Theory of Spillover Cost Pricing

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•THE SUBJECT of this paper, a theory of highway finance, derives from a collision between growth of population and a structure of production which is built upon geographical concentration. Underlying the problem is rapid development of certain technologies and relative lag in others, with the result being congestion—a differential in rates of performance. The paper examines an economic theory that derives from the problem where a spillover cost will denote the congestion of one highway user by another, where the relevant cost is loss of time and general nuisance. The text of the argument is a mathematical examination of assumptions to the proposition that highway prices (user taxes, tolls, etc.) should exceed the cost of maintaining highways, and exceed the cost for purpose of reducing the public use of such highways, a thesis recently identified with the work of British economist Walters (1). It is concluded that assumptions underlying the proposition are too improbable to serve as a foundation for public policy involving disposition of $13 billion annually, the current expenditure for highways in the United States. That Walters' thesis may be a reasonable one in light of other assumptions is beside the point of the examination, which has the purpose of considering the assumptions of his thesis for what they are. The argument pertains to a fixed highway plant which is invariably the result of industrial concentration.

THE MARGINAL COST CONTROVERSY

The problem under discussion is of interest to both the economist and the engineer, but the weakness in this alliance has been a failure of communication between the parties. Many engineers look upon highway prices as means of raising revenue or covering cost, whereas economists look upon prices mainly as rationing devices and, as Valavanis (2) observed, "only secondarily as means of raising revenue or covering costs."

By "short-run marginal (physical) cost" the economist, when considering highways, means the increase in highway maintenance and other physical costs resulting from another unit of traffic during a period—assuming full maintenance of the capital outlay. Hence, accrual for periodic resurfacing and major repair, as the accounts are treated by (engineers) Baker et al. (3), would enter the marginal cost account, whereas snow removal by comparison might not. The precise treatment of costs associated with soil failure, washout, restrained temperature warping stress (and weather in general) is not obvious.

It is instructive to note, however, that the concept of marginal cost had its origin in 18th century soil mechanics. One authority (4), for instance, determined marginal cost of a given wheel load to be "one-fourth in amount as the width of the tire is doubled," a radical measure for the time that was adopted by the State of New York (5) in 1836 for the purpose of setting highway tolls. As early as 1773 service on British turnpikes was being priced according to logical engineering methods based on such damage factors as wheel load, tire width, and horsepower (6). (For the first important American turnpike see Ref. 7.) That these early "models" are still more sophisticated than typical prescriptions (versions of the so-called ton-mile theory (8)) in use among the American states today is acknowledged. But it was from this humble origin that the theory of marginal cost as a principle of resource allocation burst

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upon the world through the imagination of French engineer Dupuit (9), who is the father of welfare economics as later developed by Marshall (10) and by other writers employing the techniques of Pareto (11).

It is the principle of marginal cost that market price should be equal to the change in cost with respect to another unit of output. Consequently, there is in principle no accommodation for the recovery of any accountable cost such as wages or rents. The only counterpart of a marginal cost is a rate of change which acts not unlike a brake upon the path of production. Increase of output in one direction is restrained by a factor (price) equal in value to the input lost for use in some other direction, with the result that resistance is equalized among all paths of production at the margin. The result is perfectly consistent with either loss or profit. And in particular, the theory of marginal cost has nothing to do with the recovery of full cost which, depending upon cost functions and level of output, may be the same as, less than, or greater than total revenue (12). The economist is only seldom concerned with the history of production; he is inconsiderate of overhead cost, which he considers to be "sunk"; and he commands an imposing theoretical machinery for moving about large chunks of humanity with the aid of two or three simplifying assumptions. If all fails, he invokes a head tax. But he is seldom concerned with that which is "sunk."

It is emphasized, however, that the basic ingredient to early experiments with marginal cost pricing was the obvious relationship between traffic and destruction, whereas given this relationship at the time, when

> . . . a heavy wagon [was] the most efficacious machine that the art of man, in its present state of science, could construct for grinding to powder the materials of our roads (13),

the policy maker faced no conceptual problem in his treatment of overhead cost because the highway was conveniently destroyed by the traffic it was designed to serve.

