Analysis of AASHO Road Test Asphalt Pavement Data by The Asphalt Institute

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• THIS REPORT is the result of the need, as foreseen by The Asphalt Institute, for independent analyses of the results of the AASHO Road Test. The Asphalt Institute took cognizance of this need in 1960 in setting up its Road Test Board of Study. The Institute reasoned that in an area so new, because of its vastly increased scope, examination of the research results from many different points of view could only help to shed still more light on the problem of adequate highway structural design.

In addition to the independent analysis of the AASHO Road Test presented here, the Road Test Board of Study was charged with developing thickness design relationships to be used in a revised edition of The Asphalt Institute's Thickness Design manual. The resulting thickness design method, distinct from but not unrelated to the work presented in this paper, is based on many sources of data besides the AASHO Road Test and has been reported elsewhere (9).

SUMMARY

This report presents a system of equations which describe all the measures of asphalt pavement performance made at the AASHO Road Test.

These measures include the Present Serviceability Index (PSI), the slope variance, the transverse profile (rutting), major cracking (Class 2 and Class 3), and roughness as indicated by the AASHO BPR-type roughometer. Each item was investigated in both the inner and outer wheelpath of each test section of the main asphalt experiment. In addition, the average serviceability index for each test section was studied. The serviceability indexes (Appendix B) used were calculated from the equations developed by The Asphalt Institute (1).

The results of our analysis as presented in this report are based on the complete data from the AASHO Road Test as released by the Highway Research Board at the time of its May 1962 special meeting in St. Louis, Mo. (2). These final results agree very well with our previous analysis of the average serviceability through the first 40 percent of the test traffic. The results of the preliminary analysis were presented at the International Conference on the Structural Design of Asphalt Pavements in August 1962 (3).

The results of our analysis are in the form of a group of equations relating performance to traffic volume, axle load, and pavement structure thicknesses. As such, they are easily used for the solution of thickness design problems or for estimating the remaining useful life of in-service highways. The effects of applied axle loads have been accounted for in a manner permitting the evaluation of mixed-traffic effects.

The effect of the Illinois environment has been evaluated by a climatic factor. This has been done to make the major part of the analysis free from any influence of the spring thaw effects of the Illinois environment. The development of the climatic factor points a way to possible evaluation of such factors for other climatic areas.

In the development of the models presented here, very deliberate efforts were made to identify and eliminate from the analysis any possible biases arising from specific conditions such as time, traffic rate, and initial pavement condition prevailing at the Road Test during testing. These factors are extraneous influences compared to the more basic engineering boundary conditions such as soil strength, materials, and methods of construction. In the course of the analyses, it became apparent that these extraneous boundary conditions would, in fact, bias the results if not accounted for.

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The methods used to test for and eliminate these sources of bias are described in the appropriate sections of this report.

The form of the equations relating thickness to applications (traffic), axle load, and the several measures of pavement performance is such as to permit solution for the minimum cost design (surface, base and subbase thicknesses) for any specific situation. This could be done by the technique of linear programing, a method of obtaining the optimum solution for large systems of linear equations. A simplified example of its use with the results of our analysis is given in the section of this report on "Minimum Cost Design."

Our models should be applicable to the analysis of satellite testing programs conducted in the various states. Of particular interest is the possibility of using our equations to describe the performance of in-service highways. This is possible because the basic parameters in the equations are independent of the age of the pavement. Thus, it would not be necessary to make any assumptions of traffic rate or condition prior to the time of starting a measurement program.

It is most important to keep in mind that the equations presented here give only a description (although a mathematical one) of what happened at the AASHO Road Test. However, a strong point in favor of the validity of our results is the fact that the deterioration rates determined by our analyses correlate very well with critical stresses and strains calculated for the pavement structures by theoretical stress-strain relationships for layered systems (4, 5).

As was pointed out earlier, there are certain real limitations on these analyses. There is no way to tell from the Road Test data how, for example, soils of different strengths will perform. To determine this will require similar data on different soils with proper measurements of the strength properties of the soils. Such extensions of the AASHO Road Test findings are urgently needed.

BASIC PERFORMANCE RELATIONSHIPS

This section describes the form of the equations used to represent the Road Test results. The development of the equations is given in Appendix A.

Present Serviceability Index

The basic equation form for the present serviceability index, P_t, average of both wheelpaths, is

$$P_t = P_o e^{-b\ W_t} \qquad (1)$$

or the logarithmic transformation

$$\ln (P_o/P_t) = b\ W_t \qquad (2)$$

in which

- P_t = present serviceability index at time, t_j;
- P_o = the same index at start of traffic ($t = 0$);
- b = deterioration rate parameter; and
- W_t = millions of accumulated load applications to time, t.

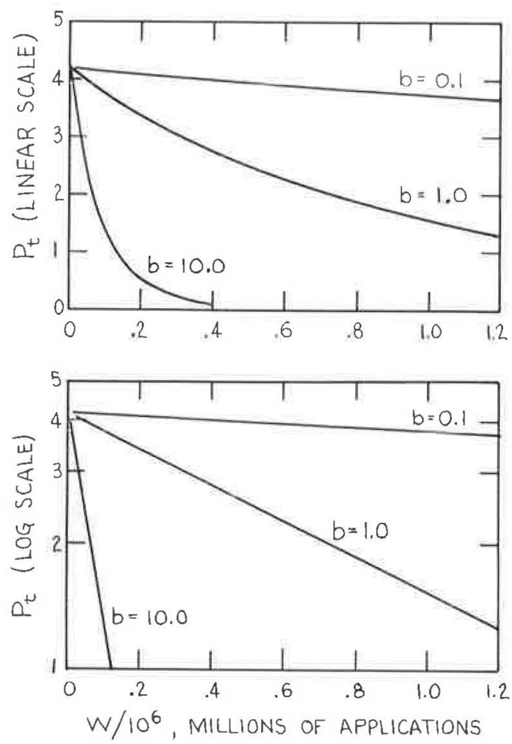

Figure 1. Basic performance relationships.

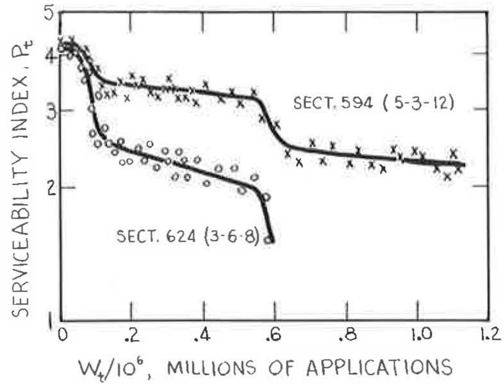

Figure 2. Typical performance data.

The shapes of these equation forms are shown in Figure 1.

In the equation, b is the deterioration rate of the pavement, dependent on thickness and strength of surface, base and subbase, subgrade soil strength, and the load applied to the pavement. Analysis of the Road Test data shows that, for a given pavement, b is constant throughout the year, except for the spring thaw periods. During these times, the deterioration rate increases to some higher value and then returns gradually to its original, nonspring thaw, value. This increase of the deterioration rate is the result of a loss of strength in the pavement structure, most likely in the subgrade soil and the granular layers. Figure 2 shows some typical Road Test data, illustrating very well this seasonal variation in the deterioration rate.

The variation of b follows a very definite and consistent pattern. The value of b can be described, at any time, i, by the equation:

$$b_i = b_o [1 + k(x_i - 2x_i^2 + x_i^3)] \tag{3}$$

in which b_o is the value of b prevailing during most of the year, x_i is the real time fraction (modulo one) of the spring thaw already passed, and k is a parameter, determined from the data, measuring the relative loss of strength in the pavement structure during a spring thaw period. An important point is that the variation of b is independent of design thickness and applied load.

If b_i is the rate of deterioration when a single load application is made, the performance equation can be written as:

$$\Delta \ln (P_o/P_t) = b_o \, v_i/10^6 \tag{4}$$

in which

$$v_i = 1 + k(x_i - 2x_i^2 + x_i^3) \tag{5}$$

If we had knowledge of the exact time, i, when each load application was made, we could sum Eq. 4:

$$\ln (P_o/P_t) = \sum_{i=1}^{t} b_o \, v_i/10^6 \tag{6}$$

As the Road Test data give load applications only in two-week counts, if the applications (again in millions) occurring in each biweekly period are denoted by n_i, Eq. 6 can be approximated by:

$$\ln (P_o/P_t) = \sum_{i=1}^{t} b_o \bar{v}_i \, n_i \tag{7}$$

in which $\bar{v}_i$ is the integral average of v_i for each two-week period ending on Index Day, i. This average assumes that the traffic rate is constant during the index period.

As b_o is constant for a given design and load, Eq. 7 may be written as:

$$\ln (P_o/P_t) = b_o \sum_{i=1}^{t} \bar{v}_i \, n_i \tag{8}$$

Here, the acceleration function, $\bar{v}_i$, operates as a weighting function on n_i, the applications during period i. This is convenient for defining

$$\text{Weighted applications} = \sum_{i=1}^{t} \bar{v}_i \, n_i = W_t* \tag{9}$$

so that we can write

$$\ln (P_o/P_t) = b_o \, W_t* \tag{10}$$

It must be kept in mind that this concept of weighted applications is only a convenience. What has really occurred is an increase of the basic deterioration rate, b_o, during the spring thaw period.

By considering the increase of b_o in this way in our analysis, we avoid any bias in the data caused by having had the traffic start in late October 1958. Most of the test sections which failed during the first spring thaw would have accumulated many more load applications before failure if traffic startup had been late June.

The weighted applications, W_t*, as defined in Eq. 9, can be interpreted as the real load applications one would require to change the serviceability from P_o to P_t for a pavement of strength b_o in an area with no spring thaw or similar acting period of accelerated deterioration. As a result, the analysis yields performance relationships for two climatic conditions: (a) the Ottawa, Ill., environment; and (b) an ideal environment (no spring thaw, etc.). Environments in most areas of this country undoubtedly fall between these two points.

If, as for design purposes, we assume a constant rate of load application, we can define an annual adversity (climatic or regional) factor, F, using N periods covering exactly one year, as:

$$F = \sum_{i=1}^{N} \bar{v}_i/N \tag{11}$$

That is, F equals the average value of $\bar{v}_i$ over the entire year.

