A Systems Engineering Model for Trip Generation and Distribution

W. L. GRECCO, Purdue University, and S. M. BREUNING, Michigan State University

This paper discusses the applicability of a systems engineering technique (linear graph theory) to urban traffic forecasting. The primary concern of the research is the analysis of the components. Synthesis of the system and its solutions were of secondary importance, but did serve to reinforce the original hypotheses. Traditionally, urban traffic forecasting models have been separated into three distinct parts—trip generation, trip distribution, and trip assignment. The research concentrates primarily on trip distribution with some work on trip generation. The problem of trip assignment is not yet included.

The techniques of systems engineering are applied to work-trip and shopping-trip distribution systems in two theoretical examples. The research confirmed two hypotheses: first, that these trip distribution systems could meet the requirements of a system solution by linear graph theory, and second, that the results of the system solution would provide acceptable trip interchanges which would compare well with other models. The results of the work indicate further that these rigorous models have the advantages of more precise definition of the parameters and their interaction.

SYSTEMS THEORY

A SYSTEM is defined as an orderly arrangement of interrelated elements acting together to achieve a specific purpose. Thus, a system must have an avowed purpose, must be free of extraneous or mathematically redundant parts, and must have the elements or components joined in an orderly fashion. System engineering problems require the use of some type of mathematical technique to achieve solutions. In this study, linear graph theory is used to solve the problem of urban traffic forecasting.

The principles of linear graph theory are stated only very briefly here. A fuller discussion of the subject is given in a previous article (1) and two of the prominent texts in the field (2, 3).

Linear graph theory is an orderly technique for formulating the mathematical characteristics of a physical system. The steps in the solution of a physical system by linear graph are shown in Figure 1.

For computation of the system characteristics, two steps are necessary; namely,

1. To establish a mathematical description of the relevant physical characteristics of the system components expressed in terms of measurements.
2. To establish in mathematical form and in terms of measurements from a knowledge of the component characteristics and their mode of interconnection, the characteristics of the system; i.e., a mathematical model of the system.

Thus the analysis of any physical system requires a mathematical description of each component part, as well as a mathematical description of how the components

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are joined to form the system. Collectively, these mathematical equations provide what is referred to as system equations. In the mathematical analysis of any given type of physical system (electrical, mechanical, thermal, hydraulic, etc.) the tie between the mathematics and the system is generally accomplished through the use of two basic measurements; the "across" or x and the "through" or y measurements.

The terminal characteristics of the component are completely described by

$$x = R\,y \qquad (1)$$

which relates the x and y measurements and is referred to as the terminal equation, which with the addition of the terminal graph of the component forms the terminal representation of the component. Components are described mathematically by relating the two measurements x and y on the component in isolation from other components. This infers that the terminal equations of the components must be independent of the system in which they are used. These measurements must be such that one is a "through" (or series) measurement called y, which when summed at the vertices must equal zero, and the other is an "across" (or parallel) measurement called x, which when summed around the circuits must equal zero. These requirements are referred to as vertex and circuit postulates and are used for the solution of the system with the aid of a systems graph constructed from component terminal graphs and a knowledge of their mode of interconnection.

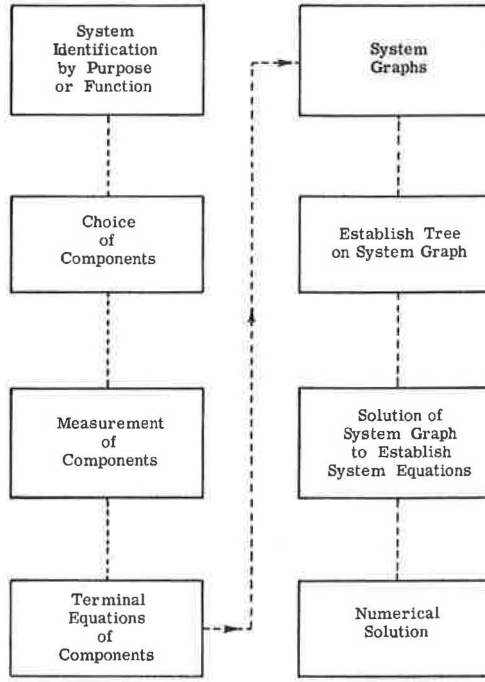

Figure 1. Steps in solution of a physical system by linear graph theory.

APPLICATION POSSIBILITIES

The work presented here is one of the earliest attempts to apply the techniques of systems theory to traffic problems, specifically to forecasting future work and shopping trips and their distribution. Two illustrative problems are presented in the Appendix.

Components

The system components which evolved are residential zones, employment zones, and shopping zones, similar to those generally established in O-D studies. Route components include the various types of streets and intersections used in traveling from an origin to a destination. These components must meet the following requirements if the techniques of linear graph theory are to be used in the systems solution:

1. The basic component must be describable mathematically by relating two valid measurements on the components.
2. When the components are arranged in a systems graph, one of the measurements taken on the component, which is noted as x, must sum to zero when the summation is made around a circuit; the other measurement, y, must sum to zero at the vertices of the systems graph.
3. The x measurement must be related to the y measurement through a linear or nonlinear function.

Measurements on Components

As a result of the analyses made to date, it was concluded that the best measurement for the through variable, y, is the flow of work or shopping trips from, to, or through the component. The measurement for the across variable, x, is a pressure-type measurement related either to some measure of desire, propensity, or trip motivation or to some measure of income and consumption assigned to trip making. The dimensions of the x and y measurements must be consistent for all components.

The y Measurement

The most logical y measurement for a traffic system of this type appears to be flow. This flow would represent the movement of persons, vehicles, or both. In the systems approach to electrical, thermal, and hydraulic systems, the y measurement represents flow of current, heat, or fluids, respectively. It seemed reasonable to assume that the y measurement for the traffic system also represents flow, inasmuch as the flow of vehicles will satisfy the criterion that the algebraic sum of y's at a vertex must equal zero.

The x Measurement

In the fields of electrical, thermal and hydraulic systems, where y is a measure of flow, the x measurement is a type of pressure differential which causes flow. The establishment of the units for the x measurement in the traffic system is not so obvious. There are no physical components on which one might measure readily a pressure differential. Because the y measurement flow is a function of the pressure differential, x, the x measurement can be evaluated in terms of its effect on the flow of trips. The development of the x measurement follows this reasoning process:

1. There is some reason for the variation in the flow of trips from several residential zones.
2. For the sake of a title, this reason is here called pressure.
3. The "pressure" term can best be described as a function of certain factors which explain the variations in the y values.
4. Two factors are used here separately to approximate the pressure term. One factor might be labeled "desire" and the other "money."
5. These can be related to more specific parameters which are subject to actual physical measurement.

