Relation Between Optimum Congestion Tolls And Present Highway User Charges

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•THE MAIN PURPOSES of this paper are to develop some rough estimates of the optimum congestion toll structure for a large urban center such as the Twin Cities (Minneapolis-St. Paul, Minn.) Metropolitan Area and to provide some even rougher estimates of the costs that result from the inability and/or unwillingness to impose such tolls. Before proceeding, however, it seems a useful precaution to recognize the problem of fiscal irresponsibility that almost invariably plagues discussions of congestion pricing. Although space permits only partial resolution of the problem, establishing that a solution is possible at all seems worthwhile.

Those who support congestion pricing for highway systems rely heavily on the fundamental proposition of economic theory that efficiency in utilizing the resources available to an economy requires that the price of goods or services be set equal to the short-run marginal costs of producing them. The short-run marginal costs of highway trips are only indirectly related to the costs incurred by highway authorities; i.e., those of maintaining existing roads and of building new ones. The short-run marginal costs are almost entirely imposed on road users by the increase of vehicle operating and time costs of trips when roads are heavily congested.

The fact that the costs borne by highway authorities are not those which determine congestion tolls has led many economists and engineers to conclude that institution of congestion-based pricing would force abandonment of the time honored and presumably desirable practice of financing highway facilities entirely out of user charges. Consider, for example, the following comment:

> Since user fees limited in this way (i.e., to the difference between short-run marginal private and social costs) may not create total revenues sufficient to attract capital to highways and in limited cases may yield more revenues than could be invested efficiently in highways, the marginal cost pricing economists tend to deny that any relation, or close relation, should exist between user fees and capital investment. The rule of self-liquidation as a general guide to efficient investment is thrust aside as unnecessary and as a substantial hindrance to efficient utilization of existing highways. What specific rule for efficient road investment is to be substituted is far from clear; presumably it would be a matter for planning according to social surplus criteria of investment, often involving subsidy expenditures (1).

This and similar charges that fiscal irresponsibility is involved in advocating congestion pricing for highway services have been strongly refuted (2, 3).

The procedures a highway authority ought to use in establishing and pricing an optimum highway system are formally identical to the process through which the economist's "competitive market" reaches long-run equilibrium. In the short run, the price of a product in a competitive market will be equated with the short-run marginal costs of production. This price need not equal the product's average production costs. If it exceeds average production costs, new producers will be attracted to the industry, thereby expanding output and lowering price. Long-run equilibrium is reached when this process of entry equalizes product price and average production costs.

Paper sponsored by Committee on Highway Taxation and Finance.

To maximize the benefits derived from an existing road network, the highway authority must levy tolls equal to the difference between short-run marginal and average congestion costs. If the resulting toll collections are greater than the total costs of the system (including, it should be emphasized, an interest charge equal to the market rate of return on capital invested in the system), expanding the system, thereby lowering both average and marginal vehicle operating costs and hence optimum tolls, is in order. A long-run optimum highway network results if this process of system expansion and toll reduction is continued to the point where network costs (again, including the market return on invested capital) equal toll collections.

Strictly speaking, a long-run optimum highway system requires that tolls equal capital costs only if the production of highway services involves constant returns to scale. Some evidence is available that substantial scale economies exist in the provision of these services. If an activity involves increasing returns to scale, economic theory suggests the desirability of subsidizing that activity. That is, in the case at hand, theory dictates that the highway network ought to be expanded beyond the point at which congestion tolls just cover highway network costs.

CHARACTERISTICS OF OPTIMUM CONGESTION TOLL STRUCTURE

The logical basis for establishing congestion tolls for highway use is the fact that the cost of a trip on a highway increases with the number of trips being taken on that highway. Specification of an optimum toll structure therefore requires quantification of the interrelationships among trip costs, traffic density, and highway characteristics. A representative vehicle operator's trip costs per mile may be written as

$$C = F(S,\ N,\ \bar{Z}) + V/S^* (S,\ N,\ \bar{Z}) \tag{1a}$$

in which V and S^* are, respectively, the value he and his passengers place on an hour's travel time and the speed at which they actually travel; F includes all other trip costs; and S, N, and $\bar{Z}$ are, respectively, the vehicle operator's desired speed, traffic volume, and a set of such highway characteristics as the number and width of lanes, curvature and grade standards, and access controls. Of the costs summarized by F, reliable information is available only on gasoline and oil consumption and tire wear. Little if any data are available on such important subjects as the effects of highway characteristics, traffic volume, and desired speeds on the frequency and severity of accidents and on driver comfort and convenience. It seems reasonable to expect that both accident and comfort and convenience costs increase with traffic density and desired speed, but because no data are available, these costs are disregarded in the following discussion. This, it should be noted, imparts a downward bias to the estimated optimum tolls.

No market exists in which travel time is bought and sold, therefore, assigning a value to it is rather difficult. Indeed, the AASHO "Red Book" (4) seems to regard objective determination of the value of travel time to be impossible and, therefore, apparently assumes a number: "a value of travel time for passenger cars of \$1.55 per hour, or 2.59 ¢ per minute, is used herein as representative of current opinion for a logical and practical value" (4, pp. 103-4).

