A Review of Soil-Pole Behavior

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The behavior of a pole embedded in soil is reviewed in the light of modern soil mechanics. A compilation and interpretation of the available published and unpublished test results provide the background information used in a discussion of the real behavior of a soil-pole system. A theoretical analysis is presented that accounts for vertical, lateral and moment loads on the pole; the nature of the previous theoretical work available in the literature is tabulated for comparison. The proposed analytical techniques also include a simple quantitative means for determining if a flexural member is rigid enough that its deflection can be described solely as a rotation.

In application of the analytical techniques to pole designs a simple means of approximating the strength and subgrade modulus of the soil is shown to be necessary. A simple static soil penetrometer test is suggested as a practical device for field use. Other variables that must be considered in developing pole design standards are the method of construction, the loading conditions and the geometrical configuration of the poles.

• THE USES or service classifications of poles are numerous; they vary from tent stakes and fence posts to supports for the heavy signs used by many highway departments and outdoor advertising companies. Lateral loads on poles may arise from wind, guy wires and/or an alignment change on power poles. Moment loads may be due to couples, lateral loads and/or eccentric vertical loads. Vertical loads may be exerted by signs, power lines, guy wires and transformers. Additional loads of consequence can arise from earthquake motions if an appreciable mass is supported by a pole. The nature of the loading on a pole can also be of some consequence; for instance, cyclic loading is usually more serious than a static loading of equal magnitude.

The purpose of this paper is to provide a review of previous theoretical and experimental work on the soil-pole problem and to explain soil-pole behavior so that realistic pole design procedures may be developed. A critical analysis of the available experimental data will provide the basis for a physical interpretation of the interaction between a pole and the soil surrounding it. Further, a comprehensive analytical procedure will be developed that accounts for the lateral, vertical and moment loads on the pole and, in addition, provides for a wide range in soil behavior. An initial inclination of the pole is also considered in the analysis. The results of tests in typical soils will be evaluated in light of the preceding analysis and discussion of soil-pole behavior. Suggestions will be given for using the information contained herein in the development of pole design standards for different pole service classifications. Finally, it will be recommended that for practical use, the strength and subgrade modulus of different soils be correlated with a simple static penetrometer test performed in the field.

STATEMENT OF PROBLEM

Poles are considered herein as essentially rigid members whose lateral deflections under load are primarily due to rotation about a point along the embedded length of the

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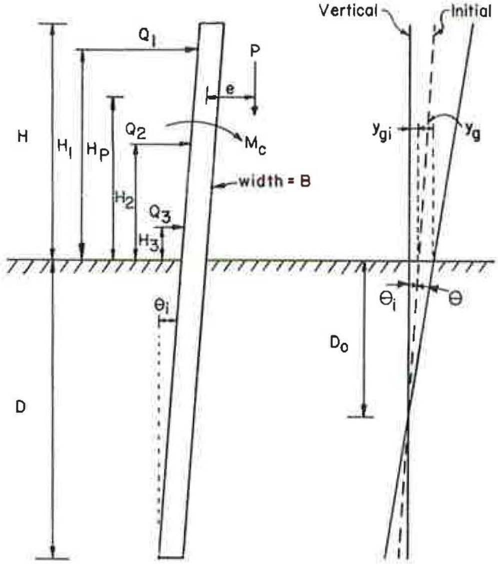

(a) Load (b) Rotation

Figure 1. Loads acting on a pole.

pole. A criterion will be given for determining when the flexural member that serves as the pole is stiff enough to be considered rigid. Because the rotational resistance of a rigid pole is governed solely by the load-deformation characteristics of the soil adjacent to the embedded portion of the pole, it is the soil that controls the behavior of a laterally loaded soil-pole system.

Figure 1a shows a pole of width B (normal to the direction of loading), embedded for a length D, and subjected to a system of lateral loads Q_1, Q_2, and Q_3 at distances H_1, H_2, and H_3 above the ground surface. For generality, a couple M_c and a vertical load P acting with an eccentricity e from the pole axis are also shown. Because an initial rotation θ_i is likely to be experienced under field conditions and because it leads to additional moments when vertical loads are present, it has been included as a variable. The pole is shown in Figure 1b as rotating an angle θ about a point at a depth D_o below the ground surface; the horizontal deflections y are measured with respect to a vertical line through the point of rotation. The initial rotation θ_i causes an initial deflection y_{gi} to exist at the ground surface; an additional deflection y_g is caused by the applied loads. Soil reactions which must be in equilibrium with the applied loads are induced by the pole rotation. The nature of the soil reactions for different soil types and loading conditions is discussed subsequently.

The scope of this paper is limited to isolated, rigid, vertical or slightly inclined poles, embedded in a homogeneous soil and subjected to vertical, lateral and moment loads. Generally poles are not placed close enough to each other that group action becomes important; however, if poles are spaced at 3 diameters or less normal to the direction of the loading, and/or 6 diameters or less parallel to the loading, group action will develop. If necessary, the effects of group action may be estimated from the work of Prakash (1).

SOIL RESISTANCE

The following discussion of soil-pole interaction represents the authors' theoretical reasoning and interpretation of the available test results. The discussion will be developed by studying the soil behavior as the lateral load on a pole is increased to failure.

Pole Rotation

Figure 2a shows a pole of width B and embedment D subjected to an increasing lateral load Q at a distance H above the ground surface. As the load Q is increased, four successive rotated positions (i to iv) are shown in Figure 2b. The net soil reaction w (force/unit of depth) is plotted versus depth in Figure 2c for the successive stages of rotation. It should be noted that w is the total reaction across the width B of the pole; it is usually considered to act uniformly across the width B, whereas in reality it represents the increase in pressure on the loaded face of the pole minus the decrease in pressure on the backside of the pole plus the shear resistance along the sides of the pole. The soil reactions for three depths (B, 2B and 3B) are plotted versus the corresponding deflections in Figure 2d. As the load is increased the point of rotation shifts downward along the pole. Furthermore, the upper point of relative

Failure

As the rotation of a pole is increased to failure, three general modes of failure may be identified. A flexural failure of the pole may occur above or below the groundline depending on the flexural properties of the pole and the nature of the loading. The elimination of a flexural failure is a simple matter once the moments along the embedded portion of the pole are known; a procedure will be given subsequently for estimating the moments. Another mode of failure is excessive rotation which may or may not be due to plastic soil behavior. Finally, a complete collapse of the pole can occur when a sufficient amount of plastic soil behavior has developed. The latter behavior has been illustrated by Krynine (3) in tests wherein the point of rotation moved downward along the pole until at large rotations it shifted to the ground surface; a wedge of soil equal to the embedded length of the pole was then forced upward as shown in Figure 4a. Appleford (4) attempted to analyze this state of total collapse by bounding the wedge with a line from the bottom of the pole inclined at an angle of 60 degrees; Williams (5) performed a similar analysis using an angle of 45 degrees.

Ultimate Soil Resistance

A discussion of the ultimate soil resistance that is available to resist the rotation of a pole will aid in understanding the soil-pole behavior at loads near the collapse load. Figure 4b shows a fully plastic distribution of soil reactions along the embedded portion of a pole (solid line). In reality, for loads near the collapse load where large rotations are observed, the actual distribution of the soil reactions is more in agreement with the dotted line in Figure 4b. It may be reasoned that collapse is impending when either the upper or lower point of maximum soil reaction reaches the fully plastic condition. Therefore, if the soil reactions and the ultimate soil resistance can be predicted, the ultimate soil resistance can be divided by a factor of safety to obtain a limit to the working values of the soil reactions.

If the loaded face of a pole is considered to be infinite in lateral extent it then becomes a wall; the ultimate soil resistance against a wall is the familiar two-dimensional passive pressure. Many investigators have assumed, mistakenly, that the ultimate soil resistance offered to a pole is the two-dimensional passive pressure. Actually the problem is three-dimensional because of the end effects at the edges of the pole; therefore, the ultimate soil resistance must exceed the two-dimensional passive pressure. At depth, a lateral translation of the pole will deform the soil in a mode similar to that for the bearing capacity of a deep footing. The problem then becomes that of the two-dimensional bearing capacity of a footing completely surrounded by soil. The ultimate soil resistance, therefore, varies with the depth below the ground surface even though the strength parameters φ and c are constant with respect to depth. In summary, the ultimate soil resistance is slightly in excess of the two-dimensional passive earth pressure at the ground surface; it increases with depth as it makes a transition to that given by the two-dimensional lateral bearing capacity.

