# Linear Programming Test of Journey-to-Work Minimization

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•THE TRANSPORTATION planner, as well as the city planner, is concerned with the spatial arrangement of activities within an urban region. The transportation planner looks for an understanding of the different arrays of human activity in order to plan a transportation system which will serve the region most efficiently. The city planner seeks to achieve a better arrangement of these activities in order to maximize the benefits accruing to urban living.

Since residential land comprises the bulk of urban development and is the base on which the bulk of urban travel is organized, residential location is a major concern of analysis of urban spatial patterns. Residential location in turn is thought to be influenced most heavily by the location of workplaces and by the length of the journey-to-work. It has generally been assumed—implicitly and frequently explicitly—that workers are not indifferent to the length of the work trip. The usual hypothesis is that persons attempt to minimize their journey-to-work in selecting among potential residences subject to a variety of other influences such as income, residential amenity, auto ownership, family characteristics, and personal preferences. There can be little doubt that the journey-to-work influences residential location. The principal question, which has not been adequately answered, is the degree of influence. To what extent does the journey-to-work influence residential location, and to what extent is journey-to-work travel traded off against other factors? One approach to understanding the magnitude of the influence of the work trip on the residential location decision is to compare the actual travel time involved in the journey-to-work with some expression of what the travel time might be under some ideal condition. This paper discusses a method by which the sensitivity to travel time can be measured. Called an index of indifference, it attempts to measure the extent to which the linkages among urban space activities (in this case, places of residences and places of work) are indifferent to time. Specific application is made to the journey-to-work in the Buffalo, N. Y. area.

## THE MEASURES

Several measures are already available by which to study spatial distributions. These vary from simple average travel times and distances to sophisticated measures of accessibility and of minima or maxima. Many of these measures are absolute, which, while useful, do not permit comparisons between different activities; that is, they do not standardize for basic differences in geographic distribution. The access measures of time (involving an exponential treatment) are difficult to interpret and to relate to alternative measures such as minima or maxima. Minimum measures tend to be hypothetical unless related in some way to actual or other hypothetical measures. What is needed is a measure that relates actual linkages to the linkages which would result if time within the region meant nothing and if time meant everything, e. g., as if a travel czar decreed that overall travel be minimized.

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Paper sponsored by Committee on Land Use Evaluation.

## Notion of Probable Interchanges

The most probable set of interchanges which would occur if travel were irrelevant to location would be a simple proportional distribution as shown by the following formula:

$$L_{ij} = \frac{A_i B_j}{\Sigma B_j} \qquad (1)$$

where $L_{ij}$ is the linkage between activity A located in zone i and activity B located in zone j. This particular measure has been used as a base against which to compare actual linkages, particularly in the gravity approach to linkages. For example, the gravity model can be expressed as:

$$L_{ij} = \frac{K A_i B_j T_{ij}^{-x}}{\Sigma B_j} \qquad (2)$$

where

K = constant of proportionality,
$T_{ij}$ = travel time between zones i and j, and
x = empirically derived constant reflecting friction of space.

This formula is identical with Eq. 1 if x = 0 and k = 1. This is the distribution which we would obtain if the friction of space measured in time units were zero. The exponent in a gravity model can be measured by the rate of change of the ratio of the actual linkage to the probable linkage per rate of change of travel time. This exponent, though useful as a measure of propensity for interaction, says nothing about the minimization of time for a set of linkages. Presumably, there is some exponent which would give the same average time as the average time for the minimum case. It probably would not satisfy the criterion that all linkages be made between the two activity types. As the exponent is further decreased (moving toward negative infinity), serious system inbalances occur and absurdity is the result.

## Notion of Minimum Time Linkage

This notion states that linkages between two activity types can be rearranged in such a way that the travel time represented by the linkages is the minimum possible. This is a system minimum rather than a series of individual minima.

