# Structural Characteristics of Reinforced Concrete Elliptical Sewer and Culvert Pipe

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• ALL WELL-ENGINEERED structures are functional in form. They embody the shape which most efficiently satisfies their end use. Most pipe have circular cross-sections inasmuch as the closed circle, encompassing a given area with the least perimeter, is hydraulically the most efficient. Sometimes, however, a pipe of different shape may be used to advantage.

One such useful shape is the ellipse. Elliptical pipe possess unique hydraulic and structural characteristics by virtue of their geometry. The characteristics of elliptical pipe with the major axis placed horizontally (HE pipe) are quite different from those gained from placing the major axis vertically (VE pipe). In effect, elliptical pipe are two distinctly different types of structures, each having its advantages and disadvantages. For example, HE pipe provides several hydraulic advantages at flows of less than full depth, and VE pipe provides greater structural strength under external earth loads.

ASTM Committee C-13 has formally adopted specifications for strength classifications and corresponding required reinforcement for HE and VE pipe; this specification, ASTM C507, has recently been issued. It will be necessary for the designer, however, to relate these 3-edge bearing test strength classes in the specification to required field strengths.

The purpose of this paper is to detail a method of structural design for these two types of pipe such that it will be possible to determine field loads under various installation conditions and relate these field loads to the conventional 3-edge bearing test strengths. Alternatively, it will be possible to relate known test strength of pipe to permissible field conditions.

## DIMENSIONS OF PIPE

The shape and wall thicknesses of HE and VE pipe have been standardized by the industry. The various sizes are given in terms of the "equivalent round" pipe, based on the elliptical pipe having approximately the same total flow capacity as the round size. These standard pipe are shown in ASTM C507. It may be noted that the term "elliptical" is a misnomer because the shape is defined by circular arcs; the approximation to an ellipse, however, is very close (Fig. 1).

## FIELD LOADS

The method of determining earth loads is identical to the Marston-Spangler theory of loads on underground conduits for trench or embankment conditions. These methods, developed over a number of years, have been compiled and summarized by Spangler in several papers (1, 2). For completeness of this paper, those methods applicable to the present subject are also reviewed here. In accordance with these theories, the method of calculating static loads in a trench installation is distinguished from that used for positive projecting embankment conditions.

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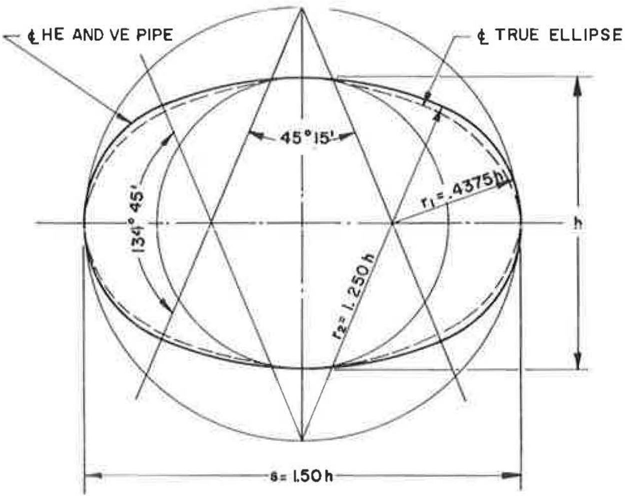



Figure 1. Basic geometry of elliptical pipe.

## TRENCH LOADS

Trench loads are computed from Marston's formula for closed conduits (3)

$$W_t = C_d w (B_d)^2 \quad (1)$$

in which

$W_t$ = trench load, lb/lin ft;
$C_d$ = trench load coefficient; and
$B_d$ = trench width, ft.

The value of the trench load coefficient, $C_d$, may be found from Figure 2.

## POSITIVE PROJECTING CONDUIT LOADS

Earth loads for positive projecting rigid conduits are computed in accordance with the Marston-Spangler expression (1, 4)

$$W_e = C_c w (B_c)^2 \quad (2)$$

in which

$W_e$ = embankment load, lb/lin ft;
$C_c$ = embankment load coefficient; and
$B_c$ = outside horizontal width of the pipe, ft.

The value of the embankment load coefficient, $C_c$, may be found from the curves of Figure 3. The use of $K\mu = 0.1924$ for the curves, results in the most conservative embankment load for the range of soil types generally encountered.

