Determination of Freezing Index Values

ARTHUR L. STRAUB and FREDERICK J. WEGMANN
Respectively, Professor of Civil Engineering and Graduate Assistant, Department of Civil Engineering, Clarkson College of Technology

Freezing index, which is used for estimating frost penetration for pavement design and for other purposes in pavement performance investigations, has been determined on a general basis for large areas of the world. However, the actual freezing index for a specific station might be very different from that estimated for the general region. The determination of freezing index for the large number of station-seasons needed to facilitate statistical treatment of data presents an overwhelming task if done by manual computing methods, and short-cut estimates reduce reliability to uselessness. There is a need for a more efficient means of determining statistically reliable values of freezing index for specific locations on a mass-production basis.

Methods of determining freezing index are described. An electronic digital computer program for finding accurate values of freezing index from daily punched card records available from the U. S. Weather Bureau and the Meteorological Branch of the Canadian Department of Transport is discussed. Problems in applying the program to the processing of masses of data are presented. Statistical treatment of the data using logarithmic-probability plotting is described for determining return period values possibly useful in application to pavement design concepts.

•COMPUTED VALUES of freezing index have provided engineers with a means of attaching numerical values to the cumulative effect of the freezing temperatures which accrue during winter seasons.

The magnitude of the value of freezing index has been correlated with the depth of frost penetration into the soil under highway and airfield pavements in areas where moderately severe winters are experienced. In this manner, freezing index has been used in pavement design procedures which take into account the factor of frost (1, 13). Freezing index is also reported in pavement performance studies as a means of documenting the severity of specific winters.

A degree day is defined as the difference between mean daily temperature and an arbitrary base temperature multiplied by the duration of one day. The base temperature of 32 F is taken when considering problems of freezing and thawing, and the degree days for any given day may be positive or negative depending on the mean temperature for that day. Degree days may be accumulated algebraically and plotted as a curve (Fig. 1). The difference between the maximum and minimum points on the cumulative curve is defined as the value of freezing index for that season.

Configurations in the cumulative degree day curve accounting for freeze-thaw cycles, and the slope of the cumulative curve have been investigated with some degree of success with regard to their correlation with pavement performance (10).

The U. S. Army Corps of Engineers in developing its use of freezing index in a pavement design method has analyzed data for wide areas of the world and determined mean values of freezing index for many stations. A value for the design freezing index may

Paper sponsored by Committee on Frost Heave and Frost Action in Soils.

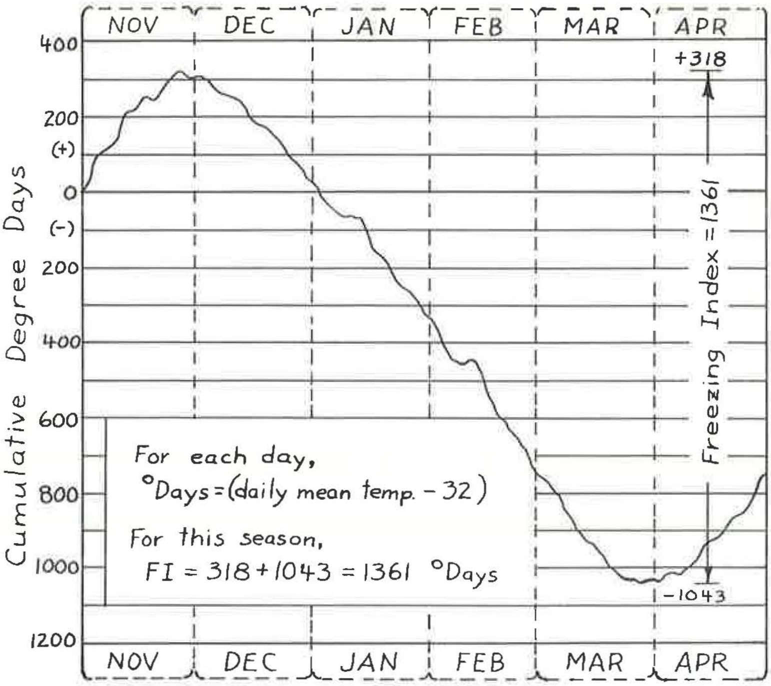

Figure 1. Determination of a freezing index value.

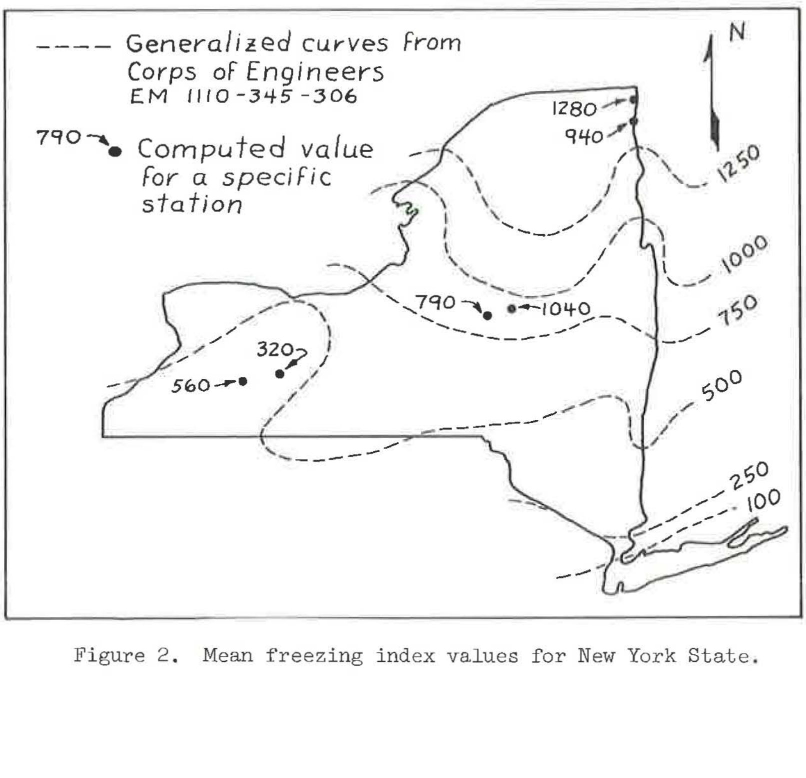

Figure 2. Mean freezing index values for New York State.