The notion also assumes that there is equal substitutability within the activities involved. For example, if work places and worker residences are the two activities, it is assumed that one job is as attractive as another and that all residences are equally attractive. The defects in this assumption are mitigated by the use of strata or classes within the activity types. It is presumed that the classes are homogeneous and that within classes equal substitutability exists.

## Notion of Actual Travel Time

The actual travel time is the travel time required by the linkages as they actually occur in the real world. Origin and destination studies are specifically designed to inventory linkages. When these linkage sets are assigned to a transportation network and the travel times are recorded, we obtain the aggregate actual travel time. This time does not include terminal time, that is, the time spent walking to and from the vehicle or waiting for a transit vehicle. Since the travel time for all three cases—minimum, probable, and actual—are calculated across the same network, the exclusion of this time seems justified.

Calculating Measure

Given the three sets of linkages and the travel time required for each set, the index follows:

$$I_i = \frac{T_A - T_M}{T_P - T_M} \qquad (3)$$

where

$I_i$ = index of indifference,
$T_M$ = minimum travel time for linkages,
$T_A$ = actual travel time for linkages, and
$T_P$ = probable travel time for linkages.

Thus, we have a measure which relates actual travel linkages to the range of travel time defined by a set of linkages in which travel time is irrelevant and a set of linkages resulting when aggregate travel time is minimized.

An index of indifference equal to zero is achieved when the linkages between the two activities are so formed that the travel time represented by this set of linkages would be exceeded by the travel time formed by all other possible sets of linkages. When the index of indifference is equal to one, the linkages are those which would result if time within a region was irrelevant to the location of activities. The index is thus based on the relation of actual linkages to two hypothetical sets of linkages: (a) a probable distribution where time has no bearing and (b) a minimum distribution where the time represented by the linkages is minimized.

## JOURNEY-TO-WORK APPLICATION

In this residential location study, the journey-to-work, or more specifically the work-home linkage, is defined as the factored total of the first reported work trip by each resident of each dwelling unit sampled in the Niagara Frontier home-interview survey. The data set was then stratified into white and nonwhite workers to isolate the probable irrational influence of the segregated real estate market on residential location.

The white data set was further stratified into three sets according to reported income class. This partitioning was done to determine whether indifference to the journey-to-work time varied by income class. The three income classes are: (a) those whose annual salary is less that $5, 000; (b) those whose salary is greater than $4, 999 but less than $8, 000; and (c) those whose salary is $8, 000 or greater.

A second stratification of the basic data was made to study the effect of auto availability on the journey-to-work time indifference. Both the white and nonwhite sets were partitioned based on whether or not the journey-to-work traveler was an auto driver. Thus, four sets were created (white drivers, white nondrivers, nonwhite drivers, and nonwhite nondrivers) for which time indifference was measured using the indifference index.

The reported work and home locations of each linkage in each set were coded to 435 geographic zones used for analysis and reference. The travel time between each zone and all other zones was obtained from a file of minimum path trees created by an assignment of present trips to the present network. The times were adjusted systematically to reflect volume.

To illustrate the variation in the distribution of income classes across the region, Figures 1 through 6 show the home locations and workplaces of the classes into which the population was divided. The relative concentrations among the higher and lower income groups are noticeable and support the common assumption of great selective ability among the former group and restricted opportunities for the latter.