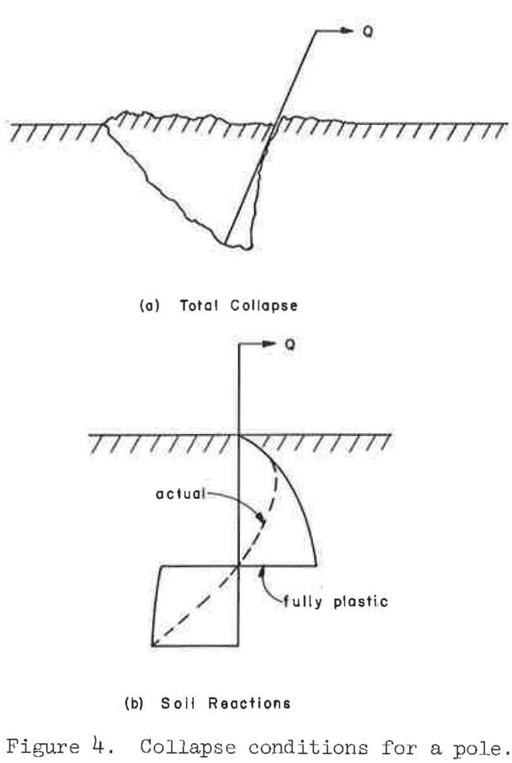

Figure 4. Collapse conditions for a pole.

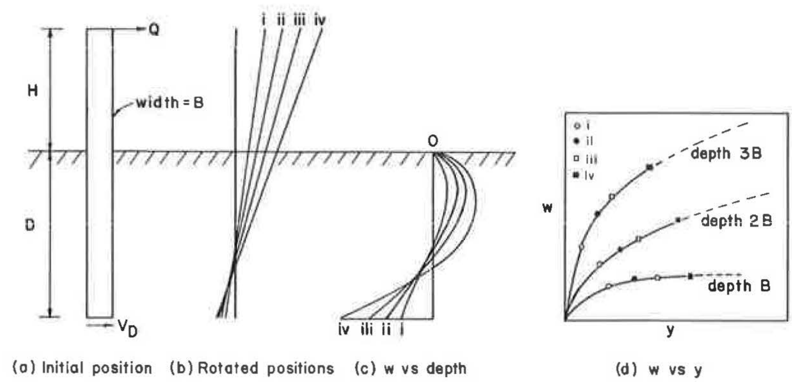

Figure 2. Pole rotation and soil reactions.

maximum soil reaction shifts downward along the pole although the same qualitative shape of the soil reaction versus depth diagram is maintained. The geometry of the pole rotation requires the largest deflections to occur at the ground surface and at the bottom of the pole; therefore, the soil resistance initially becomes plastic in these two areas before spreading downward and upwards along the pole. Generally, plastic soil resistance is observed near the ground surface even if it does not occur at the bottom of the pole. The w versus y curves in Figure 2d illustrate the foregoing discussion. At depth B plastic behavior is observed, whereas at depths 2B and 3B the soil reactions are at successively smaller percentages of the ultimate values. The ultimate soil resistance increases with depth even though the strength parameters, the cohesion c and the angle of internal friction φ, are constant with depth.

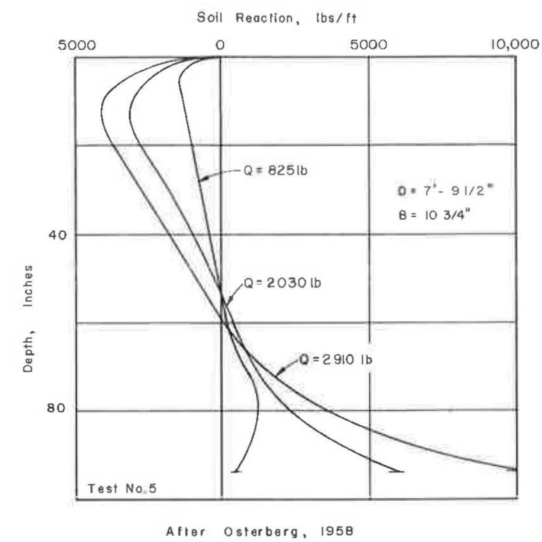

Figure 3. Soil reaction vs depth from a test in clay.

Shear on Bottom of Pole

Another element of resistance that is usually ignored in pole analyses is the sheer V_D acting along the bottom of the pole (Fig. 2a). Neglecting V_D is conservative for design purposes but the analysis of test results becomes difficult because it may or may not be present or may be present in the earlier stages of loading and practically disappears during the later stages. For laterally loaded piles V_D is ignored because piles are usually long enough that the effect of V_D cannot be detected. As the embedded length of the flexural member is decreased into the range of that for poles, V_D becomes progressively more important. Finally, for very short poles which are really block foundations, the incorporation of V_D into an analysis becomes a necessity as is also true with the vertical pressures on the bottom of the foundation. Figure 3 shows several soil reaction versus depth curves for a test on a pole in stiff clay that was reported by Osterberg (2). Under the 825-lb load, the action of V_D on the bottom of the pole is quite pronounced, whereas for higher loads its action appears negligible.

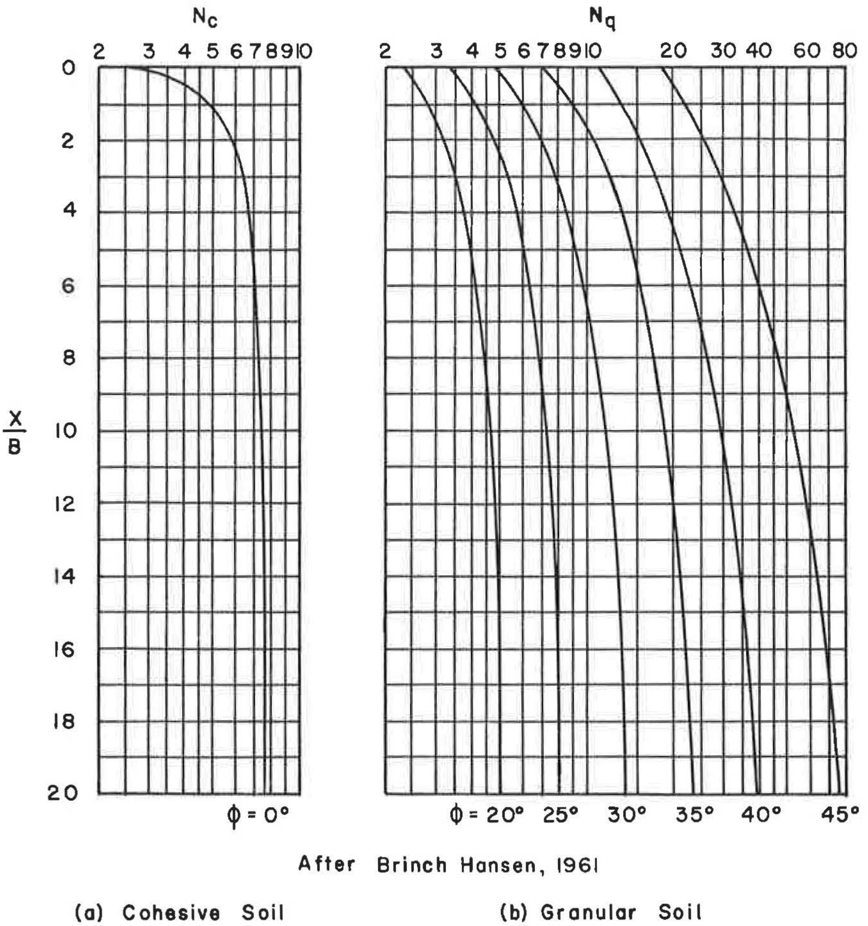

Figure 5. Bearing capacity factors vs depth.

