An Extension of Rigid Pavement Design Methods

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This paper verifies and extends certain developments in the AASHO Interim Pavement Design Guide. A choice of mathematical models is made based on studies of the AASHO and Maryland Road Tests' stress data as well as data from in-service pavements.

As an extension of the initial work, the design thickness equation is expanded to include the concrete modulus of elasticity, total traffic, and pavement continuity (jointed or continuous). A nomograph is presented that allows a quick solution to the expanded equation.

A new design chart is presented for design of the reinforcing steel in jointed reinforced pavements. In addition, a nomograph for solving bar spacing and bar size is included.

•IN 1920, A. I. Goldbeck and Clifford Older independently developed formulas for approximating the stresses in concrete pavements. The best known of these formulas is generally called the "corner formula" and was the basis for rigid pavement design for many years. Results of the Bates Road Test in 1922-23 appeared to confirm the original corner formula. In 1926, H. M. Westergaard completed his treatise on the analysis of stresses in concrete pavements (1). It was concerned with the determination of maximum stresses in slabs of uniform thickness for three load conditions under several limiting assumptions (2). The Westergaard equation for corner stresses has become the definitive design equation for portland cement concrete pavements. In this equation, Westergaard includes the following variables:

P = wheel load, in lb;
h = the thickness of the concrete slab, in in.;
μ = Poisson's ratio for concrete;
E = Young's modulus of elasticity for the concrete in psi;
k = subgrade modulus in pci; and
a = radius of area of load contact, in in.

Using this same general equation form, slightly different design equations have been developed by Spangler (3), Kelly (4), and Pickett (5). These equations are empirical or semi-empirical, but all retain the basic form of the Westergaard simplified theory.

All of these design equations are based on static loading. This is necessary because very little theory exists to describe time dependent variables such as dynamic loads.

Road Test Results Used in Design

Three large-scale road tests have been conducted involving portland cement concrete pavement—the Bates Road Test, 1922; the Maryland Road Test, 1950; and the AASHO Road Test, 1958-61. All three have added valuable information to our knowledge of concrete pavement performance. Only the AASHO Road Test, however, was large

Paper sponsored by Committee on Rigid Pavement Design.

enough to provide us with adequate information on which to base dynamic design equations. The first objective of the Road Test as outlined by the Advisory Committee (6) was:

> To determine the significant relationship between the number of repetitions of specified axle loads of different magnitude and arrangement and the performance of different thicknesses and uniformly designed and constructed asphaltic concrete, plain portland cement concrete, and reinforced portland concrete surfaces. ....

In addition to basic performance data, the AASHO Road Test also provided an opportunity to measure strains in concrete pavements under dynamic loads, and thus provide a mechanistic tie from these pavements to future designs.

Development of AASHO Design Guide

The AASHO Operating Committee on Design appointed a working subcommittee on pavement design. The job of the subcommittee was to adapt the data from the AASHO Road Test to use in design procedures for asphaltic concrete pavements and portland cement concrete pavements.

It was the unanimous opinion of the subcommittee that there are substantial factors to be considered in a design procedure that are not available as variables in the AASHO Road Test results. Four of these factors are (a) the length of test time relative to the normal life of the pavement being designed; (b) climatic and geologic differences between the conditions at the Road Test site and other geographic regions; (c) the need for a guide in designing pavement types not included in the Road Test, such as continuously-reinforced portland cement concrete pavements; and (d) expansion of the Road Test results to various other materials such as low-modulus concrete and stabilized bases.

It was decided that the AASHO Road Test performance equations should form the basis for the AASHO Rigid Pavement Interim Design Guide to add these additional factors.

The Interim Design Guide was developed as a guide for use in developing more exact design procedures. The committee was very deliberate in its efforts to provide for future improvements in the work as additional information became available. The guide (7) states that:

> The above design equations are based on fixed values for certain elements that are obviously important in the design of rigid pavement. These elements include thickness and quality of subbase environmental effects, variations in the amount of load transfer at transverse joints, and the effects of joint elimination through continuous reinforcement. It is expected the design equations will be further modified in the future as experience is gained and these elements are evaluated.

PRESENTATION OF GUIDE

Scope

It is felt that a detailed list of parameters should be incorporated into a rigid pavement structure analysis. The Rigid Guide presents a procedure that encompasses most of these parameters and allows the engineer to design the pavement structure from the subgrade up. Basically the Guide separates the design into four phases—slab dimensions, reinforcement, joints, and slab support control. The first two phases are handled by formulas and will be discussed; the latter two are not discussed.

Slab Dimensions.—The Guide's approach to pavement structure design is a combination of theoretical and empirical relations. The design parameters covered by the various theoretical analyses previously discussed are loading factor magnitude and tire pressure; support media strength; concrete properties—strength, modulus of elasticity, Poisson's ratio; and continuity (load transfer). Whereas, the final equation for the rigid pavement research phase of the AASHO Road Test encompassed the load application factor as well as the following parameters: loading factor magnitude, repetitions, and axle type. In this case, the concrete properties, subgrade support and other design factors were fixed parameters and their effect cannot be evaluated by the AASHO Road Test equation.