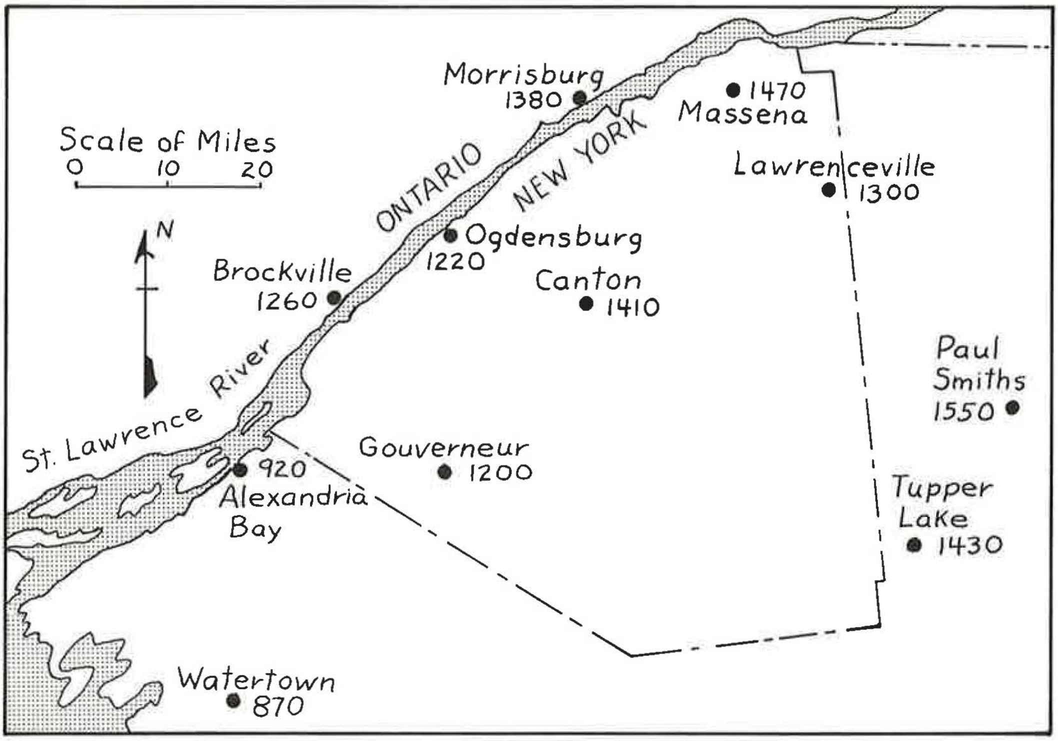

Figure 3. Mean freezing index values for St. Lawrence Co., N.Y., and surrounding area.

be found by taking the average freezing index for the three most severe winters occurring in 30 years. The value obtained is approximately equal to the 10-yr recurrence interval value.

The distribution of general values of mean freezing index as found by the Corps of Engineers (3) is shown for New York State (Fig. 2). Although general values are very useful for some purposes, it should be acknowledged that the value of freezing index for a specific location might differ greatly from the general value for the area, and that freezing index might vary widely over a small area (Figs. 2 and 3). To be applied, therefore, pavement design procedures accounting for frost penetration suggest that the design freezing index value for the specific site in question be determined (3). To be useful in very many specific design problems, therefore, a vast amount of data would have to be processed in order to make specific freezing index values for specific sites available.

The purposes of this paper are (a) to review briefly methods for computing freezing index, (b) to describe an application of an electronic digital computer to the computation of freezing indices from masses of daily weather data, and (c) to apply statistical methods to determine meaningful values of freezing index for possible applications to design problems.

GENERAL COMPARISON OF METHODS

Freezing index for a given winter season is found by computing degree day values from actual daily temperature records, and accumulating the degree days over the season beginning in fall, and ending in the spring (Fig. 4). Because the temperature records available are usually air temperature (often taken $4\frac{1}{2}$ ft above the ground), the value determined is called air freezing index (distinguished from ground freezing index).

Daily weather observations of temperature, precipitation, and other meteorological data have been recorded for a large number of stations for many years by the U. S.

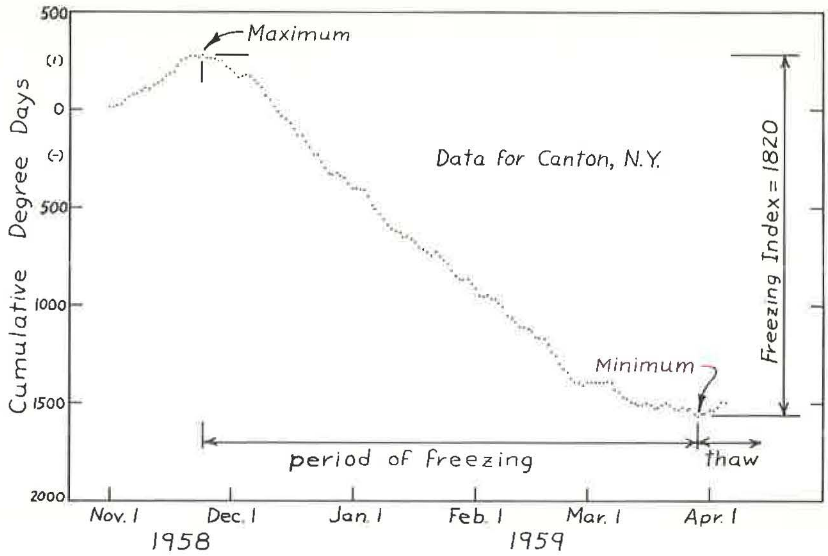

Figure 4. Determination of freezing index from daily temperature records.

Weather Bureau and the Canadian Department of Transport. Original observations which become a part of the official record are made by a relatively few first order stations and are supplemented by large numbers of individual cooperative weather observers. A variety of published summaries are issued regularly.

Manual Computation Method (Exact)

The value of freezing index for a particular season can be found by tedious manual computation beginning with tabulated records of maximum and minimum daily temperatures for the particular station as available from published weather records, such as the U. S. Weather Bureau's "Climatological Data" for state sections in monthly issues, and the Meteorological Branch of the Canadian Department of Transport's "Monthly Record." The manual process can be expedited somewhat by use of a two-man computing team. One person can mentally add the maximum and minimum daily temperatures, refer to a prepared table to find the degree day value for the day, and accumulate the values with a desk calculator. The accumulated value for each day can be called out to the second man who can tabulate and plot the points concurrently. With care given to spot checks for accuracy and allowing for necessary rest breaks, a working efficiency rate of about one manhour per freezing index determination can be achieved (Fig. 4).

