Excess Pore Pressures Which Develop During Thawing of Frozen Fine-Grained Subgrade Soils

LESLIE Y. C. YAO and BENGT B. BROMS
Respectively, Senior Engineer, Dames and Moore, New York; and Associate Professor of Civil Engineering, Cornell University

The bearing capacity of flexible pavement is generally governed by the bearing capacity of the subgrade soil which can be greatly reduced during the spring breakup period if the subgrade soil is frost susceptible. This reduction in strength is caused by an increase in water content during freezing, a decrease in soil density and incomplete dissipation of the excess pore pressures during thawing. Methods are presented by which the excess pore pressures before application of any external loads can be calculated within a subgrade during and after thawing.

•DURING FREEZING of fine-grained soils, water is drawn from the groundwater table to the freezing front. The amount of water attracted and the thickness and spacing of the resulting ice lenses that form during freezing depend mainly on rate of freezing, grain size and gradation of the subgrade soil, distance from the water table, and intensity of surcharge load (1, 2, 8, 9, 15, 18, 19).

During thawing, large amounts of excess water will be retained in the soil when the permeability of the soil is low. Several weeks may be required for full dissipation of this excess water.

The shearing strength of the subgrade soil can be greatly reduced during the spring breakup period after it has been subjected to freezing and thawing. This reduction in strength is caused by an increase in water content of the soil, a decrease in soil density, and high excess pore pressures. As a consequence, the bearing capacity of the subgrade will also be reduced and extensive damage to highway and airfield pavements may result.

The shearing strength of a subgrade generally reaches its lowest value at the beginning of the thawing period when the excess pore pressures reach a maximum. The excess pore pressures will gradually decrease with time and, consequently, the subgrade will gradually gain in strength. It is important to be able to predict this variation in shear strength and bearing capacity at any stage during the thawing period so that imposed loads (wheel or axle loads) can be maintained within safe limits.

Methods are presented by which the ultimate bearing capacity and the excess pore pressures which develop within subgrade soils can be estimated at any stage and at any depth during the thawing period.

It should be emphasized that the calculated pore pressures have not been substantiated by test data and, therefore, the limitations of the proposed method are not known. The method is presented only as a working hypothesis. Its main justification is to point out the factors which may have an important effect on the bearing capacity of subgrade soils during the thawing period.

RELATIONSHIP BETWEEN INITIAL PORE PRESSURE AND BEARING CAPACITY

The initial pore pressures will have an appreciable effect on the bearing capacity of flexible highway and airfield pavements. The difference between the maximum and minimum principal stresses at failure, $(p_1 - p_3)_f$, of a saturated subgrade soil can be

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expressed in terms of initial total vertical pressure p_o, the initial pore pressure u_o, the shear strength parameters c' and φ', the coefficient of lateral earth pressure at rest K_o and the pore pressure coefficient $\bar{A}_f$ (11, 17) as follows:

$$\frac{(p_1 - p_3)_f}{2} = \frac{c' \cos\varphi' + (p_o - u_o)\sin\varphi' \, [K_o + \bar{A}_f (1 - K_o)]}{1 + (2\bar{A}_f - 1)\sin\varphi'} \tag{1}$$

For c' equal to zero, Eq. 1 becomes:

$$\frac{(p_1 - p_3)_f}{2} = \frac{(p_o - u_o)\sin\varphi \, [K_o + \bar{A}_f (1 - K_o)]}{1 + (2\bar{A}_f - 1)\sin\varphi'} \tag{2}$$

In the derivation of Eqs. 1 and 2, it has been assumed that the soil is saturated and that no change in water content takes place during loading. If, however, an appreciable change does take place, the shear strength generally will be higher than that calculated from Eqs. 1 or 2. Thus, these equations will yield results which generally are on the safe side.

The validity of Eqs. 1 and 2 has been demonstrated mainly for remolded soils. The limitations of these equations to undisturbed soils are not known.

If the load causing failure is applied very rapidly (such as by moving traffic loads), the water content of most soils will not change appreciably. The loading rate necessary to prevent a change in water content during loading depends on the permeability and compressibility of the soil and on the boundary conditions. The permeability is the most important of these factors.

Broms (3) has proposed a method for the evaluation of the ultimate bearing capacity of flexible pavements subjected to frost action. It has been assumed in this method that a saturated soil, with a low permeability compared to the loading rate (when no or very little change in water content takes place during loading), will behave as a frictionless material with a shear strength c_u. This shear strength can be evaluated by the equation (17):

$$c_u = \tfrac{1}{2}(p_1 - p_3)_f \tag{3}$$

The corresponding net ultimate bearing capacity q_{ult}^{net}, defined as the total ultimate capacity less the overburden pressure, can then be calculated by the equation (13, 17, 20):

$$q_{ult}^{net} = c_u N_c \tag{4}$$

where N_c is a bearing capacity factor that depends on the shape and size of the loaded area as well as on the depth below the ground surface (20).

If Eqs. 3 and 4 are substituted into Eqs. 1 and 2, then:

$$q_{ult}^{net} = \frac{c' \cos\varphi' + (p_o - u_o)\sin\varphi' \, [K_o + \bar{A}_f (1 - K_o)]}{1 + (2\bar{A}_f - 1)\sin\varphi'} N_c \tag{5}$$

When c' is equal to zero, Eq. 5 becomes:

$$q_{ult}^{net} = \frac{(p_o - u_o)\sin\varphi' \, [K_o + \bar{A}_f (1 - K_o)] \, N_c}{1 + (2\bar{A}_f - 1)\sin\varphi'} \tag{6}$$

The parameters c', φ', K_o and $\bar{A}_f$ in Eqs. 1, 2, 5 and 6 can be evaluated experimentally. The apparent angle of internal friction φ' with respect to effective stresses has been found to be almost unaffected by freezing and thawing, whereas the apparent cohesion c' depends to a large extent on the rate of freezing, the number of freeze and thaw cycles, the overburden pressures, and the drainage conditions (4).

It can be seen from Eqs. 5 and 6 that the initial pore pressure u_o will have a large effect on the ultimate bearing capacity, especially for the case when the cohesion c' is equal to zero. The ultimate bearing capacity will, for this case, approach zero when the initial pore pressure u_o approaches the initial total overburden pressure p_o.