The AASHO Subcommittee for Rigid Pavement Design combined the two approaches into one equation. The parameters encompassed by the combined methods are loading factor magnitude, repetitions, tire pressure, and axle type; support media strength; concrete properties' strength, modulus of elasticity, and Poisson's ratio; continuity (load transfer); support media friction; and regional factors, i.e., weather, temperature, etc.

Reinforcement.—Steel reinforcement is placed in the slab for the purpose of holding any cracks that form in the pavement tightly closed, enabling the pavement to perform as an integral structural unit. The Guide covers the design of two basic types of reinforced concrete pavement, i.e., jointed-reinforced and continuously-reinforced. Each requires an individual procedure.

The reinforcement for the jointed concrete pavement is determined by the application of the conventional "subgrade drag theory." In essence, the formula is based on the principle of balancing the slab's resistance to movement against the tensile strength of the steel.

The design method for continuously-reinforced concrete pavement is based on the concept of balancing the internal concrete stresses developed by temperature and shrinkage against the tensile strength of steel (10).

Development of Thickness Equation

Two general approaches were open for use in the Guide to combine the Road Test equation and theory, (a) use of theoretical formuli as the basic design form modified by the load term in the final answer for repetitions, and (b) use of the Road Test equation as a valid basis adding modifications from theory for variations in physical constants.

The second approach was selected as the more valid because it depends on the Road Test results for its starting point and uses theory for determining variations in the basic equation. Also at the Road Test, failure was not defined as cracking (overstress), but as a specific reduction in serviceability that usually did not occur until after initial cracking.

After a cursory examination of the available information, the Spangler equation was selected for use in the design equation because of its simplicity and because it showed a good correlation with Road Test measurements. It was stated in the Guide that, "one point of merit in this approach is that if a better stress equation is found, it can be incorporated into the design method with very little revision. ...."

After selecting the Spangler equation for modifying concrete properties, there were two possible choices for inserting it into the general AASHO equation, (a) obtaining a ratio of the selected concrete properties to those at the AASHO Road Test and making it an additive term to the AASHO equation, or (b) modifying the term in general equation to include various concrete properties. The committee selected the first alternative and derived the following equations:

When the terminal serviceability index (p) = 2.0:

$$\log W_t = 7.35 \log (D_2 + 1) + \frac{G_t}{\beta} - 0.06 + 3.58 \log \left[\frac{S_c' \left(D_2^{0.75} - 1.132\right)}{690 \left(D_2^{0.75} - \dfrac{18.416}{Z^{0.25}}\right)} \right] \quad (1)$$

When the terminal serviceability index (p) = 2.5:

$$\log W_t = 7.35 \log (D_2 + 1) + \frac{G_t}{\beta'} - 0.06 + 3.42 \log \left[\frac{S_c (D_2^{0.75} - 1.132)}{690 \left(D_2^{0.75} - \frac{18.416}{Z^{0.25}} \right)} \right] \quad (2)$$

Discussion of Design Charts

Design nomographs that solve for the thickness of jointed-concrete pavement and the reinforcement requirements for both jointed and continuous-concrete pavement are presented in the Guide.

Thickness.—In deriving the nomograph for pavement thickness, the AASHO Road Test values for the modulus of elasticity and load transfer characteristics were fixed to solve the equation. This eliminates these factors as variables, hence the chart has variable scales only for traffic, working stress, and subgrade support. The chart, therefore, is not applicable to continuous concrete pavements or low modulus concrete pavements. Furthermore, the traffic scale is in terms of equivalent daily 18-kip single-axle load applications for a 20-yr traffic analysis. The daily traffic approach is restricting because the analysis is for fixed time period, and is difficult to use for other time periods or for evaluating the life of an existing pavement structure.

Reinforcement.—The chart solution for reinforcement in jointed pavements is in graphic rather than nomographic form. The graphic solution has variable scales for pavement thickness, slab length, and working stress; but the graph is limited to the solution for a fixed friction factor.

The chart solution for reinforcement in continuously-reinforced concrete pavement is flexible in that all the parameters involved in the design equation are included as variables on the nomograph.

DEVELOPMENT OF NEW EQUATION

The design equation developed for the AASHO Design Guide was a first attempt to utilize the AASHO Road Test data in pavement design. The equation is cumbersome and several assumptions were made early in its development (7). Other refinements were omitted from the equation that would make it a more useable formula under actual conditions.

The purposes of this investigation are to (a) simplify the design equation if possible, (b) investigate and clarify several of the assumptions made in the early development, and (c) include any additional refinements in the equation that can be developed from present data.

The equation developed herein has the following variations from the original equation: (a) the Road Test stress data are used to verify the selection of a theoretical model, (b) traffic is used as the total expected number of equivalent 18-kip application (Σ L) over the life of the pavement (design period), (c) the term for pavement continuity is evaluated and extended to continuously reinforced pavements, and (d) the use of terms for both modulus of elasticity and subgrade modulus is encouraged.