Even accepting this rather conservative value, travel time turns out to be by far the most important cost of urban travel. Therefore, it seems worthwhile to point out that a value of travel time is implicit in a driver's selection of a target speed. An increase in desired speed reduces the time costs of the trip. At the same time, however, an increase in speed increases vehicle operating and probably accident and comfort and convenience costs. Data in the "Red Book" (4, pp. 100-126) suggest that the following approximate costs per mile prevail for representative vehicles traveling on straight, level, paved rural roads where no traffic signals or stop signs exist to interrupt traffic flows:

$$\text{gasoline:}\quad \$0.30/(13.2 + 0.40S - 0.0076\,S^2) \tag{2a}$$

$$\text{oil:}\quad \$0.45/(1600 - 21S) \tag{2b}$$

$$\text{tires:}\quad \$0.0010 + 1.5 \times 10^{-8}\,S^{3.2} \tag{2c}$$

That is, on such roads, vehicle operating costs appear to be independent of traffic volume. On a very lightly traveled road where a driver can attain his desired speed, $S^* = S$ and Eq. 1a becomes

$$C = F(S, O, \bar{Z}) + V/S \qquad (1b)$$

It seems reasonable to suppose that a driver would attempt to travel at that speed which would minimize the total costs of his trip. Differentiating Eq. 1b with respect to S and setting the resulting expression equal to zero yields

$$V = S^2 \, \partial F / \partial S \qquad (3)$$

TABLE 1

TRAVEL TIME VALUES OF VEHICLE OCCUPANTS IMPLIED BY ALTERNATIVE DESIRED OPERATING SPEEDS

Speed, S (mph)	$V = S^2 \frac{\partial F}{\partial S}$ (\$/hr)
20	-0.02
30	0.13
40	0.62
50	2.06
60	7.38
70	67.82

Regarding $F(S, O, \bar{Z})$ as equal to the sum of Eqs. 2a, 2b, and 2c, Table 1 gives the travel time values implied by Eq. 3 for representative values of S. If, as the "Highway Capacity Manual" (5, p. 32) indicates, desired speeds on high-quality, straight, level highways are approximately normally distributed with mean and standard deviation of 48.5 and 8 mph, respectively, $\bar{V}$, the mean travel time value for the occupants of all vehicles, can be obtained by evaluating

$$\bar{V} = \int_{-\infty}^{\infty} S^2 \frac{\partial F}{\partial S} \, n(S; 48.5, 8) \, dS \qquad (4)$$

This value is approximately \$3.00.

The analysis and the data underlying these estimates are, to say the least, rather rough. The true standard deviation is probably larger than that computed because (a) the average driver is only dimly aware of the relationship between speed and operating costs, and (b) few people drive average cars. However, the mean of the distribution would be over- or underestimated by these procedures only if the average driver's estimates of the relationship between speed and operating costs are biased. No evidence exists on the nature of these possible biases.

Inasmuch as data in the "Red Book" suggest that vehicle operating costs on rural roads are independent of traffic volume, if the hourly output of trips on a road is conceived of as a function of three variables, $N = g(T, C, \bar{Z})$, in which T is travel time, C is operating cost, and $\bar{Z}$ represents highway characteristics, $\partial N/\partial C$ is a constant. Hence, increased traffic affects the cost of a trip over a rural road (or an urban expressway) solely by increasing the time required.

Several studies on the relationship between N and T have been undertaken. To cite just a few, Greenshields (6) and Huber (7) found an approximately linear relationship between average speed and traffic density, D; i.e., the number of vehicles occupying a mile of road at any instant of time. For conditions of "free flow," Normann (8, 5, pp. 36-43), found speed and volume to be linearly related up to the "capacity" of the highway. Greenberg (9) and Underwood (10) found that relationships of the form $D = ae^{-bS}$ and $S = ae^{-bD}$, respectively, fit the data better than did linear relationships between S and D.

If volume equals density times "space mean speed," the total distance covered by all drivers divided by total time elapsed, and density equals volume times "space mean travel time," the reciprocal of "space mean speed," traffic volume and travel time can be related as follows:

$$\text{Greenshields:} \quad N = (aT - 1)/bT^2 \qquad (5a)$$

$$\text{Normann:} \quad N = (aT - b)/T \qquad (5b)$$

$$\text{Greenberg:}\quad N = a/T\, e^{b/T} \tag{5c}$$

$$\text{Underwood:}\quad N = (a + b \log T)/T \tag{5d}$$

Although the specific form of these relationships differs, all imply that the amount of time required per vehicle-mile increases with the number of trips being taken and that there is a maximum rate at which any given highway can produce vehicle-miles. If the rate at which people attempt to make trips exceeds this capacity level, the output of vehicle-miles actually falls.

These equations suggest that the addition of a vehicle to a traffic stream will increase total travel time in two ways: (a) the occupants of the vehicle will incur travel time costs that they would not have experienced had they not chosen to make their trip, and (b) by adding to the level of congestion on the highway, the additional vehicle increases the travel times of all the remaining drivers. The marginal cost of a trip (i.e., the increase in total costs associated with an additional trip, or the decrease in costs associated with elimination of a trip) can be written

$$\frac{\partial(N\bar{V}T)}{\partial N} = \bar{V}T + N\bar{V}\frac{\partial T}{\partial N} \tag{6}$$

in which $N\bar{V}T$ is the total hourly time cost of all vehicle-miles and $\bar{V}T$ is the cost incurred by occupants of the additional vehicle; the remaining term is the cost this additional vehicle imposes on the other vehicles in the traffic stream.

The basic argument made for setting congestion tolls is that in their absence individual drivers would consider only the $\bar{V}T$ component of Equation 6. By ignoring the $N\bar{V}\ \partial T/\partial N$ component, they would tend to make trips of less value than the total cost. Only if each driver were required to pay a toll equal to $N\bar{V}\ \partial T/\partial N$ would he limit trips to those with values in excess of their costs.

