A Digital Computer Technique for Calculating Costs of Right-of-Way

WILLIAM P. SHELDON and ARTHUR L. STRAUB
Respectively, Graduate Student, Civil Engineering Department, University of Arizona, Tucson, and Professor of Civil Engineering, Clarkson College of Technology, Potsdam, New York

This article describes a digital computer program for making a land cost analysis along a highway right-of-way. The principle employed for determining a "running" (per station) cost along the centerline projection is basically one of matrix theory. Essentially, two matrices exist within the system; one used to define the location of the route centerline is superimposed on another used to assign varying costs to pieces of land in the area. Following development of the program written in FORTRAN coding, an analysis was conducted on five hypothetical lines. Limitations of the approach and possible applications are discussed.

•THE COST of buying land for highway construction can be a major item, particularly in well-settled areas, and, therefore, warrants the designer's attention, especially in early stages of route location.

Interacting components of engineering standards and construction costs are constantly working toward the detriment of each other. If engineering standards are high, they are attained only with the expenditure of much money. Conversely, if available funds are low, a sacrifice in engineering standards may result. Manpower limitations often are influential factors in attempting to provide the best engineering at the least cost. It would appear, therefore, that with the aid of high-speed electronic computer facilities for routine computation, one could justify their use in investigating many routes. From this viewpoint, should the land costs along a particular route become an important factor, a computer technique could act as a tool for investigative work.

Although beyond the scope of the present paper, over-all consideration to location and design problems would also include applications of photogrammetry such as described by Pryor (1) and Henry (5).

DEVELOPMENT OF PROGRAM

Other investigators have reported on various approaches to the problem of computing right-of-way costs of a number of trial lines using computer techniques (2, 6). The approach described here essentially uses two sets of matrices in computer memory. One set defines grid boundaries in terms of Cartesian coordinates and is used to locate segments of the line being investigated. This is superimposed on a second set of matrices which identifies the cost of the various parcels of land within the boundaries of the area under study. The cost of right-of-way for a trial line is accumulated as a running cost for each station. Comparison of the results of several trial lines can easily be made.

Assuming that a conventional system of Cartesian coordinates is established, the x, y location of points which define a route can be computed. Many programs developed for highway design purposes include this feature. The descriptive coordinates for points

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spaced at any interval (e.g., 5 ft) can be produced easily for tangent and curved portions of any route within the ordinary limits of geometric standards for alignment.

From analytical principles, the length of a line segment can be determined if the coordinates of the terminal points of the line segment are known. Accordingly, information required to compute the running costs of land along a trial route is as follows: (a) an appraised cost of the land along the route (in units of right-of-way), (b) the width of right-of-way to be secured, and (c) knowledge of the terminal points of line segments defining the route.

To accomplish compatibility between the computer and the computer user, assignment of costs can be manipulated using matrix principles. Regarding locational descriptions, the x and y coordinate axes can be assigned to the lower and left edges of the hypothetical area under study. Thus, assuming conventional conditions, negative coordinates can be eliminated. The matrix principle virtually permits perfect aggreement between computer memory and actual map conditions. Cartesian coordinates are then of significant value; they are utilized to define boundary values of rectangular blocks containing appraised cost indices. These inferences, of course, presuppose a plane coordinate network such as that currently encouraged by the U. S. Bureau of Public Roads and other organizations.

Hence, if a means exists of providing a representative display (topographic and/or land-use map) of an area through which proposed lines are to be projected, the problem can be handled by computer application. Knowing the patterns of cost indices in terms of x, y coordinates, the problem becomes one of introducing the cost of the blocks per unit of area, the x, y boundary values associated with these blocks, and the desired width of right-of-way, respectively, into the computer system.

The practical aspects of the approach developed are important; for proper execution of this technique, there are considerations that warrant further attention. First of all, knowledge of the grid patterns must be ascertained. Which grid number is associated with what x and y boundary values? What land cost index corresponds to that grid number? Also, the direction of the centerline as it progresses through the area in question must be known.