For practical design work, some estimate of an average F value must be used, because conditions such as spring thaws vary from year to year in their effect on pavements. An average value of F based on Y years observed is most easily calculated as:

$$\bar{F} = 1 + \left(\sum_{j=1}^{Y} k_j \, m_j \right) / (12YN) \tag{12}$$

in which

k_j = the value of the thaw parameter, k, for year j;
m_j = the number of observation periods in which the adverse spring thaw conditions exist in year j;
$1/12$ = the integral of $x - 2x^2 + x^3$ between x = 0 and x = 1;
Y = number of years observed; and
N = number of observation periods per year.

There are likely also large effects of subgrade soil, drainage, etc., on this adversity factor. The Road Test gives only two estimates of F, one for each complete year of testing. Satellite test programs in the various states would provide the information required to determine reasonably accurate values of F for the different areas of the country.

It follows from the definition of F that for each year

$$W_t* = F \, W_t \tag{13}$$

or for F derived from several years' data

$$W_{t^*} = \bar{F}\, W_t \tag{14}$$

Therefore, by substitution of Eq. 14, Eq. 10 can be written in terms of real applications:

$$\ln\,(P_o/P_t) = b_o\, \bar{F}\, W_t \tag{15}$$

We can now consider the adversity factor operating as a weight on b_o. The F factor is the average acceleration of b_o, as defined in Eq. 11. Hence, $\bar{F}\, b_o$ is the annual average rate of pavement deterioration. In this form, the equation is most convenient for design use because F may be incorporated directly into the relationship of b_o with factors such as thickness.

Relation of Performance to Design and Load.—The basic deterioration rate, b_o, in Eq. 15 is related to design and load as follows:

$$\ln\, b_o = a_o + a_1 D_1 + a_2 D_2 + a_3 D_3 + a_4 L \tag{16}$$

in which

D_1 = asphalt concrete surface thickness (in.),
D_2 = crushed stone base (in.),
D_3 = sand gravel subbase (in.),
L = $L_1/L_2 + a_5\,(L_2 - 1)$,
L_1 = gross axle load (kips), and
L_2 = 1 for single axles and 2 for tandem axles.

The deterioration rate, b_o, is completely independent of the age of the pavement, being a function only of the thickness and load variables.

Rewriting Eq. 16 as

$$\ln\, b_o = a_o + a_1\,[\,D_1 + \frac{a_2}{a_1} D_2 + \frac{a_3}{a_1} D_3\,] + a_4 L \tag{17}$$

we see that the term in brackets defines an equivalent thickness of surface, D_a, wherein the ratios a_2/a_1 and a_3/a_1 are the surface equivalencies of base and subbase, respectively. The coefficient, a_o, is almost certainly dependent on subgrade soil strength. Possibly a_1, a_2, and a_3 are also dependent on soil strength. Certainly a_1, a_2, and a_3 should be dependent on the respective strength properties of the surface, base and subbase.

Substituting D_a, equivalent surface thickness, for the bracketed term gives:

$$\ln\, b_o = a_o + a_1 D_a + a_4 L\,. \tag{18}$$

Combining Eq. 18 with the logarithmic transform of Eq. 15 and solving for D_a gives the design formula:

$$D_a = \frac{-\,[a_o + \ln F - \ln\ln\,(P_o/P_t) + \ln\,(W_t) + a_4 L\,]}{a_1} \tag{19}$$

The load term can be combined with the W_t term:

$$\ln\, W_t + a_4 L = \ln\,[\,W_t\, e^{a_4 L}\,] \tag{20}$$

The bracketed portion of Eq. 20 defines axle-load effects in a way that permits development of axle-load equivalencies. This is possible because the value of $W_t\, e^{a_4 L}$, regardless of the particular values of W_t and L, will require some fixed value of D_a once

P_o, P_t, and F are specified. This permits a rather straightforward evaluation of mixed traffic. A term $e^{a_4(L - L')}$ defines a load factor relating W applications of load L to W' applications of some other load L':

$$W' = W e^{a_4(L - L')} \tag{21}$$

The theory underlying this definition of axle-load equivalencies is discussed in Appendix A.

In this report, we have chosen to use the 18-kip, single-axle load as a base or reference load, defining equivalent 18-kip, single-axle loads, W_{18}, as:

$$W_{18} = W_L \; e^{a_4(L-18)} \tag{22}$$

By combining this with Eq. 19, we can write a final design equation:

$$D_a = \frac{-[a_o + \ln F - \ln \ln (P_o/P_t) + \ln (W_{18})]}{a_1} \tag{23}$$

Other Measures of Performance

Criticisms have been made of the use of the serviceability index in defining pavement failure These criticisms are based on the contention that the index is too big a melting pot, in which the various physical forms of distress lose their identity and meaning.

In a certain sense, this criticism is a valid one; knowing only that a pavement has too quickly reached an unacceptably low serviceability certainly gives no clue to the possible cause of the rapid decline. However, the serviceability index does give a good overall picture of the riding quality which the customer-taxpayer is paying for. In a sense, the serviceability index was meant to be a melting pot to provide a single measurement of the riding quality of a pavement. It is difficult, if not impossible, to analyze the index, a subjective thing, in the engineering terms of stress, strain, and strength. For this we must look at each form of pavement distress separately.

In this section of this report is given the form of the equations used to analyze, by wheelpath, the following forms of distress measured at the AASHO Road Test: (a) serviceability index, (b) slope variance, (c) rutting, (d) roughness index, and (e) cracking. In all cases, I and ϕ will be used to identify the inner and outer wheelpaths, respectively.

Basic Equation Forms.

Wheelpath Serviceability Index, P_t. —The basic equation form for the individual wheelpaths is identical to that for the average serviceability index. For the inner wheelpath:

$$P_{t,\, I} = P_o \, e^{-b_I \, W_t} \tag{24}$$

and for the outer wheelpath:

$$P_{t,\, \phi} = P_o \, e^{-b_\phi \, W_t} \tag{25}$$

The logarithmic forms of these equations used in analyzing for b_I and b_ϕ are:

$$\ln (P_o/P_{t,\, I}) = b_I \, W_t \tag{26}$$

$$\ln (P_o/P_{t,\, \phi}) = b_\phi \, W_t \tag{27}$$

P_0 is the same initial serviceability used for the average P_t analysis. The b's, of course, are the deterioration rates for the individual wheelpaths and are analogous to the b_0 defined for the average P_t performance.

Wheelpath Slope Variance, SV. —The slope variance is a statistical measure of the variability of the slope of the pavement. As such, it is a direct measure of the longitudinal roughness of a pavement. At the AASHO Road Test, a new instrument (the AASHO Profilometer) was developed to measure and record a continuous analog trace of the pavement slope. This analog trace was then sampled at 1-ft intervals to obtain point measurements of the slope, s_i. The slope variance, σ_s^2, was then calculated as:

$$\sigma_s^2 = \sum_{i=1}^{n} (s_i - \bar{s})^2/(n - 1) \tag{28}$$

For convenience, these values were scaled by a factor of 10^6 to obtain more manageable numbers. These scaled values, called SV, are the ones used in all phases of the Road Test work (including our analyses) to denote slope variance. SV = 10^6 x slope variance, according to Eq. 28.

The slope variance is by far the most important single variable influencing the serviceability index. Consequently, it was no surprise to find that the best mathematical form for analyzing slope variance was derivable from the serviceability index performance model and the serviceability index equations developed by The Asphalt Institute (1). The equations used are as follows:

for the inner wheelpath

$$\sqrt{SV_I} = \sqrt{SV_0} + b_I \, W_t \tag{29}$$

and for the outer wheelpath

$$\sqrt{SV_\phi} = \sqrt{SV_0} + b_\phi \, W_t \tag{30}$$

SV_0 is the average initial slope variance for the test section. The b's are the rates at which slope variance increased with traffic.

Wheelpath Rutting, RD. —The rut depth measures the amount of permanent deformation in the transverse profile of the pavement. At the Road Test, the rut depth was measured below the center of a 4-ft span placed across the wheelpath. The values reported are actually the average of a number of rut depth measurements made throughout the length of the test section.

Rutting plays only a secondary role in determining the serviceability index. In fact, it was not included in the index equations originally developed by the Road Test staff because the amount of rutting found on the in-service highways panel rated for serviceability was insignificant and minor in extent. As serious rutting appeared on the Road Test, some of the test sections were panel rated and rutting was found to have a significant, although secondary, effect on serviceability.

The serious rutting observed at the Road Test seems to have been a peculiar effect caused by the unnatural (though necessary) isolation of the specific axle loads. When the results of our analysis are applied to specific real traffic situations on practical sections, the level of rutting estimated at the end of, say, 20 years of traffic is very minor.

The equations found best suited for analyzing rutting are as follows:

for the inner wheelpath

$$RD_I^2 = b_I \, W_t \tag{31}$$

and for the outer wheelpath

$$RD_\phi^2 = b_\phi \, W_t \tag{32}$$

The b's are the rates of rutting in each wheelpath for the particular load and test section being studied. The initial value of RD was zero, which accounts for the lack of an RD_o term. It should be noted that the growth of rutting is a decelerating function of load applications, that is, the change in rutting (RD) per application becomes smaller with increasing load applications.

Wheelpath Roughness, RI.—Part of the AASHO Road Test measurement program involved periodic measurements of the roughness index by the AASHO roughometer, modeled after the familiar BPR roughometer. This type of instrument measures the total positive vertical displacements of the pavement surface, longitudinally, in the wheelpaths.

Theoretically, this measurement can be related to the slope variance of a pavement. However, analysis of data obtained on asphalt pavements by both methods shows only moderate correlation between the two types of measurement. This is due, it appears, to two mechanical features of the BPR-type roughometer:

1. Even on relatively smooth pavements, there seems to be extraneous vibration of the measuring wheel relative to its reference frame. As a result, there seems to be a minimum of 40 to 50 in. per mile roughness measurable on even a perfectly smooth pavement.
2. The reaction-time (inertial) characteristics of the BPR-type roughometer are such that the device is sensitive to (i. e., measures) only the higher frequency distortions in a pavement surface, filtering out, for the most part, the lower frequencies. The profilometer, on the other hand, will measure virtually all distortion frequencies.

