Viscoelastic Response of Asphalt Paving Slabs Under Creep Loading

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The results of an intensive study of the reactions of asphalt paving slabs subject to creep loads are the subject of this paper. The equipment employed in the slab research consists of a large spring base (approximately 4 ft square) inclosed in a constant-temperature cabinet. Several slabs compacted from the mixture selected for study were statically loaded on this base, and the resulting deflected shape of each was recorded against time by a system of linear variable differential transformers. Slabs were tested over a temperature range from 40 to 140 F. Each slab was subjected to stresses ranging from very low to very high to establish the linearity of its response pattern. Sufficient time was allowed between stress applications for the slab to regain its undeflected shape under the given ambient conditions.

The response of the test mixture in simple flexure was also evaluated by static bending tests on small beams (1 in. square by 12 in. long) sawed from compacted samples. The bending strains in both tension and compression were measured and used to compute a combined bending modulus as a function of time. The temperature dependence of the bending moduli was established by the application of the time-temperature superposition principle.

The time-dependent moduli derived from the simple flexure tests were used to predict the deflections which might be expected from the slab test procedures. This was done through the application of a numerical technique to classic elastic thin-plate theory. The procedures employed in this phase of the work were quite general in that no assumption of model properties was needed.

Also included in the paper is a discussion of the comparison between theoretical and measured values of slab deflections and their time dependence. Comments are also included concerning the linearity of material responses and the apparent validity of the basic theoretical assumptions underlying the work presented.

•IN THE PAST ten years considerable interest has developed in attempts to determine the time and temperature dependence of the physical properties of asphalt paving mixtures within the framework of viscoelastic theory. Ideally, the intent of this research would appear to be the development of additional analyses to broaden the scope of pavement design and permit consideration on a more rational basis of some aspects of loading which, at best, can only be qualitatively accounted for at present. Unfortunately, the extent to which this theory may be applied to a real material such as asphalt

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concrete has yet to be properly defined. Sufficient data have been published, however, to indicate in a general way, at least, that this type of analysis may be feasible. For example, previous work at the University of California has indicated that the time-dependent behavior of asphalt concrete in compression can be approximated by simple models which are combinations of linear springs and dashpots (1, 2, 3). However, results of slab tests using this simplified approach (3) indicated considerable discrepancy between predicted and measured results. Research by Pister and Monismith (4), Papazian (5), Pagen (6), and Krokosky et al. (7) would indicate the necessity for use of a more generalized representation of linear behavior, behavior analogous to considering a model with either an infinite number of Kelvin elements in series or an infinite number of Maxwell elements in parallel. However, structural problems have been very difficult to solve by using this more generalized representation of material characteristics. In the last few years, particularly with the advent of electronic computers, some of these difficulties would appear to have been overcome.

The dependence of the viscoelastic properties of asphalt concrete on temperature has been shown by Monismith (4), Pagen (6), and Krokosky (7) to obey thermorheological principles similar to those developed for polymers. Krokosky (7), however, has questioned the applicability of linear response theory and has indicated that nonlinear theory which includes functions that are stress and strain dependent are more applicable. Whereas this latter data are of interest and importance, it would seem more appropriate first to define the limits of applicability of linear behavior before complicating the analysis still further with nonlinear functions.

As noted previously, rheologic studies of the material properties are (or should be) a preliminary step, although an important one, to the structural analysis of asphalt concrete pavements. At present only elastic methods of analysis are generally available to the engineer interested in the behavior of the pavement structure, and elastic methods fail to provide a sufficiently versatile tool for the study of many problems particularly associated with, though not necessarily limited to, this type of pavement.

Some limited data have previously been published (3) showing the reactions of actual asphalt paving slabs subjected to creep loads on a foundation of constant elastic properties. These data were compared with the performance predicted by a viscoelastic analysis predicated on an assumed four-element model. Although considerable error was evident between the theoretical predictions and measured data, sufficient correlation was found to warrant further work along similar lines. The results of a more intensive study of this type are presented here.

The research consisted of three distinct operations: (a) the determination of time-dependent elastic moduli considering differences in tensile and compressive behavior for beams prepared from a typical asphalt paving mixture, using simple creep bending tests performed over a range of temperatures; (b) the measurement of time-dependent deflection profiles for a series of slabs prepared from the same test mixture and subject to creep loading on a large spring base, again over a range in temperatures; and (c) the theoretical prediction of the slab deflection profiles from the results of the simple bending tests. Such a program has the possibility of providing a reasonably rigorous test of the ability of viscoelastic theory to reflect the action of the real material.

MATERIALS

A single asphalt concrete mixture was utilized for the investigations. The actual materials used have been employed in a number of previous studies and have been described in detail elsewhere (3).

The asphalt cement used in preparing test specimens was an 85-100 penetration

TABLE 1

IDENTIFICATION TESTS ON ASPHALT

Test	Result
Penetration at 77 F, 100 g, 5 sec	96
Penetration at 39.2 F, 200 g, 60 sec	24
Penetration ratio	25
Flash point, Pensky-Martens (° F)	445
Viscosity at 275 F (SSF)	138
Heptane-xylene equivalent	20/25
Softening point, ring and ball (° F)	110
Thin film oven test, 325 F, 5 hr:	
Weight Loss (%)	0.51
Penetration Retained (%)	53
Ductility of Residue at 77 F (cm)	100+

grade material. Results of standard identification tests on this asphalt are given in Table 1.

The aggregate used in the test mixture was a crushed granite from Watsonville, Calif., with a specific gravity of 2.92. A 3/8-in. maximum size gradation conforming to the 1954 California Standard Specifications was employed for all specimens because the research was initiated before 1960. In 1960, new specifications for 3/8-in. maximum aggregate grading were prepared by California. The gradation, together with both specification limits, is shown in Figure 1. It will be noted that the mix gradation also meets the 1960 specifications for all practical purposes.

