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Travel Time—A Measure of Service and A Criterion for Improvement Priorities

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• TRAVEL time has been used as an indication of traffic congestion for some time in the Phoenix area which has grown tremendously in recent years. The first travel time study was conducted by the Arizona Highway Department in 1947. In 1956 Phoenix was selected as a pilot city by the National Committee on Urban Transportation. Evening peak-hour time data were gathered in 1957 and 1962 as a part of the continuing fact gathering effort. Travel time was obtained in accordance with national standards on all major arterial streets and selected collector streets of major importance.

The purpose of this report is to demonstrate that travel time offers a sound measure of the level of urban traffic service and can be a basic criterion for a major street improvement priority formula.

This paper compares the overall level of service as obtained by peak-hour travel time studies in 1947, 1957 and 1962. Comparison of the Phoenix street system for these years also related the level of service to population growth, increase in vehicle registration, city size and traffic volumes. Selected route segments are compared for change in average speed, vehicle delay and average daily traffic. In making these comparisons, street improvements that increased capacity are identified. Examples of these improvements are street widening, intersection widening and provision of left-turn lanes, channelization, and removal of parking.

This paper also recognizes the need to develop a simple priority formula that would aid in determining major street construction priorities in urban areas. The test formula used in Phoenix assigns major emphasis to delay rate, but also considers collisions, traffic volume and structural condition of the pavement. The formula was evaluated by comparing the relative priority rating for selected major arterial street segments, as determined by the formula, to the judgment rating of individuals. The various public works, planning, and management officials who served as raters were chosen for their familiarity and knowledge of the Phoenix street system.

This improvement priority formula is not intended to replace judgment, but could be used as an aid to develop recommended capital improvement priorities for major arterial street construction programs.

A MEASURE OF URBAN SERVICE

Travel time studies have been used for decades to show the time required to travel from one location to another. This information was useful in scheduling individual movements and later applied to the operation of mass transportation. The early traffic engineer commonly used travel time studies to show that an engineering improvement reduced the time required to go from point A to point B. Travel time studies of entire urban areas became commonplace and isochronic maps of urban areas were shown in the earliest text books concerned with traffic studies.

Travel time is easily understood by the average motorist and the "quickest way home" is a topic of conversation over the backyard fence. The motorist's desire has produced the traffic assignment diversion curves that are in widespread use. The speed of mobility for people and goods is the reason for the motor vehicle's being and travel time is a measure of service afforded by the street net.

Paper sponsored by Committee on Quality of Traffic Service.

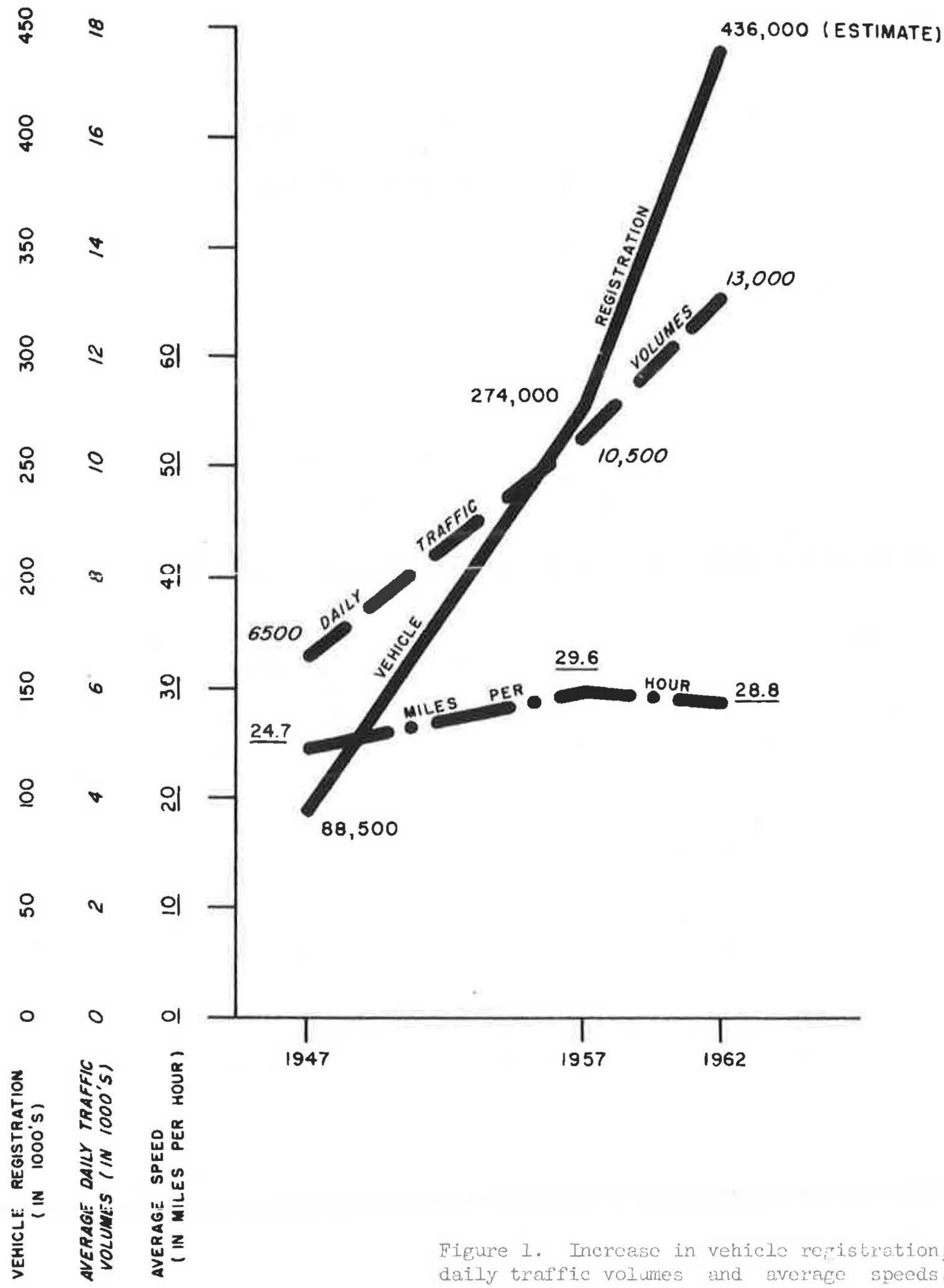


Figure 1. Increase in vehicle registration, daily traffic volumes and average speeds.

Travel time figures are easily obtainable. With a vehicle, a stop watch, and a half-hour's training, non-technical help can produce the desired study. However, because this tool is so old and so easily understood, travel time has been overlooked for more sophisticated and complicated applications. Travel time studies should be completed every two to three years for the major street and freeway network in an urban area. Thus, the trend in the overall level of service could become evident. Closer

TABLE 1

COMPARATIVE DATA FOR PHOENIX AND MARICOPA COUNTY

Year	Major Arterial Street: Peak-Hour Avg. Speed (mph)	Major Arterial Street: Avg. Daily Traffic	Maricopa County: Vehicle Registration	Maricopa County: Population	City of Phoenix: Population	City of Phoenix: Area (sq mi)
1947	24.7	6,500	88,500	270,000	95,000	12.2
1957	29.6	10,500	274,000	520,000	172,000	36.3
1962	28.8	13,000	436,000	750,000	496,400	220.3

appraisal of individual routes would result in more quickly taken remedial measures. Travel time figures can play a large part in determining construction priorities, enforcement assignments, surface mass transit routings, freeway locations and signal timing deficiencies. If travel time data were available for various urban areas, the level of service could be compared. Travel time then should become a factor in programming urban construction projects by state highway departments. It would aid in advertising street improvements, the excellence of a transportation system of a community, and could be used in support of needed legislation to obtain financing to improve and build a street and freeway system.

Phoenix Studies

Although much of the foregoing may be wishful thinking and oversimplification, certainly travel time is a sound engineering measure of the level of service of a street net. In Phoenix a comparison was made of the overall average evening peak-hour major arterial street speeds for 1947, 1957 and 1962.

The 1947 study was made by the Arizona Highway Department, Maricopa County and the Bureau of Public Roads as part of an origin and destination study. The average speed determined for the major arterial street system in 1947 was 24.7 mph. This compares with a speed of 29.6 mph after a 10-yr period of traffic engineering improvements (Table 1).

Thus, during a period of unprecedented growth, the average speed increased 4.9 mph (20%) while the major arterial average daily traffic increased from 6,500 to 10,500 vehicles. This is a 62 percent increase in traffic volume. Figure 1 shows the increases in vehicle registration, major arterial street average daily traffic, and average overall speeds found for the three travel time studies.

The average major arterial speed was slightly less in 1962 (0.8 mph) than in 1957. This is more significant in view of the continued traffic engineering improvements that have been made and even accelerated. During this period the average daily traffic volumes have increased from 10,500 to 13,000 vehicles. The present surface major arterial street system is reaching saturation.

This leads to one possible theory: when the central city of an urban area reaches a population somewhere between 400,000 and 500,000, a typical major arterial street system reaches its peak efficiency. At this point a freeway system has to be placed in operation if the downward trend in overall average street net speeds is to be prevented. The population range of this "hump" depends on many factors such as density of population, efficiency of mass transportation, the ratio of vehicle registration to population, adequacy of the street net, rights-of-way, and the ability of the city to operate an efficient street system. Figure 2 shows the isochronic drawings for the Phoenix metropolitan area for 1957 and 1962. Figure 3 shows the level of service

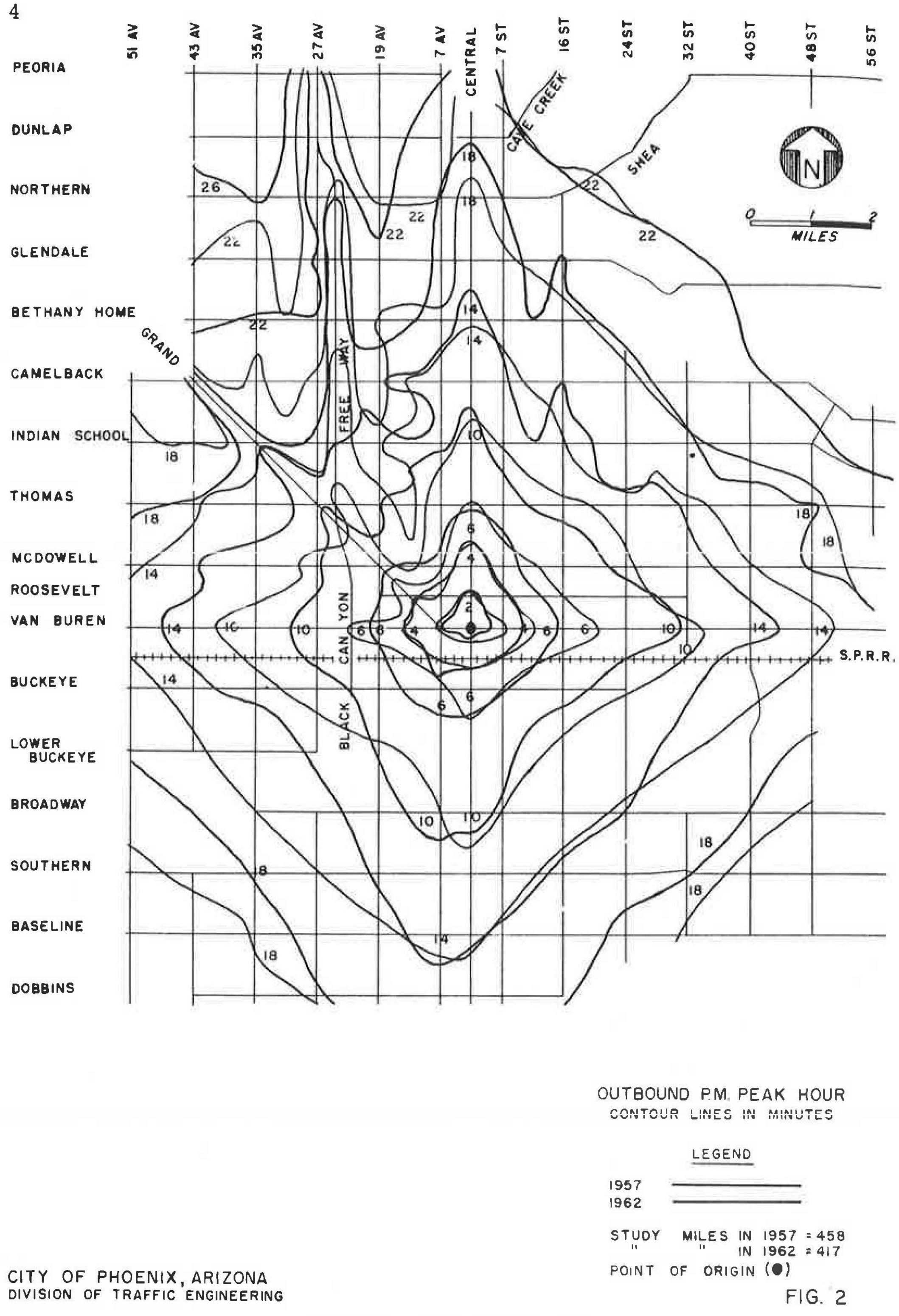


Figure 2. Time contour map.

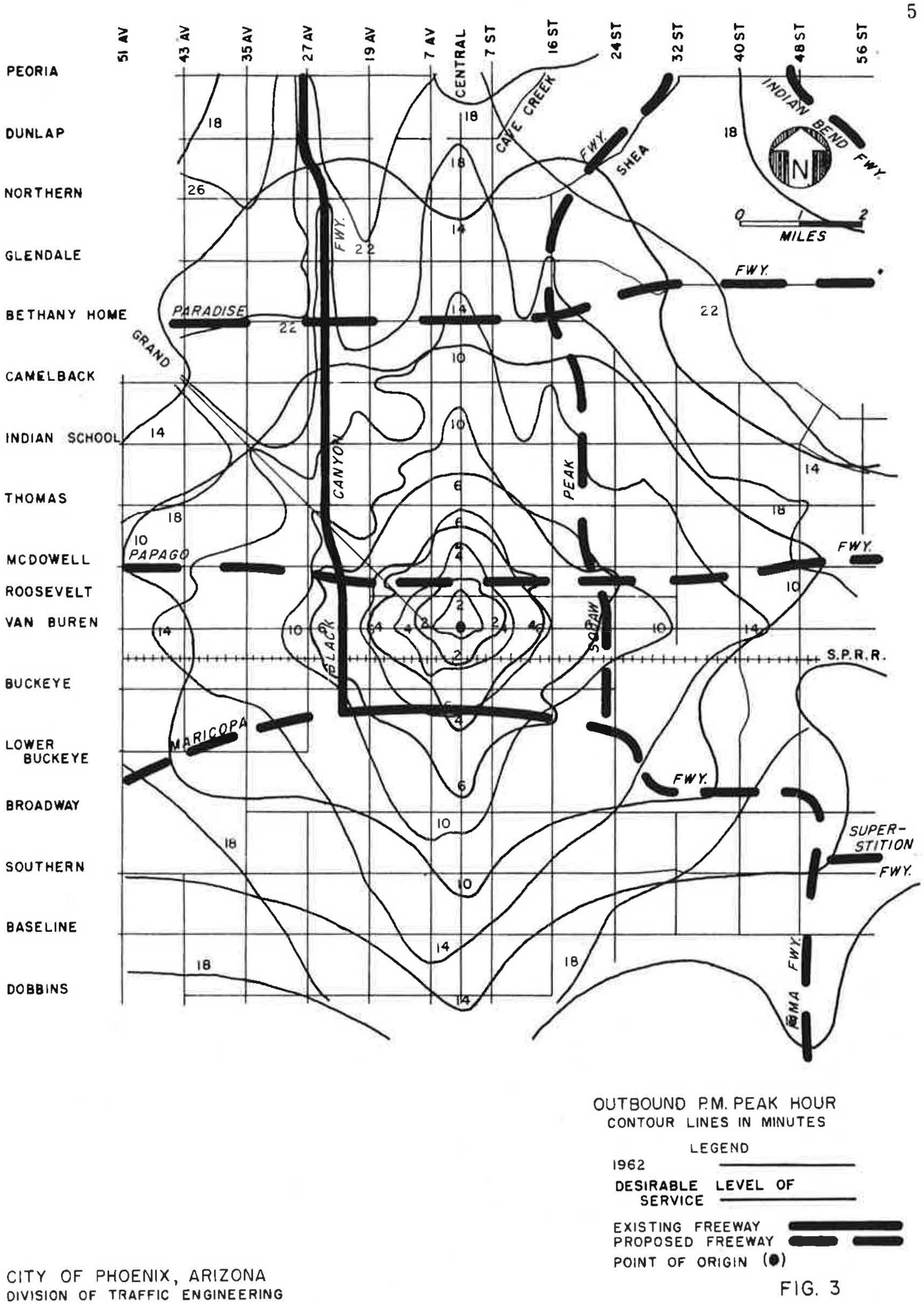


Figure 3. Time contour map.

TABLE 2

TRAVEL TIME RELATED TO ENGINEERING IMPROVEMENTS FOR SELECTED MAJOR ARTERIALS

Street Section	Average Speed		PM Peak-Hour Volume		Veh.-Min.Delay Per Hour Per Mile		Engineering Improvements
	1957	1962	1957	1962	1957	1962	
Indian School Road:							
15th Ave. to 7th Ave.	14.6	30.3	752	935	1,579	0	A, E, F
7th Ave. to 7th St.	13.4	16.3	859	1,239	2,113	2,043	C, D, E, F, G, J
7th St. to 16th St.	26.7	29.9	912	1,222	228	12	C, F, G
16th St. to 24th St.	23.2	26.5	863	1,066	509	277	C, E, F, G
24th St. to 32nd St.	23.5	29.1	774	1,175	426	71	C, D, E, F, G
McDowell Road:							
19th Ave. to 7th Ave.	14.0	24.8	587	672	111	12	A, D, E, F
7th Ave. to 7th St.	10.2	17.2	934	1,040	3,222	1,134	A, E, F, G, H
19th Avenue:							
McDowell to Thomas	22.0	32.1	421	410	139	0	A, E
Thomas to Indian School	21.1	27.3	396	503	337	101	A, E
Grand Avenue:							
7th Ave. to 19th Ave.	13.3	18.7	939	1,064	1,931	862	C, E, I
Washington Street:							
28th St. to 32nd St.	26.4	28.8	865	1,200	208	96	B, F, G

A—Street widened from 2 to 4 lanes.
B—Street channelized from 2 to 3 lanes.
C—Intersectional widening.
D—Signals installed.
E—Changed signal timing or cycle.
F—Painted left-turn channels.
G—Removed parking.
H—Raised speed limit.
I—Prohibited left turns.
J—Added right-turn lane.

which would be attained if the deficiencies in the existing major arterial system were corrected and the freeway system was completed. This desirable level of service determined for Phoenix is as follows:

Street Classification	Avg. Speed (mph)
Urban freeway	50
Major arterial:	
Normal	30
Intermediate	25
CBD	20

Definitive Travel Time Studies

In 1950 Phoenix realized that as it increased in size there was a need for a single agency to handle traffic matters. In July 1950, the division of traffic engineering was established under the direction of a traffic engineer. This division was responsible for the operation of the street system and for the propagation of traffic studies and design recommendations. The usual tools of the trade were employed: improved signal design, one-way street system, a program of parking removal, a through-street system, reversible-lane movements and channelization.

In 1957 Phoenix became one of the pilot cities in the program sponsored by the National Committee on Urban Transportation. The travel time study, in particular, gave the city administrators an opportunity to evaluate the services of the division of traffic engineering. A total of 458 miles of street was studied in 1957 for travel time in the urban area. These studies showed that during the 10-yr period (1947 to 1957) when Phoenix was growing at a faster rate than any other city over 100,000 population, the overall arterial speed had increased. This improved level of service was attained through studies and observations made at congested locations and on critical streets that were translated into physical improvements construction in the field.

TABLE 3

GROWTH OF TRAFFIC ENGINEERING IMPROVEMENTS

Year	Left-Turn Channels	Signalized Intersections	One-Way Street Length (mi)	Prohibited Parking Length (mi)	Left-Turn Prohibitions
1951	0	82	0.75	7.9	2
1957	38	146	23.1	21.7	9
1962	180	307	25.5	55	10

As an example of the work accomplished during these years, Table 2 gives certain selected sections of major arterial streets, the increases in peak-hour traffic between 1957 and 1962, the vehicle minute delay per hour per mile, and engineering improvements. Table 3 gives the growth of traffic engineering improvements in service.

Summary

Travel time is a measure of the level of service of a street system. It can be useful in determining trends in a single area and has tremendous possibilities for comparing one geographic area with another. It can be a factor in determining signal timing, needed traffic improvements and street construction priorities. In some of these fields the methods of application have not yet been developed, but there is great potential use for this easily determined and universally understood measure.

A CRITERION FOR URBAN STREET PRIORITIES

The need for a simple formula that would aid in establishing the priority for streets to be constructed in urban areas has long been recognized. Certainly such a formula would not be intended to replace judgment, but would simply be a device by which urban projects could be listed as to their relative importance.

This section is solely confined to an urban major arterial street construction priority formula. In urban street and traffic work there are several areas where priority formulas will prove useful: resurfacing programs, traffic signal installations, and major arterial street construction.

A list of major street construction projects based on a priority formula could be a significant aid to the development of a recommended capital improvement program for urban areas. A major concept in the development of a formula has been to reduce judgment in the formula to the absolute minimum and thus make the formula as factual as possible. Judgment and budgetary elements would be brought into the final selection of the actual projects for the recommended program.

In September 1960, the Highway Research Board sponsored a workshop conference on formulating highway construction programs. The results of this conference have been published and are an important contribution of the Department of Economics, Finance and Administration. A similar conference directed primarily at problems of formulating construction programs in urban areas could be a significant contribution.

The American Public Works Association transportation committee is now engaged in the study of major street construction priorities for urban areas. It is the hope of this committee that it will be able to develop a useful publication. One objective is to include several priority formulas that have been developed for urban areas.

The subcommittee on developing project priorities for transportation improvement summarized its work in Procedure Manual 10-A of the National Committee on Urban Transportation series. This procedure manual developed a technique and a suggested form for the complete evaluation of a project, including street classification; time the project is needed; and administrative, budgetary, and service considerations. The

TABLE 4

PROPOSED GUIDING PRIORITY RATING METHOD[1]
(San Diego Metropolitan Area Transportation Study)

$$\text{Priority Index} = \frac{\text{Project Cost per Vehicle-Mile}}{\text{Project Benefit Index}}$$

Project Benefit Index		Relative Weight	
Community service:			
Pattern and continuity		15	
Coordinating and timing		15	
Roadbed condition		5	
Present capacity ratio		15	
Long-range future service		10	
Subtotal			60
User benefits:			
Time saving-delay rate:			
Present	5		
5-yr future	5		
Subtotal		10	
Duration of deficiency		5	
Distance saving of improvement, 5-yr avg.		5	
Accident rate, 2 year		15	
Time to amortize investment		5	
Subtotal			40
Total			100
Project Cost			
Right-of-way plus construction per vehicle-mile (10 yr)			

[1]Priority rating index should be based on the expected improvement in deficient conditions.

balance of this paper is concerned with an effort to formulate a simple factual analysis of service considerations. This is a continuation of programs undertaken by San Diego, Calif. and Phoenix.

San Diego Effort

San Diego has been publishing an annual 6-yr capital improvement program for many years. As a part of the pilot city program of the National Committee on Urban Transportation, several efforts were made to develop a capital improvement program priority formula for major street construction. Two of the earliest formulas were based primarily on traffic data. In one of these, priority was determined by the percent capacity overload; a second combined volume, speed and delay, and accident rates into a priority formula. Both efforts were helpful, but were not the desired formula.

Table 4 gives a guiding priority rating method developed in 1958. The basic philosophy of the formula was to weight community service 60 percent and user benefits 40 percent. The final priority index brought cost into the picture by dividing the cost per vehicle-mile by the project benefit index.

In an effort to test this formula 25 projects were selected. A group of eleven people having knowledge and responsibilities in administration, planning or engineering, and who participated in the capital improvement program project selection, were asked to order the 25 projects.

As this test proceeded, it became more and more obvious that the formula itself included judgment in all of the community service benefits as well as some of the user benefits. Actually, at least 70 points out of 100 in the formula were basically judgment

TABLE 5

PHOENIX MAJOR STREET IMPROVEMENT PRIORITY, FORMULA B
(Jan. 12, 1961)

Element		Relative Weight (points)	
Community Service			
Master plan-continuity of route development		10	
Coordination and timing in relation to other projects and jurisdictions		10	
Structural condition		15	
Surface	2		
Subsurface	8		
Drainage	5		
Ratio of $\frac{\text{future (design)}}{\text{present}}$ traffic volumes		10	
Present capacity ratio		10	
Subtotal			55
User Service			
2-yr accident rate/mile + accident/mile		10	
Duration of deficiency		10	
Time saving			
Delay rate "after" less delay rate "before"		15	
Time to amortize investment		10	
Subtotal			45
Possible points			100
Highest point value = most needed facility			

ratings. Thus, the proposed priority rating formula simply provided a judgment ordering of the projects. This is essentially no different from the results obtained by the capital improvement committee using the same basic data. Therefore, the formula was not considered satisfactory.

Phoenix Formula and Test

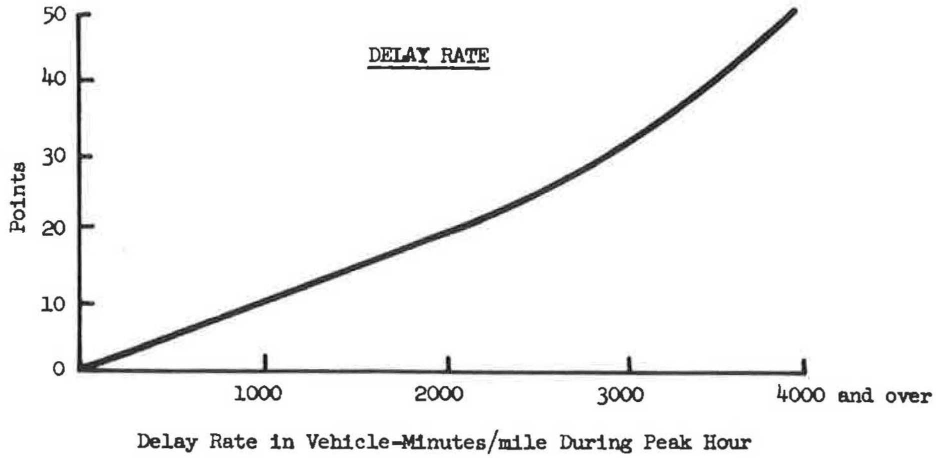
Phoenix completed a street deficiency study in December 1961 that found deficient approximately 152 out of 260 miles of major arterial streets. The estimated cost to correct the deficiencies was $54.2 million. The ever-present limitation of funds makes it essential that the priority of projects be carefully determined to insure the maximum benefit to the motoring public.

From the San Diego effort, Formula B was developed (Table 5). Again it is clear that there is a considerable amount of judgment in the elements to be rated. For this reason, Formula C (Table 6) was developed for test purposes.

Formula C reduces judgment to a minimum. In conjunction with the Major Street Improvement Priority Formula C, two rating scales were developed. These are to be used to determine the points for the delay rate and the collision index (Fig. 4). Curves were developed using existing data from Phoenix and San Diego combined with the following points of view:

1. The delay rate should give relatively few points in the lower scale of delay, but the number of points should increase more rapidly as greater delay rates are experienced.

2. Accident rates should be used but they should be tempered with the total number of accidents. If this is not done, erroneous conclusions can be drawn from either the accident rate or the use of total accidents.



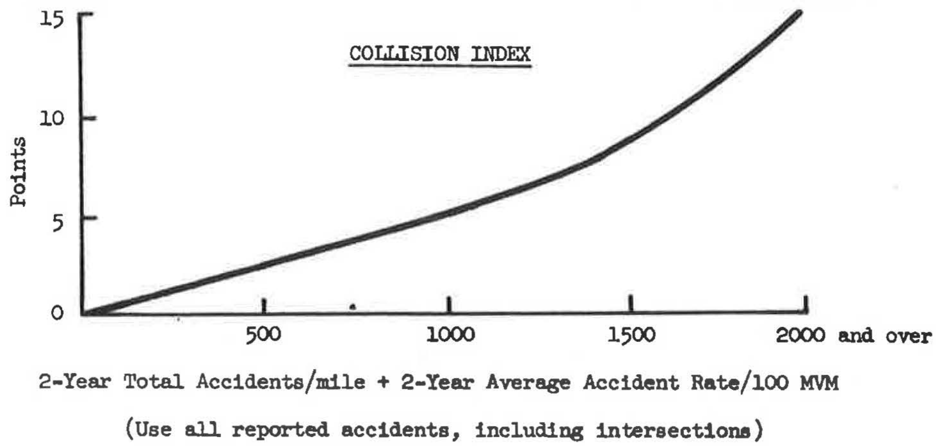


Figure 4. Major street improvement priority Formula C rating scales.

Twenty-five street segments (Fig. 5) were selected to test the formula. These segments were carefully selected to insure that they ranged from projects that had been recently completed through projects which were obviously extremely low on the priority scale. The projects that had been completed were to be rated as they existed prior to their recent improvement. Nineteen individuals having responsibility in the areas of administration, planning, public works-traffic engineering, engineering and street maintenance were asked to participate in the judgment ratings.

Test Results

Table 7 gives the result of the judgment ratings. Table 8 compares these ratings to the order of priority developed by the formula.

It is important to note that the largest deviation of 16 positions occurred on segment O, Van Buren Street, which is obviously in need of improvement. However, this 4-lane facility is presently in an intensively developed area and is fully improved. As a practical matter, significant relief will come from a nearby parallel freeway included in

TABLE 6

PHOENIX MAJOR STREET IMPROVEMENT PRIORITY, FORMULA C

Element		Relative Weight (points)
Delay rate per mile during peak hour		50
Collision index—2-yr accidents/mile plus accident rate/mile		15
Structural condition		15
Surface and subsurface	5	
Drainage	10	
Traffic - $\frac{\text{present ADT}}{2,000} + \frac{\text{future (5-yr forecast) ADT}}{\text{present ADT}}$		20
Possible points		100

Highest point value = most needed facility

the adopted major street and highway plan. This is a situation where the priority formula gave a high rating but judgment would have removed it from the construction program. This demonstrates the judgment and budgetary considerations that must be applied in the development of a capital improvement program.

Table 9 gives the specific points for each element of the formula for the 25 projects. Review of this table gives insight into the other projects where there is a significant deviation between the formula and the judgment ratings as follows:

1. Segment C, 27th Avenue project, is ¼ mile away from a completed urban freeway and the poor structural condition of the facility combined with some delay produced a higher priority by the formula. As on segment O, judgment would tend to weigh the existence of the freeway and thus lower the priority.
2. Segment D, 19th Avenue, has a low delay but a considerably higher rating on structural condition. The various raters had a widespread opinion on the relative priority of this particular project. This may well be due to its being parallel to and approximately ¾ mile away from a completed freeway.
3. Segment H, 16th Street, received a low number of delay and traffic points but a number of structural condition points. Thus, the priority formula produced a somewhat lower rating than judgment.
4. Segment N, the Van Buren project, which judgment said should be among the very earliest, received zero points on the delay rate, relatively few points on traffic, but a high number of points on structural condition. As in segment H, judgment assigned a higher position than did the formula.
5. Segment S, Indian School, showed high by the priority formula due to the relatively high delay rate and traffic points received. Judgment lowered the priority because this segment had been improved to modern 4-lane standards within the last seven years.

Few of the street segments received a high number of points for delay rate. The cause of this is not fully understood. Certainly, it is possible that the delay rate curve (Fig. 4) could be adjusted. However, the curve is based on the philosophic point of view that the relative points should increase more rapidly as the delay increases. If the shape of the curve were varied, there might well be a relatively large number of points for a relatively small amount of delay. This is not considered proper rating. The second possible cause is that congestion in Phoenix has not yet reached the point where maximum delays are the norm rather than the exception. The shape of the curve deserves further research. Perhaps a family of curves for different urban characteristics is needed.