Several authors have observed the foregoing behavior. Minikin (6) found that the ultimate soil resistance offered to poles was 2.3 to 3.4 times greater than that offered to a wall in the same soil. Krynine (7) has shown that the ultimate soil resistance was proportional to the depth for a wall and to the second power of depth for a pole. In addition, Stobie (8) measured pressures against a pole in excess of Rankine's passive pressure.

The unit lateral bearing capacity of cohesive soils ($\varphi = 0^0$ condition) can be expressed as cN_c where N_c is a dimensionless bearing capacity factor. Meyerhoff (9) has computed N_c to be as high as 11.42, whereas Brinch Hansen (10) has computed 8.14. MacKenzie's (11) tests indicate that a value between 8 and 9 is appropriate for N_c. Brinch Hansen (10), on the basis of reasonable assumptions, has computed the ultimate soil resistance against a pole as a function of depth for both cohesive and granular soils; Figure 5 shows the essential results of Hansen's theory. In this paper soils will be treated as either cohesive ($\varphi = 0^0$) or granular ($c = 0$). Correspondingly, Figure 5a shows N_c versus the depth x divided by the pole width B. The ultimate soil reaction for a cohesive soil is then expressed as

$$w_u{}^x = B c N_c{}^x \tag{1}$$

in which the superscript x indicates that w_u corresponds to the depth x for which N_c was determined. Similarly, Figure 5b shows N_q plotted versus x/B for various values

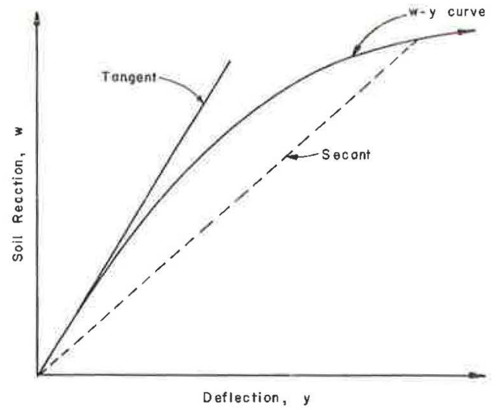

Figure 6. Soil reaction vs deflection.

of φ where N_q is a dimensionless bearing capacity factor dependent on φ. The ultimate soil reaction of a granular soil is then expressed as

$$w_u^x = Bq' N_q^x \tag{2}$$

where q' is the vertical effective stress at the depth x.

Subgrade Modulus

To calculate the soil reactions consistent with a given rotated position of the pole, some knowledge of the soil reaction versus deflection relationship (w vs y) is required. If the soil reactions can be determined under given conditions, then they may be compared to the ultimate soil resistance for the purpose of estimating the factor of safety against a total collapse of the pole.

Figure 6 shows a typical w-y curve for soil. For soil reactions of less than one-third to one-half of the ultimate soil reaction, the w-y relationship can be expressed adequately by a tangent modulus. For larger soil reactions a secant modulus is more appropriate. Because it will generally be desirable to keep the soil reactions smaller than one-third of the ultimate soil reaction, the discussion as developed herein will be concerned with the tangent modulus k (expressed in units of force/length^2)

$$k = \frac{w}{y} \tag{3}$$

If a reasonable variation of k with depth can be formulated, then a pole may be analyzed as a rigid member restrained by a series of infinitely closely spaced independent elastic springs. This is the concept of a subgrade modulus.

In using a subgrade modulus to define the soil stiffness it is assumed that the springs are independent, whereas in reality they are interrelated in a complex fashion. The error involved may be estimated by considering the contribution of Vesic (12) who extended Biot's (13) work concerning a flexible beam supported on an elastic half-space. For long relatively flexible members such as piles the error in the computed bending moments based on the subgrade modulus assumption was no more than a few percent when compared to the theory of elasticity solution. The error increased to as much as 14 percent for relatively short and essentially rigid beams. Therefore, the subgrade modulus concept has a reasonable theoretical foundation; it is believed to be more than adequate for the pole problem.

Several variations of the subgrade modulus with depth have been used in pole theories. The most useful variation was that developed by Miche (14), Titze (15) and later Palmer and Thompson (16); they expressed the modulus as

$$k_x = K\left(\frac{x}{D}\right)^n \tag{4}$$

in which K is the value of k at the bottom of the pole and n is an empirical coefficient equal to or greater than zero. Figure 7 shows the variation of k with depth for various values of n. Most authors of pole theories assumed that n = 0 for clays, or that the modulus was constant with depth, and n = 1 for granular soils, or that the modulus increased directly with depth.

The theoretical reasoning of Terzaghi (17) indicates that n is approximately unity for sands and, in addition, virtually all of the test results in granular soils can be adequately analyzed with n = 1 although there are indications that an n-value of 1.5 may be more appropriate. No conclusive test results are available for clays, but on the basis of the previous discussion concerning the w-y relationships in Figure 2d, it is likely that a value of n greater than zero is more realistic than a value of zero. A tentative value of 0.15 for n is herein suggested on the basis of the information presently available; this has the effect of including an allowance for plastic soil behavior at the ground surface. It should not be expected that a unique relationship in the form of Eq. 4 can be obtained for the variation of k with depth in clay soils because the D/B ratio, the nature of the loading, and the soil itself all affect k. However, some reasonable n-value is likely to be adequate for the majority of preloaded clays that are encountered.

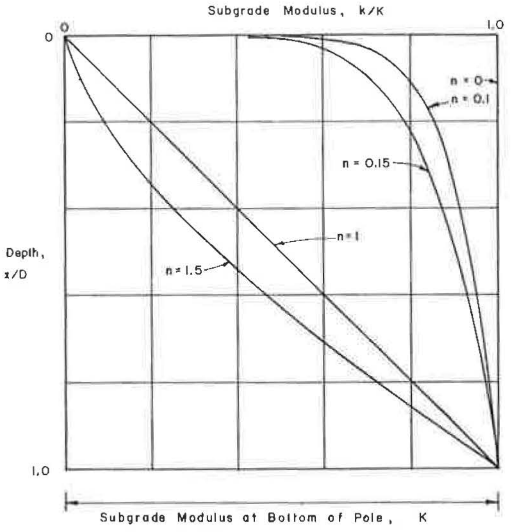

Figure 7. Variation of subgrade modulus with depth.

Terzaghi (17) has presented an extensive discussion regarding the effect of size of the loaded area on the subgrade modulus. For example, a loaded pole of width B produces soil reactions w and deflections y. If another pole of equal embedment in the same soil but of width 2B is subjected to the same loading, the soil reactions and deflections will be equal to those for the pole of width B. Although the unit soil pressures across the face of the pole with width 2B are one-half those for the pole with width B, the dimensions of the stressed volume of soil are doubled because of the size of the loaded area; therefore, the deflections at the faces of the two poles are equal. After k has been determined for a given pole its value is unchanged if the pole width B is changed.

Poles Widened at the Ground Surface

The foregoing discussion concerning the size of the loaded area was based on the assumption of linear elastic behavior for the soil. Actually, as shown in Figure 2d, plastic soil behavior is likely to develop at the ground surface. In the analysis of a pole with uniform width B this behavior will be accounted for by using an empirically predetermined elastic modulus variation with depth that really involves both the secant modulus (Fig. 6) near the ground surface and the tangent modulus at greater depths. If a pole of width B were to be widened, for example to width 2B for some depth below the ground surface, strictly elastic considerations would indicate no change in the load-deflection behavior. In actuality, the ultimate soil reaction for the enlarged portion of the pole is approximately doubled because it depends on the width, whereas the soil reaction itself is only slightly increased. This has the effect of increasing the magnitude of the secant modulus (Fig. 6) because the soil reaction is now a smaller percentage of the ultimate soil reaction than it was for an unenlarged pole section; therefore, the deflections should be reduced.