Close control of gradation in the preparation of beam specimens was accomplished by screening the granite into individual size fractions and then recombining the size fractions in the amounts necessary to produce single specimens. The slabs were prepared at the Richmond, Calif. laboratories of the California Research Corp. The aggregate for the slabs (same granite as used in the beam specimens) was taken from the stocks of that organization, where it received care in batching comparable to that employed for the beams.

PREPARATION OF TEST SPECIMENS

Two types of test specimens were employed in the research described. For determination of viscoelastic bending moduli, 3- by 2 1/4- by 12-in. beams of the test mixture were prepared by kneading compaction. The details of the technique of mixing and compacting have been discussed in previous publications (8). The design asphalt content selected was 5.1 percent by weight of aggregate, a value already used in previous investigations with the same materials (3). The average density of the compacted specimens, as determined by water displacement, was approximately 152 pcf.

As a final step in the preparation of the bending specimens, each of the compacted beams was cut into four 1- by 1- by 12-in. sections with the aid of a diamond saw under conditions permitting close control over the dimensions of each of the beams.

The slab tests utilized thin "plates" of the test mixture approximately 40 by 40 in. square and slightly more than 1 in. deep. As mentioned previously, these slabs were prepared at the Richmond laboratories of the California Research Corp., where equipment suitable for the preparation of such large-scale specimens was available. The production of these slabs has been described in detail (3). After cooling, the compacted specimens were transported to the University of California laboratories for testing.

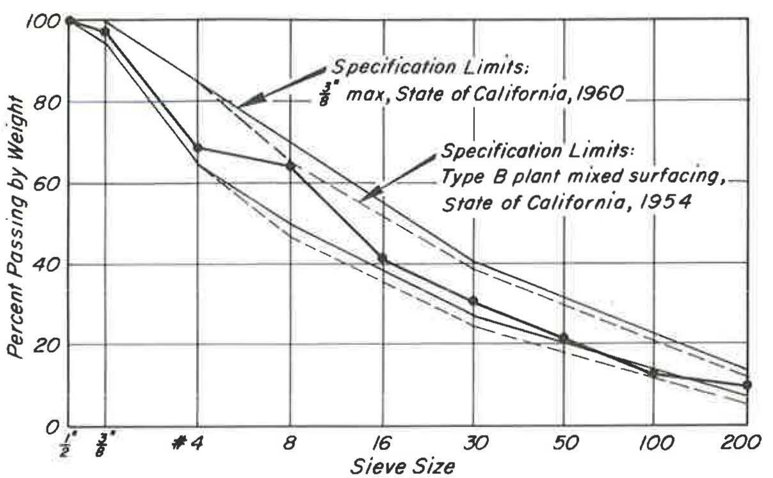


Figure 1. Grading curve, Watsonville aggregate.

It should be noted that these slabs were quite uniform in texture and thickness. Density measurements by water displacement on samples cut from the slabs after testing indicated unit weights of about 152.5 pcf, a value very close to that measured for the beam specimens.

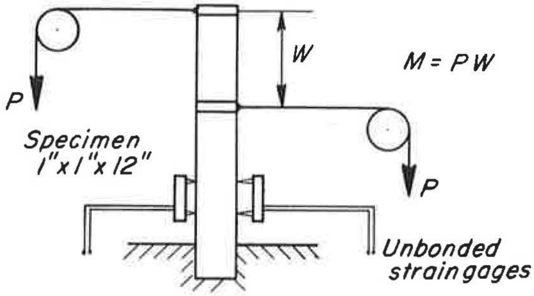


Figure 2. Schematic representation of test on slab mixture comparing creep behavior in tension and compression.

INSTRUMENTATION AND TEST TECHNIQUES

The apparatus employed in the bending creep tests is shown schematically in Figure 2. This test was performed with the beam in a vertical position, its lower end rigidly clamped against movement. A constant bending moment was then applied at the upper end of the beam by a system of pulleys and weights. Strains were measured on both the tension and compression sides of the specimen with unbonded strain gages. A continuous recording of the output of each gage was obtained through the use of an electronic strip-chart recorder. To insure uniform temperature conditions during the test, all measurements were conducted in a constant temperature room.

Two series of bending tests were performed, at 77 and 40 F, respectively. Unfortunately, the nature of the test procedure did not lend itself to measurements at elevated temperatures; the strength of a test specimen under such conditions would be inadequate to permit it to stand vertically unsupported. Each bending test was carried out for a period of 15 min, at a level of stress which induced relatively low strains (approximately 0.2 percent maximum).

The equipment used in the slab tests is shown in Figures 3 and 4; the details of its operation have been described previously (3). Approximately 1,600 coil springs, ⅞ in. in diameter, provided the elastic foundation and were so arranged that the founda-

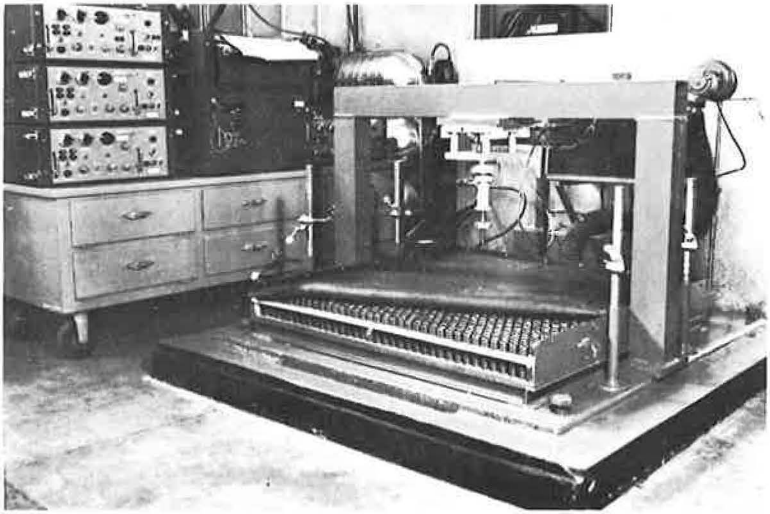
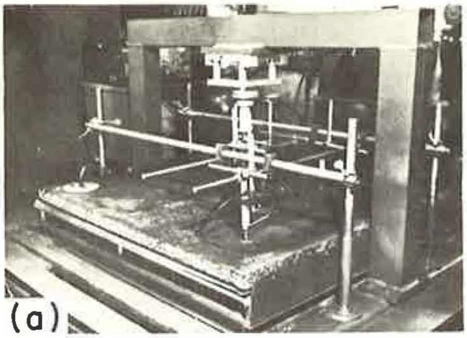


Figure 3. Slab testing device with rubber membrane stripped back to show spring base (pneumatic load cell hangs from center of reaction frame; electronic recorder for use with differential transformers appears in background).



