Study of the Reproducibility of Atterberg Limits

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The Atterberg limits have become some of the most extensively used soil tests in highway engineering. In addition, numerous earthwork design and construction specifications are set up on the basis of these limits. Frequently, the reproducibility of the Atterberg limit tests is questioned. Nevertheless, little work has been done to investigate the various aspects of operator variability.

A statistically controlled experiment was performed to investigate how well an operator can reproduce the Atterberg limits and the effects of an operator's experience on the test results. Two operators, one with considerable experience and the other with practically no experience, performed a series of liquid and plastic limit tests on three different soils. The statistical analysis of the test results revealed the following:

1. There are variations in the Atterberg limit values. However, the magnitude of the variations are relatively small. The Atterberg limits can, therefore, be regarded as reproducible from the engineering standpoint.
2. The amount of experience an operator has does affect the variations of the Atterberg limit values.
3. The plasticity index values are most variable, and the liquid limit values are least variable with the plastic limit values occupying an intermediate position.

This paper includes a discussion of the specific numerical values on which these general conclusions are based. A method based on quality control techniques is also proposed for the training of technicians in performing Atterberg limit tests.

•IN CASAGRANDE'S Atterberg limits paper (1), several factors that may cause irregularities in Atterberg limits were discussed. Shook and Fang (2) conducted an investigation on the variability that might be expected among several relatively untrained operators. They found that there was a significant difference between operators for both liquid and plastic limit values. This paper presents a study on the range and degree of operator variation in Atterberg limits values as well as the effects of operator experience on the test results.

Materials Tested

In order to cover a range of liquid limit values that are more commonly encountered, three Illinois soils were selected: a glacial till, a loess, and a glacial lake sediment. For designation convenience, the three soils are referred to as sandy silt, silt, and silty clay, respectively. The grain size distributions for these soils are shown in Figure 1 and the soil classification data are given in Table 1.

Test Procedure and Experimental Design

To provide uniform soil samples and to reduce variations among the samples, the processing and sample preparation of the three soils were undertaken with great care. After each soil was air dried, it was pulverized and thoroughly mixed in a Lancaster

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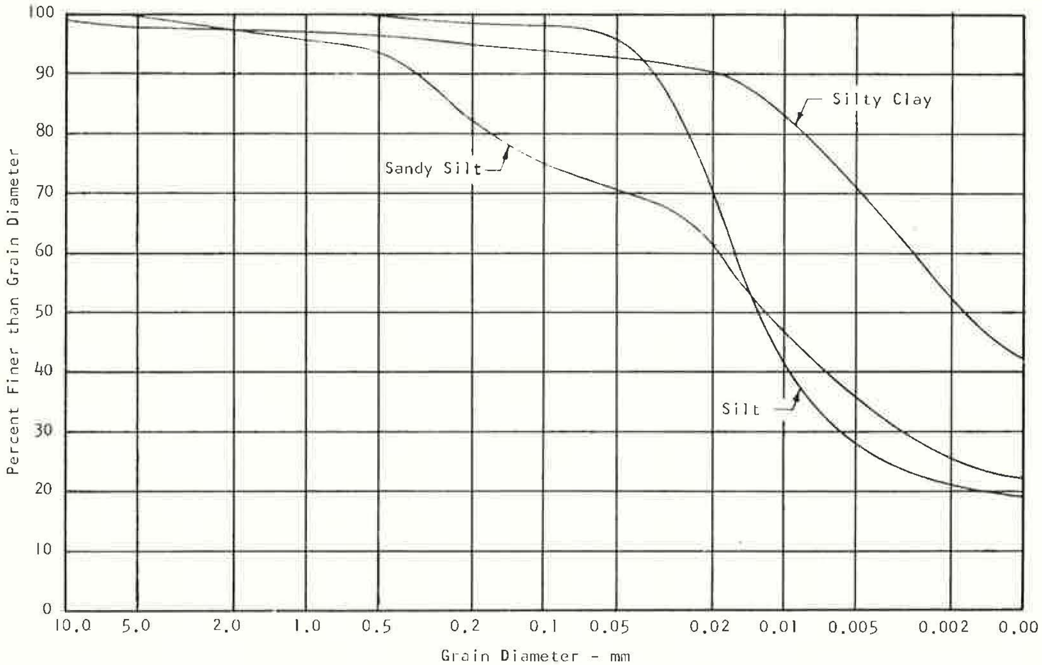

Figure 1. Grain size distribution curves.