Assuming the mean desired operating speed on high-quality, straight, level rural highways to be 48.5 mph (5, p. 32), the Greenshields, Normann, and Underwood relationships can be rearranged to yield the following relationships:

	Travel Time per Veh-Mi, T	Marginal Minus Avg. Time per Veh-Mi, (N dT/dN)	
Greenshields:	$\alpha T^2 = 0.0825\,T - 0.0017$	$T(48.5\,T - 1)/(2 - 48.5\,T)$	(7a)
Normann:	$(48.5 - 18.5\,\alpha)\,T = 1$	$T(48.5\,T - 1)$	(7b)
Underwood:	$\alpha T = 3.88 + \log T$	$17.84\,\alpha T^2/(1 - 17.84\,\alpha T)$	(7c)

in which α is the volume-capacity ratio, the ratio of actual trips per hour to the maximum number of trips per hour the highway can produce. The Normann relationship assumes also an average speed of 30 mph when a highway is used to capacity level.

The specific values for marginal and average travel times per vehicle-mile implied by these relationships vary substantially with differing volume-capacity ratios (Table 2). In all cases, the marginal travel times per vehicle-mile increase rapidly with increases in volume-capacity ratios. However, under conditions of uninterrupted flow, this effect can be ignored when determining operating costs if, as the "Red Book" suggests, vehicle operating and accident costs depend on desired rather than on realized speeds. The differences between marginal and average travel times per vehicle-mile given in Table 2 can therefore be converted into optimum tolls per mile simply by multiplying by an appropriate travel time value (Table 3).

The preceding discussion has been based on uninterrupted traffic flows of the sort that prevail on high-quality rural highways and urban freeways. Analysis of such flows is simple in comparison to the problems involved in dealing with urban arterial

TABLE 2

AVERAGE AND MARGINAL MINUS AVERAGE TRAVEL TIMES[a]

Volume-Capacity Ratio, α	Travel Times (min/mi)					
	Greenshields		Normann		Underwood	
	Aver.	Marginal Minus Aver.	Aver.	Marginal Minus Aver.	Aver.	Marginal Minus Aver.
0.1	1.27	0.04	1.28	0.05	1.28	0.05
0.2	1.30	0.07	1.34	0.11	1.34	0.11
0.3	1.36	0.15	1.40	0.18	1.40	0.20
0.4	1.41	0.23	1.46	0.26	1.47	0.30
0.5	1.46	0.31	1.53	0.36	1.55	0.45
0.6	1.51	0.39	1.60	0.47	1.66	0.69
0.7	1.57	0.59	1.69	0.61	1.80	1.07
0.8	1.69	0.96	1.78	0.77	1.98	1.75
0.9	1.90	2.22	1.88	0.99	2.26	3.42

[a]From Greenshields, Normann, and Underwood travel time relationships.

TABLE 3

ESTIMATED HIGHWAY USER TOLL

Volume-Capacity Ratio	Estimated Toll (¢/mi)					
	Travel Time Value = \$1.55/Veh-Hr			Travel Time Value = \$3.00/Veh-Hr		
	Greenshields	Normann	Underwood	Greenshields	Normann	Underwood
0.1	0.1	0.1	0.1	0.2	0.2	0.2
0.2	0.2	0.3	0.3	0.3	0.5	0.5
0.3	0.4	0.5	0.5	0.8	0.9	1.0
0.4	0.6	0.7	0.8	1.2	1.3	1.5
0.5	0.8	0.9	1.2	1.5	1.8	2.3
0.6	1.0	1.2	1.8	1.9	2.4	3.4
0.7	1.5	1.6	2.8	3.0	3.0	5.4
0.8	2.5	2.0	4.5	4.8	3.9	8.8
0.9	5.7	2.6	8.8	11.1	4.9	17.2

streets. The characteristics of traffic flows on arterials vary from complete congestion in the queues that form behind stop signs and red traffic lights, through an intermediate stage as vehicles accelerate to normal operating speeds on leaving these impediments, to the free-flow characteristics that correspond to the existing traffic volume as modified by whatever speed limits may be enforced. The characteristics are also affected by the frequency of right- and left-turn maneuvers, the amount of pedestrian traffic at intersections, the amount of curb parking, the number and spacing of bus stops and signals, and (but only for low traffic volumes) the sequencing of red and green cycles at successive traffic lights.

Few systematic studies have been undertaken of the interrelationships among volume, density, and travel time on urban arterial streets, and only one of these provides information that can be analyzed in a fashion similar to that employed with the Greenshields, Normann and Underwood relationships for rural roads and urban freeways. Coleman (11) fitted quadratic relationships to data on travel time-volume-capacity ratios observed on a sample of one- and two-way streets in several Pennsylvania cities. These relationships and the implied differences between marginal and average travel times are:

One-way streets: $$N/C = \alpha = -1.98 + 1.07T - 0.096T^2 \tag{8a}$$

$$NdT/dN = \alpha(1.07 - 0.192T)^{-1} \tag{8b}$$

TABLE 4

AVERAGE AND DIFFERENCE BETWEEN MARGINAL AND AVERAGE MINUTES PER VEHICLE-MILE ON ONE- AND TWO-WAY URBAN ARTERIAL STREETS

Volume-Capacity Ratio	Time per Mile (min)			
	One-Way Streets		Two-Way Streets	
	Aver.	Marginal Minus Aver.	Aver.	Marginal Minus Aver.
0.1	2.51	0.17	2.52	0.18
0.2	2.68	0.36	2.70	0.38
0.3	2.87	0.58	2.89	0.60
0.4	3.07	0.83	3.10	0.87
0.5	3.28	1.14	3.32	1.19
0.6	3.52	1.53	3.57	1.59
0.7	3.80	2.05	3.84	2.13
0.8	4.12	2.86	4.19	3.00
0.9	4.54	4.52	4.62	4.79