ASSUMPTIONS

In the analysis of the excess pore pressures which develop as a result of thawing of a frozen soil, the following assumptions will be made:

1. During thawing of an area of frozen ground, flow of the melted water will only take place toward the ground surface (before the soil has thawed completely). Therefore, regardless of the location of the original groundwater table, the water table will be temporarily raised during thawing. The groundwater table may be raised to the ground surface or to any pervious layer located within the thawed part of the soil. As a consequence, the subgrade soil becomes saturated or almost saturated during thawing. When the lateral extent of the subgrade is large compared to the thawing depth, the flow of water will be essentially one-dimensional. The assumption will be made that the subgrade soil is fully saturated and that the flow of melted water is one-dimensional.
2. During freezing of fine-grained soils, water is drawn to the freezing front from any available source and will form more or less continuous ice lenses or ice bands. When the ice lenses melt during thawing, part of the melted water will be absorbed by the soil located between the individual ice lenses and part will dissipate during the thawing process. However, if the thawing rate is high, most of the melted water will remain in the soil for a limited period of time in the form of more or less continuous water layers. The pore pressures which develop within these continuous water sheets will be equal to the total overburden pressure since complete separation of the individual soil particles will occur at each water sheet. It will be assumed that this pore pressure will remain constant until the water trapped within the water sheets is fully dissipated and the space between the different soil layers is completely closed.
3. After the free water trapped within the continuous water sheets has dissipated, a further change in water content of the soil will take place as the excess pore pressures gradually decrease. The resulting gradual change in water content will depend primarily on the compressibility of the soil. However, soils which have been subjected to one or several freeze-thaw cycles will be heavily preloaded by high negative pore pressures which develop within the soil during the freezing cycle (4). Consequently, the volume compressibility of frozen and thawed soils will be generally small except for swelling soils. Thus, the change in water content, which takes place as the soil reconsolidates during thawing, will also be small. It will be assumed in the following analysis that these volume changes can be neglected. Thus, the methods presented in this paper will be restricted to nonswelling types of soils.

The accuracy of these assumptions is not known and, therefore, field measurements are necessary before the proposed method can be used for design purposes.

INITIAL PORE PRESSURES

Thawing Depth and Rate of Thawing

The development of excess pore pressures within a soil depends on the rate of thawing and on the consolidation rate. If, for example, the thawing rate is high as compared to the consolidation rate, then high excess pore pressures will develop within the soil. These excess pore pressures can be evaluated by considering the factors affecting the rate of thawing and the consolidation rate.

The thawing rate can be calculated from the following general differential equation which governs the one-dimensional, unsteady heat flow:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial Z^2} \tag{7a}$$

where

Z = depth below ground surface,
t = time,
T = temperature at time t and depth Z, and
α = thermal diffusivity as defined by:

$$\alpha = \frac{K}{c\rho} \tag{7b}$$

where

K = thermal conductivity of the material,
c = specific heat of the material, and
ρ = density of the material.

Several solutions to Eq. 7a are available. Jumikis (10) has presented one which relates frost penetration depth or the thawing depth with the thermal properties of the soil. According to Jumikis (10), the thawing depth Z_o can be determined from the equation:

$$Z_o = \sqrt{\frac{2K_1 (T_s - T_f)\, t}{Q_L \gamma_w n}} \tag{8a}$$

where

K_1 = thermal conductivity of thawed soil,
Q_L = latent heat of fusion,
γ_w = unit weight of water,
n = porosity,
T_s = surface temperature, and
T_f = freezing temperature.

Eq. 8a can be rewritten as:

$$Z_o = m\sqrt{t} \tag{8b}$$

if:

$$m = \sqrt{\frac{2K_1 (T_s - T_f)}{Q_L \gamma_w n}} \tag{8c}$$

It can be seen from Eq. 8b that the depth of thawing varies as the square root of time.

The rate of thawing equal to dZ_o/dt can then be determined from the equation:

$$\frac{dZ_o}{dt} = \frac{m}{2\sqrt{t}} \tag{8d}$$

In the derivation of Eq. 8a it has been assumed that the latent heat of fusion is the only heat conducted during thawing or freezing to or from a point within a mass of soil. The amount of heat generated or consumed by a change of temperature above or below the freezing point has been neglected. The resulting inaccuracy depends on the surface temperature, the temperature in soil, water content and thermal properties of the solids. Calculations (10) have indicated that the error in thawing depths as determined by Eq. 8a is usually less than 5 percent. The amount of heat transported by the upward flow of water during thawing has also been neglected. Calculations by the authors have indicated that this heat flow is small and has no signficant effect (less than 1 percent) on the calculated thawing depth.

Eqs. 8a and 8b can be solved for the case when the quantity m is constant, that is, when the thermal conductivity of the thawed soil, the latent heat of fusion, the porosity, the surface temperature and the freezing temperature are all constant.

Excess Pore Water During Thawing

Shown in Figure 1 is a section through a typical highway pavement consisting of a wearing course, a base course and a subgrade. During thawing and the subsequent reconsolidation of the subgrade soil, its water content will decrease from w_o to w, where w_o and w are the water contents of the frozen and fully reconsolidated soil, respectively. When the subgrade is fully saturated, the unit volume decrease dV/V caused by consolidation of the soil after thawing will be equal to:

$$\frac{dV}{V} = \frac{(w_o - w)}{w_o + 1/G_s} \tag{9}$$

where G_s is the specific weight of the solid material. The unit volume change dV/V is equal to the volume of excess water expelled from the soil during thawing and the subsequent consolidation.

In addition to the volume change dV/V which is caused by a change in water content after thawing, a volume change takes place during thawing without any change in water content due to the difference in density between ice and water. This volume change does not affect the thawing rate since the transfer of heat takes place through the thawed soil.

When the depth of thawing increases from Z_o to $(Z_o + dZ_o)$, the volume of excess melted water dQ will be equal to:

$$dQ = \frac{dZ_oA\,(w_o - w)}{w_o + 1/G_s} \tag{10}$$

The rate dQ/dt by which the excess water is freed during thawing can be calculated from Eq. 10 as:

$$\frac{dQ}{dt} = \frac{dZ_o}{dt} \cdot \frac{A\,(w_o - w)}{w_o + 1/G_s} \tag{11}$$

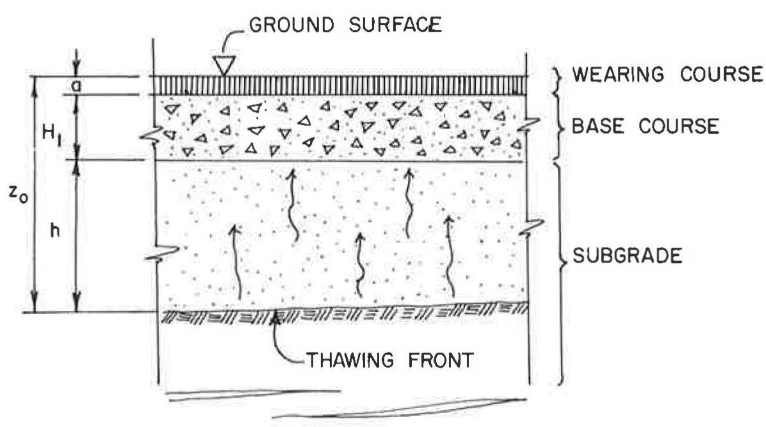

Figure 1. Cross-section through highway pavement.