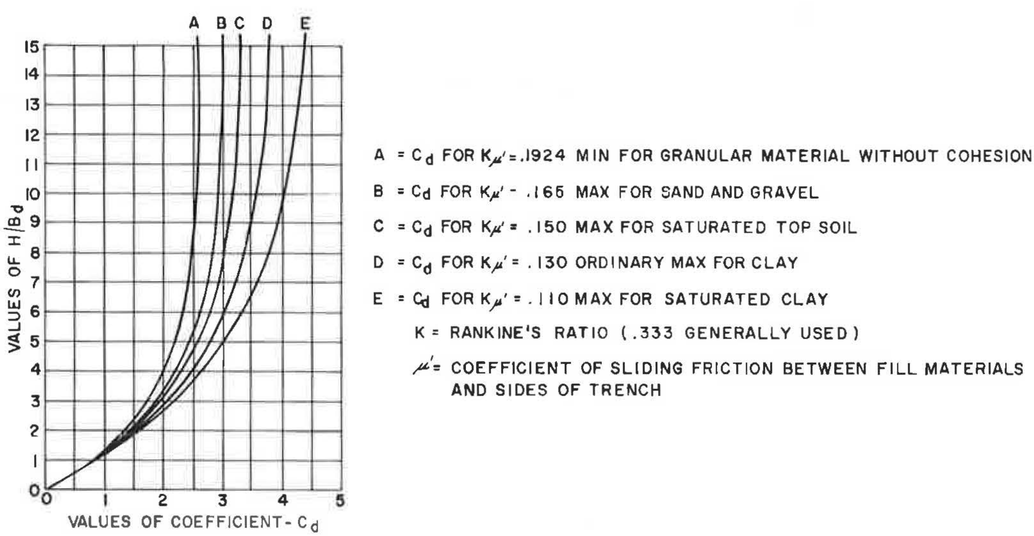
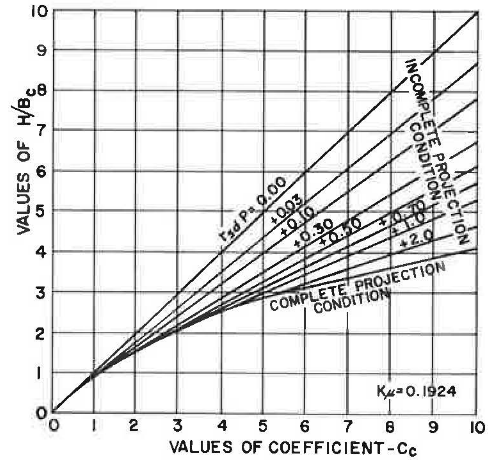



Figure 2. Computation diagram for earth fill loads on ditch conduits.

It is noted that the $C_c$ curves are dependent on the product of the settlement ratio, $r_{sd}$, times the projection ratio, p. The settlement ratio is a function of the deflection of the conduit and the relative settlement of the soil under and at the sides of the conduit (4). Analysis has indicated that, for practical purposes, settlement ratios for circular and elliptical pipe may be considered equal. For rigid culverts on foundations of ordinary soils the settlement ratio is generally chosen between +0.5 to +0.8.

In Spangler's derivation for earth load, the projecting height of the conduit above the natural ground is defined as a proportion of the conduit horizontal span, $pB_c$. It is usually more convenient to express this height as a proportion of the total vertical height of the pipe. For circular conduits, with horizontal span equal to the diameter, the height of the projection in terms of the vertical height remains $pB_c$; but for elliptical pipe, the projection height in terms of the vertical height of the pipe may be expressed as $p_eB'_c$, in which $B'_c$ is the vertical height of the pipe. Then for any specified elliptical pipe projection ratio, $p_e$, the projection ratio, p, required to find the coefficient, C, may be found by $p = 0.69p_e$ for HE pipe, and $p = 1.45p_e$ for VE pipe. This relationship is derived from the dimensions of standard elliptical pipe.




$r_{sd}$ = SETTLEMENT RATIO

p = PROJECTION RATIO

K = RANKINE'S RATIO (.333 GENERALLY USED)

$\mu$ = COEFFICIENT OF SLIDING FRICTION IN EMBANKMENT

Figure 3. Computation diagram for earth fill loads on embankment conduits.

## NEGATIVE PROJECTING AND IMPERFECT TRENCH CONDUITS

Positive projecting conduits are those installed with the top of the pipe above the natural ground surface. Embankment culverts are also sometimes installed in a subtrench with the top of the pipe below the natural ground surface. In this case, the load on the culvert should be calculated according to the theory of negative projecting conduits.

A further type of installation is the imperfect trench, which is an artificial negative projecting condition used to reduce the load on a conduit under high fills. After the embankment has been placed to several feet over the top of the pipe, a trench equal in width to that of the pipe is dug in the fill directly over the pipe. The trench is filled with loose compressible material, and the remainder of the embankment is placed. The friction developed along the sides of the trench supports some of the load of the embankment, reducing that which the pipe must support. This would be a particularly useful method of installation for HE pipe when it is necessary to place it under a high embankment. (Because of its breadth, HE pipe is less efficient structurally than circular and VE pipe.)

The formulas and coefficient diagrams for calculating loads in both these types of installation may be found in the literature (4, 5).

## LIVE LOADS

Surface live loads transmitted to the pipe are based on Holl's integration of the Boussinesq formula for a surface point load transmitted through a semi-infinite elastic solid (4). Both impact and the support given by adjacent unloaded sections of pipe should be taken into account. The general expression is

$$W_l = C_L P(1.0 + I_f) \tag{3}$$

$C_L$ may be found from

$$C_L = \frac{R_f}{L}\left[1 - \frac{2}{\pi}\left\{\left(\sin^{-1} H\sqrt{\frac{A_1^2 + B^2 + H^2}{(A_1^2 + H^2)(B^2 + H^2)}}\right) - \frac{A_1 B H}{\sqrt{A_1^2 + B^2 + H^2}}\left(\frac{1}{A_1^2 + H^2} + \frac{1}{B^2 + H^2}\right)\right\}\right] \tag{4}$$

in which

$W_l$ = live load on pipe, lb/lin ft;
$C_L$ = live load coefficient;
$R_f$ = live load reduction factor;
$P$ = wheel load, lb;
$I_f$ = live load impact factor
$A_1$ = ½ horizontal outside width of pipe, ft;
$B$ = half of effective pipe length supporting one wheel load, ft;
$L$ = effective length of pipe supporting one wheel load, ft (3 ft generally used); and
$H$ = height of earth cover over pipe, ft.