The procedures and extent of the research into the x measurement can be shown more clearly by Figure 2. Eventual acceptance of a method will depend on how well it can

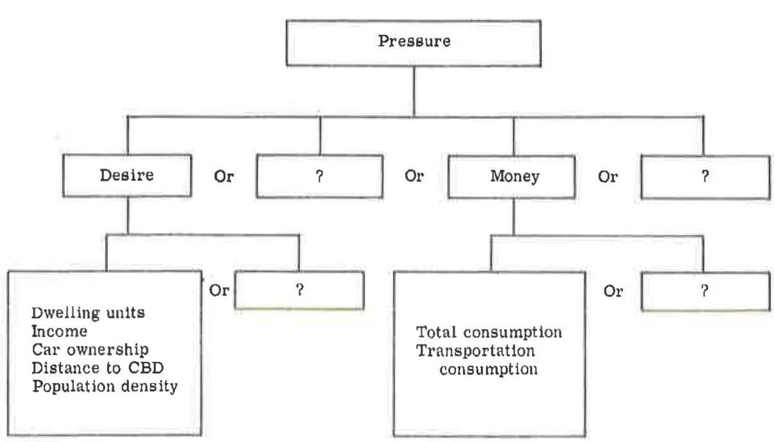

Figure 2. Organization of research on the x measurement.

predict changes in the flow of trips and on how easily the final parameters can be measured.

Desire. —A measure of desire or motivation can be related to the pressure which induces flow. The logic of this assumption can be checked by referring to Figure 3, which is a schematic representation of a simple circuit from a residential zone component, through a route component away from the zone, on through an employment component, and the return route component, for a fixed period of time, say 24 hours.

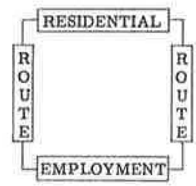

Figure 3. Simple work-trip circuit.

Considering only the single-purpose work trips made along the circuit, the desire or pressure produced in the residential zone must precisely be enough to overcome the pressure loss at each of the route components and through the employment zone. The sum of the pressure measurements will vanish around the circuit and, in so doing, will produce y trips. Some routes and employment zones will require larger pressure differentials than others. If a residential zone has a specific x value available, the circuit with the least R or resistance value will produce the greatest flow of trips to utilize the available x value. The pressure value, like that used in hydraulic systems, is equal in all directions from the residential zone. A discussion of the quantitatively descriptive parameters used to evaluate the term of desire is presented later.

Money. —From the fact that travel costs money, it can be assumed that the number of trips made is a function of the total amount of money available for transportation and the expenditure required per trip. The use of money to estimate the flow of trips has been suggested by others (4). The amount of money consumed yearly for transportation purposes has steadily increased. The approach used here assumes that the cost of travel will be minimized in terms of time and money. The measurement on x, although not necessarily equal to the money value, is related to it. A detailed explanation of the relationship between transportation consumption and the pressure, x, is presented in the next section.

Terminal Equations of Components

The terminal equations for all components have been assumed to be of the form of Eq. 1, but each term of this equation is generally a more complex expression. The following postulates concerning these terms have been established:

1. The y value shall be the flow of work or shopping trips. The y value is specified for the destination zones.
2. The x value shall be a measure of pressure which can be related to a function of desire in one case and of money spent in another. The x value is specified for the residential components.
3. The R value shall be a measure of resistance to flow, or reciprocal of attraction, which can be specified for the route, shopping, and employment components. For the employment component, R is a function of the jobs available. For the shopping area, R is an inverse function of the relative attraction of the area. The alternates presented for the establishment of R on the route component are (a) R is a function of the ratio of probable trips to actual trips, and (b) R is a function of the cost of travel through the route component.

The discussion of these postulates is presented under each of the components for which it was developed.

Residential Zone Component

Of the three possible measurements noted, only x is utilized for the residential zones.

The x Measurement--Desire. —The x measurement which represents the pressure for work trips from the residential zone will be established as a function of desire according to Eq. 1 on the following basis:

1. From existing O-D data, an equation is established for Y resident trips per dwelling unit based on income index, α; car ownership, β; distance to CBD, γ; and population density, δ;

$$Y = -A + B\alpha + E\beta + D\log\gamma - C\log\delta \tag{2}$$

2. This equation is maximized with limiting values of all parameters as forecast in the whole area for the future year. This maximum value is related to a measure of the theoretical pressure or desire by

$$X_T = Y_{max} \tag{3}$$

assuming, of course, that the equation is dimensionally adjusted.

3. If Y is developed for all trips, then the pressure or desire for work trips per dwelling units will be found by using the relation of work trips to total trips, K_W, or

$$X_W = K_W X_T \tag{4}$$

4. Using maximizing input values for all parameters except income, one can establish ΔX, the change in X due to a particular value of income, or

$$\Delta X = X_W - Y_{\alpha:\beta,\gamma,\delta} \tag{5}$$

The maximum change is found for some limiting value α; then R_α, the resistance value, can be found by equating

$$R_\alpha = \Delta X_\alpha / \Delta X_\alpha\,(\max) \tag{6}$$

A relationship for R_α versus the income, α, is shown in Figure 4, which isolates the effect of income alone on the pressure to make trips.

Similar techniques are used to establish values for R_β, R_γ, and R_δ, as shown in Figures 5, 6, and 7.

5. A subsystem of the zone parameters will be solved in order to establish each value of x_i. Schematically, this can be shown as in Figure 8, or as a system graph as in Figure 9, from which

$$x_1 = N\,(X_W) \tag{7}$$

in which N is the number of dwelling units in the zone; and R_α, R_β, R_γ, and R_δ are found from previous curves. The subsystem can then be solved for x_6 in terms of values x_1, R_α, R_β, R_γ, and R_δ. The unknown, y_6, will be solved for in the final system solution. The element number 6 can now be used in lieu of the subgraph; that is,

$$x_i = x_6 = x_1 + 1/\Sigma R^{-1}\,y_6 \tag{8}$$

The x Measurement—Money.—The x measurement, which represents the pressure for shopping trips from the residential zone, is established as a function of money or consumption allotted to the making of shopping trips, not including the money spent for purchases made. The consumption for shopping trips is found by first determining income and the percentage of income consumed. A portion of the total consumption per family is spent for transportation costs. Because this amount is distributed over the trips for many purposes, it is necessary to establish the actual amount allotted for shopping trips.

The x measurement is established on the basis of the following:

The average amount of money allotted yearly to the cost of shopping trips made from a residential zone i is C_i. Then

$$x_i = K\,C_i/300 \tag{9}$$

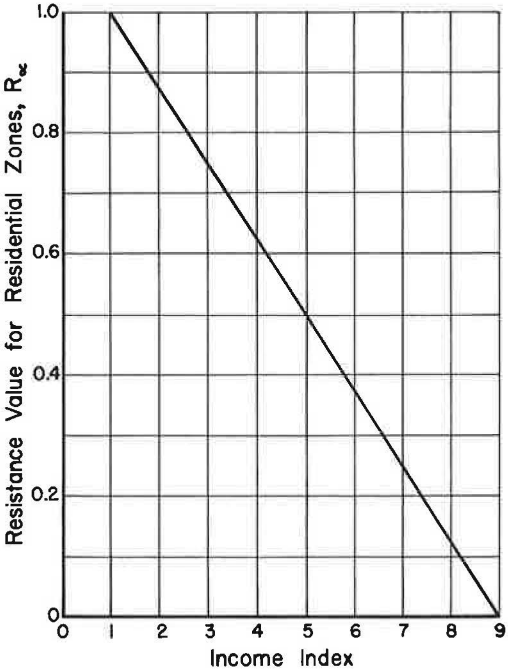

Figure 4. Relationship between resistance on residential zones and income index.

