

Equivalent H Truck Loadings	Poisson Distribution for Z = 22.0	Number of Occurrences	Design-Stress Ratio[c]	
			Full Allowance for Impact[d]	No Allowance for Impact[e]
14	0.001	3	0.883	0.833
15	0.006	18	0.903	0.849
16	0.019	57	0.923	0.855
17	0.049	147	0.943	0.881
18	0.092	276	0.962	0.898
19	0.134	402	0.982	0.914
20	0.166	498	1.002	0.930
21	0.167	501	1.022	0.946
22	0.141	423	1.042	0.962
23	0.100	300	1.061	0.979
24	0.062	186	1.081	0.995
25	0.034	102	1.101	1.011
26	0.016	48	1.121	1.027
27	0.007	21	1.141	1.043
28	0.003	9	1.160	1.060
29	0.002	6	1.180	1.076
30	0.001	3	1.200	1.092
Total	1.000	3,000		

[a]Stress effects based on continuous traffic volume of 500 vph (12,000 per day) containing 5% heavy vehicles (in excess of 13 tons gross weight).
[b]Assumed life of 50 years, resulting from 3,000 occurrences in each direction of one heavy truck in each of two adjacent lanes simultaneously.
[c]See Figure 9 for design-stress ratio equations.
[d]Q = 0.0198 H + 0.606.
[e]Q = 0.0162 H + 0.606.

The distribution in Table 2 for the 50-ft span where Z = 9. 2 was taken from Molina's tables for the Poisson equation (12). For example, in such a population of vehicles, about 101 out of each million would be expected to fall in the 8-ton H-equivalency cell. The other frequencies would be interpreted similarly. Tables 1 and 2 give the numbers of repetitions of various levels of stress with and without impact that would result from 11 million heavy trucks in 50 years taken (one at a time) on 50- and 100-ft spans, respectively.

Table 2 gives 8 tons for the smallest H-equivalency on a 50-ft span. If these vehicles are taken two at a time, one in each direction simultaneously, the smallest total H-equivalency would be 2 × 8 = 16 tons. Likewise, the average for all pairs would be twice the average or 2 × 17. 2 = 34. 4. In this case, the value of Poisson's coefficient Z = 34. 4 - 16. 0 = 18. 4. The distribution in Table 4, however, is based on Z = 18. 0 beause this is the nearest value given in Molina's tables (12). Table 5 gives the distribution for Z = 22. 0 for the 100-ft span and is interpreted similarly.

LATERAL PLACEMENT OF VEHICLES IN TRAFFIC

If each of the trucks referred to in Tables 2 and 3 were to pass over the 50- and 100-ft spans and were so positioned laterally to produce maximum stress in an interior stringer, the numbers of repetitions of various levels of stress would be as given in these tables.

If, however, these vehicles are assumed to be positioned laterally according to some logical pattern, then the numbers of repetitions of various intensities of stress would be altered accordingly.

ESTIMATED NUMBERS OF VARIOUS INTENSITIES OF STRESS IN SIMPLE-SPAN BEAM BRIDGES

Table 2 indicates that for the truck population given by the 1954 truck weight survey (Table 1), about one truck in each million heavy trucks would be expected to produce a design-stress ratio of Q = 1. 405 with impact or Q = 1. 195 without impact. In other words, on a 50-ft steel stringer bridge of H15-44 design, about one truck in a million would produce an overstress of 40. 5 percent with impact and 19. 5 percent overstress without impact.

Table 4 gives the numbers of repeated stresses in 50 years resulting from one heavy truck in each lane simultaneously on this 50-ft span. The numbers of repeated stresses resulting from a truck in each lane are quite small; the largest overstress given is 36. 0 percent with impact and 16. 0 percent without impact. The heavier individual trucks will produce higher overstresses than those that would ordinarily occur on the span at the same time.

The combined numbers of repeated stresses indicated by Tables 2 and 4 for the 50-ft span (no lateral placement) are shown in Figures 3 a and b. In these figures the numbers of repeated stresses given by Table 4 (one in each direction simultaneously) do not change the appearance of these histograms.

For this same 50-ft span, if it is assumed that the lateral placement of trucks is such that ⅓ of them produce maximum live-load stress, ⅓ produce 90 percent, and the remaining ⅓ produce 80 percent, the resulting numbers of repetitions would be as shown in Figure 3 c and 3 d.

In a similar manner, the histograms in Figure 4 for the 100-ft span show the numbers of repeated stresses without and with lateral placement of the trucks and without and with impact.

For the 50-ft span of H15-44 design, Figure 3 shows that very heavy traffic with a high percent of heavy trucks would rarely result in overstresses in excess of about 25 percent. For the 100-ft span of H15-44 design, Figure 4 shows that similar heavy traffic would rarely result in overstresses of about 20 percent.