Table 9 indicates that a good spread was obtained by collision index and structural condition ratings. However, the spread of traffic volume rating was not as broad as expected. The highest rating was 15 of 20 points—the lowest 3½. The philosophy of the traffic volume component in Formula C is to place heavy value on present vol-

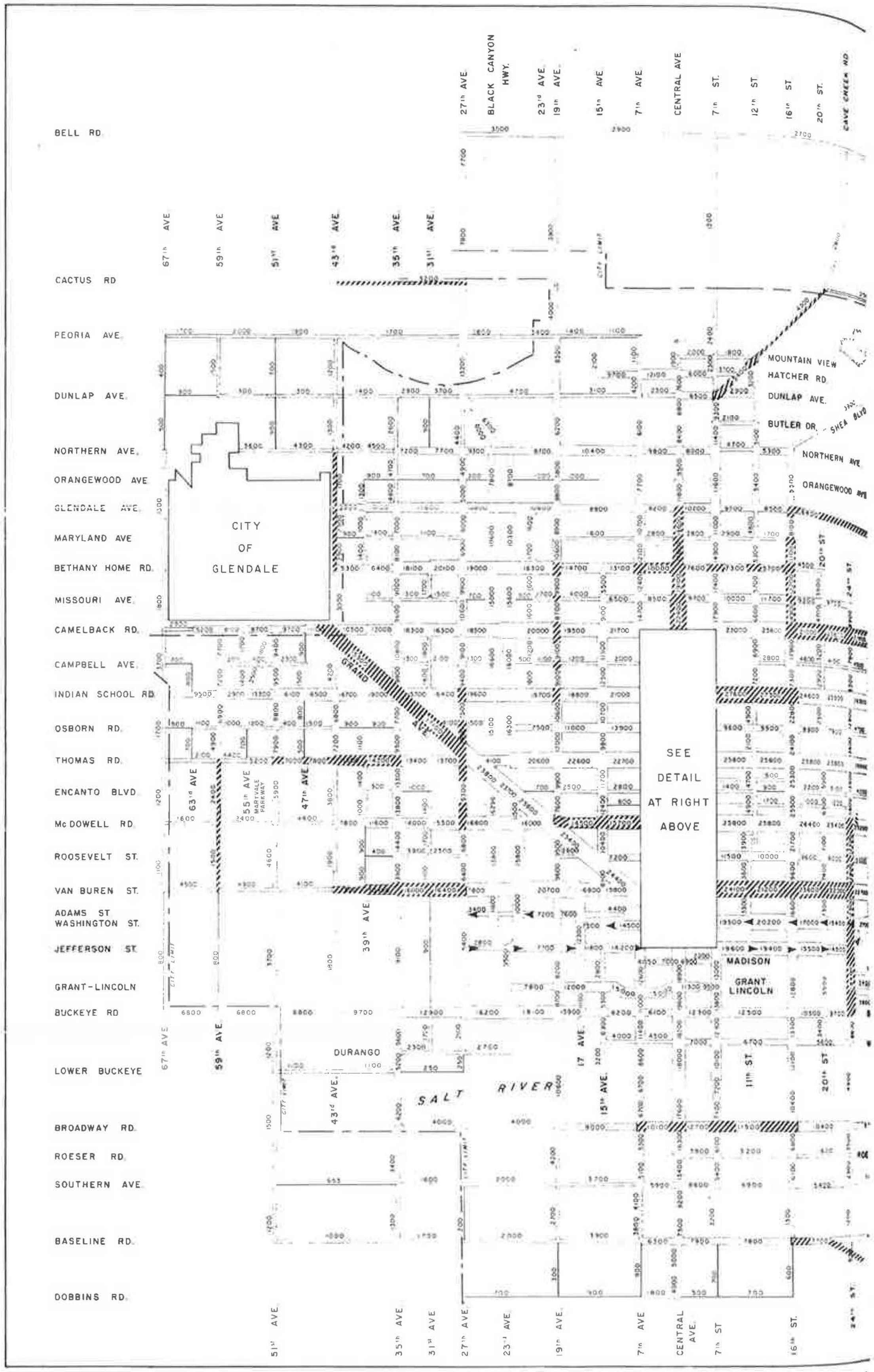
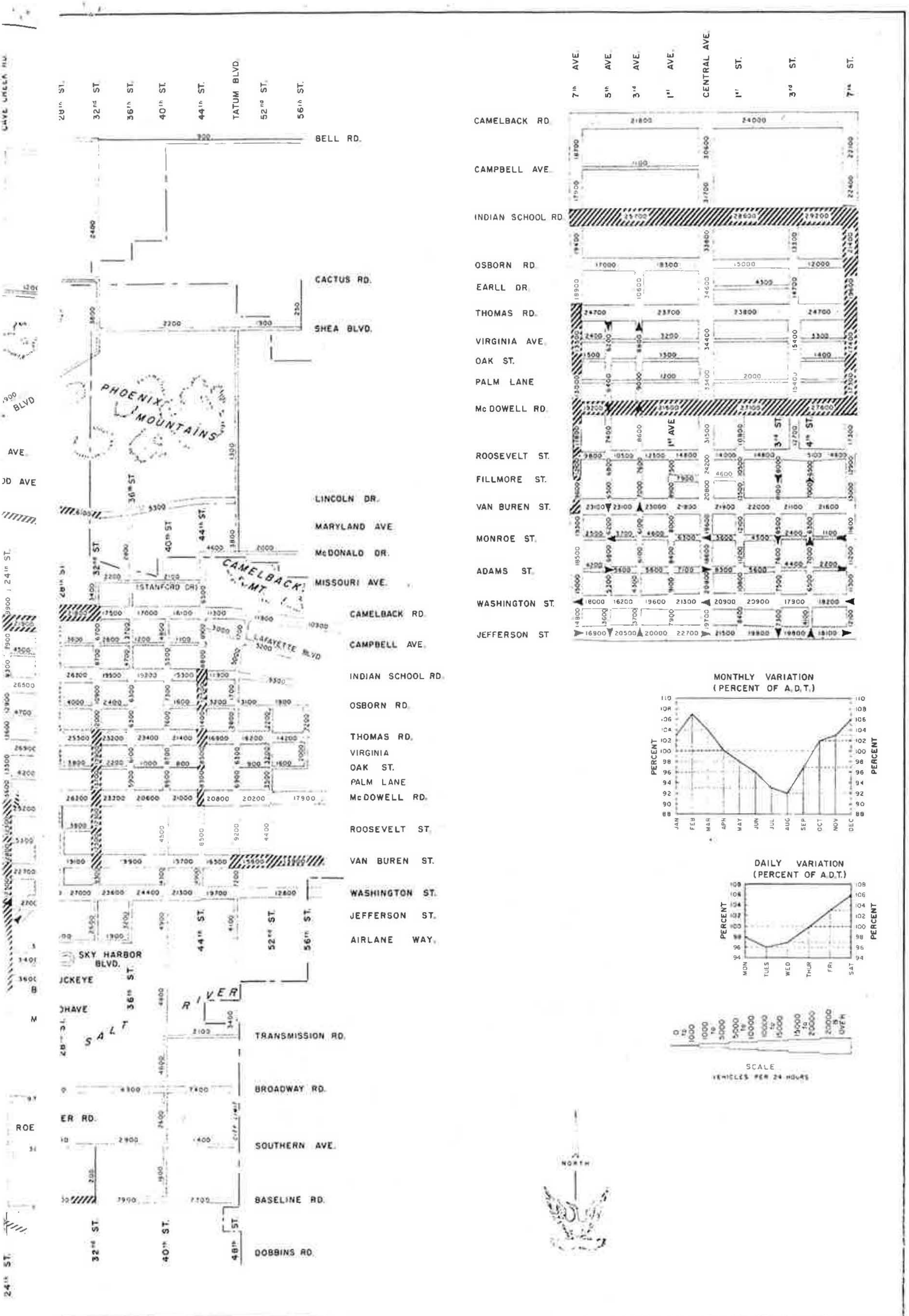


Figure 5. Selected



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street segments.

TABLE 7

PHOENIX FORMULA C JUDGMENT RATINGS

Segment	Location	Relative Order by Individual Raters																			Priority (avg.)
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
A	59th Ave. Van Buren–Thomas	22	25	23	24	24	25	25	23	22	25	20	24	25	18	19	19	24	25	25	25
B	43rd Ave. Bethany–Northern	18	20	24	20	23	24	20	18	21	17	19	23	23	24	9	22	21	21	17	22
C	27th Ave. McDowell–Ind. Sch.	17	14	8	19	14	17	13	10	16	13	10	16	15	17	4	16	16	15	9	14
D	19th Ave. Ind. Sch.–Bethany	6	5	5	7	12	12	10	7	6	21	18	15	13	13	8	3	6	13	16	8
E	7th Ave. Van Buren–Thomas	4	2	3	4	4	5	4	2	15	6	3	2	4	8	3	2	3	1	3	3
F	Central Camelback–Glendale	10	16	22	8	11	6	16	14	17	1	1	17	7	11	21	5	11	23	23	12
G	7th St. McDowell–Ind. Sch.	1	1	1	2	1	3	2	3	2	4	7	3	2	1	25	1	4	2	2	1
H	16th St. Camelback–Glendale	9	6	16	9	13	11	7	9	14	7	17	12	11	7	14	14	12	9	10	10
I	24th St. Buckeye–McDowell	7	4	4	3	5	4	5	5	3	2	2	5	3	5	2	17	5	5	5	4
J	32nd St. Van Buren–Thomas	12	15	19	5	6	18	8	11	7	5	8	7	6	6	12	13	17	8	8	7
K	44th St. McDowell–Ind. Sch.	14	9	11	18	15	19	15	12	20	9	9	21	8	12	11	12	18	11	12	13
L	Baseline 16th St.–32nd St.	24	24	21	12	22	16	23	24	13	24	21	22	21	22	13	20	25	19	22	23
M	Broadway 7th Ave.–16th St.	21	10	13	17	17	9	12	20	19	16	14	10	12	20	10	10	23	10	15	16
N	Van Buren 43rd Ave.–27th Ave.	13	11	6	14	7	7	6	15	12	19	4	8	5	14	5	11	14	7	6	6
O	Van Buren 7th St.–24th St.	23	22	20	23	20	20	18	6	4	8	13	11	17	4	24	9	19	17	20	19
P	Van Buren 48th St.–60th St.	19	19	18	22	21	8	22	17	23	18	23	20	20	23	22	21	8	16	21	20
Q	McDowell 19th Ave.–7th St.	2	3	2	1	2	1	3	1	1	3	6	4	1	3	23	7	1	4	1	2
R	Thomas 51st Ave.–35th Ave.	11	13	9	15	10	10	14	16	18	15	22	18	18	16	17	15	20	14	11	18
S	Ind. Sch. 7th Ave.–16th St.	5	17	15	21	16	21	1	4	10	12	12	6	22	2	20	6	2	6	7	9
T	Camelback 16th St.–32nd St.	25	18	10	10	9	15	19	8	11	11	11	14	9	21	16	18	10	18	19	15
U	Bethany 7th Ave.–16th St.	20	8	14	11	8	13	17	13	8	10	15	9	14	9	7	8	15	12	13	11
V	Glendale 16th St.–32nd St.	15	21	17	13	19	23	24	22	24	22	16	19	19	19	15	23	7	22	24	21
W	Cave Creek 7th St.–20th St.	8	12	12	16	18	14	9	21	9	23	24	13	16	10	1	25	13	20	14	17
X	"Q" Ave. 43rd Ave.–Black Canyon	16	23	25	25	25	22	21	25	25	20	25	20	24	25	18	24	22	24	18	24
Y	Grand Ave. Thomas–Camelback	3	7	7	6	3	2	11	19	5	14	5	2	10	15	6	4	9	3	4	5

TABLE 8

PHOENIX COMPARISON OF JUDGMENT AND FORMULA C RATINGS

Segment	Location	Judgment Priority	Formula Priority	Position Difference
A	59th Ave. Van Buren—Thomas	25	21	4
B	43rd Ave. Bethany Home—Northern	22	19	3
C	27th Ave. McDowell—Indian School	14	8	6*
D	19th Ave. Indian School—Bethany Home	8	17	9*
E	7th Ave. Van Buren—Thomas	3	6	3
F	Central Camelback—Glendale	12	12	0
G	7th St. McDowell—Indian School	1	5	4
H	16th St. Camelback—Glendale	10	15	5*
I	24th St. Buckeye—McDowell	4	7	3
J	32nd St. Van Buren—Thomas	7	9	2
K	44th St. McDowell—Indian School	13	10	3
L	Baseline 16th St.—32nd St.	23	25	2
M	Broadway 7th Ave.—16th St.	16	18	2
N	Van Buren 43rd Ave.—27th Ave.	6	11	5*
O	Van Buren 7th St.—24th St.	19	3	16*
P	Van Buren 48th St.—60th St.	20	23	3
Q	McDowell 19th Ave.—7th St.	2	1	1
R	Thomas 51st Ave.—35th Ave.	18	22	4
S	Indian School 7th Ave.—16th St.	9	4	5*
T	Camelback 16th St.—32nd St.	15	13	2
U	Bethany Home 7th Ave.—16th St.	11	14	3
V	Glendale 16th St.—32nd St.	21	24	3
W	Cave Creek 7th St.—20th St.	17	16	1
X	"Q" Ave. 43rd Ave.—Black Canyon	24	20	4
Y	Grand Ave. Thomas—Camelback	5	2	3

*Difference of 5 or more between judgment and formula order of priority.

umes and then to add the 5-yr forecast growth ratio. The 5-yr forecast is an effort to reach a balance between present and future needs in capital programming. Evidence indicates that the present volume element of the formula should be divided by 1,500 rather than 2,000. Thus, a better spread would be obtained.

The overall results from the test of Formula C are encouraging. The inconsistencies developed by the formula are either explainable or are not worse than the inconsistencies demonstrated by the spread in the individual judgment of the several raters. The lack of spread in the dalay rate point (Table 9) is cause for concern. However, it is possible that this can be explained.

Need for Broader Test

Phoenix is currently rating some 48 miles of major arterial streets included in a recently recommended capital improvement program. These streets will be rated by Formula C and combined with the 25 sections included in the first test. This broader base should provide a further evaluation of the formula's ability to differentiate between projects.

Judgment is not infallible, and therefore it is difficult at times to determine whether the formula is correct or whether the combined judgment of the raters is correct. Table 8 indicates that usually one or two raters were rather far off the mean. Several alternate efforts were made to reduce the spread of the judgment ratings. For example, the highest and lowest rater were eliminated, then the two high and two low. These efforts produced no significant difference in the judgment ratings. Table 8 also demonstrates that any one project may receive from nearly the highest to nearly the lowest

TABLE 9

PHOENIX FORMULA C

Segment	Location	Relative Weight (points)				Total Points (100 max.)	Formula Rank
		Delay Rate (50 max.)	Collision Rate (15 max.)	Structural Condition (15 max.)	Traffic (20 max.)		
A	59th Ave. Van Buren—Thomas	0	3	12	4	19	21
B	43rd Ave. Bethany—Northern	½	2	15	4½	22	19
C	27th Ave. McDowell—Ind. School	6	5	15	5	31	8
D	19th Ave. Ind. School—Bethany	1½	6	9	6	22½	17
E	7th Ave. Van Buren—Thomas	7	6	13	8	34	6
F	Central Ave. Camelback—Glendale	3½	6	7	8½	25	12
G	7th St. McDowell—Ind. School (as it was)	7½	6	13	9½	36	5
H	16th St. Camelback—Glendale	1½	6	9	7	23½	15
I	24th St. Buckeye—McDowell	7½	12	14	8½	32	7
J	32nd St. Van Buren—Thomas	2½	7	12	8	29½	9
K	44th St. McDowell—Ind. School	4	5	12	6½	27½	10
L	Baseline 16th St.—32nd St.	0	2	1	5½	8½	25
M	Broadway 7th Ave.—16th St	1	7	7	7	22	18
N	Van Buren 43rd Ave.—27th Ave.	0	6	13	7½	26½	11
O	Van Buren 7th St.—24th St.	9½	15	3	12½	40	3
P	Van Buren 48th St.—60th St.	0	2	3	9	14	23
Q	McDowell 19th Ave.—7th St. (as it was)	32	15	13	13	73	1
R	Thomas 51st Ave.—35th Ave.	0	4	8	6½	18½	22
S	Ind. School 7th Ave.—16th St.	11	8	4	15	38	4
T	Camelback 16th St.—32nd St.	3½	4	5	12	24½	13
U	Bethany 7th Ave.—16th St.	1	6	12	5½	24½	14
V	Glendale 16th St.—32nd St.	0	2	7	5	14	24
W	Cave Creek 7th St.—20th St.	½	4	15	4	23½	16
X	"Q" Ave. 43rd Ave.—Black Canyon	0	2	15	3½	20½	20
Y	Grand Ave. Thomas—Camelback	7½	15	13	9½	45	2

judgment rating. Perhaps this is the best argument of all for a major street improvement priority formula.

Summary

A major street improvement priority formula for urban areas is needed. Such a formula would be a useful tool to those responsible for developing a capital improvement program for major streets in cities. It would make possible the presentation of various projects in a relative priority list based on facts. At this point, judgment and budgetary considerations can most properly be applied to develop the capital improvement program that will provide maximum benefits to the public.

The results of the work in Phoenix and San Diego indicate that such a formula should not be too complex and should certainly minimize the judgment elements that go into it. However, this study demonstrates that one of the more difficult considerations for a priority formula to recognize and evaluate is a facility that has been improved to reasonable standards or that is near an existing or planned freeway.

It is difficult to evaluate a major street improvement priority formula because of the wide variances in judgment that have been obtained from the several studies. This emphasizes the need to develop a simple, easily applied, factual major street improvement priority formula for urban areas.

CONCLUSIONS

1. Travel time is an effective measure of level of service, both for individual routes and for urban streets and freeway systems.
2. Travel time studies of individual routes are a simple tool that can identify causes of congestion and thus lead more rapidly to needed improvements. A program of traffic engineering improvements resulted in an increase in the average speed on major arterial streets of 20 percent between 1947 and 1957, accomplished despite an increase of 62 percent in the average traffic volume.
3. Time contour maps offer a simple and reasonably accurate means of comparing the level of service of various urban areas.
4. A typical major arterial street system reaches saturation at some population level of the central city of a growing urban area. By the time this point is reached, a freeway system must be placed in operation if the overall level of service of the street system is to be prevented from declining. The population level of the central city may well be somewhere between 400,000 and 500,000 people. It appears that Phoenix has passed this "hump" as the average travel speed declined slightly between 1957 and 1962.
5. A major arterial street construction priority formula for urban areas is needed. Such a formula would not replace judgment but would be used to present various projects in a relative priority list based on factual studies. Judgment, timing and budgetary considerations can then best be applied to the priority list to develop a capital improvement program.
6. An urban major street improvement priority formula should be relatively simple and should minimize the judgment elements that go into it. A priority formula should be based on facts.
7. A major street improvement priority formula is difficult to test because of wide variances in judgment. Perhaps this conclusion is the strongest argument in favor of developing a simple, easily applied, factual major street improvement priority formula for urban areas.

An Economic Evaluation of Traffic Movement at Various Speeds

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The purpose of this study was to evaluate economic utility or cost of resources consumed by the highway transportation industry for various speeds of travel in rural and urban areas for passenger cars and commercial vehicles on 2- and 4- lane streets and highways during daytime and nighttime travel. Graphical relationships of economics of vehicle operation, values of time, safety of travel, and their sum, which is defined as the total cost of traffic movement, were drawn for the various conditions. The minimum point on each total cost curve represents that speed at which the cost of traffic movement is minimized.

Results indicated that there was a speed which minimized the cost of traffic movement for each of the various conditions considered. This speed was defined as the optimal speed. In rural areas the optimal speed was 50 mph for passenger cars and 41 mph for commercial vehicles. Optimal speeds in urban areas decreased with an increase in number of stops per mile from 41 to 29 mph for passenger cars and from 36 to 25 mph for commercial vehicles with 1 and 8 stops per mile, respectively.

The most direct application of the results is likely to be in the establishment of statewide or areawide speed limits where the limit is established so that the mean speed of the vehicles coincides with the optimal speed.

•HIGHWAY transportation is a branch of the transportation industry that consumes a large portion of America's resources, both natural and human (28). The expenditure of resources in promoting place and time utilities through highway transportation can be analyzed according to the following elements:

1. Economics of vehicle operation—expenditures incurred directly as a result of the operation of motor vehicles on street and highway systems.
2. Values of time to drivers and passengers—rate of travel has varied personal and business importance in affecting highway transportation.
3. Safety of travel—reduction of accidents has economic implications, such as decreased insurance rates, and personal bearings, such as absence of injury to ohe's self, friend, or relative.
4. Travel comfort and convenience—this service resource affects psychological attitudes of the motor vehicle occupants.

To obtain maximum benefits and services for a given investment of capital, labor, land, managerial ability, and technical innovation, proper distribution of these resources must be made among these various benefits and services (28). Therefore, it is essential that the most efficient allocation of these four resources be developed to

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enable the acquisition of optimal production of benefits and services in the highway transportation industry.

These four resources must be evaluated on a quantitative scale that allows them to be summed together in their proper proportions. This procedure is similar to the numerical system of arithmetic. A convenient method allowing the resources to be evaluated in their combined effect, and also in their proper proportion, is based on the expenditures of these resources per mile of travel, such as cents per mile. Because of the manner in which these various resources were measured, the investigation was reduced to an economic evaluation of traffic movement at various speeds.

The purpose of this study was to evaluate the economic utility or cost of the resources consumed by the highway transportation industry for various speeds of travel in rural and urban areas for passenger cars and commercial vehicles on 2- and 4-lane streets and highways during daytime and nighttime travel. This study was restricted to vehicular movement over level, tangent sections of well-paved streets and highways under free-flowing traffic conditions.

Graphical relationships of these four resource costs and their sum, defined as the total cost of traffic movement, were ascertained from economic studies of vehicular flow in rural and urban traffic areas for various types of motor vehicles. These curves were further refined for both 2- and 4-lane streets and highways and for daytime and nighttime travel conditions.

The speed that the majority of motor vehicles must travel to minimize the cost of traffic movement was obtained from the minimum points on the various total cost curves. The speed that minimizes the cost of traffic movement is defined as the "optimal speed" for the specified traffic area, vehicle type, highway type, and travel condition which the curve represents. These cost curves were representative of motor vehicle travel for roadway, traffic, and environmental conditions that are nearly ideal; that is, vehicular speeds were not limited by various physical and/or environmental factors. Because speeds of the various motor vehicles are not uniform but represent an approximate normal distribution, the optimal speed represents the mean speed of the motor vehicles (29).

Results of this investigation are likely to be useful in the development of statewide or areawide speed-zoning and in completing data that are lacking in the present road-user benefit analyses.

REVIEW OF LITERATURE

Many investigations have been made to evaluate motor vehicle movement costs at various speeds of operation. This review of literature has been confined to summarizing those studies that have attempted to measure traffic movement costs at various speeds in rural and urban areas for passenger cars and commercial vehicles on 2- and 4-lane streets and highways during daytime and nighttime travel conditions.

Operation Cost

Operation costs are defined as those direct road-user costs that are incurred as a result of the operation of motor vehicles. They can be divided into five elements: fuel, oil, tire, maintenance, and depreciation.

Fuel cost is influenced by both unit cost and consumption rate. Usually, fuel cost is approximately 40 to 50 percent of the total operation cost (6). Fuel consumption is dependent on the characteristics of the motor vehicle, speed and type of operation, road conditions, vehicle use, driving conditions, and individual driving practices (27).

"Road User Benefit Analyses for Highway Improvements" (6) disclosed some facts on fuel mileage of average on-the-road passenger cars operating at a constant speed over level, tangent sections of well-paved highways. For these conditions the fuel mileage increased with an increase in speed up to 18 miles per gallon (mpg) at 25 mph and then decreased at an increasing rate with additional increase in speed.

In a recent investigation, Claffey (5) reported fuel consumption rates at various speeds for a pickup and a dump truck, both in an empty and a loaded condition, and a passenger car. Results revealed that at the optimum speed, or the speed at which fuel consumption is at a minimum, fuel consumption of the empty pickup truck was the lowest, followed by the loaded pickup truck, passenger car, empty dump truck, and loaded dump truck.

Fuel consumption is less on 4-lane divided highways than on 2-, 3-, or 4-lane undivided highways. This differential is explained by the fact that passing maneuvers can be made with less change in speed on 4-lane highways than on either 2- or 3-lane highways. The median on a 4-lane divided highway provides a physical separation between opposing traffic, which helps to reduce the magnitude of speed changes and fuel consumption (6).

Fuel-consumption rates described in the preceding paragraphs can be used for restrictive-type vehicular operation (like that encountered along the built-up routes in urban areas), as well as for free-type vehicular operation (like that encountered along the non-built-up routes in rural areas or on fully controlled access routes in urban areas), if the additional fuel necessary for slow downs and stops is included in the total fuel consumption (6). Claffey (5) reported a linear increase in fuel consumption with an increase in speed. It was observed that the empty and loaded pickup truck consumed less fuel per stop for various approach speeds than did the passenger car, while the reverse was true for the empty and loaded dump truck. His investigation also disclosed excess fuel consumption caused by a slow down of 10 mph from various approach speeds. For both the empty and loaded conditions, the pickup and the dump trucks had definite speeds at which excess fuel consumption for a 10 mph slow down was a maximum, while excess fuel consumption for the passenger car increased at an increasing rate with an increase in speed. The four optimal approach speeds were in the 35- to 45-mph range.

In summary, fuel consumption, thus fuel cost, increased with an increase in speed beyond some optimal speed and in size of vehicle and increased with a decrease in number of traffic lanes and in freedom of vehicular operation.

Oil cost is a function of the unit price and the amount consumed. Major factors influencing oil consumption are maintenance practices, vehicle characteristics, condition of the engine and vehicle, speed of operation, vehicle equipment, road condition, weather, and driver characteristics (27).

Lane (18) related that oil mileage increased with increasing speed up to about 800 miles per quart (mpq) at 30 mph. Then, oil mileage decreased with further increases in speed to approximately 200 mpq at 70 mph.

The Washington State Highway Commission (39) observed that oil cost for a private passenger car was 0.185 cents per mile, while oil cost for commercial vehicles ranged from 0.107 cents per mile for a vehicle weighing 4,000 lb to 0.371 cents per mile for a vehicle weighing 60,000 lb.

Research has shown that oil consumption, hence oil cost, increased with an increase in speed beyond some optimal speed and in size of vehicle.

Tire cost is influenced by both initial cost and rate of wear. Rate of wear is dependent on vehicle characteristics, highway features, speed of travel, type of operation, tire maintenance, and driver habits (27).

Evans (8) reported that at 15 mph a passenger car obtained approximately 30 percent more tire mileage than at a speed of 35 mph and about 50 percent less tire mileage at 55 mph than at 35 mph.

From an investigation performed by the Washington State Highway Commission (39), tire cost for a private passenger car was found to be 0.496 cents per mile, while tire cost for commercial vehicles ranged from 0.411 cents per mile for a vehicle weighing 4,000 lb to 2.371 cents per mile for a vehicle weighing 60,000 lb.

Tire wear has been found to be less on 4-lane divided highways than on 2-, 3-, or 4-lane undivided highways. This is explained by the reduction in number of speed changes caused by passing maneuvers on 4-lane highways. The median on a 4-lane divided highway provides a physical separation between opposing traffic that also helps to reduce the magnitude of speed changes and tire wear (6).

Restrictive-type vehicular operation, peculiar to travel on city streets, greatly increases tire wear over that of the free-type vehicular operation found on rural highways and urban freeways. Moyer (25) found that a single stop and start at 35 mph wore away approximately as much rubber as a mile of travel at the same normal speed.

In summary, tire wear, thus tire cost, increased with an increase in speed beyond some optimal speed and in size of vehicle and increased with a decrease in number of lanes and in freedom of vehicular operation.

Maintenance cost, which includes cost of engine, chassis, body servicing and repairs, and lubrication, is difficult to relate to various conditions of vehicle operation. One vehicle may be given constant maintenance attention at considerable cost, yet give no better service than another vehicle receiving a minimum of maintenance. Results of hard usage at one time may not require repairs until long afterwards; therefore, it is very difficult to evaluate maintenance cost for various conditions of vehicle operation. Research has shown that maintenance cost is affected by maintenance practices, vehicle age and condition, roadway conditions, engine power and speed, speed of travel, and weather (27).

The Highway Engineering Handbook (42) prorated maintenance cost for various types of vehicles according to speed on the basis of fuel, oil, and tire cost. Therefore, maintenance cost decreases with an increase in speed up to some optimal speed and then increases with an increase in speed.

The Washington State Highway Commission (39) reported that maintenance cost for a private passenger car was 0.715 cents per mile, while the maintenance cost for commercial vehicles ranged from 4.533 to 8.845 cents per mile for vehicles weighing 4,000 to 60,000 lb, respectively.

Concerning the subject of restrictive-type vehicular operation, Wiley (40) asserted that required maintenance of brakes and clutches could be attributed to vehicle-stops. He prorated maintenance cost according to a straight-line variation between no cost at 0 mph and 0.05 cents per mile at 50 mph.

In summary, maintenance cost increased with an increase in speed, beyond some optimal speed and in vehicle size and with a decrease in freedom of vehicular operation.

Depreciation is a lessening in value of the motor vehicle due to the passage of time and/or use. That part chargeable to time is due to a loss in value because improvements have outmoded the vehicle, making it less desirable. That portion of depreciation which is a use-element cost is a function of travel rather than age. The Oregon State Highway Department suggests that two-thirds of the depreciation of passenger cars be prorated to mileage and one-third to time and that all the depreciation cost of commercial vehicles be prorated to mileage (23). The American Association of State Highway Officials (AASHO) allocated one-half of the depreciation cost of passenger cars to both mileage and time (6).

Mileage depreciation is affected by the characteristics of the motor vehicle, the highway, and the operation of the motor vehicle. Depreciation due to mileage is normally calculated on a straight-line basis; that is, to divide the initial cost of the vehicle, less salvage value, by the anticipated number of vehicle miles to be traveled by the motor vehicle (6, 23, 34, 41, 43).

AASHO recommends 1.0 cent per mile as the depreciation cost for passenger cars (6), while the Highway Engineering Handbook suggests for paved surfaces 1.0 cent per mile for passenger cars and light commercial vehicles, 1.5 cents per mile for single-unit trucks, and 2.5 cents per mile for combination vehicles (42). Therefore, depreciation cost increases with an increase in vehicle size.

Time Cost

The cost of the driver's and passenger's time must also be considered in a realistic appraisal of the economics of motor vehicle movement. There is a general acceptance that savings of time for commercial vehicles has value in direct proportion to the wages of the drivers, fixed-time costs for the vehicles, and net operating profits to the owners (6, 9, 11, 17, 19). Fewer people accept values of time, either economic or leisure, for passenger cars although it is admitted that some value is justified (6). Economic time is time gained or lost which affects the cost of production, distribution, or conservation of goods and services. This includes passenger cars of salesmen, repairmen, executives, and all who travel during working hours (10). In this case, the method of evaluation of time cost should be no different from that of commercial vehicles (6, 17).

Leisure time is time gained or lost which causes a gain or loss of convenience. It includes pleasure traffic, commuter traffic, and business traffic in those cases where gain or loss of time does not affect the cost of production, distribution, or conservation of goods and services (10). The following three methods exist to evaluate leisure-time cost for passenger cars: (a) operating-cost data, (b) the extra distance operators will travel in order to save time, and (c) arbitrary time values (34).

The theory behind the operation-cost method is that fixed cost for one hour is a measure of the value of one hour of time. This is based on the assumption that fixed cost of a passenger car continues in full effect as an element of operating cost when the vehicle is stopped or slowed down. To obtain time cost, in dollars per hour, fixed cost per mile is multiplied by the average speed of the vehicle (34).

To determine the extra distance passenger car operators travel to save time, time cost is equated to the extra mileage cost of operating the vehicle plus any toll charge divided by the time saved (34).

Many references indicate that time costs for passenger cars have been arbitrarily selected (1, 2, 3, 4, 6, 13, 14, 20, 24, 42). The most widely used method was suggested by AASHO. A time cost of $1.35 per hour per vehicle is recommended. The value resulted from an arbitrary selection of a time cost of $0.75 per person and an average of 1.8 persons per passenger car (6).

Accident Cost

Development of accident costs for a given speed, on a cost per mile basis, is the product of the traffic-accident involvement rate for the given speed and the severity of the accident at this speed. Traffic-accident involvement rates at various speeds are dependent on the characteristics of the driver, the vehicle, and the highway. Accident severity, or cost per involvement, depends on the number of persons killed and injured per involvement and the economic worth of a death, an injury, and the property damage caused by the accident.

In 1953 and 1955, the Massachusetts Department of Public Works and the Massachusetts Registry of Motor Vehicles in cooperation with the U. S. Bureau of Public Roads conducted the first comprehensive study of economic costs of motor vehicle accidents on a statewide basis (21, 22, 35, 36).

There were 1,910 passenger cars involved in accidents for every 100 million passenger-car miles of travel on Massachusetts streets and highways in 1953. These involvements were composed of 3.0 fatal-injury involvements, 467 non-fatal-injury involvements, and 1,440 property-damage-only involvements. The 1955 commercial vehicle study revealed that there were 1,412 trucks involved in accidents for every 100 million truck-miles of travel, consisting of 4.0 fatal-injury involvements, 223 non-fatal-injury involvements, and 1,186 property-damage-only involvements (21, 22).

From the Massachusetts study, 281 passenger cars were involved in accidents for every 100 million passenger-car miles traveled in rural areas, and 2,002 were involved in urban areas for the same travel rate. The cars were involved in 1.5 fatal-injury, 67 non-fatal-injury, and 212 property-damage-only accidents in rural areas, and in 3.9 fatal-injury, 511 non-fatal-injury, and 1,488 property-damage-only accidents in urban areas (35, 36).