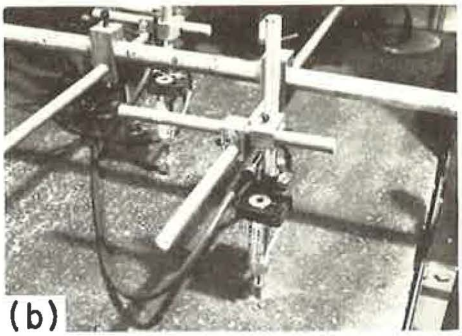


Figure 4. Slab test apparatus ready for operation: (a) Overall view of instrumented slab ready for testing, and (b) Close-up of differential transformer installation.

tion modulus of the system (k) was 200 psi per in. This value was verified by plate bearing tests. The entire foundation and slab system was designed for enclosure under an insulated cabinet equipped for accurate temperature control.

Loads for the slab tests were provided by a series of pneumatic cells, each constructed for a particular load range; a typical intermediate size is shown in Figure 3, attached to the steel frame used as a reaction member. Load for a given test was transmitted to the slab by a circular metal plate faced with rubber to simulate a flexible loaded area. The diameter of the load plate was varied with the test temperature to maintain an approximation of infinite boundary conditions for the slab.

Measurements of the deflected shapes of the loaded slabs were obtained by a series of linear variable differential transformers located along a radial line from the center of the slab, as shown in Figure 4. The outputs of these transformers were transmitted to an electronic strip-chart recorder located outside the test unit.

In performing a particular test, the slab was placed on the spring base and the transformers were positioned and calibrated. The unit was then covered and the temperature was raised to 140 F for 24 hr, to allow the slab to seat itself on the foundation. The temperature was then adjusted to the test level and the system was permitted to reach equilibrium before actual loading.

The research reported in this paper involved measurements on slabs tested at temperatures of 40, 77, and 140 F. Each slab was loaded a number of times, at successively higher levels of stress. One-half hour was allowed between loadings to permit the particular slab to recover completely (for all practical purposes) its undeformed shape. The load was maintained in each case for a period of 60 sec, and complete records were kept of variations in the deflected shape during that time.

BENDING TEST DATA

The results of the bending tests are shown in Figure 5, in which tensile and compressive strains are plotted against time for the measurements taken at both 40 and 77 F. Each curve shown represents the average of several specimens. Change in tensile and compressive properties (which become increasingly different) with time is clearly evident at both temperatures; at longer times the level of tensile strain is much greater than that of compressive strain. As is to be expected, the data indicate a material considerably stiffer at 40 F than at 77 F; however, the same general pattern of time-dependent behavior is noted.

In an earlier publication (3), a method for predicting slab performance from theoretical computations involving a four-element model was presented. The model properties employed in the computations were derived from the results of triaxial compression tests on the specimen material. However, the comparison of measured deflection profiles with those computed through theory showed considerable discrepancy.

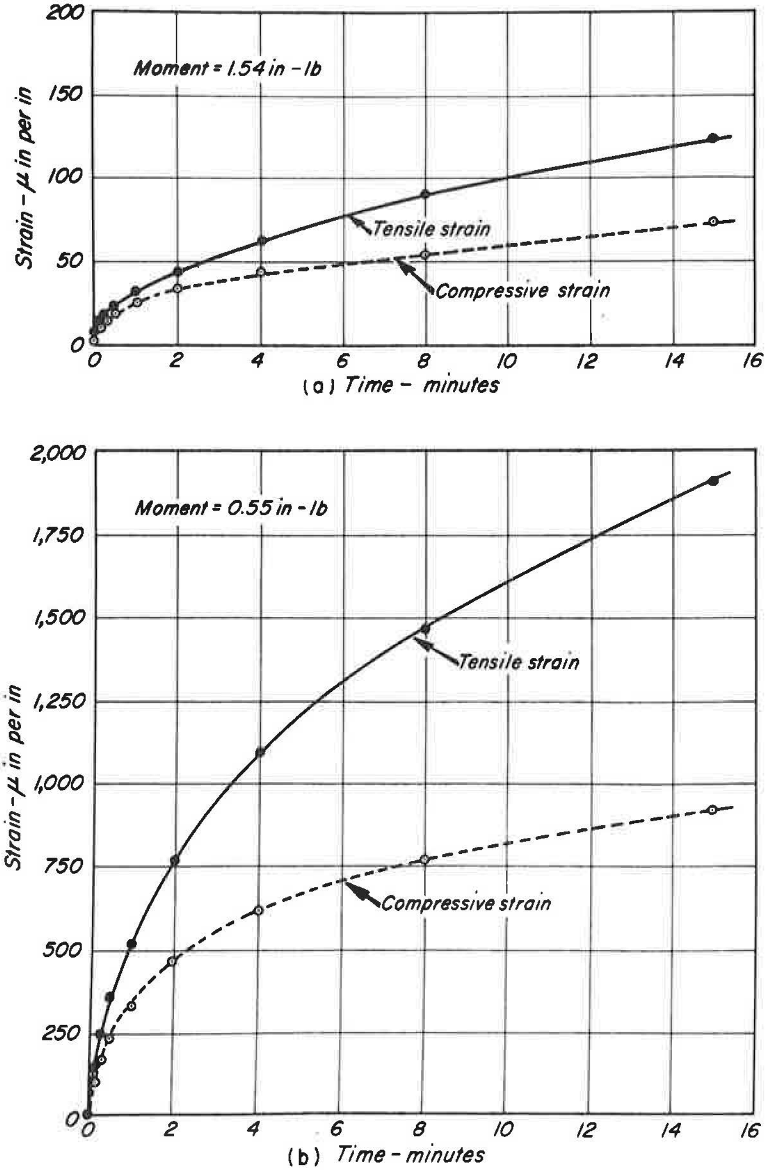


Figure 5. Comparison of tensile and compressive bending strains at (a) 40 F, and (b) 77 F.