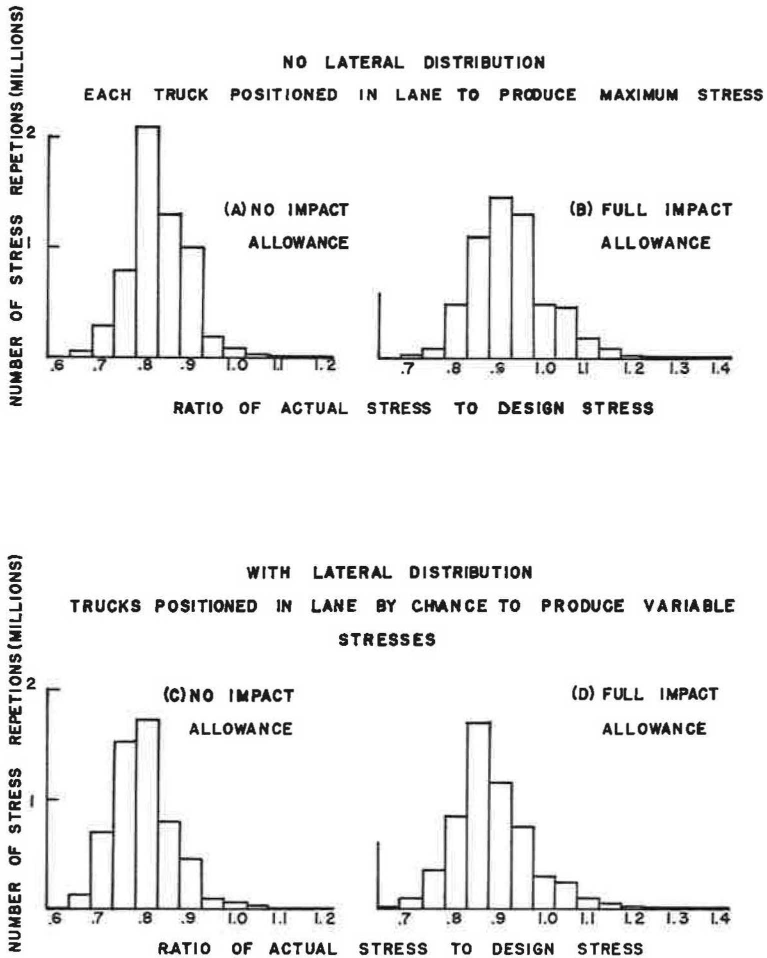


Figure 3. Number of repeated stresses produced in a 50-ft simple-span bridge of H15-44 design during an assumed useful life of 50 years (design-stress ratios based on continuous traffic volume of 500 vph containing five percent heavy trucks weighing 13 tons or more).

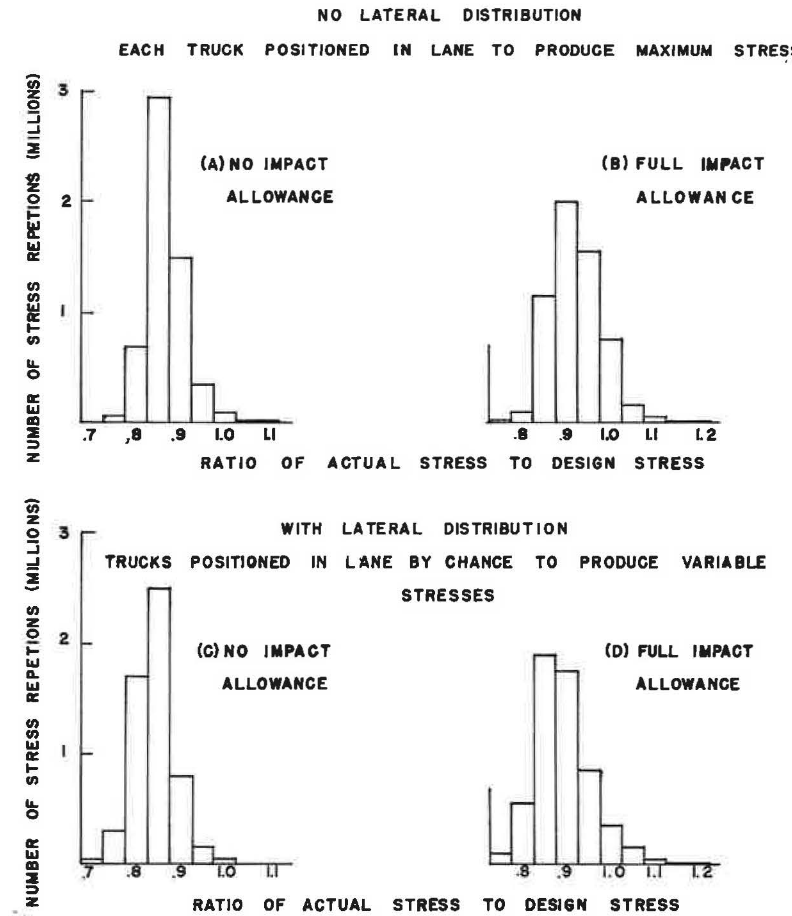


Figure 4. Number of repeated stresses produced in a 100-ft simple-span bridge of H15-44 design during an assumed useful life of 50 years (design-stress ratios based on continuous traffic volume of 500 vph containing five percent heavy trucks weighing 13 tons or more).

Appendix A

MAXIMUM BENDING STRESSES IN SIMPLE-SPAN BEAM BRIDGES

Appendix A presents a brief review of a method for estimating maximum combined bending stresses in simple-span beam bridges produced by dead load, live load, and impact. The maximum live-load and impact stresses are those resulting from any heavy vehicle type or loading found on the highway.