As a result, although both types of equipment attempt to measure the same thing, they will, in fact, measure somewhat different properties of the pavement profile because of their mechanical differences. This is not to say that one measurement is any better than the other, only that they are different. Each has its advantages and disadvantages.

The profilometer has an advantage in that it measures slope variance which, as noted, was used as the major variable in defining serviceability. Also, there is a stronger correlation of riding quality with the slope variance than with roughness index. However, the profilometer is designed to operate at a speed of about 3 mph, compared to as high as 20 mph for a roughometer. Besides the obvious economic advantages of the faster instrument, some state highway departments are greatly concerned with the safety hazards involved in operating such equipment at extremely low speeds on major highways.

The decision of which type of instrument to use on in-service pavement research or in routine maintenance/condition surveys must be made by any user agency by balancing the relative merits and demerits of the two types of instruments.

The form of the roughness-applications equation was governed by consideration of the AASHO Road Test data. This form agrees with the results obtained by Housel (6) and the theoretical relation between slope variance and roughness index developed by Painter (1):

for the inner wheelpath:

$$RI_I = RI_o + b \quad W_t \tag{33}$$

and the outer wheelpath:

$$RI_\phi = RI_o + b_\phi \; W_t \tag{34}$$

in which RI_o is the average initial roughness of the test section, and the b's are the rates of increase of roughness with traffic.

Cracking and Patching, CP.—The appearance of cracking in an asphalt surface is used by many highway engineers as a direct indication of a structural inadequacy somewhere in the pavement system. Pavement cracking was used as the principal criterion of pavement failure at the WASHO Road Test. Major cracking (Class 2 and Class 3) and patching (to cover up previously cracked areas) was found to have only a minor role in determining the serviceability (riding quality) of a pavement. This does not mean that cracking is of minor structural importance. Indications are that by the time cracking has progressed far enough to impair greatly the riding quality of a pavement, that pavement has become very rough in terms of slope variance; hence, the slope variance term accounts for most of the detrimental effects of cracking.

Cracking can be the result of two types of failure in the pavement structure:

1. Shear failure in one or more of the pavement layers caused by complete overstressing by a single load application; or
2. Fatigue failure of the asphalt surface caused by the repeated applications of loads, no one of which is necessarily even close to causing shear failure.

Neither type of failure is unique to highway pavements. Steel and concrete beams exhibit the same behavior.

In general, any pavement likely to crack because of shear failure is already woefully underdesigned; hence, the study of cracking can be limited to cases of fatigue failure. On no asphalt section at the AASHO Road Test was cracking observed until after almost a thousand load applications. Many sections exhibited no cracking at the end of the testing period after having received over 1.1 million load applications.

Because there was very little cracking observed in the inner wheelpath, it was necessary to restrict the analysis to the outer wheelpath only. The equation form is

$$\sqrt{(C + P)_{\phi}} = b_{\phi} \, W_t \tag{35}$$

In terms of C + P (cracking + patching), this is an accelerating function, i.e.,

$$(C + P)_{\phi} = b_{\phi}^2 \, W_t^2 \tag{36}$$

Relation of Other Performance Measures to Design and Load. —The rates of deterioration for the several other performance measures studied were related to design and load by equations of the same form as the average serviceability, with the addition of a term to account for wheelpath differences.

$$\ln b = a_0 + a_1D_1 + a_2D_2 + a_3D_3 + a_4L + a_6WP \tag{37}$$

in which WP = wheelpath indicating variable, with a value of 1 for the inner wheelpath and 0 for the outer wheelpath and all other terms are as defined previously for Eq. 16.

For each different performance equation, it is then possible to define an equivalent surface thickness as a function of the performance measure and equivalent 18-kip, single-axle applications, giving equations of the form:

$$D_a = \frac{-[a_0 + \ln F - f(\text{Performance}) + \ln(W_{18})]}{a_1} \tag{38}$$

The functions of performance, f(Performance), for each of the performance measures, are as follows:

Performance Measure	f(Performance)
PSI	$\ln \ln(P_0/P)$
Slope variance	$\ln(\sqrt{SV} - \sqrt{SV_0})$
Rut depth	$2 \ln RD$
Roughness index	$\ln(RI - RI_0)$
Cracking	$\frac{1}{2} \ln(C + P)$

The subscript o, as in P_o, denotes the initial value (prior to traffic startup) of that variable.

NUMERICAL RESULTS

The results presented here are all based on the complete data from the AASHO Road Test, representing a total of 1,114,000 load applications on the test pavements. The data were released to The Asphalt Institute after the Highway Research Board's St. Louis Conference on the AASHO Road Test in May 1962. Complete details of the fitting techniques used in our analyses have been described by Painter (11).

Climatic Factor

The first step in the analysis was to obtain values for the climatic factor, F, for each of the two years of testing. This was done to define the "weighted applications" scale. The average serviceability histories of all possible test sections were used for obtaining the F values. Obviously, each measure of performance could be used to define a climatic factor; however, we restricted our attention to the average serviceability so as to have only one such factor.

The average serviceability index history for each test section was fit by an equation of the form:

$$\ln P_t = a_o - a_1 - a_2 - b_o W_t \tag{39}$$

in which

a_o = fitted initial P,
a_1 = first spring drop in P in excess of traffic effect,
a_2 = second spring drop in P in excess of traffic effect, and
b_o = deterioration rate due to traffic outside spring thaw.

For each section surviving the first full year of testing (i. e., still in service on Index Day 26), an individual F value was obtained from the ratio of annual average deterioration rate over b_o, the basic deterioration rate:

$$F_1 = \frac{-a_1 - b_o W_{26}}{-b_o W_{26}} = 1 + \frac{a_1}{b_o W_{26}} \tag{40}$$

For each section still in service after two full years (on Day 52), an F value was obtained from

$$F_2 = \frac{-a_2 - b_o (W_{52} - W_{26})}{-b_o (W_{52} - W_{26})} = 1 + \frac{a_2}{b_o (W_{52} - W_{26})} \tag{41}$$

No significant effect of either structural design or load could be found to explain the variations in the F_1 or F_2 values. Accordingly, they were averaged over all sections to give $\bar{F}_1 = 3.760$ (based on 100 sections) and $\bar{F}_2 = 2.655$ (based on 44 sections). These gave values for the two spring thaw parameters (k_1 and k_2) of 174 and 119, respectively.

The overall average value of F for design purposes, computed according to Eq. 12, is $F_D = 4.0$. For each year, the design F (Eq. 12) is $F_{D,1} = 5.5$ and $F_{D,2} = 2.5$.

Table 1 lists the actual applications (W), the weighting function ($\bar{v}_t$), and the weighted applications ($W_{t*} = \sum_1^t v_t n_t$) obtained in this analysis. The relation of weighted to actual applications is shown graphically in Figure 3.

TABLE 1
WEIGHTED MEAN LOAD APPLICATIONS

Index Day	W (millions)	v_t	W_t* (millions)
1	0.0007	1.00	0.0007
2	0.0052	1.00	0.0052
3	0.0114	1.00	0.0114
4	0.0217	1.00	0.0217
5	0.0287	1.00	0.0287
6	0.0356	1.00	0.0356
7	0.0457	1.00	0.0457
8	0.0584	1.00	0.0584
9	0.0696	10.16	0.1722
10	0.0762	22.26	0.3191
11	0.0797	26.47	0.4118
12	0.0860	24.56	0.5665
13	0.0978	20.22	0.8051
14	0.1072	12.56	0.9231
15	0.1205	6.01	1.0031
16	0.1341	1.84	1.0281
17	0.1512	1.00	1.0451
18	0.1686	1.00	1.0626
19	0.1820	1.00	1.0760
20	0.1992	1.00	1.0932
21	0.2169	1.00	1.1109
22	0.2331	1.00	1.1271
23	0.2499	1.00	1.1439
24	0.2710	1.00	1.1650
25	0.2891	1.00	1.1831
26	0.3053	1.00	1.1993
27	0.3239	1.00	1.2179
28	0.3383	1.00	1.2323
29	0.3527	1.00	1.2467
30	0.3729	1.00	1.2669
31	0.3853	1.00	1.2793
32	0.4078	1.00	1.3018
33	0.4446	1.00	1.3386
34	0.4800	1.00	1.3740
35	0.5055	1.00	1.3995
36	0.5397	1.00	1.4337
37	0.5699	11.37	1.7771
38	0.6014	17.72	2.3352
39	0.6340	11.50	2.7102
40	0.6692	3.00	2.8158
41	0.7047	1.00	2.8512
42	0.7336	1.00	2.8802
43	0.7713	1.00	2.9178
44	0.8068	1.00	2.9534
45	0.8329	1.00	2.9794
46	0.8677	1.00	3.0142
47	0.9014	1.00	3.0480
48	0.9291	1.00	3.0756
49	0.9512	1.00	3.0978
50	0.9838	1.00	3.1303
51	1.0156	1.00	3.1622
52	1.0496	1.00	3.1962
53	1.0805	1.00	3.2270
54	1.0996	1.00	3.2462
55	1.1138	1.00	3.2604

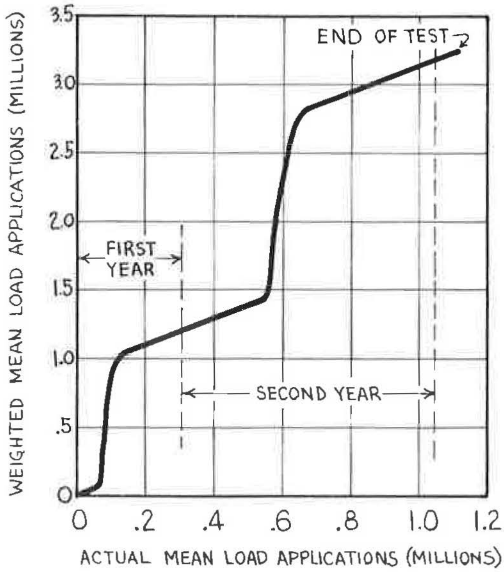

Figure 3. Weighted vs actual load applications.