TABLE 1

SOIL CLASSIFICATION

Soil Type	Passing No. 200 Sieve (%)	Liquid Limit (Avg.)	Plastic Limit (Avg.)	Plasticity Index (Avg.)	AASHO Group Classification	Group Index
Sandy Silt	73	27.6	16.2	11.4	A-6	8
Silt	98	34.5	23.0	11.5	A-6	9
Silty Clay	94	45.0	21.0	24.0	A-7-6	15

PC Mixer. Each soil was sieved through a No. 40 sieve. Using the sample splitter and standard quartering method, twenty 110-gm samples for each soil were obtained from the portion passing the No. 40 sieve. Each sample was placed in a covered paper carton.

In order to investigate whether the experience of the operator would have significant effects on the reproducibility of Atterberg limits, two operators were chosen to perform the tests. One operator had performed several hundred Atterberg limits tests while the other operator had no prior experience. Prior to the testing program, both operators were provided with copies of "AASHO Standard Specifications" (3) and "Introductory Soil Testing" (4). The inexperienced operator was also given three samples of a silty soil that he could use in familiarizing himself with the test equipment and procedure.

To eliminate the effects of day-to-day variations in conditions and operator behavior and to avoid personal bias in the interpretation of the results, the experiment was designed according to a statistical procedure. Each operator numbered his 30 samples (ten of each soil) from 1 to 30. The order of testing was determined from a table of random numbers. The liquid limit, plastic limit, and plasticity index were determined in accordance with the AASHO Standard Methods Designation T89-60, T90-56, and T91-54, respectively. A Casagrande grooving tool, however, was used in lieu of the ASTM grooving tool. Both operators used the same liquid limit device (with hard rubber base) and all the tests were conducted at the same location in the laboratory.

TABLE 2

TEST RESULTS

Soil Type	Liquid Limit		Plastic Limit		Plasticity Index	
	Experienced Operator	Inexperienced Operator	Experienced Operator	Inexperienced Operator	Experienced Operator	Inexperienced Operator
Sandy Silt	27.8	27.7	16.1	16.9	11.7	10.8
	27.6	28.0	16.0	16.7	11.6	11.3
	28.1	27.1	15.9	16.1	12.2	11.0
	27.2	28.7	15.8	17.4	11.4	11.3
	27.6	27.6	15.7	16.7	11.9	10.9
	27.2	28.0	15.0	15.6	12.2	12.4
	26.9	27.4	17.1	15.3	9.8	12.1
	27.2	27.4	16.2	16.3	11.0	11.1
	27.2	29.2	16.0	17.0	11.2	12.2
	27.6	27.2	16.5	15.3	11.1	11.9
Silt	35.0	33.8	22.3	22.5	12.7	11.3
	35.3	33.5	23.7	23.0	11.6	10.5
	35.0	33.6	23.3	23.7	11.7	9.9
	35.2	35.3	22.6	25.1	12.6	10.2
	34.5	33.2	21.3	21.9	13.2	11.3
	34.9	35.5	22.9	23.0	12.0	12.5
	33.9	32.6	24.2	21.8	9.7	10.8
	35.2	35.2	23.0	23.8	12.2	11.4
	35.3	34.0	23.2	23.8	12.1	10.2
	35.0	34.1	22.9	22.6	12.1	11.5
Silty Clay	44.7	44.5	21.8	19.9	22.9	24.6
	45.8	44.7	21.2	20.6	24.6	24.1
	44.6	44.5	21.3	20.3	23.3	24.2
	45.3	45.4	21.0	21.7	24.3	23.7
	45.8	45.1	21.1	20.6	24.7	24.5
	45.4	43.8	21.0	21.9	24.4	21.9
	45.3	45.0	20.8	20.5	24.5	24.5
	45.2	44.8	20.9	21.8	24.3	23.0
	45.8	44.6	20.7	21.8	25.1	22.8
	46.6	43.8	20.8	19.6	25.8	24.2

Analysis and Results

The complete test results are given in Table 2. The data were analyzed by the University of Illinois' Digital Computer Laboratory IBM 7094 System using various statistical analysis programs.