The procedure for estimating the total maximum stress in simple-span bridges of given construction type and design designation is accomplished in two simple steps: (a) convert any particular vehicle under consideration into its equivalent H truck loading on any span by use of the conversion coefficients given in Table 6; and (b) determine the stress-producing effects from charts similar to those in Figures 7 to 9 inclusive depending on the span and loading conditions (1, 3, 4, 6, 7).

The method for estimating the maximum combined bending stresses produced by dead load, vehicle loads and impact in simple-span beam bridges results from three simple observations.

1. It has been shown that any heavy vehicle may be converted into an equivalent H truck loading that will produce the same maximum bending moment on a given span as the particular vehicle under consideration (1, 3, 4, 6). Equivalent H truck loadings, therefore, provide a convenient means for describing or evaluating the stress-producing effects of any particular vehicle on a given span. Heavy vehicles may also be converted into any other equivalent design loading on the basis of moments, shears or any other stress function desired (1, 2).

2. It has been found, for any simple-span beam bridge of given length, that the percent of total design moment per beam caused by dead load remains about the same, irrespective of the lateral spacing of the beams (3, 6). Similarly for a given span, the percent of total design moment per beam caused by live load plus impact also remains

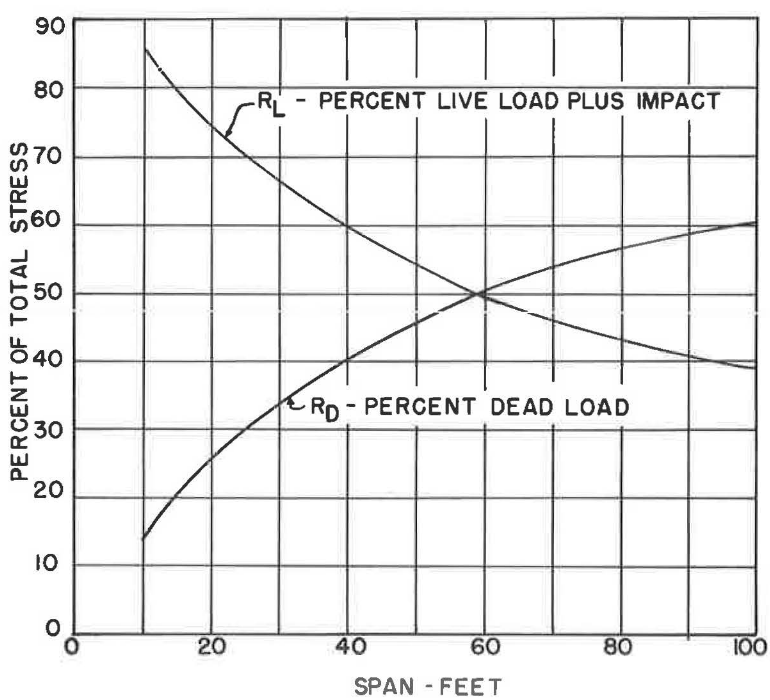


Figure 5. Estimated percent of total design stresses represented by live-load plus impact and dead-load stresses for simple-span beam bridges of H15 design.

about the same, irrespective of the beam spacing (3, 6). For a given span, therefore, it follows that the percent of total design moment caused by live and dead loads, respectively, is about the same per foot width of bridge or per 10-ft lane as it is per beam, irrespective of the beam spacing. These findings permit the live- and dead-load design moments per beam or per 10-ft lane to be generalized for simple-span beam bridges of given construction type and design designation (Fig. 5).

3. For a given bridge, the maximum dead-load stress is a fixed and definite percent of the total design stress. Also, the maximum live-load plus impact stress caused by a given vehicle will vary directly with the weight or H-equivalency of that particular vehicle. From these and preceding observations it has been shown that the total maximum stress caused by dead load, vehicle load, and impact in a given bridge may be expressed by a simple straight-line equation in which the total maximum stress is a function of the H-equivalency of the vehicle under consideration (3, 6). It is usually more convenient, though, to convert this total maximum stress into the ratio that it bears to the allowable stress for which the bridge was designed. This ratio is referred to herein as the design-stress ratio (Figs. 7 to 9).

Method for Estimating Maximum Bending Stresses

The bridges are of H15 design and consist of a non-composite deck of minimum thickness supported by unencased steel beams. It is also assumed that the supporting steel beams are so spaced that the maximum live-load bending stress produced in an interior stringer by a single vehicle in one lane only will amount to C = 75 percent of that produced by identical vehicles in each lane simultaneously. This means that if the given bridge were loaded with vehicles having identical H-equivalencies, one in each lane, the maximum live-load stress produced in a typical interior stringer would be 133 percent of that produced by only one of these vehicles in one lane only.

The reason for selecting this light type of construction is that the ratio of dead-load stresses to total design stresses is smaller than would be the case for any of the heavier types of construction, such as reinforced concrete deck girder spans. Consequently, any conclusions concerning the stress-producing effect of a given vehicle or vehicles on any particular bridge are on the conservative side. Although the discussion and examples are confined to bending moments and bending stresses in simple-span steel beam bridges of H15 design, the method is equally applicable to bridges of other construction types and design designation.

Once the percent of total design stresses caused by live load plus impact and dead load have been determined for bridges of a given type and design designation similar to those shown in Figure 5, it is convenient to consider the method for estimating total maximum bending stress in a given bridge in two parts.