Design and Load Effects

The data for each performance measure, in each wheelpath of each section, were correlated against weighted applications using the equation forms discussed previously. These equations are all of the general form:

$$g(\text{Performance})_t = b\ W_t^* \qquad (42)$$

The mathematical function, g(Performance), is the antilogarithm of f(Performance).

The b's thus obtained for all measures of performance (except cracking because of incompleteness) were then analyzed (as ln b) by routine factorial analysis methods, loop by loop, for the design, lane, and (except for average serviceability) wheelpath effects. With the possible exception of the Loop 2 analyses, no definite trends in the values of the effects could be noted. The coefficients for thickness and wheelpath effects were then averaged across all five loops.

Using the average thickness effects for each performance measure, the mean ln b values for each loop were adjusted to a common thickness (zero for all layers, for convenience) to study the load effects. This adjustment was necessary because the average thickness of each layer was different, generally, in each loop. The

TABLE 2

LAYER SUBSTITUTION RATIOS

Performance Measure	D_1	D_2	D_3	
PSIA	1	0.282	0.210	$= D_a$
PSI	1	0.291	0.171	$= D_a$
SV	1	0.313	0.200	$= D_a$
RD	1	0.145	0.046	$= D_a$
RI	1	0.245	0.167	$= D_a$
C + P	1	0.284	0.228	$= D_a$

TABLE 3

LOAD FACTORS

Axles	L_1 (kips)	L, Equivalent SA Load	Load Factor	Avg. Axle Factor
Single	2	2	0.0545	0.0545
	6	6	0.1128	0.1673
	12	12	0.336	0.375
	18	18	1.000	1.0564
	22.4	22.4	2.23	2.29
	30	30	8.88	8.90
Tandem	24	14.36	0.515	0.571
	32	18.36	1.068	1.165
	40	22.36	2.21	2.31
	48	26.36	4.57	4.74
Steering	2	2	0.0545	-
	4	4	0.0783	-
	6	6	0.1128	-
	9	9	0.1944	-
	12	12	0.336	-

lane effects were then added to account for the load differences between lanes.

The load effect for each performance measure was obtained by regression of these adjusted lane means ($a_{O,\ L}$) against the load terms shown in Eq. 16. Steering axle effects were accounted for, as shown in Eqs. 2 and 10, in accordance with the load equivalency concept developed in this analysis. The load effect was found to be virtually the same for all performance measures. Accordingly, the data were pooled and reanalyzed to obtain a single value for the effect of load.

The thickness effects are given in Table 2 as "layer substitution" ratios, relative to asphalt concrete surface.

The load effect which we obtained, expressed in terms of equivalent 18-kip, single-axle applications, is

$$W_{18} = W_L \cdot 10^{0.0790(L-18)} \tag{43}$$

with $L = L_1$ for single-axle loads or $L_1/2 + 2.36$ for tandem-axle loads.

Load factors $\left(10^{0.0790(L-18)}\right)$ for the various loads used at the Road Test are shown in Table 4. This table also contains values of average axle load factors which include the effect of the steering-axle load for each of the vehicles as used at the Road Test.

The single tandem-axle load relationship $L = L_1/2 + 2.36$ can be used to estimate the relative effect of tandem-axle loads. Consider a load, L, applied to a single axle with 2L applied to a tandem axle. Using the form of Eq. 21, we obtain

$$W_S = W_T \cdot 10^{0.079(L_T - L_S)} = W_T \cdot 10^{0.079(2.36)} = W_T \cdot 1.54 \tag{44}$$

Thus, one application of a load 2L on a tandem axle is equivalent to only 1.54 applications of load L on a single axle. For example, consider a 36-kip total load. If this is split over two single axles (18 kips each), we have, of course, two 18-kip, single-axle loads. If we put the 36 kips all on a tandem-axle combination, we have only 1.54 equivalent 18-kip, single-axle loads. In terms of asphalt pavement deterioration, therefore, it is advantageous to put loads on tandem axles rather than half loads on single axles. This type of effect does not apply to portland cement concrete pavements. In fact, from the Road Test equations developed by the Highway Research Board, 36 kips on a tandem axle is roughly equivalent to 2.5 18-kip, single-axle loads (7). (The corresponding Road Test equations for asphalt pavements give a value of about 1.4 for the 18-kip equivalency of a 36-kip, tandem-axle load.)

A possible explanation of this observed tandem-axle effect on the asphalt pavements may be that the relatively wide spacing of the single axles permits two full deflections of the pavement, with virtually complete recovery between the axles, whereas there is not complete deflection recovery between the tandem axles.

TABLE 4

DESIGN EQUATIONS[a]

Performance Measure		Constant	log W_{18}	IWP	f(Performance)
PSIA	D_a =	3.764	2.910	-	$-2.910 \log \log (P_0/P)$
PSI	D_a =	3.623	3.565	-0.938	$-3.565 \log \log (P_0/P)$
SV	D_a =	7.412	3.477	-0.737	$-3.477 \log (\sqrt{SV} - \sqrt{SV_0})$
RD	D_a =	1.617	4.208	-1.796	$-8.416 \log RD$
RI	D_a =	9.782	2.416	-0.454	$-2.416 \log (RI - RI_0)$
C+P	D =	9.403	2.931	-	$-1.466 \log (C + P)$

[a]See Eq. 43.

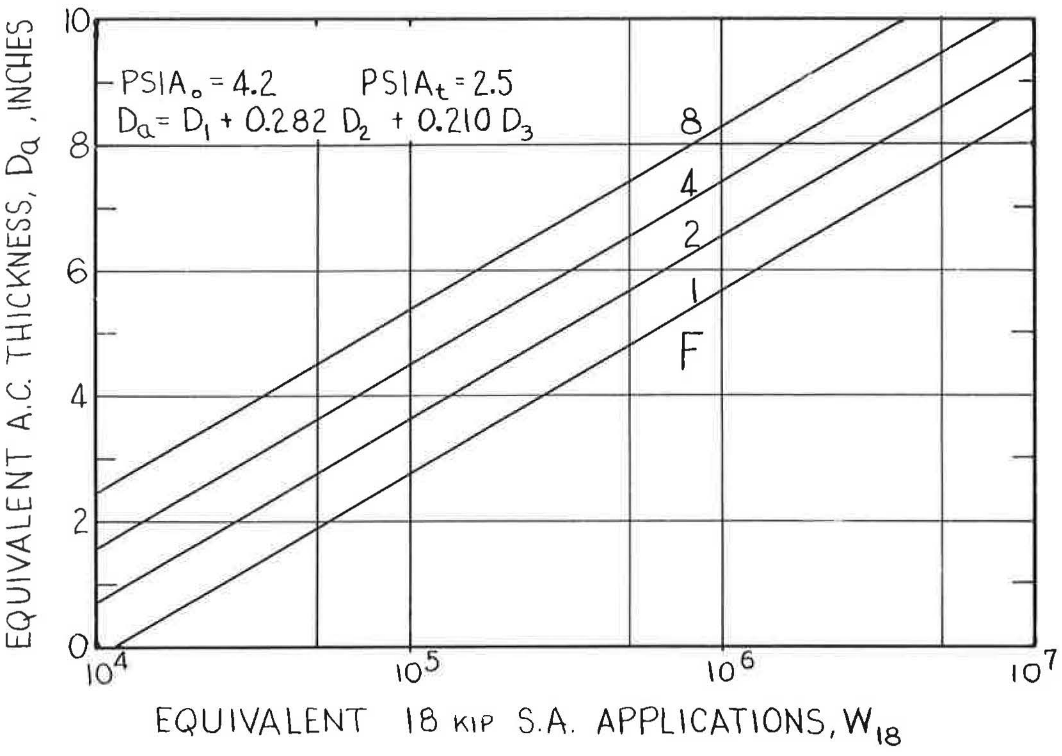

Figure 4. Design chart for average serviceability.

The final design equations for each measure of performance (according to the forms of Eqs. 23 and 28) are given in Table 4. These equations are plotted in Figures 4 through 9 for specified levels of performance. These charts are, in effect, design charts for soils of a strength of that of the AASHO embankment, with the actual level of the design thickness determined by the value of the climatic factor, F, and the total expected traffic, W_{18}.

Figures 4 and 5, for serviceability index, are based on an assumed terminal serviceability level of 2.5 and an initial value of 4.2 (Road Test asphalt pavement average). The choice of 2.5 for the terminal P_t value was somewhat arbitrary. It conforms, however, to the terminal value used by the Road Test staff in their reports.

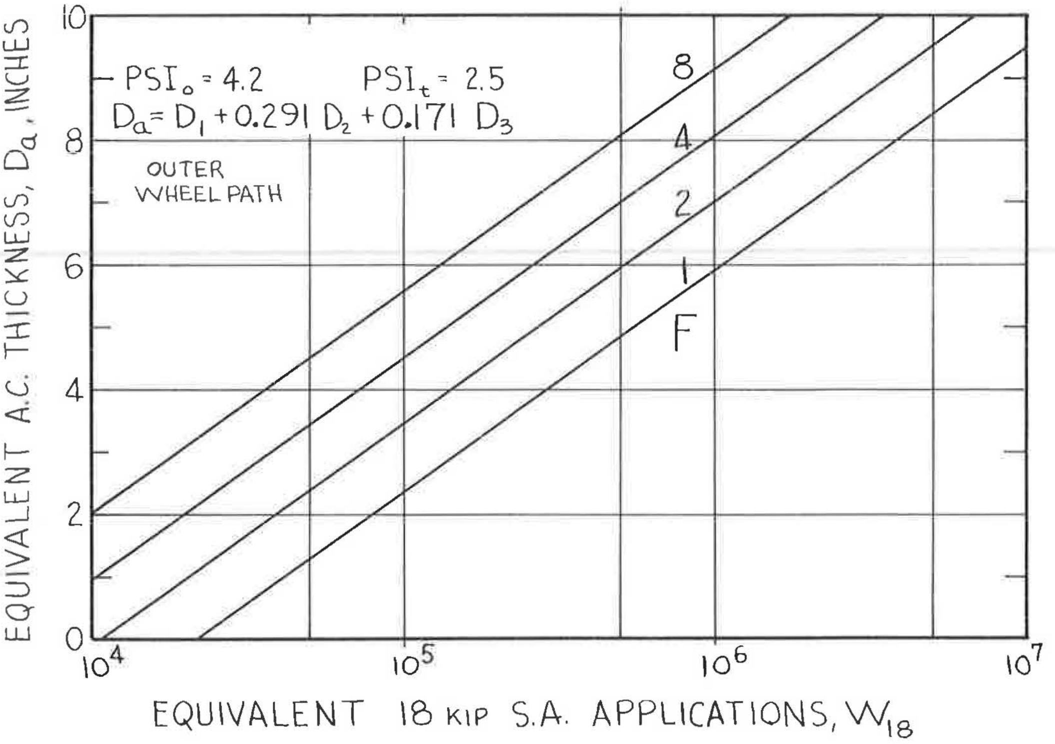

Figure 5. Design chart for outer wheelpath serviceability.