The quantity dZ_o/dt is equal to the thawing rate, which is governed by Eq. 8d. Eq. 11 can then be rewritten as:

$$\frac{dQ}{dt} = \frac{mA\,(w_o - w)}{2\sqrt{t}\,(w_o + 1/G_s)} \tag{12a}$$

or

$$\frac{dQ}{dt} = \frac{m^2A\,(w_o - w)}{2Z_o\,(w_o + 1/G_s)} \tag{12b}$$

Dissipation of Excess Pore Pressures

The thawing rate may in some cases be higher than the flow rate. When this occurs, layers or sheets of water will be occupying the locations of the former ice lens and the individual soil particles will be totally separated by these water sheets as shown in Figure 2. In this case the thawed pore water will carry the weight of the overlying soil. The resulting excess pore pressure u_{excess} will be equal to the difference between the total pore pressure u and the static (e) pore pressure u_o as defined by the equation:

$$u_{excess} = u - u_o \tag{13}$$

The static pore pressure is defined as the pore pressure when no flow of water takes place through the soil.

The excess pore pressure creates a hydraulic gradient which causes the excess pore water to flow upwards. The resulting hydraulic gradient i is equal to:

$$i = \frac{u_{excess}}{h\,\gamma_w} \tag{14}$$

where h is the length of the flow path (equal to the depth of the thawed subgrade soil for the case shown in Figure 1) and γ_w is the unit weight of water. The velocity v of this flow is governed by the Darcy law:

$$v = ki \tag{15}$$

where k is the permeability of thawed soil.

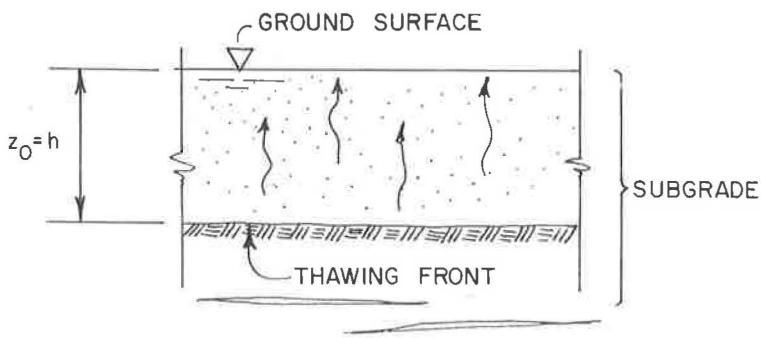

FIGURE 2 — CONDITIONS DURING THAWING (CASE 1)

Figure 2. Conditions during thawing (case 1).

The corresponding flow rate q is proportional to the available area A. Then:

$$q = Aki \tag{16}$$

If the rate of thawing is higher than the corresponding flow rate, excess pore water will accumulate in the soil and a "quick" condition will be created. This condition occurs when the seepage pressure in the soil is equal to the effective pressure, i. e., when the effective pressure becomes zero. Under this condition, the shear strength of the soil will decrease to zero in the case when the cohesion c' of the soil is equal to zero, as can be seen from Eq. 2.

Critical Depth

The thawing rate decreases with increasing thawing depth (Eq. 8d) while the flow rate remains constant. A quick condition occurs in the subgrade in the beginning of the thawing period when the thawing depth is small and the rate of thawing is high. The depth at which the behavior of the soil converts from a quick to a nonquick condition is defined as the critical depth. At the critical depth, Z_{cr}, the rate of excess water liberation (Eq. 12b) is exactly equal to the rate of excess water dissipation (Eq. 16). This occurs when:

$$\frac{m^2\,(w_o - w)}{2Z_{cr}\,(w_o + 1/G_s)} = ki \tag{17}$$

The critical depth Z_{cr} can then be calculated from Eq. 17 as:

$$Z_{cr} = \frac{m^2\,(w_o - w)}{2ki\,(w_o + 1/G_s)} \tag{18}$$

Within the critical region (where the soil is in a quick condition), the excess pore pressure will be independent of the dissipation rate and the rate of thawing (Eq. 18), and the excess pore pressure will depend only on the depth below the ground surface and the submerged unit weight of the soil (Eq. 13).

Subcritical conditions will occur below the critical depth. Within this region, the excess pore pressures will depend on the rate of thawing, the depth below the ground surface, the porosity and the permeability of the thawed soil. The general equation governing the excess pore pressures within this region can be determined by substituting Eq. 14 into Eq. 17. Then for the subcritical region:

$$u_{excess} = \frac{m^2 h\,\gamma_w\,(w_o - w)}{2kZ_o\,(w_o + 1/G_s)} \tag{19}$$

This equation will be used in the following to calculate excess pore pressures. In the derivation of Eq. 19 it has been assumed that the compressibility of the soil and the volume changes which take place due to a change in effective pressure are small and can be neglected as discussed previously. It should also be emphasized that the derived equations have not been checked by test data.

EXAMPLES

The excess pore pressures which develop during thawing within a subgrade have been investigated for three cases. It will be shown that the excess pore pressures frequently will be high and may approach the total overburden pressure.

TABLE 1
ASSUMED PROPERTIES OF SUBGRADE SOIL

Property	Value	
	Immediately After Thawing	Completion of Thawing
Water content (%)	24.0	20.0
Specific gravity of solids	2.77	2.77
Coefficient of permeability (ft/hr)	3.0×10^{-4}	-
Thermal conductivity (Btu/ft-hr-°F)	1.07	-

Case 1

In Figure 2 is shown the drainage conditions during thawing of a subgrade soil. It has been assumed in this case that the groundwater table has been raised temporarily to the ground surface. The assumed physical properties of the subgrade are listed in Table 1. Thus, it has been assumed that the water content of the soil immediately after thawing is 24.0 percent and that this water content is decreased to 20 percent after dissipation of the excess water several weeks after completion of the thawing process. In the calculations of critical depth and excess pore pressures, it has been assumed that the surface temperature has been suddenly increased to 60 F and that the average temperature of the frozen soil is 32 F.

The rate of thawing as calculated from Eq. 8b is shown in Figure 3. It can be seen that the rate of thawing decreases rapidly with increasing depth. For example, it will take 45 hr for the thaw line to reach a depth of 0.87 ft below the ground surface, and 181 hr for it to reach 1.75 ft.