An analysis of variance was conducted on all the data that were grouped into the three test categories (liquid limit, plastic limit, and plasticity index). The results of this analysis (Table 3) show that (a) for each test highly significant differences exist among the three types of soil (F value for soil types is significant at the one percent level in all three instances); (b) operator experience had a highly significant influence on the mean liquid limit values (F value for operators is significant at the one percent level for liquid limit); (c) operator experience did not cause significant differences in the mean plastic limit values (F value for operators is not significant at the five percent level for plastic limit); (d) operator experience caused significantly different results in the mean plasticity-index values (F value for operators is significant at the five percent level); (e) the magnitude of variation in all three tests for each operator and each soil is relatively small (The variance for replicates is small in all three cases); and (f) the degree of variability of the tests increases from liquid limit to plastic limit to plasticity index (The variance for replicates increases in the aforementioned order. The variance for replicates is also referred to as the residual error that is an estimate of the inherent variation for the specific test. In other words, the the standard deviation (the square root of variance) for the liquid limit, plastic limit, and plasticity index is 0.62, 0.74, and 0.80 respectively).

The following basic statistics were calculated in order to study the magnitude of variations associated with the Atterberg limits tests for each soil type and operator.

TABLE 3

ANALYSIS OF VARIANCE

Source of Variation	Degrees of Freedom	Sum of Squares	Variance	F
(a) Liquid Limit				
Total	59	3100.8325		
Soil types	2	3072.2520	1536.1260	3995.12[a]
Operators (within soil types)	3	7.8175	2.6058	6.78[a]
Replicates (within operators and soil types)	54	20.7630	0.3845	
(b) Plastic Limit				
Total	59	524.4658		
Soil types	2	493.8863	246.9432	447.68[a]
Operators (within soil types)	3	0.7925	0.2642	0.48
Replicates (within operators and soil types)	54	29.7870	0.5516	
(c) Plasticity Index				
Total	59	2160.3733		
Soil types	2	2118.4843	1059.2421	1658.13[a]
Operators (within soil types)	3	7.3930	2.4643	3.86[b]
Replicates (within operators and soil types)	54	34.4960	0.6388	

[a]Significant at the 1 percent level.
[b]Significant at the 5 percent level.

1. Mean

The mean is the arithmetic average of all the individual values. For calculating the mean, $\bar{x}$,

$$\bar{x} = \frac{\Sigma X}{N} \qquad (1)$$

in which

X = individual value, and
N = number of individual values.

2. Variance

The variance is the sum of the squared differences between the individual values and the mean divided by one less than the number of individual values. For calculating the variance, S^2,

$$S^2 = \frac{\Sigma (X - \bar{x})^2}{N - 1} \qquad (2)$$

3. Standard Deviation

The standard deviation, S, is the square root of variance.

4. Coefficient of Variation

The coefficient of variation is the ratio of the standard deviation to the mean. For calculating the coefficient of variation, V, in percentage form

$$V (\%) = \frac{S}{\bar{x}} 100 \qquad (3)$$

These values are given in Table 4. The standard deviation expresses the range of variation with respect to the mean. For the purpose of comparing the degree of variation associated with mean values considerably different in magnitude, it is often more helpful to use the coefficient of variation that expresses the standard deviation as a percentage of the mean. For ease of comparison, the coefficients of variation are shown in a bar diagram (Fig. 2).