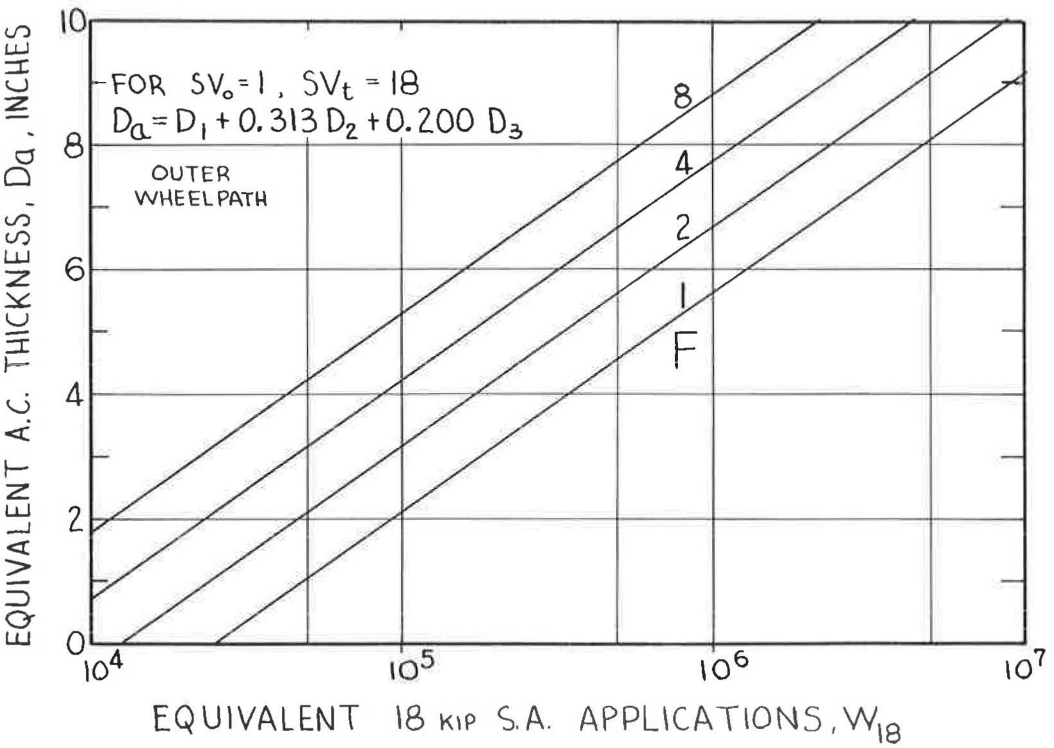

Figure 6. Design chart for slope variance.

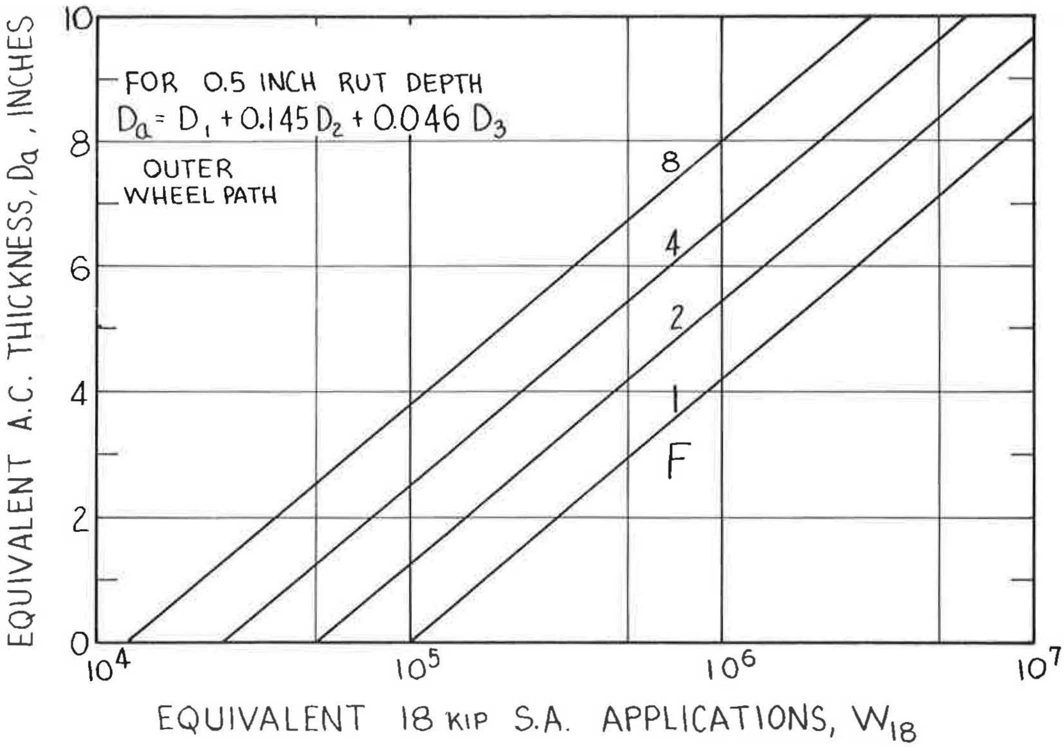

Figure 7. Design chart for rut depth.

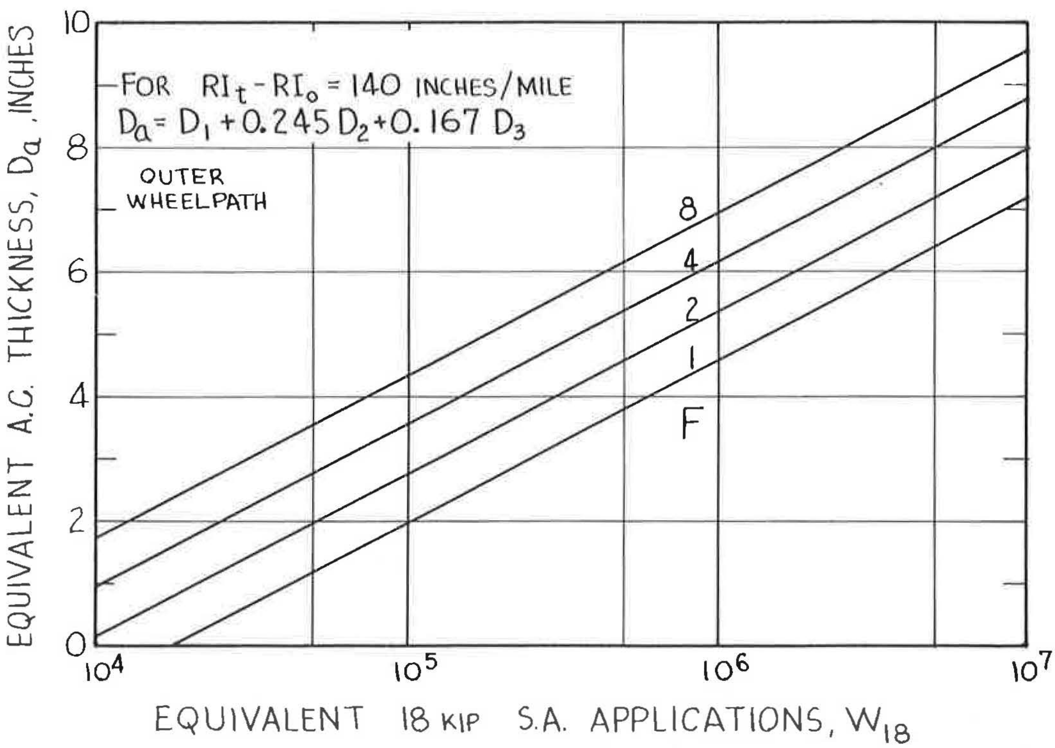

Figure 8. Design chart for roughness index.

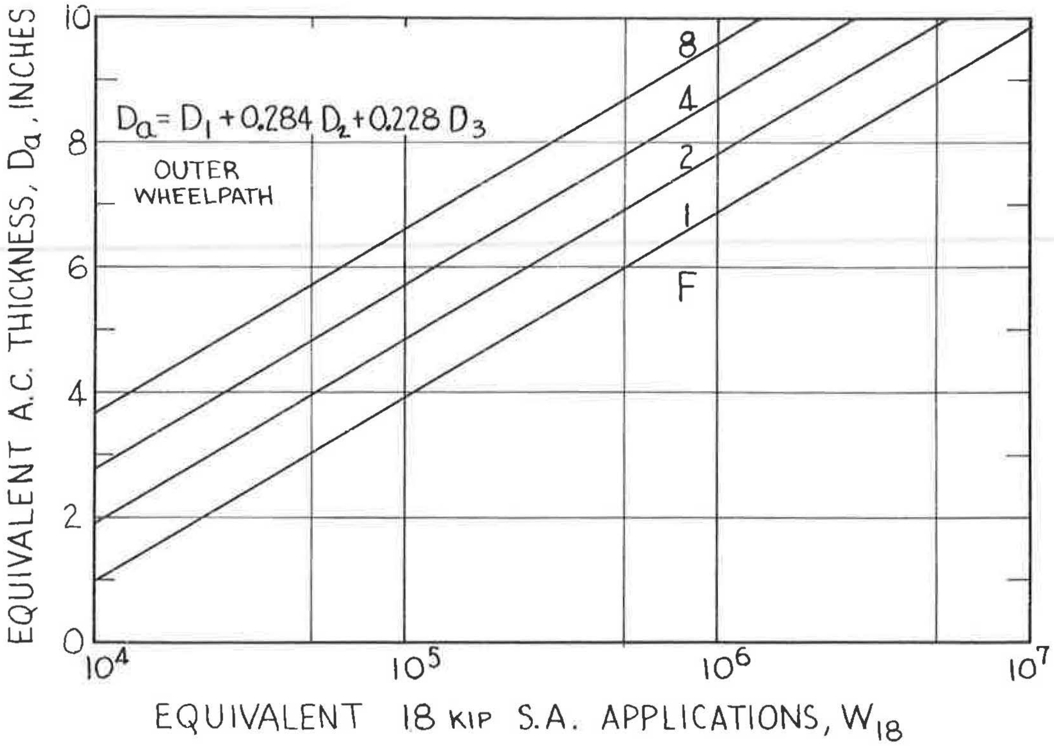

Figure 9. Design chart for 5 percent Class 2 + 3 cracking.