Two-way streets: $$N/C = \alpha = -1.90 + 1.02\,T - 0.090\,T^2 \qquad (9a)$$

$$NdT/dN = \alpha\,(1.02 - 0.180\,T)^{-1} \qquad (9b)$$

By definition, traffic volume is maximum when the volume-capacity ratio is one. The fitted relationships had maximum volumes at volume-capacity ratios of 0.904 and 1.067 for one- and two-way streets, respectively. Accordingly, the volume-capacity ratios implied by the listed formulas are, respectively, 1/0.904 and 1/1.067 times those of the computed regression relationships. The computed relationships implied mean speeds of 25.5 and 47.4 mph for the one- and two-way streets, respectively, at zero traffic volumes. The latter value clearly seems too high. For this reason the two-way street travel time values (Table 4) implied by the preceding equations are 1.08 min per mile greater than those indicated by the computed relationships.

As with rural roads, the difference between marginal and average time costs varies substantially with the volume-capacity ratio. For both one- and two-way streets, an additional trip when volume is 70 percent of capacity increases the aggregate travel time of other vehicles by approximately 3.5 times that at 30 percent of capacity. The ratio is more than 25 to 1 when comparing operation at 90 percent with operation at 10 percent of capacity.

In the Chicago Area Transportation Study, both vehicle operating and accident costs for a trip segment were negatively related to the average speed for the segment (12, Table 5). From data given in Tables 4 and 5, the accident and operating cost component of optimum congestion

TABLE 5

CHICAGO AREA TRANSPORTATION STUDY SPEED-RELATED COST PARAMETERS

Speed (mph)	Cost (¢/mi)		
	Operating	Accident	Total
5	4.80	6.75	11.55
10	3.69	4.25	7.94
15	3.10	2.70	5.80
20	2.78	1.80	4.58
25	2.57	1.15	3.72
30	2.41	0.80	3.21
35	2.36	0.55	2.91

TABLE 6

COSTS IMPOSED ON OTHER DRIVERS BY ADDITIONAL VEHICLE-MILE ON TWO-WAY URBAN ARTERIAL STREETS AND DERIVED OPTIMUM TOLL

Volume-Capacity Ratio	Imposed Costs (¢/mi): Marginal Minus Aver. Operating and Accident Costs	Imposed Costs (¢/mi): Marginal Minus Aver. Time Costs At \$1.55[a]	Imposed Costs (¢/mi): Marginal Minus Aver. Time Costs At \$3.00[b]	Optimum Toll (¢/mi): At \$1.55 (Col. 1 + Col. 2)[a]	Optimum Toll (¢/mi): At \$3.00 (Col. 1 + Col. 3)[b]
	(1)	(2)	(3)	(4)	(5)
0.1	0.2	0.5	0.9	0.7	1.2
0.2	0.5	1.0	1.9	1.5	2.4
0.3	0.8	1.5	3.0	2.4	3.8
0.4	1.1	2.2	4.3	3.4	5.5
0.5	1.5	3.1	5.9	4.5	7.4
0.6	2.0	4.1	7.9	6.1	9.9
0.7	2.5	5.5	10.7	8.0	13.2
0.8	3.3	7.8	15.0	11.1	18.3
0.9	4.7	12.4	24.0	17.1	28.8

[a] Assuming travel time value of \$1.55/hr.
[b] Assuming travel time value of \$3.00/hr.

tolls (i.e., the difference between marginal and average operating and accident costs, NdC/dN) can be estimated (Table 6). Considering operating and accident costs per vehicle-mile a function of operating speed or $C = f(S)$, and noting that, for two-way streets, $NdT/dN = \alpha/(1.02 - 0.180\,T)$, NdC/dN can be written

$$N\frac{dC}{dN} = N\frac{dC}{dS}\frac{dS}{dT}\frac{dT}{dN} = -\frac{1}{T^2}\frac{\alpha}{(1.02 - 0.18T)}\frac{dC}{dS} \tag{9c}$$

The relationship between these optimum tolls and current highway user charges can be estimated at least roughly. Federal and state excises on gasoline, averaging about 10¢ a gallon, comprise the only appreciable source of tax revenue varying directly with vehicle mileage.

On rural roads and urban freeways, the 48.5-mph driver of one of the "Red Book's" average cars would average 14.7 mi per gal and hence would pay a toll of approximately 0.7¢/mi. Assuming alternative average travel time values of \$1.55 and \$3.00 per vehicle-hour, such a driver would be paying optimum tolls at respective volume-capacity ratios of approximately 40 percent and 30 percent. At lesser volume-capacity ratios he would be paying more than the costs his trips impose on other drivers. At greater volume-capacity ratios, his trips would, in effect, be subsidized by society.

The average operating speeds underlying the city street toll estimates (Table 6) range from 13 to 24 mph at volume-capacity ratios of 90 and 10 percent, respectively. At these speeds in city traffic, one study (13) determined average gasoline consumption for a 1951 6-cylinder car to be 14 and 18 mi/gal, respectively. At the same average speeds, Eq. 2a suggests gasoline consumption rates of 17 and 18 mi/gal, respectively, or an actual tax burden of 0.5 to 0.7 ¢/mi. Even assuming the "Red Book" estimate of travel time values to be correct, Table 6 suggests that these tax rates would be optimum only for city street volume-capacity ratios of less than 10 percent. Registration fees of, for example, \$20 per passenger vehicle per year would add 0.4 and

0.1 ¢/veh-mi to the respective tax costs of 5,000- and 20,000-mi/yr drivers, raising total city street tax payments to 0.6 to 1.2 ¢/veh-mi. Even assuming $1.55 to be the correct travel time valuation, this greater tax rate would be the appropriate toll for a city street volume-capacity ratio of less than 20 percent.