The critical depth depends on the excess pore pressures within the soil and on the resulting hydraulic gradient. (At the critical depth, the excess pore pressures within the soil will be equal to the total overburden pressure.) The excess pore pressure u_{excess} within the critical depth can be determined from Eq. 13 as:

$$u_{excess} = h\gamma_{sat} - h\gamma_w = h\gamma_{sub} \tag{20}$$

where γ_{sub} is the submerged unit weight of the subgrade soil.

For this condition, the hydraulic gradient i will be equal to:

$$i = \frac{\gamma_{sub}}{\gamma_w} \tag{21}$$

The critical depth Z_{cr} (Eq. 18) can then be calculated as:

$$Z_{cr} = \frac{m^2 (w_o - w) \gamma_w}{2k (w_o + 1/G_s) \gamma_{sub}} \tag{22}$$

The critical depth will be 1.75 ft for the soil with the physical properties shown in Table 1. The time required for the thawing front to reach this depth is 181 hr or 7.6

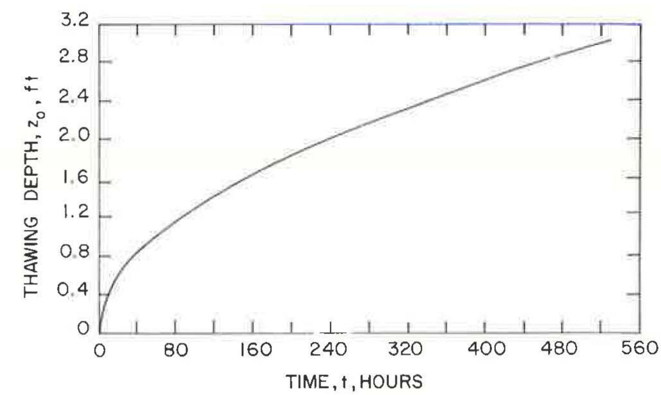

Figure 3. Relationship between thawing depth and time (case 1).

days. Thus, for 7.6 days, very high excess pore pressures will be present within the subgrade located above the thawing front and, as a result, the effective stress within the soil located above the thawing front will be reduced to zero. The corresponding hydraulic gradient is 1.063.

It should, however, be noticed from Eq. 22 that the critical depth is very sensitive to small changes of the permeability. If, for example, the permeability of the soil is equal to 1.0×10^{-4} ft/hr instead of 3.0×10^{-4} ft/hr, the critical depth will be 5.25 ft and the time required for the thaw line to reach this depth will be 68 days.

The excess pore pressure for the subcritical condition below the critical depth Z_{cr} can be determined from Eq. 19. For case 1, the depth Z_o is equal to h. For this condition:

$$u_{excess} = \frac{m^2 \gamma_w (w_o - w)}{2 k (w_o + 1/G_s)} \tag{23}$$

It can be seen from Eq. 23 that the excess pore pressure u_{excess} is constant and independent of depth. This excess pore pressure will be equal to 116.0 psf for the assumed properties of the subgrade soil (Table 1).

The corresponding hydraulic gradient within the subcritical region, below the critical depth, can be calculated from Eq. 14. This hydraulic gradient is equal to:

$$i = 1.86/Z_o \tag{24}$$

The hydraulic gradient and the resulting flow rate will remain constant as thawing takes place from the ground surface and down to the critical depth of 1.75 ft below the ground surface (Eq. 21). The pore pressure will remain constant within the soil and will be equal to the total overburden pressure as long as the thaw line is located above the critical depth, the pore pressures will decrease with increasing depth of thawing, and the hydraulic gradient and the flow rate will decrease in proportion to the thawing depth, as can be seen from Eq. 24.

The corresponding distribution of excess pore pressures is shown in Figure 4 by relationships (a) through (d). Relationship (a) illustrates the pore pressure distribution when the thawing proceeds from the ground surface down to the critical depth of 1.75 ft. Relationships (b), (c) and (d) show the distribution of excess pore pressures when the thaw line has proceeded down to a depth of 2.2, 2.6, and 3.0 ft, respectively, below the ground surface. The thaw line will reach these depths 287, 400, and 535 hr, respectively, after the beginning of the thawing period. When the thaw line has reached the depth of 3.50 ft, the excess pore pressure above the critical depth will decrease

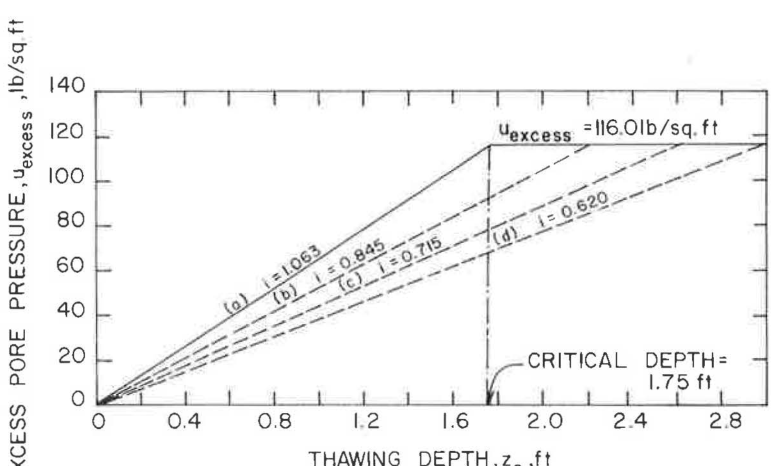

Figure 4. Relationship between excess pore pressure and thawing depth (case 1).

TABLE 2

ASSUMED PROPERTIES OF BASE COURSE MATERIAL

Property	Value
Water content (%)	12.0
Porosity	0.238
Specific gravity of solids	2.60
Thermal conductivity (Btu/ft-hr-°F)	1.80

to half its initial maximum value and the soil will have regained a large portion of its initial strength.

Case 2

In Figure 5 is shown the boundary conditions of a subgrade soil when groundwater table is not raised all the way to the ground surface. In this case, drainage takes place through a base course consisting of coarse material with a high permeability.

The same properties of the subgrade material have been chosen as for case 1 (Table 1). The assumed properties of the base and wearing course materials are given in Table 2.

The base course material located above the water table has been assumed to have a lower water content than the subgrade soil. Furthermore, it has been assumed that the thermal properties and the thawing rates are different for the two materials. (The thawing rate is frequently higher for the base course than for the subgrade material.) The equations governing the thawing rate are not continuous in this case. However, these equations can be simplified if the thickness of the base course soil is converted into an equivalent subgrade thickness. Then the base course can be treated as part of the subgrade.