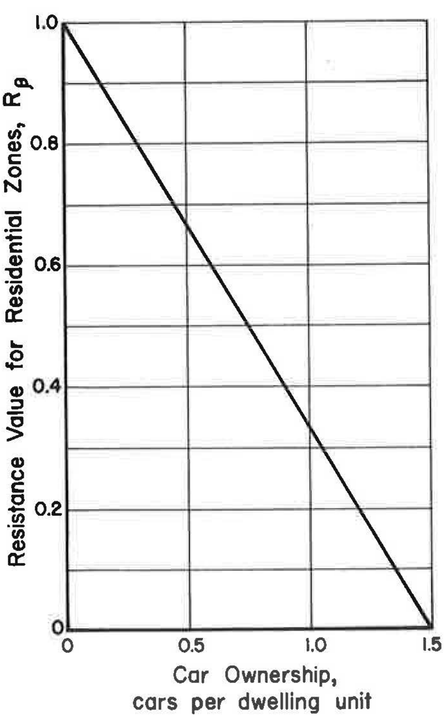

Figure 5. Relationship between resistance on residential zones and car ownership.

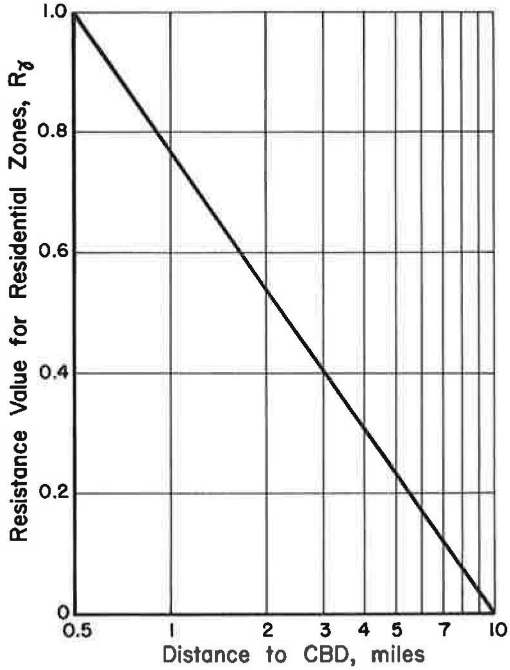

Figure 6. Relationship between resistance on residential zones and distance to CBD.

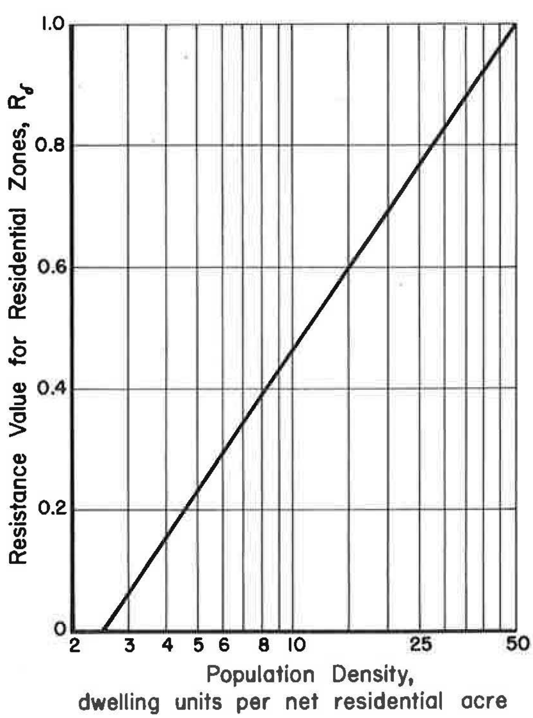

Figure 7. Relationship between resistance on residential zones and population density.

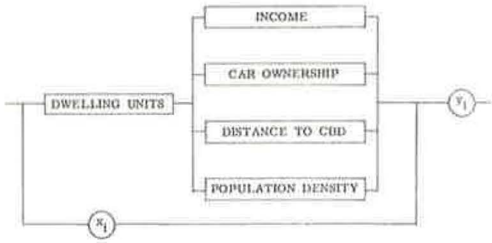

Figure 8. Schematic of zone parameter subsystem.

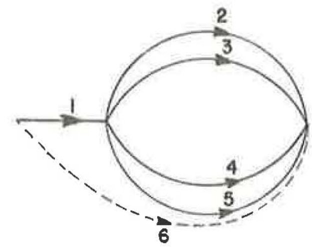

Figure 9. Typical simple system graph.

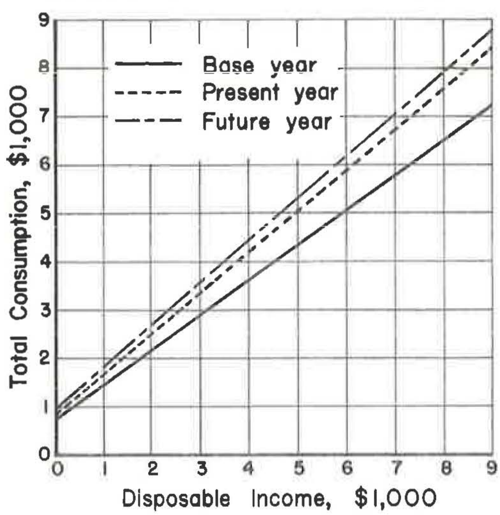

Figure 10. Relationship of consumption to disposable income (4).

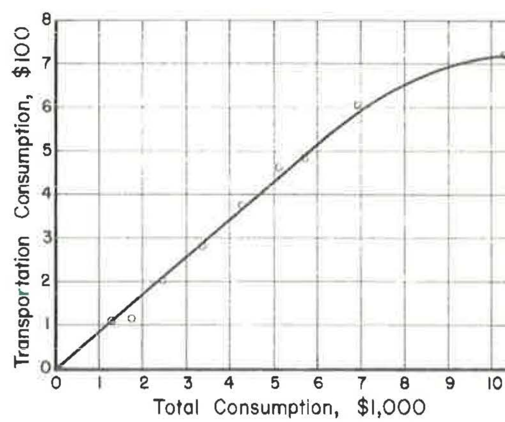

Figure 11. Relationship of transportation consumption to total consumption (5).

in which K is a constant determined to adjust for the differences in dimensions, and 300 is the number of available shopping days per year.

The value C_i can be estimated in the following manner:

1. Determine from a previously established plot of income versus consumption an estimate of the future year consumption, Z_i, based on an estimate of the future year mean income, I_i, for the zone (Fig. 10); that is,

$$Z_i = f(I_i) \tag{10}$$

2. Establish from previous data a relationship, E, between transportation consumption, z_i^t, and total consumption, Z_i, (see Fig. 11). Based on observations of trends, the yearly change in this ratio can be established; that is,

$$E_f = E_b + n \Delta E \tag{11}$$

in which E_f is the ratio of future amount spent on transportation per total consumption for the future year, E_b is the same for the base year, ΔE is the yearly change, and n is the number of years from base year to future year.