SOCIAL COSTS OF PRESENT USER CHARGES

Because the use made of both rural and urban highway networks varies substantially through time, optimum user charges also vary through time. As is indicated in Table 7, approximately 70 times as many passenger cars (or their congestion equivalents in trucks and buses) are operated on the highways of the Twin Cities Metropolitan Area between 4:30 and 5:00 p.m. as during the early morning hours. At a value for travel time of $3.00 per vehicle-hour, an optimum toll of almost 20¢/veh-mi is suggested for the afternoon peak as compared to 0.2¢ for the early morning period. Even at the "Red Book's" $1.55 per vehicle-hour travel time valuation, the peak half-hour optimum toll is on the order of 50 times that during the early morning hours. The tolls of 1¢/veh-mi imposed by current gasoline taxes and license fees fall far short of the costs of most weekday trips in this area. Inasmuch as current user fees are more than sufficient to pay for current urban highway maintenance and construction programs, Table 7 suggests that an expanded highway construction program would be in order. Indeed, the relationships developed previously and existing levels of highway use imply rates of return on new freeway construction in the area of as much as 300 percent.

TABLE 7

ESTIMATED OPTIMUM TOLLS FOR TWIN CITIES METROPOLITAN AREA

Half Hour Beginning	Aver. Traffic Density[a] (1,000 veh)	Optimum Toll (¢/mi) At $1.55/hr[b]	At $3.00/hr[b]
12:00 m	7.4	0.9	1.4
12:30 a.m.	5.0	0.8	1.3
1:00	3.9	0.5	0.8
1:30	2.5	0.3	0.5
2:00	1.4	0.2	0.3
2:30	1.1	0.2	0.2
3:00	1.0	0.2	0.2
3:30	0.9	0.2	0.2
4:00	1.4	0.2	0.3
4:30	1.8	0.3	0.4
5:00	3.0	0.3	0.5
5:30	5.9	0.7	1.2
6:00	17.3	2.0	3.3
6:30	33.5	4.0	6.5
7:00	48.3	6.1	9.9
7:30	69.4	9.1	16.1
8:00	51.9	6.6	10.8
8:30	40.3	4.9	8.0
9:00	45.6	5.6	9.2
9:30	42.2	5.1	8.2
10:00	52.6	6.7	11.0
10:30	44.1	5.4	8.8
11:00	50.0	6.3	10.2
11:30	48.1	6.0	9.8
12:00 noon	42.6	5.2	8.6
12:30 p.m.	41.4	5.0	8.2
1:00	52.0	6.6	10.9
1:30	45.8	5.6	9.2
2:00	49.3	6.2	10.1
2:30	46.6	5.8	9.5
3:00	52.5	6.7	11.0
3:30	55.0	7.1	11.6
4:00	59.6	7.8	12.8
4:30	76.9	11.5	19.1
5:00	73.2	10.6	17.6
5:30	52.7	6.7	11.0
6:00	45.1	5.3	8.7
6:30	35.3	4.2	6.8
7:00	40.0	4.9	8.0
7:30	34.8	4.2	6.7
8:00	33.4	4.0	6.5
8:30	28.9	3.4	5.6
9:00	27.9	3.3	5.4
9:30	21.8	2.6	4.2
10:00	16.3	1.9	3.2
10:30	12.7	1.5	2.4
11:00	11.6	1.4	2.2
11:30	8.6	1.0	1.6

[a]For the period July 8 to Dec. 1, 1958.
[b]Travel time value per vehicle-hour.

Although at an average travel time valuation of $3.00 per vehicle-hour, a toll of 19¢/veh-mi is suggested for afternoon rush hour traffic volumes, Table 7 does not imply that it would be desirable to charge such a toll even if it were possible to do so. If the price of peak-hour trips were increased, it seems reasonably safe to assert that the number of trips taken during the peak hour would decline. Some trips currently made then would not be taken at all, others would be shifted to off-peak hours, and still others would be consolidated. An increase in peak-hour tolls would provide a powerful stimulus toward the formation of more car pools and would quite likely lead a larger number of employers to institute staggered work hours.

If it is safe to assume that an increase in peak-hour tolls would lead to a reduction in peak-hour trips, then some of those who are currently making these trips place values on them that are less than their total costs. Thus, inability and/or unwillingness to vary user charges with the demand for trips involves a definite and perhaps substantial social cost. Determination of the magnitude of this social cost would require data on the degree to which an increase in the price of peak-hour trips would reduce their number. Unfortunately, the required data do not exist. It is possible, however, both to suggest the nature of the problem in somewhat greater detail and to draw some inferences about the orders of magnitude that may be involved.

Consider, therefore, the following simple situation. A 1-mi stretch of super highway connects two points, Here and There. The vehicle operating and time costs of a trip over this road, C, depend on the ratio of the rate at which vehicles are making trips, Q, to the capacity of the highway, Z. For 12 hr of the day the demand for trips is relatively high; for the remaining 12 hr the demand is relatively low. In both peak and off-peak periods, the number of trips taken is a function of the "price" of a trip; i.e., the vehicle operating and time costs involved plus whatever toll is imposed.