One possible reason for this disagreement was a dependence on an assumption that the slab was composed of a material whose characteristics in tension were the same as in compression. Figure 5 clearly illustrates the error involved in this approach.

Establishment of analyses of the behavior of layered systems containing asphalt concrete involving the fewest possible assumptions should, among other things, consider the representation of the asphalt layer as a material with time-dependent properties differing in tension and compression. In addition, this representation should be based on properties measured separately in tension and compression. Unfortunately, this problem, from a mathematical standpoint, cannot be solved practically; thus, a simplified approach must be utilized. One technique is to make use of the relationship developed for pure bending of beams in which the beam material behavior obeys

Hooke's law but whose modulus in tension differs from that in compression (9). For this case

$$\frac{1}{R} = \frac{\epsilon_t + \epsilon_c}{h} \tag{1}$$

in which

R = radius of curvature of beam;
$\epsilon_t + \epsilon_c$ = maximum fiber strain in tension and compression, respectively; and
h = depth of beam (rectangular cross-section).

In addition, the following expression (9) can be applied to rectangular beams of the type considered here:

$$E_r = \frac{1}{I} \frac{M}{1/R} \tag{2}$$

in which E_r is reduced modulus of material, combining properties in tension and compression; M is applied moment; and I is moment of inertia of beam.

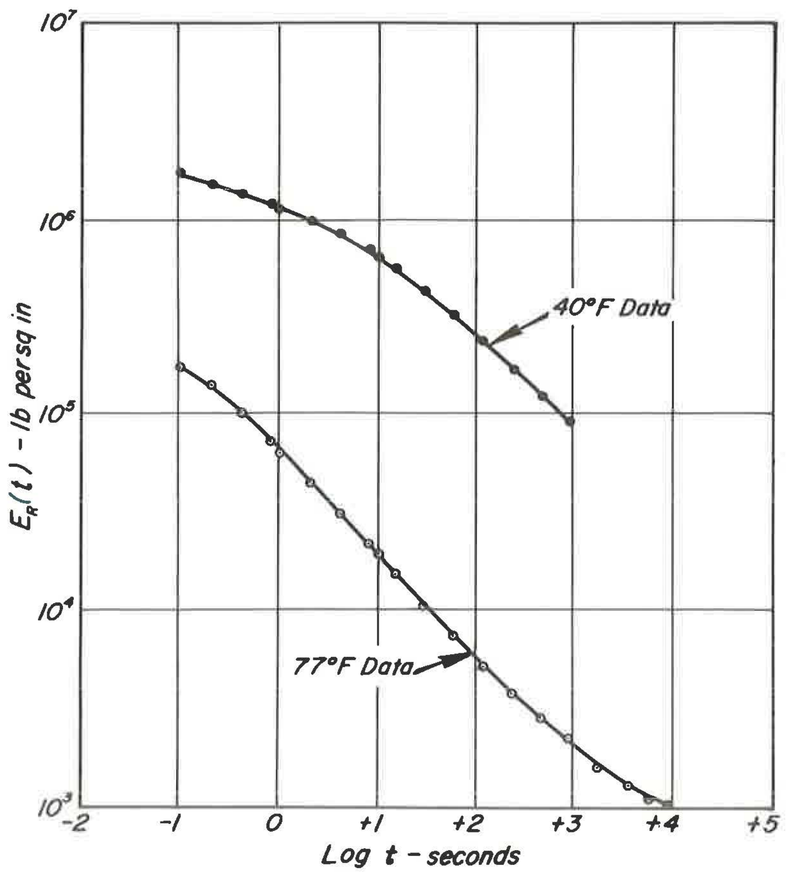


Figure 6. Reduced elastic moduli at 40 F and 77 F vs time.

As noted earlier, it is important to establish the limits, if any, of linear viscoelastic behavior for asphalt concrete. This is necessary to insure validity of certain of the techniques described below. Some data (3) have been published suggesting that the assumption of linear behavior may be acceptable at strains below about 0.3 percent. For this reason the bending test investigation was conducted at strains below this limit and the values of E_r are thus based on relatively small strains.

The data shown in Figure 5 were employed in conjunction with Eqs. (1) and (2) to produce the curves shown in Figure 6. In this figure, values of E_r for the test material at 40 and 77 F are plotted against time; again, a variation with time is evident. Due to the logarithmic character of the plots shown in Figure 6, no values for $t = 0$ (instantaneous moduli) are shown. Data were taken to supply such values, but the limitations of the recording equipment and test instrumentation employed did not permit the accuracy necessary for precise measurements at short times. Therefore, only approximate instantaneous moduli were obtained which are not included.

As has been mentioned, it was not feasible to make bending test determinations at temperatures above 77 F. This problem was handled by the time-temperature superposition principle (6, 10 and 11). In brief, the values of $E_r(t)$ shown in Figure 6 were first adjusted by a factor T_o/T, where T_o is an arbitrary reference temperature and T is a particular test temperature, both measured in absolute units. (This adjustment is based on the statistical theory of rubber-like elasticity; a full explanation of this subject is available in rheologic literature (11). For the work presented in this paper, the reference temperature T_o was selected as 298 K (77 F). A second theoretical correction factor, ρ_o/ρ, based on density changes with temperature, was neglected as being of small importance. This temperature correction shifts the curve slightly vertically.