The amount of expenditure spent on transportation for the future year can be found from

$$z_i^t = E_f Z_i \tag{12}$$

3. The $z_i{}^t$ represents the total amount spent for transportation per family in zone i. The amount spent for transportation per zone for the future year will be this factor times the number of families in zone i, or

$$Z^t = N_f \; z_i{}^t \tag{13}$$

4. Inasmuch as the primary concern is home-based shopping trips, it is necessary to determine the amount spent for these, using K_h as the weighted percentage by trip length of all trips that are home-based. Therefore,

$$Z_i{}^t = K_h \; Z^t \tag{14}$$

5. The amount of money spent for trips to shopping can be found by first establishing the ratio of the total mileage for shopping trips to the total mileage traveled for all trip purposes. This ratio is

$$K_s = \frac{k_s \; m_s}{M_{tot} \text{ (All purposes)}} \tag{15}$$

in which

$$M_{total} = (k_w \; m_w) + (k_s \; m_s) + (k_{sc} \; m_{sc}) + (k_{sr} \; m_{sr}) + (k_{em} \; m_{em}) + (k_b \; m_b) \tag{16}$$

k_w = percentage of trips to work;
m_w = average trip length to work;
k_s = percentage of trips to shopping;
m_s = average trip length to shopping;
k_{sc} = percentage of trips to school;
m_{sc} = average trip length to school;
k_{sr} = percentage of trips to social-recreation;
m_{sr} = average trip length to social-recreation;
k_{em} = percentage of trips to eat meals;
m_{em} = average trip length to eat meals;
k_b = percentage of trips to business; and
m_b = average trip length to business.

The input values for Eqs. 15 and 16 can be determined from Tables 1 and 2. Then

$$Z_i{}^{st} = K_s \; Z_i{}^t \tag{17}$$

but, by definition,

$$C_i = Z_i{}^{st} \tag{18}$$

Employment Zone Component

The y measurement, which represents the flow of work trips destined to the zone, will be specified. The number of existing work trips destined to an employment zone j can be found from the data of the O and D study. A relationship of work trips to some

TABLE 1

HOME-BASED TRIPS BY URBAN RESIDENTS IN STUDY AREAS ACCORDING TO PURPOSE[a]

Urban Area	Home-Based as % of All Linked Trips	Trips by Purpose (%)							Total Home-Based Trips per Dwelling Unit (no.)
		Work	Business	Shopping	Social-Recreational	School	Other	All Purpose	
Chicago	86.8	37.5	9.7	18.9	22.8	4.0	7.1	100.0	5.17
Detroit	87.0	41.6	8.6	13.9	20.1	6.3	9.5	100.0	4.67
Washington	91.6	43.1	9.6	14.2	12.5	9.4	11.2	100.0	4.23
Pittsburgh	87.0	37.7	21.6	14.9	13.8	12.0	—	100.0	4.21
St. Louis	91.3	37.5	8.1	17.3	21.5	6.4	9.2	100.0	4.90
Houston	91.0	33.1	8.9	17.3	18.6	10.8	11.3	100.0	5.51
Kansas City	88.2	33.4	8.8	17.2	22.7	6.0	11.9	100.0	5.14
Phoenix	85.3	25.2	10.2	19.7	20.0	11.6	13.3	100.0	4.76
Nashville	85.5	30.3	8.5	16.9	23.9	7.4	13.0	100.0	5.48
Ft. Lauderdale	86.5	27.9	15.3	24.0	22.9	0.9	9.0	100.0	2.82
Charlotte	83.9	32.2	8.0	15.6	23.8	6.6	13.8	100.0	5.56
Reno	86.5	29.2	12.7	18.1	26.3	0.5	13.2	100.0	4.88
Average	87.6	34.0	10.8	17.4	20.8	6.8	10.2	100.0	4.78

[a]After Smith (6).

TABLE 2

AVERAGE TRIP LENGTH FOR HOME-BASED TRIPS, BY PURPOSE[a]

Purpose for Home-Based Trips	Average Trip Length (mi)
Work	5.56
Shopping	3.15
School	2.97
Social-recreation	4.27
Eat meals	3.44
Business	3.71

[a]After Bevis (7).

other parameter can be determined, such as trips per acre, per job, per labor force, or other standard. It might be necessary to establish a predicting equation on the basis of multiple correlating parameters. The parameters which provide the highest correlation would then be estimated for the future year, so that a growth factor is established for each zone. The number of work trips to zone j at some future year would be solved by

$$Y_j' = G_j Y_j \tag{19}$$

Shopping Zone Component

The flow of shopping trips to each destination zone will be specified in a manner similar to the employment zones except for the use of different predicting parameters.

Route Components

The only one of the three possible measurements that can be used for the route component is the parameter R. The route component, analogous to a piece of hydraulic pipe, has no facility for generating pressure or flow. It seems logical if a long period of time is chosen; furthermore, if the arterial routes chosen as components provide no overnight parking there will not be any storage in the component. If a 24-hr period is chosen from 4 a.m. to 4 a.m., during this period the flow into the component equals the flow out of the component.

The flow of trips through a route component can be determined from a knowledge of either the pressure drop through the component or the friction encountered throughout the component. If pressure measurements for the components are in terms of either desire or money, it is difficult to vary the amount of this pressure in order to note the effect on the flow of trips. Friction can be related to such measurements as travel time or distance and these can be easily varied and measured while the relationship to flow of trips is recorded.

Several methods are proposed to establish the measure of friction through the route component which relates the pressure x to the flow y.

In several of the procedures that follow, it was assumed that the best single measurement that could quantitatively assign a value to the resistance term would be travel time. Contemporary writers on this subject have used, besides travel times, travel distances and straight-line distances from origin to destination. Of the three parameters, the straight-line distance is most easily measured, but gives the poorest indication of the friction or resistance encountered. Travel distances are a little more difficult to measure, but give a better estimate of the resistance. Travel distances can be the same for alternate routes, one having most of the travel on minor routes and the other on arterials and expressways, yet the friction established by distance alone would not show that the friction on the minor routes is much larger than on the other.

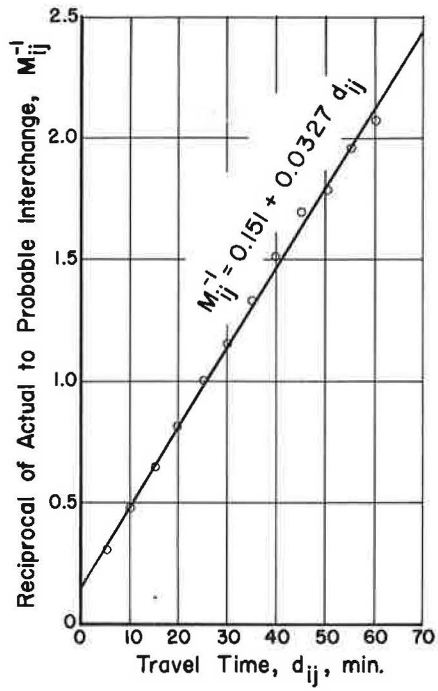

Figure 12. Relationship of travel time to actual trips per unit of probability interchange.

Probable vs Actual. —The curve used here is established using the data from an O and D study. The actual number of trips from each of the residential zones to each of the employment zones, Y_{ij}, is determined. The probable number of interchanges, P_{ij}, is calculated by assuming equal travel times from each residential zone to all the employment zones (8). A ratio, M_{ij}^{-1}, is determined; that is

$$M_{ij}^{-1} = \frac{P_{ij}}{Y_{ij}} \tag{20}$$

The value of M_{ij}^{-1} can then be plotted against travel time, d_{ij}, in minutes, as shown in Figure 12. The plot of these values can be fitted by a straight line represented by

$$M_{ij}^{-1} = 0.151 + 0.0327\, d_{ij} \tag{21}$$

It seems reasonable to find the differences between the actual interchanges and the probable values related to travel time (9).