Determining Indifference Index

The actual calculation of the indifference index is dependent on knowledge of

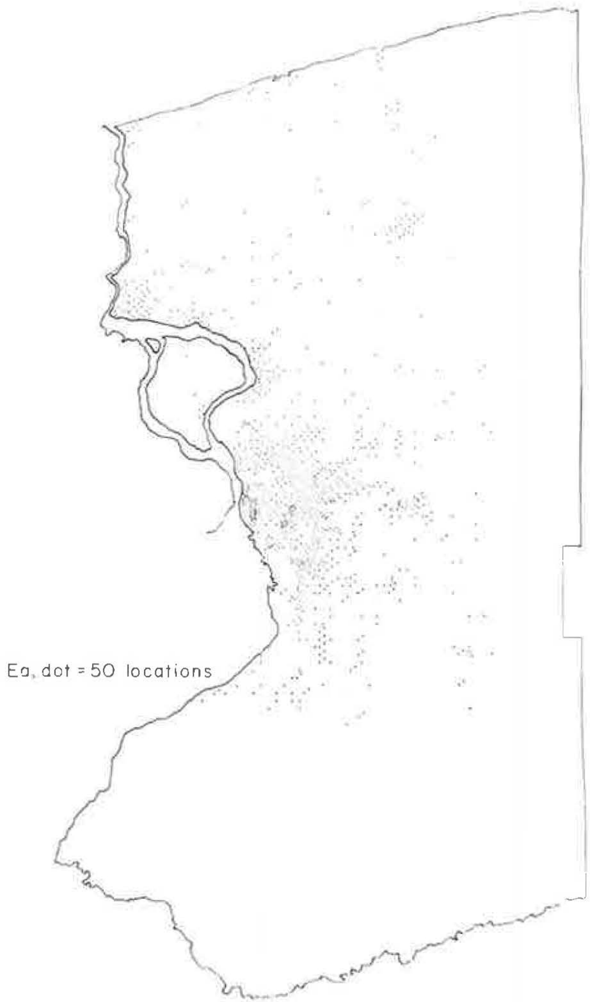


Figure 1. Residences of workers with income less than $5,000.

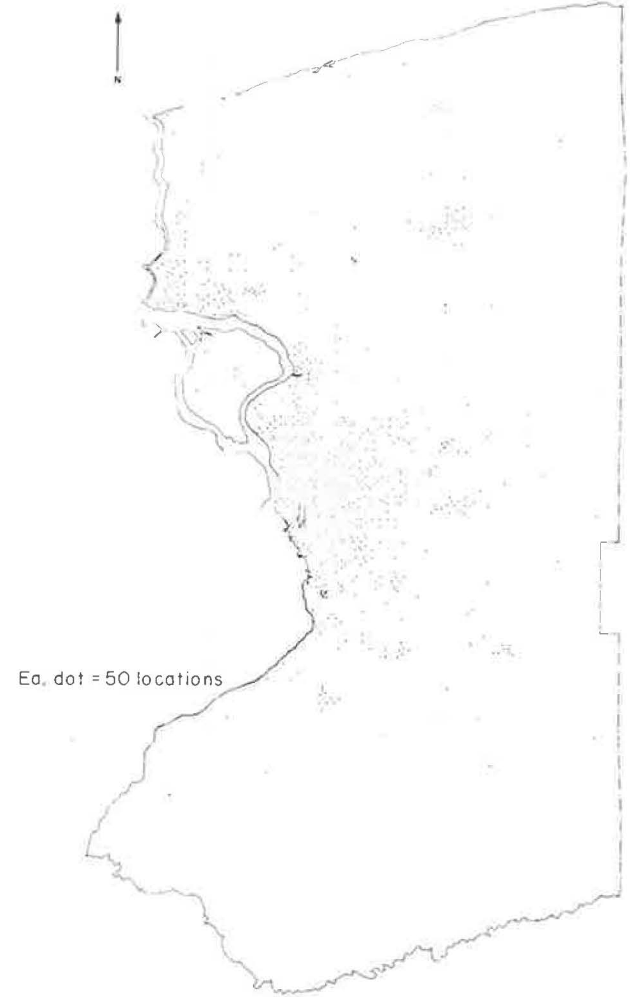


Figure 2. Workplaces of workers with income less than $5,000.