During 1957 and 1958, the U. S. Bureau of Public Roads cooperated with 11 States to conduct an investigation to obtain a better understanding of the relationship between travel speed and accidents. The study covered 3.7 billion vehicle-miles of travel on 600 miles of main, rural highways. Accident records of 10,000 drivers, together with speed observations and interviews with 290,000 drivers using these highways, provided data for the study. The study revealed that accident involvement rates for both 2- and 4-lane divided highways decreased with a decreasing rate as speed was increased to approximately 60 to 65 mph, and then the accident involvement rates increased with an increasing rate with any further increase in speed. Accident involvement rates for speeds less than 50 mph were greater on 4-lane divided highways than on 2-lane highways, as can be seen by comparing Figures 3 and 4. Above 50 mph, the 2-lane involvement rates were higher than the corresponding rates on 4-lane highways (32, 38).

The study also disclosed that in the range from 20 to 60 mph the night involvement rates were approximately twice the day rates. At speeds below 20 mph the night involvement rates were less than twice the day rates, while at speeds above 60 mph the night involvement rates were several times higher than the day rates (32, 38).

In conclusion, accident involvement rates are higher for passenger cars than for commercial vehicles, higher in urban areas than in rural areas, higher on 4-lane divided highways than on 2-lane highways at speeds less than 50 mph, and higher during nighttime than during daytime.

Accidents occurring at moderate and high speeds were more severe than those at low speeds. For example, at 40 mph there were 31 persons fatally or non-fatally injured per 100 accident-involved vehicles, while at 65 mph 70 persons were fatally or non-fatally injured per 100 accident-involved vehicles. The amount of property damage per involvement increased at an increasing rate with an increase in speed. At 20 mph property damage per involvement was $250, at 65 mph, $430 (32, 38).

Comfort and Convenience Cost

Operation, time, and accident costs are tangible costs and are easily evaluated; however, comfort and convenience cost is intangible and difficult to evaluate in relative, quantitative measures. Nevertheless, benefits gained from comfort and convenience are real and should be appraised.

The use of toll facilities is evidence that some drivers place a monetary value on comfort and convenience. People who could have driven to their destination in fewer miles and with little difference in time on a free but a more congested route have elected to pay for the privilege of traveling on the toll road. Therefore, it must be of some value to the person to drive without frequent brake application, stops and starts, or tension created by traffic or roadside interference.

Positive identification of values for assignment to various degrees of comfort and convenience is not possible because presently there are no methods available to determine unit values of the many factors entering the evaluation of comfort and convenience costs. Some of these factors are highway type, services rendered to different traffic components, type of trip being made, trip length, and degree of interference on alternate routes (6).

AASHO arbitrarily selected the following values for various degrees of comfort and convenience: free type of operation, 0 cent per vehicle-mile; normal type, 0.5 cent per vehicle-mile; restricted type, 1.0 cent per vehicle-mile. AASHO defined type of operation by the ratio of the 30th highest hourly traffic volume to the practical capacity of the roadway. The types of operation are identified from these ratios as restricted operation for ratios greater than 1.25, as normal for ratios of 0.75 to 1.25, and as free for ratios less than 0.75 (6).

PROCEDURE

Rural Highways

Rural highways are defined, for the purposes of this study, as those routes which have no or very little roadside development along their rights-of-way and where traffic-controlled intersections are a mile or more apart.

Operation Cost.—The most recent and reliable data concerning operation cost for passenger cars and commercial vehicles is in the Highway Engineering Handbook. Operation-cost data presented in this reference were developed by Winfrey (42) through adjustment, reconciliation, and trending of a large number of published reports plus personally collected data. It was assumed that unit prices of fuel in cents per gallon and oil in cents per quart were 32 and 40, 30 and 34, and 28 and 25 for passenger cars and light commercial vehicles, single-unit trucks, and combination vehicles, respectively. Price per unit of tire wear for the three vehicular groups was not stated, but tire cost per mile was based on the depth of tread and the rate of tread wear. Maintenance cost for the various vehicular groups was prorated according to speed on the basis of fuel, oil, and tire cost. Depreciation, cost which was assumed to be attributable to mileage use, was estimated at 1.0, 1.5, and 2.5 cents per mile for passenger cars and light commercial vehicles, single-unit trucks, and combination vehicles, respectively.

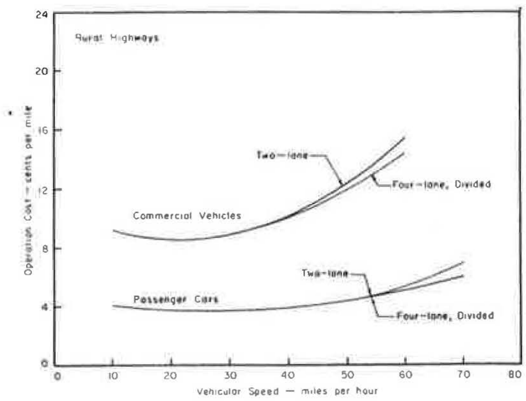

Figure 1. Operation cost vs vehicular speed, rural highways.

Total operation cost for passenger cars and commercial vehicles on 2- and 4-lane divided highways in rural areas is shown in Figure 1. Operation cost for commercial vehicles was prorated for a representative commercial traffic stream composed of 30.6 percent light commercial vehicles, 29.83 percent single-unit trucks, and 39.57 percent combination vehicles (7). Operation costs on 4-lane divided highways were obtained by using the ratio of 2-lane operation costs to 4-lane divided operation costs developed by AASHO (6). Passenger-car operation costs are a minimum at 26.5 mph for 2-lane highways and at 27.5 mph for 4-lane divided highways, whereas commercial-vehicle operation costs are a minimum at 21.0 mph for both highway types.

Time Cost.—In view of the general disagreement in the value of time for passenger cars, a conservative value is desirable so as to provide realistic and identifiable monetary benefits due to time saved. Therefore, a time cost of $0.75 per person, suggested by AASHO (6), was used in this study. The U. S. Bureau of Public Roads found on rural highways an average of 1.9 persons per passenger car (16). Therefore, the average value of time for passenger cars in rural areas was assumed to total $1.425 per hour per vehicle, or 2.375 cents per minute per vehicle.

The Highway Engineering Handbook suggests the following conservative values of time for the three groups of commercial vehicles: light commercial vehicles, $1.80 per hour; single-unit trucks, $2.10 per hour; and combination vehicles, $2.64 per hour (42). Based on a representative commercial-traffic stream in rural areas, a representative time cost for all commercial vehicles of $2.22 per hour was developed (7). Time cost per mile for passenger cars and commercial vehicles on rural highways as a function of travel speed is shown in Figure 2. Time cost is inversely proportional to the speed of the vehicle.

Accident Cost.—Accident cost for a given speed is equal to the product of the accident involvement rate for that speed and the severity of the accident at the same speed. Involvement rates for passenger cars and commercial vehicles in rural areas were developed from data provided by the U. S. Bureau of Public Roads (32, 38) and North Carolina (12).

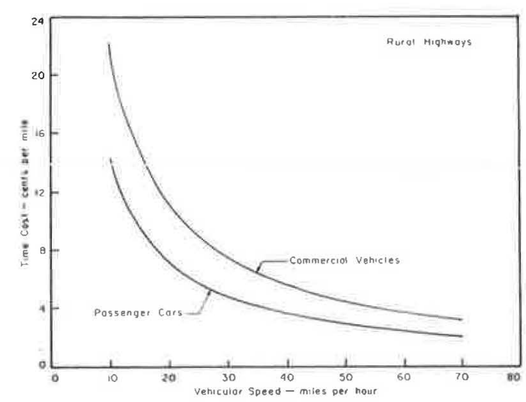

Figure 2. Time cost vs vehicular speed, rural highways.

Passenger car and commercial vehicle involvement rates on 2- and 4-lane divided rural highways and for daytime and nighttime conditions are shown in Figures 3 and 4. For each curve, there is a speed

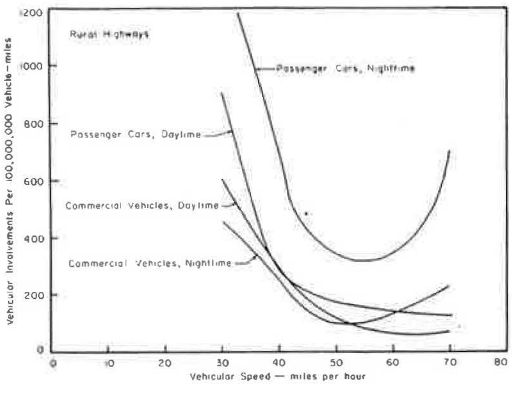

Figure 3. Vehicular involvements vs vehicular speed, 2-lane rural highways.

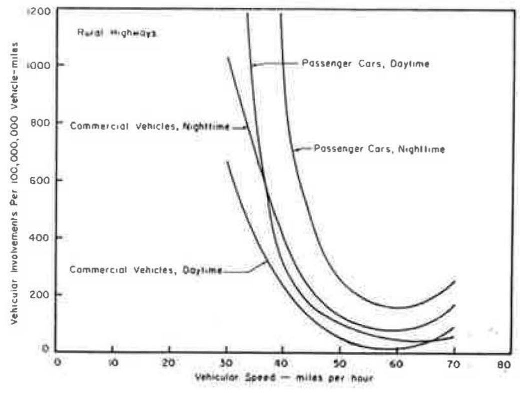

Figure 4. Vehicular involvements vs vehicular speed, 4-lane divided rural highways.

at which the involvement rate was a minimum. Speeds at which the minimum rates occurred are less for commercial vehicles than for passenger cars for each highway type and travel condition. Nighttime involvement rates are generally higher than daytime rates, and 4-lane involvement rates are higher than 2-lane rates at lower speeds, whereas the reverse is true for higher speeds. Commercial vehicle accident-involvement rates are much less than those for passenger cars at corresponding speed values.

Accident severity, expressed as cost per vehicular involvement, is dependent on the number of persons killed or injured per involvement and on the unit costs of a death, an injury, and the property damage caused by the accident. The number of persons killed or injured per passenger car and commercial vehicle involvement was developed from accident data provided by the Bureau of Public Roads (32, 38) and North Carolina (12). The National Safety Council suggested that the economic loss incurred by a death is $30,000 and that by an injury is $1,600. These values are based on wage losses, medical expenses, and overhead costs of insurance (33). Property damage per passenger car and commercial vehicle involvement for various speeds was provided by accident information collected and summarized by the Bureau of Public Roads (32, 38) Massachusetts (21, 22).

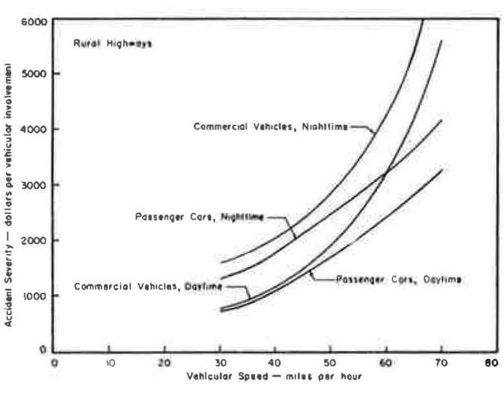

Figure 5. Accident severity vs vehicular speed, 2-lane rural highways.

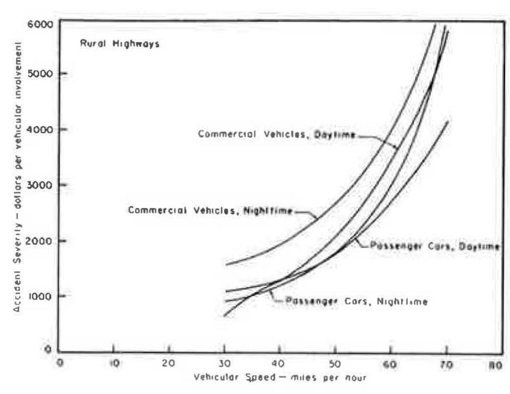

Figure 6. Accident severity vs vehicular speed, 4-lane divided rural highways.

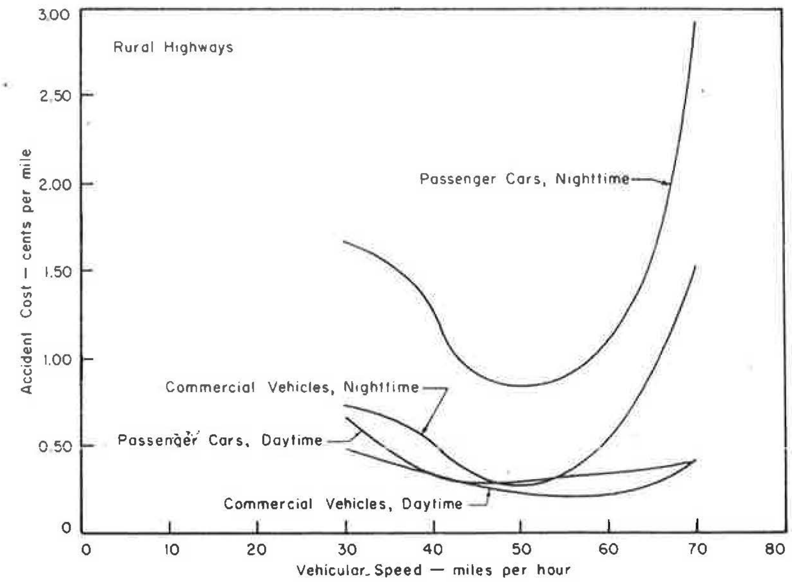

Figure 7. Accident cost vs vehicular speed, 2-lane rural highways.

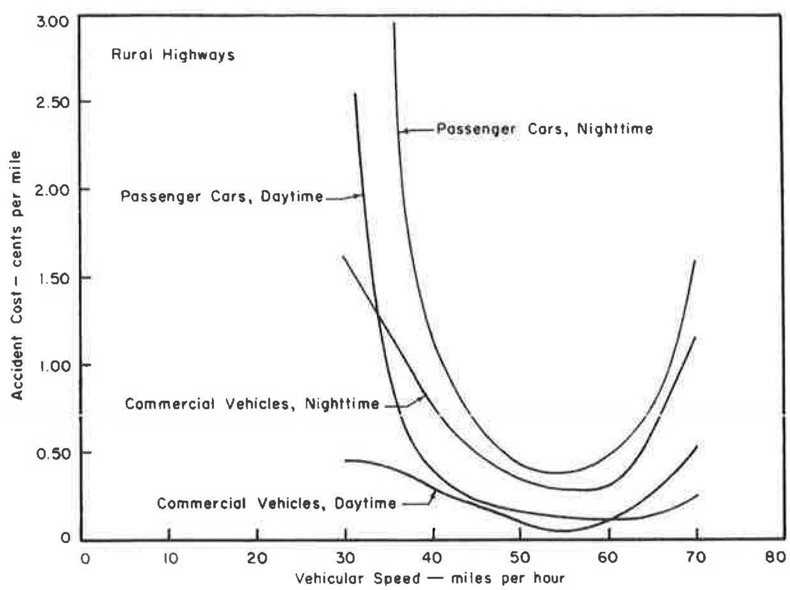

Figure 8. Accident cost vs vehicular speed, 4-lane divided rural highways.

The total accident severity of an involvement is obtained from the sum of the property-damage cost plus the product of the number of killed or injured persons and their resulting economic loss. Figures 5 and 6 show passenger car and commercial vehicle severity on 2- and 4-lane divided highways for daytime and nighttime travel. There is an increase in accident severity at an increasing rate with an increase in speed. Nighttime accident severity is higher than daytime except for passenger-car travel at low speeds on 4-lane divided highways. Accident severity is greater for commercial vehicles than for passenger cars, because the number of persons fatally injured or nonfatally injured per involvement is higher for commercial vehicles than for passenger cars.

Accident costs for various speeds on 2- and 4-lane divided highways for daytime and nighttime travel conditions are shown in Figures 7 and 8. Nighttime accident costs are higher than daytime costs throughout the speed range. In general, accident costs are less for commercial vehicles traveling in rural areas than for passenger cars under the same conditions. Optimum speeds for accident costs on rural highways are indicated in the following:

Vehicle Type	Speed (mph)			
	2-Lane		4-Lane, Divided	
	Daytime	Nighttime	Daytime	Nighttime
Passenger car	46.0	49.5	60.0	54.0
Commercial vehicle	57.5	50.0	57.0	56.5

Comfort and Convenience Cost.—Comfort and convenience cost is an intangible that is difficult to evaluate. However, it can be assumed that this cost element is higher at low speeds (driver impatience) and at high speeds (driver tension), whereas at intermediate speeds, the comfort and convenience cost is minimized in the region of driver satisfaction (30). No justifiable method of assigning values in terms of dollars and cents has been found. Therefore, comfort and convenience costs were not determined in this study.

Urban Streets

Urban streets are defined as those routes which have high or complete roadside development along their rights-of-way and where traffic controlled intersections are less than a mile apart.

Operation Cost.—The criterion used to develop operation costs for passenger cars in urban areas rests on the assumption that the same operation costs used for free-type vehicular operation can be used for restrictive-type operation if additional costs for slowdowns and stops are included in the total operation cost (6). Because this study was restricted to vehicular flow during free-flowing traffic volumes, it was assumed that the only slowdowns made by the drivers will result from complete stops for traffic-control devices. Therefore, operation costs in urban areas can be prorated, if data are available, to provide operation costs for both free-type operation at various speeds and for a normal stop from the same corresponding speeds. Operation cost for a normal stop is defined as that extra cost resulting when a typical driver decelerates from a given speed to a stop and then immediately accelerates to the same speed (5). This information is tabulated in the Highway Engineering Handbook (42).

Figure 9 shows the extra passenger car and commercial vehicle operation costs for a normal stop. Extra operation costs for commercial vehicles were prorated for an

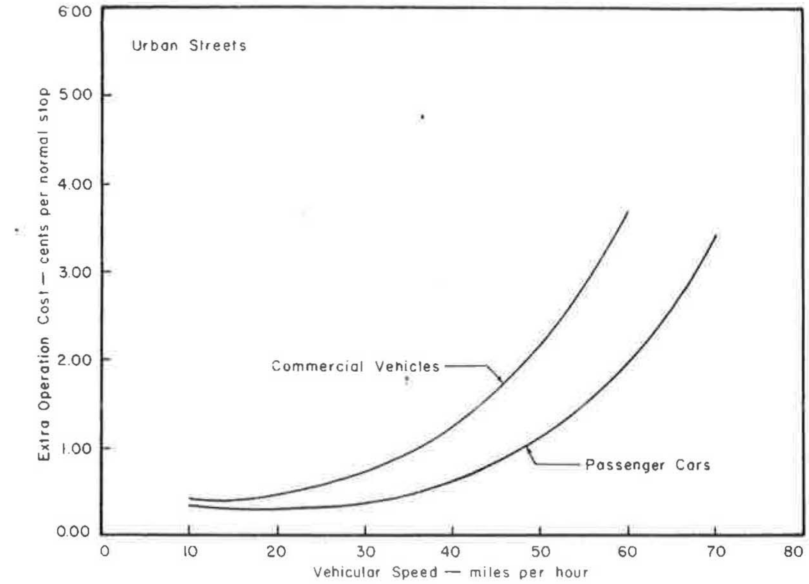

Figure 9. Extra operation cost vs vehicular speed, urban streets.

average urban, commercial-traffic stream composed of 55.77 percent light commercial vehicles, 27.98 percent single-unit trucks, and 16.25 percent combination vehicles (15). At speeds above 20 mph, extra operation costs for both passenger cars and commercial vehicles increased at an increasing rate with an increase in speed. Throughout the speed range, extra operation costs are higher for commercial vehicles than for passenger cars.

To obtain the total operation cost for a desired speed in an urban area, the operation cost for a mile of free-type operation is added to the extra operation cost per stop per mile times the number of stops per mile. Unlike highways in rural areas, operation costs were not prorated for both 2- and 4-lane streets because very few urban streets are divided by a median of adequate width to separate physically traffic movement and thus reduce operation costs. Therefore, these operation costs are applicable only to passenger car and commercial vehicle operation on all undivided urban streets.

Time Cost.—The method used to prorate passenger car time cost on urban streets was the same as that used on rural highways. The Bureau of Public Roads found that there was an average of 1.6 persons per passenger car traveling on urban streets (16). Based on the assumption that time cost equals $0.75 per person, the value of time for passenger cars in urban areas resulted in a total of $1.20 per hour per vehicle, or 2.00 cents per minute per vehicle (6).

Based on the average distribution of travel in urban areas for commercial vehicles, a representative value of time for all commercial vehicles of $2.02 per hour was evolved, somewhat less than the rural value of $2.24. This is explained by the presence of a larger percentage of light commercial vehicles (low value of time) and by a smaller percentage of combination vehicles (high value of time).

Figure 10 indicates that the extra time cost for passenger cars increased linearly with speed, whereas the extra time cost for commercial vehicles increased at an increasing rate with an increase in speed. The extra time consumed for a normal stop by a passenger car, light commercial vehicle, and a single-unit truck was obtained from a study by Claffey (5) while the extra time consumed by an average combination vehicle in performing a normal stop was abstracted from a paper by Sawhill (31).

To compute the total time cost per mile at a given speed in an urban area, the time cost per mile at the given speed is added to the extra time cost per mile multiplied by the number of stops per mile.

Accident Cost. —Unlike in the rural areas, there has been no investigation to ascertain the relationship between travel speed and accidents in urban areas. Therefore, a method was developed which tried to synthesize involvement rates at various speeds.

The 1959 national mileage (26) was proportioned to obtain urban mileage (37), urban passenger-car mileage and commercial vehicle mileage (37), and then urban daytime and nighttime passenger-car and commercial vehicle mileage (26). The absence of any valid information prevented a further breakdown between 2- and 4-lane streets. After making the assumption that the 1959 North Carolina urban speed distributions are typical for the nation (12), the daytime and nighttime mileage was distributed to the various speed groups in proportion to the number of passenger cars and commercial vehicles traveling in that speed group.

The number of fatal, non-fatal, and property-damage involvements were obtained from ratios of passenger-car involvements and commercial vehicle involvements to all-vehicle involvements. These breakdowns were developed from the 1959 North Carolina accident statistics (12) and were used to convert the 1959 all-vehicle involvement data, as estimated by the National Safety Council (26), into passenger-car and commercial vehicle fatal, non-fatal, and property-damage involvements for daytime and nighttime travel conditions.

The number of involvements divided by the number of vehicle-miles traveled resulted in the involvement rates shown in Figure 11. Daytime involvement rates are higher than nighttime rates at lower speeds, but the trend is reversed in the study conducted by the Bureau of Public Roads (32, 38). Urban involvement rates are larger than rural involvement rates, and urban minimum involvement rates occurred at lower speeds than the corresponding rural values. Furthermore, passenger-car and commercial vehicle involvement rates are very similar, except urban daytime commercial vehicle rates are greater than corresponding rates for passenger cars. The reverse is true for nighttime.

The accident severity of an involvement is obtained from the sum of the property-damage cost plus the product of the number of killed or injured persons and their resulting economic loss. The Massachusetts study (21, 22) provided the property-damage costs, the accident statistics from North Carolina (12) and the National Safety Council (26) provided the number of killed and injured at various speeds, while the National Safety Council gave the economic worth of a death and an injury (33). Both daytime and nighttime accident severity increased at an increasing rate with an increase in speed (Fig. 12). Nighttime accident severity is higher than daytime throughout the speeds considered. Accident severity for commercial vehicles in urban areas is higher than that for passenger cars in urban areas, but is lower than either in rural areas.

Accident costs, which are the products of the accident involvement rates and the accident severities, for passenger cars and commercial vehicles in urban areas are shown in Figure 13. Nighttime accident costs are less than daytime accident costs at low speeds, but daytime accident costs are less than nighttime accident costs at high speeds. Optimum speeds for accident costs on urban streets are indicated in the following:

Vehicle Type	Speed (mph)	
	Daytime	Nighttime
Passenger car	39.0	37.0
Commercial vehicle	33.5	32.0

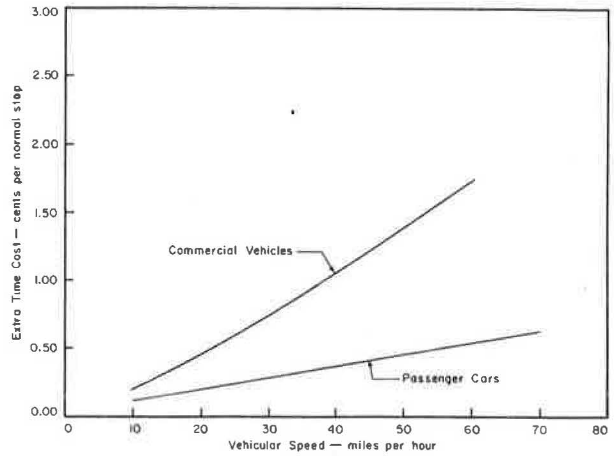

Figure 10. Extra time cost vs vehicular speed, urban streets.

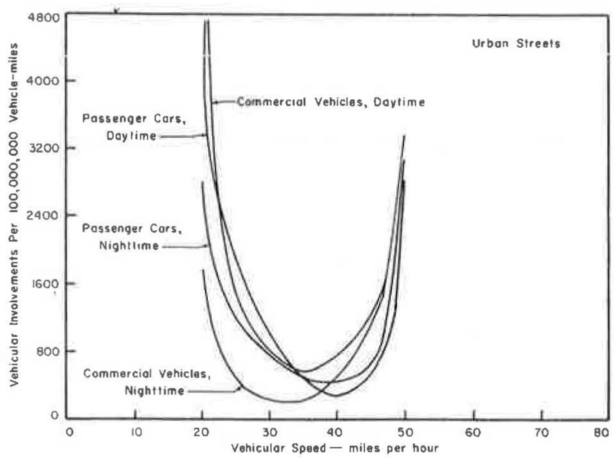

Figure 11. Vehicular involvements vs vehicular speed, urban streets.

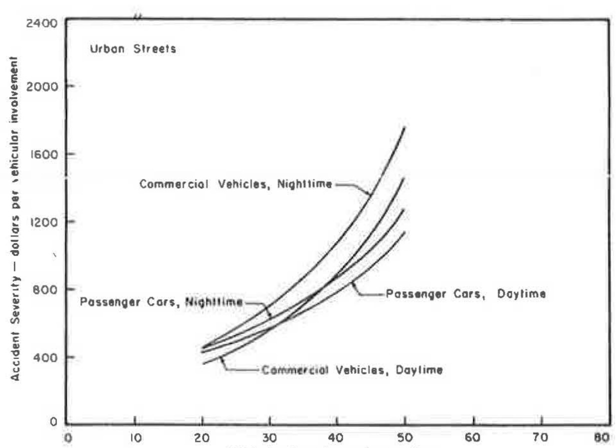

Figure 12. Accident severity vs vehicular speed, urban streets.

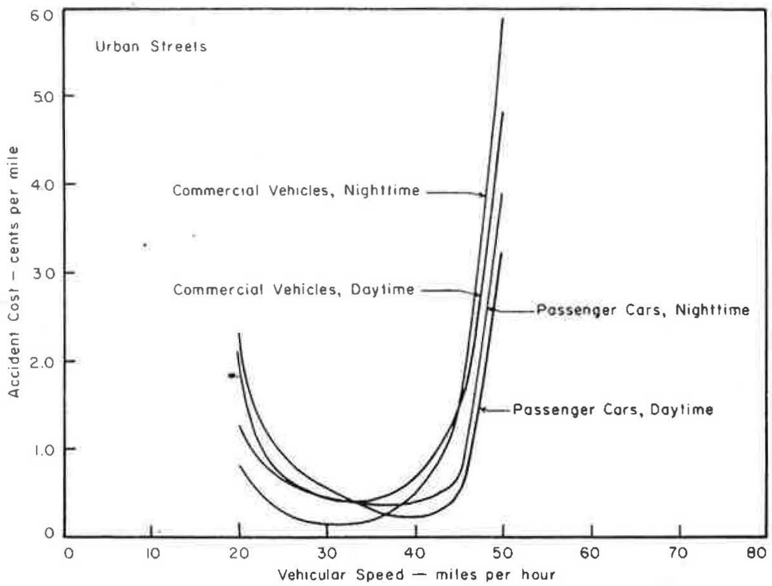

Figure 13. Accident costs vs vehicular speed, urban streets.

RESULTS

Rural Highways

Results of this investigation to determine the cost of traffic movement on rural highways are summarized in Figures 14 through 21. These diagrams represent relationships between vehicular speed and operation cost, time cost, accident cost, and total cost of traffic movement. Total cost is the arithmetic sum of these three elements. In each of the eight total cost curves, there is a travel speed at which the total traffic movement cost is minimized. Therefore, a speed that optimizes the cost of traffic movement for various motor vehicles, highway types, and travel conditions can be rationally determined. Optimum speeds for each rural condition are summarized in the following:

Vehicle Type	Speed (mph)			
	2-Lane		4-Lane, Divided	
	Daytime	Nighttime	Daytime	Nighttime
Passenger car	48.0	48.5	52.1	51.5
Commercial vehicle	40.0	41.0	41.0	44.0

At speeds both above and below these optimal points, total cost increases at an increasing rate.

For passenger cars on 2-lane highways (Fig. 14) the total cost is 7.600 cents per mile at the optimal daytime speed of 48.0 mph. Total cost of traffic movement ranged from 9.121 cents per mile at 30.0 mph down to the optimal value and then back up to 9.438 cents per mile at 70.0 mph.

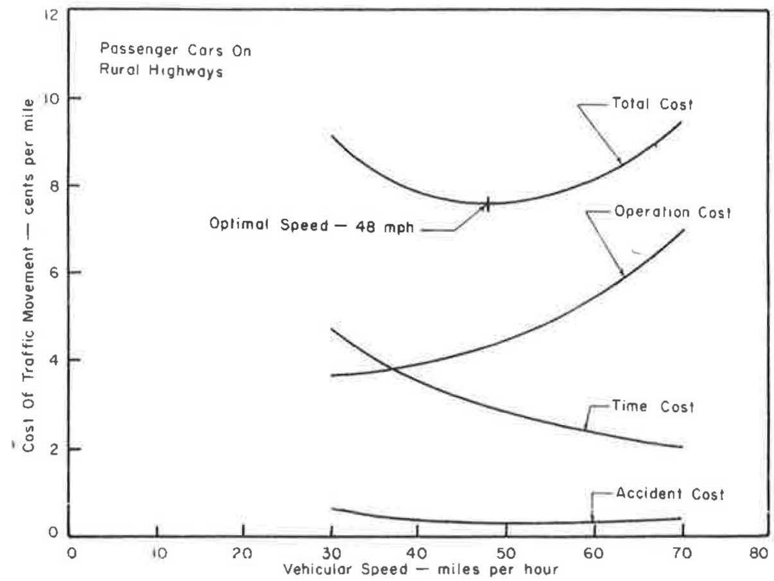

Figure 14. Cost of traffic movement vs vehicular speed, 2-lane, daytime, passenger cars on rural highways.

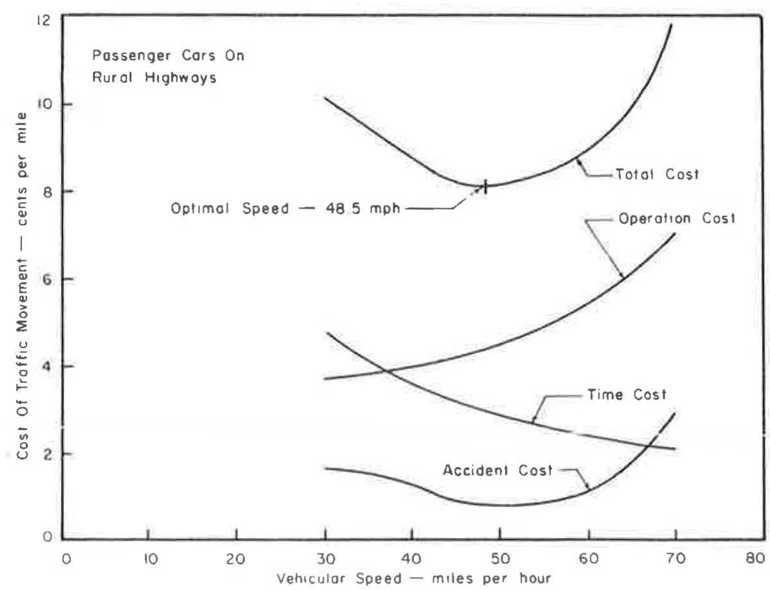

Figure 15. Cost of traffic movement vs vehicular speed, 2-lane, nighttime, passenger cars on rural highways.

Because accident cost is higher for passenger cars on two-lane rural highways for nighttime than it is for daytime, total cost of nighttime traffic movement. (Fig. 15) is slightly larger than for daytime. Figure 15 shows the total cost of traffic movement as 8.140 cents per mile at the 48.5 mph optimal speed.

In Figure 16, total cost of daytime traffic movement for passenger cars on 4-lane rural highways varied from 10.987 cents per mile at 30.0 mph down to 7.400 cents per mile at the optimal speed of 52.0 mph and then back up to 8.281 cents per mile at 70.0 mph. This high total cost at the lower speeds is the result of high accident costs in this region.