Anderson (18) and Osterberg (2) have shown large reductions in the rotation of poles that have been enlarged or otherwise stiffened at the ground surface when compared to unenlarged poles. Davisson and Gill (19) have analytically studied a similar effect for laterally loaded piles and found that reductions of 50 percent in the lateral deflection of the pile under a given load are easily obtained.

Effect of Pole Shape

Conflicting opinions have been reported in the literature on the effect of the shape of the pole on its behavior. Czerniak (20) believes that because the maximum pressure against the middle element of a circular pole is 1.57 times the average pressure on the projected area, or on a flat surface equal in width to the pole diameter, and that because a curved surface can penetrate the soil more easily than a flat surface, the effectiveness of a round pole must be lower than that of a square pole. Shilts, Graves and Driscoll's (21) experimental evidence on model poles in sands showed that a 3-in. diameter pole moved more (approximately 33 %) under a given load than a 3-in. square section. On the other hand, Williams (5) concluded from his tests on model poles that square poles of width B could withstand only about 90 percent of the overturning moment withstood by poles with a circular section of diameter B.

Davisson (22) analyzed Nakamura's (23) tests on 6-cm wide model poles embedded in sand. The shapes studied were round, square, and diamond. The diamond-shaped poles were actually square bars loaded along the diagonal rather than along a side. According to this analysis, the shape of the pole cross-section has a negligible effect on the soil resistance and pole deflections.

Effect of Cyclic Loading

Several aspects of soil behavior may influence the strength and stiffness of the soil surrounding a pole. As an example, cyclic loading may remold an undisturbed cohesive soil by the process of repeated shearing deformations. This will reduce both its strength and stiffness. However, a rest period may produce an increase in strength and stiffness depending on the thixotropic and consolidation characteristics of the soil. Similar events can occur for poles that are driven into clay soils. Matsuo (24) has shown that the lateral load resistance of driven poles increases with time after driving because of the strength increase caused by thixotropy and consolidation under the high horizontal pressures induced by driving the pole.

Cyclic loading usually causes a permanent displacement of the soil at the ground surface, thereby leaving a gap between the soil and the pole. This has the effect of reducing the subgrade modulus k to zero, regardless of its initial value. Therefore, an n-value of zero in Eq. 4 is virtually an impossibility.

Effect of Climate and Method of Construction

The soil resistance may vary considerably according to the season. Seasonal moisture variations brought on by floods, rains, frost action, droughts or other causes may exert a controlling influence on the soil resistance. For purposes of design or research, the worst soil condition should be studied.

Construction procedures also control, to a large extent, the behavior of a soil-pole system. Poles may be placed in loose or tight fitting holes that have been hand-dug or bored; they may also be driven into place. The backfill around a pole may be loose or compact or may vary considerably in thickness (horizontally), depending on the size of the hole. Good construction practice calls for a tamped or otherwise compacted fill. If the fill is thin, the load-deformation characteristics of the undisturbed soil around the pole will control the behavior. For a thick fill, the behavior may be controlled entirely by the load-deformation characteristics of the fill.

ANALYTICAL STUDIES

In the analysis of a pole it is assumed that the pole is rigid enough that its deflected shape can be described by a rotation. As the embedment of a given pole section is increased, the greater is the possibility that this assumption will become invalid. For a flexural member embedded in an elastic medium wherein k can be assumed to be constant with respect to depth (n = 0), solutions are available that define the depth to which a pole may be assumed rigid. Grandholm (25) and later Davisson and Gill (19) have suggested the following criterion: If the depth D divided by the relative stiffness factor R (Eq. 5) is equal to or less than 2, then the pole may be considered rigid.

$$R = \sqrt[4]{\frac{EI}{K}} \tag{5}$$

$$\frac{D}{R} \leqq 2 \text{ (rigid pole)} \tag{6}$$

EI represents the flexural stiffness of the pole. It is recommended that Eqs. 5 and 6 be used to approximate the limits of a pole in cohesive soils even though n is closer to 0.15 than it is to zero.

For granular soils (n = 1) Eq. 4 reduces to

$$k_x = n_h x \tag{7}$$

where n_h is the constant of the horizontal subgrade reaction equal to K/D and with units of force/length^3. Matlock and Reese (26) and Davisson (22) have proposed a criterion, similar to that given above, for poles embedded in soils for which the subgrade modulus is directly proportional to depth. It is recommended that the following equations be used for granular soils and for any other soils for which an n-value of unity is a good approximation:

$$T = \sqrt[5]{\frac{EI}{n_h}} \tag{8}$$

$$\frac{D}{T} \leqq 2 \text{ (rigid pole)} \tag{9}$$

Again, T is a relative stiffness factor.

In the following analysis the initial inclination of a pole may be of significance if a vertical load is acting on the pole. Furthermore, it is necessary that the vertical load placed on the pole does not exceed the critical vertical load P_{cr}. Because the complete analysis is facilitated if the axial load P is expressed as αP_{cr}, where α equals P/P_{cr}, the expressions for P_{cr} will be given first.

An initially vertical pole embedded for a depth D is considered to be subjected to a vertical load P at a distance H_p above the ground surface. If it is assumed that the shear V and the moment M are zero at the bottom of the pole ($V_D = M_D = 0$), and that $\sin\theta$ is approximately equal to θ, then the soil-pole system is essentially a rigid bar restrained by a moment spring. The characteristics of the moment spring are those of the soil as represented by the modulus of subgrade reaction which is considered to be governed by Eq. 4. On the basis of the foregoing assumptions Prakash (27) has derived the following equation for P_{cr}:

$$P_{cr} = \frac{KD^2}{(n+3)\,(n+2)^2\left(1+\frac{H_p}{D}\right)} \tag{10}$$

It is presumed that the vertical bearing capacity of the pole is sufficient to carry the imposed vertical load or that the pole is adequately designed as a pile.

Figure 8 illustrates the general conditions considered in the following pole analysis. In Figure 8a a pole of width B normal to the plane of the loading is embedded for a length D and unsupported for a length H. It is inclined at an angle θ_i before being subjected to the moment M_g and the horizontal load Q_g at the ground surface plus the vertical load P at a distance H_p above the ground surface. M_g is the resultant moment for all the horizontal loads and couples on the pole and includes the moment due to the eccentricity of the vertical load from the pole axis. Likewise, Q_g is the summation of all horizontal loads. The pole is shown rotated through the angle θ in Figure 8b; x is the depth coordinate measured from the ground surface and D_o is the depth to the point of rotation. Eq. 4 expresses the soil stiffness shown schematically in Figure 8c and

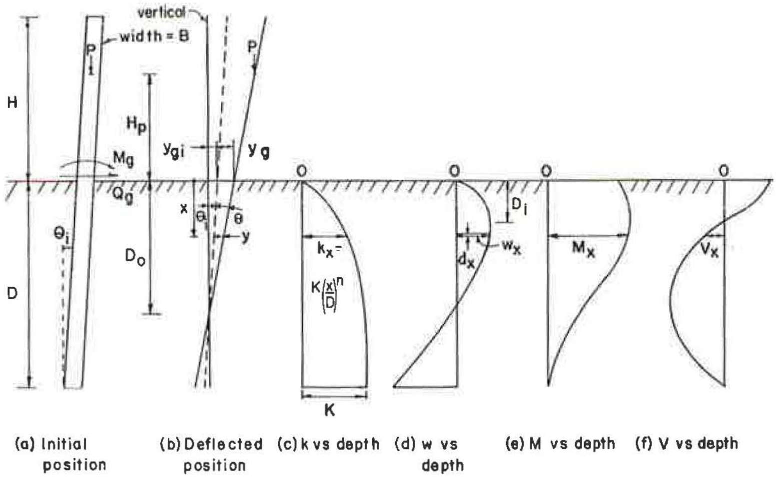

Figure 8. Variables considered in the pole analysis.