After the data were adjusted by the T_o/T factor, a corrected plot was made of $T_o/T \cdot E_r(t)$ vs log time, similar to that already shown in Figure 6 for the basic $E_r(t)$ values. By inspection, a value was selected for the dimensionless factor a_T, which would permit horizontal translation (parallel to the time scale) of the 40 F data to permit coincidence with the 77 F data. It should be noted that the shift factor may be defined as:

$$a_T = t_T/t_o \tag{3}$$

in which t_T is the time required to observe a phenomenon at temperature T, and t_o is the time required to observe the same phenomenon at some reference temperature T_o (10). A value of $\log a_T = 3.36$ was chosen for the bending data at 40 F by techniques which have been well described by Pagen (6). This number is in close agreement with those found by Monismith (12) for two other asphalt mixtures similar to the test material, as is shown in Figure 7.

Figure 8 is composite plot of $T_o/T \cdot E_r(t)$ vs log time at 77 F prepared from the shifted 40 F and the 77 F data. The curve shown represents a major portion of what might be considered a master plot of the reduced bending modulus over an extended time scale. If a_T can be defined for the test material at any temperature of interest, $E_r(t)$ for that temperature becomes available from Figure 8 by means of the time-temperature superposition technique. This concept was used to find values of $E_r(t)$ at 140 F to employ in conjunction with work on the slab deflection profiles. A value of $\log a_T = -4.0$ at 140 F was taken from Figure 7 by assuming a linear relationship between $\log a_T$ and temperature; this approach is supported by the other data shown in Figure 7 and by the work of Pagen (11).

The significance of the concept of reduced variables becomes more apparent when one considers the definition of a_T as indicated by Eq. (3) and the actual magnitude of the shift factor for 140 F as compared to 77 F. The number $\log a_T = -4.0$ indicates that a property defined at a time of 1 sec at 140 F would correspond to the same property defined at 10,000 sec (approximately 3 hr) at 77 F. In addition, the modulus (E_r) at a time of loading of 1 sec at 140 F could be obtained by entering the master curve at 77 F shown in Figure 8 at a time corresponding to 10,000 sec; this would give a value of $T_o/T \cdot E_r$ equal to 1,000 psi. Thus, E_r at 140 F corresponding to a time of loading of 1 sec would equal 1,050 psi.

SLAB TEST DATA

The data obtained from the slab test procedures were first checked for linearity by the procedure illustrated in Figure 9. This figure shows a plot of load vs deflection for one point on the surface of a slab tested at 77 F; the illustrative data are for a time of 20 sec after load application. Similar graphs were made for each point where deflections were measured at a given temperature; at each such point a family of curves was thus obtained, representing various times elapsed after the onset of loading. All curves had the general characteristics shown in Figure 9, indicating nonlinear deflections at small loads and good linearity in the higher stress ranges. Actually, the extent of nonlinearity illustrated in Figure 9 is an extreme case chosen for descriptive purposes; most of the test data exhibited this characteristic to a lesser degree.

The deviation from linearity shown in Figure 9 was thought to be a function of the test apparatus and not of the paving mixture being tested. This is supported

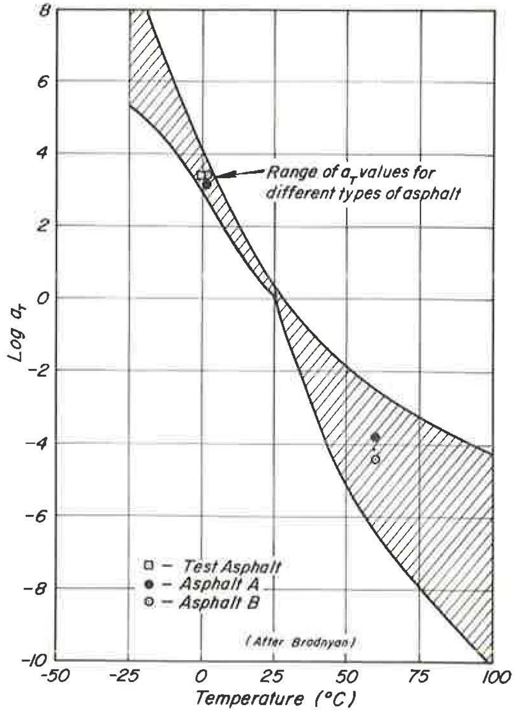


Figure 7. Temperature dependence of a_t values for various asphalts.