Equivalent Thickness. —From Eq. 8b it can be seen that the thawing rate depends only on the parameter m which depends on the thermal properties of the soil. Therefore, the thickness of the base course can be converted into an equivalent thickness H of subgrade soil by the following relationship:

$$H = \frac{mH_1}{m_1} \tag{25}$$

where

m = parameter depending on thermal properties of subgrade soil (Eq. 8c),
m_1 = parameter depending on thermal properties of base course material (Eq. 8c), and
H_1 = thickness of base course (Fig. 5).

The pavement itself will have little effect on the thawing depth since its moisture content is generally low. From Eqs. 8c and 25, the equivalent thickness H of the base course material (Table 2) can be calculated as 0.596 H_1.

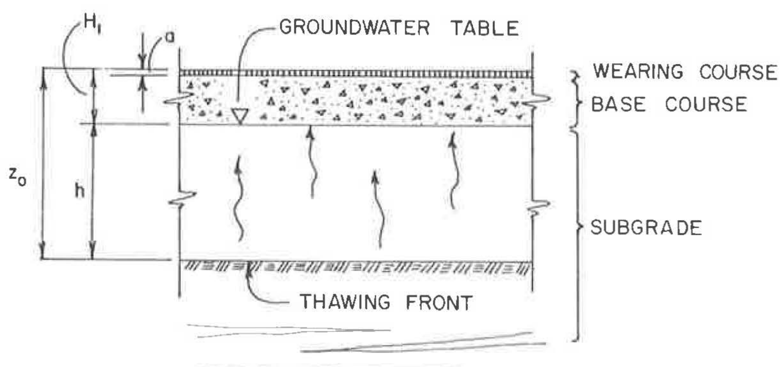

Figure 5. Cross-section through highway pavement (case 2).

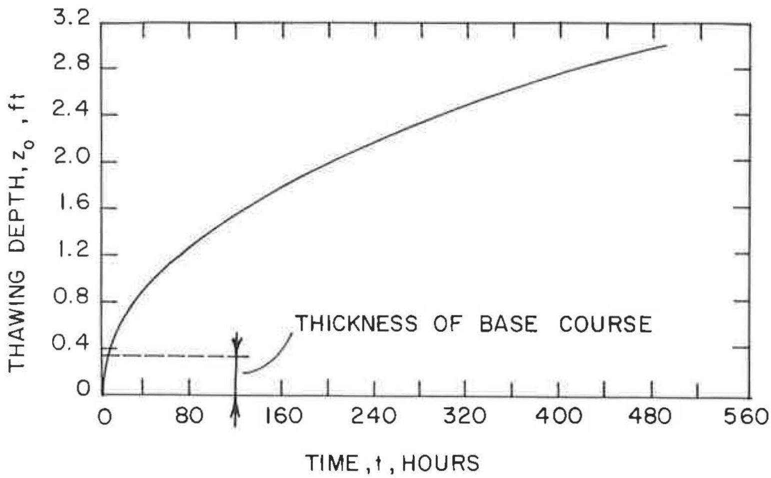

Figure 6. Relationship between thawing depth and time (case 2).

The relationship between depth of thawing and time, calculated from Eqs. 8b and 25, is shown in Figure 6. It can be seen that only a short time period is required for the thaw line to penetrate through the base course.

The critical depth can then be calculated from Eq. 18 as:

$$Z_{cr} = \frac{m^2 (w_o - w)}{2k (w_o + 1/G_s)} \left[\frac{1}{\frac{a\gamma_p}{h\gamma_w} + \frac{H_1 (\gamma_1)_{sat}}{h\gamma_w} + \frac{(\gamma_2)_{sub}}{\gamma_w}} \right] \qquad (26)$$

In this equation, a and γ_p are the thickness and unit weight, respectively, of the wearing course, and H_1 and $(\gamma_1)_{sat}$ are the thickness and saturated unit weight, respectively, of the base course material. It should be noted in Eq. 26 that Z_{cr} is the equivalent critical depth. This depth is equal to $(H + h_{cr})$.

It has been assumed in this case that the thickness of wearing course, a, is negligible and the thickness of base course, H_1, is 4.0 in. The solution to Eq. 26 yields the two critical depths, 0.216 and 0.640 ft. These two depths correspond to the actual depths of 0.549 and 0.973 ft. The two solutions of Eq. 26 indicate that critical or quick conditions will exist within the subgrade soil when the thaw line is located between 0.549 and 0.973 ft below the ground surface. For this location of the frost line, the excess pore pressures will be equal to the total overburden pressure. These results are reasonable because at shallow thawing depths, the hydraulic gradient will be high due to the imposed surcharge weight (the weight of the base and wearing courses) and a relatively short time will be required for the excess water to dissipate. The excess pore pressures will be governed by the thawing rate.

The excess pore pressure u_{excess} within critical region can be determined from Eqs. 13 and 14 as:

$$u_{excess} = a\gamma_p + H_1 (\gamma_1)_{sat} + h (\gamma_2)_{sub} \qquad (27)$$

The corresponding hydraulic gradient as calculated from Eqs. 13 and 14 is equal to:

$$i = \frac{a\gamma_p}{h\gamma_w} + \frac{H_1 (\gamma_1)_{sat}}{h\gamma_w} + \frac{(\gamma_2)_{sub}}{\gamma_w} \qquad (28)$$

Eq. 28 is only valid at the boundaries of the critical region. Within the critical region the hydraulic gradient will be constant as long as separation between the individual soil particles occurs.

It can be seen from Eqs. 21 and 28 that the hydraulic gradient when the water table is located at some depth below the ground surface is much greater than that for the case when the water table is located at the ground surface.

The excess pore pressure for subcritical conditions (Eq. 19) is equal to:

$$u_{excess} = \frac{m^2 (w_o - w) \gamma_w h}{2k (w_o + 1/G_s) (H + h)} \tag{29}$$

For the specific case when h >>H, then:

$$u_{excess} = \frac{m^2 (w_o - w) \gamma_w}{2k (w_o + 1/G_s)} \tag{30}$$

It can be seen from Eq. 30 that for large values of h, the excess pore pressure approaches a constant value. This constant value is the same as that for case 1 when the water table is located at the ground surface (Eq. 23).

The relationships between the thawing depth and time as determined from Eq. 8a are shown in Figure 6. The relationships between the excess pore pressures, hydraulic gradients and depth of thawing are shown in Figure 7. Relationships (a) through (f) show the distribution of excess pore pressures within the subgrade as the thaw line proceeds through the subgrade down to a depth of 2.93 ft below the pavement surface. Relationship (b) indicates, for example, the excess pore pressure distribution when the thaw line has reached a depth of 1.33 ft below the ground surface. The corresponding hydraulic gradient is equal to 1.55 and the excess pore pressure at a depth of 1.2 ft below the ground surface is 84.0 psf. As the thaw line proceeds from 0.549 to 0.973 ft (critical region), the pore pressure at the thaw line will be equal to the total overburden pressure. From this value, the resulting excess pore pressures can be determined by assuming that the hydraulic gradient is constant throughout the thawed portion of the subgrade. In the calculations of the excess pore pressures shown in Figure 7, it has been assumed that the soil located above the thaw line is incompressible. It is believed that the resulting error will be small.