Wiggin, Enger and Schlick have proposed live load reduction factors resulting from the adjacent unloaded sections of pipe which support the loaded sections. These factors were given for covers 2.5 ft and greater and for pipe sizes 4 in. to 60 in. To make the values a continuous function suitable for computer programing, empirical formulas approximating the values (6) are suggested as

$$R_f = 0.79 + \frac{362}{\varphi^3} \tag{5a}$$

for H between 0 and 3 ft, and

$$R_f = 0.79 + \frac{362}{\varphi^3} + 0.02(H-3) \tag{5b}$$

for H greater than 3 ft, in which $\varphi$ is the horizontal inside breadth of the pipe and $R_f$ should never be greater than 1.0.

Suggested empirical impact factors for flexible or unpaved roadways are indicated in Table 1.

The equation for $C_L$ was programed for computer analysis and values were determined for two trucks with axle width of 6.0 ft, passing within 3.0 ft of each other. The results are given in Table 2. The coefficients shown are based on the combined effect of all four wheel loads.

## LOAD FACTORS FOR ELLIPTICAL PIPE

Sewer and culvert pipe are commonly designed for strength in a 3-edge bearing test (AASHO-T33). For the same total load, this concentrated load test imposes more severe stresses in the pipe than would be induced by distributed trench or embankment loadings. For any particular combination of pipe size, wall thickness, reinforcing steel and field installation conditions, it would be possible to make comparative stress analyses to determine the test load comparable to the field load. The analysis of the indeterminate structure of reinforced concrete cross-section would, however, present a laborious task of design. To simplify design, the concept of a generalized load factor applicable to all sizes of pipe has been used for many years. The ratio of the strength of the pipe under a specified condition of field loading to its three-edge bearing test strength is called the load factor. If the load factors are known for the various field conditions, the designer is able to relate the calculated field load to a three-edge bearing test strength.

For a reinforced concrete elliptical pipe a factor of safety of 1 based on the minimum 0.01-in. crack test strength is suggested, in keeping with many years practice with circular reinforced concrete pipe. Cracks of this width are noninjurious, and small cracks in concrete will heal autogenously. Proof-of-design testing of reinforced concrete pipe requires conservatively proportioned reinforcing steel to insure that all pipe will be capable of meeting specified test strengths, thus providing, for the great majority of pipe, a margin on cracking in the field. Safety against ultimate failure is provided by the specified margin between 0.01-in. crack and ultimate strength in the 3-edge bearing test. This margin will be even greater in the field, due to the reduced shear stresses, and the build-up of supporting passive earth pressure at the sides of the pipe if the pipe were overloaded to failure in the field installation. The required 3-edge bearing strength is

TABLE 1

IMPACT FACTORS, $I_f$

| Cover, H (ft) | $I_f$ |
|---|---|
| 1.0 | 0.50 |
| 1.5 | 0.50 |
| 2.0 | 0.50 |
| 2.5 | 0.43 |
| 3.0 | 0.38 |
| 3.5 | 0.30 |
| 4.0 | 0.23 |
| 4.5 | 0.17 |
| 5.0 | 0.10 |
| 5.5 | 0.04 |
| 5.75 and above | 0 |

$$Q_{0.01} = \frac{W_d}{L_{fd}} + \frac{W_l}{L_{fll}} \tag{6}$$

in which

$Q_{0.01}$ = 3-edge bearing test load causing 0.01-in. crack, lb/lin ft;
$L_{fd}$ = load factor for earth loads;
$L_{fll}$ = load factor for live loads, 1.5 in all cases;
$W_d$ = earth load on pipe in trench or embankment, lb/lin ft; and
$W_l$ = superimposed surface live load on pipe, lb/lin ft.