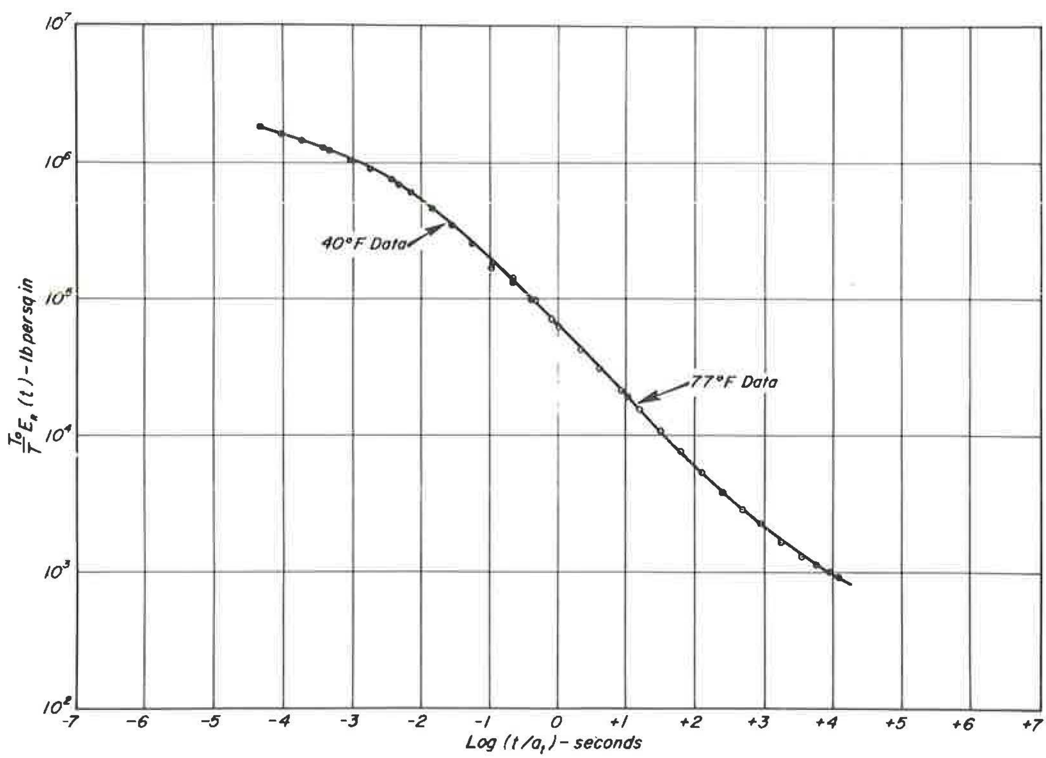


Figure 8. Reduced modulus vs time at 77 F (298 K).

by the fact that the load-deflection curves for the plate tests used to check the design foundation modulus exhibited the same type of performance. It would appear that the difficulty of securing a perfectly uniform contact between the test slab and the spring base resulted in this error. Of far more interest is the performance of the system at higher load levels, where linear responses were obtained in all cases.

To correct for the problem of seating error, all load-deflection data were adjusted to zero by the procedure illustrated in Figure 9. Typical results of this process are shown in Figure 10, where the complete load-deflection-time relationship for a particular point of measurement at 77 F is given.

From adjusted data such as are shown in Figure 10, it was possible to obtain slab deflection profiles for any load and elapsed time after load application. Inasmuch as the load-deflection relationships were linear within the test range in all cases (such profiles would change only in regard to the deflection scale for differences in load level at any particular point in time), only one magnitude of applied load at each temperature is presented herein.

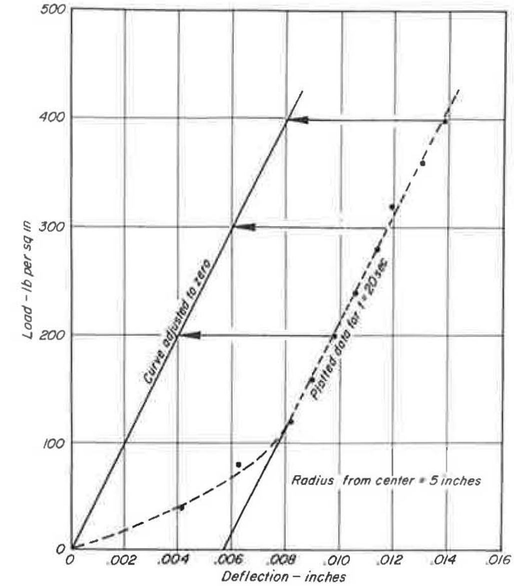


Figure 9. Typical adjustment of slab deflection data to eliminate zero error, 77 F.

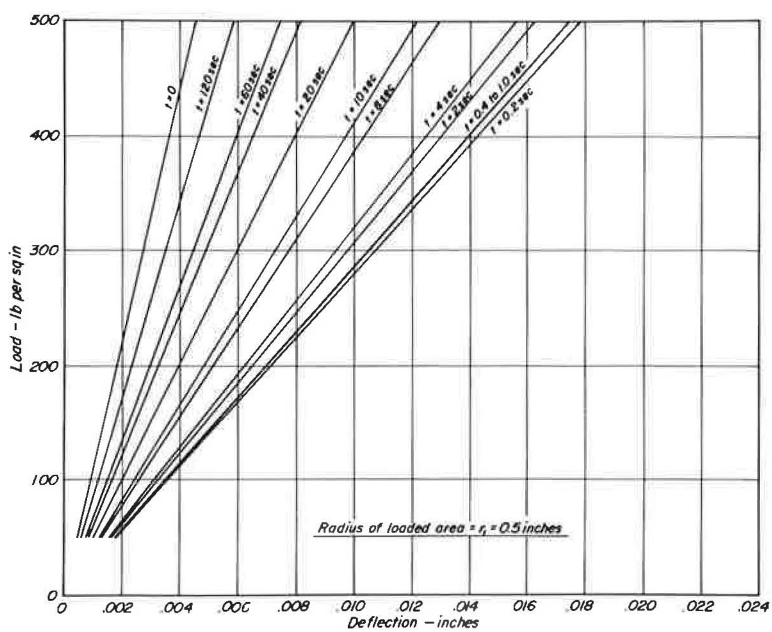


Figure 10. Typical corrected load-deflection-time relationship for point 5 in. from center of test slab, 77 F.

Figures 11 through 14 show measured deflection profiles at various times after loading for slabs at each of the three test temperatures. Also shown in each figure are the radius of the loaded area (r_1) and the intensity of stress (q_o). Figure 15 shows the deflection-time pattern for the centers of the slabs at each temperature.