The advantages of this exact manual method are as follows: (a) results are exact (by definition); (b) the plot can be used to note configurations in the cumulative degree day curve (e.g., a regularly occuring January thaw); (c) it is a convenient method for determining a few values of freezing index; and (d) it can be done using data readily available from official published records of daily temperature.

The disadvantages are that (a) the work is tedious and very boring if many seasons of data are to be processed; and (b) it is costly in manhours and is not suited to mass production of data.

Manual Computation Method (Approximate)

An approximate value for freezing index can be estimated from records of stations which experience winters with monthly mean temperatures distinctly below freezing. Values for monthly mean temperatures are often available from tabulated weather records, such as the U. S. Weather Bureau's "Climatological Data" for state sections in annual issues, and from the Meteorological Branch of the Canadian Department of

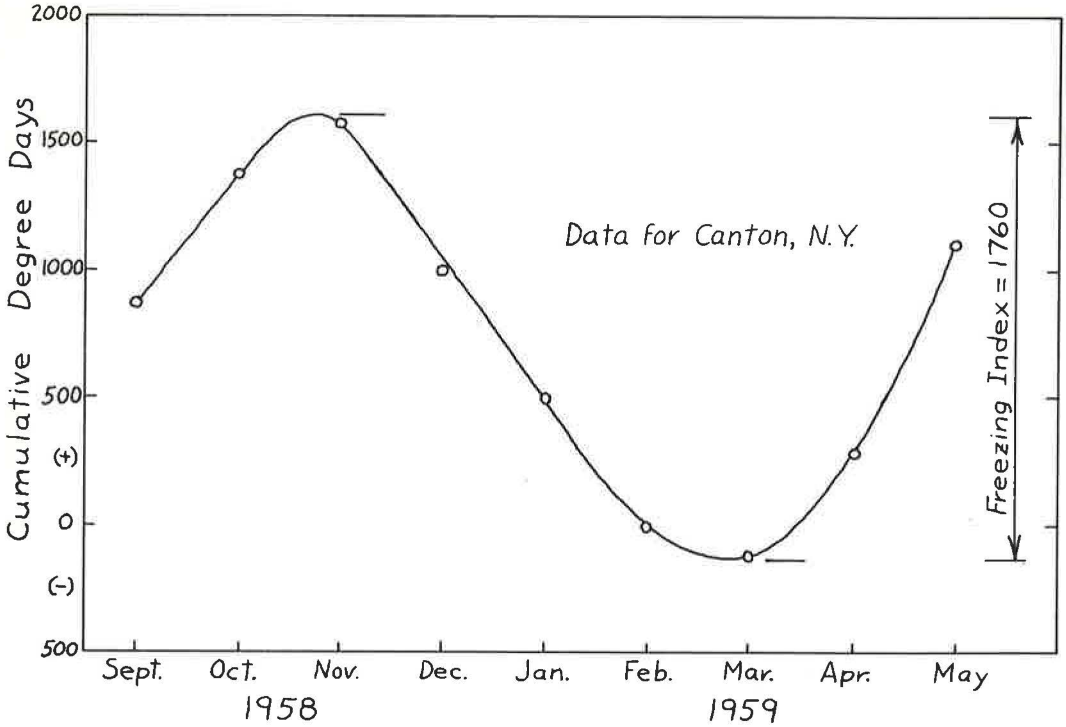

Figure 5. Determination of freezing index from monthly temperature records.

Transport's "Annual Meteorological Summary," and can be extracted for use in manually computing and plotting a rough cumulative curve (Fig. 5). The approximate value determined by this method is generally less than the exact value by an average of about 10 percent for areas with well defined winter seasons, but any given season's value might be in substantial error. The average error is less for severe winter climates. For moderate winters the error is substantially greater (10 to 40%), and it is impossible to use the method for mild winters.

The time requirement to estimate freezing index values from mean monthly temperature records is about three per manhour with reasonable care given to spot checks and efficient use of prepared data forms.

The advantages of this approximate manual method are as follows: (a) the production rate is three times the rate using daily records; (b) the method is convenient for estimating a few values; and (c) it can be done using data available from official published records of average monthly temperature.

The disadvantages are (a) the results are only approximate, and the amount of error is not known; (b) significant fluctuations in the cumulative curve during the season are masked, and often not noted; and (c) the method cannot be used for mild seasons.

Electronic-Computer Method

Because of the tedious, repetitious nature of the computations required to find freezing index, it is appropriate to consider an application of high-speed electronic digital computers to the task. Given a valid program and daily temperature data in punched card or taped form, masses of data could be processed with dispatch.

The development and use of an electronic computer were pursued because of the promise of efficiency in processing large volumes of data covering many seasons of record for scores of stations throughout a state or region (16). Quantities of values were needed before a statistical study could be made.

Beginning in 1948 and 1950, the United States and Canada, respectively, initiated major efforts to convert original observed data to punched card form in order to facilitate high speed processing of data (4). The program of conversion has been modified and developed in recent years to include film optical scanning. Current information on the status and availability of weather data in punched card and/or other form can easily be secured through correspondence.

In addition to basic sources, extensive records for selected stations are available from agricultural experiment stations at many state universities. Regional groups, such as the Regional Technical Committee for the Application of Climatology to Agriculture in the Northeast, have recently been organized in cooperation with Federal agencies to conduct a broad program of studies. Records for use in these studies extending back to about 1926 and earlier have been carefully edited and placed on punched cards and magnetic tape (6).

Editing Requirements. —Mention should be made of two essential requirements of the form of the data needed for determining freezing index from daily records by high-speed computers: complete data and chronological order. These requirements are the same as for manual computing methods. However, a lack of attention to the editing and preparation of data for the computer processing could result in inefficient and costly computer delays or invalid results which might remain undetected.

The daily temperature data for each station to be processed must be complete. Available official records occasionally are found to be incomplete for various reasons such as the inability on the part of a cooperative observer to make his observation. In editing the data, any missing temperature data must be supplied by some suitable means; e.g., estimating the temperature for the station from available values for the same date from nearby stations. If data are missing for a number of days (or weeks) the reliability of a sequence of estimated values would be open to question, and data for the entire season might better be discarded. For exacting climate studies, only those stations with reliable records extending over long periods of time should be included.