1. Determination of equivalent H truck loadings.
2. Evaluation of total maximum stress caused by a given equivalent H truck loading on a given span corresponding with specified loading conditions.

Equivalent H Truck Loadings

Any heavy vehicle may be converted into an equivalent H truck loading that will produce the same maximum bending moment on a given span as the particular vehicle under consideration. Heavy vehicles also may be converted into any other equivalent design loading on the basis of moments, shears or other stress functions on various span lengths as may be desired (1, 2, 10).

The H-equivalency of a given vehicle on a given span may be determined on an exact basis by finding the maximum moment caused by this particular vehicle on the given span and selecting the standard H truck designation in tons that would produce the same maximum moment. The procedure for any other stress function would be similar.

However, it has been found from numerous investigations of actual vehicles irrespective of the number of spacing of axles, that any normal distribution of load among the axles of a given vehicle will produce slightly less moment on a given span than the same load would produce if it were uniformly distributed over a length L equal to the wheel base length of the vehicle under consideration (4, 6). This means that the maximum moment caused by any given vehicle on a given span can be estimated quite easily and accurately, but a little on the safe side, by the moment formula resulting from the uniform load of length L and weight W on a span S (Fig. 6). This loading results in the following formula:

$$M = \frac{W}{4} \left(S - \frac{L}{2}\right) \tag{2}$$

Eq. 2 provided the basis for calculating the coefficients given in Table 6 for converting heavy vehicles of given weight and wheel base length into equivalent H truck loadings on various span lengths.

The determination and use of the coefficients given in Table 6 can be illustrated by comparing the maximum moment caused by a heavy vehicle weighing 20 tons and having a total wheel base length of 28 ft with that caused by an H 20 truck on a 50-ft simple span. According to Eq. 2, the moment caused by the heavy vehicle would be 360.0 kip-ft. This compares with a moment of 445.6 kip-ft caused by an H 20 truck on a 50-ft span. Therefore, the 20-ton heavy vehicle with 28-ft wheel base causes 80.79 percent as much moment as the H 20 truck on this 50-ft span.

This means that a vehicle, with a 28-ft wheel base, will cause 0.8079 times as much moment on a 50-ft span as a standard H truck of equal weight. It also means that a vehicle of given weight with a 28-ft wheel base will cause as much moment on a 50-ft span as a standard H truck weighing 80.79 percent as much. Therefore, this 20-ton vehicle, with a 28-ft wheel base on a 50-ft span would have an H-equivalency of $20.0 \times 0.8079 = 16.16$ tons or correspond with an equivalent H 16.16 truck on that span.

Design-Stress Relationships

As stated previously, for any simple-span beam bridge of given length, the percent of total design stress (or moment) per beam caused by dead load remains about the same, irrespective of the lateral spacing of the beams (3, 6). Similarly, for a given span the percent of total design stress per beam caused by live load plus impact also remains about the same, irrespective of the beam spacing. Therefore, for a given span it follows that the percent of total design stress caused by live and dead loads, respectively, is about the same per foot width of bridge or per 10-ft lane as it is per beam, irrespective of beam spacing. These findings permit the live- and dead-load design stresses per beam or per 10-ft lane to be generalized for simple-span bridges, or given construction type and design designation similar to those in Figure 5.

With design-stress information for simple-span bridges of given construction type and design designation, similar to that in Figure 5, it has been shown that the total maximum stresses caused by dead load, vehicle load and impact for a given bridge may be expressed by a simple straight-line equation in which the total maximum stress or design-stress ratio is a function of the H-equivalency of the vehicle under consideration (3, 6). Design-stress ratio for a given member is defined as the ratio of actual total maximum stress caused by dead load, vehicle load and impact in the member, to the total stress used for the design of that member.

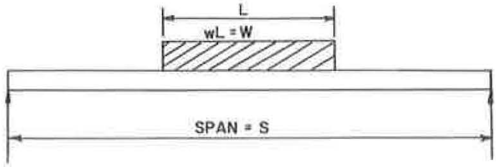


Figure 6. Maximum moment caused by a gross weight of W uniformly distributed over a length L on a span length of S.

The straight-line equation (3, 6) for determining the total maximum bending stress or design-stress ratio produced by trucks of given H-equivalency on a given span for various loading conditions is

$$Q = R_D + R_L \left[\frac{HCK'M_{H(1)}}{KM_L}\right] \tag{3}$$

The dead- and live-load ratios, R_D and

TABLE 6

COEFFICIENTS FOR CONVERTING HEAVY VEHICLE OF GIVEN WEIGHT AND WHEEL BASE LENGTH INTO EQUIVALENT H TRUCK LOADINGS ON SIMPLE SPANS