It seems reasonable to regard the net benefit of trips over this road as the sum of the prices individual travelers would pay for them minus both the total vehicle operating and time costs they incur and the costs of providing the road. Ignoring maintenance costs and assuming that constant returns to scale are involved in constructing roads, this latter cost component can be written as rkZ, in which r and k are the interest rate of relevance in valuing highway investments and the capital cost of a unit of highway capacity, respectively. This net benefit can be written as

$$B \equiv \int_0^N F(Q)\, dQ + \int_0^n f(q)\, dq - NC(N/Z) - nC(n/z) - rkZ \qquad (10)$$

in which F(Q) (f(q)) is the price that the taker of the Qth (qth) peak (off-peak) period trip would pay for it and N and n are the actual number of peak and off-peak period trips, respectively.

A beneficent highway authority would presumably want to maximize this expression by varying the toll (or tolls) charged for trips and the capacity of the highway. If different tolls can be charged for the two time periods, the benefit maximizing conditions can be found by differentiating Eq. 10 with respect to N, n, and Z, or

$$\frac{\partial B}{\partial N} = F(N) - C(N/Z) - N\frac{\partial C}{\partial N} = 0 \qquad (11a)$$

$$\frac{\partial B}{\partial n} = f(n) - C(n/Z) - n\frac{\partial C}{\partial n} = 0 \qquad (11b)$$

$$\frac{\partial B}{\partial Z} = \frac{\partial C}{\partial (N/Z)}\frac{N^2}{Z^2} + \frac{\partial C}{\partial (n/Z)}\frac{n^2}{Z^2} - rk = 0 \qquad (11c)$$

According to Eqs. 11a and 11b, to maximize benefits the "price" paid by the Nth (nth) traveler should equal the marginal cost of his trip. Inasmuch as each traveler pays the vehicle operating and time costs of his trip, C (N/Z) and C (n/Z), the "price" of the trip will equal its marginal cost only if peak and off-peak tolls equal, respectively, to $N\,\partial C/\partial N$ and $n\,\partial C/\partial n$ are charged. According to Eq. 11c, increments in capacity should be provided up to the point where the last increment yields vehicle operating and time cost savings just equal to its capital cost.

Eqs. 11a, b, and c can be rearranged to demonstrate the validity of the basic contention that optimum congestion tolls would just suffice to cover the capital costs of an optimum highway. Multiplying Eq. 11 through by Z yields the following expression for the total capital costs of an optimum facility:

$$rkZ = \frac{\partial C}{\partial (N/Z)}\frac{N^2}{Z} + \frac{\partial C}{\partial (n/Z)}\frac{n^2}{Z} \qquad (12)$$

in which $\frac{\partial C}{\partial(N/Z)} \frac{N}{Z}$ is the optimum peak period toll for the facility. A similar expression results for the off-peak period toll. Total toll collections can be obtained simply by multiplying the peak and off-peak tolls by peak and off-peak traffic levels; that is,

$$\text{Total tolls} = \frac{\partial C}{\partial(N/Z)} \frac{N^2}{Z} + \frac{\partial C}{\partial(n/Z)} \frac{n^2}{Z} \tag{13}$$

But the right-hand side of Eq. 13 equals the right-hand side of Eq. 12; i.e., optimum tolls just equal the capital costs of an optimum facility.

Even if the highway authority cannot charge different tolls during peak and off-peak periods, it would still presumably want to adjust highway capacity to satisfy Eq. 11 and to set that single toll which comes as close as possible to maximizing total benefits. That is, it would presumably want to establish a toll such that

$$\frac{\partial B}{\partial T} = \frac{\partial B}{\partial N}\frac{\partial N}{\partial T} + \frac{\partial B}{\partial n}\frac{\partial n}{\partial T} = 0 \tag{14}$$

The changes in benefits resulting from changes in the number of peak and off-peak period trips are given by Eqs. 11a and 11b, respectively. The effect of a change in the toll on peak period trips can be obtained by noting that $F(N) = T + C(N/Z)$. Differentiating this expression with respect to T gives

$$\frac{\partial F}{\partial N}\frac{\partial N}{\partial T} = 1 + \frac{\partial C}{\partial N}\frac{\partial N}{\partial T}, \quad \text{or} \quad \frac{\partial N}{\partial T} = \left(\frac{\partial F}{\partial N} - \frac{\partial C}{\partial N}\right)^{-1} \tag{15}$$