To satisfy the needs of computing, the daily records to be processed must also be arranged in chronological order. Official records are nominally reported and are published in chronological order. However, in routine processing, in including reports submitted late, and in estimating missing data, it can be expected that the required chronological order is likely to be upset. Chronological order should be checked, and if necessary, established or restored as a part of the editing and preparation of data.

Computer Program. —The IBM 1620 FORTRAN computer program is designed to perform the following operations:

1. Calculate the yearly freezing index value.
2. Determine the dates of the maximum and minimum points on the cumulative degree day curve.
3. Check to insure that the climatological data are in chronological order.
4. Check to insure that all of the climatological data used to make a determination of a freezing index value are for one and only one specific station.
5. Accumulate daily degree days for the entire year.

The computer program is adaptable to either United States or Canadian punched weather cards. Output may be either from the typewriter or from punched cards. The computer program is easily adaptable to magnetic tape facilities.*

Limitations of the Computer Program. —In order to determine the yearly freezing index value, the maximum (fall) and minimum (spring) points on the cumulative degree day curve must be clearly identified. Over many years, the winter seasons are only a small depression on the overall cumulative degree day curve. The maximum and minimum degree day points are obscured if the accumulation of degree days is begun in the summer and continued for an entire year (Fig. 6).

*A detailed description of the computer program with flow charts is available from the Highway Research Board at cost of Xerox reproduction and handling: Supplement XS-1 (Highway Research Record 68), 29 pages.

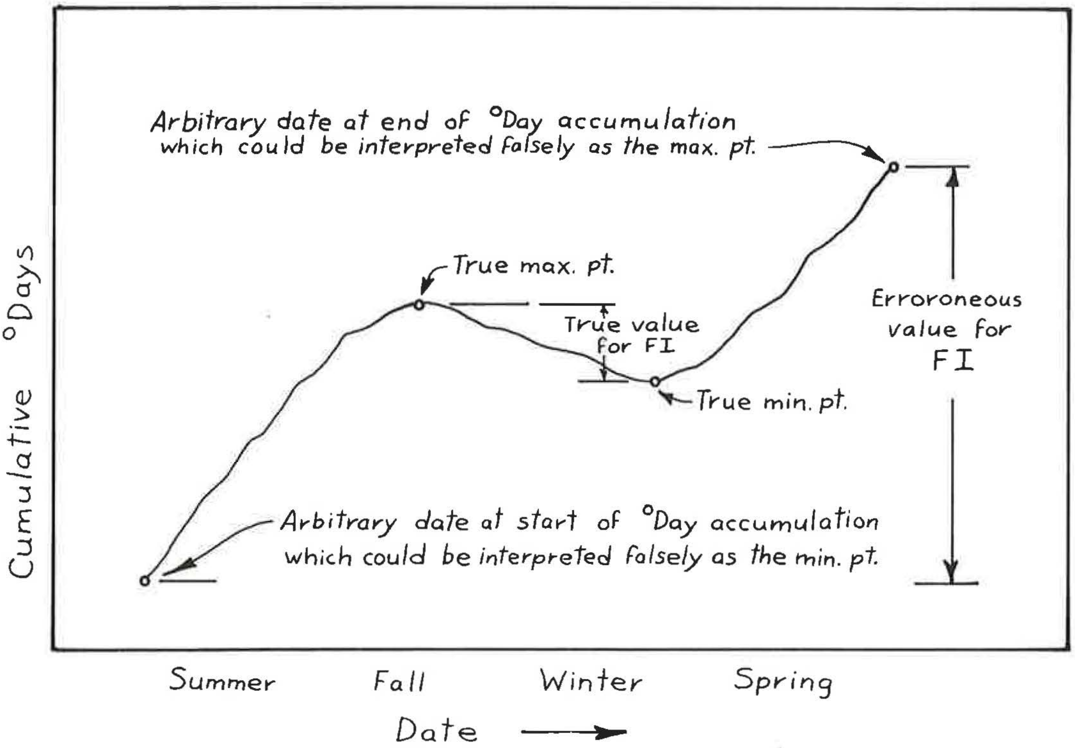

Figure 6. Effect of the incorrect selection of initial and final dates of degree day accumulation.

Thus, date restrictions are imposed to control the time of initiating and terminating the accumulation of degree days. The accumulation of degree days is reinitialized after each winter season. The restrictive dates will insure that the values on the first and last day of the cumulative degree day curve are not falsely interpreted to be the maximum and minimum degree day points. The restrictive dates define the winter seasons for the computer. These dates must be selected so that they do not obscure the maximum and minimum degree day points sought in the definition of freezing index.

The three restrictive dates are defined as follows:

1. The "initial date" must occur before the date of the maximum point. Its purpose is to initiate the degree day accumulation.
2. The "middle date" is any date between the maximum and minimum point. Its purpose is to alert the computer that the maximum point has been established and now a minimum point is to be sought.
3. The "final date" must occur after the minimum point. Its purpose is to terminate the degree day accumulation.

These three dates are illustrated in Figure 7 with example dates that can be expected to occur in a well defined winter season. The proper selection of these dates becomes a critical factor because it is necessary to estimate correctly the location of the maximum and minimum degree day points before computing the freezing index value for a station. If the restrictive dates are incorrectly defined then the resulting freezing index value is also incorrect. Errors in the selection of the restrictive dates can be determined from a scan of the tabulated results. If new restrictive dates must be selected due to an error, a rerun of the station-year is required. In areas without a well defined winter season (for example, freezing index values less than 100 degree days),

