Design of Langer Girder Bridge with Inclined Hangers

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The structure of arch-type bridges with inclined hangers was studied by the deformation method. The Abo bridge, the first Langer girder bridge with inclined hangers in Japan, was designed by the authors and construction is under way. Compared with bridges having vertical hangers, a weight saving of about 10 percent has been obtained with inclined hangers.

• THE STATICAL behavior of the Langer girder, the tied arch, and the Lohse girder bridges with inclined hangers has been under extensive investigation in the Scandinavian countries. These bridges are usually called Nielsen system bridges. The Fehmarnsund bridge recently constructed in West Germany was the first bridge of this kind outside the Scandinavian countries, and it has been reported that the chief advantage of this type of bridge is the saving in the amount of steel used in the construction. The authors have been studying the structure of the arch-type bridges with inclined hangers by the deformation method, and have set up a computer program. The Abo bridge, the first Langer girder bridge with inclined hangers built in Japan, has been designed by the authors and construction is under way. This report presents the computing procedure relative to the designing of this type of bridge.

SOLUTION BY DEFORMATION METHOD

The static equilibrium in each joint of the planar structures is expressed as shown in Eq. 1 by the deformation method:

$$\begin{bmatrix} a & b & c \\ b' & \bar{a} & \bar{c} \\ c' & \bar{c}' & d \end{bmatrix} \begin{bmatrix} u \\ v \\ \theta \end{bmatrix} = \begin{bmatrix} \bar{P} \\ \bar{Q} \\ \bar{M} \end{bmatrix} \text{ or } KU = S \tag{1}$$

where

$b', c', \bar{c}'$ = transposed matrices of b, c, and $\bar{c}$, respectively;
u, v, θ = column matrices of u_i, v_i, and θ_i elements, respectively, where u_i, v_i, and θ_i are the displacements of each joint in x and y directions and joint rotation;
$\bar{P}, \bar{Q}, \bar{M}$ = column matrices of $\bar{P}_i$, $\bar{Q}_i$, and $\bar{M}_i$, where $\bar{P}_i$ and $\bar{Q}_i$ are the external forces in x and y directions at each joint, and $\bar{M}_i$ is the external moment acting at each joint; and
K = stiffness matrix.

The submatrices contained in K are defined as follows:

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$$a = \begin{bmatrix} \nwarrow & & -a \\ & \Sigma a & \\ -a & & \searrow \end{bmatrix}, \quad \bar{a} = \begin{bmatrix} \nwarrow & & -\bar{a} \\ & \Sigma \bar{a} & \\ -\bar{a} & & \searrow \end{bmatrix}, \quad b = \begin{bmatrix} \nwarrow & & -b \\ & \Sigma b & \\ -b & & \searrow \end{bmatrix}$$

$$c = \begin{bmatrix} \nwarrow & & c \\ & \Sigma c & \\ c & & \searrow \end{bmatrix}, \quad \bar{c} = \begin{bmatrix} \nwarrow & & \bar{c} \\ & \Sigma \bar{c} & \\ \bar{c} & & \searrow \end{bmatrix}, \quad d = \begin{bmatrix} \nwarrow & & d \\ & 2\Sigma d & \\ d & & \searrow \end{bmatrix} \qquad (2)$$

where each Σ shows the diagonal elements of the matrices and the other symbols show the antidiagonal elements.

Each of the elements represented by such symbols as a, $\bar{a}$, b, c, $\bar{c}$, and d in Eq. 2 will be determined as follows by using the various dimensions of the members:

$$a_{ij} = \frac{12EI_{ij}}{l^3_{ij}} \frac{(y_j - y_i)^2}{l^2_{ij}} + \frac{EA_{ij}}{l_{ij}} \frac{(x_j - x_i)^2}{l^2_{ij}}$$

$$\bar{a}_{ij} = \frac{12EI_{ij}}{l^3_{ij}} \frac{(x_j - x_i)^2}{l^2_{ij}} + \frac{EA_{ij}}{l_{ij}} \frac{(y_j - y_i)^2}{l^2_{ij}}$$

$$b_{ij} = \left(\frac{EA_{ij}}{l_{ij}} - \frac{12EI_{ij}}{l^3_{ij}} \right) \frac{(x_j - x_i)}{l_{ij}} \frac{(y_j - y_i)}{l_{ij}}$$