To satisfy the foregoing demands, the analytical aspects of the model should be considered. Figure 1 shows a general condition that commonly exists in the field; i.e., the right-of-way passes through parcels of land of different values. Areas 1, 2, 3 and 4 represent four arbitrary patterns of cost, each area being different in its cost per unit area. Consideration of matrix theory shows that it can be applied to the solution of the problem at hand.

X, Y formation permits a convenient relationship that can be placed in computer storage.

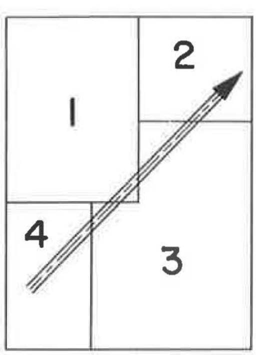

Figure 1. Right-of-way passing through parcels of land of different values.

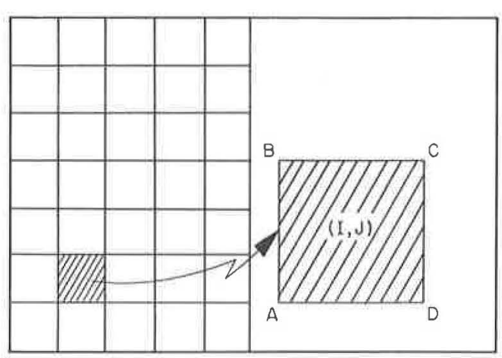

Figure 2. General matrix pattern of model (left), and enlarged I,J grid within model (right).

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A 2-dimensional array may be thought of as being composed
of horizontal rows and vertical columns. The first of the
2 subscripts then refers to the row number, running from
1 up to the number of rows, and the second to the column
number, running from 1 to the number of columns. For in-
stance, an array of 2 rows and 3 columns might be shown
in mathematical notation as
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$$\begin{matrix} A_{1,1}, & A_{1,2}, & A_{1,3} \\ A_{2,1}, & A_{2,2}, & A_{2,3} \end{matrix} \qquad (3)$$

Final development of the program then became one of establishing a core of matrices in computer memory; one matrix defined the grid boundaries, and another was superimposed to assign costs to the specific grid encompassed within these boundaries. The following procedure was used to assign i, j values to the first matrix: If I = rows and J = columns, the location of any point (i, j) uniquely defines its position with respect to a reference datum. Figure 2 (left) shows the general pattern of the system. In this instance, the hatched section represents one I, J grid within a rectangular matrix.

PHYSICAL MODEL

For purposes of convenience in trying out the technique described, a physical model was drawn with grid lines spaced at 5 in. (Actually, the grid was drawn on a topographic map and was used also for computer program development involving line description and earthwork analysis, which are beyond the scope of the present paper.) At an assumed scale of 1 in. = 50 ft, each grid square represented 250 by 250 ft of area.

The enlarged grid (Fig. 2, right) is the I, J block drawn out for further examination. For the physical model, the rectangular coordinates x, y (matrix definition and computer notation) of the corner boundaries A, B, C and D are, therefore:

$$A = 250\ (J-1),\ \text{and}\ 250\ (I-1) \qquad (1)$$

$$B = 250\ (J-1),\ \text{and}\ 250\ (I) \qquad (2)$$

$$C = 250\ (J),\ \text{and}\ 250\ (I) \qquad (3)$$

$$D = 250\ (J),\ \text{and}\ 250\ (I-1) \qquad (4)$$

This process is conducted on all grids within the matrix by iterative routine and stored in memory for subsequent use. A grid identification number is also a product of this matrix. A similar procedure was adopted for assigning the cost indices to the respective grid squares. From Eqs. 1, 2, 3, and 4 the boundary limits confine the area, permitting the designer to ascertain the location of the route centerline; hence, all he is required to specify is the I, J of the point.