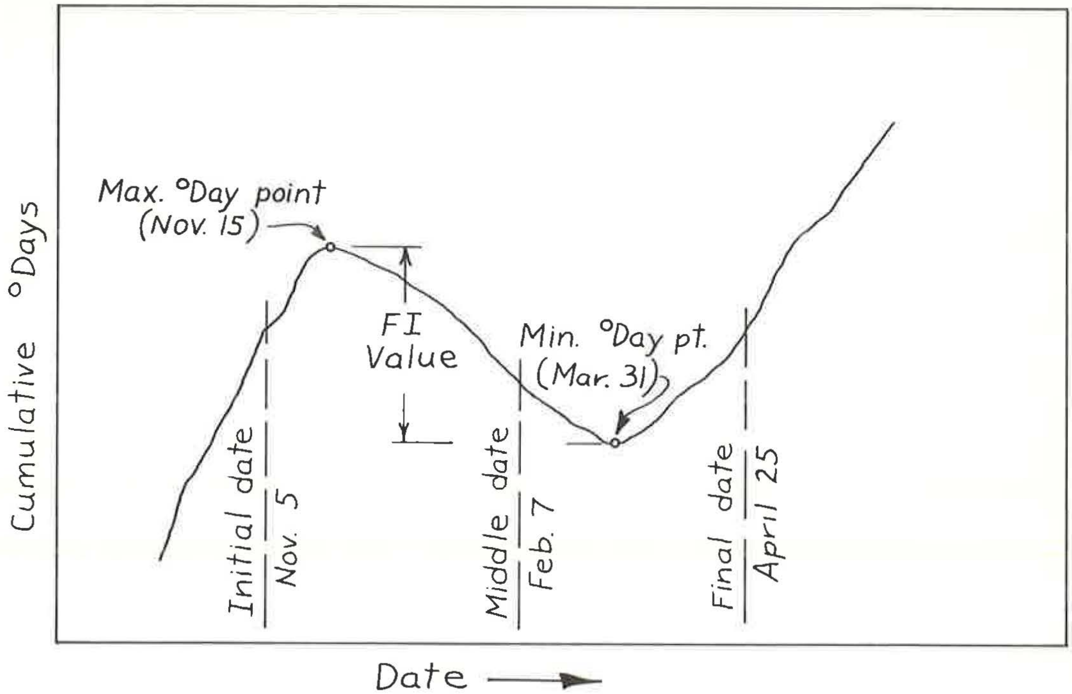

Figure 7. Examples of selected restrictive dates for computer program.

the estimation of restrictive dates becomes a serious limitation of the computer program. For these areas, the use of a sense switch will produce a daily tabulation of degree days. From this tabulation, a scan for the maximum and minimum degree day points becomes the most satisfactory way of determining the freezing index value.

A check is imposed in the program so that final cumulative degree day point never comes within five degree days of the maximum degree day point. . Thus, the selection of the final degree day restrictive date is not critical. The only requirement is that the date is selected so that it insures that the minimum degree day point will be obtained prior to this date.

In milder regions the critical restrictive dates might be different from season to season, so that a new header card and reinitiation of the computer program is required.

Program modifications could be made to avoid difficulties in the application of critical dates for use on computers with sufficient storage capacities.

Evaluation of Computer Method. —The method was applied to determine approximately 900 values of freezing indices (16). In comparison with manual methods, the use of a high-speed computer in determining freezing index values has a number of advantages and disvantages. The advantages are as follows:

1. Large volumes of data can be processed efficiently at substantial savings in man-hours. About 3 minutes per freezing index determination is required in running a full year's set of daily cards through an IBM 1620 computer. Proportionately less time would be required if the summer season cards are sorted out and not used as input.
2. Mathematically accurate values of freezing index are as easy to determine as approximations.
3. Computer program has built in checks on identity on the cumulative degree day curve.
4. Use of console sense switches facilitates modifying input and output to suit needs.
5. Punched output of daily degree day accumulation could be plotted by an automatic off line plotter which would facilitate visual reading of the cumulative degree day curve.

The disadvantages are the following:

1. Use of a computer is not suited for the determination of only a few values of freezing index.
2. For most satisfactory use, the program requires that the winter be well defined (definite period of freezing weather extending for at least two weeks). Otherwise, as in the case of a mild winter, the freezing index is ill-defined. However, by use of a console sense switch, or by a trial and error procedure, borderline cases can be handled.
3. A very careful editing and preparation of input data is necessary to insure efficient processing.

For large-scale processing of winter season data in areas where freezing index is well defined, the advantages of computer processing substantially outweigh the disadvantages, and, therefore, computer use is justified.

TABLE 1

FREEZING INDEX VALUES FOR CANTON, N. Y.

Order of Occurrence		Order of Magnitude		
Winter Season (year)	Freezing Index (deg days)	Item Order No. (m)	Freezing Index[a] (deg days)	Frequency[b] (%)
1922-23	2040	1	2290	2.56
1923-24	1280	2	2040	5.13
1924-25	1460	3	1910	7.69
1925-26	1800	4	1850	10.26
1926-27	1470	5	1820	12.82
1927-28	1300	6	1800	15.38
1928-29	1170	7	1780	17.95
1929-30	1430	8	1740	20.51
1930-31	1300	9	1730	23.08
1931-32	950	10	1680	25.64
1932-33	800	11	1660	28.21
1933-34	2290	12	1540	30.77
1934-35	1780	13	1510	33.33
1935-36	1740	14	1480	35.90
1936-37	1050	15	1470	38.46
1937-38	1440	16	1460	41.03
1938-39	1480	17	1440	43.59
1939-40	1910	18	1430	46.15
1940-41	1680	19	1430	48.72
1941-42	1290	20	1430	51.28
1942-43	1730	21	1360	53.85
1943-44	1430	22	1300	56.41
1944-45	1660	23	1300	58.97
1945-46	1510	24	1290	61.54
1946-47	1220	25	1280	64.10
1947-48	1850	26	1250	66.67
1948-49	880	27	1220	69.23
1949-50	1250	28	1170	71.79
1950-51	1060	29	1170	74.36
1951-52	1060	30	1060	76.92
1952-53	740	31	1060	79.49
1953-54	960	32	1050	82.05
1954-55	1360	33	980	84.62
1955-56	1540	34	960	87.18
1956-57	1170	35	950	89.74
1957-58	980	36	880	92.31
1958-59	1820	37	800	94.87
1959-60	1420	38	740	97.44

[a] In order of decreasing magnitude.
[b] At which given value was equaled or exceeded.

