Accuracy Requirement and Uses of Electronic-Measuring Devices for Surveying Photogrammetric Control

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•HIGHWAY DESIGN is a complex and integrated activity, and highway surveying is no exception. Different kinds of methods have to be employed simultaneously to achieve the desired results and to keep pace with the other phases of national production. These methods include surveys made in the usual manner on the ground, made by using aerial photogrammetry, and made by using electronic distance-measuring instruments; all of which are supplemented by use of high-speed electronic computers. To fulfill the demand for mass production, such methods must be harmonically organized among each other. Therefore it is necessary to have full knowledge of the nature of the methods as well as full knowledge of error propagation of the different methods. This paper contains an analysis of the methods from the error propagation point of view, providing mathematical and practical examples. All practical examples mentioned were sponsored by the Ohio Department of Highways.

AERIAL TRIANGULATION IN PRELIMINARY SURVEYING

One of the most discussed methods is aerial triangulation, which was not able to provide the required high accuracy for detailed preliminary location surveys. But relatively low accuracy is required in the reconnaissance stages of route surveys. Consequently a less accurate method, such as aerial triangulation, can be introduced to this phase of activity. An aerial triangulation method using double projection photogrammetric instruments (Kelsh type) has been investigated and published (12). This publication contains information about strip aerial triangulation by use of a double projection instrument. The photographs were taken at a scale of 500 feet per inch using a K-12 aerial camera.

The photography strip contained six stereoscopic models and was controlled at the beginning and at the end of the strip. The average standard residual error of a point was found to be ±1.0 feet and the average vertical error was about ±0.6 feet. This favorable error propagation makes it possible to compute the maximum bridging distance by use of the following empirical equation.

$$mp = h \sqrt{N}\, mp_o \tag{1}$$

in which

- mp = standard residual position error (in meters);
- h = flight height (in kilometers);
- N = number of stereoscopic models;
- mp_o = standard residual position error in meters for $N = 1$, which can be computed using the equation: $mp_o = m_o M_M$ in which M_M is the scale of the stereoscopic model, and m_o = standard error of observation.

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Using the preceding equation, an example can be computed. If the desired horizontal accuracy is not greater than ±2 feet and the mapping scale is 200 feet per inch, the number of stereoscopic models usable for the aerial triangulation would be about six, and the bridged distance would be about ten miles. Two conclusions can be drawn. First, the employment of aerial triangulation would provide about 40 to 60 percent savings in the ground control measurements required. Second, the derived mathematical equations indicate 1:10,000 relative accuracy would be suitable for ground surveyed control, because the previously mentioned ±2-ft residual error in a distance of ten miles is representing about 1:26,000 relative accuracy. But before the final answer would be given to the question, how accurately should the ground surveyed control be measured, it is necessary to analyze the surveying for right-of-way acquisition and determine what maximum accuracy is needed.

MEASUREMENT BY PHOTOGRAMMETRY FOR RIGHT-OF-WAY ACQUISITION

Several attempts have been made in the United States, as well as in foreign countries, to employ photogrammetric methods to solve right-of-way problems. Most of the methods can be classified as graphical, which have their limitations. A map compiled by photogrammetric means usually contains three categories of errors (including photographic, plotting, and measurement) which get into the right-of-way data when measurements are made from the map. If a computational method is employed, the measurements required for right-of-way computation are not obtained from the photogrammetrically compiled map, but are measured on the stereoscopic model in a precision photogrammetric instrument. In this case, the measurements are free from map plotting errors as well as from the errors in measurement, and the photographic errors can be corrected for mathematically.

This possibility was investigated and the mathematical conception of the method has been published (13). In this method the stereoscopic model coordinates have been observed and corrected by use of the following equations:

$$X_P = 0.5\,(X''_P + X'_P) + 0.5\left[\frac{X'_P}{b}\left(\frac{X''_P - X'_P}{2}\right) + \frac{X''_P}{b}\left(\frac{X''_P - X'_P}{2}\right) + \frac{X''_P - X'_P}{2}\right] \tag{2}$$

$$Y_P = 0.5\,(Y''_P - Y'_P) + 0.5\left[\frac{Y'_P}{b}\left(\frac{Y''_P - Y'_P}{2}\right) + \frac{Y''_P}{b}\left(\frac{Y''_P - Y'_P}{2}\right) + \frac{Y''_P - Y'_P}{2}\right] \tag{3}$$

in which X_P and Y_P are the correct stereoscopic model coordinates of a point, and X'_P X''_P and $Y'_P Y''_P$ are the coordinates measured on the normal and on the pseudo stereoscopic model.