Model Selection

In order to select a model for combining theory with Road Test performance data, the Road Test strain data (2) were compared with various modifications of the Westergaard theory. The table in Figure 1 gives the equations that were examined and the correlation obtained. It can be seen that Spangler's equation fits the data as well as any of the more complicated equations.

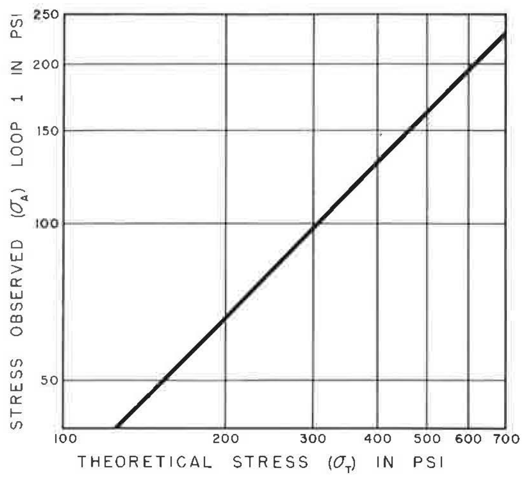
It should be noted that the data fitting does not support nor deny the theoretical formulation of ℓ (radius of relative stiffness), because none of the factors involved in the radius of relative stiffness, because none of the factors involved in the radius of relative stiffness, i.e., E, k, or μ, were varied at the Road Test in a manner allowing proper analysis.

After considering the fit of the data, the Spangler equation was selected. Figure 1 shows the correlation between Spangler, Westergaard, Pickett, and the Road Test stresses as calculated from corner load strains, Loop 1, AASHO Road Test (2). The following equation was selected as a result of the correlation.

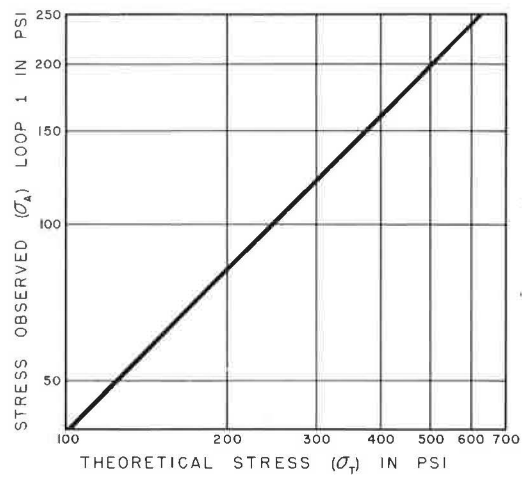
$$\log \sigma_{18} = 1.010 \log \sigma_{sp} - 0.521 \tag{3}$$

in which:

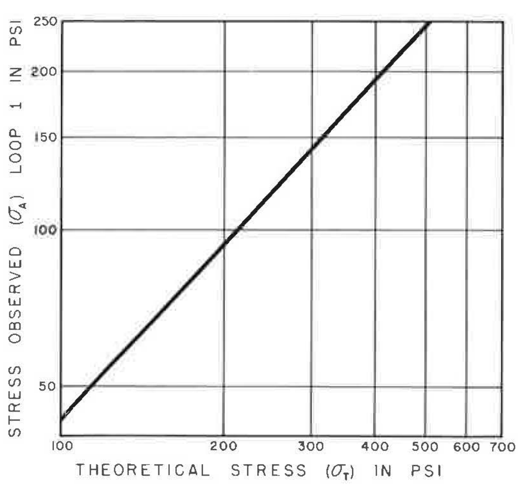
σ_{18} = stress calculated from strains measured under an 18-kip single-axle vibratory load on Loop 1, AASHO Road Test, psi.



PREDICTED BY SPANGLER'S CORNER EQUATION



PREDICTED BY PICKETT'S CORNER EQUATION



PREDICTED BY WESTERGAARD'S CORNER EQUATION

EQUATION FOR PREDICTING CORNER STRESS OBSERVED AT THE AASHO ROAD TEST IN TERMS OF A THEORETICAL STRESS:

$$\sigma_A = A\sigma_T^{\,B}$$

WHERE:
σ_A = STRESSES OBSERVED LOOP 1 - AASHO ROAD TEST
σ_T = THEORETICAL STRESSES
A & B = CORRELATION CONSTANTS
r^2 = CORRELATION COEFFICIENT

	A	B	r^2
SPANGLER'S EQUATION	.301	1.010	0.999
PICKETT'S EQUATION	.389	1.006	0.999
WESTERGAARD'S CORNER EQUATION	.309	1.078	0.999

Figure 1. Theoretical stresses compared with observed stresses on Loop 1, AASHO Road Test.

σ_{sp} = stress predicted by the Spangler equation for a 9,000-lb wheel load (18-kip single-axle), psi.