The completed manuscript consisted of a 5-by-7 rectangular grid arrangement. The matrix pattern, therefore, consisted of 35 equal squares. A display and documentation of the final model is shown in Figure 3. Each grid square contained integer data for identification and computational purposes. For an example grid square in Figure 3, (2, 2) represents the subscripts assigned to the example grid square for identifying the I, J of the complete matrix, and [200] represents the cost index, in dollars per unit of area, of the area in which the example grid square is located (an arbitrarily chosen index to depict land value). In this study, an acre was the parameter of areal extent.

The program, as written, was capable of varying the costs at the will of the user. Cost indices used in the study for depicting appraised land values were \$10,000, \$200, \$60, and \$20 per acre for settled and varying levels of marginal lands, respectively. These indices were assigned arbitrarily to the grid squares to represent a patchwork pattern of land value. The boundaries of the patchwork pattern used in the study are shown by the sinuous lines in Figure 3.

The mechanical operation for computing costs per station is fundamental. The programed instructions compute the cost for each segment by multiplying the width of way (which also may be varied if desired) by the running length of 5 ft, converting this into

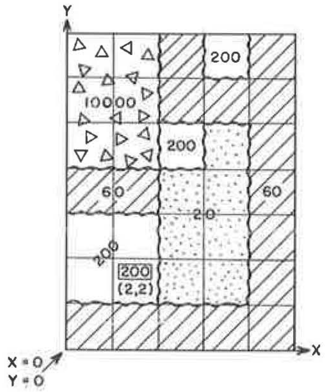

Figure 3. Documentation of cost assignment to grid squares.

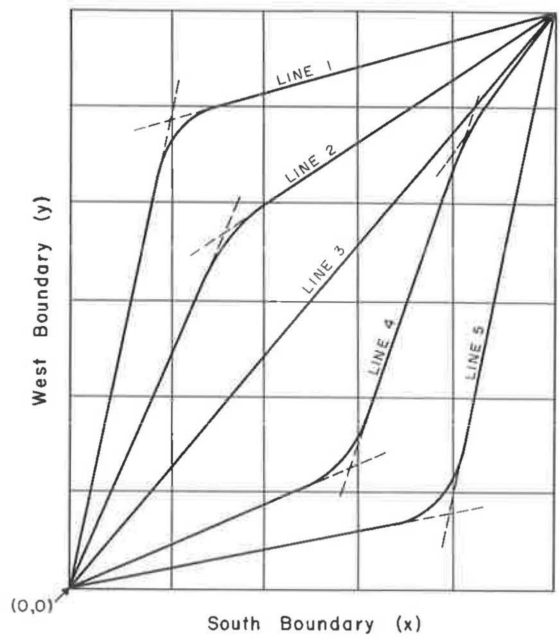

Figure 4. Locations of lines chosen for investigation.

acres, and multiplying the area by the cost index applicable for the particular grid unit in which the line segment lies. The cost for each segment is accumulated for each station and continues over the entire length of the line.

INPUT REQUIRED

The following parameters must be furnished for execution of the program:

1. Appraised or estimated land costs for each grid square;
2. Beginning and ending station and their respective coordinates for each line segment (output from related program describing route);
3. Width of right-of-way between terminal points for each line segment;
4. Total cost (initial) accumulated up to the beginning station; and
5. Line identification number.

EXAMPLE PROBLEM

For trial purposes, five lines were arbitrarily selected for investigation. The general locations of these lines are shown in Figure 4. The assumed right-of-way width was 300 ft, and the cost indices for land were assumed to be those already shown in Figure 3. The beginning station for each line was assumed to be zero, and the accumulated cost up to the beginning of each line was taken to be zero dollars.

The digital computer used for this investigation was an IBM 1620 Data Processing System accompanied by an IBM 1622 card reader-card punch unit. The average execution time for the five trial routes was 2.5 min.