$$c_{ij} = \frac{6EI_{ij}}{l^2_{ij}} \frac{(y_j - y_i)}{l_{ij}}$$

$$\bar{c}_{ij} = \frac{6EI_{ij}}{l^2_{ij}} \frac{(x_j - x_i)}{l_{ij}}$$

$$d_{ij} = \frac{2EI_{ij}}{l_{ij}} \qquad (3)$$

The sectional forces of each member are determined from the displacements, after computing the inverse matrix of K, as follows:

$$M_i = c_{ij} (u_j - u_i) - \bar{c}_{ij} (v_j - v_i) + d_{ij} (\theta_j + 2\theta_i) \frac{(x_j - x_i)}{| x_j - x_i |}$$

$$N_i = \left\{ a_{ij} \frac{(x_j - x_i)}{l_{ij}} + b_{ij} \frac{(y_j - y_i)}{l_{ij}} \right\} (u_j - u_i) +$$

$$\left\{ b_{ij} \frac{(x_j - x_i)}{l_{ij}} + \bar{a}_{ij} \frac{(y_j - y_i)}{l_{ij}} \right\} (v_j - v_i) +$$

$$\left\{ c_{ij} \frac{(x_j - x_i)}{l_{ij}} - \bar{c}_{ij} \frac{(y_j - y_i)}{l_{ij}} \right\} (\theta_j + \theta_i)$$

$$Q_i = \left\{ b_{ij} \frac{(x_j - x_i)}{l_{ij}} - a_{ij} \frac{(y_j - y_i)}{l_{ij}} \right\} (u_j - u_i) +$$

$$\left\{ \bar{a}_{ij} \frac{(x_j - x_i)}{l_{ij}} - b_{ij} \frac{(y_j - y_i)}{l_{ij}} \right\} (v_j - v_i) -$$

$$\left\{ \bar{c}_{ij} \frac{(x_j - x_i)}{l_{ij}} + c_{ij} \frac{(y_j - y_i)}{l_{ij}} \right\} (\theta_j + \theta_i) \qquad (4)$$

COMPUTATIONAL PROCEDURE

The block diagram in Figure 1 shows the process of solution of the problem. A number is assigned to each of the joints. If there are 2n joints, the numbers will be 0, 2, 4, . . . , 2n in the lower joints and 1, 3, 5, . . . , 2n - 1 in the upper joints as shown in Figure 2.

The following input data are needed:

1. The projections $(x_j - x_i)$ and $(y_j - y_i)$ of the member $\overline{ij}$ in the x and y directions, respectively;

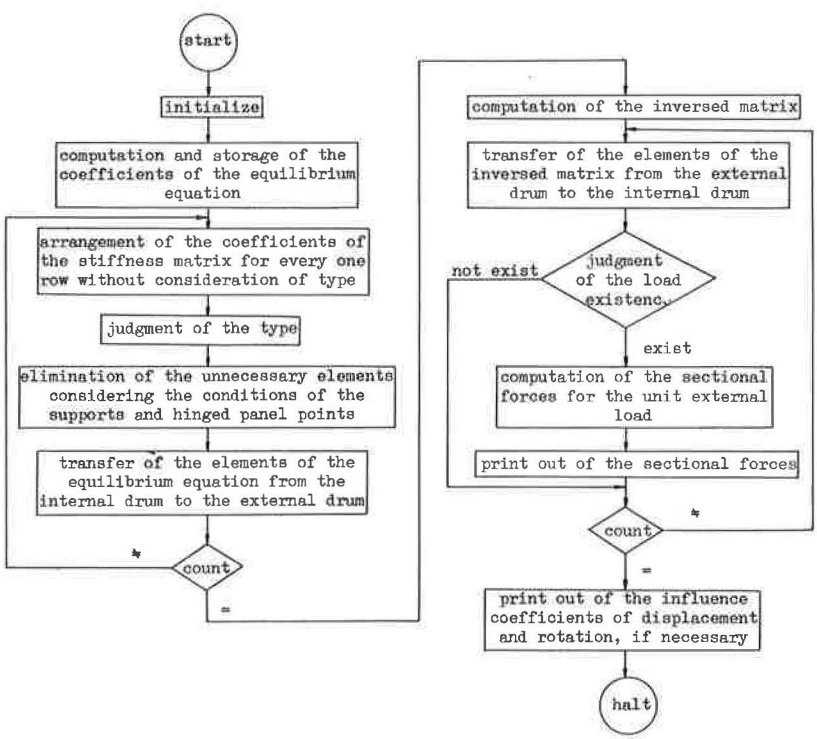

Figure 1. Block diagram for computer analysis.