Also shown in Figures 11 through 15 are deflection-time relationships predicted from the results of the bending tests. These computed values were obtained by applying the values of $E_r(t)$ at each test temperature to a known solution for the deflections of an infinite elastic plate on a Winkler foundation. This problem has been discussed in detail (13), hence only the resulting formulations need to be repeated here:

$$w(r) = \frac{q_o}{k} \int_0^{\infty} \frac{1}{\left[\left(\frac{\ell_o}{r_1}\right)^4 \lambda^4 + 1\right]} J_1(\lambda)\, J_o\left(\frac{\lambda}{r_1} r\right) d\lambda \qquad (4)$$

in which

λ = an integration variable,

$\ell_o^{\,4} = \frac{1}{k} \frac{E h^3}{12(1 - \mu^2)}$,

E = elastic modulus of slab,

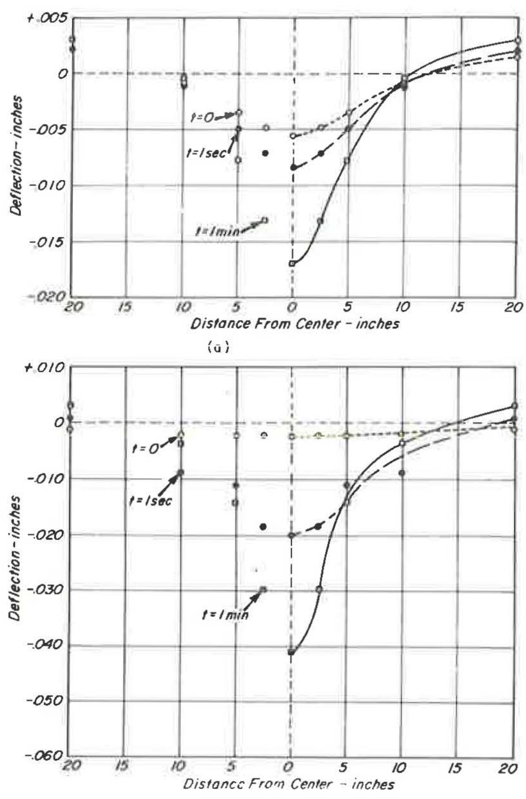


Figure 11. Deflection profiles at various times after load application at 40 F, q_o = 500 psi, r_1 = 0.5 in.: (a) theoretical, and (b) measured.

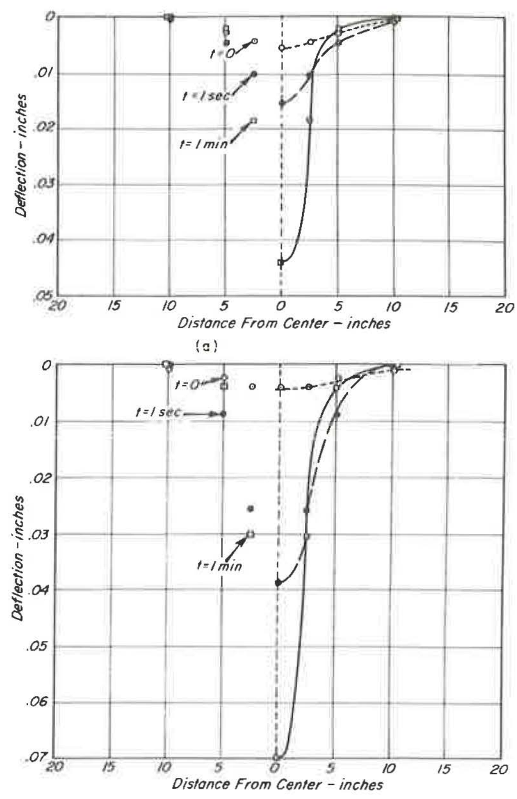


Figure 12. Deflection profiles at various times after load application at 77 F, q_o = 250 psi, r_1 = 0.5 in.: (a) theoretical, and (b) measured.

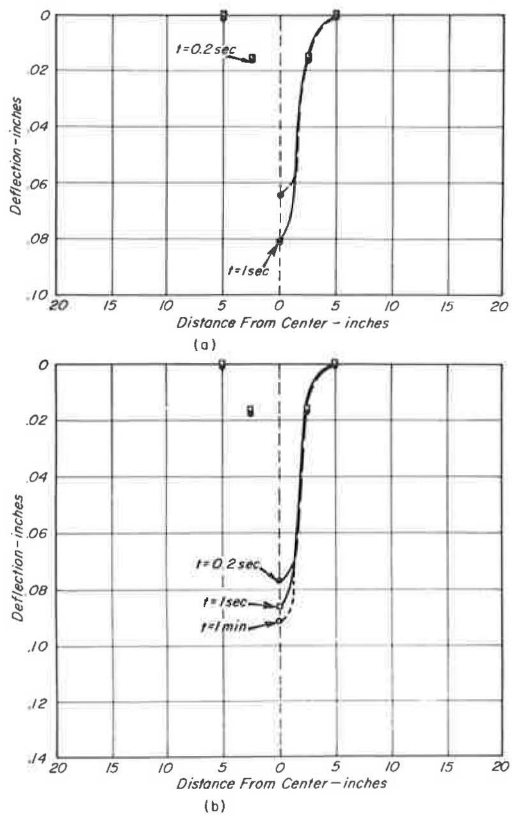


Figure 13. Deflection profiles at various times after load application at 140 F, q_o = 50 psi, r_1 = 1.0 in.: (a) theoretical, and (b) measured.

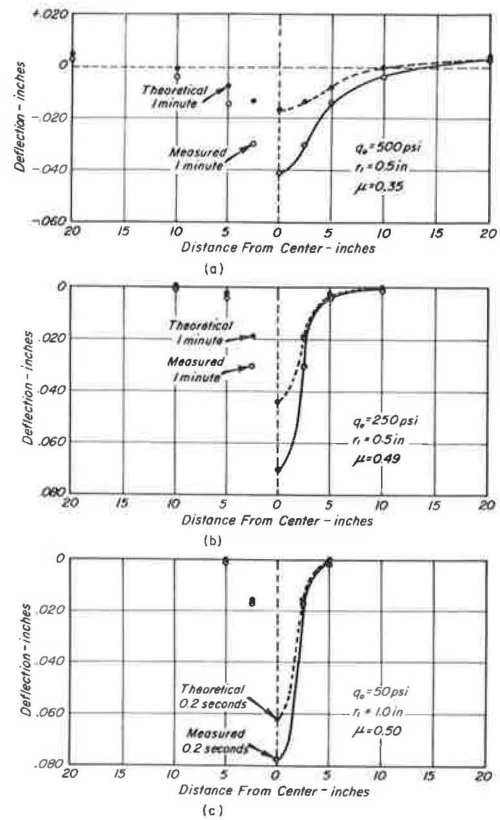


Figure 14. Typical comparisons of theoretical and actual deflection profiles: (a) at 40 F, (b) at 77 F, and (c) at 140 F.

h = slab thickness,
μ = Poisson's ratio of slab material,
r = distance from center of load, and
r_1 = radius of loaded area.