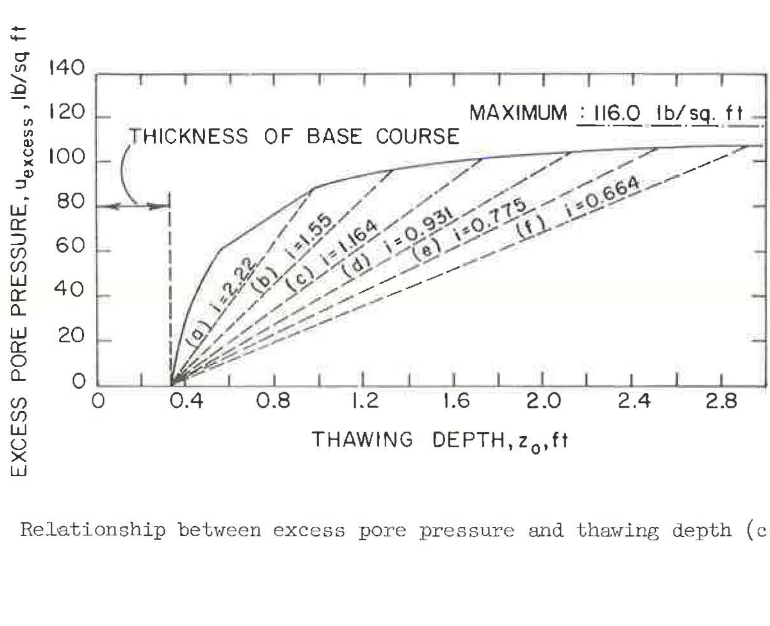

Figure 7. Relationship between excess pore pressure and thawing depth (case 2).

Effect on Ultimate Bearing Capacity. —The ultimate bearing capacity is affected by the initial excess pressures (Eqs. 5 and 6). This excess pore pressure (caused by incomplete dissipation) affects the initial effective pressures in the soil. The effect of initial pore pressure on the bearing capacity of a flexible pavement has been investigated for the subgrade shown in Figure 5. It has been assumed that the average condition in the subgrade is represented by an element located at a depth of 2.0 ft below the ground surface. The time required for the thaw line to reach this depth is 200 hr (Fig. 6). The corresponding excess pore pressure u_{excess} when the thaw line has just reached this depth is equal to 104 psf (Fig. 7). When the thaw line reaches a depth of 2.5 ft (after 325 hr), the excess pore pressure at a depth of 2.0 ft is 80 psf. Thus, at the depth of 2.0 ft, the excess pore pressures will decrease during this time by 24 psf. Similarly, when thawing depth reaches 2.9 ft, the excess pore pressure at a depth of 2.0 ft will be equal to 70 psf.

If it is assumed that the cohesive strength c' is equal to zero, and that the parameters φ', $\bar{A}_f$, K_o and N_c remain constant during thawing, the ultimate bearing capacity will be proportional to the initial effective pressure $\bar{p}_o$. For the conditions shown in Figure 5, the initial effective pressure at a depth of 2.0 ft below the ground surface will be equal to 53, 77 and 87 psf after 200, 325 and 450 hr of thawing. Thus, the bearing capacity of a subgrade soil during thawing will depend on time. After 450 hr of thawing, the bearing capacity will be 1.64 (87/53) times the bearing capacity after 200 hr of thawing. The bearing capacity of a subgrade soil will reach a minimum at the beginning of the spring breakup period and will regain slowly its original strength as the initial excess pore pressures dissipate.

Case 3

In this example, the thickness of the pavement, a, and the thickness of the base course, H_1, are each assumed to be equal to 4.0 in. The unit weight of the wearing course material has been taken as 150 pcf and its moisture content has been assumed to be small. The assumed properties of the base course and of the subgrade are shown in Tables 1 and 2.

The critical depth is governed by Eq. 26 and no real solution exists to this equation. This indicates that critical conditions will not exist within the subgrade during thawing.

The relationship between the depth of thawing and time is shown in Figure 8. It can be seen that only a very short time will be required for the thaw line to penetrate through the pavement because of its negligible latent heat of fusion. The thaw line will have penetrated to a depth of 2.49 ft after 240 hr or 10 days of thawing.

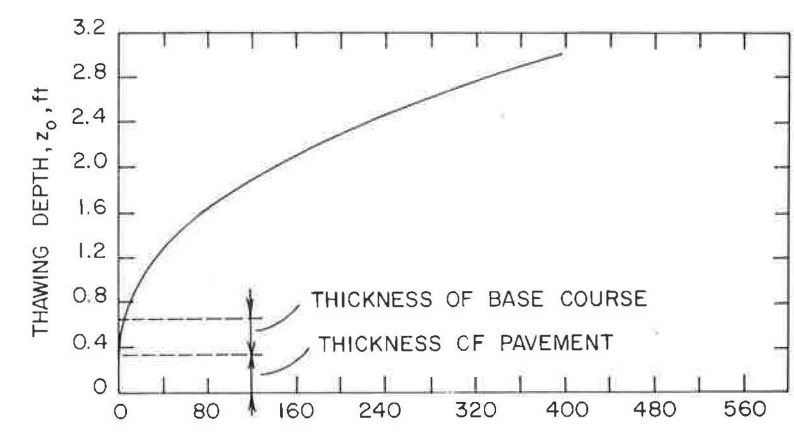

Figure 8. Relationship between thawing depth and time (case 3).

The corresponding excess pore pressures and the resulting hydraulic gradient can be computed from Eqs. 14 and 29 and are plotted in Figure 9. It can be seen that no excess pore pressures will exist within the base course or subgrade until the thaw line has penetrated through the base course. As the thaw line progresses through the subgrade, excess pore pressures will build up rapidly. The maximum pore pressure, which occurs at the thaw line, will increase with increasing depth of thawing. For example, the maximum excess pore pressure is equal to 85 psf at a depth of 1.2 ft below the ground surface. This maximum will occur 32 hr after the beginning of the thawing period and the excess pore pressure will decrease rapidly as the thawing depth increases.

DRAINAGE AND CONSOLIDATION AFTER COMPLETION OF THAWING

The equations developed in preceding sections governing the buildup and dissipation of pore pressures apply only when continuous thawing of the subgrade soil takes place and before complete thawing of the subgrade has occurred. As pointed out earlier, the water table will be temporarily raised during thawing and flow of water will only take place in the upward direction. (The soil located below the thaw line is still frozen.) However, when the soil is completely thawed, the original water table will govern the pore pressure distribution in the soil. The flow of water will be reversed and will change from that of seepage to that of drainage.