Table 1 summarizes the results of five trial lines. The output format in the computer program was designed so that the results were arrayed in the following manner:

Station	Total Cost, Dollars
xx	xxxxx. xx

Figure 5 is presented to give a pictorial display of the land cost for right-of-way acquisition for the total study. Values in Figure 5 are those excerpted from Table 1.

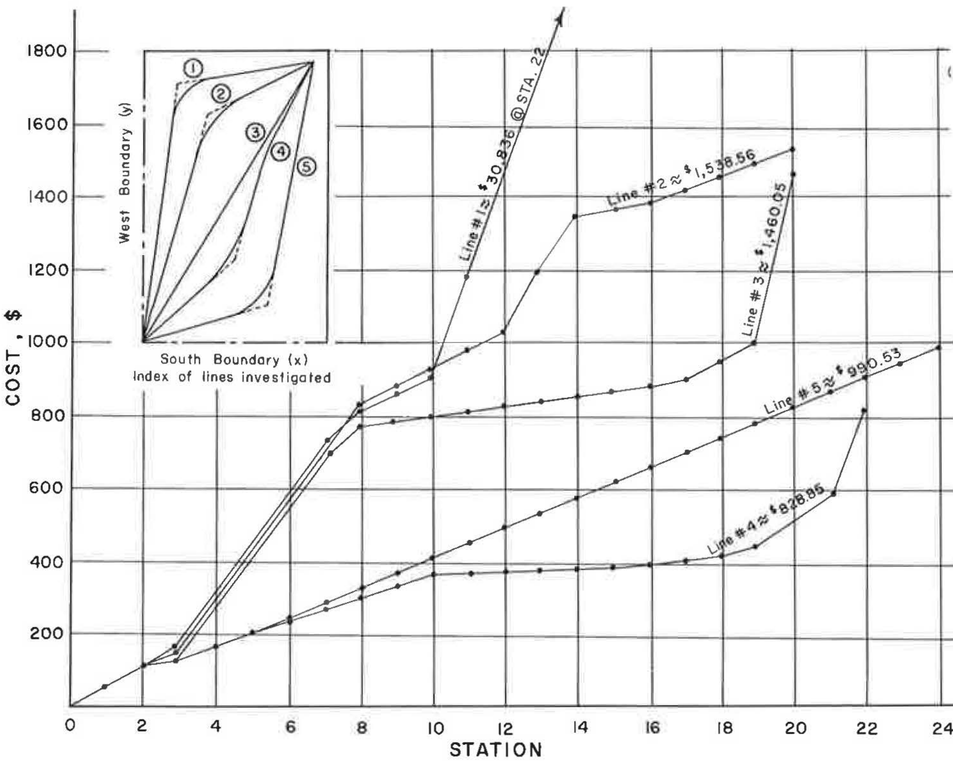

Figure 5. Plot of right-of-way costs by station for alternate routes.

DISCUSSION OF RESULTS

From either Table 1 or Figure 5 the total accumulated cost as well as the pattern of cost over each of the five trial lines can easily be determined. It would appear that the right-of-way costs for Line 4 are most favorable in comparison to costs of the other lines. The cost of right-of-way for Line 1 is highest as the result of passing through a relatively short section of highly valued land. Sudden changes in the slopes of any of the curves in Figure 5 indicate where sudden changes occur in the value of the land through which the lines pass. Given such station-cost patterns, minor shifts in some lines could be noted which would result in costs more favorable than any trial line investigated thus far.

LIMITATIONS OF APPROACH AND SUGGESTIONS FOR FURTHER DEVELOPMENT

It is well to consider some of the limitations of the approach outlined here, especially as they would affect the validity and accuracy of the results.