TABLE 2

| Equivalent Round Pipe Size (in.) | Height of Earth Cover (ft) | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| (a) HE pipe | | | | | | | | | | | | | | |
| 18 | 0.235 | 0.161 | 0.109 | 0.078 | 0.058 | 0.045 | 0.035 | 0.029 | 0.020 | 0.015 | 0.011 | 0.009 | 0.007 | 0.006 |
| 24 | 0.245 | 0.184 | 0.130 | 0.095 | 0.072 | 0.056 | 0.044 | 0.036 | 0.025 | 0.019 | 0.014 | 0.011 | 0.009 | 0.007 |
| 27 | 0.249 | 0.193 | 0.140 | 0.104 | 0.079 | 0.061 | 0.049 | 0.040 | 0.028 | 0.021 | 0.016 | 0.012 | 0.010 | 0.008 |
| 30 | 0.251 | 0.201 | 0.149 | 0.112 | 0.085 | 0.067 | 0.053 | 0.043 | 0.031 | 0.023 | 0.017 | 0.014 | 0.011 | 0.009 |
| 33 | 0.253 | 0.207 | 0.156 | 0.119 | 0.091 | 0.072 | 0.057 | 0.047 | 0.033 | 0.025 | 0.019 | 0.015 | 0.012 | 0.010 |
| 36 | 0.255 | 0.214 | 0.165 | 0.127 | 0.098 | 0.077 | 0.062 | 0.051 | 0.036 | 0.027 | 0.021 | 0.016 | 0.013 | 0.011 |
| 39 | 0.256 | 0.218 | 0.171 | 0.133 | 0.104 | 0.082 | 0.067 | 0.055 | 0.039 | 0.029 | 0.023 | 0.017 | 0.014 | 0.011 |
| 42 | 0.256 | 0.222 | 0.176 | 0.139 | 0.109 | 0.087 | 0.071 | 0.058 | 0.041 | 0.031 | 0.024 | 0.019 | 0.015 | 0.012 |
| 48 | 0.257 | 0.227 | 0.184 | 0.147 | 0.117 | 0.094 | 0.077 | 0.064 | 0.046 | 0.035 | 0.027 | 0.021 | 0.017 | 0.014 |
| 54 | 0.258 | 0.231 | 0.191 | 0.156 | 0.126 | 0.102 | 0.084 | 0.070 | 0.051 | 0.038 | 0.030 | 0.023 | 0.019 | 0.015 |
| 60 | 0.259 | 0.234 | 0.196 | 0.162 | 0.133 | 0.109 | 0.090 | 0.076 | 0.055 | 0.042 | 0.033 | 0.026 | 0.021 | 0.017 |
| 66 | 0.259 | 0.236 | 0.200 | 0.168 | 0.139 | 0.115 | 0.096 | 0.081 | 0.059 | 0.045 | 0.036 | 0.028 | 0.022 | 0.018 |
| 72 | 0.259 | 0.237 | 0.203 | 0.172 | 0.144 | 0.120 | 0.101 | 0.085 | 0.063 | 0.049 | 0.038 | 0.030 | 0.024 | 0.020 |
| 78 | 0.259 | 0.238 | 0.206 | 0.176 | 0.148 | 0.125 | 0.105 | 0.090 | 0.067 | 0.052 | 0.041 | 0.032 | 0.026 | 0.021 |
| 84 | 0.259 | 0.239 | 0.208 | 0.179 | 0.152 | 0.128 | 0.109 | 0.094 | 0.070 | 0.055 | 0.043 | 0.034 | 0.028 | 0.023 |
| 90 | 0.259 | 0.240 | 0.209 | 0.181 | 0.155 | 0.132 | 0.113 | 0.097 | 0.074 | 0.058 | 0.046 | 0.036 | 0.029 | 0.024 |
| 96 | 0.259 | 0.240 | 0.210 | 0.183 | 0.157 | 0.135 | 0.116 | 0.100 | 0.177 | 0.060 | 0.048 | 0.038 | 0.031 | 0.026 |
| 102 | 0.260 | 0.241 | 0.211 | 0.184 | 0.159 | 0.137 | 0.119 | 0.103 | 0.079 | 0.063 | 0.050 | 0.040 | 0.032 | 0.027 |
| 108 | 0.260 | 0.241 | 0.212 | 0.186 | 0.161 | 0.139 | 0.121 | 0.105 | 0.082 | 0.065 | 0.052 | 0.042 | 0.034 | 0.028 |
| (b) VE Pipe | | | | | | | | | | | | | | |
| 36 | 0.247 | 0.187 | 0.134 | 0.098 | 0.074 | 0.057 | 0.046 | 0.037 | 0.026 | 0.019 | 0.015 | 0.011 | 0.009 | 0.007 |
| 39 | 0.249 | 0.194 | 0.141 | 0.104 | 0.079 | 0.061 | 0.049 | 0.040 | 0.028 | 0.021 | 0.016 | 0.012 | 0.010 | 0.008 |
| 42 | 0.251 | 0.199 | 0.147 | 0.110 | 0.084 | 0.065 | 0.052 | 0.042 | 0.030 | 0.022 | 0.017 | 0.013 | 0.010 | 0.009 |
| 48 | 0.253 | 0.208 | 0.156 | 0.119 | 0.091 | 0.072 | 0.057 | 0.047 | 0.033 | 0.025 | 0.019 | 0.015 | 0.012 | 0.010 |
| 54 | 0.255 | 0.215 | 0.166 | 0.128 | 0.100 | 0.079 | 0.063 | 0.052 | 0.037 | 0.028 | 0.021 | 0.016 | 0.013 | 0.011 |
| 60 | 0.256 | 0.221 | 0.174 | 0.136 | 0.107 | 0.085 | 0.069 | 0.057 | 0.040 | 0.030 | 0.023 | 0.018 | 0.014 | 0.012 |
| 66 | 0.257 | 0.225 | 0.180 | 0.143 | 0.113 | 0.091 | 0.074 | 0.061 | 0.044 | 0.033 | 0.025 | 0.020 | 0.016 | 0.013 |
| 72 | 0.258 | 0.228 | 0.186 | 0.149 | 0.119 | 0.096 | 0.079 | 0.065 | 0.047 | 0.035 | 0.027 | 0.021 | 0.017 | 0.014 |
| 78 | 0.258 | 0.231 | 0.191 | 0.155 | 0.125 | 0.102 | 0.083 | 0.069 | 0.050 | 0.038 | 0.030 | 0.023 | 0.018 | 0.015 |
| 84 | 0.258 | 0.233 | 0.194 | 0.160 | 0.130 | 0.106 | 0.088 | 0.073 | 0.053 | 0.040 | 0.031 | 0.025 | 0.020 | 0.016 |
| 90 | 0.259 | 0.235 | 0.197 | 0.164 | 0.134 | 0.110 | 0.092 | 0.077 | 0.056 | 0.043 | 0.033 | 0.026 | 0.021 | 0.017 |
| 96 | 0.259 | 0.236 | 0.200 | 0.167 | 0.138 | 0.114 | 0.095 | 0.080 | 0.059 | 0.045 | 0.035 | 0.028 | 0.022 | 0.018 |
| 102 | 0.259 | 0.237 | 0.202 | 0.170 | 0.142 | 0.118 | 0.099 | 0.083 | 0.061 | 0.047 | 0.037 | 0.029 | 0.023 | 0.019 |
| 108 | 0.259 | 0.238 | 0.204 | 0.173 | 0.145 | 0.121 | 0.102 | 0.086 | 0.064 | 0.049 | 0.039 | 0.030 | 0.024 | 0.020 |