Wheel Base (ft)	Span (ft)								
	20	30	40	50	60	70	80	90	100
4	1.1250	1.1354	1.0984	1.0636	1.0541	1.0541	1.0470	1.0416	1.0373
6	1.0625	1.0948	1.0695	1.0548	1.0453	1.0386	1.0336	1.0297	1.0277
8	1.0000	1.0543	1.0406	1.0324	1.0269	1.0231	1.0202	1.0179	1.0161
10	0.9375	1.0137	1.0117	1.0099	1.0086	1.0076	1.0067	1.0061	1.0055
12	0.8750	0.9732	0.9828	0.9875	0.9903	0.9921	0.9933	0.9942	0.9949
14	0.8125	0.9326	0.9539	0.9651	0.9719	0.9766	0.9799	0.9824	0.9844
16	0.7500	0.8921	0.9250	0.9426	0.9536	0.9611	0.9665	0.9706	0.9738
18	0.6875	0.8515	0.8960	0.9202	0.9352	0.9456	0.9530	0.9587	0.9632
20	0.6250	0.8110	0.8671	0.8977	0.9169	0.9301	0.9396	0.9469	0.9526
22	–	0.7704	0.8332	0.8753	0.8986	0.9146	0.9262	0.9351	0.9420
24	–	0.7299	0.8093	0.8528	0.8802	0.8991	0.9128	0.9232	0.9314
26	–	0.6893	0.7804	0.8304	0.8619	0.8836	0.8993	0.9114	0.9208
28	–	0.6488	0.7515	0.8079	0.8436	0.8681	0.8859	0.8995	0.9103
30	–	0.8082	0.7226	0.7855	0.8252	0.8526	0.8725	0.8877	0.8997
32	–	–	0.6937	0.7631	0.8069	0.8371	0.8591	0.8759	0.8891
34	–	–	0.6648	0.7406	0.7885	0.8216	0.8456	0.8640	0.8785
36	–	–	0.6359	0.7182	0.7702	0.8061	0.8322	0.8522	0.8679
38	–	–	0.6070	0.6957	0.7519	0.7906	0.8188	0.8404	0.8573
40	–	–	0.5781	0.6733	0.7335	0.7751	0.8054	0.8285	0.8468
42	–	–	–	0.6508	0.7152	0.7596	0.7919	0.8167	0.8362
44	–	–	–	0.6284	0.6969	0.7441	0.7785	0.8049	0.8526
46	–	–	–	0.6059	0.6785	0.7286	0.7651	0.7930	0.8150
48	–	–	–	0.5835	0.6602	0.7131	0.7517	0.7812	0.8044
50	–	–	–	0.5611	0.6418	0.6976	0.7383	0.7694	0.7938
52	–	–	–	–	0.6235	0.6821	0.7248	0.7575	0.7833
54	–	–	–	–	0.6052	0.6666	0.7114	0.7457	0.7727
56	–	–	–	–	0.5868	0.6511	0.6980	0.7338	0.7621
58	–	–	–	–	0.5685	0.6356	0.6846	0.7220	0.7515
60	–	–	–	–	0.5503	0.6201	0.6711	0.7102	0.7409

NOTE: The H-equivalency of any vehicle of given weight and wheel base length on a given span, is equal to the weight of the vehicle times the conversion coefficient for that span corresponding to the given vehicle's wheel base length. For example, a 20-ton vehicle with a 28-ft wheel base on a 50-ft span would have an H-equivalency of 20.0 x 0.8079 = 16.16 tons or correspond with an equivalent H 16.16 truck loading.

R_L, respectively, are given by charts similar to Figure 5. For any particular span, design-stress ratios for a given situation are equal to the dead-load ratio plus some straight-line function of the live-load ratio. The numerator of the fraction in the bracket represents the actual moment plus impact (if any) produced by the vehicle under consideration, and the denominator represents the live-load plus impact moment used for design. If the actual moment for a given situation is greater than the design moment, then the design-stress ratio Q will be greater than 1.000. Conversely, if it is less, then Q will be less than 1.000.

Incidentally, the equations for Q at the bottom of Figures 7 to 9 are based on substituting values of R_D and RR_L from Figure 5 into Eq. 3.

To find the H-equivalency required for a given situation to produce a specified value of Q,

$$H = \frac{KM_L(Q-R_D)}{CK'M_{H(1)}R_L} \tag{4}$$

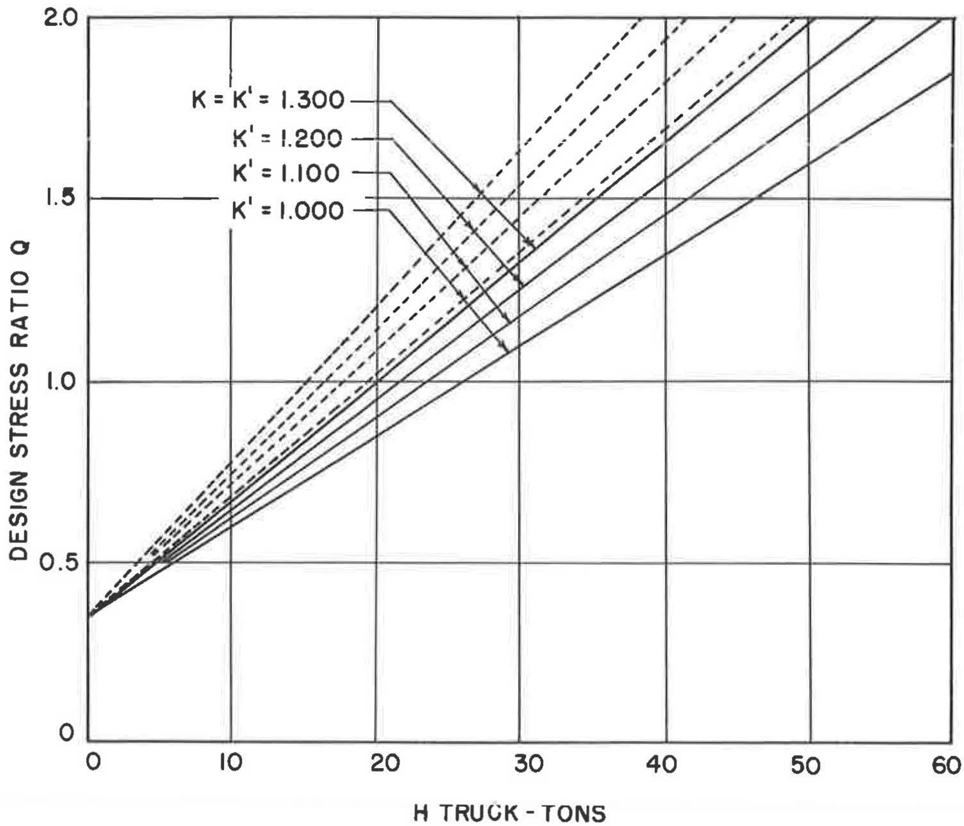
Estimating Maximum Bending Stresses Caused by Equivalent H Trucks