From Table 4 and Figure 2 the following observations can be made:

1. Liquid limit
   a. Experienced operator. —The range of variation in liquid limit increases with the increasing plasticity of the soil. (Standard deviation increases with in-

TABLE 4

SUMMARY OF STATISTICAL DATA

Soil Type	Operator	Index Property	Mean	Standard Deviation	Coefficient of Variation
Sandy Silt	Experienced	Liquid Limit	27.4	0.36	1.31
		Plastic Limit	16.0	0.54	3.38
		Plasticity Index	11.4	0.71	6.20
	Inexperienced	Liquid Limit	27.8	0.67	2.41
		Plastic Limit	16.3	0.74	4.52
		Plasticity Index	11.5	0.59	5.15
Silt	Experienced	Liquid Limit	34.9	0.43	1.24
		Plastic Limit	22.9	0.79	3.43
		Plasticity Index	12.0	0.94	7.81
	Inexperienced	Liquid Limit	34.1	0.96	2.83
		Plastic Limit	23.1	1.01	4.35
		Plasticity Index	11.0	0.79	7.20
Silty Clay	Experienced	Liquid Limit	45.4	0.59	1.29
		Plastic Limit	21.0	0.32	1.52
		Plasticity Index	24.4	0.82	3.37
	Inexperienced	Liquid Limit	44.6	0.52	1.16
		Plastic Limit	20.9	0.86	4.12
		Plasticity Index	23.7	0.90	3.78

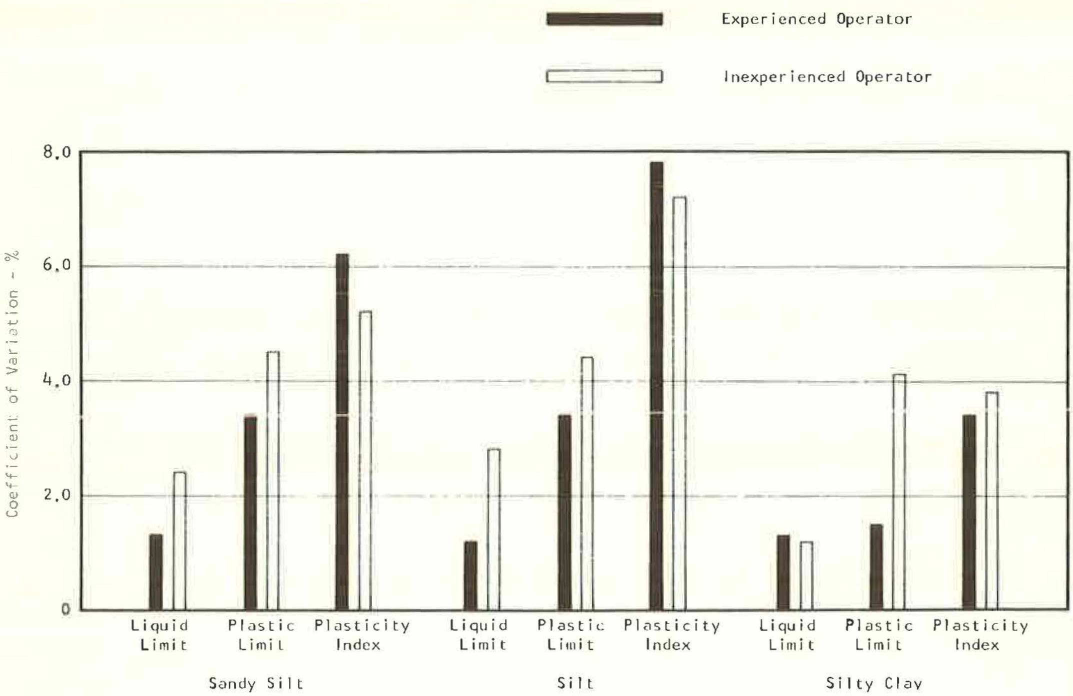

Figure 2. Coefficient of variation.

creasing mean values.) However, the relative degree of variation in liquid limit is practically the same irrespective of the plasticity of the soil (the coefficients of variation remain unchanged for all three soils tested). In other words, an experienced operator can be expected to produce a nearly constant degree of variation in liquid limit values for soils varying from low to moderately high plasticity.

b. Inexperienced operator. —Both the range and relative degree of variation in liquid limit for the silty clay are smaller than that for the other two soils.