Figure 17 shows total cost of traffic movement to be higher for nighttime travel than for daytime travel (Fig. 16) throughout the entire speed range. At the optimal speed of 51.5 mph, total cost of traffic movement is 7.660 cents per mile.

For commercial vehicles on 2-lane rural highways for daytime travel (Fig. 18), the total cost of traffic movement is 16.083 cents per mile at the optimal speed of 40.0 mph, whereas at 30.0 mph and 60.0 mph, total cost of traffic movement is 16.851 cents per mile and 19.199 cents per mile, respectively.

Total cost of traffic movement for commercial vehicles on 2-lane rural highways for nighttime travel (Fig. 19) did not vary appreciably from daytime travel (Fig. 18). At the optimal speed of 41.0 mph, the total cost of traffic movement is 16.220 cents per mile.

For commercial vehicles on 4-lane rural highways for daytime travel (Fig. 20), total cost of traffic movement varied from 16.764 cents per mile at 30.0 mph down to 15.900 cents per mile at the optimal speed of 41.0 mph and then up to 18.212 cents per mile at 60.0 mph.

For commercial vehicles on 4-lane rural highways for nighttime travel (Fig. 21) the cost of traffic movement was observed to be slightly higher than daytime total cost. At the optimal speed of 44.0 mph total cost of traffic movement is 16.320 cents per mile, while at 30.0 mph and 60.0 mph total cost is 17.932 cents per mile and 18.451 cents per mile, respectively.

It is evident that the commercial vehicle total costs are approximately twice the passenger car total costs. Total costs for nighttime travel are consist-

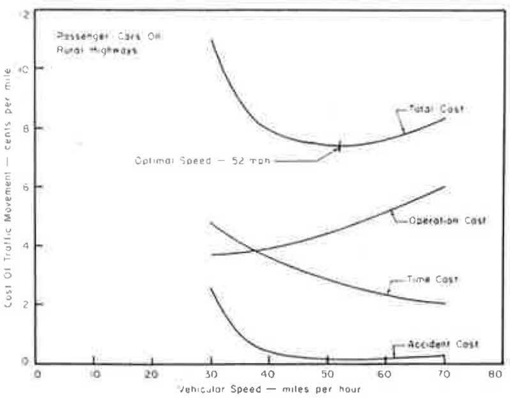

Figure 16. Cost of traffic movement vs vehicular speed, 4-lane divided, daytime, passenger cars on rural highways.

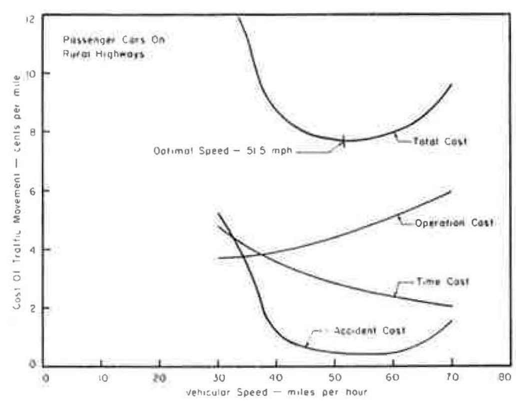

Figure 17. Cost of traffic movement vs vehicular speed, 4-lane divided, nighttime, passenger cars on rural highways.

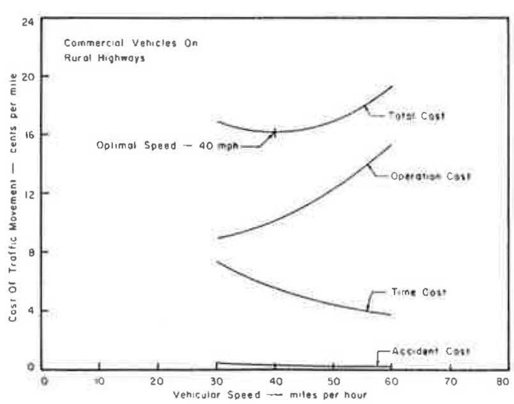

Figure 18. Cost of traffic movement vs vehicular speed, 2-lane, daytime, commercial vehicles on rural highways.

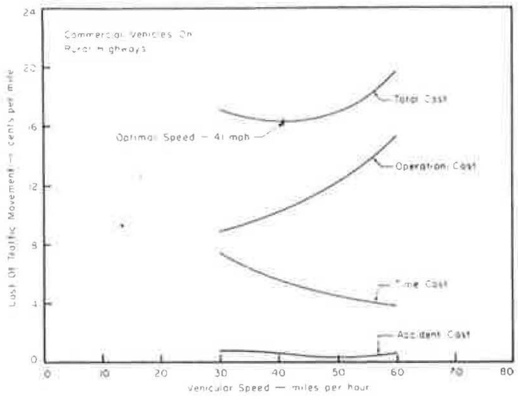

Figure 19. Cost of traffic movement vs vehicular speed, 2-lane, nighttime, commercial vehicles on rural highways.

ently higher than for daytime travel. Also, total costs for 2-lane highways are higher than for 4-lane divided highways except at lower speeds where very high accident costs on 4-lane divided highways produce higher total costs.

Urban Streets

Figures 22 through 25 show the results of this study of vehicle costs on urban streets. These figures depict cost of traffic movement for various speeds, vehicle types, stops per mile, and travel conditions. For each of the various stops per mile, an optimal speed minimizing cost of traffic movement was found.

For passenger cars on urban streets for daytime travel (Fig. 22), optimal total costs of traffic movement ranged from 7.080 cents per mile at an optimal speed of 42.0 mph for 0 stops per mile to 18.420 cents per mile at an optimal speed of 27.0 for 16 stops per mile.

For passenger cars on urban streets for nighttime travel (Fig. 23), optimal total costs of traffic movement ranged from 7.300 cents per mile at an optimal speed of 41.5 mph to 18.240 cents per mile at an optimal speed of 24.5 mph for 0 stops per mile and 16 stops per mile, respectively.

Total cost of traffic movement for commercial vehicles on urban streets is approximately 1.75 times larger than for passenger cars on urban streets. For commercial vehicles on urban streets for daytime travel (Fig. 24), optimal total costs of traffic movement varied from 12.580 cents per mile at an optimal speed of 37.5 mph for 0 stops per mile to 24.117 cents per mile at an optimal speed of 25.0 mph for 8 stops per mile.

For commercial vehicles on urban streets for nighttime travel (Fig. 25), total costs are less than total costs for daytime travel for each of the various stops per mile. Optimal total costs of traffic movement ranged from 12.420 cents per mile at an optimal speed of 37.5 mph to 23.730 cents per mile at an optimal speed of 25.0 mph for 0 stops per mile and 8 stops per mile, respectively.

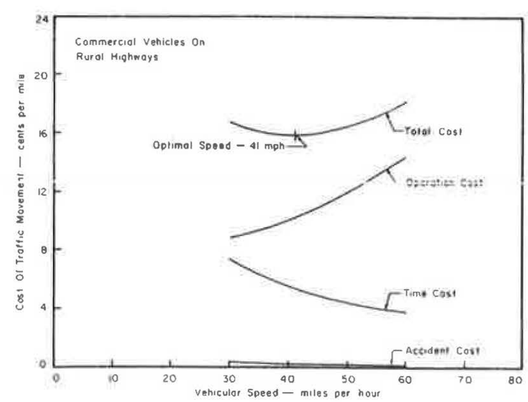

Figure 20. Cost of traffic movement vs vehicular speed, 4-lane divided, daytime, commercial vehicles on rural highways.

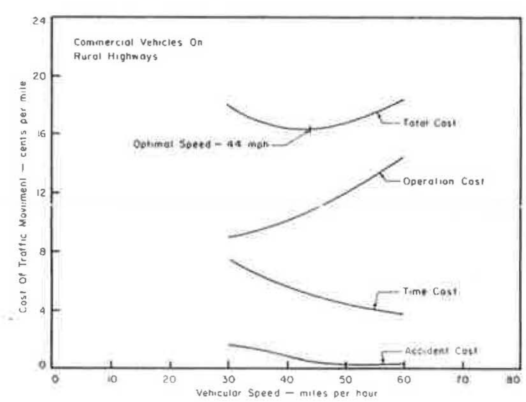

Figure 21. Cost of traffic movement vs vehicular speed, 4-lane divided, nighttime, commercial vehicles on rural highways.

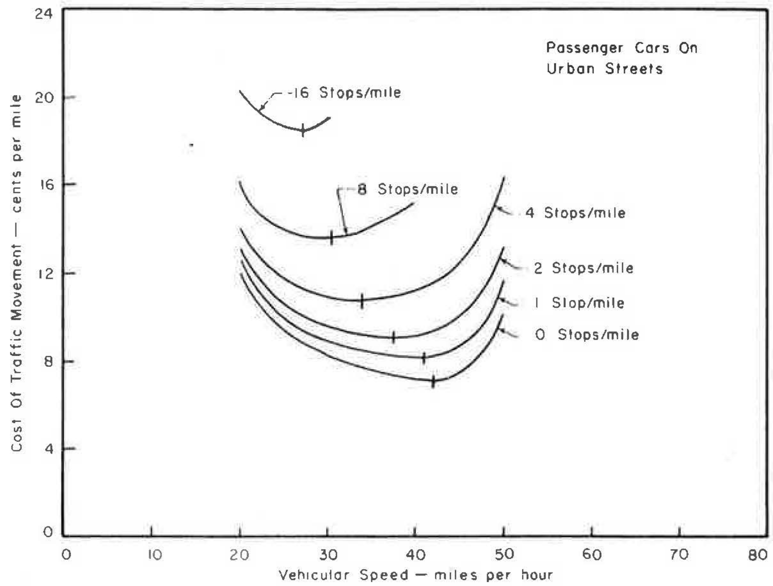

Figure 22. Cost of traffic movement vs vehicular speed, daytime, passenger cars on urban streets.

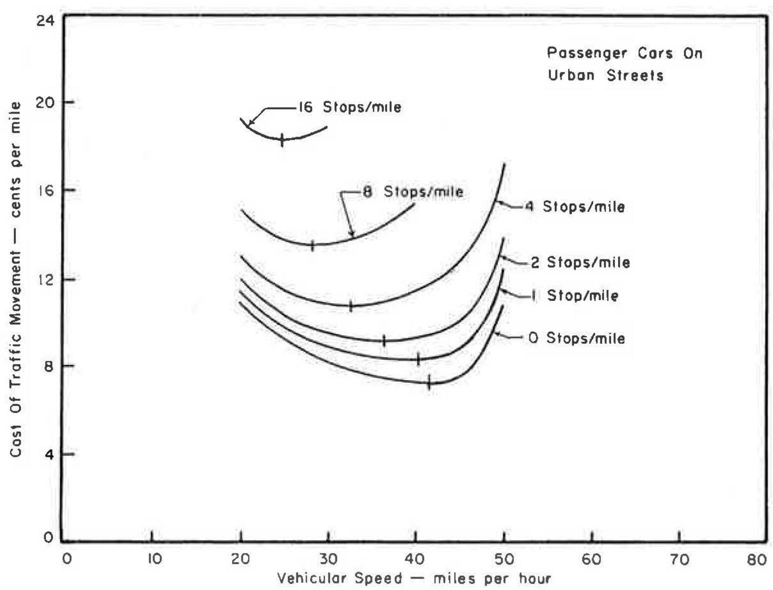

Figure 23. Cost of traffic movement vs vehicular speed, nighttime, passenger cars on urban streets.

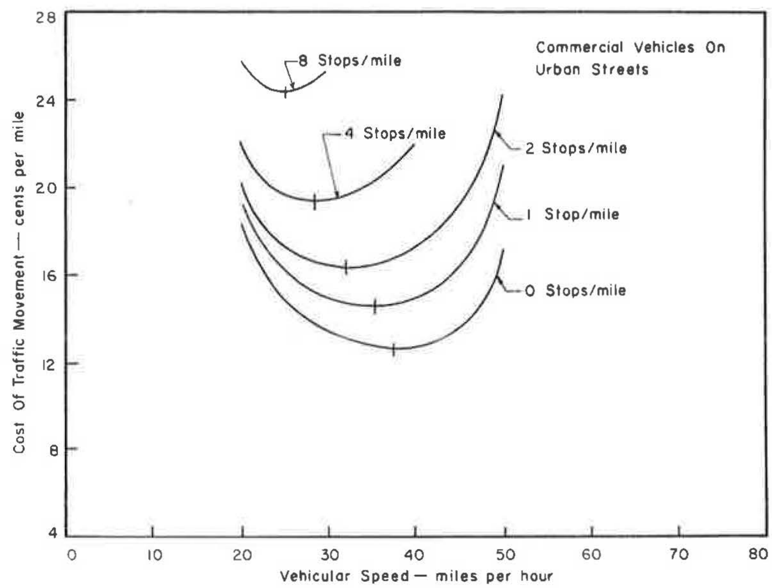

Figure 24. Cost of traffic movement vs vehicular speed, daytime, commercial vehicles on urban streets.

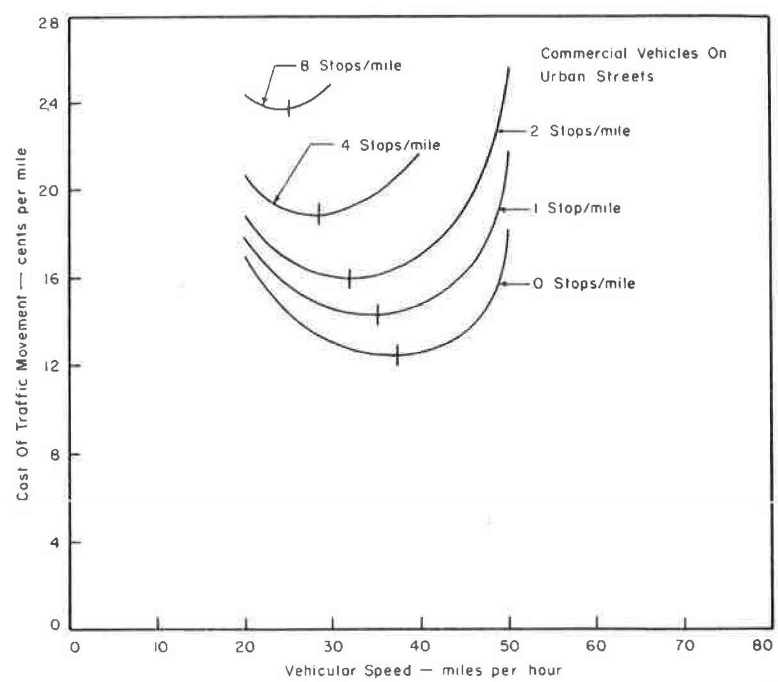

Figure 25. Cost of traffic movement vs vehicular speed, nighttime, commercial vehicles on urban streets.

The following relationships between optimal speeds and stops per mile were established:

Passenger car, daytime:

$$Y = 41.0 - 11.63 \log X \tag{1}$$

Passenger car, nighttime:

$$Y = 40.5 - 13.29 \log X \tag{2}$$

Commercial vehicle, daytime and nighttime:

$$Y = 35.5 - 11.63 \log X \tag{3}$$

in which

Y = optimal speed in mph, and
X = number of stops per mile.

Figure 26 shows optimal speeds for stops ranging from 1 to 16 for passenger cars on urban streets for daytime and nighttime travel, and Figure 27 shows optimal speeds for stops varying from 1 to 8 for commercial vehicles on urban streets for daytime and nighttime travel.

A method was developed to estimate the number of stops due to traffic signals a motor vehicle made over a certain distance in an urban area, by assuming that under free-flowing urban traffic conditions, the probability of being stopped at any given traffic signal was inversely proportional to the ratio of green time to cycle time (G/C). The probable number of stops per mile for various numbers of traffic signals and G/C ratios is shown in Figure 28. Of course, the number of interruptions by stop signs per mile must be added to the value in Figure 28 before the total number of stops by the motor vehicle can be estimated.

To illustrate the procedure for obtaining optimal speed on an urban street, assume that for a 1-mi section of the given street there are 4 intersections. One of these intersections is regulated by a stop sign and the other three by traffic signals having G/C ratios of 0.60, 0.50, and 0.40, respectively. The number of probable stops per mile caused by the three traffic signals with an average G/C ratio of 0.50 is 1.5 stops per mile (Fig. 28). After the extra stop for the stop-sign-controlled intersection is added, the total probable number of stops for the street is 2.5. The optimal speeds obtained are 36.4 and 35.2 mph, respectively, for passenger cars during daytime and nighttime travel (Figs. 26 and 27). Commercial vehicles for both daytime and nighttime travel have an optimal-speed value of 30.9 mph.

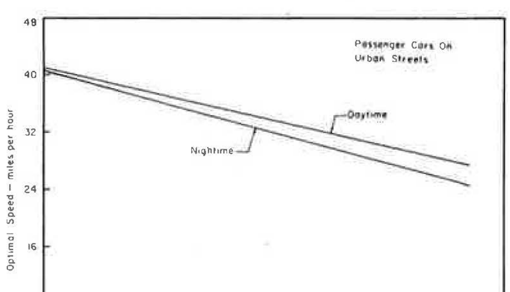

Figure 26. Optimal speed vs number of stops, passenger cars on urban streets.

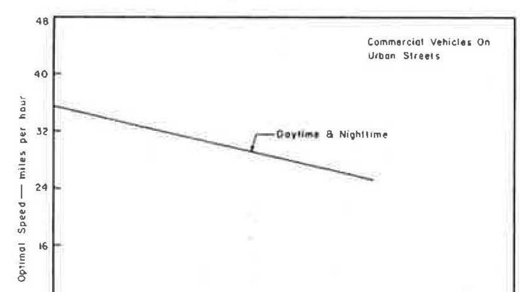

Figure 27. Optimal speed vs number of stops, commerical vehicles on urban streets.

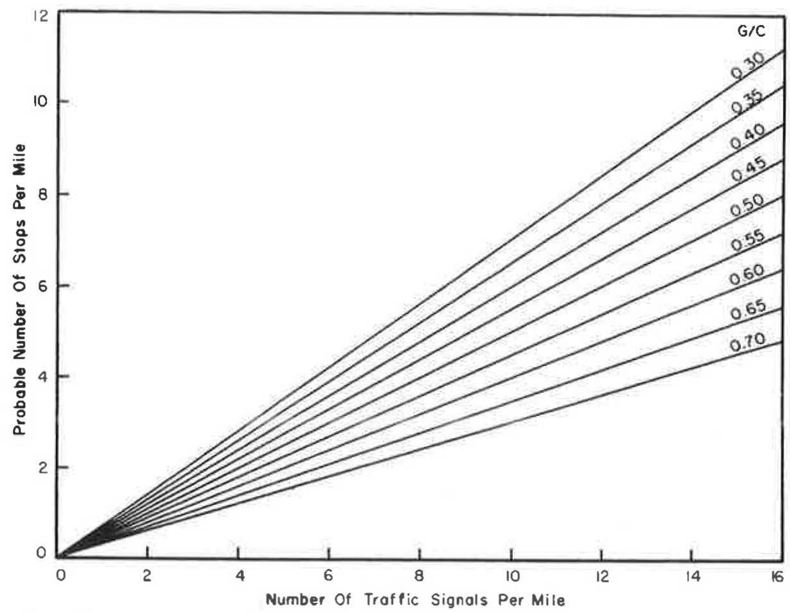

Figure 28. Probable number of stops vs number of traffic signals.

CONCLUSIONS

The most important conclusion that was drawn from this investigation is that a vehicle-operating speed does exist which minimizes cost of traffic movement for each of the various conditions considered. It is also concluded that differences up to 11 mph were observed between the optimal speeds of passenger cars and commercial vehicles, whereas there were lesser differences between optimal speeds on 2- and 4-lane rural highways and even smaller differences between daytime and nighttime optimal speeds.

Application of the results of this study, which are the consolidated results of many published articles on the subject of traffic movement costs, will probably be restricted to the establishment of statewide or areawide maximum or minimum speed limits. At this time, it is not possible to speed zone for specific locations using these results because the data necessary to make the analysis are not available for a micro analysis.

Data developed in this investigation can also be used to help complete data lacking in the present road-user benefit analyses. In the past, commercial vehicles and accident costs were omitted from the analyses because of the lack of available data. Highway engineers now have a broader knowledge of the actual benefits received by the road-user through highway improvements.

SUGGESTION FOR FURTHER RESEARCH

One difficulty encountered in this evaluation was the lack of adequate data on the values of time and accident involvement rates. An acceptable value of time for passenger car leisure trips should be ascertained and an urban accident study similar to the one conducted in rural areas by the Bureau of Public Roads should be initiated (32, 38). Driver comfort and convenience should also be studied to determine its proper place in appraising the actual cost of traffic movement. When more data become available, it is suggested that individual cost elements, along with total cost of traffic movement, be re-evaluated.

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REFERENCES

1. Beakey, John, and Crandall, F. B., "The Effect of Surface Type, Alignment and Traffic Congestion on Vehicular Fuel Consumption." Tech. Bull. No. 17, Oregon State Highway Dept. (1944).
2. Bevis, Howard W., "The Application of Benefit-Cost Ratios to an Expressway System." HRB Proc., 35:63-75 (1956).
3. Bellis, Wesley R., "Costs of Traffic Inefficiencies." Proc., Inst. of Traffic Eng., 29-32, (1952).
4. Bone, A. J., "Travel-Time and Gasoline-Consumption Studies in Boston." HRB Proc., 31:440-456 (1952).
5. Claffey, Paul J., "Time and Fuel Consumption for Highway-User Benefit Studies." Public Roads, 31:1, 16-21 (April 1960).
6. Committee on Planning and Design Policies, "Road User Benefit Analyses for Highway Improvements." AASHO (1952).
7. Dimmick, Thomas B., "Traffic and Travel Trends, 1956." Public Roads, 29:11, pp. 253-265 (Dec. 1957).
8. Evans, R. D., "Maximum Mileage Possibilities of Existing Tires and Available Tire-Building Materials." HRB Proc., 22:53-62 (1942).
9. Gibbons, John W., and Proctor, Albert, "Economic Costs of Traffic Congestion." HRB Bull. 86, 1-25 (1954).
10. Giffin, Harold W., "Some Observations on the Value of Time Saved to Motorists." HRB Proc., 28:53-56 (1948).
11. Green, Forest, "The Value of Time Saved by Commercial Vehicles as a Result of Highway Improvements." U. S. Bureau of Public Roads (unpublished).
12. Griffin, Ollis D., Private communication. (March 9, 1961).
13. Haikalis, George, and Hyman, Joseph, "Economic Evaluation of Traffic Networks." HRB Bull. 306, 39-63 (1961).
14. Hall, Edward M., and George, Stephen, Jr., "Travel Time—An Effective Measure of Congestion and Level of Service." HRB Proc., 38:511-529 (1959).
15. Hitchcock, S. T., Private communication. (Aug. 11, 1960).
16. Hitchcock, S. T., Private communication. (March 15, 1961).
17. Johannesson, S., "Cost of Traffic Delays." Civil Engineering, 3:3, 149-151 (March 1933).
18. Lane, Paul S., "Controlling Oil Consumption in Passenger Car Engines." SAE Jour., 58:11, pp. 18-22 (Nov. 1950).
19. Lawton, Lawrence, "Evaluating Highway Improvements on Mileage-and-Time-Cost Basis." Traffic Quarterly, IV:1, pp. 102-125 (Jan. 1950).
20. May, A. D., Jr., "Economics of Operation on Limited-Access Highways." HRB Bull. 107, 49-62 (1955).
21. McCarthy, James F., "Economic Cost of Traffic Accidents in Relation to the Vehicle." HRB Bull. 263, 23-39 (1960).
22. McCarthy, James F., "The Economic Cost of Traffic Accidents in Relation to the Vehicle." Public Roads, 31:2, 44-48 (June 1960).

23. McCullough, C. B., and Beakey, John, "The Economics of Highway Planning." Tech. Bull. No. 7, Oregon State Highway Dept. (1938).
24. Moyer, R. A., "Rural Mail Carrier Motor Vehicle Operating Costs on Various Types of Road Surfaces." HRB Proc., 17:53-78 (1937).
25. Moyer, R. A., and Tesdall, G. L., "Tire Wear and Cost on Selected Roadway Surfaces." Bull. 161, Iowa Eng. Exp. Sta. XLIV:9 (1945).
26. "Accident Facts." National Safety Council (1960).
27. Oppenlander, J. C., "Analysis of Highway Motor Vehicle Operation Costs." Univ. of Illinois (August 1959, unpublished).
28. Oppenlander, J. C., "A Theory on Vehicular Speed Regulation." Univ. of Illinois (June 1960, unpublished).
29. Oppenlander, J. C., Bunte, W. F., and Kadakia, P. L., "Sample Size Requirements for Vehicular Speed Studies." HRB Bull. 281, 68-86 (1961).
30. Roden, Norman C., "A Determination of the Optimum Operating Speed for Passenger Cars on Rural Highways." Univ. of Illinois, Thesis (1960).
31. Sawhill, Roy B., and Firey, J. C., "Motor Transport Fuel Consumption Rates and Travel Time." HRB Bull. 276:35-91 (1960).
32. Solomon, David, "Speed and Accidents on Rural Highways in the United States of America." Fifth International Study Week in Traffic Engineering, London, World Touring and Automobile Organization (1960).
33. "Traffic Safety Memo No. 113." National Safety Council (July 1960).
34. Tucker, Harry, and Leager, Marc C., "Highway Economics." International Textbook Co. (1942).
35. Twombly, Bernard B., "Economic Cost of Traffic Accidents in Relation to the Highway." HRB Bull. 263, 1-22 (1960).
36. Twombly, Bernard B., "The Economic Cost of Traffic Accidents in Relation to the Highway Systems." Public Roads, 31:2, 39-43 (June 1960).
37. U. S. Bureau of Public Roads. Table VM-1 (1958).
38. U. S. 86th Congress, 1st Session, "The Federal Role in Highway Safety." H. D. No. 93 (1959).
39. Washington State Highway Commission, "Washington Motor Vehicle Operating Cost." (1952-1953).
40. Wiley, T. T., "Cost of Passenger Automobile Stops." Civil Engineering, 5:5, 284-286 (May 1935).
41. Winfrey, Robley, "Automobile Operating Cost and Mileage Studies." Bull. 106, Iowa Eng. Exp. Sta., XXX:8 (1931).
42. Winfrey, Robley, "Highway Economics." Highway Engineering Handbook, McGraw-Hill (1960).
43. Winfrey, Robley, "Statistics of Motor Truck Operation in Iowa." Bull. 114, Iowa Eng. Exp. Sta., XXXI:52 (1933).

Multivariate Analysis of Vehicular Speeds

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This investigation was concerned with the multivariate analysis of vehicular speeds on 2-lane rural highways. The objective was to gain a better understanding of traffic-stream characteristics through the formulation and evaluation of mathematical models describing the time-rate of traffic flow.

The concept of traffic flow was founded on the assumption that vehicular speeds are a function of various travel conditions present in and adjacent to the traffic stream. Thus, various types and levels of travel features produce different average spot speeds. The total effect of these resistance variables determines the speed characteristics of a particular highway location. Average speed was qualitatively described as a hyperplane to account for the many variables that significantly influence vehicular speeds.

The description of this traffic-flow concept in the language of mathematics was accomplished through the development of multiple linear regression equations. Two mathematical models were devised to relate mean spot speed as a function of generated factors in one equation and of travel-restriction variables in the other expression. These regression models were restricted by the assumption of linearity in both the variables and the parameters.

To evaluate and verify the proposed mathematical relationships, it was necessary to collect sufficient data on many variables influencing vehicular speeds. Directional traffic flow on 2-lane rural highways was studied with measurements of 49 variables representing various driver, vehicle, roadway, traffic, and environmental conditions. Study locations in Illinois were operated at random during the months of June through August 1961. Multivariate analysis of mean values of the selected variables was accomplished by computer programs for factor analysis and multiple linear regression and correlation analysis.

A verification study performed on ten study sites indicated that two multiple linear regression equations can be used to estimate mean spot speeds with a reasonable degree of confidence.

•THE MOVEMENT of people and goods on the highway and street systems is a segment of the transportation industry that consumes considerable national resources, both natural and human. Highway transportation produces the desired place utility, within the limitations of the specified time utility, for many persons and large quantities of various commodities.

The general functions of highway facilities are to provide for the expedient movement of relatively large volumes of motor vehicles, to furnish access to various forms of land use, and to serve as routes on which certain drivers and passengers desire to

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travel. It is essential that these purposes are accomplished with a high degree of safety and at acceptable levels of comfort and convenience. Vehicular speed is an important consideration in highway transportation because the rate of vehicle movement has significant economic, safety, time, and service (comfort and convenience) implications to both the motoring and the general public (16). To this end, effective operation and control of vehicular traffic on various highways and streets have often necessitated the regulation of motor-vehicle speeds.

When the speed of motor-vehicle operation is not limited by any driver, vehicle, roadway, traffic, and environmental (considered collectively as travel) characteristics, the individual driver is free to select his desired rate of travel. A wide speed range results when conditions permit road users to select their own speeds (10). The vehicle operator, however, must often drive under travel conditions that restrict his freedom of operation. Numerous investigations have concluded that various restrictions on the flow of traffic produce spot-speed characteristics of the non-free-flowing traffic that are less than those corresponding speed values indicative of free-flowing conditions (14). In many instances, the relationship between the average spot speed and the increasing measure of a specific traffic-stream restriction, with all other variables constant, is approximately a straight line with a negative slope. These functional relations are valid only for the constant conditions of the fixed parameters.

No comprehensive analysis has been performed to permit the correlation of average time-mean speeds on a given traffic facility with various travel conditions because:

1. Many variables may have a significant influence on vehicular speeds;
2. The amount of this effect on the rate of travel may depend on both the character and the magnitude of various restrictions;
3. Different combinations of travel conditions may produce modifications that vary from the composite result obtained from the individual evaluation of each restriction (known as interaction); and
4. Some variables may be present with varying degrees of significance at different levels of traffic operation.

It is readily apparent that a realistic evaluation of spot-speed characteristics necessitates the appraisal of those driver, vehicle, roadway, traffic, and environmental conditions that determine or significantly modify vehicular speeds.

The purposes of this investigation were to develop a conceptual theory on the time-rate of traffic flow under various types and levels of travel conditions, to formulate mathematical models representative of this theory, to evaluate quantitatively these mathematical expressions through designed experiments, and to test the validity of these functional models.

Solutions to mathematical models for various highway types (2-lane, 3-lane, and multi-lane) in general traffic areas (business, residential, intermediate, and rural) permit:

1. The reliable estimation of vehicular-speed characteristics on any existing highway or street location;
2. The determination of the influence on traffic operation occasioned by proposed improvements in existing roadway and/or traffic features (improved alinement, adequate lateral clearance, access control, traffic-control device, etc.);
3. The evaluation of the design of new highway facilities for expected operational conditions;
4. The development of reasonable speed regulations for adverse roadway and/or environmental characteristics (restricted sight distance, road roughness, limited visibility, icy pavement, etc);
5. The ascertainment of advisory speed limits where roadway and/or traffic conditions govern the rate of vehicular movement (horizontal alinement, roadside development, traffic volume, percentage of commercial vehicles, etc.); and
6. The accurate simulation of traffic-stream characteristics in highway planning studies incorporating traffic-flow model analysis.

Both theoretical and practical considerations, in their proper perspective, were applied for a scientific approach to the analysis of the speeds at which drivers operate their motor vehicles on roadways under prevailing traffic and environmental conditions. Thus, a better understanding of traffic-stream characteristics has been gained through the comprehensive appraisal and evaluation of vehicular speeds under actual travel conditions.

REVIEW OF LITERATURE

The subject of vehicular speeds has occupied a prominent position in the literature of highway and traffic engineering. Numerous quantitative and qualitative analyses, appraisals, and discussions have appeared in the literature to provide a better understanding of vehicular-speed characteristics (15). In many problems encountered in the planning, design, and operation of a highway transportation system, a knowledge of the characteristics of motor-vehicle speeds is imperative if sound engineering decisions are to be realized.

Vehicular Speed

This review was confined to those articles on vehicular-speed characteristics that appeared to be a definite contribution to the disciplines of highway and traffic engineering. Although many publications on vehicular speed were reviewed, only those articles that were pertinent to the subject of this investigation were used.

To execute their role of providing highway transportation services of high quality at a minimum cost, highway and traffic engineers require a knowledge of spot-speed characteristics at many highway and street locations. The requisites for data on spot speeds can be briefly summarized with the following examples: speed-trend studies, analysis of problem locations, determination of traffic controls, geometric design applications, highway and traffic planning considerations, studies of traffic-stream characteristics, guides in enforcement, before-and-after studies, and traffic safety investigations.

Speed Fundamentals.—The classic definition of vehicular speed is "the rate of movement of traffic, or of specified components of traffic, expressed in miles per hour" (10). In reality, the velocity (a vector) of a motor vehicle is the ratio of its displacement (a vector) to the time interval (a scalar) in which the displacement occurred. The traffic engineer, however, is generally only interested in the magnitude and not the direction of the velocity vector. As a result, the speed (a scalar) of a moving vehicle is defined as the ratio of the length of traveled path (a scalar) to the elapsed time (a scalar). Speeds and not velocities of highway traffic were measured and analyzed in this study.