In Eq. 3, the design-stress ratio Q is a linear equation. For any given member of a bridge the change in Q varies directly with the values of H, C, and K' in Eq. 3 (Figs. 7 to 9).

On a 50-ft span, for example, Figure 8 shows that one equivalent H30 truck in each lane simultaneously (C = 1.00) with full allowance for impact would result in a maximum design-stress ratio, Q = 1.50. The maximum stress produced in one of the

C = .75 or 1.00 K' = Varies Span Length = 30'

One vehicle in one lane (C = .75) with varying allowance for impact ———
One vehicle in each lane (C = 1.00) with varying allowance for impact ---



Design Stress Equations for C = 0.75

K = K' = 1.300; Q = .0329H + .340
K' = 1.200; Q = .0305H + .340
K' = 1.100; Q = .0279H + .340
K' = 1.000; Q = .0254H + .340

Design Stress Equations for C = 1.00

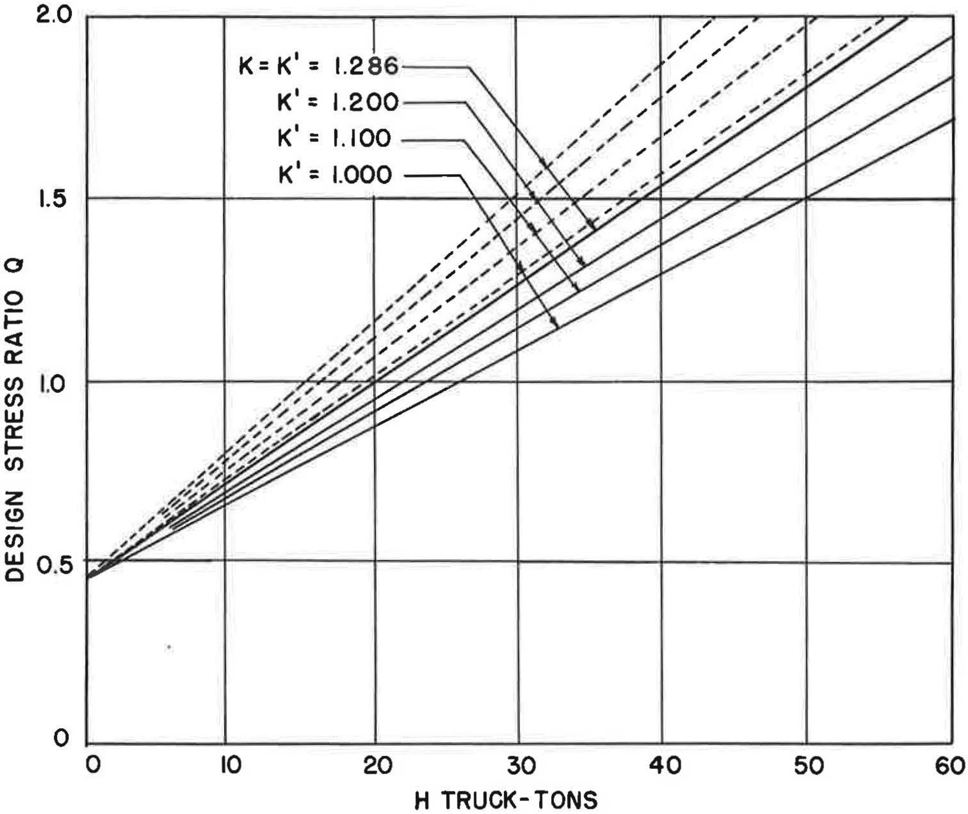
K = K' = 1.300; Q = .0439H + .340
K' = 1.200; Q = .0406H + .340
K' = 1.100; Q = .0372H + .340
K' = 1.000; Q = .0338H + .340

NOTE: K = 1.00 + Impact Allowance Required by AASHO Specs.
K' = 1.00 + Impact As Specified (Actual)

Figure 7. Design-stress ratio produced by equivalent H trucks on simple-span bridges of H15 design consisting of non-composite concrete deck supported by steel stringers.