Money Value. —The following approach to R does not use travel time alone as the controlling factor. This technique follows from an early attempt to establish an x_{ij}, or pressure value, on the route component. The subscripts ij are used to designate the route component from a zone of residence i to a zone of shopping j. The pressure drop along route ij was called x_{ij} and was related to the total cost of making the trip per vehicle, C_{ij}. The factor K was used to adjust the equation dimensionally and y_{ij} represents the actual flow.

$$x_{ij} = K\, C_{ij}\, y_{ij} \tag{22}$$

The value of C_{ij} would be the sum of the products of vehicle costs per mile times the number of miles and travel time costs per minute times the number of minutes for the route from i to j, or

$$C_{ij} = c_v m_{ij} + c_t d_{ij} \tag{23}$$

in which c_v is the total operating cost per vehicle mile; m_{ij} is the length of the route, in miles; c_t is the time cost per vehicle mile per minute of travel, assuming an occupancy of one person per vehicle; and d_{ij} is the travel time per vehicle per route, excluding terminal time. Inasmuch as the y_{ij} of actual flow in the component is the same y_{ij} required in the system solution, the equation can be further reduced and

$$R_{ij} = \frac{x_{ij}}{y_{ij}} \tag{24}$$

$$x_{ij} = K\ C_{ij}\ y_{ij} \tag{25}$$

$$R_{ij} = K\ C_{ij} \tag{26}$$

This last equation makes it evident that for route components the parameters which establish x are confounded in y and that specified x's are not possible. Specified R values of the route components can be determined on the basis of money.

SUMMARY

The findings can be summarized with regard to the components and the system solution. The postulates previously established are utilized in the solution of two illustrative problems (see Appendix). The most pertinent findings from this research pertain to the selection, measurement and terminal equations of the components.

Component Selection

A work trip distribution system contains workers, jobs, and some facility to bring the workers to the jobs. It was determined in this study that the best components would be the residential zone component, the route component, and the employment component. For the shopping trip distribution system, a shopping area zone component was used in lieu of the employment zone. In actuality, these two zones could be the same one.

Measurements

Two measurements are necessary on each component: one, y, must sum to zero at vertices of the system graph; the other, x, must sum to zero around the circuits of the system graph. Analogous to the other fields of system analysis, it can be established that the proper y measurement should be the flow of work trips or shopping trips. The x measurement was assumed to be a pressure-type measurement which is a causative influence on the flow and diminishes around the circuit.

The fact that the amount of travel varies from person to person is generally accepted. It might be reasoned that the variation can be tied to a set of circumstances which influence travel. In an effort to be more specific, two possible measurements were proposed which could be related to trip making. It was hypothesized that the trip interchange is a function of demand, as expressed in a need or willingness to travel. This x measurement was designated as desire. As the amount of desire increases, the number of trips also increases. This relationship between desire and trips is also influenced by the relative attraction of the trip and the friction or deterrent factors encountered. The number of trips made is based on the amount of desire available for trip making modified by the attraction of the destination and friction incurred on the route of travel.

The second expression for the pressure-type x measurement is money. The amount of money available for travel can be determined for each residential zone. The flow of trips must then be large enough to use up the money available.

Terminal Equations

A basic equation which utilizes the foregoing postulates can be stated: the flow of trips equals the pressure measurement divided by the resistance encountered, or $y = x/R$. This can also be related to a relative attraction term, G, or $y = G x$.

For expediency in the formulation process, the basic equation is retained, even though values for x and R or G cannot be directly measured. The x measurement in terms of desire is related to parameters such as income, car ownership, distance to the CBD, and population density. The x measurement in terms of money is related to disposable income, transportation consumption, and specifically the consumption utilized for the making of shopping trips.

For the employment and shopping zones, the flow or y measurement is used throughout the research. This serves as a control volume that will properly adjust the magnitudes of the trip interchange.

The route components are related to the friction term, R. The best single predictor of R for the route components seems to be travel time. When the pressure measurement, x, is related to money, it seems advisable to express R as a function of travel costs in terms of time and vehicle costs.

System Solution

The illustrative problems in the Appendix are presented to test on a theoretical basis the application of the postulates in a system solution. In summary, the results of these illustrative solutions show that:

1. The most basic solution of future zonal interchanges occurs when the estimated trips from each residential zone, y_i, and the estimated trips to each employment zone, y_j, are specified. The R value on the route components is established as a function of an empirically established ratio of probable to actual trip interchanges.
2. The use of the pressure measurement in terms of desire provides reasonable interchanges and the trip origins at each residential zone closely approximate those found in a multiple regression equation.
3. The use of money as a function of the x measurement for residential zones and the R measurement on the route component provide results which are balanced and reasonable.

CONCLUSIONS

The justification for further research into the application of systems engineering techniques to urban traffic forecasting can be evaluated on the basis of the following conclusions:

1. Models of this type have many advantages. The parameters which influence travel patterns can be better understood through testing and evaluation in a mathematical model. A procedure for keeping the model up to date can be devised which will make periodic tests and adjustments.
2. The complex interaction of persons, vehicles, facilities and jobs in the work trip distribution system cannot be simply stated in equation form without the use of a formulation technique such as linear graph theory. This is just as true for a shopping trip distribution system.
3. Through the use of systems theory, it is possible to establish a mathematical model of the relevant physical characteristics of the system components in terms of measurements.
4. A mathematical model of each system can be formulated in terms of the characteristics of the components and their mode of interconnection.
5. The systems approach provides for a balanced flow between inputs and outputs of the system. Other models in use generally require an iterative process to produce this balance. The iteration procedures used are aimed primarily at achieving a balance of flows. The true system's effect or the interaction of the component parts is placed second in importance to the balance of flow.

6. The system engineering model is much more flexible than iteration-type models for establishing parameter values from empirical data.

7. The eventual goal of the traffic forecaster is a theoretical model that can be used in any urban area, independent of a previous O and D study. The models proposed by others have not had much success in their application to other areas. Although it has yet to be substantiated, it is proposed that a general model, which predicts the pressure for the flow of trips from any residential zone, can be established by the techniques of systems engineering.

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Appendix

SAMPLE PROBLEMS

PROBLEM 1

Problem Statement

Given Information—

For the hypothetical city "Red Cedar," the following information is established from a current O and D survey and a special sampling survey:

1. The present interzonal work trips, t_{ij}.
2. The present travel times between zones, d_{ij}.
3. Present and future evaluation of employment zones to establish relative attraction values on the basis of number of jobs and/or other parameters.
4. Probability interchange established on the basis of the relative attraction of employment zones assuming equal travel times, P_{ij}.
5. The actual trips per unit of probability interchange, $M_{ij} = Y_{ij} / P_{ij}$.
6. Curve plot or equation for $1/M_{ij}$ versus d_{ij} (Fig. 12).
7. Present estimates of income, car ownership, distance to CBD, and population density.
8. Forecast of these parameters for each zone, i, for the future year.
9. Forecast of area-wide limits or ceilings on these parameters for the future year.
10. Distribution of trips by purpose, where K_W is the work trip factor.
11. Existing land-use data.
12. Future land-use forecasts.