the soil reactions develop as shown in Figure 8d. The general shapes of the moment and shear diagrams versus depth are shown in Figure 8e and f, respectively. Assuming that $\sin\theta$ is approximately equal to θ and that the shear and moment are zero at the bottom of the pole ($V_D = M_D = 0$), the following expressions have been developed by Prakash (27):

$$\frac{D_o}{D} = \frac{\frac{M_g}{Q_g D} + \frac{n+2}{n+3} - \frac{\alpha}{(n+2)(n+3)} + \frac{\alpha \theta_i K D^2}{(n+3)(n+2)^2 Q_g}}{\frac{n+2}{n+1}\frac{M_g}{Q_g D} + \frac{\alpha \theta_i K D^2}{(n+1)(n+2)(n+3) Q_g} + 1} \tag{11}$$

$$y_g = \frac{(n+2) Q_g \frac{D_o}{D}}{KD\left(\frac{n+2}{n+1}\frac{D_o}{D} - 1\right)} \tag{12}$$

$$\theta = \frac{y_g}{D_o} \tag{13}$$

$$w_x = K\left(\frac{x}{D}\right)^n y_g \frac{D_o - x}{D_o} \tag{14}$$

With the foregoing equations the moments and shears along the embedded portion of the pole can be computed. To check the upper and lower points of relative maximum soil reaction against the ultimate soil resistance, an expression for the depth D_1 (Figure 8d) is needed. The upper point of maximum soil reaction occurs at

$$D_1 = \frac{n}{n+1} D_o \tag{15}$$

whereas the lower maximum is at the depth D.

It should be noted that the principle of superposition was assumed valid in the derivations. This follows directly from the assumption of linear elastic soil behavior. Because these equations are intended for use in the range of small rotations, as is desirable under working conditions, the soil behavior should be nearly elastic; therefore, the principle of superposition becomes a reasonable approximation.

For granular soils and for any other soils for which an n-value of unity is reasonable, Eqs. 10, 11, 12, and 14 reduce to the following equations on substitution of n = 1 and $K = n_h D$

$$P_{cr} = \frac{n_h D^3}{36\left(1 + \frac{H_p}{D}\right)} \quad (n = 1) \tag{16}$$

$$\frac{D_o}{D} = \frac{\frac{M_g}{Q_g D} + \frac{3}{4} - \frac{\alpha}{12} + \frac{\alpha \theta_i n_h D^3}{36\, Q_g}}{\frac{3}{2}\frac{M_g}{Q_g D} + \frac{\alpha \theta_i n_h D^3}{24\, Q_g} + 1} \quad (n = 1) \tag{17}$$

$$y_g = \frac{3\, Q_g \frac{D_o}{D}}{n_h D^2 \left(\frac{3}{2}\frac{D_o}{D} - 1\right)} \quad (n = 1) \tag{18}$$

$$w_x = \frac{3\, Q_g}{D\left(\frac{3}{2}\frac{D_o}{D} - 1\right)} \left(\frac{D_o}{D} - \frac{x}{D}\right) \frac{x}{D} \quad (n = 1) \tag{19}$$

On substitution of the proper depths in Eq. 19 the two maximum soil reactions may be expressed as follows:

$$w_{D1} = \frac{3\, Q_g}{D\left(\frac{3}{2}\frac{D_o}{D} - 1\right)} \left(\frac{D_o}{2D}\right)^2 \quad (n = 1) \tag{20}$$

$$w_D = \frac{3\, Q_g}{D\left(\frac{3}{2}\frac{D_o}{D} - 1\right)} \left(\frac{D_o}{D} - 1\right) \quad (n = 1) \tag{21}$$

Similarly, for preloaded cohesive soils for which an n-value of 0.15 is tentatively offered

$$P_{cr} = \frac{K D^2}{14.6\left(1 + \frac{H_p}{D}\right)} \quad (n = 0.15) \tag{22}$$

$$\frac{D_o}{D} = \frac{\dfrac{Mg}{Q_g D} + 0.683 - \dfrac{\alpha}{6.78} + \dfrac{\alpha \theta_i K D^2}{14.6 Q_g}}{1.87 \dfrac{Mg}{Q_g D} + \dfrac{\alpha \theta_i K D^2}{7.80 Q_g} + 1} \quad (n = 0.15) \tag{23}$$

$$y_g = \frac{2.15 Q_g \dfrac{D_o}{D}}{KD\left(1.87 \dfrac{D_o}{D} - 1\right)} \quad (n = 0.15) \tag{24}$$

$$w_x = \frac{2.15 Q_g}{D\left(1.87 \dfrac{D_o}{D} - 1\right)} \left(\frac{D_o}{D} - \frac{x}{D}\right)\left(\frac{x}{D}\right)^{0.15} \quad (n = 0.15) \tag{25}$$

$$w_{D1} = \frac{2.15 Q_g}{D\left(187 \dfrac{D_o}{D} - 1\right)} \left(0.870 \frac{D_o}{D}\right)\left(0.130 \frac{D_o}{D}\right)^{0.15} \quad (n = 0.15) \tag{26}$$

$$w_D = \frac{2.15 Q_g}{D\left(1.87 \dfrac{D_o}{D} - 1\right)} \left(\frac{D_o}{D} - 1\right) \quad (n = 0.15) \tag{27}$$

The preceding equations are tractable once the quantity D_o/D is determined. If θ_i is taken as zero, Eq. 11 can be solved by the use of graphs similar to Figure 9a for the solution of Eq. 17 and Figure 9b for the solution of Eq. 23. A somewhat more complicated nomograph can be developed that will account for any of the parameters that it is desired to vary. Because poles are generally considered to be subjected to some standard loading and because large quantities of similar poles are used, a nomograph can be developed for each standard pole type. This approach will greatly reduce the necessary calculations when a large number of similar poles are under consideration.

REVIEW OF THEORETICAL DEVELOPMENTS

The development of adequate pole theories has been a rather slow process. A pole foundation involves most of the uncertainties that are present in large and very important projects; these uncertainties may be grouped under the headings of soil properties and the interaction between the soil and the foundation when a load is applied. Therefore, the desire for a simple pole design procedure is analogous to the desire for a magic wand to aid in the solution of foundation engineering problems. Simple design procedures can be developed for a limited range of variables, but their development requires a broad understanding of the problem.

The earlier theories tried to predict the lateral load necessary to overturn a pole; it was usually assumed that the ultimate soil resistance was equal to the two-dimensional passive pressure. This resulted in very conservative expressions for the overturning force. Although Carpentier (28), Demogue (29) and Stobie (8) recognized the error involved in using the two-dimensional passive pressure, other authors continued to develop and publish a proliferation of erroneous theories.