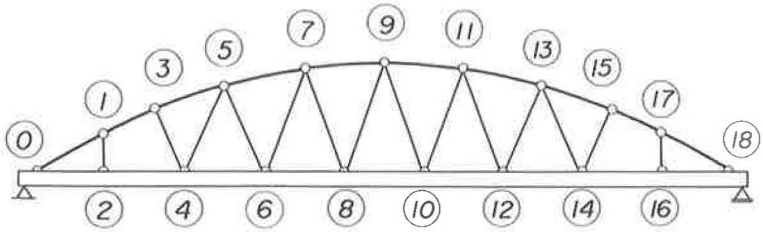

Figure 2. Example of number assignment to panel point.

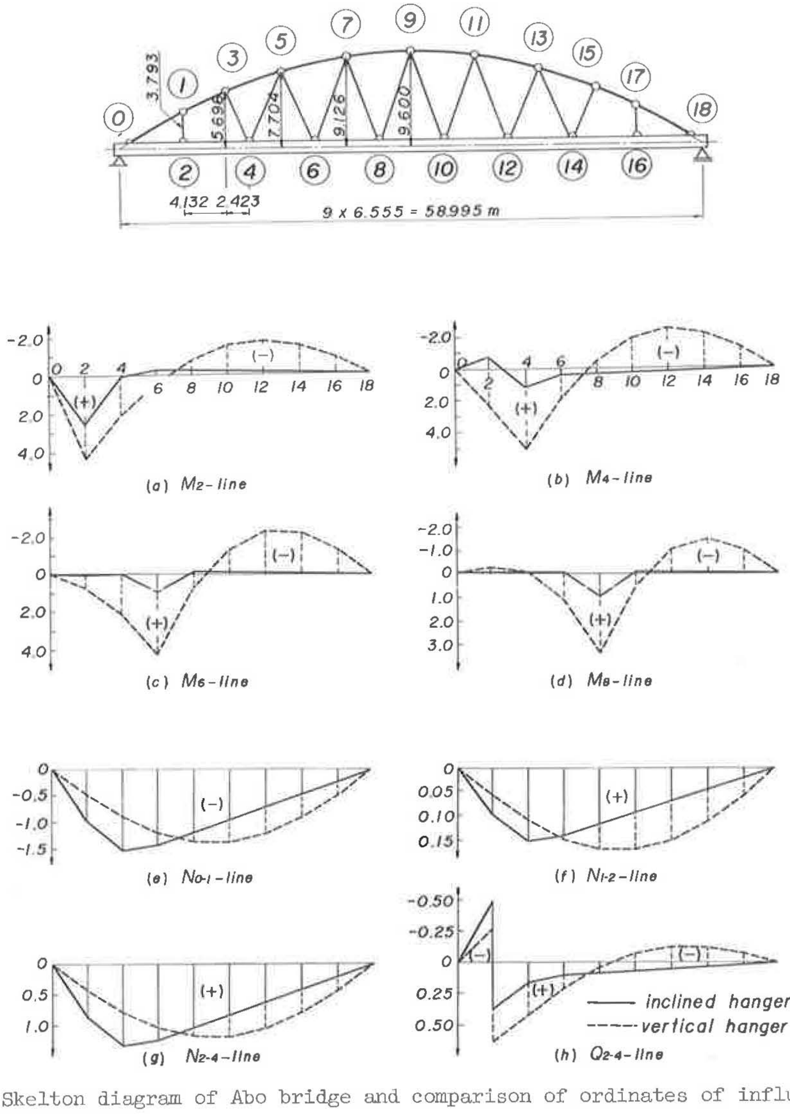

Figure 3. Skelton diagram of Abo bridge and comparison of ordinates of influence lines.