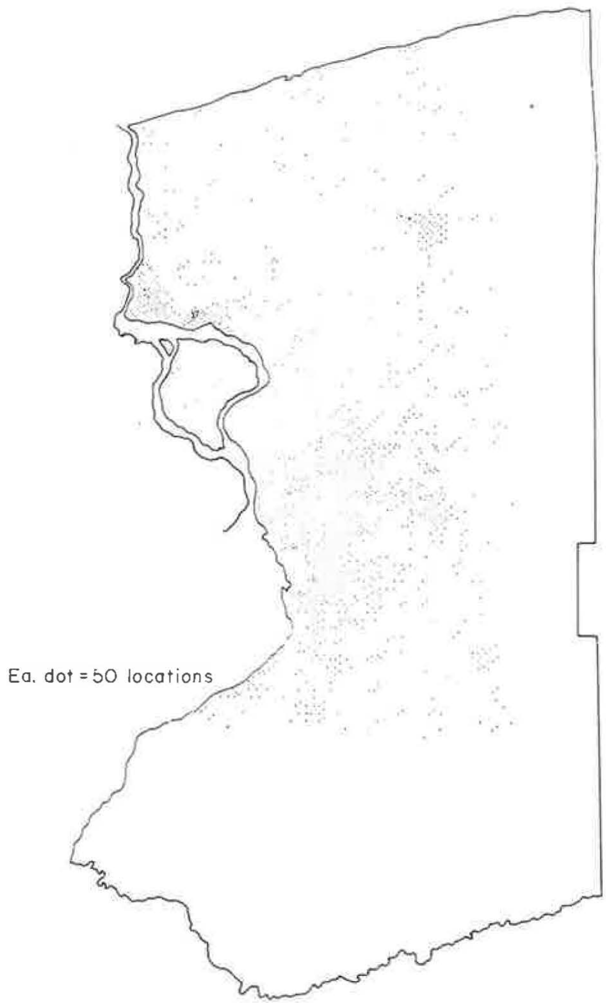


Figure 3. Residences of workers with income of $5,000 to $7,999.

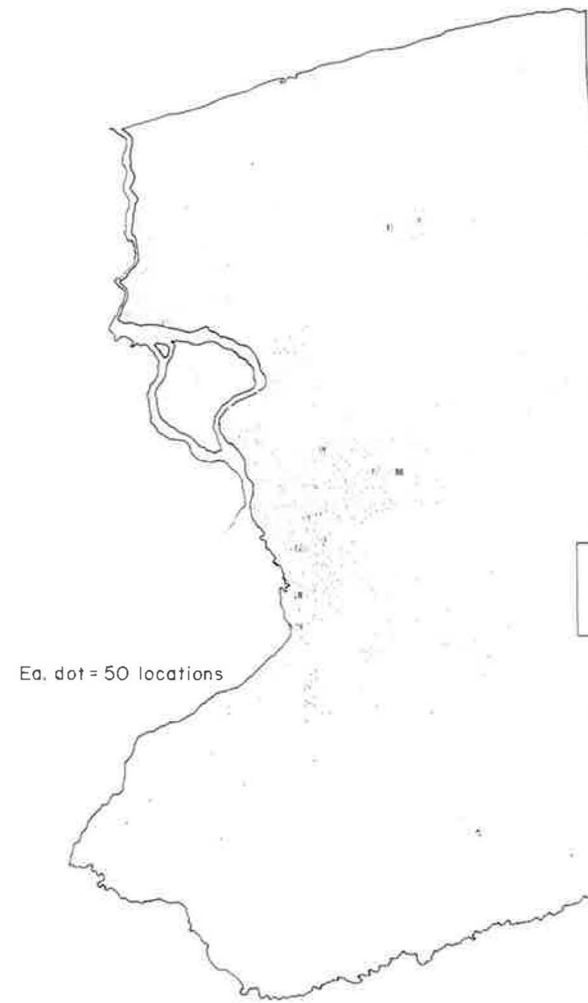


Figure 4. Workplaces of workers with income of $5,000 to $7,999.