C = .75 or 1.00 K' = Varies Span Length = 50'

One vehicle in <u>one</u> lane (C = .75) with varying allowance for impact —
One vehicle in <u>each</u> lane (C = 1.00) with varying allowance for impact -



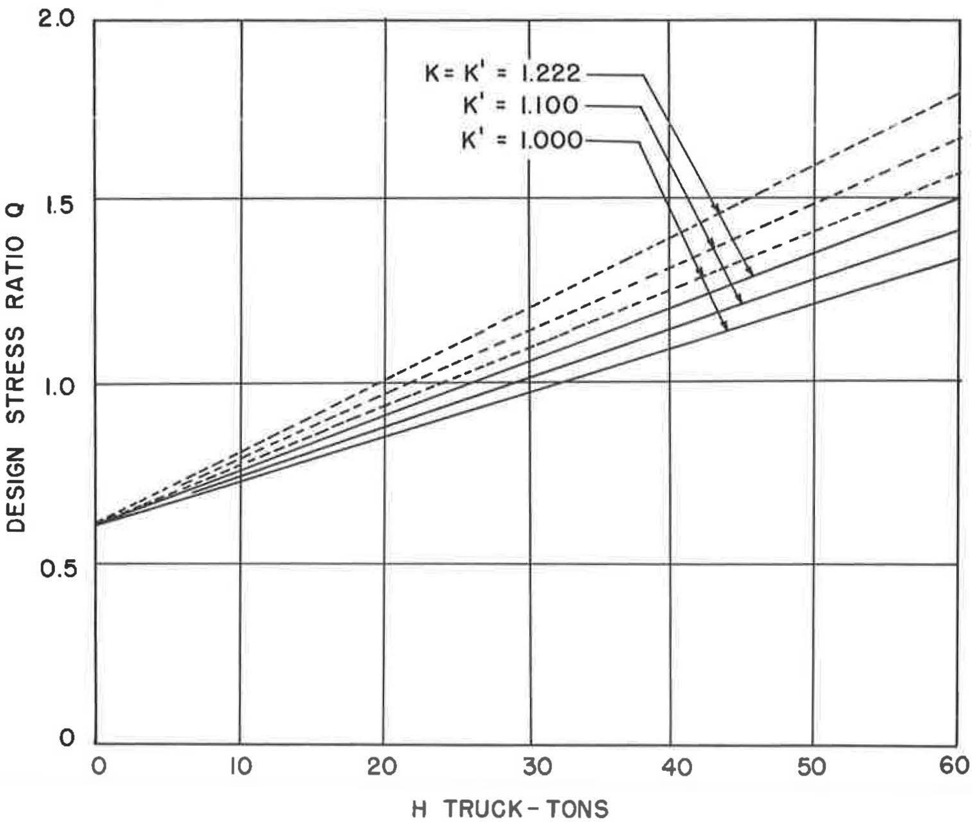
Design Stress Equations for C = 0.75	Design Stress Equations for C = 1.00
K = K' = 1.206; Q = .0270H + .460	K = K' = 1.286; Q = .0360H + .460
K' = 1.200; Q = .0252H + .460	K' = 1.200; Q = .0336H + .460
K' = 1.100; Q = .0231H + .460	K' = 1.100; Q = .0308H + .460
K' = 1.000; Q = .0210H + .460	K' = 1.000; Q = .0280H + .460

NOTE: K = 1.00 + Impact allowance required by AASHO Specs.
K' = 1.00 + Impact as specified (actual)

Figure 8. Design-stress ratio produced by equivalent H trucks on simple-span bridges of H15 design consisting of non-composite concrete deck supported by steel stringers.

C = .75 or 1.00 K' = Varies Span length = 100'

One vehicle in <u>one</u> lane (C = .75) with varying allowance for impact ——
One vehicle in <u>each</u> lane (C = 1.00) with varying allowance for impact ---



Design Stress Equations for C = 0.75	Design Stress Equation for C = 1.00
K = K' = 1.222; Q = .0149H + .606	K = K' = 1.222; Q = .0198H + .606
K' = 1.200; Q = .0146H + .606	K' = 1.200; Q = .0194H + .606
K' = 1.100; Q = .0134H + .606	K' = 1.100; Q = .0178H + .606
K' = 1.000; Q = .0122H + .606	K' = 1.000; Q = .0162H + .606

NOTE: K = 1.00 + Impact allowance required by AASHO Specs.
K' = 1.00 + Impact as specified (actual).

Figure 9. Design-stress ratio produced by equivalent H trucks on simple-span bridges on H15 design consisting of non-composite concrete deck supported by steel stringers.

interior steel stringers by such a loading would be 150 percent of the basic allowable design stress, or an overstress of 50 percent. However, if the speed of these equivalent H30 trucks were reduced to about 5 mph, which would result in little or no impact, the maximum amount of overstress in an interior stringer would be reduced to about 27 percent.

Similarly, on a 50-ft span, Figure 8 shows that one equivalent H30 truck in one lane only (C = 0.75) with full allowance for impact would result in a maximum design-stress ratio, Q = 1.28, or an overstress of about 28 percent. However, if the speed of this equivalent H30 truck were reduced so as to result in little or no impact, the maximum amount of overstress in an interior stringer would amount to less than 8 percent.

Summary

The preceding discussion shows that the maximum combined bending stresses (design-stress ratios) caused by dead load, live load and impact in simple-span beam bridges may be estimated rather quickly in two simple steps.