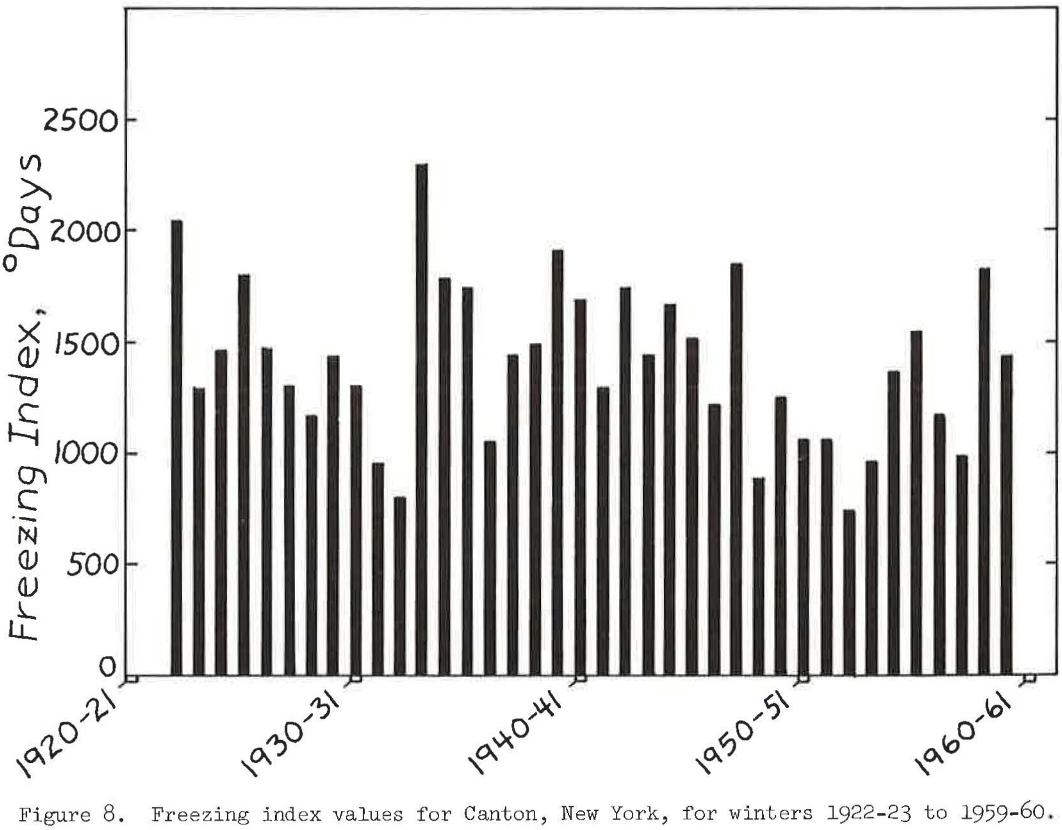

Figure 8. Freezing index values for Canton, New York, for winters 1922-23 to 1959-60.

OBTAINING A MEANINGFUL DESIGN VALUE OF FREEZING INDEX

With natural fluctuations in climate the values of freezing index vary widely for a given location from season to season, as given in Table 1 and shown in Figure 8 for Canton, N. Y. Values available for the winter of 1922-23 through 1959-60 range from a minimum of 740 occurring in 1952-53, to a maximum of 2290 in 1933-34. The average of all 38 seasons is 1403 degree days.

Acknowledging the existence of widely fluctuating values of freezing index, the question remains as to what value might be selected for applications to design. A mean value of freezing index does not account for the occurrence of the more severe winters of special concern. The extreme value for a period of study might be too conservative and/or unreliable. Linell (12) in discussing this matter suggested that a more significant design freezing index value would be that which occurs about one year in ten, particularly in areas of low mean freezing index. This design value can be approximated by computing the average air freezing index of the three coldest winters in the latest 30 years of record and is the current design practice of the Corps of Engineers (3, 13). However, it has been found that there is only a fair correlation between mean winter season temperatures and freezing index computed from daily records (15). At least five or six coldest seasons must be selected for computing freezing indices to be sure of finding the highest three values which could be averaged for use in design.

Return Period Value for Design

As experience is gained and refinements are made in engineering practice, more attention can be paid to bringing in considerations based on calculated risk. Concepts of design life are based on this approach, although as applied to pavement designs allowing for climate effects a designer must depend on his experience and judgment in estimating the risks involved. Although neither the only factor nor the most important factor, it is suggested that return period values of freezing index might be worthy of further consideration in future refinements in design methods accounting for seasonal frost. If there is justification in pavement design procedures to include the factor of freezing index to account for frost penetration, there is justification to refine freezing indices to more accurate and statistically reliable values.

A return period value is one which is expected, on the average, to be equaled or exceeded only once in a stated interval. It is particularly useful in describing naturally occurring phenomena which appear to fluctuate widely by chance. For example, after an analysis of data for Canton, N. Y., it can be stated that a freezing index value of 1950 is expected, on the average, to be equaled or exceeded only once in ten years, a value of 2100 only once in 20 years, and a value of 2260 only once in 50 years. The stated values are not to be interpreted as forecasts for a certain time, but rather they are the most probable largest values to occur within the time periods stated.

Determining Return Period Values

Although other and more sophisticated statistical methods can be applied in computing the probabilities of the occurrence of a certain value, the relatively simple graphical method using probability paper for analyzing return period values for stream flows under flood and drought conditions is of special interest (7, 8). An example of probability paper is shown in Figure 9. With some modification, this method is used widely today in the field of hydrology. Mockus (14) discussed this concept and method of determining a flood frequency value. Burchinal and Dickerson (2) apply the method in determining rainfall probability. It can be adapted directly for use in freezing index analysis, as by Legget and Crawford (11). In applying the graphical method, it is desirable to have at least 30 points for each station, but with the application of electronic computation, the quantities of data necessary would be readily available for use.