From the definitation of vehicular speed, it is readily evident that two distinct types of speed measures can be derived to express the rate of traffic movement. The first type of speed is time-mean speed or spot speed, which is the instantaneous speed of a motor vehicle at some given location on a roadway. Time-mean speed is generally calculated as the average of several spot-speed observations at the particular highway location; that is, time-mean speed = $\sum_{1}^{n} S_i/n$, in which S_i = spot speed of the i-th vehicle and n = number of vehicles that comprise the sample of speed observations.

The other evaluation of speed is space-mean speed or travel speed, which is the speed over a specified section of highway. Space-mean speed is computed as the specified travel distance divided by the average time of several trips over this highway section; that is, space-mean speed = $dn/\sum_{1}^{n} t_i$, in which d = travel distance, n = number of trips that comprise the sample of time observations, and t_i = travel time of the i-th trip. It is apparent that time-mean speed is usually greater than space-mean speed because travel time includes stops and delays as well as running time. Space-mean speed approaches time-mean speed as a limit when the time for stops and delays approaches zero. Space-mean speeds are a function of the density of vehicles on the highway, whereas time-mean speeds are related only to the number of vehicles passing a given point on the roadway.

Spot-Speed Studies.—The exact details of conducting a spot-speed study, such as location and time of the study, types of vehicles observed, and sample size, are predicated on the purpose of the survey and on the desired precision of the speed statistics. However, the basic purpose of the spot-speed study is to estimate the instantaneous-speed distribution of motor vehicles passing a particular roadway location under conditions prevailing at the time of the study.

Two basic techniques are utilized for the measurement of motor-vehicle spot speeds. One procedure involves a determination of an instantaneous speed; the other technique approximates an instantaneous speed by measuring the time required for the observed vehicle to traverse a short measured course.

The former method uses a radar meter which operates on the Doppler principle: a radio wave reflected from a moving target has its frequency changed in proportion to the speed of the vehicle (19). Because of convenience, ease of operation, and high degree of accuracy, the radar meter is presently being used by many highway departments, traffic engineering units, law enforcement agencies, and research groups in the measurement of vehicular speeds.

The other method of ascertaining spot speeds is to measure the time interval required by a vehicle to travel between two points separated by a known distance (13). Techniques of timing vehicles over short distances have been widely employed by highway and traffic engineers in establishing the distributions of vehicular speeds at highway and street locations.

Analyses of Spot-Speed Data.—Data collected in a spot-speed study constitute an array of speed values describing the rates of traffic movement under conditions present at the time of the survey. As the results of many field studies have illustrated, the frequency distribution of spot-speed data very closely approximates the normal curve. Several authors report the results of using the following statistical tests to verify a normal distribution in the spot-speed population: chi-square test, moment test, percentile method for testing normality, and normality testing using probability paper (1, 7, 18). All these statistical techniques indicate that spot-speed data significantly conform to a normal distribution. It may be inferred that the population of vehicular spot speeds and any representative and random sample of this population have a normal probability distribution. Therefore, three important characteristics of spot-speed data are apparent: the central tendency is described by the arithmetic mean, the variability of speed data is measured by the standard deviation, and the shape of the frequency distribution is accurately represented by the normal curve.

The equation for the normal probability curve depends on two values: population mean and population standard deviation. Because sample mean and sample standard deviation are unbiased estimates of their respective population counterparts, the distribution of spot speeds at a specific roadway location can be described by two values, the mean and the standard deviation of the spot-speed sample data.

Other properties or measures of the frequency distribution of spot speeds are of special significance in speed regulation, as well as in determination of design speed (14). Among these descriptive devices are various percentile values (such as 85th-percentile speed, and 15th-percentile speed), mode, median, pace, frequency-distribution curve, and cumulative-frequency curve (13).

Variables Influencing Spot-Speed Characteristics.—Spot speeds of motor-vehicle traffic are affected by many conditions or variables present at the instant when the speed of an individual vehicle is observed at a particular roadway location. The number of possible items that have significant influences on vehicular speeds is exceedingly large; however, the literature contains results of studies on spot-speed characteristics for only a few conditions. The selection of variables that significantly modify vehicular spot speeds was, no doubt, predicated on a priori considerations of traffic-stream characteristics (17).

Driver.—The subject of road-user influence on spot-speed characteristics assumes a minor role in the research activities of highway and traffic engineers. The absence of literature in this specialized field is probably due to lack of interest on the part of psychologists and to engineers' deficiency in an understanding and knowledge of the discipline of psychology.

In summarizing driver variables, trip distance has the most significant influence on spot-speed characteristics, while passengers in the car and the sex of the driver alter driving speeds to a lesser extent. From the discontinuities evident in the literature on driver characteristics, it is reasonable to assume that driver variables influence vehicular speeds to different degrees in various parts of the country.

Vehicle.—The influence of the motor vehicle on spot-speed characteristics has been limited to consideration of several variables. These vehicle variables were normally features of the automobile or commercial vehicle that were readily observable or easily measured. Most articles on this subject have been written by highway and traffic engineers. Automotive engineers apparently devote little time and attention to the performance of vehicles and their respective characteristics under actual travel conditions.

Type of vehicle (passenger car, single-unit truck, combination truck, or bus) and age of the vehicle appear to have predominant effects on spot speeds of highway motor vehicles. A further subdivision of single-unit trucks and combination trucks by gross weight is feasible in evaluating spot-speed characteristics.

Roadway.—Actual speeds adopted by motor-vehicle operators are greatly affected by various aspects of the roadway. Different rates of travel result from the driver's attempt to evaluate roadway conditions in order to select a safe speed. Numerous engineering surveys have been conducted to evaluate quantitatively the influences of roadway features on spot-speed characteristics.

In a recapitulation of roadway characteristics, vehicular spot speeds are most significantly influenced by functional classification, curvature, gradient, length of grade, number of lanes, and surface type. Other elements of interest are geographic location, sight distance, lane position, lateral clearance, and frequency of intersections.

Traffic.—Vehicular speeds are controlled to various degrees by actual characteristics of traffic streams and by operational techniques and devices designed to regulate traffic flows. Considerable attention has been devoted to this subject of highway research.

Vehicle volume and traffic density exert pronounced influences on spot-speed characteristics. Percentage of commercial vehicles, passing maneuvers, opposing traffic, and access control are also important variables that should be considered in evaluation of traffic-stream characteristics.

Environment.—The operation of motor vehicles on highways is subject to various influencing conditions that are cyclic or random in occurrence. These variables are independent of the driver, vehicle, roadway, and traffic elements previously discussed and are presented under the general classification of environment. Little attention has been devoted to research on environmental variables because they are difficult to control and to express in terms of quantitative measures.

Environmental variables consisting of time and weather present important considerations that must be appraised in the actual evaluation of spot-speed characteristics.

Although many studies have been conducted to assess the influences of various travel conditions on vehicular speeds, few investigators have applied the techniques of statistical inference in the evaluation of their experimental findings.

Multivariate Analysis Techniques

Several statistical techniques are available for analyzing the relationships among many variables. Selection of the proper statistical tool is predicated on the nature and purpose of the investigation. The mathematical models developed in this qualitative explanation and quantitative evaluation of vehicular speeds were representative of equations for multiple estimating. Solutions to these equations were found through the application of factor analysis and multiple linear regression and correlation analysis to experimental data.

The remaining sections of this literature review were limited to brief descriptions of factor analysis and multiple linear regression and correlation analysis and to discussions of highway and traffic engineering studies that have employed these statistical procedures.

Factor Analysis.—Factor analysis has been used as an analytical technique mainly by behavioral scientists. The principal concern of factor analysis is to resolve a set of variables linearly into a smaller number of factors; that is, to attain a parsimonious description of observed data. As a result, factor analysis permits a simple interpretation of a given array of data and affords a fundamental description of the particular set of variables analyzed (8). Theories and methods of factor analysis are fully presented in textbooks (5, 8).

Goldstein and Mosel (6) conducted an inventory study in 1955 to determine the factors underlying drivers' attitudes. Factor-analysis procedures were applied to a 186-item attitude inventory designed to measure 14 aspects of driver attitudes, which were considered to cover the domain. This set of variables was reduced to five factors: attitude toward competitive speed, attitude toward other users of the roadway, attitude toward policemen, attitude toward the vehicle, and a general attitude of care or concern for safety. The implications of these findings are discussed under the consideration of driver characteristics. Another factor-analytic investigation was an evaluation of roadway and accident data (20). In an attempt to explain the occurrence of traffic accidents on 2-lane rural highways, factor-analysis solutions were applied to the number of accidents and to 13 measurable roadway variables. These variables were generated into four factors identified as capacity, traffic conflict, modern roads, and roadside structures.

Multiple Linear Regression and Correlation Analysis.—Regression and correlation analysis has been employed by various researchers in different disciplines to evaluate the functional relationships and the significance of these relationships among experimental variables, although statistical procedures have infrequently been applied in engineering investigations. Regression analysis is concerned with the establishment of numerical relationships between study variables, whereas the measurement of the degree of relationship established between the variables under consideration is known as correlation analysis (4). The regression techniques used in this investigation of traffic-stream characteristics were limited to the derivation of multiple linear regression equations. This is a first-degree equation representing the relationship between a single dependent variable and two or more independent variables. The subject of multiple linear regression and correlation analysis is completely described in textbooks on statistics (2, 3).

The Chicago Area Transportation Study in 1958 applied multiple correlation analysis to ascertain the functional relationships between average overall travel speed and 13 independent variables reasoned to have significant influence on traffic operation (12). Multiple linear equations were developed, with varying degrees of success, for vehicle travel on expressways, rural highways, and urban streets with and without parking. In 1960, multiple linear regression expressions were derived to relate the influences of five independent variables, which were believed to be important elements of roadway friction, on the criterion variables of travel time and fuel consumption for traffic movement on urban facilities (11).

It is evident that these multivariate analysis techniques have received little application in the evaluation of traffic-stream characteristics, which are influenced by the presence and action of many variables.

THEORY

The subject of traffic-flow theory has been formulated only to a very limited extent. This lack of application of theoretical considerations to traffic movement is largely explained by the extreme complexity of vehicular traffic and traffic problems, by the concentration of technical efforts to upgrade quickly an inadequate highway transportation system, and by the general absence of research and engineering personnel concerned mainly with developing the theory of traffic flow.

Various theoretical concepts have been expressed as mathematical models to describe the complex phenomenon of traffic flow. These methods of quantitatively depicting traffic flow are classified under the following general approaches: statistical models, car-following concept, queueing theory, traffic-network analyses, computingmachine simulation studies, mathematical experiments, and distribution-function theories (9).

The motion of vehicular traffic is not only governed externally by the physical laws of nature, but it is further complicated internally by driver behavior. Thus, the theory of traffic flow must evolve from the combined application of the knowledge afforded by both human-behavioral and physical sciences to the man-machine system of highway transportation.

Because the a priori knowledge of the theory of traffic flow is rather limited, considerable understanding of traffic-stream characteristics can be gained through the study of statistical models. The following presents the development of a conceptual model that describes the speed of vehicular movement and the formulation of this theory into mathematical models that may be evaluated by statistical techniques.

Conceptual Model

The concept of time-rate of traffic flow was predicated on the assumption that vehicular speeds are a function of various travel conditions present in and adjacent to the traffic stream. As a result, different average spot speeds ensue under various types and levels of travel features. The sum effect of these resistance variables produces the speed characteristics peculiar to a given highway or street location.

When no restrictions are present to impede traffic flow, high average speeds result. As the magnitude of influence of restrictive variables increases, the net effect is to reduce the average rate of vehicle movement. This concept is graphically described by the simplified traffic-flow model (Fig. 1) where the fundamental speed-volume relation is depicted as a closed region delimited by upper and lower boundaries. The upper limit is indicative of free-flowing traffic, and any modifications in spot-speed characteristics are occasioned only by traffic volumes. Congested conditions of traffic-stream operation are represented by the lower limit, which is essential to the preservation of the continuity of traffic movement; that is, the equality (volume is the product of speed and density) must be maintained.

This simplified model is restricted only by a single variable. A particular value of this impedance to the rate of traffic flow is represented by one of the dashed lines (Fig. 1). At a specific traffic volume, the average vehicular speed for a particular type and level of traffic-flow restriction is represented by a point falling on or within the boundaries of the region.

To simulate the various conditions of actual traffic flow, this conceptual model was expanded to incorporate those travel conditions that significantly influence vehicular speeds. Average speed is now represented by a hyperplane of class one in n + 1 dimensional space, where n is the number of variables restricting the rate of traffic movement. In addition to the two limits specified in the simplified model, boundaries were formed by the inequalities indicative of the range in actual values that the various travel restrictions can possibly assume. These limits were designated as hyperplanes of class one in n + 1 dimensions. Intersections of limiting hyperplanes with the speed hyperplane form boundary hyperplanes of class two in n dimensions. This generalization produced a polyhedron as the geometric representation of the comprehensive traffic-flow model. For a particular combination of driver, vehicle, roadway, traffic, and environmental conditions, the average spot speed is defined by a point within or on the surface of this closed space.

Mathematical Models

To evaluate the comprehensive traffic-flow model in the language of mathematics, it was necessary to describe this concept in terms of mathematical models. The conceptual model can be summarized as the statistical estimation of the functional relationships between mean speeds and various driver, vehicle, roadway, traffic, and environmental variables that significantly control these rates of traffic movement. The description of this concept as a mathematical model for the estimation of vehicular speeds suggested equations for multiple estimating.

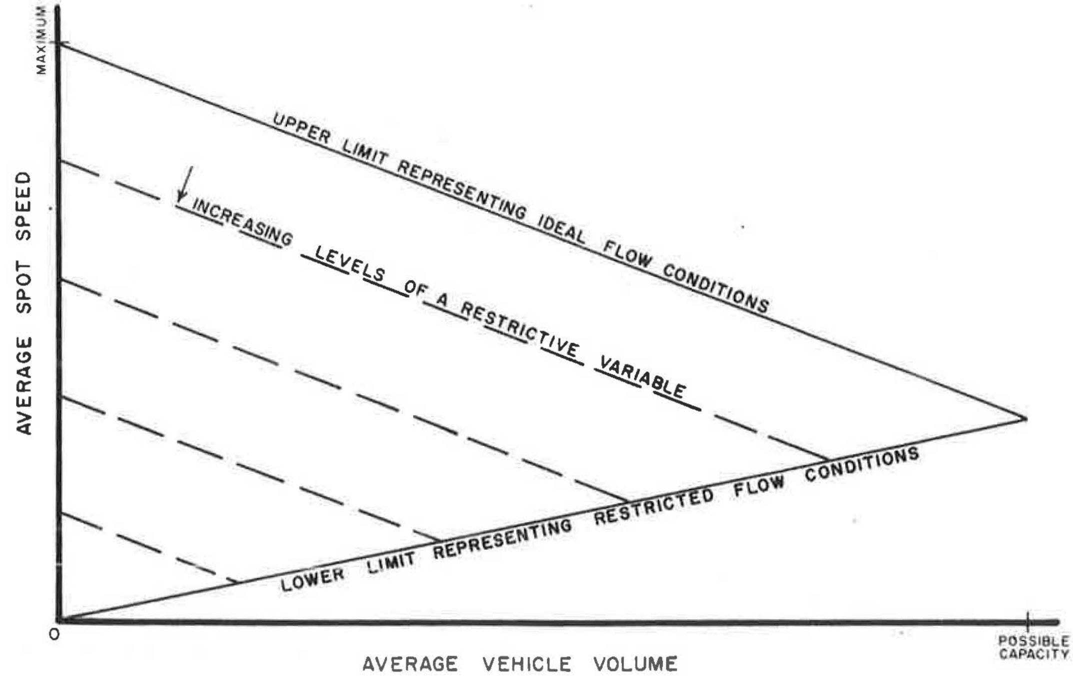

Figure 1. Schematic representation of the simplified traffic-flow model.

This inference problem was best formulated as multiple linear regression equations. The selection of first-degree expressions was based on the many linear relationships existing between average spot speeds and various travel restrictions (17). The two forms of multiple linear regression equations postulated in this multivariate analysis of vehicular speeds are

$$S_1 = \bar{S} + s\,(c_1 F_1 + c_2 \cdot F_2 + \ldots + c_m F_m + c\,U) \qquad (1)$$

in which

S_1 = mean spot speed;
$\bar{S}$ = grand mean of spot speeds;
s = standard deviation of spot speeds;
c_j = common factor coefficient ($j = 1, 2, \ldots, m$);
$F_j = \sum_{1}^{n} e_{ij} Z_i + K_j$ = common factor ($i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$);
e_{ij} = standard regression coefficient for j-th factor score ($i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$);
Z_i = independent variable ($i = 1, 2, \ldots, n$);
K_j = residual variable for j-th factor score ($j = 1, 2, \ldots, m$);
c = unique factor coefficient;
U = unique factor;
m = number of common factors; and
n = number of independent variables.

$$S_2 = a + b_1 X_1 + b_2 X_2 + \ldots + b_n X_n + Q \qquad (2)$$

in which

S_2 = mean spot speed;
a = intercept;
b_i = net regression coefficient ($i = 1, 2, \ldots, n$);
X_i = independent variable ($i = 1, 2, \ldots, n$);
Q = residual variable; and
n = number of independent variables.

The mean spot speeds in these regression models are defined as the dependent variables which are functionally related to generated factors in the first equation and to travel-restriction variables in the second relationship. In addition, these mathematical models are founded on the assumption that the relation of the dependent variable to each independent variable is linear.

Although the limiting conditions are unimportant in this inference problem, the following inequalities were specified as limiting hyperplanes of class one to complete the mathematical description of the traffic-flow concept:

$$S_{max} = a + \sum_{1}^{n} b_i X_i + Q \qquad (3)$$

$$S_{min} = V/D \qquad (4)$$

$$g_i \leq X_i \leq h_i \qquad (5)$$

in which

S_{max} = maximum mean spot speed for ideal travel conditions;
a = intercept;
b_i = net regression coefficient ($i = 1, 2, \ldots, n$);
X_i = independent variable ($i = 1, 2, \ldots, n$);
Q = residual variable;
S_{min} = minimum mean spot speed for continuity of flow;
V = traffic volume;
D = traffic density;
g_i = lower limit of i-th independent variable ($i = 1, 2, \ldots, n$);
h_i = upper limit of i-th independent variable ($i = 1, 2, \ldots, n$); and
n = number of independent variables.

Thus, the hyperplane of class one representing mean spot speed is limited by these boundary hyperplanes of class one.

A concept of the rate of traffic flow has been expressed in terms of two mathematical models in the form of multiple regression equations. These expressions are restricted by the assumption of linearity in both the parameters and the variables.

PROCEDURE

In addition to the theoretical considerations this study was concerned with the quantitative evaluation of the proposed regression models and with the validity of these mathematical expressions as equations for estimation. This part of the report describes the experimental design, the conduct of studies, and the data analysis that were necessary for accomplishing the evaluation and verification of the regression equations.

A priori considerations of traffic-stream characteristics indicated that these mathematical models should be solved for each type of highway or street (2-lane and multilane) in the different traffic areas (business, residential, intermediate, and rural). This statistical analysis was limited to vehicular movement on 2-lane rural highways.

A rural area was defined as any area where the number of residential, commercial, and industrial buildings along the highway was less than 10 per mile and where the number of crossroads and driveways was less than 20 per mile. The minimum length of highway considered was 1 mile.

Design of Experiment

To evaluate and verify the proposed regression models, it was necessary to collect data on many variables deemed to have a significant influence on vehicular speeds. Determination of the study variables was predicated on information afforded by the literature review and on available personnel, equipment, and time. The following variables were measured in evaluating directional traffic flow on 2-lane, rural highways located throughout Illinois:

1. Female driving passenger car, percent;
2. Out-of-state passenger car, percent;
3. Passenger-car driver accompanied by one or more passengers, percent;
4. Light truck (two axles with single tires), percent;
5. Single-unit truck (two or more axles with dual tires on one or more axles), percent;
6. Truck combination (tractor with one or more trailers), percent;
7. Commercial bus, percent;
8. Degree of curve, deg;
9. Total central angle per mile (measured for 1 mile in advance of the speed site), deg per mile;
10. Rate of superelevation, ft per ft;
11. Reciprocal of test-car speed, mph^{-1};
12. Gradient, percent;
13. Length of grade (measured from the PI to the speed site), ft;
14. Total algebraic rise and fall per mile (measured for 1 mile in advance of the speed site), ft per mile;
15. Minimum sight distance, ft;
16. Lane width, ft;
17. Shoulder width, ft;
18. Presence of curb or gutter;
19. Number of commercial roadside establishments, such as restaurants, service stations, motels, and taverns, per mile (counted on both sides of the roadway for ½ mile before and ½ mile beyond the speed site), no. per mile;
20. Number of friction points, including at-grade intersections, at-grade railroad crossings, pedestrian crossings, and school crossings, per mile (counted for ½ mile before and ½ mile beyond the speed site), no. per mile;
21. Number of access points, including all intersections, driveways, and other points of access to various forms of land use, per mile (counted on both sides of the roadway for ½ mile before and ½ mile beyond the speed site), no. per mile;
22. Total traffic volume, vph;
23. Mean headway, sec;
24. Vehicle in platoon at time of speed measurement, percent;
25. Vehicle passed other vehicles at time of speed measurement, percent;
26. Vehicle met opposing traffic at time of speed measurement, percent;
27. Opposing traffic volume, vph;
28. Directional distribution (observed volume divided by total volume), percent;
29. Minimum speed limit (regulatory or advisory), mph;
30. Presence of centerline pavement markings that were obviously visible to drivers;
31. Presence of no-passing-zone pavement markings that were applicable and obviously visible to drivers;
32. Presence of edge-line pavement markings that were applicable and obviously visible to drivers;
33. Accident rate for 1960 (total number of fatal, personal-injury, and property-damage-only accidents counted for ½ mile before and ½ mile beyond the speed site), no. per mi per yr;

34. Monday;
35. Tuesday;
36. Wednesday;
37. Thursday;
38. Friday;
39. 7:31 AM to 10:00 AM;
40. 10:01 AM to 12:30 PM;
41. 12:31 PM to 3:00 PM;
42. 3:01 PM to 5:30 PM;
43. Clear;
44. Cloudy;
45. Drizzle;
46. Rain;
47. Wet pavement;
48. Presence of large advertising signs; and
49. Mean spot speed, mph.

Except for certain specified items, these variables were applicable only for traffic in the observed direction of travel; that is, only one-way traffic movement was evaluated.

Because of the many variables involved and the lack of experimental control over certain variables, it was difficult to develop speed-site criteria that included the range of values indicative of each modifying condition. Criteria on traffic volume, degree of curve, gradient, and lane width were specified to permit a partial factorial design in the selection of study sites. In addition, the following stipulations were observed in the location of speed sites:

1. Each study site was as homogeneous as possible for a distance of ½ mile before and ½ mile beyond the site;
2. A speed site on a horizontal curve was located near the center of the curve;
3. A study location was selected near the middle of a grade having a minimum length of 400 ft;
4. Shoulder width ranged from 0 to 20 ft;
5. Locations with and without curbs or gutters were chosen;
6. Roadside development ranged from 0 to 10 buildings per mile;
7. Speed sites had regulatory speed limits ranging from 35 to 65 mph in 5-mph increments;
8. Sites were selected with and without centerline, no-passing-zone, and edge-line pavement markings; and
9. Study locations with and without large advertising signs were specified.

A total of 469 study sites provided a random sample of traffic flow on 2-lane, rural highways in Illinois for the evaluation of the postulated mathematical models. Ten additional speed sites were studied to obtain data for verification of the multiple linear regression equations.

Conduct of Studies

The actual collection of the data at these 479 locations involved both field and office studies. Certain measurements were obtained from the drawings and records available in the district offices of the Illinois Division of Highways. The remaining items were measured through the conduct of field studies. Study locations were operated randomly in regard to time of day and day of week during the months of June, July, and August 1961. Observations were not taken at nighttime and on Saturdays, Sundays, or holidays.

Each study was limited to one direction of travel. The radar speedmeter was located adjacent to the lane of traffic being studied and was pointed toward the oncoming vehicles. The speedmeter and observers were concealed from the view of approaching drivers. Because it was necessary to record the amount of traffic traveling in the opposite direction during the study, a recording traffic counter was placed near the site. The counter was always located far enough beyond the speed site so that drivers, traveling in the direction under observation, did not notice it until after their speeds were measured.

A sample size of 150 vehicles was computed to be sufficient for the evaluation of traffic-flow conditions at each location (18). Information was recorded for all motor vehicles except school buses, motorcycles, and motor scooters. To collect the desired data for each element of the sample, two observers were required for the operation of a study site. Observer 1 was charged with recording the speed and the traffic conditions present for each observed vehicle. Characteristics of the vehicle and its occupants were noted by observer 2. The matching of each vehicle was controlled by having each observer record whether or not the vehicle was a commercial truck or bus.

Analysis of Data

The data were processed to evaluate the proposed mathematical models. Figure 2 summarizes the analytic procedure.

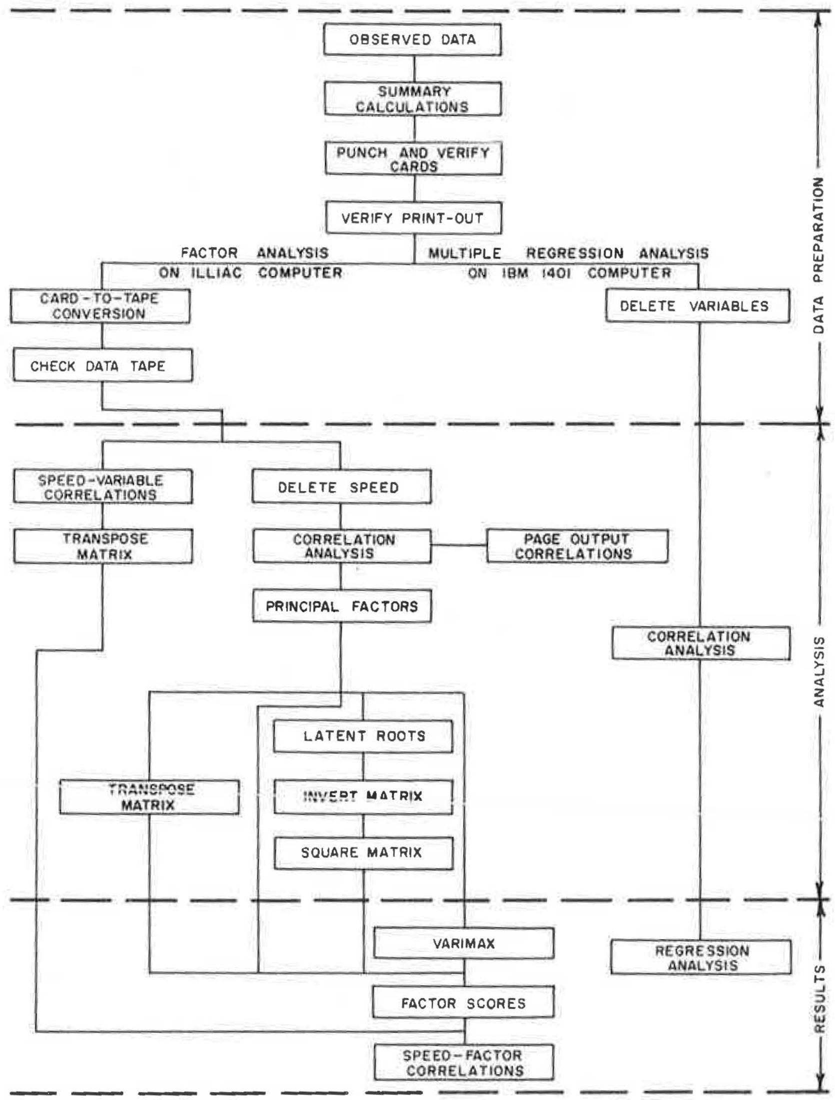

Figure 2. Flow diagram for data processing.

Mean values for each study were calculated for those variables that were a function of the observed traffic. These average measures and the remaining variables that were constant during the study were coded and verified on IBM punch cards. Further verification of the punch cards was accomplished by comparing printouts with the original data sheets.

The factor analysis was first performed to provide an exploratory appraisal of traffic-stream characteristics. A card-to-tape converter was employed to prepare a data tape as input for the University of Illinois Illiac computer. Accuracy of this original tape was confirmed by a data-tape checking routine performed on this digital computer (22).

Because speed was not included in the generation of factors, a revised data tape was prepared on this computer with the speed column deleted from the data matrix (23). The input requirements for the factor-analysis problem necessitated the calculation of a product-moment correlation matrix for the remaining 48 variables, with means and standard deviations computed as additional output (29). These descriptive statistics are given in Table 6 (Appendix). A computer routine (27) afforded the convenient page outputs of these correlation coefficients (Table 8, Appendix).

Orthogonal factors were obtained by the principal-axes solution performed on the Illiac computer (28). Points of the study variables are represented in n dimensional space and are contained in a common-factor space of only m dimensions. For normal populations the loci of these points of uniform frequency density are concentric, similar, and similarly situated ellipsoids having m dimensions. The axes of these ellipsoids correspond to the factors generated in the principal-factor solution. These axes are selected by choosing a set of factors in decreasing order of their contribution to the total communality. Generation of the first factor is accomplished by a successive-approximation solution of the following relationship expressed in matrix notation:

$$R\, p_1 = y_1\, p_1 \qquad (6)$$

in which

R = original correlation matrix;
p_1 = column vector of coefficients for the first factor; and
y_1 = eigenvalue of R for the first factor.

The first factor is determined by this iterative procedure so that its contribution to the communalities of the variables is the maximum possible; that is, maximization of the expression

$$y_1 = \sum_{1}^{n} p_{i1}^2 \qquad (7)$$

Thus, first-factor coefficients in the factor pattern are calculated from the largest latent root to account for the maximum amount of the total communality. After this operation is completed, the first-factor residual correlations are obtained according to the following matrix equation:

$$R_1 = R - p_1\, p_1' \qquad (8)$$

in which

R_1 = matrix of first-factor residual correlations;
R = original correlation matrix; and
p_1 = column vector of coefficients for the first factor.

By repetition of this iterative analysis on the residual-correlation matrix, the second factor found is independent of the first factor and provides the maximum contribution to the residual communality. This technique is continued until the contribution of additional factors to an explanation of the variable variance is negligible (8). Thus, the original correlation matrix was reduced to a factor matrix that is contained in a space of smaller dimension than the number of variables.

Determination of the common-factor space is not dependent on the selected coordinate frame of reference. Therefore, it is desirable to transform the coordinate axes to facilitate interpretation of the factor solution. This rotation of axes was accomplished on the Illiac computer by the varimax method (31). The varimax-rotation concept is founded on the principle that the factor has the greatest interpretability when the variance is a maximum. This transformation is accomplished by extending the vectors representing the variables to unit length in the common-factor space, carrying out the rotations to maximize the variance, and then bringing the vectors back to their original length. This multiple-factor solution ideally affords both simple structure and factorial invariance (8).

The next step in the analysis was the evaluation of factor scores; that is, coefficients were developed to express the common factors in terms of the study variables. This computation was performed by solving the following equation expressed in matrix notation:

$$E = AP' \, Y^{-2} P \tag{9}$$

in which

E - factor-score matrix;
A = varimax matrix;
P = principal-factor matrix; and
Y = diagonal matrix of latent roots.

The various transposition, inversion, and multiplication of matrices were performed on the Illiac computer (24, 25, 26).

The final phase of the factor analysis was to correlate vehicular speed with the common factors. A multiple linear regression equation was developed for estimating spot speed as a function of the important factors of traffic flow by evaluating the following matrix equation:

$$c = Er' \tag{10}$$

in which

c = column vector of regression coefficients;
E = factor-score matrix; and
r = row vector of correlation coefficients for speed correlated with the other variables.

The row vector of correlation coefficients (Table 7, Appendix) was calculated by inputing the original data tape with the computer routine for product-moment correlations (29). Computer programs permitted the matrix transposition and multiplication necessary for solution (25, 26).

After the results of the factor analysis were appraised, it was possible to obtain another multiple linear regression expression for predicting vehicular speed in terms of those variables that significantly influence spot-speed characteristics. New punch cards containing the selected variables were reproduced from the original cards. These data constituted the input for a mean, standard deviation, and product-moment correlation analysis using the IBM 1401 computer (21). Finally, a regression routine for this computer provided the net regression coefficients, the coefficient of multiple correlation, and the standard error of estimate for this inference problem (30).

RESULTS

Solutions to the proposed mathematical models describing the rate of traffic flow on 2-lane highways in rural areas are presented and discussed according to the statistical techniques used in their development. These evaluations of the multiple estimating equations were accomplished by factor analysis and multiple linear regression analysis. Finally, the results of this spot-speed inference were tested with a verification study of the established functional relationships.