The correction of the coordinates in the stereoscopic model by use of the equations leads to the result which indicated about 40 to 50 percent improvement in accuracy.

Eight different stereoscopic models were examined, and the results indicate the increase in accuracy is about 45 percent in both X and Y coordinates as well as in Z. These photographs were taken of the camera testing area of U.S.C. & G.S. in northern Ohio with a Fairchild F-501 (6-in. focal length) camera at the scale of 1:12,000 (1,000 ft per inch), and the stereoscopic model scale was 1:5,000. Targets were placed on all of the examined points before the photographs were taken, and their ground position coordinates were obtained from U.S.C. & G.S. in State Plane Coordinate System of Ohio. The measurement of coordinates of the control points in the stereoscopic models was done with Wild Autograph Model A7, and orientation of the models was accomplished by use of optical-mechanical orientation methods in the usual manner.

The residual position error of a point was found to be ±19 cm or 0.62 ft without correction, and about 13 cm or 0.43 ft with correction. The average residual error in elevation was found to be 1:7,000 of the photography flight height without correction and 1:13,000 after using the correction equations.

From these results a conclusion can be drawn. It is possible to achieve high accuracy by use of computational photogrammetric methods. Also this accuracy is suitable for solution of right-of-way problems, especially when it is realized the scale of these experimental photographs was 1,000 feet to one inch and the desirable scale for right-of-way measurement and description purposes would be 200 feet per inch. One could expect the maximum error to be reduced to 0.2 of a foot if the photography scale is 200 feet to one inch.

Such a result must be further analyzed because the residual error introduced by photogrammetry combines with the error in ground surveyed control.

Derivation of Error Propagation Equations

By having the correction equation, the error propagation in coordinates of the stereoscopic model can be derived. According to the error propagation law of the least-square theory, the standard error of a function is

$$\mu_X = \sqrt{\left(\frac{\partial X}{\partial x}\mu_x\right)^2 + \left(\frac{\partial X}{\partial y}\mu_y\right)^2 + \left(\frac{\partial X}{\partial z}\mu_z\right)^2} \tag{4}$$

if the function is $X = F(xyz)$, in which x, y, and z are the variables.

Substituting the partial derivatives obtained from the correction equation the final equations in simplified form become

$$\mu_{X_P} = \sqrt{(\mu_{X''_P})^2 + \left(\frac{X''_P}{2b}\mu_{X''_P}\right)^2 + \left(\frac{X'^2_P}{2b}\mu_{X'_P}\right)^2 + \left(\frac{X''^2_P - X'^2_P}{4b^2}\mu_b\right)^2} \tag{5}$$

$$\mu_{Y_P} = \sqrt{(\mu_{Y''_P})^2 + \left(\frac{Y''_P}{2b}\mu_{Y''_P}\right)^2 + \left(\frac{Y'^2_P}{2b}\mu_{Y'_P}\right)^2 + \left(\frac{Y''^2_P - Y'^2_P}{4b^2}\mu_b\right)^2} \tag{6}$$

and the standard position error of a point in a stereoscopic model is

$$P = \sqrt{\mu X_P^2 + \mu Y_P^2} \tag{7}$$

Using such examples the following result has been obtained: $\mu_{X_P} = \pm 10$ microns, $\mu_{Y_P} = \pm 12$ microns, and $\mu_{m_P} = \pm 16$ microns.

Multiplying the standard position error by the 1:5,000 scale of the stereoscopic model the average standard position error is $\mu_P = \pm 8$ cm or about ± 0.26 ft.

Upon analyzing the average standard position error, it is evident such an error is much smaller than the average standard residual position error computed by using the actual errors. The difference is about ± 6 cm or 0.2 ft approximately. This is a clear indication a measurable error exists in the final result, which is due to causes other than photogrammetry. Consequently the result had to be given further analysis.