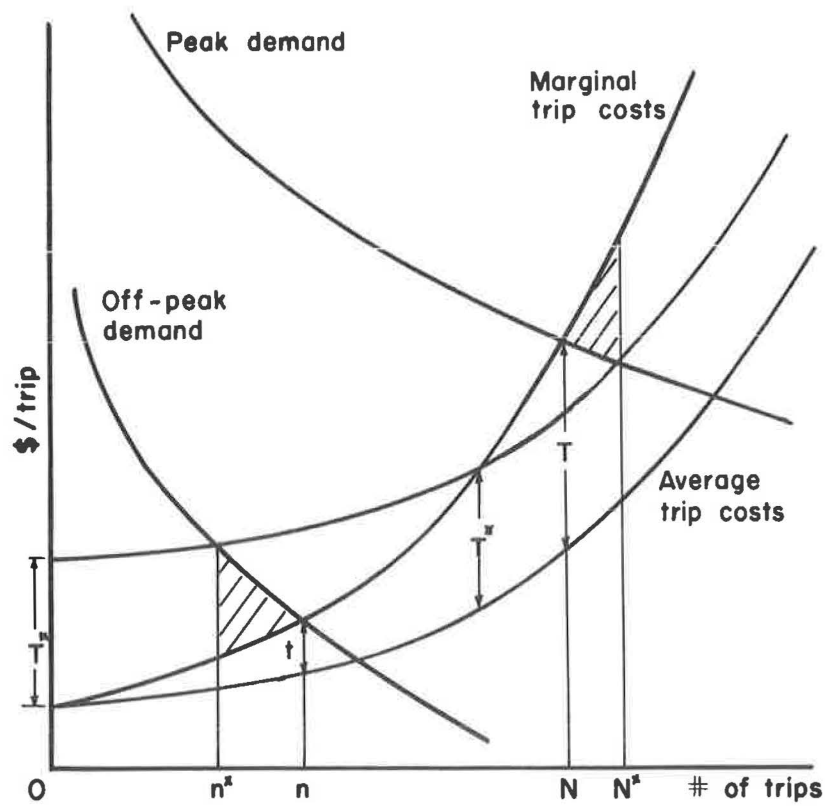

Figure 1.

Off-peak period trips may be treated similarly. Considering T* the best single toll, Eqs. 11a, 11b, 14, and 15 imply that this toll must satisfy the condition

$$\frac{T^* - N\frac{\partial C}{\partial N}}{\frac{\partial C}{\partial N} - \frac{\partial F}{\partial N}} + \frac{T^* - n\frac{\partial C}{\partial n}}{\frac{\partial C}{\partial n} - \frac{\partial f}{\partial n}} = 0 \tag{16}$$

i.e., that T* ought to be set somewhere between the optimum peak and off-peak hour tolls.

Figure 1 provides a graphic illustration of the problem. Tolls of T and t resulting in N and n trips in the peak and off-peak periods, respectively, maximize the benefits to be derived from the highway. If only a single toll is possible, greater than and fewer than the optimum number of trips would be taken in the peak and off-peak periods, respectively. That is, during the off-peak period, an additional n - n* trips could be taken having values greater than their costs. The loss of benefits involved in not allowing these trips to be made is represented by the lower shaded area. Similarly, during the peak period N* - N trips would be made involving values less than their costs. The upper shaded area represents the loss of benefits involved in these additional trips. How large these two benefit losses would be depends on the elasticities of the two demand curves and the distance between them, as well as on the elasticity of the vehicle operating and time costs relationship. The smaller the difference between peak and off-peak hour demands and the more nearly perpendicular these demand relationships are, the smaller will be the loss in benefits resulting from charging a single toll. The loss in benefits will also be smaller the more nearly horizontal the C(Q/Z) relationship is.

In principle, the difference between the maximum benefits attainable on the road between Here and There when variable tolls are and are not charged can be estimated by substituting demand and cost relationships in Eqs. 10 through 16. Unfortunately, even the quite simple travel time-volume-capacity relationships lead to formidable computational problems. The benefit estimates summarized in the following were therefore based on an even simpler relationship between volume-capacity ratios and vehicle operating and time costs:

$$C(Q/Z) = 6.2¢ + bQ/Z \tag{17}$$

in which 6.2¢ is the travel time cost per mile of the occupants valuing their time at \$3.00 an hour traveling in a vehicle at 48.5 mph. Vehicle operating costs are ignored on the assumption that these costs are independent of volume-capacity ratios on rural roads and urban freeways. Two alternative values of b (3.8¢ and 6.2¢) were employed. The former value derives from the Highway Capacity Manual's estimate (5, p. 32) that the average speed on a high-quality rural road is 30 mph at a volume-capacity ratio of 1. The latter value is based on Greenshield's implication that average travel time at a volume-capacity ratio of 1 is double that at a volume-capacity ratio of zero.

Analysis of highway department construction estimates suggests that the average cost of four-lane urban expressways in Minnesota is approximately \$1.2 million per mile (14). If such a highway has a capacity of 7,200 vph and if the interest rate appropriate to valuing highway investments is 10 percent, the hourly cost is approximately 0.2¢ per unit of freeway capacity (i.e., per vehicle-mile per hour).

For computational ease, linear demand as well as linear cost relationships was assumed in the benefit calculations. That is, both the peak and the off-peak demand relationships assumed were of the form P = C - DN, in which P is the price (the toll plus the time costs) at which N trips would be made. For each of the two cost functions, benefit calculations were made for nine pairs of demand relationships. The specific parameter values used (Table 8) were chosen to reflect differences both within and between periods in demand elasticities and differences between demand levels in peak and off-peak periods.

TABLE 8

SPECIFIC PARAMETER VALUES FOR BENEFIT COMPUTATIONS

Peak Period Demand		Off-Peak Demand		Group Numbers
C	D	C	D	
			0.00004	A1, D1
0.2	0.00002	0.1	0.00002	A2, D2
			0.00001	A3, D3
			0.00040	B1, E1
2.0	0.00020	1.0	0.00020	B2, E2
			0.00010	B3, E3
			0.00040	C1, F1
2.0	0.00020	0.1	0.00020	C2, F2
			0.00010	C3, F3

TABLE 9

OPTIMUM TOLL ON HYPOTHETICAL FREEWAY

Group	Toll Type[a]	Optimum Toll (¢/mi)		
		Subgroup 1	Subgroup 2	Subgroup 3
(a) Time Cost per Mile = 6.2¢ + 3.8¢ x volume/capacity				
A	T*	0.83	0.72	0.70
	T	1.18	1.15	1.08
	t	0.18	0.32	0.51
B	T*	0.87	0.80	0.84
	T	1.16	1.07	0.86
	t	0.28	0.52	0.83
C	T*	0.79	0.61	0.42
	T	1.19	1.19	1.19
	t	0.01	0.02	0.05
(b) Time Cost per Mile = 6.2¢ + 6.2¢ x volume/capacity				
D	T*	1.05	0.91	0.87
	T	1.50	1.47	1.38
	t	0.23	0.40	0.63
E	T*	1.10	1.01	1.07
	T	1.48	1.37	1.09
	t	0.36	0.66	1.05
F	T*	1.02	0.77	0.54
	T	1.52	1.52	1.52
	t	0.02	0.03	0.06

[a]T*, T, and t are, respectively, the optimum single toll, the optimum peak period toll, and the optimum off-peak toll for highways of optimum size.