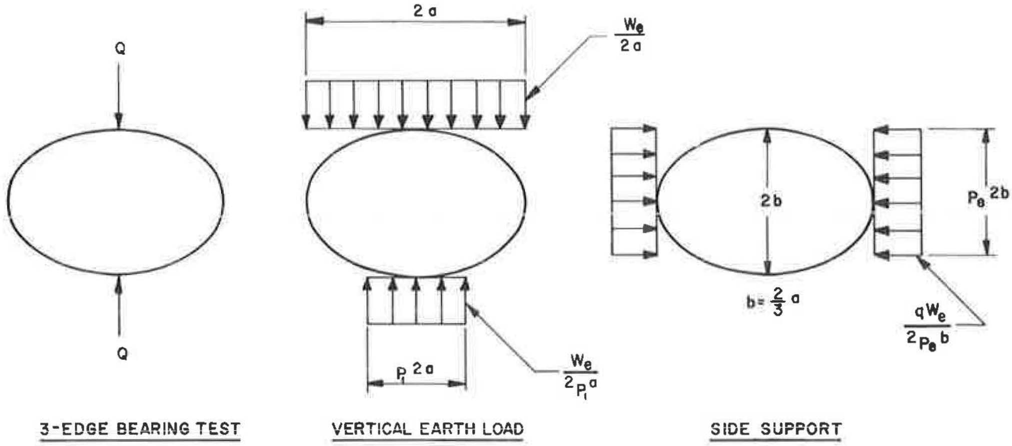



Figure 4. Load conditions for HE pipe.

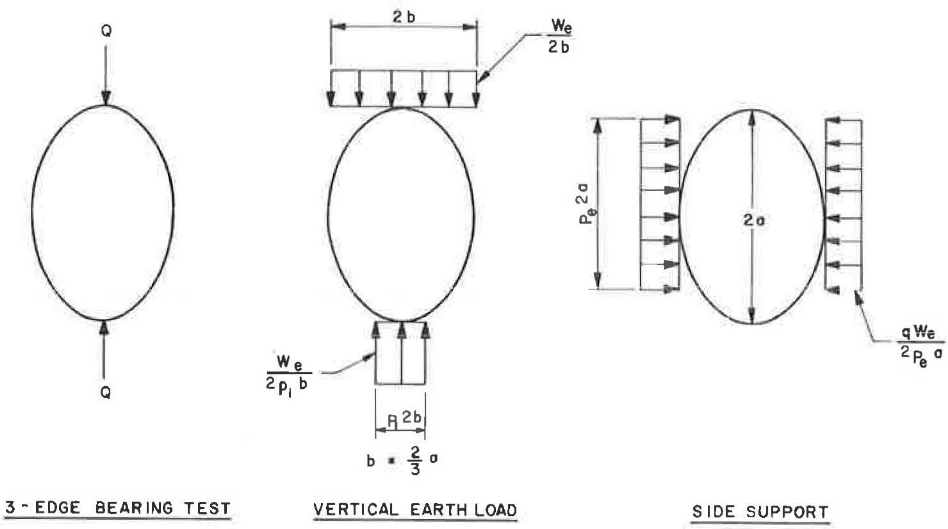



Figure 5. Load conditions for VE pipe.

## GENERAL METHOD OF DERIVATION OF LOAD FACTORS

Load factors for any given conditions of field installation may be found by comparing the critical tensile stresses developed in the field and those induced in a 3-edge bearing test. The load factor is the ratio of the field load to test load producing the same critical stress.

The stress analysis of elliptical pipe follows the general methods used by Spangler (7) in deriving load factors for circular rigid conduits. The pipe is considered an elastic beam of constant and homogeneous cross-section, indeterminate to the third degree, from which by conventional moment-area principles, the redundant moment, thrust and shear forces may be determined. Table 3 indicates the resulting coefficients for moment, thrust and shear for the various test and field loading conditions shown in Figures 4 and 5.