To Find—

The problem is to forecast future interzonal work trip movements when specified x's or "pressure drivers" are given for the residential zones, specified y's or "through drivers" are given for the employment zones, and resistance values are given for the route components.

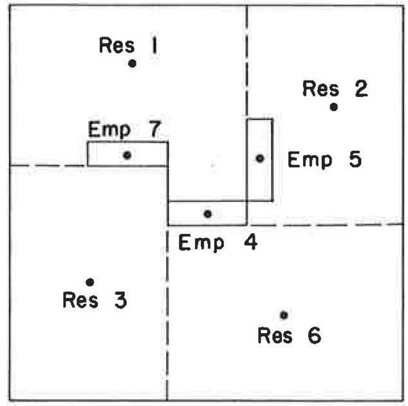


Figure 13.

Schematic of Physical System

The schematic of the physical system involved is given in Figure 13.

Input Values and How Established

The following input values are necessary for solution of this sample problem by linear graph theory:

1. The value of trip desire or pressure, x_i, for each residential zone, i, is required. The values will be established in the following manner:

(a) From existing O and D data for income index α, car ownership β, distance to the CBD γ, and population density δ, an equation will be established for Y, resident trips per dwelling unit. For this example,

$$Y = -0.1958 + 0.0008\,\alpha + 4.6480\,\beta + 1.7288 \log \gamma - 0.5464 \log \delta \tag{27}$$

Eq. 27 is then maximized with limiting values of all parameters as forecast in the whole area for the future year. The limiting values used to maximize Eq. 27 are income index, 9; car ownership, 1.5 cars per dwelling unit; distance to CBD, 100 (tenths of miles); population density, 2.5 (dwelling units, in tenths, per net acre). This maximum value of Y is defined as the theoretical pressure or desire,

$$X_T = Y_{max} \tag{28}$$

The pressure or desire for work trips per dwelling unit is found by using the previously defined factor, K_w, in

$$X_w = X_T\, K_w \tag{29}$$

(b) Using maximizing inputs for Eq. 27 on all parameters except income, α, will develop a relationship between discrete values of income and $Y_{\alpha:\beta,\gamma,\delta}$. A curve plot or equation is then established for r_α. The decrease in X from the maximum value is related to the coefficient r_α by

$$\left[X - Y_{\alpha:\beta,\gamma,\delta}\right] = \left[r_\alpha X\right] \tag{30a}$$

or

$$r_\alpha = \frac{\left[X - Y_{\alpha:\beta,\gamma,\delta}\right]}{X} \tag{30b}$$

The value of R_α is found by normalizing r_α (see Fig. 4) and should serve to isolate the effect of income on pressure and trips made.

(c) Similar techniques are used to establish curves or equations for R_β, R_γ, and R_δ (see Figs. 5, 6, and 7).

(d) A subsystem of the zone parameters is solved in order to find each x_i value. This can be shown either schematically (Fig. 8) or as a system graph (Fig. 9), and Eqs. 7 and 8 apply. The summary of solution of input x values and $(\Sigma G_i)^{-1}$ is given in Table 3.

2. Y_j values for each employment zone are determined by methods previously defined.

3. The input values for the route components are R_{ij}^{-1} or G_{ij} values as given in Table 4.

TABLE 3

COMPONENT INFORMATION NECESSARY TO SOLVE THE LINEAR GRAPH

Element No.	Component	Travel Time (min)	R_{ij}^{-1} or G_{ij}	Work Trips, y
1	Employment (I-7)	—	—	2,000
8	Street	10	2.092	
9	Street	17	1.414	
10	Street	20	1.242	
11	Street	10	2.092	
12	Street	14	1.642	
13	Street	10	2.092	
18	Street	10	2.092	
19	Street	14	1.642	
20	Street	17	2.092	
21	Street	14	1.642	
22	Street	10	2.092	
23	Street	14	1.642	
26	Residential (R-1)	—	—	—
27	Residential (R-2)	—	—	—
28	Employment (I-4)	—	—	2,000
29	Employment (I-5)	—	—	4,000
30	Residential (R-3)	—	—	—
31	Residential (R-6)	—	—	—

TABLE 4

SUMMARY OF SOLUTION OF INPUT x VALUES AND $(\Sigma G_i)^{-1}$ FOR SUBGRAPH OF RESIDENTIAL ZONES

Item or Factor	Res. 1 (26)	Res. 2 (27)	Res. 3 (30)	Res. 4 (31)
No. dwelling units	1,050	1,010	818	715
Income index	8	7	5	6
R_α	0.118	0.250	0.500	0.368
Car ownership	1.3	1.1	0.8	0.9
R_β	0.133	0.266	0.466	0.400
Distance to CBD	8.0	3.0	2.0	4.0
R_γ	0.075	0.401	0.538	0.306
Population density	10	20	30	20
R_δ	0.462	0.694	0.828	0.694
Factor K_x	0.22	0.22	0.22	0.22
Factor K_w	0.348	0.348	0.348	0.348
x_i (total)	-773	-742	-600	-525
$(\Sigma G_i)^{-1}$ or R_e	0.0317	0.0855	0.1387	0.1007

Terminal Representation of the Component

1. Residential zones i will have a computed x value

   26 Specified x's

2. Employment zones j will have specified y's or "through drivers"

   1 Specified y's

3. Route components are given in equation form where

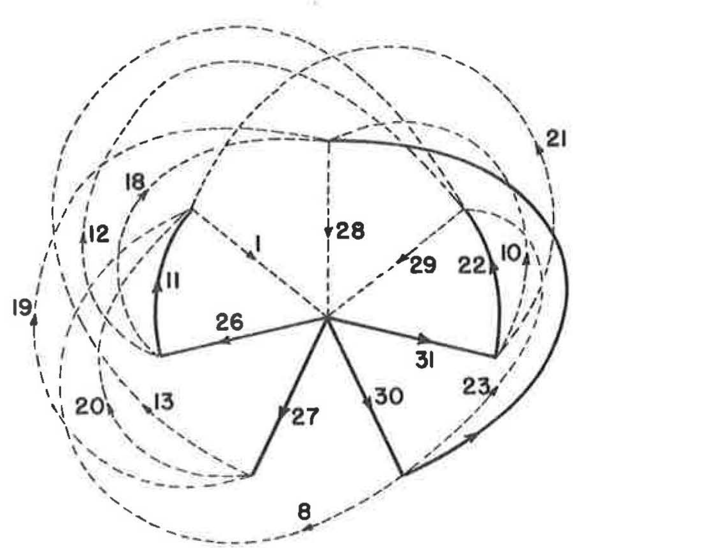
$$R = \frac{x}{y} \tag{31}$$

System Graph and Tree

The system graph and tree is shown in Figure 14. The specified flows or y's at the employment zones are placed in the chord set; the residential zones are placed in the branches. Figures 8 and 9 have been replaced by the equivalent element (element 1) in the system graph (Fig. 14).

Formulation in Cut-Set Equations

Symbolically the cut-set equation can be shown without the Y_{b-1} term because there are no specified x variables.