THE PARADOX OF OVERHEAD COST

With the introduction of modern concrete pavement, however, the economist was confronted by a more subtle relationship because, like many other materials, concrete has a fairly well-defined limiting unit stress below which millions of repetitions of stress will not cause the material to fail. Under specified conditions it is possible to describe a set of relationships that would be consistent in principle with zero marginal cost or, worse still, negative marginal cost, for when stress from load is less than limiting unit stress, the repetition of stress is "actually beneficial and strengthens the concrete" (14, 15). These suggestions are contained in modern analysis of materials.

But the problem for the economist was evident enough by 1923 for Clark (16) to frame his famous "paradox of overhead cost":

> Here the paradox of overhead cost assumes an extreme form. Before a road is built, it is rational to say that the traffic which benefits should bear the overhead cost and that if it cannot bear it, the outlay is probably not justified. But once a well-paved road is built, reasonable use costs nothing at all, and any charge which limits the amount of such traffic would result in unused capacity and the loss described by the phrase 'idle overhead.'

Conversely, if charges were set equal to the cost of reasonable use, total revenue would be less than total cost because reasonable use costs nothing at all, just the reverse of the early highway problem. Such is the paradox. It must be added, however, that solving the problem of "excess capacity" on an isolated feeder road would only increase congestion in the trunkline, the problem of "chasing rainbows" in terminology of engineers.

But the problem was apparently resolved by Hotelling (17), who in 1938 demonstrated that "everyone can be made better off" under a policy of effectively no user taxes at all. The overhead cost might be recovered through an income or head tax.

Resulting controversy (18) involving "second best" solutions is not dealt with here, although the rationale of the Hotelling thesis is examined later on. Related theory under general treatment of public utilities is found in a paper by Montgomery (19).

The engineer could not help but reply, however, that careful inspection of Hotelling's resolution indicates the solution of a paradox without a paradox to resolve. To measure excess capacity as conceived by J. M. Clark, one must first have a measure of highway capacity simpliciter. Yet the latter is merely a generic term relating to the ability of a roadway to accommodate traffic (20) so that without additional information, one can only assert, for example, that the "capacity" of a modern two-lane highway is in some sense any value one may care to select between, say, 10 and 2, 000 motorcars per hour. But if one may adopt any value within this range, how shall he determine a unique value for purposes of defining excess capacity which is a factor in producing the paradox of overhead cost? Indeterminacy would suggest the need for at least an additional relationship.

This relationship is found, of course, in average traffic speed, which is an index of quality of service and which varies inversely with number of vehicles per hour or, in Clark's terminology, the "amount of traffic." Consequently, any charge which "limits the amount of such traffic" will result in better service, an improvement which Clark identified, in effect, with the creation of "unused capacity and the loss described by the phrase, 'idle overhead.'" But if such is the case, the highway problem is now reversed because "excess capacity" is the very product which consumers demand, a shift anticipated by Pigou (21, 22, 23) in his treatment of congestion prior to Hotelling's rehabilitation of Dupuit. This brings one to the present period of the marginal cost controversy.

THE PARADOX OF SPILLOVER COST

Accordingly, it has been argued by Walters (1) and others that the proper price for highway service will be the sum of the short-run marginal (physical) cost and the consumer spillover cost, which jointly constitute the social marginal cost. Under the paradox of overhead cost it was assumed that, in any case, the charge for highway service would not exceed average total physical cost—or simply unit cost—as allocated during a period. But this assumption is now in doubt, presenting the policy maker with a new "paradox" in highway finance. Before a road is built, to paraphrase Clark, it has been reasonable to think that the charge for service should approach (total) unit cost as an upper limit. But once a road is built the spillover cost of its use during a period may exceed the revenue from charges equal to unit cost during the period, in which case any charge which fails to exceed total physical cost will result in resource misallocation. It has been convenient to think that roads were subsidized when user assessments failed to meet full cost. It will now be less convenient to think that roads are subsidized when assessments fail to exceed full cost. Such are the subtleties of economic analysis.