TABLE 1

NORMAL FORCES OF UPPER CHORD MEMBERS AND HANGERS

Member	Dead Load	Live Load	Impact	Total
N_{01}	-123.455 t	-51.375 t	-9.453 t	-183.283 t
N_{13}	-117.651	-48.960	-9.009	-175.620
N_{35}	-117.839	-49.038	-9.023	-175.900
N_{57}	-117.569	-49.213	-9.005	-175.839
N_{79}	-115.489	-48.432	-8.911	-172.832
N_{12}	12.601	5.244	0.965	18.810
N_{34}	11.111	4.604	0.847	16.562
N_{45}	2.396	11.367	3.990	18.293
		-7.381	-2.591	-7.917
N_{56}	12.514	10.452	3.669	26.635
		-6.758	-2.372	-0.370
N_{67}	8.385	11.082	3.890	23.357
		-5.844	-2.051	-2.026
N_{78}	9.250	9.996	3.509	22.755
		-6.392	-2.244	-2.161
N_{89}	8.803	10.749	3.773	23.325
		-6.524	-2.290	-2.652

TABLE 2

BENDING MOMENTS AND NORMAL FORCES OF STIFFENING GIRDER

Member	Dead Load	Live Load	Impact	Total
(a) Applied Loads Producing Maximum Bending Moment				
M_2	31.293 tm	40.545 tm	7.460 tm	79.298 tm
M_4	36.044	27.167	4.999	68.210
M_6	15.435	16.260	2.992	34.687
M_8	12.897	15.942	2.933	31.772
N_{02}, N_{24}	106.858 t	40.401 t	7.434 t	154.693 t
N_{46}	109.997	40.523	7.456	157.976
N_{68}	112.062	25.574	4.706	142.342
N_{8-10}	112.345	20.065	3.692	136.102
(b) Applied Loads Producing Maximum Normal Force				
N_{02}, N_{24}	106.858 t	44.468 t	8.182 t	159.508 t
N_{46}	109.997	45.521	8.376	163.894
N_{68}	112.062	46.336	8.526	166.924
N_{8-10}	112.345	46.131	8.488	166.964
M_2	31.293 tm	10.501 tm	1.932 tm	43.726 tm
M_4	36.044	23.925	4.402	64.371
M_6	15.435	15.538	2.859	33.832
M_8	12.897	14.647	2.695	30.239

2. The cross-sectional area A_{ij} and the moment of inertia I_{ij} of each member;
3. Numbers of the members at each joint and the assigned numbers at the other end of each member;
4. Applied loads;
5. Total numbers of the members;
6. Total numbers of the joints including the supports;
7. Total numbers of the pinned joints excluding the supports; and
8. Smallest number assigned at the pinned joints.

The output data consist of the ordinates of the influence lines of bending moment M at all rigid joints, of normal force N and shearing force Q of all rigidly jointed members, and of normal force N of all pin-jointed members.

This computational procedure has been programmed for the NEAC-2203 computer of Nagoya University, and is applicable to the Langer girder, the tied arch, and the Lohse girder with inclined or vertical hangers up to 15 panels. The details have been previously presented (1).

DESIGN OF ABO BRIDGE

Dimensions of the bridge include span length, 58.995 m; effective width, 6.0 m; specified load, 2nd class of Japan Standard Specification for Steel Highway Bridges (1962); distance between stringers and crossbeams, 2.3 and 6.555 m, respectively; slab, 15 cm thick of reinforced concrete; pavement, 5 cm thick of concrete. The assumed cross-sectional area and moment of inertia of the members are for upper chord members, A = 137 sq cm for 0-1, 17-18, and A = 123 sq cm for the others; for lower chord members, A = 223.7 sq cm and I = 834,865 cm^4 for all members; and for hangers, A = 51.2 sq cm for members 1-2 and 16-17 and A = 40.15 sq cm for the others.

Comparison of the influence lines of the sectional forces of certain members is shown in Figure 3. The normal forces of the upper chord members and hangers due to dead loads and live loads are given in Table 1. The bending moments and normal forces of the lower chord members due to dead loads and live loads are given in Table 2.

The quantities of material necessary for the bridge are computed as follows: 26.344 tons for main girder, 16.263 tons for upper chord members, 6.694 tons for hangers, 2.418 tons for portal, 1.472 tons for sway bracings, 9.844 tons for floor beams, 15.258 tons for stringers, 6.048 tons for lateral bracings, and 2.072 tons for shoes, totalling 86.413 tons (240.9 kg/sq m). The steel weight of the same type of bridge with the vertical hangers would be 95.2 tons, and, therefore, a saving of about 10 percent in the weight of steel has been achieved.