Branches (b-2) (26, 27, 30, 31, 9, 11 & 22)
Chords (c-1) (8, 10, 12, 13, 18, 19, 20, 21 & 23)
Chords (c-2) (1, 28 & 29)

Figure 14. System graph and tree, problem 1.

$$\begin{bmatrix} U & A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} Y_{b-2} \\ Y_{c-1} \\ Y_{c-2} \end{bmatrix} = 0 \tag{32}$$

By multiplying the matrix through,

$$\begin{bmatrix} U & A_{21} \end{bmatrix} \begin{bmatrix} Y_{b-2} \\ Y_{c-1} \end{bmatrix} + A_{22}\, Y_{c-2} = 0 \tag{33}$$

The Y values can be established as an explicit function of the X values and the x term is the specified x value from the previously noted subgraph:

$$\begin{bmatrix} Y_{b-2} \\ Y_{c-1} \end{bmatrix} = \begin{bmatrix} G_{b-2} \\ G_{c-1} \end{bmatrix} \begin{bmatrix} X_{b-2} \\ X_{c-1} \end{bmatrix} - \begin{bmatrix} G_{b-2} \\ 0 \end{bmatrix} \begin{bmatrix} x_1 \\ 0 \end{bmatrix} \tag{34}$$

This value can be substituted in Eq. 33, or

$$\begin{bmatrix} U & A_{21} \end{bmatrix} \begin{bmatrix} G_{b-2} \\ G_{c-1} \end{bmatrix} \begin{bmatrix} X_{b-2} \\ X_{c-1} \end{bmatrix} - \begin{bmatrix} U & A_{21} \end{bmatrix} \begin{bmatrix} G_{b-2} \\ 0 \end{bmatrix} \begin{bmatrix} x_1 \\ 0 \end{bmatrix} + A_{22}\, Y_{c-2} = 0 \tag{35}$$

The X_{c-1} term can then be expressed in terms of X_{b-2}, or

$$X_{c-1} = A_{21}{}^{T}\, X_{b-2} \tag{36}$$

and the final form becomes

$$\begin{bmatrix} G_{b-2} + A_{21}\, G_{c-1}\, A_{21}{}^{T} \end{bmatrix} \begin{bmatrix} X_{b-2} \end{bmatrix} - \begin{bmatrix} A_{21}\, G_{b-2}\, x_1 \end{bmatrix} + A_{22}\, Y_{c-2} = 0$$

Final Solution

The work trip interchanges found from this computation are given in Table 5. The details of the final solution have been omitted from this example, because the procedure is a straightforward matrix multiplication.

Discussion of Results

The value of desire, as a measurement of the pressure x, was found by maximizing the multiple regression equation for Y, given in the section on input values. The primary specified x_1 value for work trips for all residential zones was found to be 0.734 per dwelling unit. The final x values for each residential zone varied according to the number of dwelling units and the resistance factors.

The theoretical pressure for each zone is reduced because the parameters of income, car ownership, and so forth are different from those which would optimize the pressure for trip making.

TABLE 5

RESULTS OF ILLUSTRATIVE PROBLEM 1

Origin	Destination	Work Trips (no.)			
		Systems Engineering		Multiple Regression	Adjusted Total
Zone 26	Zone 1	873		—	
	Zone 28	965		—	
	Zone 29	1,165		—	
	Total		3,003	2,940	3,014
Zone 27	Zone 1	590		—	
	Zone 28	536		—	
	Zone 29	1,207		—	
	Total		2,333	2,180	2,235
Zone 30	Zone 1	322		—	
	Zone 28	281		—	
	Zone 29	737		—	
	Total		1,340	1,310	1,343
Zone 31	Zone 1	215		—	
	Zone 28	218		—	
	Zone 29	891		—	
	Total		1,324	1,373	1,408
Total			8,000	7,803	8,000

The trips from each residential zone computed in this problem are compared with the trips generated by each zone, using the multiple regression equation for Y with specific parameter values. The comparison (Table 5) shows that the total trips by the Y equation are only 7,803, whereas the stated input trips to the employment zones total 8,000.

Trips generated by the multiple regression equation are then adjusted proportionately to agree with the total destinations estimated at the employment zone.

PROBLEM 2

Problem Statement

Given Information—

For a given hypothetical city "Grand River," the following information is established from a current O and D survey and a special sampling survey:

1. The present interzonal trips, t_{ij}.
2. The present travel times between zones, d_{ij}.
3. Present and future evaluation of the shopping zones to establish relative attraction values on the basis of number of employees, retail square feet, and/or other parameters.
4. Probability interchange established on the basis of the relative attraction of shopping zones assuming equal travel times, P_{ij}.
5. The actual trips per unit of probability interchange, $M_{ij} = Y_{ij} / P_{ij}$.
6. Curve plot or equation for $1/M_{ij}$ versus d_{ij}.
7. Present and future estimates of income, consumption and their relationship (Fig. 10).
8. Relationship for transportation consumption, present and future, by yearly change.
9. Families per zone, present and future.
10. Percentage of trips for each purpose and their mean length (Tables 1 and 2).
11. Existing land-use data.
12. Future land-use forecasts.
13. Additional requirements on the route components, such as (a) length of each route, in miles; (b) average speed over each route component; (c) average vehicle costs per mile (Table 6); and (d) average time costs per hour.

TABLE 6

ESTIMATED COST OF OPERATING A MOTOR VEHICLE[a]

Item	Cost (¢/mi)	Percent of Total
Costs, excluding taxes:		
Depreciation	2.54	26.0
Repairs, maintenance	1.72	17.6
Replacement tires and tubes	0.18	1.8
Accessories	0.14	1.4
Gasoline, except tax	1.45	14.9
Oil	0.19	2.0
Insurance	1.29	13.2
Garaging, parking, tolls, etc.	1.08	11.1
Sub-total	8.59	88.0
Taxes and fees:		
Gasoline	0.70	7.2
Registration	0.10	1.0
Titling and property	0.10	1.0
Oil	0.01	0.1
Auto, tires, parts, etc.	0.26	2.7
Sub-total	1.17	12.0
Total operating cost	9.76	100.0

[a]After Smith (6).

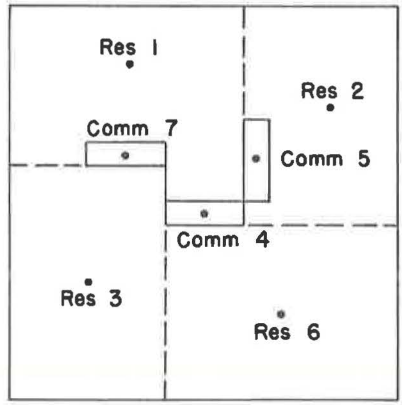


Figure 15.

To Find—

The problem is to forecast future interzonal shopping trip movements when x values for the residential zones are specified on the basis of money available for shopping trips, y values are specified for the destination zones as shopping trips, and the route components have specified resistance value, R, established on the basis of the money spent on traveling the route.

Schematic of the Physical System

The schematic of the physical system involved is given in Figure 15.