PURE THEORY OF SPILLOVER COST PRICING

It is in order to inquire of the theory underlying the spillover cost solution. Imagine an economy of m consumers, each of whom supplies homogeneous labor (or resources) and exists upon a diet of highway travel and some other composite good so that the utility function, U, of the jth consumer ($j = 1, 2, \ldots, m$) might take the form

$$U_j = U_j(x_j, y_j, z_j; x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_m) \qquad (1)$$

in which x_j denotes the number of vehicle trips of defined length made by the jth consumer during a period; y_j, the units of the composite good consumed; and z_j, the units of labor (or resources) supplied by the jth consumer during the same period, assuming that

$$\frac{\partial U_j}{\partial x_i} \begin{Bmatrix} > \\ < \end{Bmatrix} 0 \text{ for } \begin{Bmatrix} i = j \\ i \neq j \end{Bmatrix} \quad i, j = 1, 2, \ldots, m \qquad (2)$$

while

$$\frac{\partial U_j}{\partial y_j} > 0 > \frac{\partial U_j}{\partial z_j} \tag{3}$$

within the range of the argument where it is also assumed that if the consumer were to lose successive units of one good, he would require in exchange even greater increments of the other good in order to remain just as well off ($dU_j = 0$) as before, the standard convexity premise.

It is also assumed that the consumer will act so as to maximize the value of Eq. 1 subject to the income barrier

$$z_j w - x_j p_x - y_j p_y = 0 \tag{4}$$

in which w denotes wage (income) per unit of labor (resources); p_y, the price of the composite good per unit; and p_x, the cost or price of a vehicle trip—the cost to the consumer, which includes vehicle expenses, and may or may not include a highway toll or tax, but does not include a factor for the cost of time and congestion which is rather treated under the form of Eq. 2 for $i \neq j$. It is assumed that p_x, p_y, and w are constant—the same for all consumers—and regarded as parameters by the consumers who are to operate homogeneous vehicles, use homogeneous fuel, provide homogeneous labor and, in general, to live in a state of pure competition as defined by the textbooks. Accordingly, one can form the function

$$U_j^* = U_j(x_j, y_j, z_j; x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_m) + \gamma_j(z_j w - x_j p_x - y_j p_y) \tag{5}$$

and set its partial derivatives equal to zero

$$\frac{\partial U_j^*}{\partial x_j} = \frac{\partial U_j}{\partial x_j} - \gamma_j p_x = 0 \tag{6}$$

$$\frac{\partial U_j^*}{\partial y_j} = \frac{\partial U_j}{\partial y_j} - \gamma_j p_y = 0 \tag{7}$$

$$\frac{\partial U_j^*}{\partial z_j} = \frac{\partial U_j}{\partial z_j} + \gamma_j w = 0 \tag{8}$$

$$\frac{\partial U_j^*}{\partial \gamma_j} = z_j w - x_j p_x - y_j p_y = 0 \tag{9}$$

giving the necessary conditions for a maximum value of Eq. 5, in which γ_j is the Lagrange multiplier—defined as the change in utility with respect to income for the jth consumer who, since he has no control over the traffic of others, will regard

$$x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_m \tag{10}$$

as parameters and increase highway travel to the point where utility derived from another trip is equal to the cost of the trip

$$\frac{\partial U_j}{\partial x_j} = \gamma_j p_x \tag{11}$$

with the same being "true" for all consumers so that, in general, one obtains

$$\frac{p_x}{p_y} = \frac{\partial U_j/\partial x_j}{\partial U_j/\partial y_j} \qquad j = 1, 2, \ldots, m \tag{12}$$

all of which may be derived from Eqs. 6 and 7 and the relevant assumptions. Because Eq. 12 is consistent with any form of human behavior, the second-order conditions for a maximum of Eq. 5 are baroque, unless the jth consumer is disposed to mistake happiness for misery. In any case, one does not assume that the consumer will take into account the congestion he imposes upon others.