The reported findings were predicated on the assumption that the sample data were randomly selected from normal populations. In addition, homogeneity of variance was assumed for the study variables. Descriptive statistics of the study variables are tabulated in the Appendix. Means and standard deviations are given in Table 6, and product-moment correlation coefficients are given in Tables 7 and 8.

To simplify presentation of the research results, the variables and factors are identified, respectively, by numbers and letters. Each variable is listed by a number in the discussion of the design of experiment, and each factor is noted by a letter in the evaluation of the factor analysis.

Factor Analysis

The principal-axes solution provides a mathematically unique and highly desirable factorization of a correlation matrix (8). The correlation matrix (Table 8, Appendix) was factorized by this technique with unities inserted in the main diagonal of the matrix. Generated factors having latent roots or eigenvalues greater than 1.00 were considered to offer significant contributions to the total variance of the variables. The value of 1.00 for the terminal latent root has been arbitrarily established by convention in the application of factor analysis to multivariate studies. This criterion resulted in the determination of the 17 principal factors (Table 9, Appendix). The contribution of these factors to the total variance of the variables is given in Table 10 (Appendix). The 17 common factors that were determined to be significant accounted for 68 percent of this variance. A parsimonious description of the 48-dimensional space representing the original variables was obtained by the common-factor space of 17 dimensions. This factor matrix reproduces in some reasonable sense the original correlation matrix. The coordinate axes of this reduced space are the common factors, and the original variables can be expressed linearly in terms of these factors.

Rotation of the factor pattern was necessary to facilitate interpretation of the common factors. The principal-axes solution was transformed into a more desirable multiple-factor solution by the varimax method. Although an infinite number of rotations was possible from one coordinate system to another without any effect on the adequacy of the solution, the rotated-factor matrix (Table 1) provided a good approximation to orthogonal simple structure. Plus or minus signs on the factor coefficients indicate, respectively, the increasing or decreasing presence of the study variables in the composition of the generated factors. An interpretative name and a brief description, along with the important component variables and their respective factor coefficients, are listed for the following 17 common factors describing traffic flow on 2-lane rural highways:

A. Stream friction (traffic-stream elements that impede vehicular movement):
   22. Total volume, +0.8957
   23. Mean headway, -0.7037
   24. Vehicle in platoon, +0.8009
   26. Opposed vehicle, +0.7659
   27. Opposing volume, +0.9008
   33. Accident rate, +0.6524

B. Horizontal resistance (horizontal features of the roadway that control the rate of traffic movement):
   8. Degree of curve, +0.8304
   9. Total central angle, +0.5787
   10. Superelevation, +0.6922
   11. Test-car speed, +0.5038
   15. Min. sight distance, -0.4416
   25. Passing vehicle, -0.4961
   29. Min. speed limit, -0.7907

TABLE 1

ROTATED-FACTOR MATRIX

Vari-able	Factor								
	A	B	C	D	E	F	G	H	I
1	+0.0029	-0.0404	-0.5727	-0.2073	+0.1429	-0.0825	-0.0116	-0.0496	-0.0942
2	-0.0043	-0.0498	+0.7687	-0.0527	-0.0029	-0.0241	+0.0971	+0.0806	+0.0199
3	-0.1266	+0.0102	+0.5044	-0.0683	-0.0234	-0.1227	+0.1064	+0.0865	-0.0248
4	-0.3884	-0.0066	-0.4900	+0.0113	+0.0728	+0.1610	+0.0393	+0.0963	-0.0079
5	-0.1494	+0.0536	-0.0134	-0.0528	-0.0080	+0.0888	-0.0261	+0.1625	+0.0280
6	+0.0291	-0.1284	+0.6193	+0.0364	-0.0875	+0.1281	-0.0444	-0.3048	-0.0421
7	+0.0639	+0.0429	-0.0110	-0.0286	-0.0261	+0.0902	+0.0678	+0.0791	-0.0946
8	-0.1794	+0.8304	+0.0215	-0.0025	+0.0763	-0.0094	-0.0032	-0.0731	-0.0355
9	-0.1008	+0.5787	+0.0865	-0.0343	-0.0011	+0.1185	-0.0366	+0.1552	+0.1434
10	-0.1232	+0.6922	+0.0112	+0.0400	-0.1497	+0.0296	+0.0897	-0.0900	+0.0099
11	-0.0253	+0.5038	-0.0883	-0.0413	-0.0942	-0.0824	-0.0395	-0.0895	-0.0448
12	+0.0146	+0.0125	-0.0102	+0.0028	-0.0010	+0.0020	+0.0745	-0.0235	+0.8247
13	-0.0983	-0.1177	+0.1205	+0.0425	-0.0195	-0.0310	-0.1370	+0.0190	+0.0936
14	-0.0119	+0.0750	+0.0792	-0.0323	-0.0089	-0.0723	-0.0323	+0.0320	+0.7421
15	+0.0118	-0.4416	+0.1251	-0.0319	+0.0536	+0.0082	+0.0233	-0.0991	-0.4468
16	+0.2512	-0.0243	+0.1330	-0.1274	-0.0176	-0.0148	+0.0522	-0.2319	+0.0409
17	+0.0495	-0.0299	+0.1227	-0.0417	-0.0590	+0.0617	-0.1308	-0.5865	-0.0321
18	-0.0733	-0.0729	+0.1050	+0.0400	-0.0436	+0.0825	-0.0981	+0.7538	+0.0028
19	+0.2064	-0.0232	-0.0155	-0.0568	+0.8184	+0.0517	-0.0541	+0.0150	-0.0053
20	+0.0936	+0.1245	-0.0002	-0.0562	+0.5659	-0.0179	-0.0760	-0.1294	+0.0001
21	+0.2757	-0.0170	-0.2127	-0.0378	+0.7526	+0.0374	-0.0351	-0.0069	+0.0057
22	+0.8957	-0.1940	+0.0231	-0.0597	+0.1837	-0.0569	-0.0138	-0.0336	-0.0064
23	-0.7037	+0.2110	-0.1008	+0.0800	-0.1853	+0.1253	-0.0899	+0.0125	+0.0308
24	+0.8009	+0.0378	+0.0837	+0.0468	+0.0156	+0.0943	-0.2235	+0.0086	+0.0810
25	-0.0240	-0.4961	+0.2457	-0.0914	-0.0462	+0.0361	+0.0156	-0.0289	-0.1302
26	+0.7659	-0.0762	-0.0518	+0.0595	-0.0312	+0.0366	-0.1380	-0.2199	+0.0492
27	+0.9008	-0.1877	+0.0222	-0.0566	+0.1745	-0.0663	-0.0023	-0.0424	-0.0081
28	-0.0970	-0.0439	+0.0477	-0.0394	+0.0692	+0.1122	-0.1074	+0.0688	-0.0210
29	-0.0378	-0.7907	+0.0071	+0.0728	-0.1883	-0.0389	+0.0074	+0.0839	+0.0419
30	-0.0316	-0.1208	+0.0054	+0.0497	+0.0708	-0.0532	-0.0454	+0.1567	-0.0411
31	-0.1144	+0.4549	-0.0994	+0.0400	-0.0796	-0.0794	-0.0351	+0.5209	+0.0575
32	-0.0159	+0.6530	+0.1102	+0.1028	-0.0203	+0.0277	-0.0044	+0.2080	-0.0258
33	+0.6524	+0.0638	-0.0568	-0.0468	+0.1772	+0.0344	-0.0142	+0.0596	-0.0553
34	-0.0100	+0.0278	-0.0982	-0.0456	-0.0456	-0.0666	-0.0633	-0.0229	-0.0270
35	-0.0349	-0.0064	+0.0604	-0.0313	+0.0074	+0.0331	+0.0062	+0.0161	-0.0171
36	-0.0861	-0.0260	+0.0137	+0.1114	+0.1224	-0.0308	-0.0275	+0.0646	+0.0313
37	+0.0011	+0.0531	+0.0648	-0.0722	-0.0912	+0.0048	+0.0199	-0.0841	+0.0664
38	+0.1300	-0.0542	-0.0458	+0.0436	+0.0158	+0.0608	+0.0637	+0.0352	-0.0613
39	+0.0157	-0.0290	+0.0050	-0.0625	-0.0289	-0.0252	+0.0418	-0.0530	+0.0197
40	-0.1242	+0.0484	+0.0060	+0.0449	+0.1060	+0.8292	-0.0505	-0.0028	-0.0438
41	-0.0411	-0.0362	+0.0389	+0.0193	+0.0124	-0.8831	-0.0527	+0.0332	+0.0446
42	+0.2552	+0.0143	-0.0790	-0.0258	-0.1611	+0.0553	+0.1156	+0.0200	-0.0234
43	+0.1417	+0.0562	-0.0309	-0.2925	+0.0578	-0.0108	-0.9152	-0.0072	-0.0336
44	-0.1115	-0.0579	+0.0263	-0.1601	-0.0606	+0.0099	+0.9483	+0.0089	+0.0054
45	-0.0410	+0.0395	-0.0047	+0.8046	-0.0606	+0.0230	+0.0226	+0.0867	-0.0638
46	-0.0644	-0.0600	+0.0275	+0.4780	+0.0967	-0.0303	-0.0584	-0.1371	+0.2049
47	-0.0438	+0.0249	-0.0228	+0.9394	-0.0167	+0.0180	+0.0668	+0.0444	+0.0313
48	+0.1552	+0.0859	+0.0753	+0.0469	+0.4158	-0.0646	+0.2213	+0.1669	-0.0804

TABLE 1

ROTATED-FACTOR MATRIX (Continued)

Variable	Factor J	K	L	M	N	O	P	Q
1	+0.0815	+0.0389	-0.3309	+0.1881	-0.0217	-0.1653	+0.0003	-0.1604
2	-0.1250	-0.0176	-0.1417	+0.0262	+0.1133	-0.1178	-0.0306	-0.1993
3	+0.0795	+0.0702	-0.5542	+0.2019	+0.0119	-0.0666	-0.1113	-0.1338
4	-0.0913	-0.0855	+0.1277	-0.0317	+0.1214	-0.1498	+0.1821	-0.0893
5	-0.0344	-0.0988	+0.2216	+0.0873	+0.0138	+0.0836	+0.6404	+0.1189
6	+0.0508	-0.0542	+0.0592	-0.1832	-0.1033	-0.2256	+0.1566	+0.0550
7	+0.0955	-0.1244	-0.0571	-0.0121	-0.0210	-0.0763	+0.1335	-0.6845
8	+0.0405	+0.0152	-0.0660	+0.0729	-0.0176	+0.1472	+0.0277	+0.0338
9	+0.0315	+0.0556	-0.0319	+0.0904	+0.0432	+0.1614	-0.1260	+0.0336
10	-0.0228	-0.0245	+0.0095	+0.0003	-0.0749	-0.0871	+0.1730	-0.0517
11	-0.0603	+0.0004	+0.0742	+0.2349	-0.1845	-0.1087	-0.0956	-0.0607
12	-0.0079	-0.0260	-0.0289	-0.0234	-0.0327	-0.0001	-0.1021	-0.0239
13	-0.0231	+0.1477	+0.1393	+0.0052	+0.0696	+0.2645	-0.1378	-0.6135
14	+0.0605	+0.0877	+0.0811	+0.0936	+0.1322	+0.0041	+0.1722	+0.0822
15	+0.0194	+0.1950	+0.0887	+0.1781	+0.1666	+0.1547	+0.1006	+0.0481
16	-0.1454	+0.0230	-0.0052	-0.5754	-0.1991	-0.0203	-0.1450	-0.1690
17	+0.1847	+0.1748	+0.0873	+0.0375	-0.0115	-0.0359	+0.0073	+0.0735
18	+0.0885	+0.0289	+0.0464	+0.0223	-0.1163	-0.0587	+0.0913	+0.0089
19	-0.1108	-0.0462	-0.0176	-0.0768	-0.0034	-0.0832	-0.0086	-0.0094
20	+0.0973	+0.0698	-0.0991	-0.0014	-0.2264	+0.2363	+0.0499	+0.2060
21	-0.0511	-0.0987	+0.0292	+0.0340	-0.0429	+0.0151	-0.0691	-0.0600
22	+0.0025	-0.0011	+0.0174	-0.0338	+0.0232	-0.0121	-0.0379	-0.0347
23	-0.0623	+0.0041	-0.0385	+0.0726	+0.0604	+0.0622	+0.2186	+0.0933
24	-0.0162	-0.0370	+0.0603	-0.0088	+0.0630	+0.0378	-0.0372	+0.0107
25	+0.0975	+0.1301	-0.0460	+0.0402	-0.1195	+0.1391	+0.0405	-0.0652
26	-0.0407	-0.0589	+0.0481	-0.0257	+0.0543	+0.0294	+0.1181	+0.0771
27	-0.0039	+0.0133	+0.0083	-0.0423	+0.0126	-0.0157	+0.0555	-0.0708
28	+0.1115	-0.0519	+0.1520	+0.0580	+0.0380	+0.0263	-0.6690	+0.2079
29	-0.0198	+0.0293	+0.0386	-0.0933	-0.1017	-0.1056	-0.0632	+0.0018
30	+0.0639	+0.1182	-0.0228	-0.7147	+0.2076	+0.0843	+0.0603	+0.0387
31	+0.0945	+0.0432	-0.0713	-0.2043	+0.1472	-0.0470	-0.0165	-0.0250
32	+0.0032	+0.0979	+0.0493	-0.2605	+0.0100	-0.1057	+0.0593	-0.0063
33	+0.0836	+0.0858	-0.0509	-0.0014	-0.1403	-0.0625	+0.0503	+0.0160
34	+0.1873	+0.1223	-0.1209	-0.1521	+0.1340	+0.7539	+0.0341	-0.0631
35	-0.8859	+0.1529	-0.0394	-0.0151	+0.1356	-0.1210	+0.1171	+0.0858
36	+0.0952	-0.8455	+0.0418	+0.1717	+0.1418	-0.0529	+0.0417	-0.0585
37	+0.4763	+0.3567	-0.0211	-0.1597	+0.3128	-0.5768	-0.0639	+0.0219
38	+0.0528	+0.1882	+0.1415	+0.1712	-0.7579	+0.0508	-0.1204	+0.0147
39	+0.0626	+0.0289	+0.8522	+0.0738	-0.1162	-0.1429	+0.0028	-0.0773
40	-0.0193	-0.0256	-0.3974	-0.0650	-0.1536	-0.0450	+0.0767	+0.0120
41	+0.0243	-0.0252	-0.2916	-0.0650	-0.0925	+0.0440	+0.1097	+0.0663
42	-0.0876	+0.0461	+0.0336	+0.1183	+0.5582	+0.1904	-0.3091	-0.0264
43	+0.0125	+0.0072	-0.0087	-0.0429	+0.0081	+0.0218	-0.0274	+0.0131
44	-0.0168	-0.0185	+0.0096	+0.0058	+0.0071	-0.0116	+0.0231	+0.0090
45	+0.1092	-0.2360	-0.0418	-0.0860	-0.0468	+0.0158	-0.0348	+0.1086
46	-0.1517	+0.4007	+0.0609	+0.2726	+0.0144	-0.0641	+0.0716	-0.2482
47	-0.0482	+0.0713	-0.0165	+0.0424	-0.0202	-0.0114	+0.0176	-0.0409
48	+0.1685	+0.0475	+0.0806	+0.0067	+0.1567	-0.2032	-0.0065	+0.0094

31. No-passing zone, +0.4549
32. Edge line, +0.6530

C. Long-distance travel (drivers on long trips are evident with the pronounced presence of male drivers, out-of-state cars, passengers in the automobiles, and long-haul commercial trucks):
1. Female driver, -0.5727
2. Out-of-state car, +0.7687
3. Passenger in car, +0.5044
4. Light truck, -0.4900
6. Truck combination, +0.6193

D. Inclement weather (certain adverse weather conditions encountered in highway travel):
43. Clear, -0.2925
45. Drizzle, +0.8046
46. Rain, +0.4780
47. Wet pavement, +0.9394

E. Marginal friction (variables along the margin of the roadway that interrupt traffic flow):
19. Roadside establishment, +0.8184
20. Friction point, +0.5659
21. Access point, +0.7526
48. Advertising sign, +0.4158

F. Time of day (although not completely defined, indicative of variations in traffic-stream characteristics for different periods of the day):
40. 10:01 to 12:30, +0.8292
41. 12:31 to 3:00, -0.8831

G. Relative darkness (contrast in light intensity would probably reflect nighttime travel conditions):
43. Clear, -0.9152
44. Cloudy, +0.9483

H. Lateral restriction:
16. Lane width, -0.2319
17. Shoulder width, -0.5865
18. Curb or gutter, +0.7538
31. No-passing zone, +0.5209
32. Edge line, +0.2080

I. Vertical resistance (influence of vertical alignment on traffic):
12. Gradient, +0.8247
14. Total rise and fall, +0.7421
15. Min. sight distance, -0.4468

J. Day of week (daily variation in traffic-stream behavior is evident, but the precise pattern is not discernible):
35. Tuesday, -0.8859
37. Thursday, +0.4763

K. Day of week (representing additional variation within the week, not fully explained because of few variables):
36. Wednesday, -0.8455
37. Thursday, +0.3567

L. Home-to-work travel (characteristic of male drivers traveling alone in the early morning):
1. Female driver, -0.3309
3. Passenger in car, -0.5542
39. 7:31 to 10:00, +0.8522
40. 10:01 to 12:30, -0.3974
41. 12:31 to 3:00, -0.2916

M. Obsolete pavement (representative of narrow, unmarked, inadequate pavements that are no longer constructed):

16. Lane width, -0.5754
30. Centerline, -0.7147
31. No-passing zone, -0.2043
32. Edge line, -0.2605

N. Work-to-home travel (home-bound trips in late afternoon for every weekday except Friday):
37. Thursday, +0.3128
38. Friday, -0.7579
42. 3:01 to 5:30, +0.5582

O. Day of week (further variation in travel characteristics among different days of the week is broadly evident):
34. Monday, +0.7539
37. Thursday, -0.5768

P. Local-business travel (single-unit trucks in the opposite direction of the major traffic flow account for the delivery and service operations of local businesses—usually completed by late afternoon):
5. Single-unit truck, +0.6404
28. Direct. distribution, -0.6690
42. 3:01 to 5:30, -0.3091

Q. Local-service road (absence of commercial buses and long grades; primarily affording access to various forms of land use):
7. Commercial bus, -0.6845
13. Length of grade, -0.6135

The multiple-factor solution (Table 1) was readily interpretable because many common factors were significantly loaded with a sufficient number of variables. However, several orthogonal factors, particularly those concerned with weekday variations in traffic-stream conditions, were not completely defined by the few variables present.

Although the factor coefficients (Table 1) permit the evaluation of a variable in terms of the common factors, it was necessary to compute the factor-score matrix (Table 2) to express the generated factors as functions of the original variables. Thus, a factor can be quantitatively determined in standard-score measure from a multiple linear equation stated in terms of the variables with the respective factor scores as standard regression coefficients.

Mean spot speed was not included as a variable in the principal-axes solution, the varimax rotation, and the development of factor scores. Because it was desired to correlate mean speed with the common factors, speed was not used in the generation of these factors. Consequently, the factor pattern was not determined to any degree by this external variable.

The results of the linear correlation of mean speed with the 17 common factors are given in Table 3. The factors of horizontal resistance, long-distance travel, marginal friction, vertical resistance, and obsolete pavement displayed correlation coefficients that were significant at the 5 percent level. Because the statistical analysis was performed with standard scores, these correlation coefficients are also the standard regression coefficients for the different factors. Therefore, Eq. 11 was written to estimate mean spot speeds in terms of those common factors that had significant coefficients of regression:

$$S_1 = 41.42 + 7.269\,(-0.7487\,F_B + 0.1227\,F_C - 0.2677\,F_E - 0.1157\,F_I - 0.1360\,F_M) \quad (11)$$

in which

S_1 = mean spot speed in mph; and
F_j = significant common factor.

Eq. 11 represents the evaluation of Eq. 1 proposed in the theoretical analysis. The unique factor with its coefficient of 0.5664 was not included in this relationship. Unique-

TABLE 2

FACTOR-SCORE MATRIX

Vari-able	Factor								
	A	B	C	D	E	F	G	H	I
1	+0.0052	-0.0260	-0.3227	-0.0741	+0.0143	-0.0510	-0.0104	-0.0558	-0.0396
2	-0.0134	+0.0053	+0.3780	-0.0489	+0.0410	-0.0222	+0.0163	+0.0569	-0.0034
3	-0.0196	+0.0016	+0.2327	-0.0486	+0.0138	-0.0633	+0.0216	+0.0524	-0.0149
4	-0.0887	-0.0352	-0.2503	+0.0041	+0.0492	+0.0701	+0.0028	+0.0015	-0.0013
5	+0.0114	-0.0066	+0.0400	-0.0565	-0.0019	+0.0414	-0.0052	+0.1170	+0.0182
6	-0.0261	-0.0120	+0.2854	+0.0222	-0.0082	+0.0435	-0.0450	-0.1814	-0.0302
7	+0.0376	+0.0245	-0.0519	-0.0297	-0.0641	+0.0526	+0.0118	+0.0150	-0.0609
8	-0.0253	+0.2401	+0.0570	-0.0041	+0.0621	-0.0212	+0.0186	-0.0859	-0.0597
9	+0.0066	+0.1529	+0.0838	-0.0389	+0.0055	+0.0878	+0.0012	+0.0932	+0.0677
10	+0.0200	+0.2040	+0.0036	+0.0041	-0.0808	-0.0056	+0.0442	-0.0940	-0.0340
11	+0.0217	+0.1600	-0.0348	-0.0232	-0.0704	-0.0726	-0.0243	-0.0572	-0.0603
12	-0.0032	-0.0407	-0.0284	-0.0251	-0.0009	+0.0382	+0.0526	-0.0339	+0.5353
13	-0.0540	-0.0242	+0.0530	+0.0170	+0.0181	-0.0011	-0.0797	-0.0044	+0.0575
14	+0.0058	-0.0243	+0.0456	-0.0384	+0.0188	-0.0096	-0.0050	+0.0195	+0.4640
15	-0.0143	-0.0978	+0.0881	+0.0110	+0.0684	+0.0099	+0.0106	-0.0150	-0.2645
16	+0.0098	+0.0277	+0.0117	-0.0485	-0.0347	-0.0222	+0.0451	-0.1591	+0.0288
17	-0.0293	+0.0159	+0.0454	+0.0248	+0.0035	+0.0316	-0.0601	-0.3572	-0.0142
18	+0.0369	-0.0555	+0.0766	-0.0355	-0.0524	+0.0555	-0.0671	+0.5204	-0.0097
19	-0.0592	-0.0076	+0.0335	+0.0047	+0.4234	-0.0148	-0.0020	-0.0198	+0.0010
20	-0.0509	+0.0328	+0.0608	-0.0020	+0.3052	-0.0293	+0.0120	-0.0776	+0.0175
21	-0.0241	-0.0047	-0.0635	+0.0127	+0.3641	-0.0053	+0.0173	-0.0260	+0.0165
22	+0.2090	-0.0175	-0.0069	-0.0078	-0.0107	-0.0045	+0.0307	+0.0399	+0.0013
23	-0.1401	+0.0206	-0.0302	+0.0212	-0.0251	+0.0555	-0.0699	-0.0417	+0.0160
24	+0.2148	+0.0369	+0.0344	+0.0309	-0.0986	+0.0904	-0.0800	+0.0627	+0.0434
25	-0.0206	-0.1321	+0.1182	-0.0464	-0.0067	+0.0371	+0.0056	+0.0319	-0.0463
26	+0.2047	+0.0079	-0.0487	+0.0508	-0.1153	+0.0497	-0.0353	-0.0934	+0.0352
27	+0.2162	-0.0154	-0.0120	-0.0077	-0.0187	-0.0125	+0.0359	+0.0321	-0.0010
28	-0.0730	-0.0132	+0.0625	-0.0089	+0.0704	+0.0804	-0.0516	+0.0659	-0.0068
29	-0.0222	-0.2357	-0.0429	+0.0301	-0.0930	-0.0134	-0.0251	+0.0859	+0.0615
30	-0.0543	-0.0432	-0.0185	+0.0404	+0.0719	-0.0325	-0.0145	+0.0562	-0.0351
31	+0.0195	+0.1020	-0.0448	-0.0081	-0.0511	-0.0399	-0.0261	+0.2972	-0.0080
32	+0.0224	+0.1887	+0.0595	+0.0360	-0.0091	-0.0075	-0.0014	+0.0998	-0.0744
33	+0.1726	+0.0396	-0.0396	-0.0128	-0.0082	+0.0326	+0.0273	+0.0890	-0.0379
34	+0.0023	+0.0064	-0.0104	-0.0175	-0.0073	+0.0271	+0.0230	-0.0101	+0.0119
35	+0.0023	+0.0109	+0.0230	-0.0202	+0.0054	-0.0221	-0.0079	+0.0040	-0.0313
36	-0.0288	-0.0233	+0.0454	+0.0339	+0.0545	-0.0300	-0.0405	-0.0176	+0.0345
37	-0.0152	+0.0059	-0.0378	+0.0010	-0.0100	+0.0031	-0.0270	-0.0695	+0.0047
38	+0.0415	+0.0000	-0.0160	+0.0019	-0.0421	+0.0212	+0.0556	+0.1012	-0.0216
39	-0.0225	+0.0177	+0.0366	-0.0332	+0.0168	-0.0414	+0.0203	+0.0094	-0.0136
40	-0.0019	-0.0163	-0.0239	+0.0056	+0.0019	+0.4947	-0.0145	-0.0137	+0.0193
41	-0.0398	-0.0102	+0.0186	+0.0225	+0.0415	-0.5469	-0.0469	-0.0086	+0.0030
42	+0.0963	+0.0209	-0.0369	-0.0028	-0.0917	+0.0995	+0.0735	+0.0247	-0.0196
43	-0.0105	+0.0113	+0.0100	-0.1089	-0.0219	-0.0221	-0.4620	+0.0254	-0.0271
44	+0.0157	-0.0039	-0.0028	-0.1131	+0.0008	+0.0322	+0.4961	+0.0058	+0.0247
45	+0.0078	-0.0025	-0.0116	+0.3841	-0.0115	+0.0008	-0.0140	-0.0039	-0.0595
46	-0.0300	-0.0245	-0.0100	+0.2465	+0.0977	-0.0360	-0.0550	-0.1121	+0.1027
47	+0.0088	-0.0062	-0.0369	+0.4549	+0.0278	-0.0068	+0.0028	-0.0333	-0.0133
48	+0.0011	+0.0284	+0.0581	+0.0354	+0.2377	-0.0551	+0.1130	+0.0914	-0.0759

TABLE 2

FACTOR-SCORE MATRIX (Continued)

Vari-able	Factor							
	J	K	L	M	N	O	P	Q
1	+0.0655	+0.0445	-0.2633	+0.1220	-0.0106	-0.1980	+0.0208	-0.1642
2	-0.0951	-0.0445	-0.0490	+0.0629	+0.0745	-0.0276	-0.0256	-0.1119
3	+0.0569	+0.0376	-0.3316	+0.1947	-0.0007	-0.0426	-0.0575	-0.0743
4	-0.0477	-0.0315	+0.0594	-0.0664	+0.0923	-0.1288	+0.0952	-0.0986
5	+0.0160	-0.0207	+0.1373	+0.0814	+0.0210	+0.0968	+0.4719	+0.1121
6	+0.0486	-0.0689	+0.0459	-0.1237	-0.0565	-0.1123	+0.1129	+0.0438
7	+0.0760	-0.1298	-0.0565	-0.0212	-0.0413	-0.0731	+0.0943	-0.5512
8	+0.0171	+0.0200	-0.0094	+0.0412	-0.0064	+0.1043	+0.0074	+0.0166
9	+0.0142	+0.0448	+0.0110	+0.0826	+0.0249	+0.1393	-0.1016	+0.0399
10	-0.0210	-0.0224	+0.0059	-0.0160	-0.0619	-0.0552	+0.1107	-0.0656
11	-0.0619	-0.0124	+0.0366	+0.1460	-0.1324	-0.0968	-0.0827	-0.0755
12	-0.0033	-0.0498	-0.0427	-0.0180	-0.0506	+0.0401	-0.0816	-0.0205
13	-0.0253	+0.0418	+0.1011	-0.0138	+0.0348	+0.1693	-0.1117	-0.4560
14	+0.0658	+0.0688	+0.0336	+0.1012	+0.0889	+0.0437	+0.1427	+0.0920
15	+0.0318	+0.1672	+0.0668	+0.1529	+0.1669	+0.1024	+0.1056	+0.0858
16	-0.1375	-0.0653	+0.0075	-0.4525	-0.1804	-0.0063	-0.1250	-0.1643
17	+0.1420	+0.1036	+0.0306	+0.0169	+0.0371	-0.0400	+0.0304	+0.0467
18	+0.0869	+0.0589	+0.0564	+0.0666	-0.1142	-0.0099	+0.0655	+0.0497
19	-0.0708	-0.0044	+0.0200	-0.0615	+0.0430	-0.0443	-0.0228	+0.0049
20	+0.0817	+0.0758	-0.0323	-0.0061	-0.1228	+0.1773	+0.0476	+0.1752
21	-0.0322	-0.0444	+0.0323	+0.0071	+0.0092	+0.0102	-0.0612	-0.0459
22	-0.0047	-0.0053	-0.0109	+0.0233	+0.0238	-0.0023	+0.0217	-0.0132
23	-0.0282	+0.0244	-0.0209	+0.0276	+0.0491	+0.0355	+0.1267	+0.0610
24	-0.0188	-0.0322	+0.0179	+0.0490	+0.0515	+0.0392	+0.0092	+0.0152
25	+0.0872	+0.0778	-0.0261	+0.0457	-0.0786	+0.1105	+0.0545	-0.0178
26	-0.0311	-0.0451	-0.0131	+0.0174	+0.0541	+0.0190	+0.1287	+0.0504
27	-0.0044	+0.0053	-0.0210	+0.0185	+0.0160	-0.0057	+0.0934	-0.0405
28	+0.0503	-0.0430	+0.1334	+0.0284	+0.0322	+0.0286	-0.5153	+0.1623
29	-0.0049	+0.0039	-0.0003	-0.0710	-0.0892	-0.0790	-0.0428	+0.0082
30	+0.0408	+0.0797	+0.0299	-0.5152	+0.1170	+0.0538	+0.0374	+0.0387
31	+0.0603	+0.0543	-0.0121	-0.1225	+0.0588	-0.0412	-0.0213	-0.0123
32	-0.0054	+0.0757	+0.0682	-0.1795	-0.0109	-0.0591	+0.0253	-0.0030
33	+0.0676	+0.0747	-0.0509	+0.0398	-0.0905	-0.0406	+0.0786	+0.0216
34	+0.1277	+0.0751	-0.0460	-0.1011	+0.0768	+0.5148	+0.0573	-0.0232
35	-0.6565	+0.1044	-0.0225	-0.0072	+0.1139	-0.0681	+0.0431	+0.0782
36	+0.0583	-0.5872	+0.0278	+0.0952	+0.0845	-0.0279	-0.0072	-0.0976
37	+0.3680	+0.2734	-0.0334	-0.0749	+0.2391	-0.4306	-0.0070	+0.0277
38	+0.0459	+0.1139	+0.0772	+0.0956	-0.5394	+0.0528	-0.0850	+0.0151
39	+0.0485	+0.0087	+0.5438	+0.0128	-0.0714	-0.0658	-0.0188	-0.0496
40	+0.0252	-0.0069	-0.2713	-0.0337	-0.0763	+0.0069	+0.0367	-0.0071
41	-0.0103	-0.0228	-0.1688	-0.0512	-0.1121	-0.0369	+0.1061	+0.0445
42	-0.0886	+0.0369	+0.0236	+0.1225	+0.4012	+0.1330	-0.2079	+0.0049
43	-0.0020	-0.0112	-0.0104	-0.0086	+0.0130	-0.0504	-0.0127	-0.0219
44	-0.0090	-0.0085	+0.0160	-0.0109	-0.0173	+0.0670	+0.0178	+0.0351
45	+0.0816	-0.1403	-0.0208	-0.0816	-0.0306	+0.0051	-0.0481	+0.0708
46	-0.0830	+0.2879	+0.0118	+0.1973	+0.0614	-0.0636	+0.0556	-0.1545
47	-0.0190	+0.0781	-0.0187	+0.0174	+0.0124	-0.0191	-0.0042	-0.0169
48	+0.1354	+0.0801	+0.0807	+0.0271	+0.1361	-0.1218	+0.0058	+0.0433

TABLE 3

CORRELATION OF MEAN SPEED WITH THE FACTORS

Factor	Correlation Coefficient
A	-0.0868
B	-0.7487*
C	+0.1227*
D	-0.0104
E	-0.2677*
F	-0.0157
G	+0.0699
H	-0.0103
I	-0.1157*
J	+0.0018
K	+0.0460
L	+0.0517
M	-0.1360*
N	-0.0086
O	-0.1034
P	-0.0590
Q	-0.0646

*Significant at the 5 percent level.

ness is composed of the specificity and unreliability contributed by the variables and cannot be determined from sample data.