The effects of a sudden lowering of the groundwater table as a result of complete thawing on the dissipation of excess pore pressures after completion of thawing is illustrated by the following example. A cross-section through a subgrade soil is shown in Figure 10. It has been assumed that the water table is located during thawing at the ground surface while after completion of thawing, the permanent water table is located at the distance H_a below the ground surface. (The location of the permanent groundwater table is frequently governed by a stratum with a high permeability. This stratum may be located at some distance below the depth of maximum frost penetration.)

Consider first a soil element located at a distance Z_1 below the ground surface. Z_1 is assumed to be smaller than H_a. Before completion of the thawing process, the static pore pressure at Z_1 will be equal to $\gamma_w Z_1$. If the distance $(H_a - Z_1)$ is less than the height of capillary rise for the subgrade soil, the static pore pressure at Z_1 after completion of drainage will be equal to $-(H_a - Z_1)\ \gamma_w$. The increase in excess pore

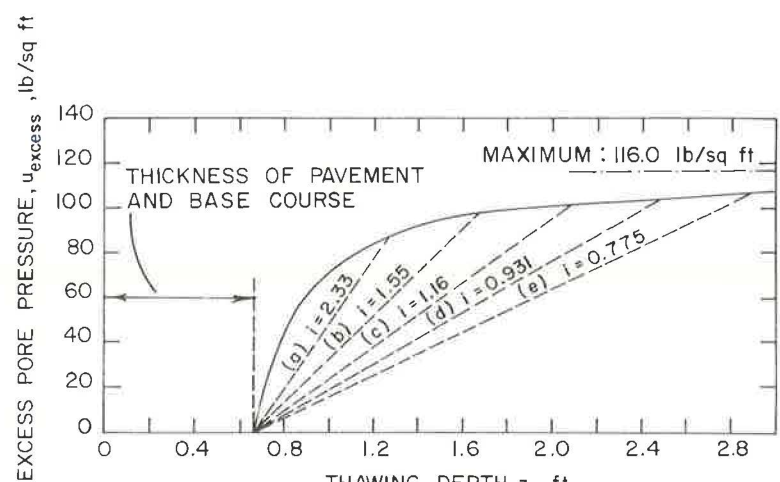

Figure 9. Relationship between excess pore pressure and thawing depth (case 3).

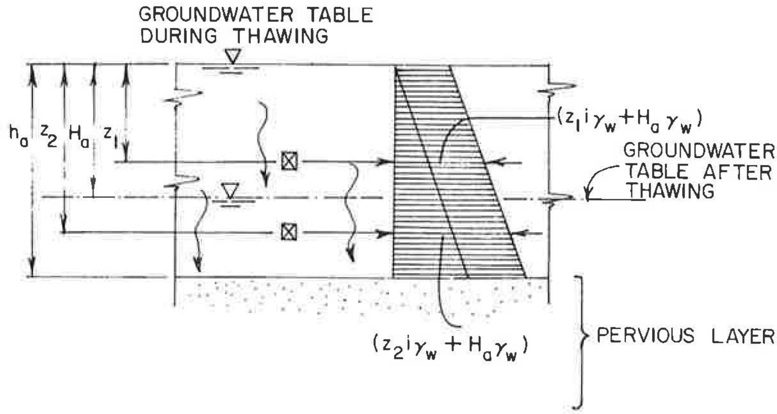

Figure 10. Distribution of excess pore pressure.

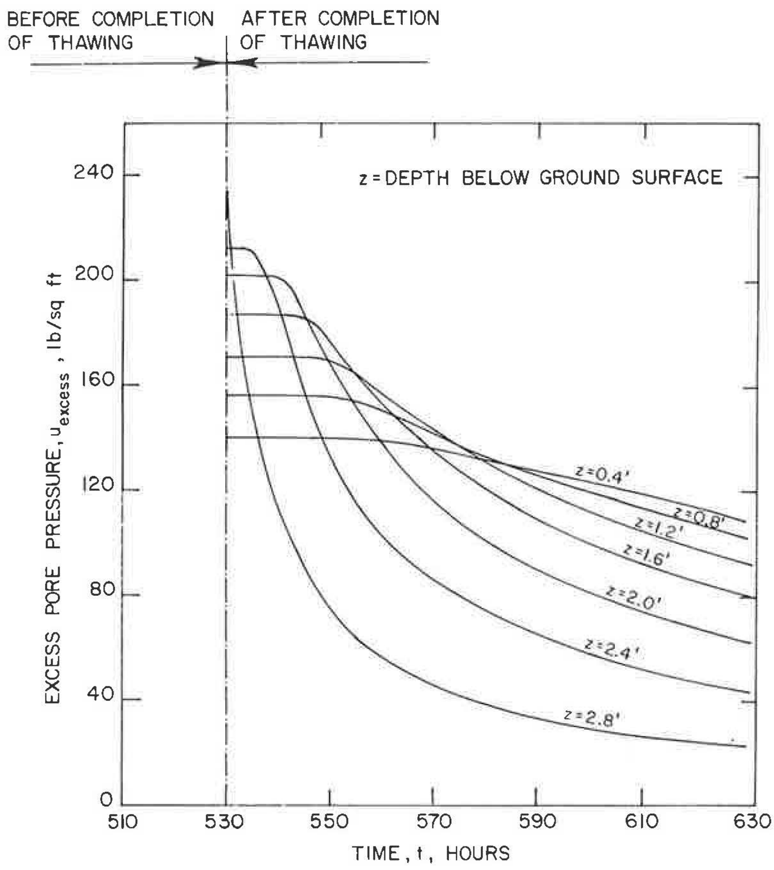

Figure 11. Dissipation of excess pore pressure.

pressure at the completion of thawing will be equal to the difference in static pore pressures before and after completion of thawing. This increase is equal to:

$$\Delta u_{excess} = Z_1 \gamma_w + (H_a - Z_1)\, \gamma_w = H_a \gamma_w \qquad (31a)$$

Consider next a soil element located a distance Z_2 below the ground surface where Z_2 is larger than H. Before drainage, the static pore pressure at a distance Z_2 from the surface will be equal to $Z_2 \gamma_w$. The static pore pressure at the same level after completion of thawing will be equal to $(Z_2 - H)\, \gamma_w$. Therefore, the resulting change in excess pore pressure is equal to:

$$\Delta u_{excess} = Z_2 \gamma_w - (Z_2 - H_a)\, \gamma_w = H_a \gamma_w \qquad (31b)$$

From these two examples it can be seen that the change in excess pore pressure during completion of thawing is independent of depth and is equal to $H_a\, \gamma_w$. It should be pointed out, however, that the total pressure is not changed at completion of thawing because no additional load is imposed.