1. Convert the heavy vehicle (or axle group load within the vehicle) into its equivalent H truck loading on a given span by use of the appropriate coefficient in Table 6.
2. With the H-equivalency found in the first step, an estimate of the bend stresses caused by it on the given span (steel stringer bridges of H15-44 design) may be read directly from the appropriate chart (Figs. 7 to 9) depending on the span length and loading conditions.

Appendix B

NOTATION

A = average number of vehicles per hour in any one designated direction, or total traffic in both directions, as specified.

C = coefficient representing the fractional part of the total live-load stress in a given member produced by one or more lanes loaded. C = 1.00 if a stringer bridge is loaded with identical vehicles, one in each lane and so placed as to produce maximum stress. For a steel stringer bridge, if one vehicle in one lane only would produce 75 percent as much stress in an interior stringer as identical vehicles in each lane, it would mean that C = 0.75.

D = average speed of traffic in designated direction.

E = number of events or trials between occurrences of vehicle groups as defined.

G = group of vehicles as defined.

H = equivalent H truck in tons. For example, if a given vehicle produces the same maximum moment (or other stress function) in a given member as a standard H truck weighing 23.6 tons, it would be rated as an equivalent H 23.6 truck loading, in which case H = 23.6 tons. H also represents one heavy freight vehicle.

I = impact fraction (maximum 0.30 or 30%) as determined by the AASHO Formula $I = 50/(S + 125)$ in which S = length in feet of the portion of the span which is loaded to produce the maximum stress in the member.

I' = impact fraction assumed in connection with the determination of the stress-producing effects of any given vehicle under consideration. For example, if the speed of a given vehicle were limited to 5 mph, this impact fraction might be considered so small as to be negligible, in which case I' might be assumed equal to zero. Depending on traffic and conditions, therefore, the impact fraction could be assumed at any reasonable value between zero and the full impact allowance, I, as defined by the AASHO design specifications.

K = $(1.00 + I)$ = coefficient by which the design live-load moment (shear, or other stress function) is multiplied to obtain the live-load plus impact moment (shear, or other stress function) used for design. Thus, $K\,M_L$ would be equal to the live-load plus impact moment used for design. Similarly, $K\,V_L$ would be equal to the live-load plus impact shear used for design.

K' = $(1.00 + I')$ = coefficient by which the live-load moment (shear, or other stress function) produced by a given vehicle is multiplied to obtain the live-load plus impact moment (shear, or other stress function) produced on a given span or in a given member by the vehicle under consideration. Thus, $K'M_H$ would be equal to the live-load plus impact moment produced on a given span by any particular vehicle having an H-equivalency of H tons.

M_D = dead-load moment as included in total design moment.

M_L = live-load moment as included in total design moment.

M_T = moment used for design or total design moment.

M_H = moment in an interior stringer (or other member) resulting from equivalent H trucks weighing H tons each. Likewise, M_H represents the moment for one lane produced by equivalent H truck weighing H tons.

$M_{H(1)}$ = moment for lane produced by a standard H truck weighing 1 ton.

P = general term indicating probability that an event will occur as specified.

Q = design-stress ratio — ratio of total actual stress to total design stress in any particular member or part of a given highway bridge.

R_D = (M_D/M_T) = ratio of dead-load moment M_D (shear, or other stress function) to total moment M_T used for design. In terms of shear this ratio would be $R_D = (V_D/V_T)$, and for other stress functions it would be similar.

R_L = $(K\,M_L/M_T)$ = ratio of live-load plus impact moment, $K\,M_L$(shear, or other stress function), used for design to the total design moment, M_T, or total moment (shear, or other stress function) used for design. In terms of shear, this ratio would be $R_L = (K\,V_L/V_T)$, and for other stress functions it would be similar.

S = span length or that portion of span which is loaded to produce maximum stress in the member under consideration in feet.

T = time interval between occurrences of certain specified events.

V = vehicle interval between occurrences of certain specified events. V may also be used to describe shear as a stress function.

X = length of section or distance along highway (distance interval), in feet, within which the grouping of vehicles is to occur.

Z = average number of vehicles expected within a specified length of X feet or a specified time of t seconds, based on total traffic in both directions. For a specified length of X feet, $Z = AX/5280D$; for a specified time of t seconds, $Z = At/3600$.

$P(2H, X;2)$ = probability of the group, 2H, occurring within X feet in each of the two directions.

$P(G, X;a/2)$ = probability of the group, G, occurring within X feet in any manner in either or both directions.

$E(n, X;2)$ = number of events between occurrences of n vehicles in each of two directions within X feet.

$V(G, X;a/2)$ = vehicle interval between occurrences of the group, G, in any manner in either or both directions within X feet.

$T(G, X;a/2)$ = time interval between occurrences of the group, G, within X feet in either or both directions.

e = exponential base, 2.718, 281. . . .

f = unit stress in psi or as may be defined. f_D = unit stress resulting from dead load; f_L = unit stress resulting from live load; f_T = maximum total design stress; and f_H = stress resulting from vehicle or vehicles weighing H tons each.

n = number of vehicles in a group or sequence but unassigned as to class or type.

t = time interval in seconds within which the grouping of vehicles is to occur.

z = average number of vehicles expected within a length of X feet or a time of t seconds in one designated lane, based on the number of vehicles per hour, (R_1), and average speed of vehicles, D, in that lane.

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