A discussion has been previously presented of the assumptions underlying Eq. (4) and the relation of these assumptions to the type of test reported herein (3). From the viewpoint of this paper, the two most important are those of (a) material properties of plate, the same in tension and compression, and (b) infinite boundary conditions. It is believed that the first of these two assumptions was satisfied by using the reduced moduli E_r in the computations. A discussion of the validity of the second will be dealt with later.

To these two should be added a third assumption, the dependence of Poisson's ratio, μ, on time of loading. In the previous work (3) and in the investigation discussed herein, μ has been assumed constant (time independent) at a particular temperature. Some data were presented previously (3) indicating μ to be essentially constant at high temperatures in a measurable time range; at low temperatures the data indicated μ to be somewhat time dependent. This could cause some differences between theory and measurement at low temperatures because the simple beam creep test is essentially an analog computer for the development of $E_r(t)$. Because of the dimensions of the beam, however, Poisson effects do not influence the test results. In the plate (asphalt slab), on the other hand, the Poisson effect is present because of the biaxial state of stress.

As has been already mentioned, Eq. (4) was used with values of $E_r(t)$ to predict the slab deflection profiles. To obtain deflections for a particular temperature and time after load application, the appropriate E_r was selected from the results of the bending tests. The evaluation of the infinite integral was done numerically, using an IBM 7090 computer. Values for μ were taken from earlier work with the same material (3).

An inspection of Figures 11 through 14 indicates some disagreement between theoretical and measured deflection-time data. The disagreement is most severe at 40 F and least at 140 F. However, the essential ability of the viscoelastic approach to predict the changing shapes of the test slabs is also quite evident, particularly at the higher temperatures. Inasmuch as a purely elastic analysis would provide no means for predicting such changes in shape with time, the relative value of the two analytic systems is obvious.

It should be noted that the use of the modulus $E_r(t)$ as reported here considerably improved the predicted data in comparison with that published earlier, wherein only compression data were utilized. The improvement was most dramatic at 140 F. This phenomenon was to be expected because an assumption of the same behavior in tension and compression for asphalt paving mixtures at high temperatures is naturally poor.

The disagreement between theoretical and measured profiles found at low temperatures was disappointing. However, inspection of these data indicates that the assumption of infinite slab boundaries was not entirely valid (Fig. 11). Theoretical analysis of the problem from the viewpoint of free boundary conditions, an assumption in line with actual test conditions, may possibly improve the comparison. Also, further investigation of the time dependence of μ may be worthwhile.

It is interesting to note that the above approach is similar to that proposed by various Shell researchers (14, 15), wherein a modulus corresponding to a particular time of loading is substituted in an existing elastic solution.

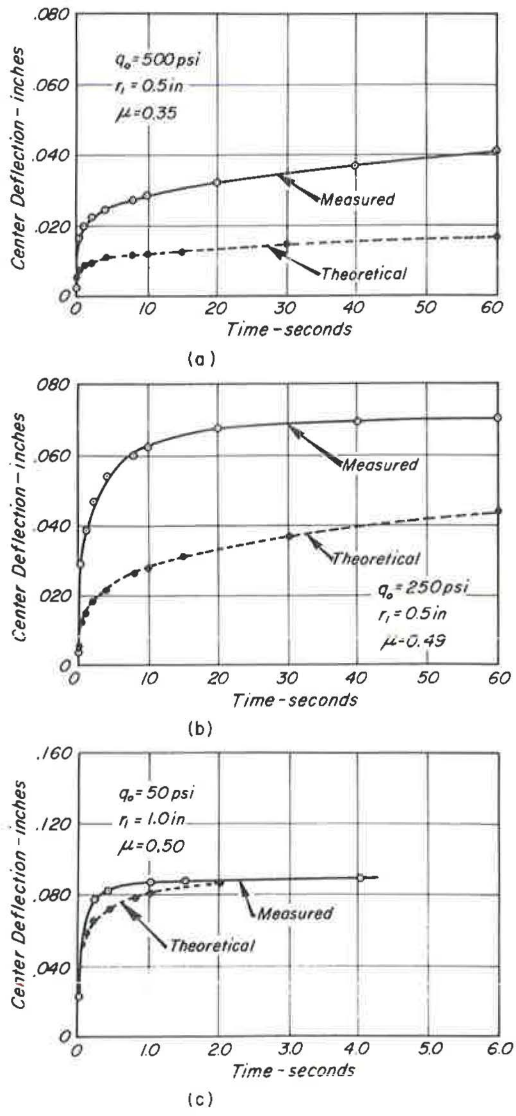


Figure 15. Comparison of measured deflections under centers of loaded slabs with deflections predicted from viscoelastic theory: (a) at 40 F, (b) at 77 F, and (c) at 140 F.

SUMMARY AND CONCLUSIONS

The work presented in this paper represents an evaluation of the usefulness of viscoelastic techniques in the analysis of the performance of structures prepared from asphalt paving mixtures. To provide adequate laboratory control of test conditions, the structure involved was necessarily simple; a similar analysis for a real flexible pavement would require a far more complex theoretical approach. Also, the comparison given here between measured slab performance and that predicted from theory was by no means perfect. Despite these drawbacks, it is felt that the results of this work cannot help but indicate that the viscoelastic approach provides an additional tool for analysis and design of asphalt concrete pavements. Continued investigations along such lines would seem definitely indicated.

ACKNOWLEDGMENTS

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