This indicates that an inexperienced operator can reproduce the liquid limit value of a moderately highly plastic soil better than he can reproduce those of soils with low to moderately low plasticity.

c. Experienced operator vs inexperienced operator.—For soils with low to moderately low plasticity, the variation in liquid limit for the inexperienced operator is higher than that for the experienced operator. However, the amount of experience seems to have little effect on the variation in liquid limit of moderately highly plastic soil.

2. Plastic limit

a. Experienced operator.—Both the range and relative degree of variation in plastic limit for the silty clay are smaller than that for the other two soils. In other words, an experienced operator can reproduce the plastic limit value of a moderately highly plastic soil better than he can reproduce those of soils with low to moderately low plasticity.

b. Inexperienced operator.—The relative degree of variation in plastic limit is about the same for all three soils. Thus, for any soil within the range of plasticity investigated, an inexperienced operator can be expected to produce an approximately similar degree of variation in plastic limit.

c. Experienced operator vs inexperienced operator.—Operator experience affects the variation in plastic limit because the experienced operator has a narrower range and a smaller degree of variation for all three soils. Consequently, an experienced operator can be expected to reproduce the plastic limit values better than an inexperienced operator.

3. Irrespective of operator experience, the variations in the plastic limit test are generally larger than those in the liquid limit test.

4. The plasticity index values are generally more variable than either the liquid limit or plastic limit values.

5. A detailed examination of the magnitude of standard deviation and coefficient of variation reveals the following:

a. Range of standard deviation.—Liquid limit, 0.36 to 0.96; and plastic limit, 0.32 to 1.01.

b. Range of coefficient of variation.—Liquid limit, 1.16 to 2.83; and plastic limit, 1.52 to 4.52.

Actually, these values are relatively small. This fact is shown in Figure 3 (a plot on the AASHO plasticity chart of the range of test results obtained by each operator for each soil). It is obvious that the results of the two operators show a small amount of variation. Thus, the liquid limit and plastic limit can be regarded as reproducible from the engineering standpoint.

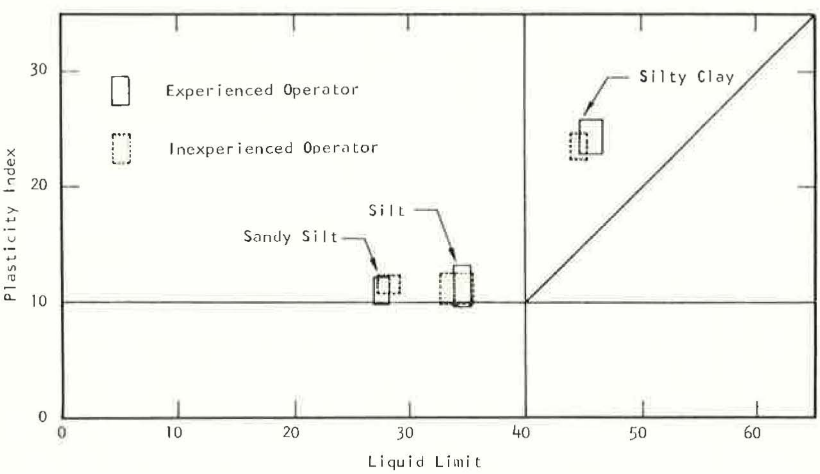

Figure 3. Range of test results.

TABLE 5
"T" VALUES

Soil	Liquid Limit	Plastic Limit	Plasticity Index
Sandy Silt	1.62	1.04	0.31
Silt	2.54[a]	0.45	2.66[a]
Silty Clay	3.37[b]	0.65	1.66

[a]Significant at the 5 percent level.
[b]Significant at the 1 percent level.