Modifying the Road Test Equation

A study by W. R. Hudson and F. Scrivner (8) showed excellent correlation between observed stresses at the Road Test, slab thickness, and log W, i.e., the number of load applications carried. To extend the study and obtain a correlation of the form needed in this work, a correlation of the term (D + 1) with observed corner load stresses on the Road Test Loop 1 was developed (Fig. 2). The resulting equation (Eq. 4) has a coefficient of determination (r^2) of 0.999.

$$\log (D + 1) = 1.995 - 0.517 \log \sigma_{18} \qquad (4)$$

Substituting Eq. 3 into Eq. 4 gives:

$$\log (D + 1) = 1.995 - 0.517 (1.010 \log \sigma_{sp} - 0.521) = 2.264 - 0.522 \log \sigma_{sp} \qquad (5)$$

In a preliminary report (11) the Road Test equation is developed in terms of ΣL (accumulated equivalent 18-kip single-axle loads).

The equation becomes:

$$\log \Sigma L = 7.35 \log (D + 1) - 0.06 + \frac{G}{\beta'} \qquad (6)$$

in which

$$G = \frac{4.5 - P_t}{3.0}$$

$$\beta' = 1 + \frac{1.624 \times 10^7}{(D + 1)^{8.46}}$$

$$\log \rho = 7.35 \log (D + 1) - 0.06$$

P_t = serviceability at end of time, t.

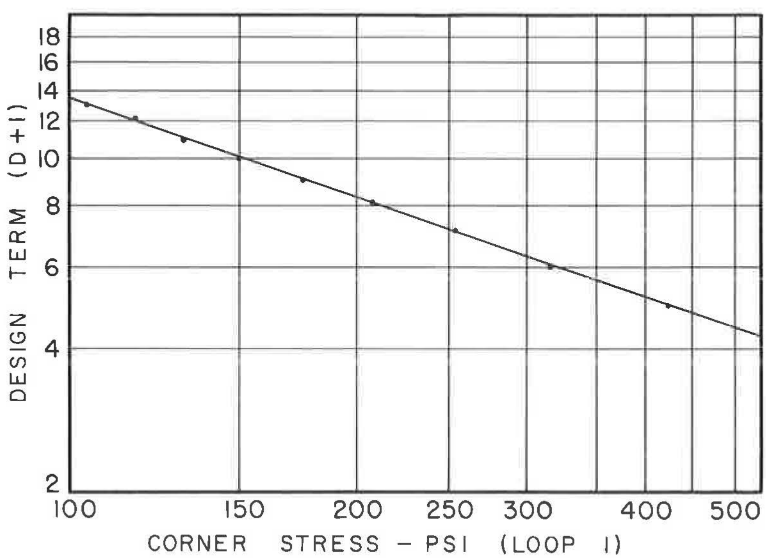


Figure 2. Correlating design term (D+1) with corner load stresses on Loop 1.

In this equation β' is a curvature parameter, and ρ is a design function as shown when the equation is in the form:

$$G = \beta' \left[\log \Sigma L - \log \rho\right] \tag{7}$$

This being the case, and because (D + 1) exerts a large influence on log ΣL through the ρ term and only a weak influence through the β term, it was decided to substitute σ for (D + 1) in the ρ term only. Therefore, substituting Eq. 5 into Eq. 6 gives:

$$\log \Sigma L = 7.35 \left[2.264 - 0.522 \log \sigma_{sp}\right] - 0.06 + \frac{G}{\beta} \tag{8}$$

This equation obtains for pavements of a fixed strength, S_c, (28 day) for AASHO Road Test pavements was constant at 690 psi ± random variations. Previous design equations have relied on the σ/S_c ratio as the measure of adequate design. Work done for the AASHO Interim Rigid Pavement Design Guide related this ratio to pavement life in terms of log ΣL. This can be stated as follows:

> It can be assumed that log ΣL is a function of the σ/S_c ratio; and that when an increased σ is matched by an increased S so that the ratio $\sigma x/S_x$ remains equal to the ratio σ/S_c, no change in ΣL would result. Therefore, the rate of change of ΣL as S_c changes is inversely proportional to the rate of change of log ΣL as σ changes.

Inserting strength into Eq. 8 as such an inverse ratio with the fixed strength of the Road Test pavements (690 psi) the following is obtained:

$$\log \Sigma L = 7.35 \left[2.264 - 0.522 \log \left(\frac{\sigma_{sp}{}^{690}}{S_x}\right)\right] - 0.06 + \frac{G}{\beta} \tag{9}$$

The Spangler equation for stress has the form,

$$\sigma_{sp} = \frac{J\,P}{D^2}\left(1 - \frac{a_1}{\ell}\right) \tag{10}$$

Substituting the full Spangler equation, σ_{sp}, expanding and combining terms obtains:

$$\log \Sigma L = -9.483 - 3.837 \log \left(\frac{J}{S_x D^2}\left[1 - a_1/\ell\right]\right) + \frac{G}{\beta} \tag{11}$$

in which

$$\ell = \left[\frac{Z\,D^3}{12\,(1 - \mu^2)}\right]^{0.25}$$

In order to simplify the design equation and without damage to the theory, Poisson's ratio (μ) is fixed at a value of 0.20, resulting in a simplied form for the radius of relative stiffness: $\ell = (ZD^3/11.52)^{0.25}$. Taking $a_1 = a\sqrt{2}$ and substituting for ℓ and a_1, Eq. 11 becomes

$$\log \Sigma L = -9.483 - 3.837 \log \left(\frac{J}{S_x D^2}\left[1 - \frac{2.61a}{Z^{1/4} D^{3/4}}\right]\right) + \frac{G}{\beta} \tag{12}$$

in which

ΣL = number of accumulated equivalent 18-kip single-axle loads
J = a coefficient dependent on load transfer characteristics or slab continuity