For the numerically achieving absolute orientation of the measured stereoscopic models, three control points have been used. The average distance between these points is about 1,300 meters. According to the U.S.C. & G.S. information, the coordinates of ground surveyed control points have been determined from second-order traverses attaining a relative accuracy of 1:25,000, which represents 5.2 cm or about 0.16-ft error at the distance of 1,300 meters between control points. This relative error corresponds well to what was obtained by theoretical consideration.

The conclusion can be drawn that the standard position error of a photogrammetrically determined point includes the errors of photogrammetric procedures as well as the errors of the ground survey. Therefore the ground control cannot be assumed as practically errorless as is the custom in common practice today.

The error propagation equations, including both ground and photogrammetric surveying, have been derived but do not represent high practical values because they are too complicated. They can be replaced by more practical equations, and the following is recommended for practical use:

$$\mu_P = \sqrt{\left(\frac{d}{2} M_M\right)^2 + \left(D R_C\right)^2} \qquad (8)$$

in which μ_P is the standard position error of a photogrammetrically measured point, d is the diameter of the measuring mark of the instrument in space at the stereoscopic model scale, M_M is the scale of the stereoscopic model, D is the average distance between points, and R_C is the relative accuracy of the control points.

This equation has been checked, and proved to be accurate within about ± 10 percent for Wild Autograph Model A7, used by an experienced instrument operator. By using the equation, if the standard position error is given by a specification, as in the case of right-of-way surveying, the relative accuracy required in the control survey can be precomputed for the desired photography scale.

ERROR PROPAGATION IN PHOTOGRAMMETRY AND ELECTRONIC SURVEYING

The error propagation in photogrammetry is homogeneous or uniform, which means if a photogrammetric error is ± 0.3 feet the error remains the same regardless of the distance between points.

It is a well-known fact the magnitude of an error in conventional surveying increases with the distance between points of measurement. Therefore the specifications usually given in relative accuracy, such as one in ten thousand, etc., are ideal to the nature of conventional surveying.

But the two errors, namely the error of photogrammetry and the error of conventional ground surveying, are completely different in nature and the final result is consequently unfavorable. The most important part of this situation is that the combination of these two kinds of errors can result in systematic errors, which cannot be avoided within a desired economy, because higher degrees of accuracy would unfavorably influence the surveying cost, unless an entirely new method is used. This new method is electronic surveying.

Electronic surveying is based on the fact the velocity of the electro-magnetic waves and of light rays is known. Thus the elapsed time during which the rays travel from one point to another and return is measured, and the distance computed from such measurements. Consequently, the error of an electronically measured distance is composed of the error of the velocity of the electro-magnetic waves or light rays and the error in time measurement. The error of the velocity is dependent upon the metrological circumstances, which can be corrected by the measurement of temperature, barometric pressure, and the humidity. After the correction, the residual error can be assumed to be constant. The error in the time measurement depends on the instrument in use and for a particular instrument it is constant. Consequently, the error in an electronically measured distance is constant regardless of its length. In other words, the error propagation in electronic surveying is homogeneous, such as in the photogrammetry. It is clear the combination of electronic surveying and photogrammetry is ideal.

ELECTRONIC SURVEYING TRAVERSE COMPUTATION WITH LEAST SQUARE ADJUSTMENT

Preliminary surveying executed by photogrammetry indicates a relative accuracy of 1:10,000 is suitable for ground surveying. Right-of-way application indicates the

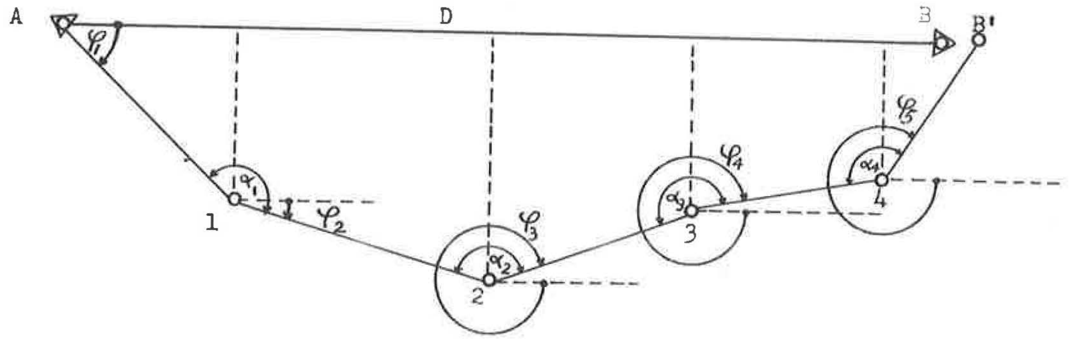

Figure 1.