Such capacity for role taking could be ascribed only to a larger fiduciary having a welfare function, W, on the order of

$$W = W(U_1, U_2, \ldots, U_m) \tag{13}$$

which might take any form, say

$$W = U_1 + U_2 + \ldots + U_m \tag{14}$$

depending on the sentiments of the fiduciary who, in any case, would be restrained by a production barrier which, given

$$X = x_1 + \ldots + x_m; \quad Y = y_1 + \ldots + y_m; \quad Z = z_1 + \ldots + z_m \tag{15}$$

might take the form

$$Z - Z(X, Y) = 0 \tag{16}$$

in which minimum aggregate amount, Z, of labor required to produce the social product is stated as a function of the aggregate product produced (and consumed) during a period. Suppose the case in which the fiduciary wished to maximize some given form of Eq. 13 subject to Eq. 16. Then a maximum value would be sought

$$W^* = W\Big[U_1(x_1, y_1, z_1; x_2, \ldots, x_m) \ldots, U_m(x_m, y_m, z_m; x_1, \ldots, x_{m-1})\Big] + \varphi[Z - Z(X, Y)] \tag{17}$$

the partial derivatives of which are set equal to zero

$$\frac{\partial W^*}{\partial x_j} = W_j \frac{\partial U_j}{\partial x_j} + \sum_{\substack{i=1 \\ i \neq j}}^{m} W_i \frac{\partial U_i}{\partial x_j} - \varphi Z_x = 0 \tag{18}$$

$$\frac{\partial W^*}{\partial y_j} = W_j \frac{\partial U_j}{\partial y_j} - \varphi Z_y = 0 \tag{19}$$

$$\frac{\partial W^*}{\partial z_j} = W_j \frac{\partial U_j}{\partial z_j} + \varphi = 0 \tag{20}$$

$$\frac{\partial W^*}{\partial \varphi} = Z - Z(X, Y) = 0 \tag{21}$$

$$j = 1, 2, \ldots, m$$

yielding the necessary conditions for a maximum value of Eq. 17 and in which φ is the Lagrange multiplier and $\partial W/\partial U_j$ is denoted by W_j and $\partial Z/\partial X$ by Z_x, etc.

By replacing the summand or spillover cost factor in Eq. 18 by S_j and, considering only the center group of terms in Eqs. 18 and 19, these are rearranged so as to bring the correlative assertion into view

$$\frac{\partial U_j/\partial x_j}{\partial U_j/\partial y_j} = \frac{\varphi Z_x - S_j}{\varphi Z_y} \tag{22}$$

in which the Lagrange multiplier expresses the change in social utility with respect to labor; Z_x (etc.) expresses the change in labor, Z, with respect to X; and S_j expresses the change in social utility with respect to the congestion created by another vehicle trip—with the result that all terms are reducible to social utility or, more precisely, to the utility or welfare of the fiduciary who is assumed to have elected Eq. 14 as the form of his own utility function.

Accordingly, it follows under the rationale of Eq. 12 that the fiduciary will select a new set of market prices (r_x, r_y) for units of X and Y so that

$$\frac{r_x}{r_y} = \frac{Z_x - S_j/\varphi}{Z_y} \tag{23}$$

the right side of which is now expressed in units of labor and is equivalent (in terms of labor) to Pigou's solution (18)

$$\frac{\text{Producer's Marginal Cost X + Consumer's Spillover Cost X}}{\text{Producer's Marginal Cost Y}} = \frac{\text{Marginal Social Cost of X}}{\text{Marginal Social Cost of Y}} \tag{24}$$

assuming pure competition and absence of all side effects except the presence of S_j, for which it is also assumed that