TABLE 1

RESULTS OF LAND COST ANALYSIS[a]

Station	Cost ($)				
	Line 1	Line 2	Line 3	Line 4	Line 5
1	41.32	41.32	41.32	41.32	41.32
2	82.64	82.64	82.64	82.64	82.64
3	167.35	152.89	123.96	123.96	123.96
4	305.09	290.63	237.60	165.28	165.28
5	442.83	428.37	375.34	206.61	206.61
6	580.57	566.11	513.08	247.93	247.93
7	718.31	703.85	650.82	285.12	289.25
8	822.31	841.59	763.77	298.89	330.57
9	863.63	926.30	777.54	312.67	371.90
10	904.95	967.63	791.32	326.44	412.32
11	7,107.43	995.17	805.09	340.21	453.64
12	13,994.48	1,046.14	818.86	353.99	494.96
13	20,887.52	1,183.88	832.64	367.76	536.28
14	27,768.56	1,321.62	846.41	381.54	577.61
15	30,547.60	1,353.79	860.19	395.31	618.93
16	30,588.92	1,373.27	884.98	409.08	660.25
17	30,630.24	1,414.59	926.30	422.86	701.57
18	30,671.56	1,455.91	967.62	458.67	742.90
19	30,712.88	1,497.24	1,008.95	499.99	784.22
20	30,754.20	1,538.56	1,460.05	541.32	825.54
21	30,795.52	—	—	582.64	866.86
22	30,836.84	—	—	828.85	908.18
23	—	—	—	—	949.51
24	—	—	—	—	990.83

[a]Costs varying (grids assigned different costs); constant right-of-way approximately 300 ft selected in card data; land cost approximately 0 + 00 = $0.

Any approach to the accurate estimation of cost for right-of-way for trial lines, including the one outlined here, is limited by the accuracy of the appraised or assumed cost of the land being studied. Accuracy is needed on the dollar amounts, as well as the specific locations of the parcels or buildings involved. It is not a question of value of an entire property which might include some portions of high value and others of low value. It is a question of the value of the particular strip that would be taken by the right-of-way. A building on a given property is either taken or not taken depending on the specific location of the building and the specific location of the right-of-way. Any automated system of data processing for right-of-way costs would depend on a systemati and efficient method of identifying the location and values of land and improvements whether the data are derived from assessors' maps or records, airphotos, or field inspection. The system used should strive for accuracy and, at the same time, should avoid arousing public concern prematurely. The problems of arriving at sufficiently accurate and detailed values of land and improvements are formidable and are beyond the scope of this paper. For purposes here, it is assumed that the required data are available for analysis.

Given adequate data on land costs, however, there is a mathematical limitation of the grid matrix approach which should be noted. The boundaries of the areas of different cost indices must theoretically coincide with the grid network. Skew orientation of cost boundaries, or spacing of cost boundaries between the grid network, cannot be permitted. This limitation might not be serious in a general study where generalized boundaries of land values could be defined to coincide with a grid system. Actually, however, the boundaries of land of different values are likely to be oriented in any fashion, and often include curved or irregular sides. Buildings are frequently odd-shaped and set in many orientations. If an accurate accounting of land and building costs is to be made by the grid matrix approach, the grid network must be divided finely enough to permit reasonably accurate approximations of actual locations of land areas and buildings by the matrix notation. The limit of the computer memory storage capacity limits the fineness of the grid and the areal extent of the problem which can be undertaken in a one-step solution.

In possible application, a coarser grid might be used in rural areas and a finer grid in settled or urban areas. Because input cost figures are based on unit area, special attention should be given to the problem arising when the right-of-way cuts through part of a building, when, in fact, all of the building must be taken. This, again, relates to the matter of the fineness of the grid network required to attain needed accuracy.

Whereas the input data are based on appraised or estimated value, experience shows what additional percentage might be added to account for damages and adjustments normally encountered in various areas classified by land use. Refinements of this sort could make the resulting figures accurate enough to be suitable for use in preliminary engineering studies.

In addition to using results to assist in preliminary route studies, once a route is chosen, the data derived from such a study could be filed and used later for review and comparison with the results of the actual cost of acquisition. Large deviations between the two could be spotted easily and could be checked to determine whether the discrepancies were due to errors in original estimating or to impropriety in the business dealings.