In the development of the serviceability index concept (8), it was found that, on the average, the rating panel members considered serviceabilities less than about 2.5 to be unacceptable for heavy-duty highways. Implicit in this finding is the possibility that lower terminal values may be acceptable on secondary roads. More research is needed along these lines to establish the relation between minimum acceptable serviceability and road type (usage patterns, etc.). Chastain and Burke (9) have shown that 2.5 is a fair average terminal value for main-line highways being "retired" by overlaying or reconstruction in Illinois.

For all other performance measures (Figs. 6 through 9) values were also chosen arbitrarily so as to give approximately 2.5 serviceability if all terminal values are reached. There is no support, similar to that referenced for serviceability index, for any of the individual values used.

It must be kept in mind that the D_a's obtained from the design equations (Table 4 or Figs. 4 through 9) are only equivalent to each other in terms of an all-asphalt pavement structure. For example, 10-in. D_a from the PSIA equation does not represent the same real structural section (with base and subbase courses) as does 10 in. of D_a from the rut depth equation. This is because the layer substitution ratios are, in general, different for all the different measures of performance. It is recommended that when the full set of equations is to be used, the total D_a be allocated among surface, base, and subbase by a linear programing study.

COMPARISON OF RESULTS WITH DATA

Predicted values of log W were compared with the observed values at a particular "terminal" level of performance for each performance measure. The standard errors in log W and the corresponding error in D_a are shown in Table 5, together with the terminal (and initial) values used for each performance measure. For all test sections,

TABLE 5

ERROR SUMMARY

Performance Measure	Initial	Terminal	Standard Errors	
			log W	D_a
PSIA	4.2	2.5	0.247	0.72
PSI	4.2	2.5	0.297	1.06
SV	1.0	18.0	0.335	1.17
RD	0	0.5	0.389	1.64
RI	70	210	0.364	0.88
C + P (owp), %	0	5	0.359	1.05

the observed log W was adjusted to account for the section's initial performance measure not having been exactly the value used in this comparison. (The initial values used here are approximately the average initial values for each measure at the Road Test.) The amount of adjustment in log W is determined by the models used to analyze the data, but is independent of the fitted parameters.

The errors given in Table 5 include only the data on sections which reached the "terminal" level of performance during the two years of traffic testing. The serviceability errors are less than the corresponding errors reported by the Road Test staff (7) or in the analysis by Shook and Finn (10). The Road Test report (7) shows a mean absolute error of 0.23 (standard error approximately 0.30) in log W, considering all levels of terminal serviceability. Shook and Finn (10) report a standard error of 0.35 in log W and 2.2 in their thickness factor, corresponding to about 0.9 in the D_a

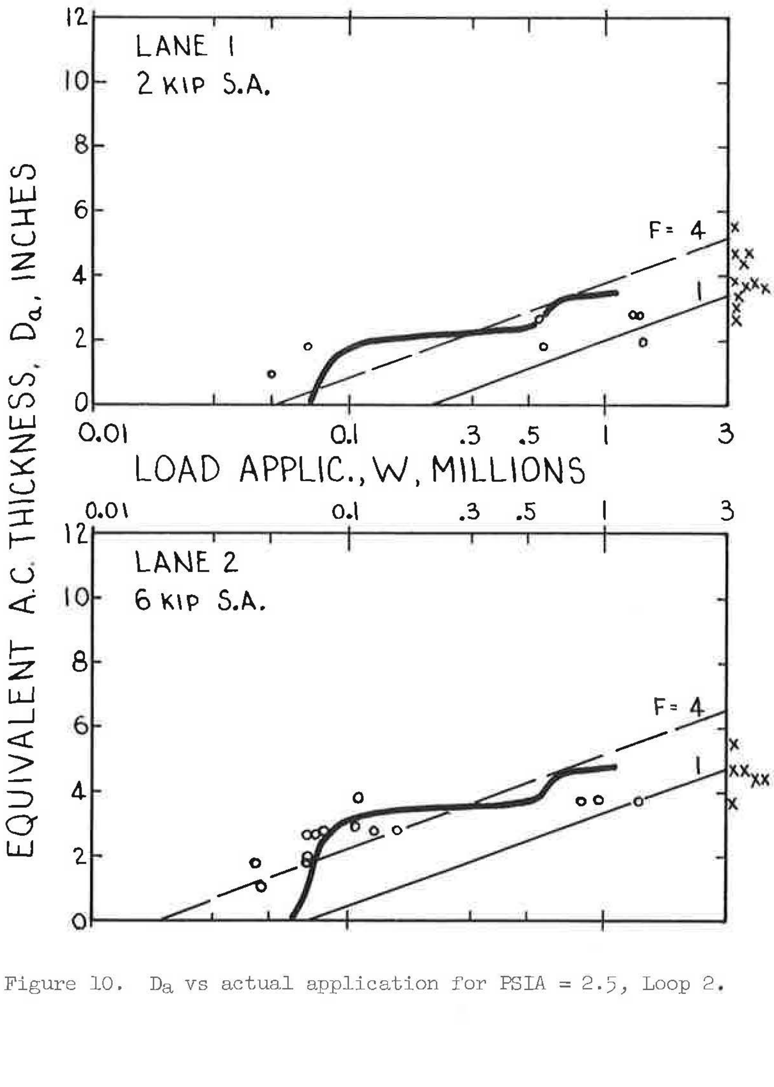

Figure 10. D_a vs actual application for PSIA = 2.5, Loop 2.

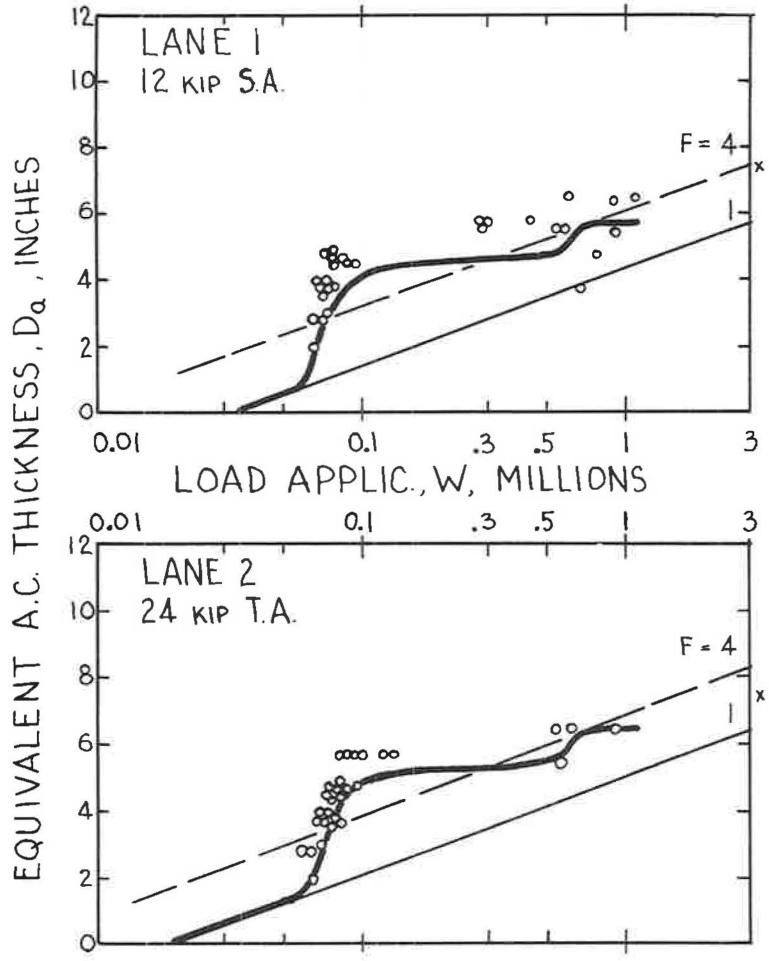

Figure 11. D_a vs actual application for PSIA = 2.5, Loop 3.