relative accuracy of a ground survey should be about 1:25,000 to 1:50,000. To reduce the number of field measurements it is evident the different accuracy requirements must be combined into one, so each field measurement can be used for different purposes. This combination requires: first, the use of permanent station markers on which appropriate targets are centered before photography, and second, a universal measuring system which could be already available in the State Plane Coordinate System. The accuracy, or more specifically, the correct orientation of a measurement can be increased by computation. Least square adjustment is suitable for this purpose.

A suggested method for traverse computation by the least square method is subsequently given. It may be assumed that a traverse is measured between A and B triangulation points (Fig. 1), and the plane coordinates of these triangulation points were known previously. During the traverse surveying procedure accidental errors are introduced in the separate measurements as well as in the angle measurements. Due to these errors, the terminal B point is dislocated to B'. If the traverse has previously been computed or an azimuth has been measured, the φ_1 angle (which is the azimuth of AB line) can be obtained by computation or measurement. Knowing the angle φ_1, the other azimuth angles, such as $\varphi_2 \varphi_3$, etc., can be computed.

One condition equation can be written such that the sum of the projected traverse segments to the AB direction is equal to the D distance, computed from the plane coordinates of triangulation points A and B; mathematically,

$$d_1 \cos \varphi_1 + d_2 \cos \varphi_2 + d_3 \cos \varphi_3 + d_4 \cos \varphi_4 + d_5 \cos \varphi_5 = D \tag{9}$$

Actually this equality does not exist because of the introduced small accidental errors. A small correction must, therefore, be added to the traverse segments and the angles to fulfill the condition equation, as follows:

$$\begin{aligned}&(d_1 + V_1) \cos (\varphi_1 + \Delta\varphi) + (d_2 + V_2) \cos (\varphi_2 + \Delta\varphi + \Delta\alpha_1) + (d_3 + V_3) \cos \\ &(\varphi_3 + \Delta\varphi + \Delta\alpha_1 + \Delta\alpha_2) + (d_4 + V_4) \cos (\varphi_4 + \Delta\varphi + \Delta\alpha_1 + \Delta\alpha_2 + \Delta\alpha_3) + (d_5 + V_5) \\ &\cos (\varphi_5 + \Delta\varphi + \Delta\alpha_1 + \Delta\alpha_2 + \Delta\alpha^3 + \Delta\alpha_4) = D = \sqrt{(X_A - X_B)^2 + (Y_A - Y_B)^2}\end{aligned} \tag{10}$$

in which

$\Delta\varphi$ = correction to the initial azimuth,
$\Delta\alpha$ = corrections to the measured angles, and
V = corrections in the traverse segments.

Realizing that

$$\cos (\varphi + \Delta\varphi) = \cos\varphi \cos\Delta\varphi - \sin\varphi \sin\Delta\varphi \tag{11}$$

and because $\Delta\varphi$ is very small, thereby making $\cos \Delta\varphi = 1$ and $\sin \Delta\varphi = \Delta\varphi$:

$$\cos(\varphi + \Delta\varphi) = \cos\varphi - \Delta\varphi \sin\varphi \tag{12}$$

Substituting these into the condition equation the coefficients of $\Delta\varphi$ become zero.