$$S_1 = S_2 = \ldots = S_m \tag{25}$$

in which the value of S_j is negative in sign by Eq. 2 for $i \neq j$. For the special case $S_j = 0$ relating to the "wide road" of Pigou, which is very wide, and where the price of road service exceeds the money value of Z_x ($= w\,Z_x$) by some amount attributed to overhead cost, the paradox of overhead cost is obtained in terms of Eq. 23. If this difference is removed by setting $r_x = w\,Z_x$, the solution identified with Hotelling is obtained. For the present case in which $S_j \neq 0$ (but negative in value) and

$$r_x = w(Z_x - S_j/\varphi) \quad ; \quad r_y = w(Z_y) \tag{26}$$

it follows that the market value of goods produced will exceed the income from production, indicating the need of a negative head tax (subsidy) in order to exhaust the product.

Will a small railroad obtain the same subsidy as a large trucking firm with or without a utility function? The subsidy cannot be permitted to vary with use of highways because the consumer might discount the road price by the amount of the subsidy and, apart from some time preference, go about his business as before. Will a small railroad without a utility function obtain the same subsidy as a consumer with or without a motorcar? Will trucks be required to pay for congesting themselves? Will trucks be required to pay for congesting the motorcar after trucks have purchased sufficient additional road space to permit the motorcar to travel at the same rate before and after the injection of trucks (according to the incremental cost theory of the Bureau of Public Roads)? It has always been obvious that the theory of pure competition posed a threat to the General Motors Corporation, but never quite so obvious as at the present.

But other and subversive properties of the toll theory remain to be considered and pertain to its operational status simply as a matter of conception apart from feasibility which, at this level of precision, is rather beside the point. Going back to Eq. 19 and, considering only the center group of terms, rearrange these so as to bring the following into view

$$W_j \frac{\partial U_j}{\partial y_j} = \varphi Z_y \qquad j = 1, 2, \ldots, m, \tag{27}$$

for which, by assumption of Eq. 14,

$$W_1 = \ldots = W_j = \ldots = W_m = 1 \tag{28}$$

so that Eq. 27 becomes

$$\frac{\partial U_1}{\partial y_1} = \ldots = \frac{\partial U_m}{\partial y_m} = \varphi\, Z_y \tag{29}$$

which, in conjunction with Eq. 7, yields

$$\gamma_1 = \ldots = \gamma_m \tag{30}$$

the assertion that marginal utility of income must be identical for all consumers in order to maximize welfare of the fiduciary. The marginal utility of trips must be identical for all consumers; the marginal utility of composite good, identical; the marginal (dis)utility of labor, identical; and the marginal utility of income must be identical for all consumers in order to maximize welfare of the fiduciary. Welfare of fiduciary approaches its maximum as human differences approach zero and, at the optimum, vanish—yielding a theory of welfare with the property of human variation factored out.

If these results are too extreme, one may select another explicit form for the welfare function. But let there be no mistake about the issue. Until a given form of Eq. 13 is adopted, the theory of tolls is short as many equations as the number of consumers minus one. Advocates of the theory can appraise many features of their system but not the feature at issue (i.e., welfare). It will also be obvious that specification of Eq. 13 embraces implications that transcend any theory of highway prices.

Other problems for the policy maker may be noted. It is not true that labor receives the value of its marginal product as strictly required by the theory; not true that prices are equal to marginal cost as required by the theory; not true that producers are too small to affect the price; not true that commodities are homogeneous, and so on. But, in general, it is not true that the American economy will satisfy the assumptions of the theory of spillover cost pricing. And if it will not, it simply will not, a negation made explicit by Chamberlin (24).

It will also be obvious that one cannot eschew the assumptions of the model under an argument where the test of the model is power of prediction. No prediction is involved. The assumptions are the thing.