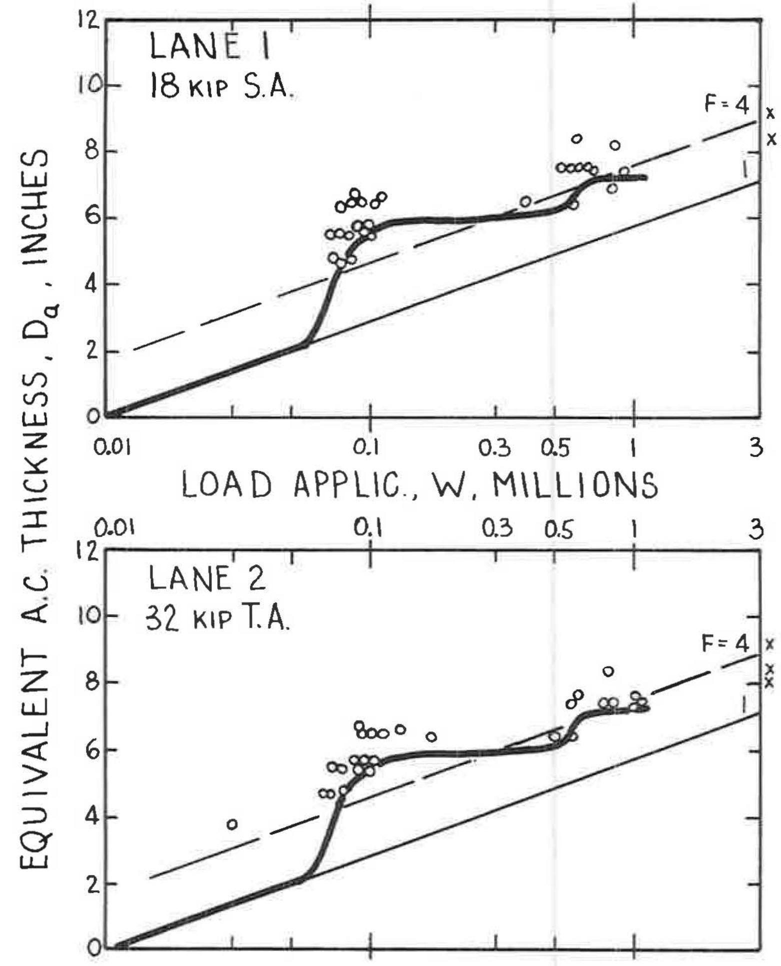

Figure 12. D_a vs actual application for PSIA = 2.5, Loop 4.

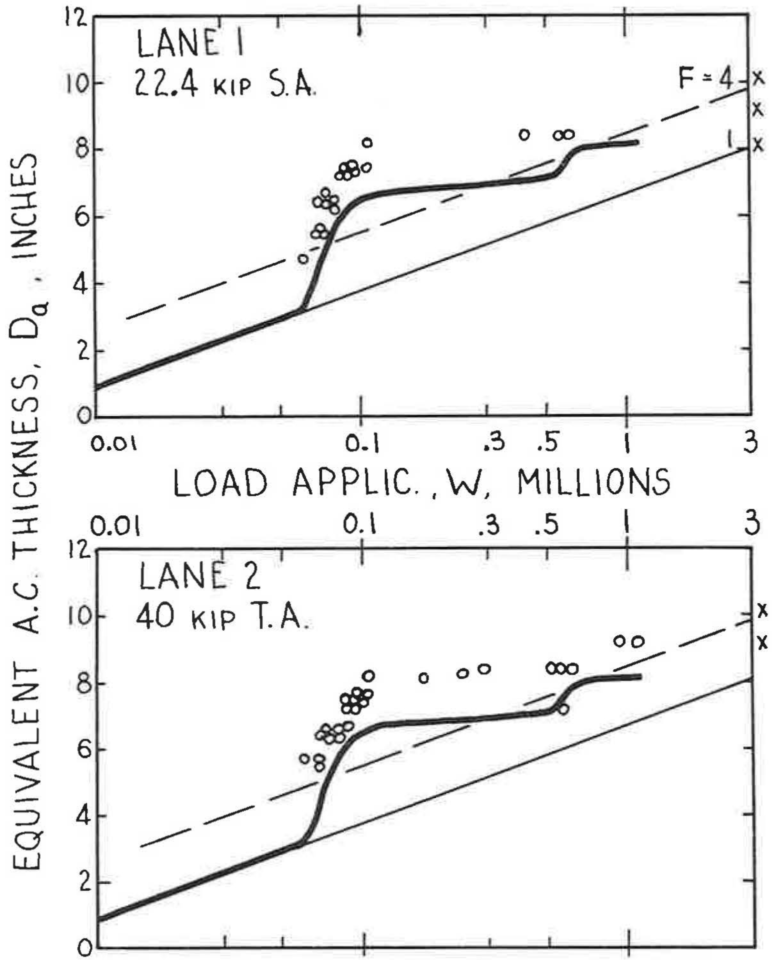

Figure 13. D_a vs actual application for PSIA = 2.5, Loop 5.

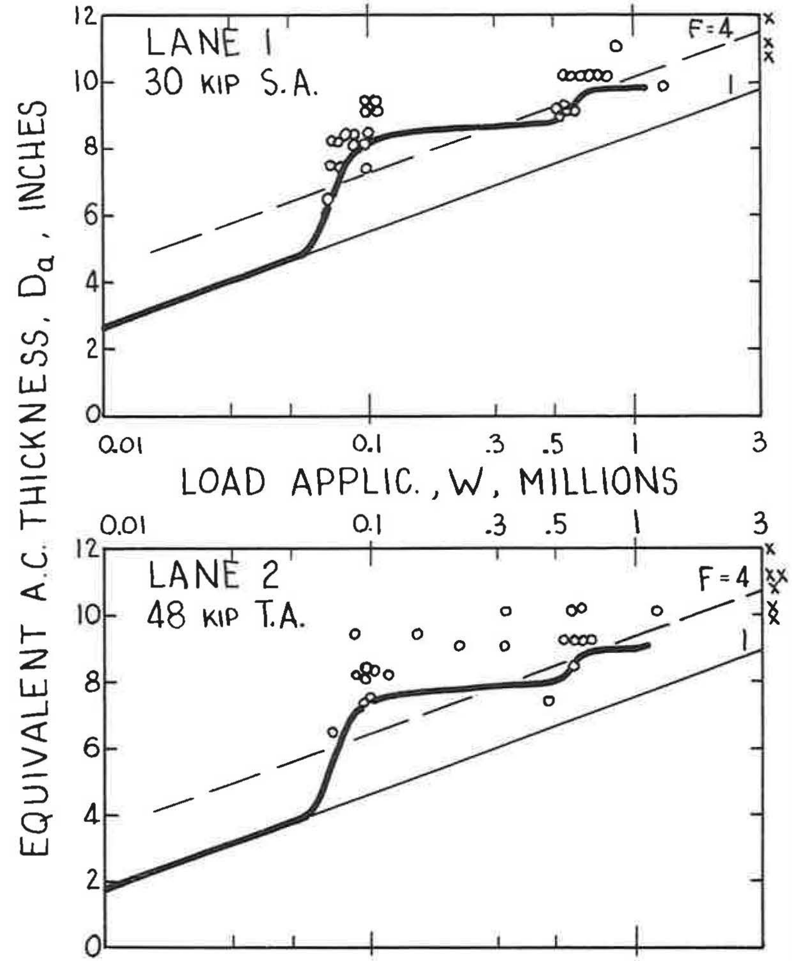

Figure 14. D_a vs actual application for PSIA = 2.5, Loop 6.

defined here. These other analyses of the Road Test data were restricted to average serviceability; hence, no comparisons can be made of the errors for the other performance measures.

Figures 10 through 14 show the PSIA data for each of the traffic lanes, plotted as adjusted actual applications to a terminal serviceability of 2.5, with the equivalent asphalt concrete thickness (D_a) as the ordinate. On each of these plots, straight lines are drawn for F values of 1 (W = weighted applications) and 4.0 (two-year average design value). In addition, a curved line is drawn which represents the conversion of the weighted applications vs D_a (F = 1) line to unweighted (actual) applications. In each graph, this curved line follows the data clusters very closely. We feel that this close fit of the curved lines to the data is a strong point in favor of the models used in this analysis, particularly for our method of weighting the applications.

As can be seen in Figures 10 through 14, quite a few points have large errors in log W because the section reached a 2.5 serviceability level just at the end of the first spring thaw, whereas the fitted actual applications (curved) line indicates the section should have "survived" until the following spring. In our model, a very small difference in pavement strength is decisive in determining whether such a section reaches 2.5 at the end of one spring thaw or the beginning of another. If we delete from the error analysis all points for which this behavior is apparent, we obtain a standard error in log W (for average serviceability) of about 0.12. This is probably a more realistic estimate of error for pavements built for a longer life than the two years of the Road Test.

All other performance measures show similar comparisons of the observed points and fitted lines.

MINIMUM COST DESIGN

The set of design equations which we have developed from the AASHO Road Test data is ideally suited for determining minimum cost structural designs. For a given amount of traffic (service life), each performance measure equation defines a required structural strength (which we call equivalent asphalt surface thickness, D_a; in the Road Test staff analysis, as utilized in the AASHO interim guide, it is called Structural Number) as a function of the amount of deterioration to be permitted at the end of the service life. The problem becomes one of choosing that combination of surface, base, and subbase thicknesses that will satisfy all the desired restraints (terminal values of performance measures) at minimum cost.

It should be apparent that merely specifying a required D_a to meet some performance criterion still permits an infinity of actual designs, i.e., values of D_1, D_2, and D_3, which will give the required D_a. One technique which can be used for selecting the minimum cost design from the large number of possible designs is linear programing. This is a mathematical method for "searching" through a system of linear equations having more than one feasible solution to find the one solution that minimizes (or, if desired, maximizes) an objective equation. Commonly, this objective is an economic one: costs are to be minimized or profits maximized. As a result, the objective has been traditionally called the cost function. This name fits our objective perfectly.

Linear programing is a mathematically rigorous tool for solving our type of problem. Use of this tool requires sound engineering practice to set up the problem, i.e., to decide on the proper restraints and obtain the pertinent economic data. To illustrate the use of this technique for optimum pavement design, we have solved five somewhat simplified problems, using our analysis of the Road Test data to define a design method. Common to all problems are the six performance measure design equations, as shown in Table 4 with D_a replaced by $D_1 + a_2D_2 + a_3D_3$. These are coupled with a cost function equation and ten restraint equations.

The cost function equation was, in all cases,

$$1.0\ D_1 + 0.38\ D_2 + 0.28\ D_3 = \text{Cost} \tag{45}$$

The coefficients 1.0, 0.38, and 0.28 are costs of an inch of thickness of surface, base, and subbase, respectively, made relative to the per inch cost of a.c. surface. These figures were obtained from the actual costs reported for a highway project in California's Mendocino County, as tabulated by the Pacific Coast Division of The Asphalt Institute. It is in these cost figures that the major simplification in our examples occurs. The costs are here assumed constant regardless of the layer thickness called for in solving the problem. This is equivalent to assuming that each layer can or will be put down in one lift. Obviously, the time cost per inch is determined both by the number of lifts required and the total thickness of each layer. This type of complication is still amenable to linear programing solution, but was not deemed necessary for these illustrative examples.