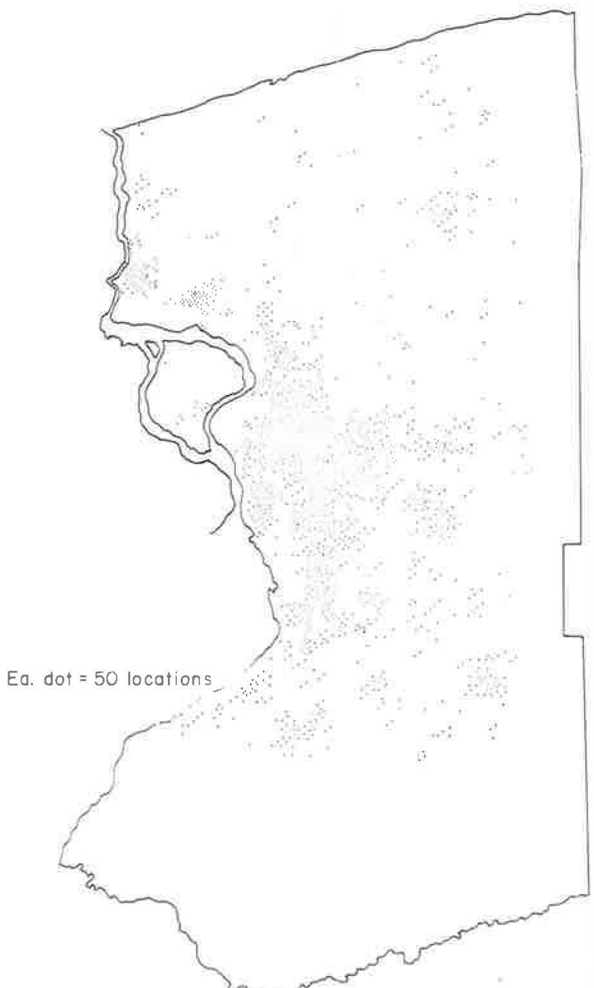


Figure 5. Residences of workers with income of $8,000 and over.

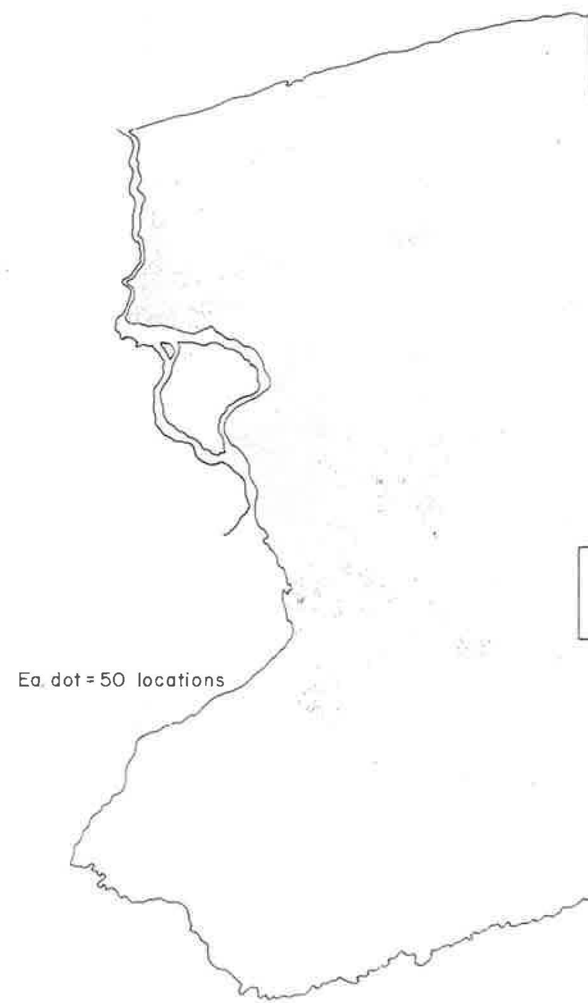


Figure 6. Workplaces of workers with income of $8,000 and over.

three points on the continuum representing the extent of indifference: complete indifference, absolute system minimization, and the world as it exists.

Complete substitutability of homes was assumed within each set. That is, when calculations were made for a specific set, all homes were considered to be equally acceptable to all workers.