CONCLUSIONS

This article has elaborated on a computer approach for estimating right-of-way costs along a route and has possible applications in the investigation of trial lines during preliminary route location studies. Widths of right-of-way and appraised costs of land parcels through which the route passes can be varied as they are input parameters of the program. Using high-speed computer facilities for routine calculations, the engineer is permitted to investigate a wide range of alternate routes. The neat, orderly output can be checked with a minimum of effort and is a self-explanatory document which can be filed for later use. The logic of the approach is sound from the engineering standpoint, and gives satisfactory mathematical results. Further method investigation and program refinements are warranted.

The program described here has been limited only to right-of-way cost determination. In practice, consideration should be given to all significant variables and conditions involved in route design (4). To be most useful and efficient in actual automated location and design problems, the determination of right-of-way costs should be integrated with the determination of line, grade, earthwork, and any other item of construction and operating costs which can be quantified.

ACKNOWLEDGMENT

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Appendix

COMPUTER PROGRAMING TERMS, LISTING AND FLOW CHART

Definitions of Programing Terms

COST (I, J): Represents cost indices, stored in memory; designed to furnish the cost patchwork pattern by arraying index costs for zones in a row and column arrangement

XORIG[1]: Beginning station X-ordinate

YORIG[1]: Beginning station Y-ordinate

XFINL[1]: Ending station X-ordinate

YFINL[1]: Ending station Y-ordinate

BSTA[1]: Beginning station

ESTA[1]: Ending station

WOROW: Width of right-of-way chosen

CSTT: Initial cost (at BSTA)

LIDN: Line identification number

RATE: Represents area secured for land acquisition for 5-ft increment of centerline, in acres, for a given WOROW

[1] From related program, this input data are either punched on cards or placed on tape to be read in at 5-ft intervals.