S_X = modulus of rupture of concrete at 28 days (psi)
D = nominal thickness of concrete pavement (inches)
Z = E/k
E = modulus of elasticity for concrete (psi)
k = modulus of subgrade reaction (psi/inch)
a = radius of equivalent loaded area = 7.15 for Road Test 18-kip axles

$$G = \frac{P_O - P_t}{3} = \frac{4.5 - P_t}{3}$$

$$\beta = 1 + \frac{1.624 \times 10^7}{(D+1)^{8.46}}$$

At this point, a so-called life term must be inserted into the design equation. The life term will simply serve to modify the life of a pavement section as predicted by Road Test equation (a 2-yr test). Studies of existing pavements in Texas and Illinois, among others, have established this fact. A substitution of the Road Test values for parameters in Eq. 12 would reduce it back to the basic Road Test equation. Performance studies now being conducted in Texas have indicated that the logarithm of the predicted applications obtained by the Road Test equation must be reduced by a factor of 0.896. The AASHO Subcommittee on Rigid Pavement Design in effect reduced the logarithm of the predicted applications by a factor of 0.935 by using a safety factor (0.75 of the concrete strength for a working stress). Although the use of a safety factor to reduce the working stress is satisfactory, the use of a life term was adopted because future results of performance studies will undoubtedly provide a better estimate of the true factor and such values can be used to replace the trial value.

In determining the magnitude of the life factor both the Design Guide and the Texas performance studies were given equal consideration and an average factor of 0.9155 was selected.

Application of the life factor to the right side of Eq. 12 gives:

$$\log \Sigma L = -8.682 - 3.513 \log \left(\frac{J}{S_X D^2} \left[1 - \frac{2.61a}{Z^{1/4} D^{3/4}} \right] \right) + 0.915 \frac{G}{\beta} \qquad (13)$$

Only one term in Eq. 13 has not been evaluated adequately, the continuity or J term. The selection of a value, J, for design purposes must now be postulated on the basis of limited data. The J value for the jointed pavements on the Road Test is automatically fixed at the value of 3.2 that was used in all correlation work. For the present, this value shall be assumed to apply for all jointed-concrete pavements with adequate load transfer. A J value of 2.2 was selected for continuously-reinforced concrete pavements based on comparisons of previous design procedures and performance studies. This value also gives answers that are compatible with the recommendations in the AASHO Design Guide.

Graphical Solution

Using this equation, it is particularly hard to solve for concrete pavement thickness D. It is a very simple matter, however, to program this equation on a computer and solve for ΣL using all combinations of the other variables. The resulting output is useful in the form of tables. These tables can be combined graphically into a very useful nomograph (Fig. 3). The nomograph is for a final serviceability level of 2.5. Evaluation of terminal serviceability throughout the United States has shown that an acceptable level for the final or terminal condition of an Interstate pavement is 2.2 - 2.5. The Texas Highway Department has settled on 2.5 for use in design of such pavements. For design of lower class roads a terminal serviceability of 1.5 is felt to be satisfactory.

Use of the Nomograph.—The examples on the chart show how typical design problems may be handled. Certain information is normally fixed by the conditions at the site or by arbitrary choice.