The real-world value was obtainable quite simply by summarizing the travel times for the defined journey-to-work trip file. Stated mathematically:

$$T = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} X_{ij} \tag{4}$$

where

$T$ = total travel time,
$a_{ij}$ = travel time from zone i to zone j,
$X_{ij}$ = trips from zone i to zone j,
m = number of work zones, and
n = number of residence zones.

Complete indifference was simulated by allocating workers to homes on a proportional basis. Total travel time, T, was as above, but interchanges were redefined as:

$$X_{ij} = \frac{J_i H_j}{\sum_{j=1}^{n} H_j} \tag{5}$$

where

$X_{ij}$ = trips from zone i to zone j,
$J_i$ = workplaces in zone i,
$H_j$ = homes in zone j, and
n = number of home zones.

Absolute system minimization was obtained using a form of linear programming commonly known as the transportation problem. The solution to the transportation problem allocates linkages between supply points and demand points in a manner that minimizes the total cost of the linkage. This technique requires that the total number of supplies and demands be equal and that all supplies and demands be used (Eq. 4), so that:

$$\sum_{j=1}^{n} X_{ij} = J_i$$

and

$$\sum_{i=1}^{m} X_{ij} = H_j$$

with all terms as previously defined.

The supplies and demands in the journey-to-work study were, respectively, workers at place of work and homes. Therefore, the transportation problem solution was an allocation of workers to homes which minimized the total travel time.

The data processing required to determine the points for each indifference index for the real world and the complete indifference points were performed on the Studies' own computer installation, an IBM 1401, using locally written programs. Each minimization was performed with a modified version of the Dennis Transportation Code, as distributed by SHARE for use on an IBM 7090/94.

## JOURNEY-TO-WORK RESULTS

Table 1 gives the results of the analysis of journey-to-work by income class. The index of indifference ranges from 0.39 to 0.63 for the income classes and is 0.39 for all classes. This indicates that persons are not indifferent to travel time, but it does not demonstrate that they minimize aggregate travel time. Since an index value of 0.5 is midway from complete minimization to complete indifference to travel time, the results suggest that minimization is a potent influence.

This conclusion is reinforced by consideration of the nature of the travel minimizing linkage set. Like any ideal construct, the time minimization linkages are quite unrealistic. The mathematical solution is such that workers in each zone select residences on the average in only two of the 435 zones. Actually, workers in each zone are linked with residences in a sizable percentage of all zones. By contrast, the proportional allocation which links workers in each zone with residences in every zone is a much better representation of actual linkage patterns. The results also show that indifference to travel time increases with income. This is as expected.

Table 1 also gives the results of the analysis by race and auto availability classifications. The index of indifference is relatively stable for white drivers, white nondrivers, and nonwhite drivers and the value of the index approximates the previous average. However, nonwhite nondrivers are shown to be relatively indifferent to travel time.

The stratification by race was made originally because it was presumed that the effective location markets were different. This notion can be tested by examining the range between the travel time required for the minimum linkages and for the probable linkages. If either or both the distribution of workplaces and places of residence is constricted, the range will be relatively narrow. In the extreme case, if all residential opportunities were limited to one zone, the travel time required by the minimum