The ten restraints imposed on the system in each problem are shown in Table 6, together with the optimum designs which resulted. Certain auxiliary information obtained from the solutions is also included in Table 6; this consists of the range over which the cost figures could vary before the optimum solution would change, the minimum cost arrived at, and the terminal values estimated for all the performance measures for the optimum design. The costs are relative and so do not represent any total number of dollars. In the five problems, only certain of the restraints were changed. The restraints listed in Table 6 are grouped accordingly.

Case 1 may be considered the base case, as the other four problems each involved a change in just one of the four varied restraints away from Case 1. In all cases, the

TABLE 6

OPTIMUM (MINIMUM COST) DESIGNS

Performance Measures	Initial Values	Case 1 (Base)	Case 2	Case 3	Case 4	Case 5
		(a) Restraints				
Fixed						
$F \cdot W_{18}$	-	40×10^6	40×10^6	40×10^6	40×10^6	40×10^6
PSIA	4.2	≥ 2.5	≥ 2.5	≥ 2.5	≥ 2.5	≥ 2.5
PSI, owp	4.2	≥ 2.0	≥ 2.0	≥ 2.0	≥ 2.0	≥ 2.0
SV, owp	1.0	≥ 36.0	≥ 36.0	≥ 36.0	≥ 36.0	≥ 36.0
RI, owp, in./mi	50	≤ 200	≤ 200	≤ 200	≤ 200	≤ 200
D_1, in.	-	≥ 3	≥ 3	≥ 3	≥ 3	≥ 3
Varied						
RD, owp, in.	0	≤ 0.5	≤ 0.5	≤ 0.75	≤ 0.5	≤ 0.5
C + P, owp, %	0	≤ 20	≤ 100	≤ 20	≤ 20	≤ 20
D_2, in.	-	≥ 4	≥ 4	≥ 4	≥ 0	≥ 4
D_3, in.	-	≥ 6	≥ 6	≥ 6	≥ 6	≥ 0
		(b) Solution				
D_1, in.	-	10.0	10.0	8.9	10.6	10.3
D_2, in.	-	4.0	4.0	4.0	0	4.0
D_3, in.	-	6.0	6.0	6.0	6.0	0
Cost ranges D_1	-	0-2.379	0-2.379	0-1.186	0-2.379	0-2.379
D_2	-	0.18-0.38	0.18-0.38	0.33-0.38	0.18-0.38	0.18-0.38
D_3	-	0.05-0.28	0.05-0.28	0.18-0.28	0.05-0.28	0.05-0.28
Minimum relative cost	-	13.07	13.07	11.93	12.27	11.69
Limiting performance measure	-	RD, owp	RD, owp	PSI, owp	RD, owp	RD, owp
Terminal values of						
PSIA	-	3.81	3.81	3.30	3.61	3.40
PSI, owp	-	2.91	2.91	2.00	2.50	2.36
SV, owp	-	5.7	5.7	15.6	10.0	12.7
RD, owp, in.	-	0.50	0.50	0.68	0.50	0.50
RI, owp, in./mi.	-	55	55	64	57	60
C + P, owp, %	-	1.2	1.2	6.9	2.8	6.4

solution calls for the minimum thicknesses of base and subbase. This appears to be related to the fact that the relative cost coefficients of these materials are higher than their "layer substitution" coefficients in any of the performance design equations. If sufficiently different cost figures were used, this pattern would be expected to change. Case 2 gave a solution identical to Case 1 because it involved relaxing the percent cracking tolerated, which was not the variable restricting Case 1.

In Case 3 the rut depth tolerance which was binding on Case 1 was eased in an amount sufficient to make the outer wheelpath serviceability criterion control the optimum solution. This resulted only in decreasing the required surface thickness by 1.1 inches. In Cases 4 and 5, the minimum restrictions were removed on D_2 and D_3, respectively. The maximum $\frac{1}{2}$-in. rut limitation again controlled the optimum designs, which are somewhat lower in relative cost than the base case.

The five cases shown here were calculated in less than one minute on a high speed computer, at a cost of about ten dollars.

These cases are intended merely to indicate what can be done with a set of consistent design equations reflecting different criteria of pavement performance to determine minimum cost designs. The engineer must decide on the design method (equations), the acceptable terminal levels of performance, and must, of course, obtain reliable cost information. This method of finding optimum design should be a great help to the highway engineer.

CONCLUSIONS

In the course of this analysis, deliberate efforts were made to detect and eliminate any biases that might be caused by: (a) changes in the traffic rate during the testing period, (b) differences in the severity of the two spring thaw periods, (c) differences in initial condition of the test sections, (d) times of traffic startup and stopping, and (e) the relatively short (two years plus) testing period.

The need for eliminating any such biases can be stated simply: highways are not designed to last for only two years, rather more like twenty. Unfortunately for the research engineer, projects such as the AASHO Road Test cannot be run for 15 to 20 years; therefore, we must make our way carefully through a very short "data time" in an attempt to discern the long-range trends of real interest. We feel this has been accomplished in our analysis.

Of particular importance is the ability to use this analysis to assist in the minimum cost allocation of structural strength between the several pavement layers. This analysis represents, perhaps, the first use of a consistent set of mathematical models for this purpose.

For full utilization of the results of the AASHO Road Test, whether in economic optimization of pavement design or in more routine structural design, it will be necessary to extend the Road Test findings beyond the engineering limitations of the Road Test. That is, further testing must be done under different climatic conditions, with soils of other strengths, and different methods of construction. The satellite tests being proposed by the various states should help greatly to provide the necessary data.

The equations presented in this paper are not being offered as final results for design purposes. Before this would be possible, it will be necessary to further verify the models used and even then to add on a safety factor to reduce the probability that a pavement will become unserviceable before its design lifespan. (As the equations now stand, there is a 50 percent chance that a pavement falls short of its design life.)

The Asphalt Institute offers these results as an alternative means of evaluating and describing asphalt pavement performance.

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Appendix A

DEVELOPMENT OF PERFORMANCE EQUATIONS

Average serviceability, P, is used here to illustrate the performance equation development. The development of all equations followed the same lines.

The entire analysis is predicated on the assumption that asphalt pavement performance for a particular pavement, p, can be expressed by an equation of the form:

$$\Delta_{t_2 - t_1} f(P)_p = \sum_{t = t_1}^{t_2} \sum_{1} \zeta\, ltp \tag{46}$$

This equation states that the change from time t_1 to t_2 in the value of a function of P can be expressed as the sum over that time interval of the effects ζ of all loads l, which effects are dependent also on the time of load application and the pavement considered. The time interval t_1 to t_2 can be considered without loss of generality to be an interval such as the Index Periods used at the Road Test. In this equation we assume that the effects ζ for different loads operate independently of each other.

The object of our analysis was to describe the ζ in terms of design p, time t, and load l. The time dependency of ζ as used here means the seasonal variation of ζ, not something related to the age or condition of the pavement. The parameters ζ_{ltp} are considered to be always positive in value, in the sense that they always contribute to deterioration of the pavement, whether that deterioration be of a form such as a decrease in serviceability index or an increase in slope variance. Actually, the condition $\zeta_{ltp} < 0$ is imposed.

To transform the Eq. 46 into a form consistent with the data from the Road Test, we will consider the change in F(P) from t = 0 to t = t, that is, from the startup of traffic until time t. We assume that the seasonal (t) variations of ζ can be described by the simple function

$$\zeta_{ltp} = \beta_{lp} \mu_t . \tag{47}$$

For the Road Test biweekly Index Periods with their traffic counts N_{lt} in each lane (load), we obtain

$$\sum_{t=t_1}^{t_2} \zeta_{ltp} = \beta_{lp} N_{lt} \bar{\mu}_t \tag{48}$$

in which $\bar{\mu}_t$ is some suitable average value of $\bar{\mu}_t$. It was found in the analysis that the load dependency of β_{lp} was describable by

$$\beta_{lp} = \beta_p e^{\alpha L} \tag{49}$$

Combining these results with Eq. 46 yields the equation

$$f(P)_{pt} - F(P)_{p0} = \beta_p \sum_1 e^{\alpha L} \sum_{t=0}^{t} N_{lt} \bar{\mu}_t \tag{50}$$

in which the summation over t on the right side leads directly to our definition of weighted applications W*. The expression $\sum_1 e^{\alpha L} \sum_t N_{lt} \bar{\mu}_t$ or its equivalent $e^{\alpha L} W_l^*$ immediately defines the equivalencies of different loads in terms of their effects on pavement condition. Two different axle loads, L_1 and L_2, will cause the same deterioration in a pavement if their counts (traffic rates) are related by $e^{\alpha L_1} W_1 = e^{\alpha L_2} W_2$, so that

$$\frac{W_2}{W_1} = e^{(L_2 - L_1)} \tag{51}$$

Choosing L_1 to be some reference load (18 kips), we can relate the effect of W_2 applications of any load to a "deterioration equivalent" number of reference load axle applications by Eq. 51.

Eq. 50 also defines the "mixed-traffic theory" of our analysis. The equation shows clearly the way in which the effects of different loads are to be combined, again through the expression $\sum_1 e^{\alpha L} W_{lt}$. By using 18 kips as a reference axle load we obtain the result

$$\text{total } W_{18} = \sum_1 e^{\alpha (L - 18)} W_l \tag{52}$$

This method of handling mixed-traffic effects is a direct consequence of our basic model (Eq. 46), especially of the assumption of independence of axle application effects. Support for this assumption is obtained from the Road Test data which showed almost independent action of tandem-axle wheels. Theoretical considerations lend support to the idea that heavy axles spaced 20 or more feet apart act independently.

Appendix B

SERVICEABILITY INDEX EQUATION

The development of the equation used to estimate serviceability index for this analysis is described fully in Ref. 1. The equation is

$$\ln \frac{P}{5} = -0.1615 \sqrt{SV} - 0.4967\ RD^2 - 0.00278 \sqrt{C+P} \tag{53}$$

A tabular solution of this equation is also given in Ref. 1.