The horizontal-resistance factor had the most pronounced influence on spot-speed characteristics. On the other hand, the almost negligible effect of the stream-friction factor on mean speeds is probably because the traffic volumes did not exceed the practical capacity of the roadway. The amount of traffic flow was, therefore, not great enough to modify significantly the rate of vehicular movement. The long-distance-travel factor was positively related to speed, whereas the remaining factors restricted the rate of traffic flow.

The precision of this multiple estimate was measured by a standard error of estimate equal to 4.12 mph, and the degree of correlation for this multivariate analysis was expressed by a multiple correlation coefficient of 0.824, which was significant at the 5 percent level. Thus, approximately 68 percent of the variation in vehicular speed on 2-lane rural highways was explained by these five factors. This equation afforded a reasonable evaluation of the proposed regression model which functionally relates time-mean speeds to the generated factors.

To evaluate the selected factors, multiple linear regression expressions were developed from the results of the factor-score analysis. These five equations were expressed in terms of those study variables that predominantly accounted for the generation of a particular factor:

$$F_B = 0.2401\ Z_8 + 0.1529\ Z_9 + 0.2040\ Z_{10} + 0.1600\ Z_{11} - 0.0978\ Z_{15} - 0.1321\ Z_{25} - 0.2357\ Z_{29} + 0.1020\ Z_{31} + 0.1887\ Z_{32} \quad (12)$$

$$F_C = -\ 0.3227\ Z_1 + 0.3780\ Z_2 + 0.2327\ Z_3 - 0.2503\ Z_4 + 0.2854\ Z_6 \quad (13)$$

$$F_E = 0.4234\ Z_{19} + 0.3052\ Z_{20} + 0.3641\ Z_{21} + 0.2377\ Z_{48} \quad (14)$$

$$F_I = 0.5353\ Z_{12} + 0.4640\ Z_{14} - 0.2645\ Z_{15} \quad (15)$$

$$F_M = -\ 0.4525\ Z_{16} - 0.5152\ Z_{30} - 0.1225\ Z_{31} - 0.1795\ Z_{32} \quad (16)$$

in which

F_j = common factor; and
Z_i = significant variable.

Values of the different variables must be reduced to standard-score form for solution. This reduction is accomplished by the following relationship:

$$Z = (X - \overline{X})/s \tag{17}$$

in which

Z = standard score;
X = observed value;
$\overline{X}$ = mean of variable; and
s = standard deviation of variable.

Means and standard deviations of the study variables are given in Table 6 (Appendix). Thus, a technique is available to evaluate the common factors in standard-score units so that an estimate of the mean speed for any highway location can be derived from the speed-factor expression.

Multiple Linear Regression and Correlation Analysis

An exploratory insight into traffic-stream behavior was gained by the factor-analysis study. A better understanding, both qualitatively and quantitatively, of those broad categories that described traffic flow and influenced spot speeds permitted a knowable selection of independent variables for the evaluation of the second regression model. Study variables were chosen for the multiple linear regression and correlation analysis in compliance with the following criteria:

1. Each significant factor (horizontal resistance, long-distance travel, marginal friction, vertical resistance, and obsolete pavement) was represented by at least one variable;
2. The variables selected for the respective common factors were to have high factor coefficients;
3. These study variables had low intercorrelations;
4. Values of the variables selected were readily obtainable from engineering records and drawings; and
5. A parsimonious description was desired for the time-rate of traffic flow.

Out-of-state car, truck combination, degree of curve, gradient, minimum sight distance, lane width, roadside establishment, and total volume were the variables specified for multiple correlation with mean spot speed.

The results of this multivariate analysis are summarized in Table 4. The second regression model for estimating vehicular speeds was evaluated according to the following:

$$S_2 = 39.34 + 0.0267\ X_2 + 0.1396\ X_6 - 0.8125\ X_8 - 0.1126\ X_{12} + 0.0007\ X_{15} + 0.6444\ X_{16} - 0.5451\ X_{19} - 0.0082\ X_{22} \tag{18}$$

in which

S_2 = mean spot speed in mph; and
X_i = independent variable.

This relationship provides a quantitative description of Eq. 2, which was developed from theoretical considerations. The coefficient of multiple correlation was 0.788 and was significant at the 5 percent level. A measure of 4.47 mph for the standard error of estimate approximated the closeness between estimated and observed values. These eight variables, therefore, accounted for about 62 percent of the variation in the rate of vehicle operation on 2-lane rural highways. The inference expression developed by the factor-analysis procedure was slightly more precise than the multiple linear equation evaluated for the selected variables. The influence of certain variables on spot speeds was combined into a single factor, and this composite representation afforded by the common factors probably provided a more accurate explanation of the variation in mean speed.

The net regression coefficients for the chosen variables were all significant at the 5 percent level. Positive relationships were established between mean speed and out-of-state car, combination truck, minimum sight distance, and lane width. The remaining

TABLE 4

RESULTS OF MULTIPLE LINEAR REGRESSION AND CORRELATION ANALYSIS

Intercept = 39.34 mph

Multiple Correlation Coefficient = 0.788*

Standard Error of Estimate = 4.47 mph

Variable	Net Regression Coefficient	Standard Error
2	+0.0267*	0.0137
6	+0.1396*	0.0510
8	-0.8125*	0.0345
12	-0.1126*	0.0596
15	+0.0007*	0.0001
16	+0.6444*	0.1874
19	-0.5451*	0.0838
22	-0.0082*	0.0015

* Significant at the 5 percent level.

variables (degree of curve, gradient, roadside establishment, and total volume) were related to spot speed in a negative manner. Positive coefficients of regression represent increases in mean speeds; negative values indicate speed reductions.

The results of the multiple regression analysis showed that degree of curve, associated with the horizontal-resistance factor, exerted the greatest influence on determining the average rate of traffic movement. This finding is analogous to the factor-analysis results which demonstrated the pronounced effect of horizontal resistance on mean vehicular speeds.

Although stream friction did not appear to be an important factor in the correlation with mean spot speed, total volume was included as an independent variable in the multiple linear regression and correlation analysis. This decision was predicated on the importance attached to traffic volume in the technical literature dealing with traffic-stream characteristics. The inclusion of total volume in this multivariate analysis produced a net regression coefficient that was significantly and negatively related to mean speed. This relationship is in agreement with the general concept postulated for the speed-volume pattern in the development of the theory.

The solution to the second regression model provided a reasonable and efficient evaluation of the functional relationship between mean spot speed and eight selected variables that significantly influenced traffic flow on 2-lane rural highways.

Model Verification

The final purpose of this investigation was to test the validity of the two multiple linear regression equations proposed for describing the rate of traffic movement. The ten study sites not included in the factor analysis and the multiple linear regression analysis permitted an empirical comparison between observed and estimated mean speeds. These results are given in Table 5, where the actual mean speeds are compared with the calculated mean speeds derived from the common factors in Eq. 11 and from the independent variables in Eq. 18.

The rather small differences between the variable-estimate mean and the observed mean and between the factor-estimate mean and the observed mean indicated that the two multiple equations were fairly reliable for predicting mean spot speeds. In no case were these differences significant at the 5 percent level. However, there was a tendency for the estimated mean speed to be less than the observed value. This discrepancy could be attributed to the small number of speed sites used in this verification of the evaluated regression models.

The time-mean speed estimates calculated from the speed-factor equation were more precise than those computed from the speed-variable expression. This variation was explained by the higher degree of correlation existing between speed and the factors

TABLE 5

RESULTS OF VERIFICATION STUDY

Study No.	Speed - mph				
	Observed Mean	Variable		Factor	
		Estimated Mean	Difference (Est.-Obs.)	Estimated Mean	Difference (Est.-Obs.)
1	50.62	45.00	-5.62	46.18	-4.44
2	51.82	46.46	-5.36	53.01	+1.19
3	51.79	43.79	-8.00	53.00	+1.21
4	43.29	40.72	-2.57	40.39	-2.90
5	36.96	38.51	+1.55	35.61	-1.35
6	32.37	37.59	+5.22	35.52	+3.15
7	42.74	40.56	-2.18	39.63	-3.11
8	51.56	47.63	-3.93	47.44	-4.12
9	50.53	46.24	-4.29	46.17	-4.36
10	47.74	47.42	-0.32	46.55	-1.19

than between speed and the variables. The difference in the standard errors of estimate for the variable and the factor equations was 0.35 mph.

In summary, the two multiple linear regression equations, evaluated from actual measurements of driver, vehicle, roadway, traffic, and environmental conditions, can be used as inference devices to estimate mean spot speeds with a reasonable degree of confidence. Slight adjustment in the intercepts may be justified to account for variations in speed patterns attributable to local conditions and to the annual increase in the rate of motor-vehicle operation.

CONCLUSIONS

The conclusions inferred from this multivariate analysis of vehicular speeds are valid only for those drivers and vehicles sampled at the selected roadway locations under the prevailing traffic and environmental conditions. However, the real benefits of research are derived through generalizations deduced for the entire population of motor-vehicle drivers. As a result, the following conclusions concerning the traffic-stream characteristics of 2-lane rural highways were abstracted:

1. The concept of time-rate of traffic flow was qualitatively described as a hyperplane to account for the many variables that influence vehicular speeds. The combination of the speed hyperplane and the limiting hyperplanes produced a polyhedron as the geometric representation of the comprehensive traffic-flow model.
2. The average speed of traffic movement under various types and levels of travel conditions was formulated as multiple linear regression models. Two regression models were postulated with mean speed related to generated factors in the first equation and to travel variables in the second equation.
3. The rate of traffic flow was largely determined by the five factors identified as horizontal resistance, long-distance travel, marginal friction, vertical resistance, and obsolete pavement. The horizontal-resistance factor accounted for the greatest variation in time-mean speeds.
4. Variations in mean spot speeds were statistically explained to an acceptable degree by variations in the eight variables defined as out-of-state car, truck combination,

degree of curve, gradient, minimum sight distance, lane width, roadside establishment, and total volume. The most pronounced influence on speed characteristics was caused by degree of curve.

5. Two multiple linear equations were developed to relate mean spot speed to five generated factors and to eight travel-restriction variables. Both expressions were verified as suitable for the reliable estimation of mean speeds.

6. The application of multivariate analysis techniques (factor analysis and multiple linear regression and correlation analysis) was essential for appraising and evaluating the complex phenomenon of traffic flow. These analytic devices provided an exploratory appraisal of traffic-stream characteristics, afforded a parsimonious and accurate description of vehicular speeds, and permitted the development of inference equations for the reliable estimation of mean spot speeds.

SUGGESTIONS FOR FURTHER RESEARCH

As the conduct of this research study progressed, it became readily apparent that various phases in the area of traffic-stream characteristics required comprehensive evaluation by both theoretical and applied investigations. The following suggestions are offered as possibilities for further research:

1. The proposed multiple regression models should be evaluated for multi-lane highways in rural areas and for 2-lane and multi-lane highways and streets in business, residential, and intermediate areas. It is quite unlikely that the common factors developed in these analyses will be different from those describing traffic flow on 2-lane rural highways. However, the influences of these factors on speed characteristics will, no doubt, assume different proportions for the various facilities. This change in degree of significance will also be evident in the correlations of mean speed with selected travel-restriction variables. Different sets of variables will describe the rate of vehicle movement for the various combinations of highway type and traffic area.

2. Detailed analyses of specific factors will yield refined information concerning traffic-stream characteristics. The actual composition of a factor can be investigated to ascertain the relative importance of the variables that comprise this factor. These micro-analyses will be greatly aided by the application of analysis of variance and analysis of covariance techniques to traffic-stream data.

3. Additional effort should be expended on the realistic determination of the common factors describing the movement of highway vehicles. This elaborate study is mandatory if the quantitative evaluation of a factor is to measure its qualitative connotation with an acceptable degree of accuracy.

4. Consideration of non-linearity in the parameters and/or the variables may offer increased precision in the estimation of speed statistics. The two regression models were restricted to multiple linear functions and did not consider curvilinear expressions and joint functional relationships.

5. Analysis of data by individual speed observations may prove valuable in promoting the understanding of driver behavior. The variation in speed practices for the individual driver must be evaluated in the appraisal of traffic-stream characteristics.

6. As improvements in techniques and devices for recording driver, vehicle, roadway, traffic, and environmental conditions are realized, the study of additional variables that presently cannot be evaluated may further refine the knowledge of speed characteristics.

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REFERENCES

1. Berry, D. S., and Belmont, D. M., "Distribution of Vehicle Speeds and Travel Times." Proc., Second Berkeley Symp. on Math. Statistics and Probability, Univ. of Calif. Press, 589-602 (1951).
2. Croxton, F. E., and Cowden, D. J., "Applied General Statistics." 2nd Ed., Prentice-Hall (1955).
3. Ezekiel, M., and Fox, K. A., "Methods of Correlation and Regression Analysis, Linear and Curvilinear." Wiley (1959).
4. Ferber, R., "Statistical Techniques in Market Research." McGraw-Hill (1949).
5. Fruchter, B., "Introduction to Factor Analysis." Van Nostrand (1954).
6. Goldstein, L. G., and Mosel, J. N., "A Factor Study of Drivers' Attitudes, with Further Study on Driver Aggression." HRB Bull. 172, 9-29 (1958).
7. Greenshields, B. D., and Weida, F. M., "Statistics with Applications to Highway Traffic Analyses." Eno Foundation, 150-160, 173-187 (1952).
8. Harman, H. H., "Modern Factor Analysis." Univ. of Chicago Press (1960).
9. Herman, R., "Mathematical Theory of Traffic Flow." Proc., Inst. of Traffic Engs., 67-79 (1960).
10. "Highway Capacity Manual." U. S. Bureau of Public Roads, 17, 27-63 (1950).
11. Kaneko, E. T., "Effect of Roadway Frictions on Vehicle Operating Characteristics." Mich. State Univ., M. S. thesis (June 1960).
12. Keefer, L. E., "The Relation Between Speed and Volume on Urban Streets." Chicago Area Trans. Study (1958, unpublished).
13. "Manual of Traffic Engineering Studies." 2nd Ed., Assoc. of Casualty and Surety Co's., 114-123 (1953).
14. Matson, T. M., Smith, W. S., and Hurd, F. W., "Traffic Engineering." McGraw-Hill, 45-62 (1955).
15. "Motor Vehicle Speeds, Annotated." HRB Biblio. 27 (1960).
16. Oppenlander, J. C., "A Theory of Vehicular Speed Regulation." HRB Bull. 341, 77-91 (1962).
17. Oppenlander, J. C., "Multivariate Analysis of Vehicular Speeds." Univ. of Illinois, Ph.D. thesis (June 1962).
18. Oppenlander, J. C., Bunte, W. F., and Kadakia, P. L., "Sample Size Requirements for Vehicular Speed Studies." HRB Bull. 281, 68-86 (1961).
19. "Traffic Engineering Handbook." 3rd Ed., Inst. of Traffic Engrs. (In press).
20. Versace, J., "Factor Analysis of Roadway and Accident Data." HRB Bull. 240, 24-32 (1960).

Computer Programs

21. "Correlation Analysis," C 4, Statistical Services Unit, University of Illinois.
22. "Data Tape Checking Routine," KSL 5.90, Digital Computer Laboratory, University of Illinois.
23. "Delete Rows and Columns from a Matrix," KSL 5.50, Digital Computer Laboratory, University of Illinois.
24. "Inversion of Symmetrical Matrix," KSL 5.11, Digital Computer Laboratory, University of Illinois.
25. "Matrix Multiplication with or without Rescaling," M 28, Digital Computer Laboratory, University of Illinois.
26. "Matrix Transposition with or without Rescaling," KSL 5.30, Digital Computer Laboratory, University of Illinois.
27. "Page Output Correlations," KSL 5.57, Digital Computer Laboratory, University of Illinois.
28. "Principal Axis Factors Solution," KSL 1.11, Digital Computer Laboratory, University of Illinois.

29. "Product Moment Correlations, Variance-Covariances, Means, and Standard Deviations for Use with Magnetic Drum," K 8, Digital Computer Laboratory, University of Illinois.
30. "Regression Analysis," R 3, Statistical Services Unit, University of Illinois.
31. "Varimax Rotation of Factors," KSL 1.80, Digital Computer Laboratory, University of Illinois.

Appendix

TABLE 6

MEANS AND STANDARD DEVIATIONS OF THE STUDY VARIABLES

Variable	Mean	Standard Deviation
1	21.169	06.494
2	19.794	15.887
3	57.077	10.803
4	08.253	03.578
5	08.859	04.674
6	06.175	04.500
7	00.174	00.400
8	05.540	06.300
9	054.06	060.76
10	.03476	.03298
11	.01564	.02478
12	0.0360	3.6511
13	0644.6	0500.4
14	001.27	050.07
15	1279.7	1289.9
16	10.477	01.163
17	08.396	04.455
18	0.3625	0.4807
19	01.213	02.598
20	03.170	04.636
21	12.625	09.037
22	219.00	148.35
23	049.02	033.34
24	19.120	15.432
25	00.259	00.692
26	09.905	10.652
27	108.94	073.16
28	50.266	04.781
29	52.239	13.089
30	0.9488	0.2203
31	0.4989	0.5000
32	0.6333	0.4819
33	04.753	05.685
34	0.1919	0.3938
35	0.1770	0.3816
36	0.1940	0.3954
37	0.2495	0.4327
38	0.1876	0.3904
39	0.1727	0.3780
40	0.4072	0.4913
41	0.3262	0.4688
42	0.0938	0.2916
43	0.5885	0.4921
44	0.3539	0.4782
45	0.0405	0.1971
46	0.0170	0.1295
47	0.0767	0.2662
48	0.2814	0.4497
49	41.419	07.269

TABLE 7

CORRELATION OF SPEED WITH THE OTHER VARIABLES

Variable	Correlation Coefficient
1	-0.0483
2	+0.1387
3	+0.0624
4	-0.0497
5	-0.0959
6	+0.2240
7	+0.0220
8	-0.7044
9	-0.4635
10	-0.4258
11	-0.3015
12	-0.0950
13	+0.1036
14	-0.1528
15	+0.3325
16	+0.1133
17	+0.0495
18	+0.0789
19	-0.2159
20	-0.1976
21	-0.2776
22	+0.0244
23	-0.1043
24	-0.2044
25	+0.3225
26	-0.0520
27	+0.0242
28	+0.0221
29	+0.7828
30	+0.1054
31	-0.3319
32	-0.3274
33	-0.1829
34	-0.0155
35	+0.0287
36	-0.0595
37	+0.0485
38	-0.0059
39	+0.0953
40	-0.0642
41	-0.0213
42	+0.0189
43	-0.1112
44	+0.1239
45	-0.0271
46	+0.0063
47	-0.0364
48	-0.0854

TABLE 8

CORRELATION MATRIX

	1	2	3	4	5	6	7	8	9	10	11	12	13
1		-3044	0371	1406	-0086	-2464	0567	0069	-0886	-0446	0809	-0327	-0433
2	-3044		4690	-2125	-1047	3763	0883	-0603	-0320	-0207	-0300	0280	0937
3	0371	4690		-2288	-1553	1572	0721	0453	0821	0387	-0422	0078	0395
4	1406	-2125	-2288		0971	-2676	0357	0223	0153	0913	0295	-0355	-0690
5	-0086	-1047	-1553	0971		0351	0132	0732	0116	1237	0073	-0247	-0431
6	-2464	3763	1572	-2676	0351		0031	-1173	-1765	-0196	-0746	-0194	-0421
7	0567	0883	0721	0357	0132	0031		-0012	0309	0277	-0153	-0243	0830
8	0069	-0603	0453	0223	0732	-1173	-0012		4745	4927	3725	-0013	-0774
9	-0886	-0320	0821	0153	0116	-1765	0309	4745		2712	1925	1024	-0021
10	-0446	-0207	0387	0913	1237	-0196	0277	4927	2712		2653	0026	-0733
11	0809	-0300	-0422	0295	0073	-0746	-0153	3725	1925	2653		0379	-0549
12	-0327	0280	0078	-0355	-0247	-0194	-0243	-0013	1024	0026	0379		0609
13	-0433	0937	0395	-0690	-0431	-0421	0830	-0774	-0021	-0733	-0549	0609	
14	-0782	0353	0046	0067	0675	-0140	-0830	0675	1102	0834	0229	4015	0072
15	0088	0450	0471	-0155	-0393	0856	-0319	-2804	-2061	-3116	-1591	-2930	0485
16	-0775	0697	-0207	-1509	-1317	2036	0263	-0745	-0213	-0218	-0657	0205	0189
17	-0182	0250	-0041	-0890	-1023	2198	-0586	-0456	-0642	0380	0025	0113	-0014
18	-0560	0810	0706	1020	1127	-0424	0501	-0743	0589	-0155	-0538	-0071	0030
19	0862	0178	-0564	0329	-0234	-0323	0190	-0409	-0588	-1280	-0595	0049	-0825
20	0704	-1279	-0538	-1172	-0242	-0414	-0507	1671	0887	-0330	0364	-0034	-1042
21	2347	-1151	-1393	0380	-0627	-1447	0251	-0370	-0598	-1336	-0152	0004	-0695
22	0526	0177	-0952	-3228	-1597	0402	0164	-2972	-2166	-2509	-1234	0121	-0060
23	-0332	-1227	0259	3545	2329	-0351	-0405	2862	2165	2556	0938	-0146	0133
24	-1111	0610	-1280	-2516	-1213	0630	0067	-1283	0216	-1198	-0463	0010	-0128
25	-0688	1413	0953	-1067	-0324	1132	0054	-2796	-1652	-3206	-1820	-0892	1282
26	-0554	-0925	-1946	-2336	-1055	0725	0005	-1714	-1006	-1510	-0636	0059	-0722
27	0500	0206	-0957	-3142	-1365	0511	0380	-2939	-2177	-2390	-1162	0100	0063
28	-0356	-0557	0036	-0802	-1319	-0464	-0795	-0175	0729	-1017	-0340	-0111	-0088
29	-0326	0675	-0253	-0097	-0722	1617	-0239	-7648	-4141	-4476	-2593	0080	0753
30	-0644	0126	-0517	-0034	-0332	0463	0037	-0880	-0945	-1581	-2275	-0162	0313
31	-0022	-0056	0313	1281	0710	-2352	0365	3160	2848	2458	1152	0221	-0327
32	-1034	0305	0033	0036	0231	0224	0128	4166	3084	4346	2662	-0010	-0432
33	0893	-0429	-0891	-1783	-0896	0161	0732	-0543	-0523	-0497	-0151	-0008	-0952
34	0212	-0683	0093	-0375	-0046	-1424	-0308	0677	0270	-0074	-0148	-0233	0986
35	-0320	1185	-0075	0557	0981	0085	-0334	-0295	0059	0210	-0222	0185	-0253
36	0115	-0077	-0400	1251	0783	-0414	0482	-0010	-0021	-0121	-0230	-0109	-0333
37	-0064	0693	0772	-0334	-0918	1323	0126	-0090	0394	0318	-0008	0363	-0367
38	0054	-1159	-0471	-1063	-0688	0306	0010	-0284	-0745	-0361	0609	-0237	-0004
39	-1032	-0624	-2821	0760	0939	0700	0090	-0878	-0497	0042	0715	0239	0342
40	0318	0070	0621	1079	0390	0681	0237	0805	0853	0760	-0123	-0621	-0488
41	0384	0364	1832	-1204	-0409	-0816	-0515	0104	-0407	-0372	-0304	0148	-0091
42	0185	0105	-0335	-0868	-1217	-0742	0311	-0386	-0138	-0737	-0231	0499	0524
43	0764	-1017	-1043	-0910	-0133	-0047	-0490	0308	0366	-0701	0462	-0554	0548
44	-0128	1169	1102	0781	0065	-0109	0432	-0425	-0365	0454	-0460	0297	-0741
45	-1230	-0478	-0040	0278	0239	0273	0010	0421	0225	0589	-0010	0345	-0480
46	-0555	0275	-0044	0152	-0101	0169	0251	-0242	-0386	0089	-0043	0481	1387
47	-1288	-0291	-0274	0423	-0015	-0112	-0050	0328	0175	0611	0012	0389	0356
48	0321	0227	0033	-0601	-0155	-0243	-0027	0067	0041	0033	-0025	-0111	-0195

Note: These correlation coefficients are scaled by 10^4.

TABLE 8

CORRELATION MATRIX (Continued)

	14	15	16	17	18	19	20	21	22	23	24	25	26
1	-0782	0088	-0775	-0182	-0560	0862	0704	2347	0526	-0332	-1111	-0688	-0554
2	0353	0450	0697	0250	0810	0178	-1279	-1151	0177	-1227	0610	1413	-0925
3	0046	0471	-0207	-0041	0706	-0564	-0538	-1393	-0952	0259	-1280	0953	-1946
4	0067	-0155	-1509	-0890	1020	0329	-1172	0380	-3228	3545	-2516	-1067	-2336
5	0675	-0393	-1317	-1023	1127	-0234	-0242	-0627	-1597	2329	-1213	-0324	-1055
6	-0140	0856	2036	2198	-0424	-0323	-0414	-1447	0402	-0351	0630	1132	0725
7	-0830	-0319	0263	-0586	0501	0190	-0507	0251	0164	-0405	0067	0054	0005
8	0675	-2804	-0745	-0456	-0743	-0409	1671	-0370	-2972	2862	-1283	-2796	-1714
9	1102	-2061	-0213	-0642	0589	-0588	0887	-0598	-2166	2165	0216	-1652	-1006
10	0834	-3116	-0218	0380	-0155	-1280	-0330	-1336	-2509	2556	-1198	-3206	-1510
11	0229	-1591	-0657	0025	-0538	-0595	0364	-0152	-1234	0938	-0463	-1820	-0636
12	4015	-2930	0205	0113	-0071	0049	-0034	0004	0121	-0146	0010	-0892	0059
13	0072	0485	0189	-0014	0030	-0825	-1042	-0695	-0060	0133	-0128	1282	-0722
14		-1337	-0163	-0316	0035	-0593	-0393	-0469	-0332	0672	0448	-0875	0214
15	-1337		-0018	0996	-0363	0064	-0293	-0203	1248	-0904	-0671	2974	0611
16	-0163	-0018		0478	-0877	0807	0266	0282	2788	-3051	1509	0957	1851
17	-0316	0996	0478		-2423	-0371	0562	-0422	0685	-0311	1048	0995	1398
18	0035	-0363	-0877	-2423		-0448	-0172	-0541	-0840	0907	-0723	0173	-1667
19	-0593	0064	0807	-0371	-0448		3534	6615	3372	-2521	2169	-0526	1469
20	-0393	-0293	0266	0562	-0172	3534		3190	1640	-1400	0474	-0605	0862
21	-0469	-0203	0282	-0422	-0541	6615	3190		3511	-2939	2622	-0791	2360
22	-0332	1248	2788	0685	-0840	3372	1640	3511		-7125	6717	0853	6492
23	0672	-0904	-3051	-0311	0907	-2521	-1400	-2939	-7125		-4897	-1191	-4041
24	0448	-0671	1509	1048	-0723	2169	0474	2622	6717	-4897		-0182	6487
25	-0875	2974	0957	0995	0173	-0526	-0605	-0791	0853	-1191	-0182		-0047
26	0214	0611	1851	1398	-1667	1469	0862	2360	6492	-4041	6487	-0047	
27	-0266	1299	2760	0670	-0856	3322	1656	3385	9839	-7054	6556	0883	6588
28	-0503	0104	0050	0415	0182	0221	-0196	0348	0054	-0374	0254	0031	-0943
29	-0630	2397	0397	0266	1336	-1112	-1402	-1390	0825	-1523	-0793	2665	-0052
30	-0032	0518	1528	-0423	0543	0488	-0165	0032	0184	0145	0319	0494	0367
31	0870	-3004	-1212	-1692	2322	-0343	-1416	-0704	-2091	1747	-0221	-2293	-2219
32	0892	-2341	0165	-0762	1228	-0431	0614	-0947	-1156	1413	-0084	-3379	-1112
33	-0484	0039	1525	0508	-0109	2629	1669	2683	5379	-4032	4571	-0426	3894
34	-0194	0407	0201	0040	-0295	-0816	0825	-0091	-0178	0113	-0021	-0100	-0268
35	-0028	0267	0349	-0400	-0358	0565	-0628	-0302	-0315	0843	-0356	-0119	0027
36	0239	-0383	-1412	-1320	0675	0448	-0250	0836	-0379	0595	-0219	-0691	-0158
37	0663	-0582	0679	0836	-0144	-0587	-0967	-1478	0082	-0560	-0146	0027	-0087
38	-0753	0362	0135	0761	0125	0467	1107	1178	0781	-0921	0753	0887	0501
39	0622	0430	-0043	1074	0192	-0483	-0971	0177	0348	-0948	0272	-0132	0418
40	-1075	-0404	-0053	-0241	0340	0856	0650	0315	-1383	1563	-0480	0203	-0795
41	0629	-0306	0200	-0548	-0232	-0238	0538	-0541	0134	-0189	-0674	0378	-0225
42	-0006	0617	-0177	-0106	-0448	-0433	-0702	0109	1664	-1099	1539	-0779	1159
43	0146	-0182	0454	0987	0176	1270	1401	1158	1512	-0698	2565	-0255	1831
44	-0321	0367	0081	-0809	-0479	-0968	-1138	-0951	-0995	0206	-2529	0540	-1742
45	-0527	-0643	-1114	-0790	0925	-0710	-0589	-0525	-1093	0801	-0355	-0559	-0415
46	1431	0314	-0329	0437	-0308	-0171	-0226	-0091	-0406	0673	0132	-0174	0104
47	-0070	-0544	-1026	-0868	0825	-0760	-0711	-0598	-1027	1001	-0247	-0537	-0180
48	0085	-0240	-0295	-0738	0212	2132	0578	1923	1746	-2338	0353	-0603	0062

Note: These correlation coefficients are scaled by 10^4.

TABLE 8

CORRELATION MATRIX (Continued)

	27	28	29	30	31	32	33	34	35	36	37	38	39
1	0500	-0356	-0326	-0644	-0022	-1034	0893	0212	-0320	0115	-0064	0054	-1032
2	0206	-0557	0675	0126	-0056	0305	-0429	-0683	1185	-0077	0693	-1159	-0624
3	-0957	0036	-0253	-0517	0313	0033	-0891	0093	-0075	-0400	0772	-0471	-2821
4	-3142	-0802	-0097	-0034	1281	0036	-1783	-0375	0557	1251	-0334	-1063	0760
5	-1365	-1319	-0722	-0332	0710	0231	-0896	-0046	0981	0783	-0918	-0688	0939
6	0511	-0464	1617	0463	-2352	0224	0161	-1424	0085	-0414	1323	0306	0700
7	0380	-0795	-0239	0037	0365	0128	0732	-0308	-0334	0482	0126	0010	0090
8	-2939	-0175	-7648	-0880	3160	4166	-0543	0677	-0295	-0010	-0090	-0284	-0878
9	-2177	0729	-4141	-0945	2848	3084	-0523	0270	0059	-0021	0394	-0745	-0497
10	-2390	-1017	-4476	-1581	2458	4346	-0497	-0074	0210	-0121	0318	-0361	0042
11	-1162	-0340	-2593	-2275	1152	2662	-0151	-0148	-0222	-0230	-0008	0609	0715
12	0100	-0111	0080	-0162	0221	-0010	-0008	-0233	0185	-0109	0363	-0237	0239
13	0063	-0088	0753	0313	-0327	-0432	-0952	0986	-0253	-0333	-0367	-0004	0342
14	-0266	-0505	-0630	-0032	0870	0892	-0484	-0194	-0028	0239	0663	-0753	0622
15	1299	0104	2397	0518	-3004	-2341	0039	0407	0267	-0383	-0582	0362	0430
16	2760	0050	0397	1528	-1212	0165	1525	0201	0349	-1412	0679	0135	-0043
17	0670	0415	0266	-0423	-1692	-0762	0508	0040	-0400	-1320	0836	0761	1074
18	-0856	0182	1336	0543	2322	1228	-0109	-0295	-0358	0675	-0144	0125	0192
19	3322	0221	-1112	0488	-0343	-0431	2629	-0816	0565	0448	-0587	0467	-0483
20	1656	-0196	-1402	-0165	-1416	0614	1669	0825	-0628	-0250	-0967	1107	-0971
21	3385	0348	-1390	0032	-0704	-0947	2683	-0091	-0302	0836	-1478	1178	0177
22	9839	0054	0825	0184	-2091	-1156	5379	-0178	-0315	-0379	0082	0781	0348
23	-7054	-0374	-1523	0145	1747	1413	-4032	0113	0843	0595	-0560	-0921	-0948
24	6556	0254	-0793	0319	-0221	-0084	4571	-0021	-0356	-0219	-0146	0753	0272
25	0883	0031	2665	0494	-2293	-3379	-0426	-0100	-0119	-0691	0027	0887	-0132
26	6588	-0943	-0052	0367	-2219	-1112	3894	-0268	0027	-0158	-0087	0501	0418
27		-1268	0808	0240	-2127	-1060	5390	-0200	-0201	-0491	0074	0813	0380
28	-1268		0287	-0330	0211	-0729	0189	0084	-1024	0418	0408	0041	0647
29	0808	0287		0841	-2814	-3549	-1033	-0151	0039	-0365	-0101	0596	0554
30	0240	-0330	0841		1930	1445	-0237	0886	0063	-0818	0892	-1114	-0987
31	-2127	0211	-2814	1930		3966	-0488	0552	-0493	0280	0850	-1300	-1062
32	-1060	-0729	-3549	1445	3966		-0035	-0336	0051	-0182	0809	-0422	-0385
33	5390	0189	-1033	-0237	-0488	-0035		0050	-0339	-0972	0043	1218	0099
34	-0200	0084	-0151	0886	0552	-0336	0050		-2260	-2391	-2809	-2342	-1224
35	-0201	-1024	0039	0063	-0493	0051	-0339	-2260		-2275	-2673	-2228	-0345
36	-0491	0418	-0365	-0818	0280	-0182	-0972	-2391	-2275		-2829	-2358	-0245
37	0074	0408	-0101	0892	0850	0809	0043	-2809	-2673	-2829		-2771	0494
38	0813	0041	0596	-1114	-1300	-0422	1218	-2342	-2228	-2358	-2771		1272
39	0380	0647	0554	-0987	-1062	-0385	0099	-1224	-0345	-0245	0494	1272	
40	-1358	0078	-0705	-0241	-0026	0815	-0059	-0402	0477	0103	-0366	0240	-3787
41	0193	-0949	0425	0790	0879	-0461	-0009	0882	-0367	0266	-0228	-0548	-3179
42	1486	0554	-0215	0415	0007	-0131	-0014	0846	0232	-0284	0342	-1172	-1470
43	1463	0578	-0854	0417	0025	0110	1275	1104	-0209	-0499	-0085	-0309	-0306
44	-0947	-0523	0590	-0304	-0162	-0381	-0948	-0889	0306	0089	0370	0097	0393
45	-1091	-0067	0268	0477	0761	0666	-0557	-0177	-0669	1453	-0685	0120	-0367
46	-0402	-0161	0655	-1189	-0656	-0022	-0493	-0642	0683	-0646	0002	0632	0269
47	-0960	-0570	0425	-0057	0647	0865	-0593	-0388	0342	0408	-0552	0255	-0258
48	1836	-0213	-0781	0162	0203	1122	1882	-0762	-0417	0406	0994	-0336	0151

Note: These correlation coefficients are scaled by 10^4.