Input Values and How Established

The necessary input values for the solution of this sample problem by linear graph theory are:

1. The x_i on the basis of amount of money, in dollars, to be spent for the total shopping trips made from the residential zone i. The following equation is the condensation of all factors discussed in detail in the text:

$$x_i = \frac{K}{300} \frac{k_s m_s}{\Sigma(km)} K_h N_f (E_b + n \Delta E) Z_i \tag{37}$$

in which

- K = constant to adjust differences in dimensions;
- 300 = number of shopping days per year;
- Σ (km) includes:
- k_w = percentage of trips to work;
- m_w = average trip length to work;
- k_s = percentage of trips to shopping;
- m_s = average trip length to shopping;
- k_{sc} = percentage of trips to school;
- m_{sc} = average trip length to school;
- k_{sr} = percentage of trips to social-recreation;
- m_{sr} = average trip length to social-recreation;
- k_{em} = percentage of trips to eat meals;
- m_{em} = average trip length to eat meals;
- k_b = percentage of trips to business;
- m_b = average trip length to business;

K_h = weighted percentage, by trip length, of all trips that are home-based;
N_f = number of families in the zone;
E_b = ratio of amount spent on transportation per total consumption for base year;
ΔE = yearly change;
n = number of years from base to future year; and
Z_i = total consumption per base year.

A summary of the solution leading to values of x by the foregoing procedures is given in Table 7.

2. The input y_i values for each shopping zone are determined by methods previously defined in the text.

3. The resistance value on the route components is also established on the basis of cost of travel on the route, as explained in the text. The combined equation is

$$R_{ij} = K (c_v m_{ij} + c_t d_{ij}) \tag{38}$$

TABLE 7

SUMMARY OF SOLUTION TO x INPUT VALUES FOR RESIDENTIAL ZONES

Item	Res. Zone 1 (26)	Res. Zone 2 (27)	Res. Zone 3 (30)	Res. Zone 6 (31)
(a) Present Year				
Average disposable income per dwelling unit ($)	5,300	6,300	4,300	4,300
Total consumption, Z ($)[a]	5,300	6,150	4,400	4,400
Transp. consumption, Z^t ($)[b]	455	530	375	375
Ratio, $Z^t/Z = E_b$	0.0859	0.0862	0.0852	0.0852
(b) Future Year Estimate				
Average disposable income per dwelling unit ($)	5,700	6,500	4,800	4,600
Total consumption, Z ($)[a]	5,900	6,600	5,100	4,900
Ratio, Z^t/Z ($E_f = E_b + n\ E$)	0.1002	0.1005	0.0995	0.0995
Transp. consumption, Z^t ($)[b]	591	663	507	488
Percent for shopping trip, K_s	0.107	0.107	0.107	0.107
Shopping trip consumption, Z^{st} ($)	63.2	70.9	54.3	52.2
Number of dwelling units	750	510	442	464
Total shopping trip consumption ($1,000)	47.5	36.2	24.0	24.2
$x_i = K(C_i/300)^c$	39.6	30.2	20.0	20.2

[a]From Figure 7. [b]From Figure 8. [c]Assume K = 0.25.

TABLE 8

SUMMARY OF SOLUTION OF R_{ij} BY SUM OF VEHICLE AND TIME COSTS

Route No. (element)	Length, m_{ij} (mi)	Vehicle Cost, $c_v m_{ij}$[a] ($)	Travel Time, d_{ij} (min)	Time Cost, $d_{ij} c_t$[b] ($)	Total Cost, C_{ij} ($)	Resistance, R_{ij}, = $K C_{ij}$[c]
8	3.3	0.325	10	0.225	0.550	0.3850
9	6.8	0.664	17	0.382	1.046	0.7322
10	8.0	0.781	20	0.450	1.231	0.8617
11	3.3	0.325	10	0.225	0.550	0.3850
12	4.7	0.458	14	0.315	0.773	0.5411
13	3.3	0.325	10	0.225	0.550	0.3850
18	3.3	0.325	10	0.225	0.550	0.3850
19	4.7	0.458	14	0.315	0.773	0.5411
20	3.3	0.325	10	0.225	0.550	0.3850
21	4.7	0.458	14	0.315	0.776	0.5411
22	3.3	0.325	10	0.225	0.550	0.3850
23	5.6	0.546	14	0.315	0.861	0.6027

[a]Vehicle cost per mile assumed at $0.0976 (6).
[b]Time cost per hour assumed at $1.35 (6).
[c]K assumed to be 7×10^{-3}.

in which

K = constant for all route components to adjust for differences in dimensions;

c_v = total operating cost per vehicle mile;

m_{ij} = length of the route, in miles;

c_t = time cost per vehicle per minute, assuming an occupancy of one person per vehicle; and

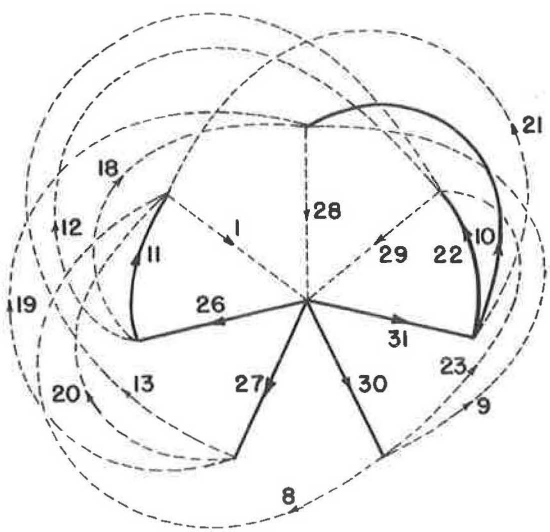
d_{ij} = travel time per vehicle per route, excluding terminal time.

A summary of the solution leading to values of R_{ij} for the route components on the basis of vehicle and time costs is given in Table 8.

System Graph and Tree

Figure 16 shows the system graph and tree used for the solution. The specified x values on the residential components were placed in the branches (b - 1) and the specified y's from the shopping zones were placed in the chord set (c - 2).

Inasmuch as the solutions require only mechanical substitution in equations, they are omitted and only the results are presented (Table 9).



Branches (b-1) Elements (26, 27, 30 & 31)
Branches (b-2) Elements (10, 11 & 22)
Chords (c-1) Elements (8, 9, 12, 13, 18, 19, 20, 21 & 23)
Chords (c-2) Elements (1, 28 & 29)

Figure 16. System graph and tree, problem 2.

Discussion of Results

The two sample problems are for systems having different trip purposes, and no effort was made to compare the two methods on like systems. The pressure-type x measurement of problem 2 is dependent only on income and consumption. In contrast to this, problem 1 uses income as well as other parameters such as car ownership, population density, and distance from the CBD. It is anticipated that these parameters are related to income, but the extent of the correlation has not yet been established.

The establishment of R_{ij} for the route components in problem 2 should provide a better estimate of the resistance offered. Whether the increased data collection is warranted by an improved estimate is subject to further evaluation.

TABLE 9

RESULTS OF ILLUSTRATIVE PROBLEM 2

Origin	Destination	Shopping Trips (no.)	
Zone 26	Zone 1	111	
	Zone 28	137	
	Zone 29	129	
	Total		377
Zone 27	Zone 1	86	
	Zone 28	80	
	Zone 29	157	
	Total		323
Zone 30	Zone 1	60	
	Zone 28	45	
	Zone 29	83	
	Total		188
Zone 31	Zone 1	43	
	Zone 28	38	
	Zone 29	131	
	Total		212
Total			1,100