TABLE 1
JOURNEY-TO-WORK ANALYSIS

| Class | No. Workers | Total Travel Time (hr) | | | Avg. Travel Time (min) | | | Indifference Index |
|---|---|---|---|---|---|---|---|---|
| | | Indifference | Actual | Minimum | Indifference | Actual | Minimum | |
| (a) Income[a] | | | | | | | | |
| 1 | 51,242 | 21,820.9 | 9,973.8 | 3,223.0 | 25.5 | 11.7 | 3.8 | 0.36 |
| 2 | 108,696 | 50,806.6 | 25,358.0 | 9,059.3 | 28.0 | 14.0 | 5.0 | 0.39 |
| 3 | 92,750 | 32,595.4 | 24,498.8 | 10,709.1 | 21.1 | 15.8 | 6.9 | 0.63 |
| Total | 252,688 | 117,379.4 | 59,830.6 | 22,411.0 | 27.9 | 14.2 | 5.3 | 0.39 |
| (b) Race and Auto Availability | | | | | | | | |
| White driver | 247,099 | 87,140.2 | 45,827.9 | 18,954.9 | 21.1 | 11.1 | 4.6 | 0.39 |
| White nondriver | 45,536 | 12,846.1 | 6,625.8 | 2,545.6 | 16.9 | 8.7 | 3.3 | 0.40 |
| Nonwhite driver | 10,616 | 2,565.2 | 1,839.1 | 1,341.0 | 14.4 | 10.3 | 7.6 | 0.41 |
| Nonwhite nondriver | 6,919 | 3,254.7 | 1,716.9 | 755.6 | 11.8 | 9.1 | 6.6 | 0.65 |
| (c) Sampled and Factored Linkages | | | | | | | | |
| Sample | 9,236 | 3,254.7 | 1,716.9 | 715.9 | 21.1 | 11.1 | 4.6 | 0.394 |
| Factor | 247,099 | 87,140.2 | 45,827.9 | 18,954.9 | 21.1 | 11.1 | 4.6 | 0.394 |

[a]Does not include samples with nonreported income.

linkages would equal that required by the probable linkages. A dispersion index constructed as the ratio of the difference between probable linkage travel time and minimum linkage travel time to probable linkage travel time measures this influence. If dispersion is small (minimum and probable times are equal) the ratio approaches zero. As dispersion becomes large the ratio approaches one. The dispersion values are 0.78 for white drivers, 0.81 for white nondrivers, 0.47 for nonwhite drivers, and 0.44 for nonwhite nondrivers. The differences between the white and nonwhite values are sufficiently large to support the assumption of difference in the dispersion of spatial opportunities of the two groups.

The possible influence of auto availability is less clear. It makes no significant difference among the white group. However, it appears to make a great deal of difference among the nonwhite group since there is no significant difference in the dispersion index for nonwhite drivers and nonwhite nondrivers. The results suggest that other factors, not controlled in this analysis, provide the correct explanation. Chief among these might be the household status of the worker, i.e., whether head of the household or secondary worker such as a working wife.

The effect of using the factored number of work-home linkages instead of the actual sampled number in the transportation problem was questioned. Two indifference indexes were determined for the white driver set, one using the factored number of linkages and one using the actual sample number (Table 1). The two indexes are equal.

## SUMMARY

This paper has described an index which permits meaningful comparisons of spatial arrays of urban activity. The index simultaneously standardizes a spatial distribution of linkages against the time minimum and time indifferent linkage sets.

The use of this index has been illustrated with journey-to-work data from the Niagara Frontier Transportation Study. The analysis showed that people overall are not indifferent to time. They do not, however, organize their linkages so as to expend the minimum amount of time in travel, although they tend to exhibit more minimization than indifference.

When the data were stratified into income classes, those in the highest income class tended to be indifferent to travel time as measured by the index. Both the middle- and low-income groups appeared more sensitive to travel time than the high-income class, although the lowest income group tended to be only slightly less indifferent than the middle-income class. When the data were stratified by race and auto availability, inconclusive results were obtained.

Additional analysis of the home and workplace linkages are planned with special emphasis on the assumption of equal substitutability of jobs and residences. The distributions will be standardized for differences in family size, car ownership, occupation and industry of employment, and other socio-economic differences beyond income itself.

The use of the index will be extended beyond the area of the journey-to-work. Analysis of market areas of different activity types, such as retailing and recreation areas, may profit from the application of this measure. Another linkage which may be studied is that of school-home.

The ability to plan for the arrangement of human activities in a region is heavily dependent on the planner's understanding of the way the region is currently organized and, more importantly, his understanding of why the region is so arranged. It is felt that this index will be useful in measurement and, therefore, verification of theories of urban spatial structure.