TABLE 3

MOMENT, THRUST AND SHEAR COEFFICIENTS FOR PIPE

| Loading Condition | | At Invert | | | At Top | | | At Springlines | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $M_A$ | $P_A$ | $V_A$ | $M_B$ | $P_B$ | $V_B$ | $M_S$ | $P_S$ | $V_S$ |
| (a) HE Pipe | | | | | | | | | | |
| 3-edge bearing test | | 0.297 | 0 | 0.500 | 0.297 | 0 | -0.500 | -0.203 | -0.500 | 0 |
| Vertical earth load $p_1$ = | 0.707 | 0.1414 | -0.0410 | 0 | 0.1229 | 0.0410 | 0 | -0.1545 | -0.500 | -0.0410 |
| | 0.500 | 0.1717 | -0.0621 | 0 | 0.1296 | 0.0621 | 0 | -0.1619 | -0.500 | -0.0621 |
| Side support $p_e$ = | 0.9 | -0.0786q | -0.4469q | 0 | -0.0826q | -0.5531q | 0 | 0.1009q | 0 | -0.0025q |
| | 0.7 | -0.0709q | -0.3320q | 0 | -0.0946q | -0.6680q | 0 | 0.1123q | 0 | -0.0460q |
| | 0.5 | -0.0523q | -0.2158q | 0 | -0.0976q | -0.7842q | 0 | 0.0916q | 0 | -0.2158q |
| | 0.3 | -0.0288q | -0.1079q | 0 | -0.0848q | -0.8921q | 0 | 0.0432q | 0 | -0.1079q |
| (b) VE Pipe | | | | | | | | | | |
| 3-edge bearing test | | 0.227 | 0 | 0.500 | 0.227 | 0 | -0.500 | -0.106 | -0.500 | 0 |
| Vertical earth load $p_1$ = | 0.707 | 0.1152 | -0.0166 | 0 | 0.0996 | 0.0166 | 0 | -0.0835 | -0.500 | -0.0166 |
| | 0.500 | 0.1376 | -0.0250 | 0 | 0.1043 | 0.0250 | 0 | -0.0874 | -0.500 | -0.0250 |
| Side support $p_e$ = | 0.9 | -0.1446q | -0.4466q | 0 | -0.1514q | -0.5534q | 0 | 0.1240q | 0 | -0.0022q |
| | 0.7 | -0.1311q | -0.3281q | 0 | -0.1749q | -0.6719q | 0 | 0.1398q | 0 | -0.0421q |
| | 0.5 | -0.0946q | -0.2074q | 0 | -0.1799q | -0.7926q | 0 | 0.1127q | 0 | -0.2074q |
| | 0.3 | -0.0496q | -0.0989q | 0 | -0.1518q | -0.9011q | 0 | 0.0493q | 0 | -0.0989q |

Note: From the tabulated coefficients, C, above, the bending moments, thrusts and shears may be found from the following relationships:

$M = C\,a\,W$
$P = C\,W$
$V = C\,W$

in which M = bending moment at section, in.-lb/lin ft,
P = thrust at section, lb/lin ft,
V = shear at section, lb/lin ft,
W = total vertical load on pipe, lb/lin ft,
= Q for 3-edge bearing test,
= $W_e$ for vertical earth load and side support, and
a = half of major axis of pipe (to ℄ of wall), ft.

$q = \frac{pK}{C_c}\left(\frac{H}{B_c} + \frac{p}{2}\right)$ in which q = active lateral pressure as proportion of vertical load,
p = projection ratio ($p = 0.69p_e$ for HE and $1.45\ p_e$ for VE),
K = Rankine ratio (use 0.333),
$C_c$ = embankment load coefficient,
H = height of earth cover over pipe, ft, and
$B_c$ = outside horizontal width of pipe, ft.

Wall sections of maximum bending moment may then be analyzed for stresses, and from a ratio of test to field stresses the general expression for embankment load coefficients, corresponding to Spangler's notation is

$$L_f = \frac{A}{N - xq} \tag{7}$$

in which

$L_f$ = embankment load factor,
A = factor corresponding to pipe shape,
N = factor corresponding to distribution of vertical load and vertical reaction,
x = factor corresponding to the area of vertical projection of the pipe in which active lateral pressure acts, and
q = ratio of the total active lateral pressure to the total vertical load.

q may be found from

$$q = \frac{pK}{C_c}\left(\frac{H}{B_c} + \frac{p}{2}\right) \tag{8}$$

K and $C_c$ are defined in Figure 3, and p, H, and B are defined in Figure 6.

The resulting factors to be used in the embankment load factor equation are given in Table 4. Embankment installation conditions corresponding to Class B and Class C bedding are shown in Figure 6.

In trenches, side support is disregarded. The parameter x in Eq. 7 is therefore zero, and the load factor becomes a constant for each class of bedding. The theoretical determination of trench load factors, using this equation, then yields for Class C (ordinary) bedding, 1.75 and 1.66 for HE pipe and VE pipe, respectively, and for Class B (first class) bedding, 2.12 and 1.98 for HE pipe and VE pipe, respectively.

However, because of the correspondence of these theoretical factors to trench load factors as determined by test at Iowa State University for circular pipe, it is suggested