Example Determination

Recent developments in the theory of extreme values, such as reported by Gumbel (5), could have been applied to the problem at hand. However, for present purposes,

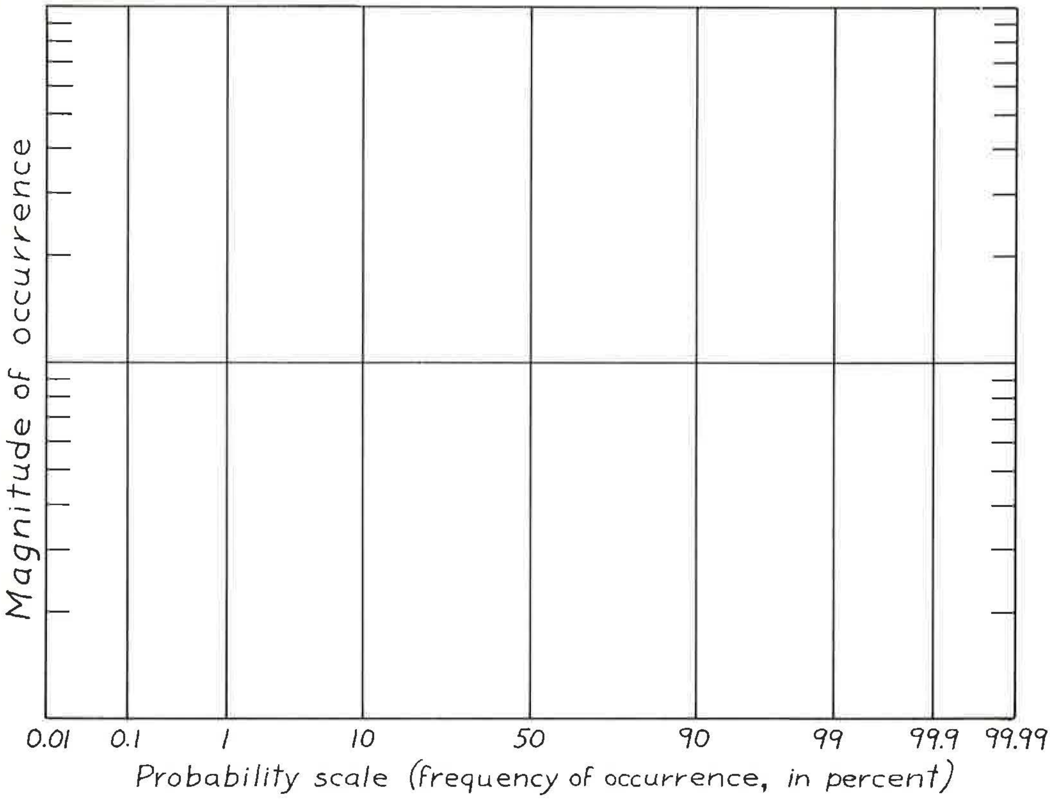

Figure 9. Sample of probability paper.

it is sufficient to illustrate the point by using a relatively simple plot of data on logarithmic-probability paper to determine return period values of freezing index for Canton, N. Y. The data for the 38 seasons are placed in order of magnitude (Table 1), and then the frequency at which a given value was equaled or exceeded is computed. In the example, the Kimball method (9) is applied to compute frequencies, although other methods would yield very similar results. The frequency is found from

$$F = \left(\frac{m}{n + 1}\right) 100 \qquad (1)$$

in which

m = the order number;
n = the total number of items; and
F = the percent of years during which the freezing index is equal to or greater than the freezing index of the order item m.

The computed frequencies are the plotting positions on a probability scale (percent) as the abscissa and with freezing index on the ordinate. In the example (Fig. 10), a line of best fit is drawn by eye through the plotted points. The equation of the line could be found and tests for goodness of fit could be made, but in practical application, for purposes at hand, these steps are not necessary and might even be misleading if not properly applied.

Probability paper of this type is available commercially with ordinates laid out both in logarithmic and arithmetic scales. The choice would depend on how the data plotted;

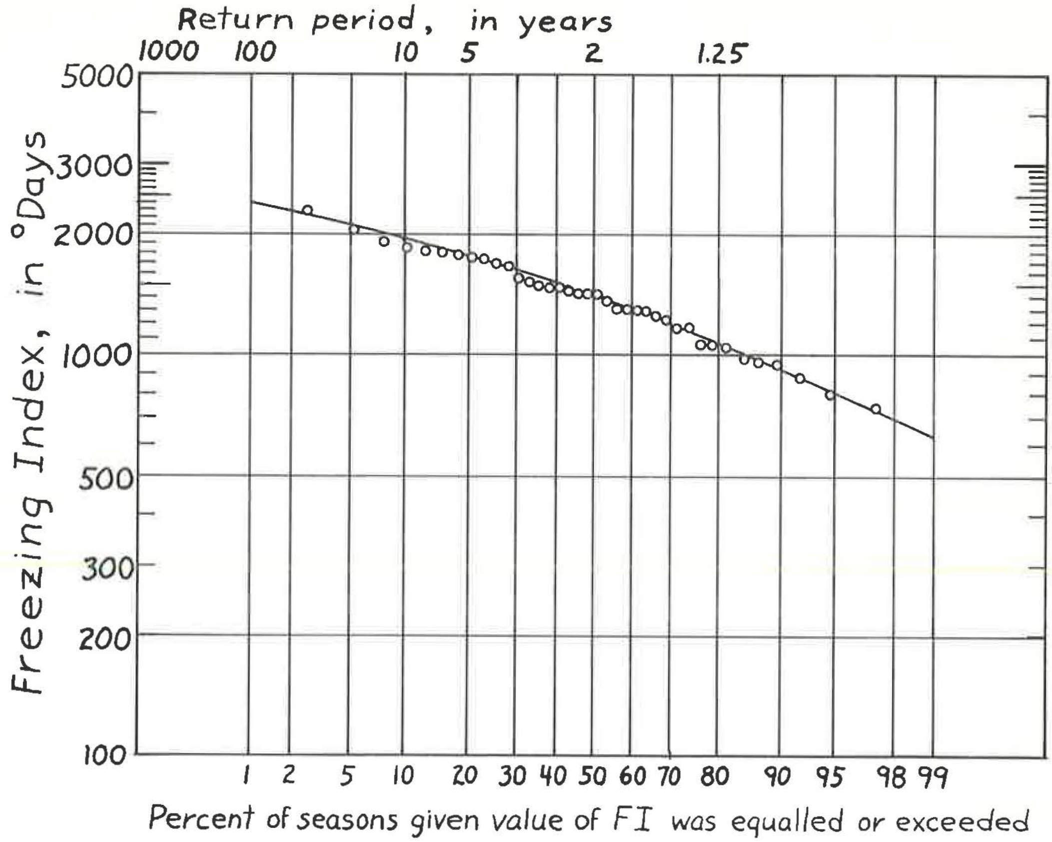

Figure 10. Logarithmic probability plot of freezing index data for Canton, New York, for winters 1922-23 to 1959-60.

it is desirable for the plot to be as nearly a straight line as might be convenient to facilitate drawing the line of best fit by eye in practical application. Values should be selected from near the upper end of the line which represents the more severe winters, but curve extrapolation should be done with caution as to reliability.