TABLE 8

CORRELATION MATRIX (Continued)

	40	41	42	43	44	45	46	47	48
1	0318	0384	0185	0764	-0128	-1230	-0555	-1288	0321
2	0070	0364	0105	-1017	1169	-0478	0275	-0291	0227
3	0621	1832	-0335	-1043	1102	-0040	-0044	-0274	0033
4	1079	-1204	-0868	-0910	0781	0278	0152	0423	-0601
5	0390	-0409	-1217	-0133	0065	0239	-0101	-0015	-0155
6	0681	-0816	-0742	-0047	-0109	0273	0169	-0112	-0243
7	0237	-0515	0311	-0490	0432	0010	0251	-0050	-0027
8	0805	0104	-0386	0308	-0425	0421	-0242	0328	0067
9	0853	-0407	-0138	0366	-0365	0225	-0386	0175	0041
10	0760	-0372	-0737	-0701	0454	0589	0089	0611	0033
11	-0123	-0304	-0231	0462	-0460	-0010	-0043	0012	-0025
12	-0621	0148	0499	-0554	0297	0345	0481	0389	-0111
13	-0488	-0091	0524	0548	-0741	-0480	1387	0356	-0195
14	-1075	0629	-0006	0146	-0321	-0527	1431	-0070	0085
15	-0404	-0306	0617	-0182	0367	-0643	0314	-0544	-0240
16	-0053	0200	-0177	0454	0081	-1114	-0329	-1026	-0295
17	-0241	-0548	-0106	0987	-0809	-0790	0437	-0868	-0738
18	0340	-0232	-0448	0176	-0479	0925	-0308	0825	0212
19	0856	-0238	-0433	1270	-0968	-0710	-0171	-0760	2132
20	0650	0538	-0702	1401	-1138	-0589	-0226	-0711	0578
21	0315	-0541	0109	1158	-0951	-0525	-0091	-0598	1923
22	-1383	0134	1664	1512	-0995	-1093	-0406	-1027	1746
23	1563	-0189	-1099	-0698	0206	0801	0673	1001	-2338
24	-0480	-0674	1539	2565	-2529	-0355	0132	-0247	0353
25	0203	0378	-0779	-0255	0540	-0559	-0174	-0537	-0603
26	-0795	-0225	1159	1831	-1742	-0415	0104	-0180	0062
27	-1358	0193	1486	1463	-0947	-1091	-0402	-0960	1836
28	0078	-0949	0554	0578	-0523	-0067	-0161	-0570	-0213
29	-0705	0425	-0215	-0854	0590	0268	0655	0425	-0781
30	-0241	0790	0415	0417	-0304	0477	-1189	-0057	0162
31	-0026	0879	0007	0025	-0162	0761	-0656	0647	0203
32	0815	-0461	-0131	0110	-0381	0666	-0022	0865	1122
33	-0059	-0009	-0014	1275	-0948	-0557	-0493	-0593	1882
34	-0402	0882	0846	1104	-0889	-0177	-0642	-0388	-0762
35	0477	-0367	0232	-0209	0306	-0669	0683	0342	-0417
36	0103	0266	-0284	-0499	0089	1453	-0646	0408	0406
37	-0366	-0228	0342	-0085	0370	-0685	0002	-0552	0994
38	0240	-0548	-1172	-0309	0097	0120	0632	0255	-0336
39	-3787	-3179	-1470	-0306	0393	-0367	0269	-0258	0151
40		-5767	-2667	0141	-0327	0498	-0086	0544	-0073
41	-5767		-2239	0181	-0205	-0046	0137	-0127	-0107
42	-2667	-2239		-0133	0371	-0290	-0424	-0378	0100
43	0141	0181	-0133		-8851	-2457	-1575	-3285	-0836
44	-0327	-0205	0371	-8851		-1521	-0975	-0794	0821
45	0498	-0046	-0290	-2457	-1521		-0271	7126	0397
46	-0086	0137	-0424	-1575	-0975	-0271		4569	-0458
47	0544	-0127	-0378	-3285	-0794	7126	4569		0154
48	-0073	-0107	0100	-0836	0821	0397	-0458	0154	

Note: These correlation coefficients are scaled by 10^4.

TABLE 9

PRINCIPAL-FACTOR MATRIX

Vari-able	Factor								
	A	B	C	D	E	F	G	H	I
1	+0.0466	+0.0996	-0.4694	-0.0410	-0.0725	-0.2221	+0.1789	-0.1425	+0.0570
2	+0.0321	-0.2005	+0.6037	-0.1189	+0.2857	+0.0228	+0.2605	+0.0800	+0.1677
3	-0.1231	-0.1435	+0.4156	-0.1940	+0.3647	-0.1759	+0.2931	-0.2558	+0.2065
4	-0.3672	-0.0356	-0.4458	+0.2149	-0.1181	-0.0126	+0.0593	+0.2737	-0.0601
5	-0.2254	+0.0204	-0.1676	+0.1043	-0.0373	+0.1247	-0.0172	+0.2710	+0.0518
6	+0.1327	-0.2406	+0.4580	-0.1236	+0.1448	+0.4335	+0.1087	+0.0837	+0.0150
7	+0.0097	+0.0167	+0.0730	+0.0876	+0.0558	-0.0496	+0.1530	+0.0447	-0.0787
8	-0.4685	+0.6524	+0.0876	-0.1336	-0.0135	+0.0683	+0.0741	-0.2517	-0.0098
9	-0.3450	+0.4772	+0.1385	-0.1072	+0.0408	+0.0099	-0.0045	-0.0346	+0.0052
10	-0.4431	+0.4710	+0.2137	-0.0351	-0.1588	+0.1589	+0.0689	-0.0844	-0.0625
11	-0.2269	+0.3856	+0.0207	-0.0872	-0.2330	+0.1535	+0.0008	-0.1955	+0.0436
12	-0.0296	+0.1011	+0.2064	+0.0528	-0.1978	-0.1032	-0.1500	+0.2496	+0.4887
13	+0.0048	-0.1596	+0.0750	-0.0935	+0.0789	-0.0443	-0.2202	-0.0139	+0.1309
14	-0.0895	+0.1339	+0.2081	-0.0350	-0.1998	-0.0835	-0.2214	+0.3308	+0.4789
15	+0.2353	-0.4505	-0.1553	-0.0703	+0.0707	+0.0662	+0.0628	-0.1796	-0.1710
16	+0.3228	+0.0232	+0.2497	-0.1721	+0.0196	+0.0638	+0.0871	+0.0445	-0.0589
17	+0.1593	-0.0659	+0.0742	-0.2932	-0.1572	+0.3823	-0.0821	-0.1955	+0.0045
18	-0.1511	-0.0464	+0.0160	+0.1971	+0.2900	-0.1394	-0.0354	+0.3319	-0.0330
19	+0.4022	+0.3012	-0.2459	+0.2249	+0.2419	-0.0051	+0.3472	+0.1049	+0.2809
20	+0.2066	+0.3237	-0.2313	+0.0008	+0.1868	+0.0656	+0.1579	-0.2509	+0.2644
21	+0.4307	+0.3400	-0.3736	+0.2695	+0.0969	-0.0510	+0.2487	+0.0025	+0.2330
22	+0.8942	+0.1858	+0.1175	+0.0896	-0.0607	-0.0926	+0.0034	+0.0147	-0.0660
23	-0.7621	-0.0699	-0.1726	-0.0576	+0.0779	+0.1381	-0.0977	+0.0407	+0.0338
24	+0.6684	+0.3454	+0.1938	+0.0521	+0.0208	+0.0746	-0.2116	+0.0937	-0.1286
25	+0.2083	-0.5005	+0.0094	-0.1237	+0.1552	+0.0665	+0.0547	-0.1037	+0.0377
26	+0.6815	+0.2149	+0.0929	+0.0620	-0.1159	+0.1050	-0.2106	+0.0108	-0.1271
27	+0.8872	+0.1852	+0.1289	+0.0984	-0.0635	-0.0855	+0.0100	+0.0189	-0.0521
28	+0.0146	-0.0241	-0.0858	-0.0950	+0.0181	+0.0256	-0.0452	-0.0149	-0.1140
29	+0.2443	-0.7295	-0.0548	+0.1025	+0.0389	-0.0465	-0.1411	+0.1542	+0.0419
30	+0.0577	-0.0642	+0.0750	-0.0438	+0.2892	-0.2667	-0.0843	+0.2807	-0.2427
31	-0.3815	+0.3895	+0.1191	+0.0324	+0.1360	-0.3444	-0.0699	+0.2303	-0.1661
32	-0.3123	+0.5266	+0.3076	+0.0198	+0.0704	+0.0251	+0.0240	+0.1098	-0.1655
33	+0.5569	+0.3387	+0.0478	+0.0963	-0.0111	+0.0071	+0.0841	-0.0450	-0.0687
34	-0.0117	+0.0475	-0.1453	-0.2487	+0.1587	-0.2877	-0.2687	-0.2892	-0.1472
35	-0.0425	-0.0562	+0.0348	+0.0418	+0.0584	+0.1369	+0.1088	+0.1934	+0.1670
36	-0.0883	+0.0212	-0.1482	+0.3079	+0.0689	-0.1378	+0.0294	+0.1495	+0.0006
37	-0.0094	+0.0005	+0.3239	-0.2118	-0.1386	-0.0290	+0.0954	+0.2650	-0.1538
38	+0.1531	-0.0149	-0.0963	+0.1328	-0.1333	+0.3281	+0.0292	-0.3424	+0.1551
39	+0.0705	-0.0712	-0.0446	+0.0602	-0.5642	+0.3025	-0.1202	+0.2592	-0.0037
40	-0.1487	+0.0729	-0.1103	+0.1452	+0.4684	+0.4751	+0.3361	+0.0582	-0.1663
41	+0.0076	-0.0390	+0.0976	-0.1587	+0.0691	-0.5774	-0.1895	-0.2784	+0.3763
42	+0.1470	+0.0322	+0.0868	-0.0675	-0.1689	-0.2643	-0.1058	+0.0135	-0.3200
43	+0.2524	+0.3007	-0.3113	-0.5097	+0.3790	+0.1899	-0.3901	+0.2227	+0.0637
44	-0.1741	-0.2820	+0.2060	+0.1863	-0.4627	-0.2571	+0.6132	-0.1075	-0.0832
45	-0.1768	-0.0096	+0.1776	+0.6237	+0.2042	+0.0190	-0.3368	-0.1985	-0.1965
46	-0.0471	-0.0866	+0.1521	+0.2994	-0.0422	+0.1986	-0.2689	-0.1471	+0.3644
47	-0.1840	-0.0427	+0.2498	+0.7197	+0.1279	+0.0882	-0.3887	-0.2497	-0.0161
48	+0.1556	+0.1815	+0.0874	+0.2381	-0.0173	-0.1724	+0.2971	+0.0811	+0.0032

TABLE 9

PRINCIPAL-FACTOR MATRIX (Continued)

Vari-able	Factor							
	J	K	L	M	N	O	P	Q
1	+0.1263	+0.0756	-0.0616	-0.0409	-0.0242	-0.3461	+0.3489	-0.0655
2	-0.0754	-0.0620	+0.0057	+0.3018	+0.0562	+0.0130	-0.0933	-0.0083
3	+0.0736	-0.1036	-0.0969	+0.1632	-0.0695	-0.1561	+0.0987	+0.0953
4	+0.0052	+0.1454	+0.0113	-0.0309	+0.1164	-0.0832	+0.1259	-0.0936
5	-0.4143	+0.0117	+0.2020	+0.0453	-0.0001	+0.1848	+0.1429	+0.3322
6	-0.1959	-0.1935	+0.0108	-0.1227	+0.1451	+0.0992	+0.0400	-0.0512
7	+0.0839	+0.0859	+0.2181	+0.3684	+0.0358	+0.0413	+0.4787	-0.2753
8	-0.0605	+0.0039	+0.0199	+0.0500	+0.0972	+0.0824	-0.0445	+0.0300
9	+0.1461	+0.0257	-0.0589	+0.1372	-0.0853	+0.1116	-0.1205	+0.1138
10	-0.1582	+0.0322	+0.0777	-0.0015	+0.0019	-0.0164	+0.1203	-0.0665
11	-0.0023	-0.0912	+0.0744	+0.1387	-0.0882	-0.1857	-0.0105	-0.1121
12	+0.2184	+0.0923	-0.3203	-0.1313	-0.1870	+0.2198	+0.0874	-0.0636
13	+0.2816	+0.2341	+0.1856	+0.4277	+0.1471	+0.1906	+0.0969	-0.2199
14	+0.0496	+0.0527	-0.1926	-0.0713	-0.0254	+0.1685	+0.1043	+0.2434
15	-0.0887	+0.0527	+0.1129	+0.1574	+0.2448	-0.0399	-0.0983	+0.2922
16	+0.0027	+0.1953	+0.1131	-0.2587	-0.0455	+0.1734	-0.0399	-0.4827
17	+0.0188	-0.0423	-0.1262	-0.1483	+0.2801	+0.0195	+0.1127	+0.0386
18	+0.1494	-0.1650	+0.3525	+0.1231	-0.3320	-0.0326	-0.0481	+0.2527
19	+0.0505	+0.0133	+0.0229	-0.0336	+0.2830	+0.0460	-0.1678	-0.0520
20	+0.0107	-0.0449	+0.0451	-0.2066	+0.1231	+0.2301	-0.1158	+0.1428
21	+0.1070	+0.0025	-0.0114	+0.0363	+0.2056	+0.0752	-0.0975	-0.0717
22	-0.0403	+0.0301	-0.0016	+0.0519	-0.0678	-0.0268	+0.0402	+0.0474
23	-0.1059	+0.0780	-0.0576	-0.0399	+0.0551	+0.0065	+0.0412	+0.0831
24	-0.0407	+0.0183	-0.0853	+0.1239	-0.1275	-0.0015	+0.0222	+0.0791
25	+0.0541	-0.0096	+0.1193	+0.0438	-0.0403	+0.1196	+0.0529	+0.1254
26	-0.2139	+0.0780	-0.1400	-0.0257	-0.0547	-0.0101	+0.1242	+0.0615
27	-0.0924	+0.0664	+0.0433	+0.0476	-0.0580	-0.0232	+0.1044	+0.0593
28	+0.4465	-0.2890	-0.2311	+0.0589	-0.0315	+0.0166	-0.4514	-0.0747
29	+0.0951	-0.0342	+0.0350	-0.1296	-0.1549	-0.0653	+0.0102	-0.0607
30	+0.0166	+0.1954	+0.1819	-0.3902	+0.2080	+0.1900	-0.0859	-0.1179
31	+0.1248	+0.0015	+0.1875	-0.0243	-0.0938	-0.0724	-0.0243	+0.0370
32	-0.0022	+0.0415	+0.2380	-0.0821	+0.0363	-0.0161	-0.0763	-0.0349
33	+0.0221	+0.0034	+0.1223	-0.0492	-0.1357	-0.0709	+0.1049	+0.1227
34	+0.0297	+0.2721	+0.0205	+0.0241	+0.0460	+0.4809	+0.0492	+0.1449
35	-0.3947	+0.5475	-0.0520	+0.0956	-0.0797	-0.3452	-0.4041	-0.0915
36	-0.2887	-0.5207	-0.3256	+0.3083	+0.0584	+0.1738	+0.1243	-0.2216
37	+0.3724	-0.1493	-0.0119	-0.3027	+0.3532	-0.3370	+0.2426	+0.1573
38	+0.2356	-0.1167	+0.3731	-0.0945	-0.4192	+0.0499	-0.0494	-0.0066
39	+0.0822	-0.2091	+0.3597	+0.1609	+0.1545	+0.1401	-0.1719	-0.0437
40	+0.1806	+0.2091	-0.2779	-0.1012	-0.2059	+0.0802	+0.2284	+0.0574
41	-0.3285	-0.1949	+0.2140	-0.2205	+0.0405	-0.1672	+0.0105	-0.0931
42	+0.1171	+0.2320	-0.3421	+0.3164	+0.0815	-0.0478	-0.1789	+0.1097
43	-0.0073	-0.1396	+0.0440	+0.0884	+0.0110	-0.1625	+0.0204	-0.0546
44	-0.0283	+0.1397	-0.0249	-0.0647	-0.1014	+0.2114	-0.0342	+0.0621
45	-0.0279	-0.1847	-0.0898	-0.1448	+0.0565	+0.0274	-0.0126	-0.0761
46	+0.1753	+0.2959	+0.0614	+0.1233	+0.2468	-0.2051	+0.0681	+0.0941
47	+0.0664	+0.0981	-0.0119	-0.0703	+0.1459	-0.1062	-0.0020	+0.0006
48	+0.1194	-0.1385	+0.1188	+0.0028	+0.2781	-0.0468	-0.0807	+0.1512

TABLE 10

CONTRIBUTIONS OF THE 17 PRINCIPAL FACTORS
(Unities in Diagonal of Correlation Matrix)

Factor	Eigenvalue	Percent of Total Variance	Cum. Percent of Total Variance
A	5.42	11.29	11.29
B	3.53	7.36	18.65
C	2.40	4.99	23.64
D	2.17	4.51	28.15
E	1.86	3.88	32.03
F	1.92	4.00	36.03
G	1.95	4.06	40.09
H	1.65	3.44	43.53
I	1.54	3.21	46.74
J	1.41	2.94	49.68
K	1.43	2.97	52.65
L	1.35	2.81	55.46
M	1.34	2.80	58.26
N	1.20	2.51	60.77
O	1.21	2.51	63.28
P	1.17	2.45	65.73
Q	1.02	2.12	67.85

Discussion

DAVID SOLOMON, U. S. Bureau of Public Roads.—The author is to be complimented on a substantial contribution to an understanding of the relationship between speed and numerous other variables. These questions then arise: Is the procedure valid for use, and if so where? Which procedure is more desirable for practical application, the regression analysis or the factor analysis? How can the procedure be used?

The factor analysis is slightly more accurate in its ability to predict speeds. The standard error of estimate for the factor analysis is 4.1 mph compared to 4.5 mph for the regression analysis. Thus, about two-thirds of the estimates of average speed would be within 4.1 or 4.5 mph. Similarly, the multiple correlation coefficient was 0.82 for the factor analysis and 0.79 for the regression analysis.

In the 10 test studies used to validate the data, the average speed was always estimated to within 5 mph in the factor analysis; whereas in the regression analysis, 4 of the ten studies exceeded 5 mph. If 4 mph is chosen as the criterion, the factor analysis showed that 3 of the 10 studies exceeded this value. In the regression analysis, 5 of the 10 studies exceeded the criterion. Again the factor analysis produced a slightly more accurate prediction of average speeds.

This greater accuracy was obtained at a price, however, and the price was the requirement, in the factor analysis, that 21 variables be measured to insert in the 5 factors employed. In the regression analysis, only 8 variables were required. Moreover, the 8 variables needed in the regression analysis were more easily obtainable because 4 were geometric design elements usually available from plans, 3 were volume or classification counts usually available in state highway departments, and the final variable was the number of commercial establishments which is frequently available from maps or easily obtained in the field. By way of contrast, the factor analysis required the obtaining of such variables as the percent of vehicles passing at the time of speed measurement, and the reciprocal of the test car speed.

It is seen, therefore, that the regression analysis with its 8 easily measured variables is a simpler technique for application. However, the question still remains as to whether the equation for the regression analysis is valid in other states, or in other areas, or for other highways. This points up the desirability of validating the equation by selecting a few dozen study sites in other states where the 8 variables and the average speeds could be measured. The measured speeds could then be compared with the average speed as computed from the regression analysis. This is certainly a much easier process than trying to repeat the regression analysis itself. The results of such studies will show whether the procedures can be used elsewhere or whether some modification is required. These validation studies are relatively easy to do and could well be incorporated into a regular program of spot-speed studies.

If the validation studies indicate that the equations can be used elsewhere, the next question is how can the equations be used. One obvious use for such procedures would be in speed zoning a section of highway without actually measuring the speeds of traffic. Thus, conceivably an entire highway network could be speed zoned from design and traffic data available in many highway department offices.

Another and related use of the equation would be to predict future speeds depending on changes in some of the variables with time. For example, changes in hourly volume or the percent of out-of-state traffic will produce a corresponding change in average speed. In the case of commercial establishments, the change in average speed is particularly pronounced. According to the equation, for each increase of 10 commercial roadside establishments per mile the average speed of traffic will decrease 5 mph. This has important implications in two directions: (a) If it is desirable to permit these commercial establishments without limit, speed zoning may need to be revised every few years to take account of rapid commercial growth; (b) if it is desirable to maintain high average speeds for traffic, it may be desirable to zone the roadside to inhibit the location of commercial establishments along the highway. Obviously, the ultimate in this regard is the freeway.

The regression equations might also be a tool for design purposes. These equations provide estimates of the average speed. However, the average speed is not ordinarily used for design purposes but rather some percentile indicative of the speeds of the percentage of drivers who travel at speeds somewhat above the average. If the standard deviation of speeds for any selected site is known then it is possible from the estimate of the average speed to closely approximate a cumulative speed distribution. An earlier paper by Oppenlander, Bunte, and Kadakia, "Sample Size Requirements for Vehicular Speed Studies," showed for example, that the standard deviation for 2-lane rural highways was nearly always between 7.5 and 10 mph. Thus, for example, if the equation predicts an average speed of 50 mph on a certain highway section and it is assumed that the standard deviation is 9 mph, the 85 percentile speed will be approximately 60 mph and the 95 percentile speed approximately 68 mph. The latter study, incidentally, showed a slight tendency for the standard deviation to decrease as traffic volume increased on 2-lane rural highways.

BRUCE D. GREENSHIELDS, University of Michigan.—The paper demonstrates the importance of applying the statistical method of variance analysis to the study of vehicular speeds. By this method the author has reduced 49 variables, 42 of these having to do with the highway and 7 with traffic, to 8 which account for 68 percent of the spot-speed variations observed at 469 locations. This indicates that with further investigation it should be possible to predict speeds from the controlling highway and traffic factors.

Although it is agreed that the method of analysis of speed data is the best extant, do spot speeds furnish the best data for obtaining the correlation between vehicle speeds and highway and traffic factors? Continuous speed records of traffic streams over considerable distances would seem preferable.

As the individual driver, who collectively makes up the traffic stream, moves over a highway he meets a succession of highway and traffic events to which he responds by changing the speed or direction of his car. There are two methods of recording traffic stream flow that seem to provide satisfactory data.

One of these methods consists of using a pacing car equipped with a special device to record vehicle motions and traffic and highway events. The pertinent vehicle motions consist of speed, change of speed, and change of direction. The device permits the recording of a number of simultaneous traffic and highway events. There is no definite limit on the number of events that may be recorded. They may be digitally and/or photographically recorded.

The other method that furnishes suitable data on traffic stream performance is aerial time-lapse photographs. Transferring speed data from the films is tedious but it does permit simultaneous observation of several vehicles. Speed and change of speed can be obtained to an accuracy of about 1 mph.

The highway factors that affect traffic flow behavior in general consist of (a) the geometry, (b) the surface condition, and (c) the appearance of the highway. A device has been designed (but not constructed) to obtain these highway characteristics at any reasonable speed on 35-mm film.

The outlined methods would, it is believed, furnish more complete and more effective data than spot speeds. The same drivers would be observed as they met a multiplicity of traffic and highway events.

The statistical analyses would be the same as described by the author. The more intricate field recording devices required in the alternate methods mentioned would furnish a more exact solution to the problem of estimating traffic behavior from traffic and highway causes.

These suggestions are not intended to detract from the importance of the paper. The author is to be congratulated on his presentation of a new approach to the solution of a most important highway problem.

Sample Size Determination for Spot-Speed Studies at Rural, Intermediate, and Urban Locations

J. C. OPPENLANDER, Department of Civil Engineering, University of Illinois

•IN THE estimation of traffic characteristics by a sampling technique, the design of experiment requires the determination of an adequate and economical sample size. The evaluation of speed characteristics is accomplished by a sampling survey and a statistical analysis. A statistical procedure for sample size determination was developed and previously presented (2). This information made it possible to design a spot-speed study with a sample size that is statistically acceptable.

The equation for mimimum sample size was derived, and graphical solutions for this expression were also presented (2). The determination of a sample size for a spot-speed survey is predicated on a knowledge of the standard deviation of vehicular speeds at the study location. The other variables in the sample size expression are selected in accordance with the desired precision of the spot-speed study.

This measure of speed variability can be obtained from the results of previous speed surveys. However, if this quantity is not available, then a reliable estimate of standard deviation permits the use of the equation for determining sample size. The purpose of this study was to supplement the findings of the previous investigation by analyzing the standard deviations of spot speeds for 2- and 4-lane highways in rural, intermediate, and urban areas. The determination of sample size requirements can be greatly facilitated by the availability of standard deviation estimates that accurately describe the variability of spot speeds for various highway types in different traffic areas.

PROCEDURE

Spot-speed data were collected in the summer of 1960 to develop reliable estimates of standard deviations of vehicular speeds. The following numbers of study locations were chosen to provide information for various highway types in different traffic areas of Illinois.

1. Rural area: (a) 2-lane highway, 60; and (b) 4-lane highway, 50.
2. Intermediate area: (a) 2-lane highway, 42; and (b) 4-lane highway, 42.
3. Urban area: (a) 2-lane highway, 47; and (b) 4-lane highway, 40.

The following definitions were adopted to permit the delineation of the three traffic areas:

1. A rural area was any area where the number of residential, commerical, and industrial buildings along the highway was less than 10 per mile and where the number of crossroads and driveways was less than 20 per mile;
2. An intermediate area was any area where the number of residential, commerical, and industrial buildings along the highway was greater than 10 per mile but less than 100 per mile and where the number of crossroads and driveways was greater than 20 per mile; and
3. An urban area was any area where the number of residential, commerical, and industrial buildings along the highway was greater than 100 per mile.

The minimum lengths of highway considered were 1, ½, and ¼ mile, respectively. The speed sites were located on level, tangent roadway sections where traffic conditions were not influenced by the presence of intersections.

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Vehicular speeds were measured during the daytime for low volume conditions. It was considered desirable to evaluate the standard deviations of spot speeds during periods of low traffic flow in order to approach the maximum standard deviations occurring at the various study locations (2). A radar speedmeter was located adjacent to the lane or lanes of traffic being studied and was pointed toward the oncoming vehicles. After the spot speeds of 100 vehicles were obtained, the same procedure was repeated for the other direction of travel with the speedmeter relocated on the opposite side of the highway. This procedure provided a composite sample of 200 observations at each study site. The speedmeter and the observers were concealed from the view of approaching drivers. The average annual daily traffic volume (ADT) for each spot-speed site was obtained from information published by the Illinois Division of Highways (3).

The standard deviation of spot speeds was calculated for each location. Means and standard deviations of these standard deviations were obtained on the IBM 650 computer for the various combinations of traffic areas and highway types. To correlate standard deviation of vehicular speeds with average annual daily traffic volume, a regression routine for this computer provided the regression coefficients and the coefficients of correlation (1).

RESULTS

The results of the regression and correlation analyses are given in Table 1. Except for 4-lane highways located in intermediate and urban areas, the correlation coefficients were not significantly different from zero at the 5 percent level. However, these significant variations in standard deviation of spot speeds were explained to a limited degree by the variations in ADT volume. These linear equations, consequently, offer no advantage in estimating standard deviations for sample size determination. In the first investigation a significant linear relationship was established between standard deviation and ADT for 2-lane rural highways. The standard deviations of vehicular speeds were independent of traffic volumes for 4- and 6-lane rural highways (2).

Because standard deviation was generally independent of ADT for the locations studied, the statistics in Table 2 provide reasonable and proper estimates of standard deviation for computing sample size requirements in the experimental design of spot-speed studies. As indicated by the low standard errors of estimate, the average values produce sample sizes that are statistically adequate. Average standard deviations plus one or two standard errors of estimate are tabulated for use in studies requiring precise speed statistics. Average standard deviations minus one or two standard errors

TABLE 1

RESULTS OF REGRESSION AND CORRELATION ANALYSES STANDARD DEVIATION OF SPEED VERSUS AVERAGE ANNUAL DAILY TRAFFIC VOLUME

Traffic Area	Highway Type	Intercept (a)	Slope (b)	Correlation Coefficient (r)	r^2
Rural	Two-lane	6.14	-0.0193	-0.136	0.0185
Rural	Four-lane	4.36	-0.0021	-0.032	0.0010
Intermediate	Two-lane	5.34	-0.0012	-0.012	0.0001
Intermediate	Four-lane	3.07	0.0170	0.366*	0.1340
Urban	Two-lane	6.63	-0.0202	-0.247	0.0610
Urban	Four-lane	1.04	0.0299	0.464*	0.2153

*Significant at the 5 percent level.

TABLE 2

STANDARD DEVIATIONS OF SPOT SPEEDS FOR SAMPLE SIZE DETERMINATION

Traffic Area	Highway Type	Average Standard Deviation (mph)	Standard Error of Estimate (mph)	Average Standard Deviation ± One Standard Error of Estimate (mph)	Average Standard Deviation ± Two Standard Errors of Estimate (mph)
Rural	Two-lane	5.31	0.41	5.72 - 4.90	6.13 - 4.49
Rural	Four-lane	4.16	0.38	4.54 - 3.78	4.92 - 3.40
Intermediate	Two-lane	5.28	0.46	5.74 - 4.82	6.20 - 4.36
Intermediate	Four-lane	5.25	0.45	5.70 - 4.80	6.15 - 4.35
Urban	Two-lane	4.81	0.46	5.27 - 4.35	5.73 - 3.89
Urban	Four-lane	4.88	0.49	5.37 - 4.39	5.86 - 3.90

of estimate are listed for the design of speed studies that are limited in scope by economic considerations. In general, the average standard deviations provide sample sizes that are both statistically sufficient and economical.

The results of this study were developed to augment the findings of the first report on sample size determination. The statistics shown in this paper permit the reliable estimation of a standard deviation of vehicular speeds if this value is not known from a previous spot-speed survey. Finally, the minimum sample size requirement can be calculated from the theoretical expression presented in the first report (2).

The average standard deviations (Table 2) ranged from 4.16 to 5.31 mph for the 6 combinations of traffic areas and highway types. Because this variability in the measures of speed dispersion was limited, an average standard deviation of 5.0 mph is suggested as a rule-of-thumb value for spot speeds on any highway type in any traffic area.

ACKNOWLEDGMENTS

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REFERENCES

1. Croxton, F. E., and Cowden, D. J., "Applied General Statistics." 2nd Ed., Prentice-Hall (1955).
2. Oppenlander, J. C., Bunte, W. F., and Kadakia, P. L., "Sample Size Requirements for Vehicular Speed Studies." HRB Bull. 281, 68-86 (1961).
3. "Traffic Characteristics on Illinois Highways." Illinois Division of Highways (1961).