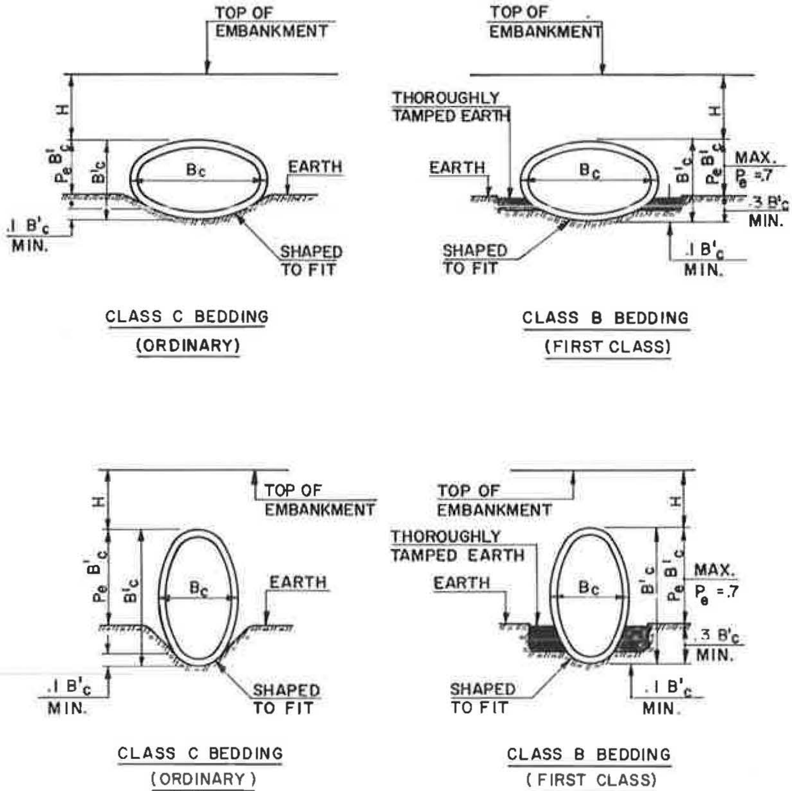



Figure 6. Installation of HE and VE pipe in embankments.

that the test values used be for Class C (ordinary) bedding—1.5, and for Class B (first class) bedding—1.9.

Figure 7 shows details of trench installations corresponding to Class B and Class C bedding.

## LOAD FACTORS FOR SUPERIMPOSED SURFACE (LIVE) LOADS

The load transmitted to a buried pipe by a concentrated load at the ground surface is considered to act as a uniformly distributed load over a selected effective length of pipe. In actuality the intensity of vertical pressure is highest at the crown of the pipe. This has little effect on the load factor, however, because maximum field moments occur at the invert where the bearing pressure does remain uniform, thus maintaining approximately the same maximum bending moment as occurs under vertical earth load. In deriving load factors for circular pipe, Spangler (7) found a very narrow range of load factor for superimposed loads varying from 1.51 for 1 ft of fill to a maximum of 1.70 for relatively high fills.

TABLE 4

LOAD FACTOR COEFFICIENTS

| Type Pipe | A | N: Bedding Class | N: Factor | X: Projection Ratio $p_e$ | X: Projection Ratio p | X: Factor |
|---|---|---|---|---|---|---|
| | | B | | 0.9 | 0.62 | 0.421 |
| | | (first class) | 0.630 | 0.7 | 0.48 | 0.309 |
| HE | 1.337 | | | 0.5 | 0.34 | 0.268 |
| | | C | | 0.3 | 0.21 | 0.146 |
| | | (ordinary) | 0.763 | 0 | 0 | 0 |
| | | B | | 0.9 | 1.30 | 0.718 |
| | | (first class) | 0.516 | 0.7 | 1.01 | 0.639 |
| VE | 1.021 | | | 0.5 | 0.72 | 0.457 |
| | | C | | 0.3 | 0.43 | 0.238 |
| | | (ordinary) | 0.615 | 0 | 0 | 0 |

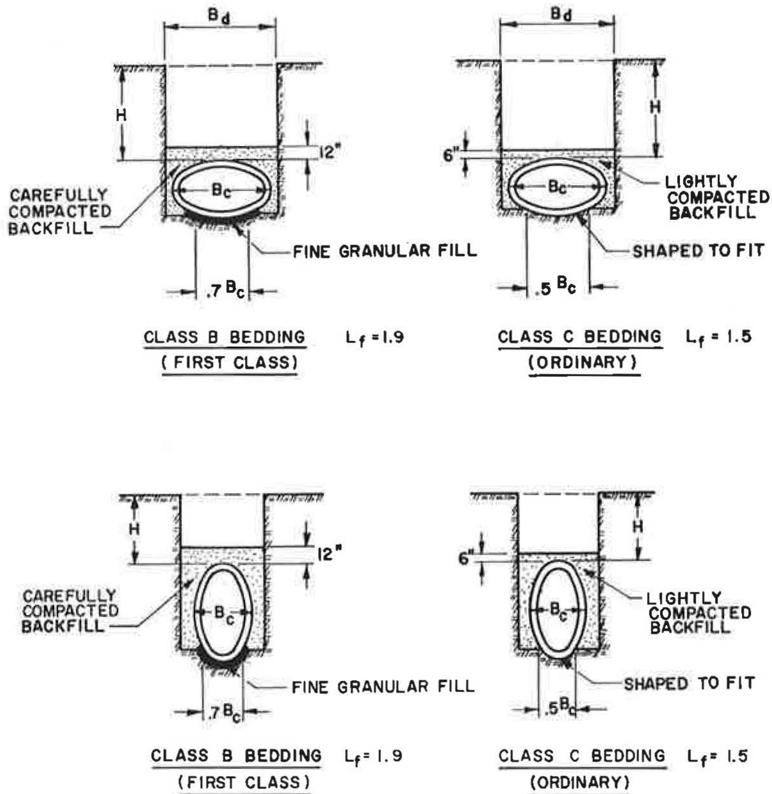



Figure 7. Installation of HE and VE pipe in trenches.

The load factor did not increase with higher class of bedding because the increased bending moment at the top compensated for the reduction in moment at the invert. Embankment installation likewise had no effect, because surface loads do not induce appreciable accompanying increases in side support. Accordingly, Spangler recommended a constant value of 1.5 for all superimposed surface loads.

In view of the correlation of trench load factors for HE, VE and circular pipe, the identical constant load factor of 1.5 for surface loads on each of these types of pipe appears reasonable.