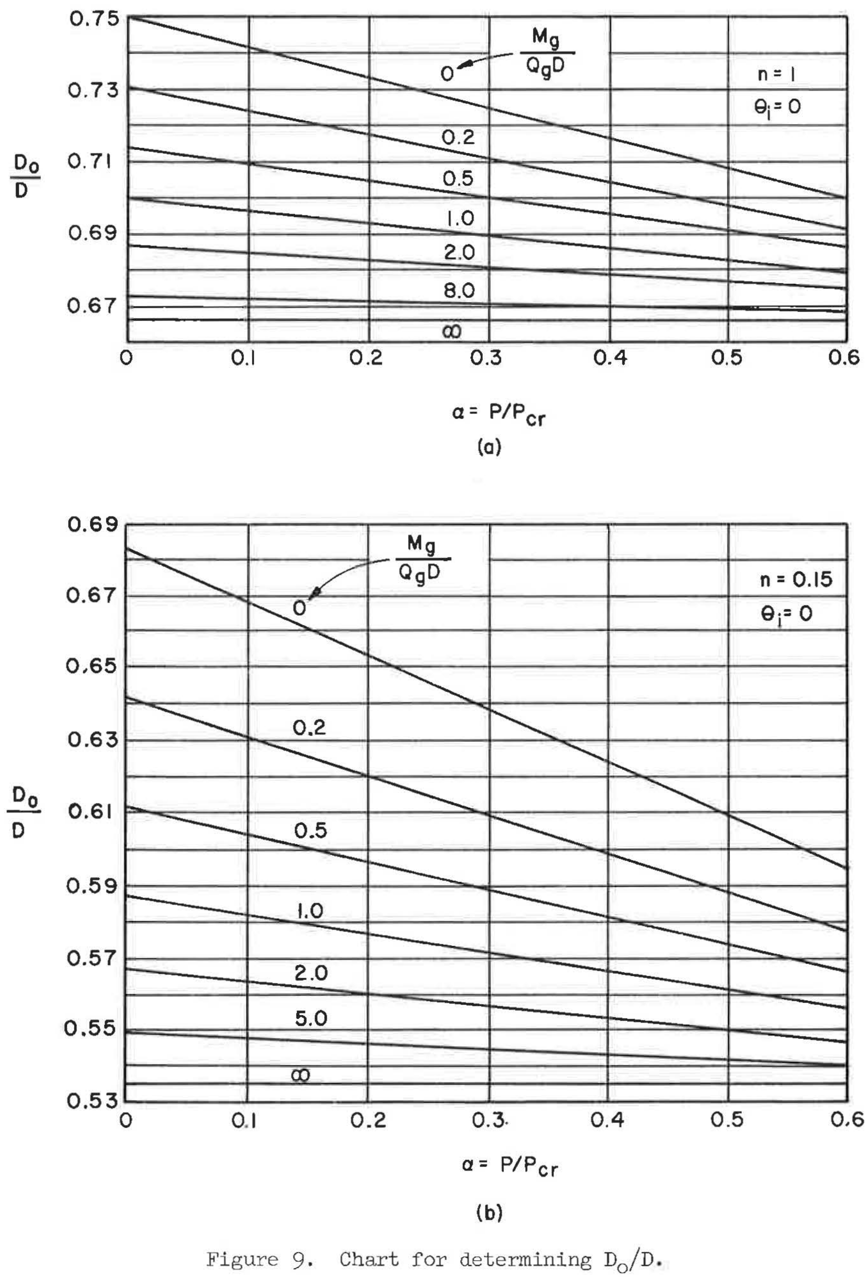

Figure 9. Chart for determining D_o/D.

More realistic analyses of the force required to overturn a pole were developed on the basis of a wedge of soil that became plastic during loading. Seiler (30) and Drucker (31) were among the earliest to use this approach. Analyses concerned with predicting the load deformation characteristics of the soil-pole system have usually been based on the assumption that the soil stiffness can be represented by a subgrade modulus. Grandholm (25) was the earliest to indicate when a flexural member is stiff enough to be considered rigid in a pole analysis. Rowe (32) was among the first to use a realistic variation of the subgrade modulus with depth, whereas Brinch Hansen (10) gave the first unified picture of the variation of the ultimate soil resistance with depth. Anderson

TABLE 1

THEORETICAL CONTRIBUTIONS

Author	Remarks
Carpentier (28)	Parabolic distribution of soil reactions assumed.
Schutz (35)	Linear variation of soil pressures.
Baldini (36)	
Stotzer (37)	
Goodrich (38)	
Grandholm (25)	Defined a pole using subgrade modulus technique.
Williams (5)	Solved for point of rotation and soil reactions.
Wilcoxen (39)	
Jager (40)	Usefulness of enlarged pole sections at ground surface noted.
Seiler (30)	Empirical adjustment of theory to fit tests.
Wolff (41)	
Drucker (31)	
Raes (42)	
Demogue (29)	
Griffith (45)	
Abbet (44)	
Minikin (45)	
O'Neil (46)	Considered non-uniform pole sections.
Fordham (47)	
Kohler (48)	
Pender (49)	
Gray (50)	
Lummis (51)	
Minikin (6)	
Terzaghi (17)	Realistic soil properties.
Rowe (32)	Realistic soil properties.
Robbins (52)	
Nelidov (53)	
Czerniak (20)	
Anderson (18)	Considers poles enlarged at the ground surface.
Matlock and Reese (26)	Define a pole. General subgrade modulus techniques.
Prakash (27)	Includes vertical load and initial rotations.
Greene (33)	Dimensional analysis techniques.
Kent (34)	Non-linear subgrade modulus.
Brinch Hansen (10)	Variation of ultimate soil resistance with depth.

(18) took into account various shapes for the loaded face of the pole; his analysis indicates that pole shapes widened at the ground surface are remarkably efficient in reducing the pole rotation under a given load. Greene (33) has used dimensional analysis techniques combined with test results to develop empirical expressions for the load versus rotation relationship, whereas Kent (34) has used a nonlinear expression for the subgrade modulus in an attempt to arrive at the same relationship.

Table 1 lists the various authors of theories applicable to poles in a generally chronological order; remarks are listed for selected pertinent references. A review of the literature indicates that pole theories are so simple that most authors develop and publish their own original work quite unaware that exactly the same thing has been done many times before.

REVIEW OF EXPERIMENTAL RESULTS

A considerable amount of experimental work has been performed on poles by a large number of authors; however, most of the test data cannot be analyzed because the pertinent properties of the soil are unknown. In most cases, the soil is not even adequately described. Therefore, the majority of the experimental data is useful only for the aspects of soil-pole behavior that can be physically observed. This observed behavior has been used in the discussion of soil resistance previously given. Table 2 gives a summary of the experimental sources known to the authors.

Sandeman (54) performed the first set of pole-pile tests reported in the literature. Wooden poles were embedded to various depths in sand, clay and fill composed of ashes. The results indicated that the ultimate load a pole could withstand increased with embedment until a depth was reached where the structural strength of the pole limited the loading. It was not until fifty years later that Stobie (8) performed a series of tests, measuring soil reactions, that proved equally useful. Nakamura's (23) tests, besides indicating the pole shape effect previously cited, illustrated that progressively increasing deflections are observed for poles embedded in sand when subjected to repeated loads. Rifaat (59) performed tests in sands with soil reaction measurements. He found that an n-value of unity in Eq. 4 expressed the variation of subgrade modulus with depth; however, it was necessary to consider the shear V_D along the bottom of the pole in the analysis.

Matsuo (24) found that the deflections for a given lateral load on a pole decreased with the time after driving of the pole in clay. Minikin (6, 45) experimentally observed the difference in the unit ultimate soil resistance between poles and walls. He proposed coefficients for the two-dimensional passive pressure formulas to account for the three-dimensional behavior of poles. Anderson (62) experimentally demonstrated the large reduction in the deflections of a pole enlarged at the ground surface when compared to a prismatic pole.

Loos and Breth (64) presented the first test results using SR-4 gages to indicate the moments along the embedded portion of the pole. Similar techniques have been used by Wilkins (65), Walsenko (71) and Osterberg (2, 67). The bending moment diagrams may be double differentiated to obtain an approximate curve of soil reaction versus depth. Tests of this type are generally suitable for analysis if a sufficient quantity of closely spaced SR-4 gages are employed.

ANALYSIS OF TEST RESULTS

Virtually all test data available for either piles or poles indicate that an n-value of unity in Eq. 4 is appropriate for granular soils and normally loaded cohesive soils. Davisson (22) and Prakash (1, 27) have collected ample evidence to support this statement. An example of the differences between the theoretically computed and the experimentally observed moments for a pole in sand is shown in Figure 10. Wilkin's (65) tests were analyzed by using the observed moment, lateral load, and deflection at the ground line in Eqs. 17 and 18 to obtain n_h. Then Eq. 19 was used in the calculation of the theoretical moments shown by the dashed lines (Fig. 10). The comparison between these observed and computed moments is typical of all test data analyzed in the foregoing manner for granular soils and normally loaded cohesive soils.