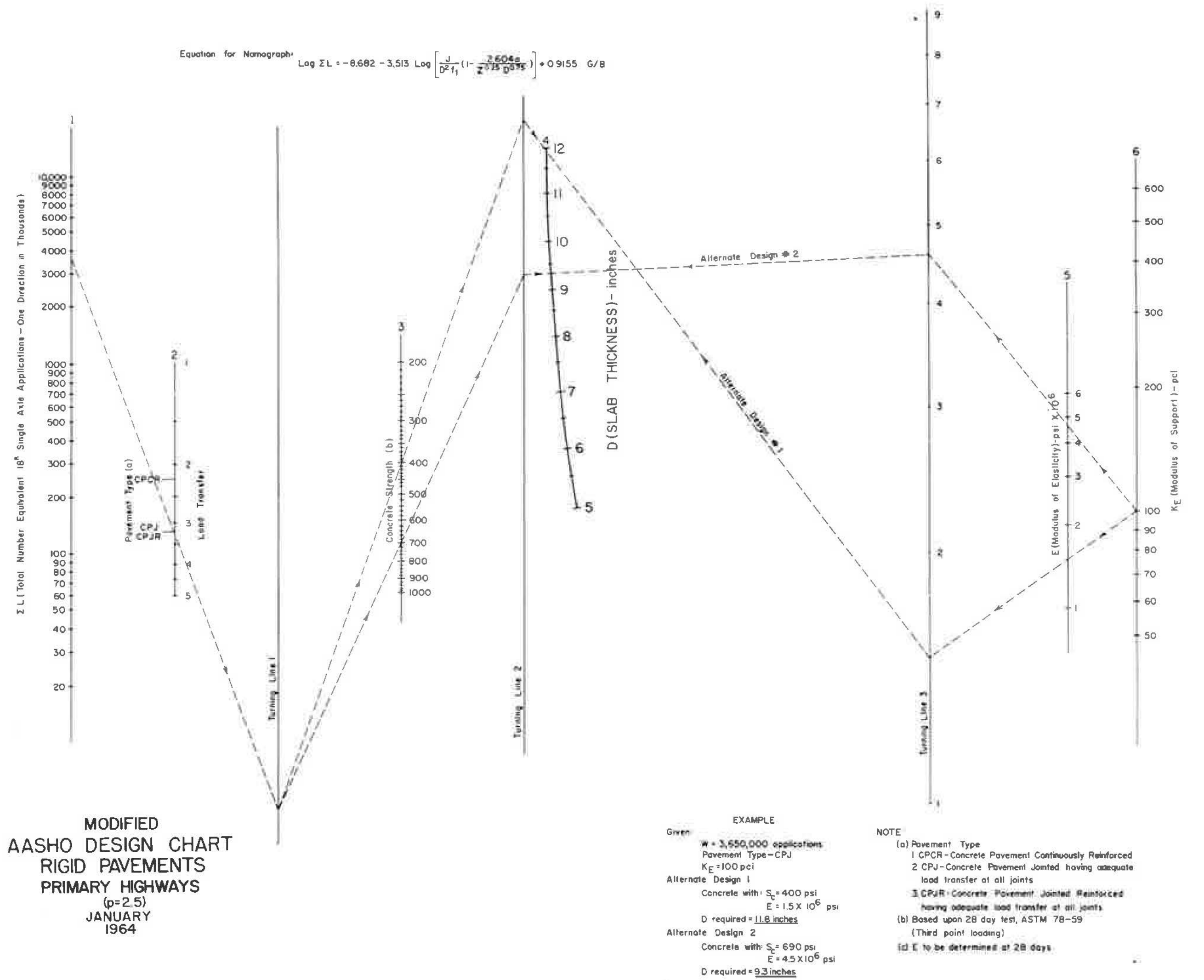


Figure 3. Rigid pavement design nomograph.

1. ΣL = 3,650,000 applications is an estimate of the traffic to be carried during the life of the proposed pavement. It should be established by statistical prediction procedures from study of past loadometer and traffic count data. Texas Highway Department methods may be found in (9).

2. k_E = 100 pci is established by the existing subgrade plus some evaluation of the improvement that will be gained by the subbase (12).

3. Pavement type, CPJ, jointed plain concrete pavement with load transfer at the joints. This factor may be chosen by the designers and varied for several different designs. Often, however, the choice is dictated by other existing factors as assumed for this example.

With these factors provided, it is appropriate to establish the value of each factor on its respective scale and proceed as follows: (a) mark ΣL on scale 1; (b) mark pavement type on scale 2; (c) mark k_E value on scale 6; (d) taking E = 1.5 in anticipation of using low-modulus shell concrete for the first trial, mark 1.5 on scale 5 as shown; (e) connect the points on scale 1 and scale 2 projecting the line to a point on turning line 1; (f) select a trial concrete strength (400 psi) and mark it on scale 3; (g) connect this point on scale 3 to the intersection on turning point 1 and extend it to a point on turning line 2; (h) transferring over the scales 5 and 6, connect the points on these scales and project to turning line 3; and (i) connect turning points 2 and 3 to establish the required thickness D = 11.8 in. on scale 4.

It may be desirable to check alternate designs. Another example is shown on the design chart, which using different concrete characteristics and following the same procedures yields D = 9.3 in.

MODIFICATION OF REINFORCEMENT DESIGN

Jointed-Reinforced Concrete Pavement

The friction factor was not included as a variable in the Design Guide nomograph for determining the reinforcement in jointed-concrete pavement. The nomograph was solved for a fixed friction factor of 1.5. This was an adequate premise during the period when sand cushion blankets were used between the pavement and the subbase, but the current trend toward crushed stone or stabilized subbase emphasizes the need for considering friction factor in design. Experiments performed by the Texas Highway Department have shown friction factors in excess of two, therefore, if the Guide's nomograph were used to design the reinforcement for a high friction subbase, an inadequate design would result.