In the analysis of variance, it was found that there are highly significant differences in mean liquid limit values as determined by an experienced and an inexperienced operator. No significant differences are found in mean plastic limit values obtained by the two operators. However, the results of the analysis of variance will not reveal whether the significantly different mean values are caused by one soil or all three soils. In order to answer this question, the "t-test" (5) can be used to determine whether there is significant difference between two means—such as the mean liquid limit values of the sandy silt determined by two operators. Various "t" values comparing the mean values determined by the operators in each soil type were calculated. These data are given in Table 5. It can be seen that the mean liquid limit values obtained by these two operators are most significantly different for silty clay, significantly different for silt, and not significantly different for the sandy silt. No significant difference between the mean plastic limit values determined by the two operators is indicated for any of the three soils. The mean plasticity index values are only significantly different for the silt.

CONCLUSIONS

A statistically controlled experiment was performed to investigate an operatior's ability to reproduce the Atterberg limits and the effects of an operator's experience on the test results. Two operators, one with considerable experience and the other with practically no experience, performed a series of liquid and plastic limit tests on three different soils. The statistical analysis of the test results revealed the following:

1. There are variations in the Atterberg limit values. However, the magnitude of the variations as determined on the same equipment and according to the specified method and procedure is relatively small. The Atterberg limits can, therefore, be regarded as reproducible from an engineering standpoint.
2. The operator's experience does affect the variations of the Atterberg limit values. An experienced operator can be expected to reproduce the test results better than an inexperienced operator.
3. The plasticity index values are generally most variable, and the liquid limit values are least variable. The plastic limit values occupy an intermediate position.

ACKNOWLEDGEMENT

Gratitude is expressed to Eugene R. Wilkinson and Richard Miller who performed the two series of tests. Appreciation is also expressed to Samuel G. Carmer, Assistant Professor of Biometry, University of Illinois, for his assistance on the statistical analysis.

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Discussion

ROBERT D. KREBS, Associate Professor of Civil Engineering, Virginia Polytechnic Institute—It was found in our laboratory that a single operator could obtain appreciable differences in liquid limit. This was dependent on whether the soil was moistened or dried between successive trials and on the amount of spatulation employed. This was checked with an air-dried kaolinite soil that was mixed with water to above its liquid limit and cured for three days. As shown in Figure 4, a flow curve was established by successively drying the cured soil between blow-count determination until fifty blows were required to close the groove. Water was added at this point, between blow-count determinations and a second flow curve resulted. It was also found that after prolonged mixing, the cured soil appeared to gain strength with spatulation without changing water content. This phenomenon is illustrated by the horizontal line in Figure 5. The four points were determined within one minute at a nearly constant water content. Finally, with additions of water, the determinations progressed along a flow curve with a normal appearance. This strength gain phenomenon may be responsible for the effect shown in Figure 4.

Even though the soil was vigorously mixed with water (more mixing than one might routinely employ), cured several days, and thoroughly remolded before determination of the liquid limit, it is evident that the soils propensity to adsorb water increased during the course of the determination. This may be due to the progressive breakdown of structural "domains" within the soil. These domains, as described by Aylmore and Quirk (6), consist of clusters of clay particles. As the soil and water are mixed, it may appear that thorough mixing has occurred; however, if spatulation is continued, the domains may break down thereby allowing individual clay particles to adsorb more