The total excess pore pressure will then be equal to the sum of the initial excess pore pressure before completion of thawing and increase of excess pore pressure which occurs at completion of thawing.

The distribution of excess pore pressures during thawing has been discussed in the previous section. This excess pore pressure can be calculated if the corresponding hydraulic gradient is known. At the depth Z, the excess pore pressure during thawing is equal to $iZ\, \gamma_w$. The total excess pore pressure after completion of thawing can then be evaluated as:

$$u_{excess} = iZ\gamma_w + H_a \gamma_w \qquad (32)$$

The dissipation of this excess pore pressure can be calculated by the Terzaghi consolidation theory (20). A numerical method proposed by Gibson and Lumb (7) utilizing this theory permits the determination of the excess pore pressure at any given location and at any given time.

The dissipation of excess pore pressure has been calculated for the soil conditions assumed for case 1 (Fig. 2). It has been assumed that h_a and H_a are 3.0 and 2.0 ft, respectively; that a pervious layer is located 3.0 ft below the ground surface and that no excess pore pressure exists within this pervious stratum at any time.

It can be seen from Figure 4 that the excess pore pressure during thawing is equal to 116.0 psf at the thawing front and that this pore pressure is independent of the depth of thawing. The corresponding hydraulic gradient within the thawed soil just as the thawing front reaches the depth of maximum frost penetration after 530 hr is equal to 0.620. The resulting excess pore pressures can then be calculated from Eq. 23. The dissipation of this excess pore pressure has been calculated by the numerical method mentioned previously. The results from these calculations are presented in Figure 11 where the excess pore pressure has been plotted as a function of time. The excess pore pressure decreases rapidly with time after completion of thawing, especially close to the pervious stratum. For example, at the depth of 2.4 ft below the ground surface, the excess pore pressure 580 hr after the initiation of the thawing is about 30 percent of the excess pore pressure after 530 hr of thawing.

It can be seen from Figure 11 that the dissipation of excess pore pressure due to the sudden lowering of water table after completion of thawing is considerably faster than that before complete thawing of the soil. Therefore, it is expected that the soil will regain its initial strength in a relatively short time once the subgrade soil is completely thawed. However, it should be noted that the dissipation of excess pore pressures is affected by the location of the pervious stratum which governs the stationary groundwater table. The time required for dissipation of excess pore pressures will increase rapidly with increasing distance of the pervious layer from the ground surface. The reason for the relatively long time required for dissipation of pore pressure at the depth of 0.4 ft below the ground surface is the location of the pervious layer. This layer is assumed to be located 3 ft below the ground surface.

SUMMARY

Methods have been developed by which excess pore pressures which develop during thawing of subgrade soils can be calculated. These methods are based on the premise that the rate of thawing and rate of liberation of excess pore water decrease with increasing thawing depth and the flow rate remains constant. The thawing rate can be calculated from the general differential equation which governs the one-dimensional unsteady heat flow. The dissipation of the resulting excess pore pressures can be calculated by Darcy's law. The use of these methods has been demonstrated by three examples.

It has been shown that the excess pore pressures which develop during thawing may be very high and that certain time periods exist, usually at the beginning of the thawing period, during which the subgrade is in a quick condition. When this occurs, the pore pressure is equal to the total overburden pressure and the resulting effective stress in soil approaches zero. Therefore, the bearing capacity of the subgrade may be very low during the spring months at the beginning of the thawing period. It has been shown that the excess pore pressures which develop in the subgrade during thawing decrease rapidly with increasing thickness of the base course. This indicates that the stability of a road may be increased considerably during thawing by the presence of a pervious base course.

The dissipation of excess pore pressures after completion of the thawing process has also been investigated. There is an increase in pore pressure (in addition to the excess pore pressure created during thawing) due to the sudden lowering of the temporarily perched groundwater table to the original groundwater table. The dissipation of these pore pressures can be calculated by the Terzaghi consolidation theory. The pore pressures decrease rapidly with time for those stratum located close to a pervious layer, and the soil will regain its initial strength within a relatively short time once the subgrade soil is completely thawed.

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Appendix

NOTATIONS

A = area (sq ft);

$\bar{A}_f$ = pore pressure coefficient;

a = thickness of pavement, ft;

c = specific heat;

c_u = apparent cohesion determined by undrained triaxial or direct shear tests (psf);

c' = apparent cohesion with respect to effective stresses determined from consolidated-undrained triaxial or direct shear tests (psf);

G_s = specific gravity of solids;

H = equivalent thickness (ft) (Eq. 25);

H_a = location below ground surface of permanent groundwater table (ft);

H_1 = thickness of base course (ft);

h = length of flow path (ft);

i = hydraulic gradient;

K_o = coefficient of lateral earth pressure at rest;

K, K_1 = coefficient of thermal conductivity (Btu/ft-hr-°F) (Eq. 7b);

k = permeability of soils (ft/hr);

m, m_1 = $\sqrt{\dfrac{2K_1(T_s - T_f)}{Q_L \gamma_w n}}$ (Eq. 8c);

n = porosity;

N_c = bearing capacity factor;

p_o = initial total pressure (psf);

$\bar{p}_o$ = initial effective pressure (psf);

p_1 = major total principal stress (psf);

p_3 = minor total principal stress (psf);

q_{ult}^{net} = net ultimate bearing capacity, (psf);

Q = excess melted water (cu ft);

Q_L = latent heat of fusion (Btu/lb);

S_r = degree of saturation (%);

t = elapsed time from beginning of thawing period (hr);

T = temperature at time t and depth Z (°F);

T_s = surface temperature (°F);

T_f = freezing temperature (°F);

u = total pore pressure (psf);

u_o = static or initial pore pressure (psf);

u_{excess} = excess pore pressure (psf);

V = total volume (cu ft);

v = flow velocity (ft/hr);

w = moisture content of frozen soil (%);

w_o = moisture content of thawed soil (%);

Z, Z_1, Z_2 = depth (ft);

Z_{cr} = critical depth (ft) (Eq. 18);

Z_o = depth of thawing (ft) (Eq. 8a);

φ' = apparent angle of internal friction with respect to effective stresses measured by consolidated-undrained triaxial or direct shear tests (deg);

γ_p = unit weight of pavement material (pcf);

γ_{sat} = saturated unit weight of soil (pcf);

γ_{sub} = submerged unit weight of soil (pcf);

γ_w = unit weight of water (pcf);

α = thermal diffusity (sq ft/hr); and

ρ = density of material (pcf).