To simplify the preceding equations, the following notation can be written:

$$\begin{aligned}
d_2 \sin\varphi_2 + d_3 \sin\varphi_3 + d_4 \sin\varphi_4 + d_5 \sin\varphi_5 &= N_1 \\
d_3 \sin\varphi_3 + d_4 \sin\varphi_4 + d_5 \sin\varphi_5 &= N_2 \\
d_4 \sin\varphi_4 + d_5 \sin\varphi_5 &= N_3 \\
d_5 \sin\varphi_5 &= N_4 \\
\cos\varphi_1 &= a_1 \\
\cos\varphi_2 &= a_2 \\
\cos\varphi_3 &= a_3 \\
\vdots \quad &\quad \vdots
\end{aligned}$$

Using these new symbols, the equation becomes;

$$a_1 V_1 + a_2 V_2 + a_3 V_3 + a_4 V_4 + a_5 V_5 + \Delta\alpha_1 \frac{N_1}{\rho} + \Delta\alpha_2 \frac{N_2}{\rho} + \Delta\alpha_3 \frac{N_3}{\rho} + \Delta\alpha_4 \frac{N_4}{\rho} + W = 0 \tag{13}$$

in which

$\rho = 206264.8$, and
$W = (d_1 \cos\varphi_1 + d_2 \cos\varphi_2 + \ldots + d_5 \cos\varphi_5) = D.$

In this problem, angles and distances are adjusted at the same time. Thus it must be assumed the measured angles and distances are weighted differently. Weights for measured distances are

$$P_{d_1} = \frac{C^2}{\mu_{d_1}^2 + 1}, \quad P_{d_1} = \frac{C^2}{\mu_{d_2}^2}, \text{ etc.}$$

and the weights for measured angles are

$$P_{\alpha_1} = \frac{C^2}{\mu_{\alpha_1}^2}, \quad P_{\alpha_2} = \frac{C^2}{\mu_{\alpha_2}^2}, \text{ etc.}$$

in which μ^2's are the standard error of individual measurements, and C^2 is an arbitrarily chosen number depending on the computational convenience. Combining these equations, the normal equation is

$$\left(\frac{\left[aa \right]}{\left[P_d \right]} + \frac{\left[\frac{NN}{SS} \right]}{\left[P_\alpha \right]} \right) k + W = 0 \tag{14a}$$

From the normal equation

$$k = \frac{-W}{\frac{\left[aa \right]}{\left[P_d \right]} + \frac{\left[\frac{NN}{SS} \right]}{\left[P_\alpha \right]}} \tag{14b}$$

(In these equations the brackets represent the sum of the products.)

By knowing k, the corrections are computed in the following way:

$$V_1 = \frac{a_1}{P_{d_1}} k \qquad \Delta\alpha_1 = \frac{\frac{N_1}{\rho}}{P_{\alpha_1}} k$$

$$V_2 = \frac{a_2}{P_{d_2}} k \qquad \Delta\alpha_2 = \frac{\frac{N_2}{\rho}}{P_{\alpha_2}} k$$

$$\vdots \qquad\qquad\qquad \vdots$$

$$V_n = \frac{a_n}{P_{d_n}} k \qquad \alpha_n = \frac{\frac{N_n}{\rho}}{P_{\alpha_n}} k$$

The corrected angles and traverse distances are computed algebraically by adding the appropriate correction to the corresponding distances and angles. Because the corrected angles and distances are known, the traverse can now be computed with no discrepancy between points B and B'.

By employing such a computation procedure and the electronic surveying, a relative accuracy of 1:100,000 can be achieved easily which fulfills any requirement.

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions result from the experiments and mathematical considerations.

1. Employment of electronic surveying in connection with the establishment of control for photogrammetric purposes is ideal.
2. Permanent use of station markers and targeting them before photography is taken are recommended for electronically measured traverses.
3. Use of State plane coordinate system is recommended.
4. A consequence of utilizing recommendations numbered 1, 2, and 3 will be, within five to ten years, a large amount of field control established throughout a State conforming to the accuracy of second-order triangulation. The increase in control points will occur as rapidly as precision surveying is done photogrammetrically without having any more than the minimum number of field surveyed measurements. Consequently, the use of this procedure will be highly economical.
5. Further investigations are required in: (a) aerial triangulation for precise surveying, (b) application of numerical photogrammetry for right-of-way land acquisition, and (c) specifications for ground surveying in which the new advanced methods are to be employed. Such research could be combined into one large research project and could result in a manual for modern highway surveying and an evaluation of methods and instruments.

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