In Table 8, groups A, B, and C were distinguished from groups D, E, and F in that, for the former set, the more elastic cost curve was assumed in the benefit computations. That is, the computations for these groups were made under the assumption that the slope of the average time cost curve was 3.8¢, whereas those for the latter group assumed it to be 6.2¢. Speaking roughly, groups A and D and the remaining groups involve relatively elastic and inelastic peak period demands, respectively. The ratios of off-peak to peak demands are relatively high for groups B and E and relatively low for groups C and F. Finally, within each of the alphabetical groups, subgroups 1, 2, and 3, respectively, involve highly elastic, moderately elastic, and inelastic off-peak demand schedules.

Initially, two sets of benefit computations were undertaken for each of these 18 subgroups. First, the hypothetical authority in charge of the hypothetical highway was allowed to vary the size of the highway but was constrained to establish only a single toll applicable to both time periods. The toll-highway size combination that maximized net benefits was determined. Next, the authority was allowed to levy different tolls on peak and off-peak users. The benefit maximizing combination of tolls and highway size was again found. Net benefits as defined by Eq. 10 were computed under both pricing systems.

The optimum single tolls developed in this fashion ranged from 0.4 to 1.1¢, the highest tolls generally being associated with the highest ratios of peak to off-peak hour demand elasticities. When the authority was allowed to vary tolls between periods, the optimum peak period tolls ranged from 0.9 to 1.5¢/veh-mi. In this case, the highest tolls were associated with the highest ratios of peak to off-peak period traffic.

These toll estimates seemed reasonable enough. However, several of the remaining conclusions were rather surprising. First, the hypothetical authority designed highways that seemed to be considerably more lavish than those currently being built. Whereas current urban expressway design standards call for peak-hour volume-capacity ratios of 70 to 80 percent, the hypothetical authority's highways all had peak period volume-capacity ratios in the 20 to 30 percent range. Second, the optimum single-toll highways were only 0.1 to 3 percent larger than their two-toll counterparts rather than substantially larger, as had been expected. Third, and most surprising, the inability to vary tolls with highway demand had practically no effect on maximum attainable highway benefits. The ratio of single-toll to two-toll net benefits ranged from 99.6 to 99.9999 percent.

An important qualification should quickly be added here. The revenues derived from optimum variable tolls on an optimum highway system would just cover the capital costs of that system. However, the general principle easily can be established that the revenues from benefit maximizing single tolls will fall short of the capital costs of any optimum highway system, and a subsidy will be necessary, if the elasticity of the off-peak demand for trips is equal to or greater than that of the peak demand. Inasmuch as the bulk of peak-hour trips are work trips, there is probably considerably less elasticity during this period than at off-peak hours when the demand is for pleasure, shopping, and other trip purposes. Because this proved true in the cases studied, maximizing benefits while charging a single toll required subsidizing the highway. In the most extreme cases, the subsidy amounted to almost two-thirds of the highway's total cost (Table 10).

TABLE 10

SUBSIDY REQUIRED TO MAXIMIZE BENEFITS IF A SINGLE TOLL IS LEVIED

Group	Subsidy		
	Subgroup 1	Subgroup 2	Subgroup 3
A	0.77	0.72	0.77
B	0.88	0.89	0.99
C	0.67	0.51	0.37
D	0.76	0.71	0.74
E	0.87	0.89	0.99
F	0.67	0.52	0.37

These unanticipated results suggested that two further possibilities are worthy of exploration: (a) that these results would not hold for highways with design volume-capacity ratios more nearly equal those presently being built; (b) that they would not hold if the highway authority were forced to be self-supporting. Time did not permit exploring this latter possibility. As for the former, a final set of benefit computations was run in which the size of the highway was restricted to levels that would yield peak period volume-capacity ratios of 60 to 80 percent. These restrictions reduced maximum possible benefits by as much as 10 to 15 percent, increased optimum peak period tolls to the 3 to 4.5¢ range, and made the highway self-supporting even when a single toll was charged. However, the ratios of maximum possible benefits in the one- and two-toll cases were still uniformly in excess of 99 percent.

CONCLUSIONS

It is, of course, redundant to say that the results obtained from the foregoing analysis depend on the assumptions made and that these assumptions may be grossly unrealistic. Still, it seems reasonable to hazard two fundamental generalizations:

1. Inasmuch as congestion tolls of more than 6¢/veh-mi are associated with the traffic levels prevailing during the greatest part of the day, current gasoline and vehicle license taxes aggregate to only about 1¢/veh-mi, and these taxes appear more than adequate to cover current highway construction and maintenance programs, it seems reasonably safe to assert that the present highway network of the Minneapolis-St. Paul area, and probably most other major urban centers, is grossly underdeveloped.
2. A considerable social loss is unquestionably involved in current highway utilization patterns. The fact that user charges do not vary with the demand for highway services undoubtedly contributes to this loss. However, the apparent inadequacy of present urban highway networks may well be of far greater importance.

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