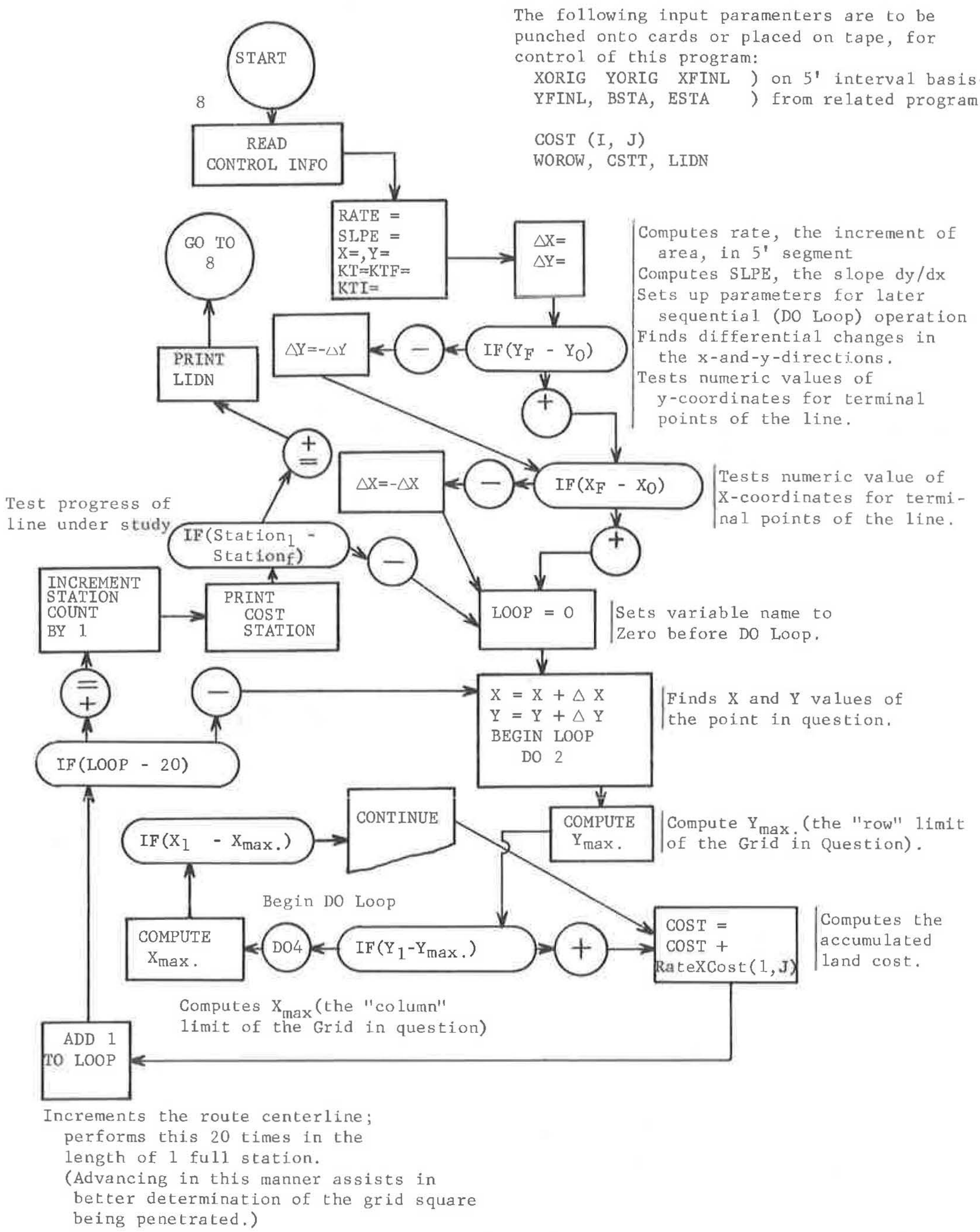

Figure 6. Flow chart.

SLPE: Defines slope of route under investigation (divides total change in Y-direction by the corresponding X-direction)

DELX: Represents differential change in X-direction, incremental length divided by conventional slope function

DELY: Represents the differential change in Y-direction for a given increment in length, equal to absolute value of slope multiplied by d(X)

LOOP: Parameter used for later sequential operation within system

YMAX: Row limit of grid in matrix under investigation

XMAX: Column limit of grid in matrix under investigation

Program Listing

```
DIMENSION COST(7, 5)
8 DO 1 I = 1, 7
DO 1 J = 1, 5
1 READ7, COST(I, J)
READ7, XORIG, YORIG, XFINL, YFINL, BSTA, ESTA
READ7, WOROW, CSTT, LIDN
RATE = 5. *WOROW/43560.
X = XORIG
Y = YORIG
KTI = BSTA
KT = KTI
KTF = ESTA
SLPE = (YFINL - YORIG)/(XFINL - XORIG)
DELX = 5./SQR(1. +(SLPE)**2)
DELY = ABS(SLPE*DELX)
IF(YFINL - YORIG) 10, 11, 11
10 DELY = - DELY
11 IF(XFINL - XORIG) 12, 13, 13
12 DELX = - DELX
13 LOOP = 0
6 X = X + DELX
Y = Y + DELY
DO 2 I = 1, 7
YMAX = I*250
IF (Y - YMAX) 3, 3, 2
2 CONTINUE
3 DO 4 J = 1, 5
XMAX = J*250
IF(X - XMAX) 5, 5, 4
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4 CONTINUE
5 CSTT = CSTT + COST(I, J)*RATE
LOOP = LOOP + 1
IF(LOOP - 20) 6, 7, 7
7 KT = KT + 1
TCOS = CSTT
JSTA = KT
PRINT7, JSTA, TCOS
IF(KT - KTF) 13, 9, 9
9 PRINT7,
PRINT7, LIDN, LIDN, LIDN, LIDN
GO TO 8
END
```