Given a plot such as Figure 10, it is possible to select frequency values for any return period within reasonable limits of the data.

As with any study of historical events, the analysis depends on what has happened in the past. From the analysis of past records, it can be stated, for example, that a freezing index value of 2100 had a return period of 20 years corresponding to a frequency of 5 percent. The assumption is made that, unless strong evidence of climate change can dictate otherwise, it is probable that a freezing index of 2100 or greater will occur at Canton sometime during the coming 20-yr period. Values for any other return period can similarly be found from Figure 10, although extrapolation beyond 40 years for the data available would be less certain as to reliability.

The plot of data would also be useful in comparing a given season's freezing index with the general distribution. For example, if a given season produced a freezing index of 1830, it can be said that a value of at least this magnitude is expected to occur within any 7-yr period.

The lower end of the curve might be equally useful in other applications. For example, at the lower end there is a 98 percent probability that the freezing index will be equal to or greater than a value of 700, or a value less than 700 will probably not occur more than once in a 50-yr period.

Choice of Return Period Value

The choice of the proper return period for design purposes is beyond the scope of the present paper but is a subject which might be worthy of further study. An understanding of climate effects on pavement performance is far from complete, and design procedures remain arbitrary in the manner of considering the effects of certain factors, such as soil and frost penetration. The factor of frost penetration alone can be estimated from freezing index which is easily computed, and a freezing index value with a return period corresponding to the design life of the pavement might prove to be the proper value to select.

Additional Applications

In addition to the application of return period values of freezing index, it is suggested that the concept of frequency of occurrence and return period values might also find useful application in studying other factors considered important in frost action studies. For example, soil moisture of different layers in and beneath the pavement vary considerably as precipitation and drainage conditions change. Accurate measures of soil moisture could be analyzed to account for the frequency and return period values. Also, temperature gradients within the soil vary widely over the season as air-temperatures vary. Accurate measures of the changes in slope of the temperature gradient could be analyzed to account for the frequency and return values. The growth of ice lenses in frost susceptible soil depends on the combination of freezing temperatures and available moisture, and possibly an analysis of the expected frequency of certain temperature gradients and soil moisture would contribute to a better understanding of frost action. The time unit for an analysis of frequency need not be one year, as in the case of the freezing index example, but can be any appropriate unit, such as an hour or a day. An intensive study of soil moisture and temperature gradients or any other factors, of course, would depend on an ample supply of reliable data.

CONCLUSION

Two principal conclusions were drawn: (a) the electronic digital computer program and method presented can be used efficiently to process large quantities of daily weather records in determining accurate values of freezing indices; and (b) given large quantities of results, the application of probability paper can produce meaningful values for possible use in design.

ACKNOWLEDGMENTS

This investigation was largely conducted during two summers while the authors were in the employ of the New York State Department of Public Works and engaged in research activities supported by the U. S. Bureau of Public Roads. Personnel of the Department's Bureau of Physical Research participated in developing the electronic data processing program and furnished considerable assistance in preparing the data cards. The Bureau of Electronic Data Processing cooperated in the development of the final program and processed a large volume of data.

Cooperation and assistance was received from the Department of Civil Engineering, Clarkson College of Technology. The following individuals were especially helpful: Professor Robert D. Larsson, for initial program concepts; Dr. John M. Perry, Director of the Computing Center, for initial program development; Mr. William G. Austin, for manual computations in early stages of development; Mrs. Anne Serabian, for manuscript typing; and Mrs. Betty Lyman, for duplication of masters.

REFERENCES

1. Armstrong, Malcolm D., and Csathy, Thomas I., "Frost Design Practice in Canada." Highway Research Record No. 33, 170-201 (1963).
2. Burchinal, J. C., and Dickerson, W. H., "Rainfall Probability and Its Applications." Bull. 454T, West Virginia Univ. Agric. Exp. Sta. (March 1960).

3. Corps of Engineers, U. S. Army, "Engineering and Design, Pavement Design for Frost Conditions." EM 1110-345-306 (May 1962).
4. Cudbird, B. S. V., "Modern Techniques in Canadian Climate Data Processing." Meteorological Branch, Canadian Department of Transport, Cir. 3558 (Nov. 1961).
5. Gumbel, Emil J., "Statistical Theory of Extreme Values and Some Practical Applications." Applied Mathematics Series No. 33, U. S. National Bureau of Standards, pp. 12-15 (Feb. 1954).
6. Havens, A. V., and McGuire, J. K., "Spring and Fall Low-Temperature Probabilities." The Climate of the Northeast, Bull. 801, New Jersey Agric. Exp. Sta., Rutgers University (June 1961).
7. Hazen, Allen, "Storage to Be Provided in Impounding Reservoirs for Municipal Water Supply." Trans. ASCE, Vol. 77 (1914).
8. Hazen, Allen., "Flood Flows." John Wiley, Inc. (1930).
9. Kimball, Bradford F., "Assignment of Frequencies to a Completely Ordered Set of Sample Data." Trans., American Geophysical Union, XXVII (Dec. 1946).
10. Kübler, Georg, "Influence of Meteorologic Factors on Frost Damage in Roads." Highway Research Record No. 33, 217-261 (1963).
11. Legget, R. F., and Crawford, C. B., "Soil Temperatures in Water Works Practice." Jour., American Water Works Assoc., 44:10, 923-939 (Oct. 1952).
12. Linell, Kenneth A., "Frost Design Criteria for Pavements." HRB Bull. 71, 18-31 (1953).
13. Linell, K. A., Hennion, F. B., and Lobacz, E. F., "Corps of Engineers' Pavement Design in Areas of Seasonal Frost." Highway Research Record No. 33, 76-136 (1963).
14. Mockus, Victor, "Selecting a Flood-Frequency Method." Trans., American Soc. of Agric. Engineers (1960).
15. Straub, Arthur L., "Climatological Studies Project Report." Bureau of Physical Research, N. Y. State Dept of Public Works, 1960 (unpublished).
16. Straub, A. L., and Wegmann, F. J., "Computation of Freezing Index Values for New York State—Methods and Results." Bureau of Physical Research, N. Y. State Dept. of Public Works, 1962 (unpublished).