## CONCRETE CRADLE LOAD FACTORS

Concrete cradle load factors for round pipe have been determined principally by test, several series of experiments having been conducted at Iowa State and Ohio State Universities (8, 9). Apparently, the mechanism of failure is associated with "springing" of the pipe from the cradle. For this reason, perhaps such load factors should be determined by test rather than by stress analysis. This suggests a line of future research.

## DETERMINATION OF STEEL REINFORCEMENT

Once the designer has, through the use of the proper load factor, determined the required 3-edge bearing test D-load to meet a specific field installation, he must then select the proper amount of steel reinforcement such that the pipe will have the strength to meet the design D-load. Standard D-load classifications and designs are specified in ASTM C507. The 3-edge bearing test loads in pounds per linear foot of pipe are found by multiplying those D-loads by the inside horizontal span of the pipe, in feet. Several examples of the use of load factors are given in the Appendix.

## CONCLUSION

The use of load factors in design is a simple method of avoiding a laborious stress analysis for each installation. Having once performed the computation and expressed the results in terms of load factors, the exercise need not be performed again. By a proof-of-design test, the designer is assured that the pipe will be capable of sustaining the stresses in the field that he has determined implicitly through the use of load factors.

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# *Appendix*

Examples

No. 1: A 48-in. equivalent round VE culvert is to be placed in a trench. The cover over the top of the pipe is 15 ft. The soil is sand and gravel and weighs 110 pcf. The bedding is considered ordinary. What are the required D-load strength and pipe class?

Step 1. Compute earth load from

$$W_t = C_d\, w\,(B_d)^2 \tag{8a}$$

if

$w = 110$ pcf,
$B_d = B_c + 2 = 4.11 + 2 = 6.11$ ft,
$H = 15$ ft,
$H/B_d = 15/6.11 = 2.455$,
$C_d = 1.6$, and
$K_\mu = 0.165$ (Fig. 2).

This yields

$$W_t = 1.6 \times 110 \times (6.11)^2 = 6{,}570 \text{ lb/lin ft} \tag{8b}$$

Step 2. Compute required 3-edge bearing strength from

$$Q_{0.01} = W_d/L_{fd} \tag{9a}$$

in which

$W_d = W_t$, and
$L_{fd} = 1.5$ (ordinary bedding)

which yields

$$Q_{0.01} = 6,570/1.5 = 4,380 \text{ lb/lin ft} \tag{9b}$$

Step 3. Compute D-load and class from

$$\text{D-load} = Q_{0.01}/\text{horiz. inside span, ft} = 4,380/3.19 = 1375\text{D} \tag{10}$$

Therefore, the pipe used must be Class VE IV (ASTM C507).

No. 2: A 96-in. equivalent round HE pipe is to be placed in a roadway embankment. The cover over the top of the pipe will be 4 ft. Live load will be equivalent to H-20 truck loading on a flexible pavement. The culvert will have first class bedding in ordinary soil and will be projecting 0.7 of its vertical height above the natural ground; soil weight is 120 pcf.

Step 1. Compute embankment earth load from

$$W_e = C_c w B_c^2 \tag{11a}$$

if

$$\begin{aligned} w &= 120 \text{ pcf}, \\ B_c &= 11.63 \text{ ft}, \\ H &= 4 \text{ ft}, \\ H/B_c &= 4/11.63 = 0.344, \\ p_e &= 0.7, \\ p &= 0.69 p_e, \\ p &= 0.69 \times 0.7 = 0.483, \\ r_{sd} &= 0.8, \text{ and} \\ C_c &= 0.4 \text{ (Fig. 3)}. \end{aligned}$$

This yields

$$W = 0.4 \times 120 \times (11.63)^2 = 6,500 \text{ lb/lin ft} \tag{11b}$$

Step 2. Compute live load from

$$W_l = C_L P (1.0 + I_f) \tag{12a}$$

in which

$$\begin{aligned} C_L &= 0.183 \text{ (Table 2)}, \\ P &= 16,000 \text{ lb (rear wheel, H-20 truck), and} \\ I_f &= 0.23 \text{ (Table 1)}. \end{aligned}$$

Therefore

$$W_l = 0.183 \times 16,000 \times 1.23 = 3,600 \text{ lb/lin ft} \tag{12b}$$

Step 3. Compute required 3-edge bearing strength from

$$Q_{0.01} = \frac{W_d}{L_{fd}} + \frac{W_l}{L_{fll}} \tag{13}$$

in which

$$\begin{aligned} W_d &= W_e, \\ L_{fll} &= 1.5, \\ Q_{0.01} &= \frac{6,500}{2.46} + \frac{3,600}{1.5} = 5,040 \text{ lb/lin ft, and} \\ L_{fd} &= \frac{A}{N - xq}. \end{aligned}$$

If

$$\begin{aligned} A &= 1.337 \text{ (Table 5)}, \\ N &= 0.630 \text{ (Table 5)}, \\ x &= 0.369 \text{ (Table 5), and} \\ q &= \frac{pK}{C_c}\left(\frac{H}{B_c} + \frac{p}{2}\right) = \frac{0.483 \times 0.333}{0.4}(0.344 + 0.242) = 0.236 \end{aligned}$$

then

$$L_{fd} = \frac{1.337}{0.63 - 0.369 \times 0.236} = 2.46.$$

Step 4. Compute D-load and Class from

$$\text{D-load} = Q_{0.01}/\text{horiz. inside span in ft} = 5{,}040/10.05 = 501\text{D} \qquad (14)$$

Therefore, the pipe used must be Class HE-A (ASTM C507).