The only tests in preloaded cohesive soils suitable for analysis are those by Osterberg (2, 67). Unfortunately, these tests are obscured by variations in the

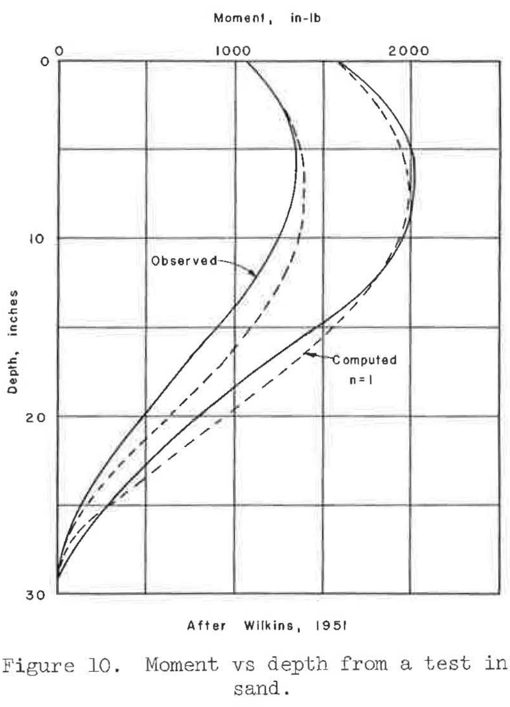

Figure 10. Moment vs depth from a test in sand.

TABLE 2

EXPERIMENTAL STUDIES

Author	Soil Type	Remarks
Sandeman (54)	Sand, clay, ashes	First published tests.
Winchester (55)	Gravel, sand	
Stieves (56)		
Carpentier (28)		
Chardin (57)		
Browne and Fontaine (58)	Hard clay	
Stobie (8)	Sand, clay	First pressure measurements.
Krynine (3)	Clay	
Nakamura (23)	Sand	Effect of pole shape and repeated loads.
Rifaat (59)	Sand	Pressure measurements, effect of V_D.
Agatz (60)		
Raes (42)	Sand	
Matsuo (24)	Clay	Effect of remolding soil.
Minikin (45)	Sand, clay	Realistic appraisal of ultimate soil resistance.
Fordham (47)	Sand	
Osipovich (61)		
Gray (50)		
Schilts, Graves, and Driscoll (21)	Sand, clay	
Anderson (62)		Effect of poles enlarged at the ground surface.
Gruyter and Schieveen (63)		
Loos and Breth (64)	Sand	Moments determined from SR-4 gage measurements. Illustrated transition from rigid pole to flexible pile behavior.
Minikin (6)	Sand, clay	Realistic appraisal of ultimate soil resistance.
Wilkins (65)	Sand	Moments determined from SR-4 gage measurements.
Williams (5)	Sand	
Marjerrison (66)	Rock, pumice	
(Rutledge) Osterberg (67)	Clay	Moments determined from SR-4 gage measurements.
Caswell and Andrews (68)	Gravel, clay	Use of gravel to improve clay backfills.
Rowe (32)	Sand	Realistic appraisal of subgrade modulus.
Mors (69)		
Lazard (70)	All types	Statistical results of over 200 tests.
Steel (71)	Gelatin	Model studies to find D_0.
Walsenko (72)	Sand	Moments determined from SR-4 gage measurements.
Osterberg (2)	Clay	Moments determined from SR-4 gage measurements.
Behn (73)	Sand, clay	Short-term and long-term tests.
Kent (34)	Sand	
Greene (33)	Sand	
Christensen (74)	Sand	Concerned with ultimate load.

properties of the clay and the technique of placing the pole. The pole was a 10.75-in. diameter aluminum pipe equipped with SR-4 gages. Installation was made by augering a 2-in. oversize hole in the stiff clay soil; the pole was placed, plumbed and a sand backfill was vibrated, by tapping the pole, into the space between the pipe and the sides of the hole. Therefore, the tests do not reflect entirely the behavior of the clay, but are a combination of the behavior of sand and clay. One test was selected by the authors as being the most representative of the clay; this test was analyzed assuming various values for n and applying the same techniques used in analyzing Wilkin's tests. An n-value of 0.15 was selected as reasonable. Figure 11 shows the observed moments (solid line) compared to the calculated moments (dashed line) for n = 0.15. It is on the basis of this test combined with reasoning that a tentative value for n of 0.15 is recommended.

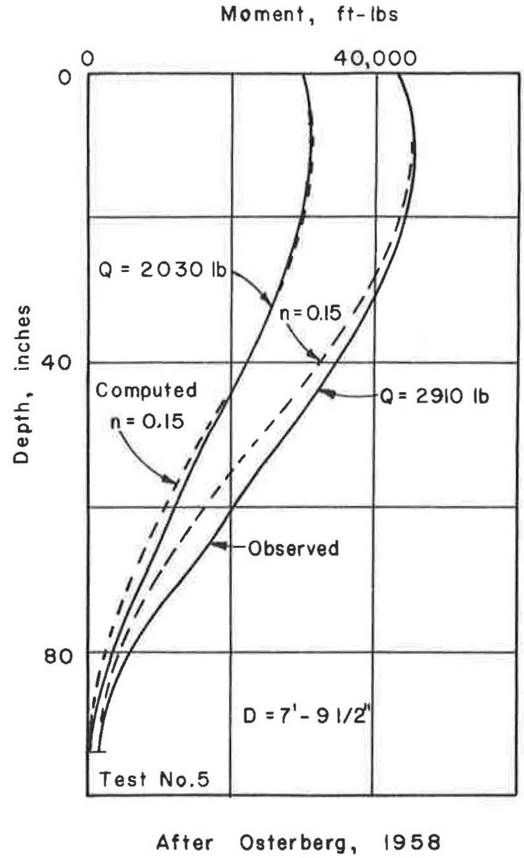

Figure 11. Moment vs depth from a test in clay.

No experimental evidence was available to verify the expression for the critical vertical load P_{cr} given in Eq. 10. To rectify this, a simple test was performed on a pole in sand for which it is known that n = 1 is a good assumption. The pole was a 0.5-in. diameter, 12-in. long steel rod embedded 3 in. A lateral load test was performed and analyzed for n_h in the same manner as Wilkin's tests. Using the n_h value determined experimentally in Eq. 16, a critical load of 16.7 lb was obtained. Three vertical load tests were performed by placing weights on top of the pole; failure occurred twice at loads of 16 lb and once at a load of 18 lb, or an average of 16.7 lb. It is not suggested that the excellent agreement obtained between test and theory is indicative of the accuracy of Eq. 10; it is only suggested that Eq. 10 is a reasonable approximation of the actual critical vertical load.

APPLICATION

Because poles are generally relatively inexpensive structures used in large quantities over wide geographical areas, there is considerable pressure for a simple design procedure that will produce answers at a glance. Economics will not allow the use of refined design procedures involving extensive soil exploration. The foregoing discussion makes it clear, however, that no panacea in the form of a rule of thumb or a chart is likely to be found. It is not the purpose of this paper to provide a specific design procedure, but rather to review the aspects of soil-pole behavior that occur and must be accounted for, and to provide an adequate theoretical framework within which design procedures may be either developed or evaluated.

There are several features concerning the use of poles that will allow simple design procedures to be developed for a specific range of working conditions. For example, in a given series of pole installations the physical characteristics of the pole (EI, H) and the loading (P, M_g, Q_g) are known and can be standardized to give constant values.

TABLE 3

RECOMMENDED VALUES OF n_h FOR SANDS

Author	n_h (pci)					
	Loose		Medium		Dense	
	Dry	Submerged	Dry	Submerged	Dry	Submerged
Terzaghi (17)	9.4	5.3	28	19	75	45
Rowe (75)	8.1	—	—	—	65	—

Furthermore, the allowable initial inclination (θ_i) can be specified. Then the theoretical expressions can be evaluated in terms of the depth of embedment (D) and the soil properties (c, φ, k_x). In addition, the method of construction is likely to be known and it may be predetermined if the desired soil properties will be those of the undisturbed soil, for example, as it would be with a concrete pole cast in a drilled hole, or those of the backfill. In the latter case, the soil properties can be controlled by the construction procedure. At this point two alternates are available. The soils likely to be encountered may be grouped under 4 or 5 headings, with conservative soil properties assigned to each, and the depth of embedment computed; then it is merely necessary to identify the soil at the location of any pole as belonging to one of these groups in order to obtain the depth of embedment. A more versatile but also more involved procedure involves the use of a hand-operated static penetrometer in the soil controlling the behavior of the pole at any given location. With this technique any desired number of typical soil classifications could be incorporated. The pertinent soil properties could be assigned to each group and the depth of embedment expressed as a function of the penetrometer record. Obviously, correlations would have to be developed between the penetrometer readings and the strengths and subgrade moduli of the selected soil groups. This appears to be easily accomplished in view of the success with static penetrometers in Europe.