Of course, some stringent features of the model might be relaxed with gains in empirical correspondence by the introduction of an "efficiency unit" for labor, thus relaxing the homogeneity assumption for labor, and permitting wages (say per hour) to vary with "efficiency." But precisely what shall be relaxed and what shall be held firm? How will assumptions not relaxed appear to our sense of logical balance in the absence of assumptions that are relaxed? For instance, according to the rationale of

Eq. 23, the only remaining problem for the policy maker is to summarize the findings expressed for this purpose in units of labor by Eq. 23. But will a unit of labor paid $60 an hour in the production of commercial advertisement count the same as a unit of labor paid only $3 an hour in the production of primary education? Given the "efficiency unit," the answer may be that a unit in commercial advertisement should count 20 (= 60/3) times as much as a unit in primary education—the rationale being that abandonment of homogeneity postulate for labor does not involve abandonment of assumption that factors of production receive the social value of their marginal product. But the problem at this point is too obvious. The policy maker might even reply that assumptions underlying the proper price for highway service are a greater social liability than the traffic.

There is finally a problem of theoretical discretion. Perhaps in a free society (and a wealthy one), what is done may not prove to be so important as the reasons given for what is done. Consumers are to be charged for the time delays they cause each other according to the magnitude of S_j. But as St. Clair has remarked, it is like "adding insult to injury first to recognize that time delay is a cost to the consumer, and then to say that he must pay, in order to maximize his own benefits, a tax that will cause the cost to leap from the average to the marginal point."

REFERENCES

1. Walters, A. A. Econometrica (1961).
2. Valavanis, S., "Traffic Safety from an Economist's Point of View." Quart. Jour. Econ., p. 477 (Nov. 1958).
3. Baker, R. F., Chieruzzi, R., and Bletzacker, R. W., "Highway Costs and Their Relationship to Vehicle Size." Chapt. 9. Ohio State Univ. Eng. Exper. Station (1958).
4. Penfold, C., "Husbandry." Sec. iv, p. 22. Baldwin and Cradock, London (1840). (Deposit Harvard Law Library).
5. Revised Statutes of The State of New York (Packard Edition), Ch. XVIII, p. 599 (1836).
6. 13 George III, C. 84, English Statutes at Large, Vol. 30 (1773).
7. Statutes at Large of Pennsylvania, Vol. XIV, pp. 287-288. Harrisburg Publ. Co. (1794).
8. Grubbs, C. M., "Problems of Highway Cost Allocation." Nat. Tax Jour. (1964).
9. Dupuit, J. (translation) International Economic Papers, No. 2, pp. 81-110 (1962).
10. Marshall, A., "Principles of Economics." 8th Ed., Ch. 13.
11. Pareto, V., "Cours d'Economie Politique." Vol. 1, p. 20; Vol. 2, p. 90. Lausanne (1897).
12. Samuelson, P., "Foundations of Economic Analysis." p. 242. Harvard Univ. Press (1955).
13. Fry, J. S., "An Essay on the Construction of Wheel Carriages." p. 14. Manchee, London (1820).
14. "Concrete Pavement Design." p. 30. Portland Cement Assoc. (1951).
15. "Fatigue of Concrete." Ill. Dept. of Highways, Springfield, Ill. (1934).
16. Clark, J. M., "Studies in the Economics of Overhead Cost." p. 304. Univ. of Chicago Press (1923).
17. Hotelling, H., "The General Welfare in Relation to Problems of Taxation." Econometrica, pp. 242-269 (1938).
18. Friedman, M., "Essays in Positive Economics." pp. 100-113. Univ. of Chicago Press (1953).
19. Montgomery, R. H., "Government Ownership and Operation of the Railroads." Ann. Amer. Acad. Pol. Sc., 201: 137-145 (1939).
20. "Highway Capacity Manual." U. S. Govt. Printing Off., Wash., D. C. (1950).
21. Pigou, A. C., "The Economics of Welfare." p. 194.
22. Knight, F., "Fallacies in the Interpretation of Social Cost." Quart. Jour. Econ., pp. 582-606 (1924).
23. Beckmann, M., McGuire, C. P., and Winsten, C. B., "Studies in the Economics of Transportation." pp. 83-87. Yale Univ. Press (1956).
24. Chamberlin, E., "Theory of Monopolistic Competition." Harvard Univ. Press, 213 pp. (1934).