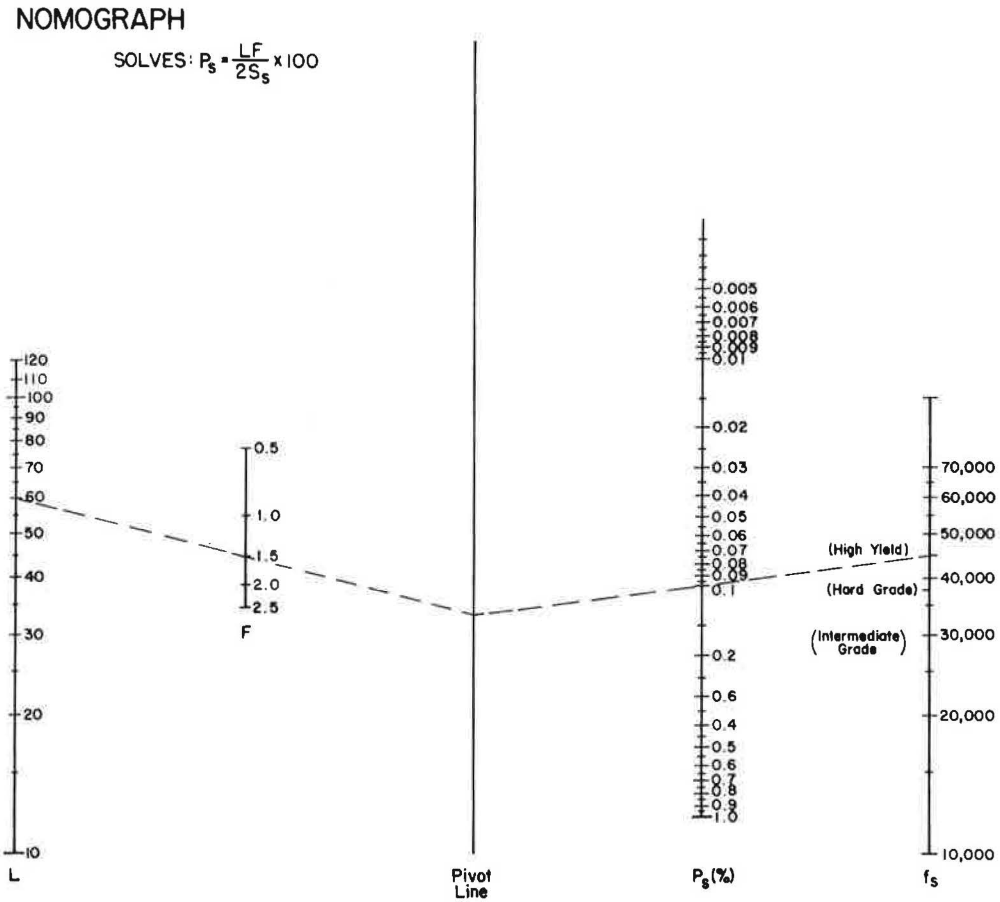
In addition to inserting friction factor into design, it is felt that the solution for the reinforcing requirements could best be expressed as a percentage in lieu of the current concept of using the area of steel per ft of slab width. The latter designation is satisfactory for estimating purposes, but is difficult to comprehend from a design standpoint. Furthermore, when the solution is expressed as a percentage, the values are comparable and compatible with the solutions obtained with continuously-reinforced concrete pavement. By simply changing the designation of several expressions in the Guide, the answer for "subgrade drag" would be in percentages as follows (7):

$$P_s = \frac{L\,F}{2\,f_s} \times 100 \qquad (14)$$

in which

P_s = required steel percentage, percent.
L = length of slab between joints, ft.
F = friction factor of subbase.
f_s = allowable working stress in steel, psi.

Figure 4 shows a nomograph for solving Eq. 14. Note the flexibility provided, in that the working stress can be varied between wide limits in addition to including friction factor as a variable. The inclusion of a complete scale for working stress in lieu of several fixed values allows the designer to apply any desired value.



Example Problem:

$L = 60$ ft
$F = 1.5$
$f_s = 45{,}000$ psi

Answer: $P_s = 0.2$ %

Where:

P_s = Required steel percentage — %.
L = Length of slab between joints — feet.
F = Friction factor of subbase.
f_s = Allowable working stress in steel — psi. (0.75 of yield strength recommended, the equivalent of safety factor of 1.33)

Figure 4. Distributed steel requirement for light reinforced jointed-concrete pavements.

In addition, the designer has added flexibility using the two scales on the right to either select the steel type or grade and determine the resulting required steel percentage, or select an optimum steel percentage and determine the steel type.

Steel Size and Spacing Requirements

The solution for jointed-reinforced concrete pavement is expressed as a percentage. The next step in design after determining the steel percentage is to determine the bar spacing and size needed to fulfill the required percentage. The equation for solving for bar spacing is as follows:

$$y = \frac{A_b}{D \ P_s} \times 100 \qquad (15)$$

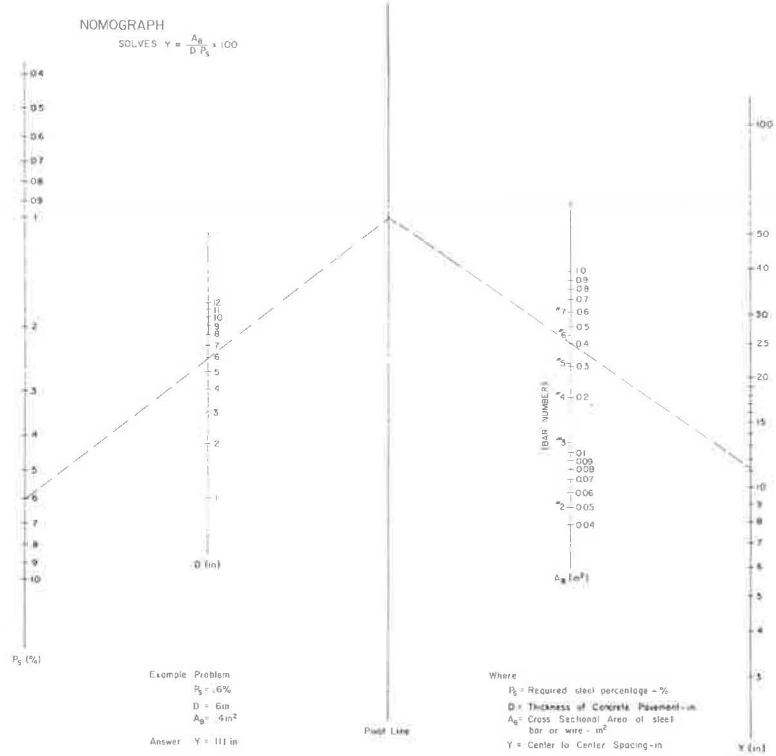


Figure 5. Steel spacing for reinforced concrete pavement.

in which

y = bar or wire spacing, center to center, in.
A_b = cross-sectional area of bar or wire, sq in.
D = pavement thickness, in.
P_s = required steel percentage, percent.

Figure 5 shows a nomograph solution of this equation. Using the variable scales on the right side of the nomograph, several combinations of bar spacings and sizes that meet the steel percentage requirements can be readily obtained.

SUMMARY

Conclusions

The following conclusion can be draw from this work: (a) Based on an analysis of stresses "observed" at the AASHO Road Test, the Spangler simplification of the Westergaard stress equations fits the Road Test pavements. The use of this equation as a stress model in design is therefore justified. (b) A design equation relating load applications to pavement design factors including modulus of elasticity and slab continuity can be developed through the relationship of stress to slab thickness and load applications observed at the Road Test. (c) The complicated design equation involving load applications, modulus of rupture, modulus of elasticity, slab continuity, modulus of subgrade reaction, thickness of slab, and pavement performance can be usefully displayed as a nomograph using general computer solutions of the equations. (d) The evaluation of all variables and constants are reasonably well founded except for the value of life term and slab continuity. Continued observations on existing pavements will help verify these effects. (e) By use of a series of nomographs, the steel reinforcing requirements, i.e., bar size and spacing, for the design conditions of either jointed-reinforced concrete pavement or continuously-reinforced concrete pavement can be determined with several simple manipulations. (f) The design charts developed herein allow the designer to consider numerous variables that were not accounted for in previous design methods. Hence, more flexibility is afforded the designer.

Needed Research

The design methods reported herein are intended to represent the best use of available knowledge concerning portland cement concrete pavements. They are presented as empirical approximations of the true phenomena involved. Continuing research into various aspects of this problem is being carried on.

Powerful computational techniques along with the wealth of experimental data that is being accumulated should advance pavement performance knowledge. Specifically, additional information is needed to evaluate a variable termed "subbase quality" (Q). This variable is related to the load carrying capacity, but must also evaluate the ability of the subgrade to maintain its integrity under repeated load applications. The search should also continue to develop a meaningful environment factor ($\overline{RF}$), a function of weather and other environmental conditions. This term would of course include the curling and warping effects of temperature and moisture differentials.

In addition to these two variables, not included in the design equation developed herein, a great amount of work remains for the verification of the following parameters:

1. J, a function of slab continuity, load conditions, and jointing procedures.
2. ℓ, radius of relative stiffness, a function of E, K, and D. The present application of these factors is based on elastic theory. It can immediately be noted that K is far from elastic and additional study is warranted.
3. Log ΣL, several satellite studies designed to extend and verify the AASHO Road Test equations are in various stages of planning at the present time. Such studies are vital to the solution of this problem.

Method of Proposed Research

In addition to Road Test satellite studies, which are considered to be vital to the solution of this problem, at least two other avenues of research must be exploited.

There is an immediate need for the development of more adequate and versatile methods of analysis permitting the extension of the available solutions past the simplified special-case solutions developed by Westergaard in 1925. Particular attention is needed on dynamic loadings.

A second need is that of developing additional information concerning the effects of dynamic loads on the so-called elastic material properties. There is sufficient proof available from the Road Test to indicate that such a study is both physically and economically feasible by employing a vibrating loader similar to that introduced at the Road Test (2, 8).

The Road Test strain-performance studies provide a basis for extending the Road Test performance equations to include additional design variables, for example: (a) modulus of elasticity of concrete (E), (b) flexural strength of concrete (S_c), (c) joint type and arrangment, (d) subbase and subgrade characteristics (k), and (e) slab loading conditions (continuity).

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