In using the theoretical expressions a minimum factor of safety of 3, as defined earlier, is recommended to keep the pole rotations in the range where they can be predicted, and also to insure an adequate margin against failure. Repetitive loading has been shown to induce a progressively increasing rotation; this factor may be accounted for by increasing the factor of safety in the design analysis. Unfortunately, the necessary increase in the factor of safety is presently unknown.

The design approach recommended above would be greatly enhanced if the results of two or more series of carefully conducted tests in granular and preloaded cohesive soils were to become available. Each test series should be in a carefully controlled soil whose pertinent physical properties are known. One test in each series should be carried to failure. Static short-time loading and long-time loading should then be applied to a pole equipped with SR-4 gages for the purpose of determining the moments on the embedded portion of the pole. Then repetitive loading of various degrees of severity should be applied in one or more instrumented pole tests. With this information, the variation of subgrade modulus with depth in cohesive soils could be determined and the required factor of safety under repeated load service conditions expressed as a function of the severity of the repeated load.

In using the analytical procedures presented herein, some notion of the magnitude of the subgrade moduli likely to be encountered will be helpful. For granular soils Terzaghi (17) and Rowe (75) have published realistic values for n_h (Table 3). In addition, the authors have observed n_h-values for very loose submerged sands as low as 1.5 pci under repetitive loading and over 100 pci for dry, very dense sand under static loading. In soft, essentially normally loaded clays n_h-values of 2 pci under static loading and 1.0 pci under repetitive loading have been observed. For very soft, or-

TABLE 4

RECOMMENDED VALUES OF K FOR CLAYS

q_u (tons/ft^2)[a]	Range of K (psi)	Recommended K (psi)
1 - 2	463 - 926	694
2 - 4	926 - 1,390	1,390
> 4	> 1,852	2,780

[a]Unconfined compressive strength.

ganic silt n_h-values ranging from 0.4 pci to 1.0 pci have been reported by Peck and Davisson (76).

Very little information exists concerning K-values for piles and poles in preloaded clays. Terzaghi's (17) recommended values have been converted to the system used in this paper (Table 4). It should be noted that Terzaghi's recommendations are conservative and can be presumed to include an allowance for consolidation under static long time loading.

A large quantity of empirical data regarding the subgrade modulus may be obtained economically by performing pole tests in the following manner:

1. Use Tables 3 and 4 to estimate the values of n_h or K for the soils in which the tests will be performed.
2. Design the test pole to be rigid using Eqs. 5 through 9.
3. Install the pole and displace it upwards slightly to eliminate the possible effects of V_D.
4. Determine D and the height above the ground surface to the lateral load.
5. Measure the lateral load Q, and the deflection at the ground surface y_g, and the pole rotation θ.
6. Assume that Eq. 4 applies and determine n from Eq. 11 and K from Eq. 12.
7. Check step 2 to be sure that the pole is rigid.
8. Estimate or measure the soil strength parameters (c, φ) and describe the soil, preferably by the AASHO and Unified Soil Classification Systems.

This test procedure will provide the soil parameters necessary for a complete pole analysis. It can be used as a means of collecting data for developing a design procedure, either for soils in general or for a selected soil type in particular.

SUMMARY AND CONCLUSIONS

It has been shown that the behavior of a soil-pole system is governed by the properties of the soil, and therefore, by any factors affecting these properties. For example, repeated loading leads to progressively increasing deflections which indicates a reduction in the subgrade modulus. Also, seasonal changes in the moisture content of cohesive soils can greatly affect their properties. In addition, the method of construction can determine whether the pertinent soil properties are those of the backfill or those of the undisturbed soil.

The pertinent soil properties are the shear strength (φ, c) and the subgrade modulus along with its variation with respect to depth. An n-value of unity in Eq. 4 has been shown reasonable for granular soils and normally loaded cohesive soils, whereas an n-value of 0.15 has been tentatively suggested for preloaded cohesive soils. Although the soil behavior is nearly elastic in the recommended working range, the subgrade modulus has been empirically adjusted somewhat to account for the plastic soil behavior which usually occurs near the ground surface.

Expressions have been given in terms of the flexural stiffness of the pole and the soil stiffness (subgrade modulus) which determine if a pole is rigid enough to meet the assumption that the pole deflections can be described by a rotation. Furthermore, a comprehensive analytical technique is presented that accounts for the moment and shear loads at the ground line in addition to the vertical load and any initial rotation that the pole may have. The analysis allows virtually any variation of subgrade modulus with the depth to be considered. The variation of the ultimate soil resistance with depth is also presented and a technique involving a factor of safety is recommended to insure an adequate margin against overturning; it also insures that the soil reactions are in a range where they can be expressed as a linear function of the deflections.

Recommendations have been given for the test programs necessary for furnishing the information needed to develop general design procedures within the framework of this paper. Repeated load tests in both sands and clays are recommended; it is hoped that the factor of safety used in design can be related to the severity of the repeated loading. Tests are needed in preloaded cohesive soils to help define more precisely the variation of the subgrade modulus with depth.

The reduced deflections under a given load that are observed for a pole section that is enlarged near the ground surface, when compared to an unenlarged pole, are explained. Expressions indicating the behavior of these pole types can be developed in a manner similar to the expressions given in this paper. Similarly, expressions can be developed for other imposed conditions of restraint.

Two general design approaches are outlined. Both involve grouping the soil under a selected number of headings and choosing conservative soil properties for each group. Designs can then be prepared for the different standard poles and standard loadings where the depth of embedment is a function of the soil type. The simplest procedure involves only 4 or 5 soil types with a single conservative depth of embedment for each type. A more versatile procedure would make the depth of embedment a function of the static soil penetrometer record within each soil type.

Finally, it is hoped that this paper can serve as a source of reference material on pole behavior and provide a framework within which pole design procedures can be evaluated and developed.

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Appendix

NOTATION

Note: The units are expressed in terms of force F and length L.

Symbol			Units
B	=	width of pole normal to the loading	L
c	=	cohesion	FL^{-2}
D	=	embedded length of pole	L
D_o	=	depth to point of rotation	L
D_1	=	depth of upper point relative maximum w	L
e	=	eccentricity of P from pole axis	L
E	=	Young's modulus for the pole	FL^{-2}
H	=	unsupported length of the pole	L
H_p	=	distance from ground surface to P	L
I	=	moment of inertia of pole cross-section	L^4
k	=	subgrade modulus	FL^{-2}
k_x	=	k as a function of depth	FL^{-2}
K	=	value of k at depth D	FL^{-2}
M_c	=	moment couple	FL
M_g	=	resultant moment at ground surface	FL
n	=	coefficient governing k_x	
n_h	=	constant of horizontal subgrade reaction, K/D for n = 1	FL^{-3}
N_c	=	bearing capacity factor for cohesion	
N_q	=	bearing capacity factor for overburden pressure	
P	=	vertical load	F
P_{cr}	=	critical vertical load	F
q'	=	vertical effective stress	FL^{-2}
Q	=	horizontal load on pole	F
Q_g	=	resultant horizontal load at ground surface	F
R	=	relative stiffness factor for n = 0, $\sqrt[4]{EI/K}$	L
T	=	relative stiffness factor for n = 1, $\sqrt[5]{EI/n_h}$	L
V	=	shear in the pole	F
V_D	=	shear in the pole at depth D	F
w	=	soil reaction	FL^{-1}
w_u	=	ultimate soil reaction	FL^{-1}
w_x	=	w as a function of depth	FL^{-1}

x	=	depth coordinate from ground surface	L
y	=	horizontal deflection of pole	L
y_g	=	y at ground surface	L
y_{gi}	=	y at ground surface due to θ_i	L
α	=	P/P_{cr}	
θ	=	pole rotation	
θ_i	=	initial pole rotation	
φ	=	angle of internal friction	