DISCUSSION

Normal stresses of the hangers due to dead loads are always positive. The member stress N_{34} is always positive for live loads, but N_{45} - N_{89} are alternate stresses, even though the negative magnitude is very small except for N_{45}. If the negative stresses in the inclined hangers are erased by some means, rods or cable wires may be used as the hanger members. In the Fehmarnsund bridge in West Germany, use of cable hangers is made possible by the additional dead weight of concrete blocks. In Sweden, rod hangers are used without the aid of any dead weight in particular. If the combined stresses become negative in some rod hangers under the dead and live loads, the rod hangers can no longer support the force. Hence, the behavior of the bridge system will change under the corresponding loads, and a special computation will be required. Therefore, pipe members are being used for the inclined hangers of the Abo bridge so that the compressive forces are taken care of by these members as well. That is, a saving in the weight of steel used is expected in the stiffening girder due to the reduction in the bending moment applied to the girder.

According to a Swedish bridge engineer, however, use of the stiff hangers instead of flexible hangers will change the characteristics of the Nielsen system bridge because the hangers will then function for any kind of load, which is not the case with the original Nielsen system.

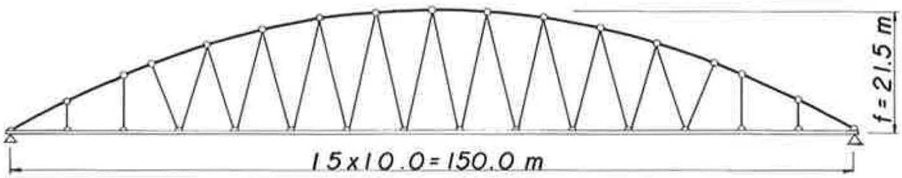

Figure 4. Skelton Diagram of Shitoku bridge.

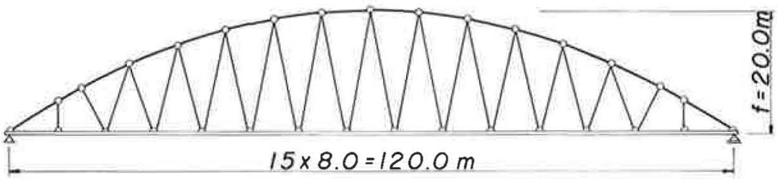

Figure 5. Skelton diagram of Shin-Ishikari bridge.

The span length of the Abo bridge is small, and, therefore, the compressive forces are caused in the inclined hangers. If the span length were longer, the compressive forces would not be caused in the inclined hangers except for N_{45}, and in such a case the cable wires or rods would be used. Because this particular bridge is the first Langer girder bridge with inclined hangers to be constructed in Japan, the pipe members have been adopted for the inclined hangers. The authors, however, are inclined to think that the bridge is not a good specimen of this special type of Langer girder bridge.

DESIGN EXAMPLES

The amount of steel used for a Langer girder bridge (Fig. 4) with a span length of 150 m and an effective width of 5.5 m will be 383.3 tons with the inclined hangers; the same bridge with the vertical hangers will require 432.8 tons of steel. The saving in the steel used in the case of the former will amount to about 11 percent.

A total of 237.2 tons of steel will be needed for a Langer girder bridge (Fig. 5) with a span length of 120 m and an effective width of 8.0 m with inclined hangers for upper and lower chord members, and hangers. With vertical hangers, the amount required would be 258.1 tons. The saving in the amount of materials by use of inclined hangers is apparent.

CONCLUDING REMARKS

As already stated, the authors have demonstrated the automatic computational process by the deformation method for the analysis of the Langer girder bridge and also have shown the several comparative designs of the various bridges. A weight saving of about 10 percent has been obtained for the bridges with inclined hangers compared with those having vertical ones.

The computer program is also applicable without any difficulty to the Lohse girder, the Vierendeel girder, and the tied arch with the inclined or vertical hangers.

REFERENCE

1. Kojima, Hiroyuki and Naruoka, Masao. Analysis of Nielsen System Bridge by Digital Computer. Prelim. Pub., 7th Cong. Int. Assoc. for Bridge and Struct. Eng., pp. 65-74, 1964.