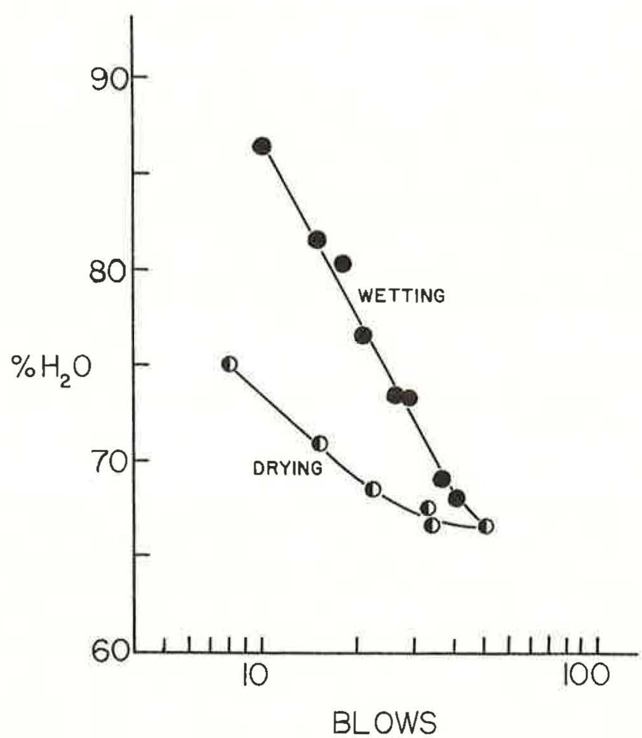

Figure 4. Flow curves for a kaolinitic soil for conditions of drying and wetting during liquid limit determination.

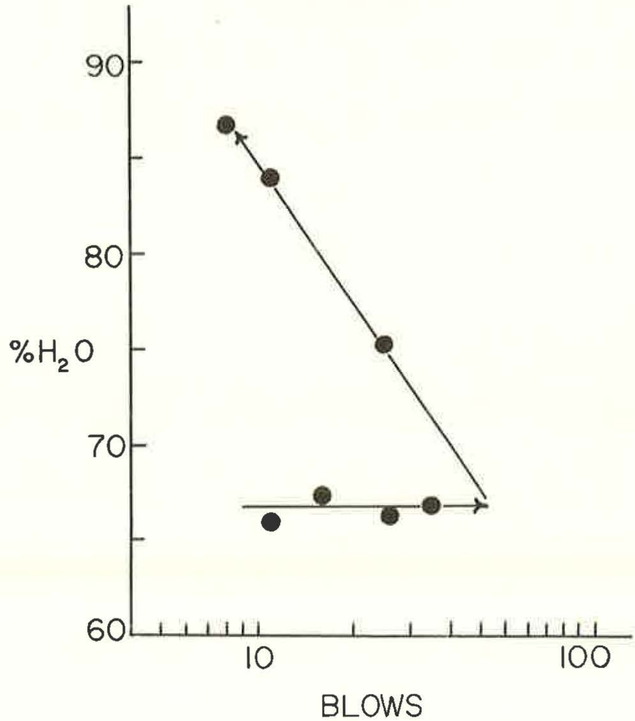

Figure 5. Flow curve for a kaolinitic soil showing the effect of spatulation without drying during liquid limit determination.

water. In this way, the amount of effective lubricating water surrounding each particle or domain is decreased. Because there was no significant change in liquid limit of the kaolinitic soil with curing times varying from one to fifty days, spatulation is apparently an important part of this process. Air drying soil prior to testing probably enhances the formation of domains and increases their resistance to breakdown.

It seems probable that error from this source may account for much of the operator variation found by the authors. If so, the variation can be reduced by curing the soil at near the liquid limit, spatulating with or without slight drying until well below the liquid limit, and finally establishing the flow curve with successive additions of water.

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THOMAS K. LIU and THOMAS H. THORNBURN, Closure—Mr. Krebs has presented some interesting and unusual results on the influence of testing procedure on the liquid limit values of a kaolinitic soil. Our purpose was to study the operator variation in Atterberg limits values associated with the current AASHO Standard Procedures.

Further investigations of the test procedure appear to be in order on the basis of Mr. Krebs' data. If his findings are confirmed for other clay minerals and especially for soils with lower liquid limits, the standard procedures may need revision. Even with such revisions, however, it may be anticipated that a certain amount of operator variability will remain. If the testing procedure becomes more involved, the operator variability, very possibly, will become greater. Only investigations such as those reported in this paper provide a valid basis for the estimation of inherent variability of a procedure—regardless of the reasons for this variability.