A Critical Review of Present Knowledge of the Problem of Rational Thickness Design of Flexible Pavements

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A study of theoretical knowledge of the structural behavior of flexible pavements is reported. The limitations of present concepts of the elastic layer theory are discussed and means for developing better theoretical relationships are suggested. Several concepts of elastic models including two- and three-layer systems were studied. The hypothesis that flexible pavements are perfectly elastic appears to be valid for limited numbers of safe loads of short duration. However, for slow-moving or static loads large enough to nearly overstress the pavement, a rheologic model is more applicable. It was also determined that pavement layers lacking tensile strength may account for a large part of the difference between calculated and measured deflections and stresses. Also, there may be merit to an approach using an equivalent modulus of elasticity, E, for the system. In a discussion of Poisson's ratio it is concluded that although this parameter has little effect on calculated stresses, a value of 0.5 seems to be most appropriate for soils and pavements. A discussion of the effect of pavement rigidity on pavement deflections indicated that the deflection factor is most important. The layered system model of flexible pavements should be expanded to include the condition of zero tensile modulus of the component materials.

A well-documented history of the development of theoretical solutions for the design of flexible pavements for the period 1906 through 1962 is presented. At least five different theoretical approaches to the design of flexible pavements have been developed.

•FOR THE LAST two years the São Paulo State Highway Department has been engaged in a research study of pavement deflections in several state highways, employing Benkelman beams and load bearing tests. The research program is still under way at this time. Some 12,000 individual measurements have already been made covering 1,200 km (750 mi) of roads. The test results obtained so far have been statistically analyzed and reported (1), but final conclusions are not yet available. In a parallel project a bibliography survey was made of present knowledge on the structural behavior of flexible pavements, seeking to provide a theoretical framework to be used in the analysis and interpretation of experimental data. This survey has been already completed (2), and includes in unified and comprehensive form, a critical review of most of the proposed theories on the problem and of experimental studies published by several authors. It was later felt that this report might be of enough interest to other engineers engaged in pavement research to warrant its publication as a separate study. This paper is an abridgment of the latter report.

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BACKGROUND

A quarter of a century has elapsed since the development of the CBR method, the first correlation-based empirical method of thickness design of flexible pavements (3). Twenty years have passed since the presentation of the elastic layered theory, the first wholly theoretical analysis of the flexible pavement (4). These two opposite but converging approaches to the same problem still stand today, though much improved. In the meantime, more than three dozen different methods have been proposed, both of semiempirical and semitheoretical conception. The problem is still far from solved, but a few definite trends can be recognized, at least on a worldwide scale.

The CBR method has gained an increasing reputation and has won the confidence of design engineers the world over. Its advance and improvement have been continuous and steady, largely due to the remarkable contribution of the U.S. Army Corps of Engineers (5). A review of the technical literature of several countries and of several organizations in the U. S. indicates that the use of the CBR method and its modifications far exceeds the use of all the other non-CBR methods combined. Starting as a purely empirical method, the CBR has evolved, due to experience, research, and theoretical analysis of the correlation curves, into what could be very properly called a rational method. It is rational in the sense of having reason and understanding, of properly relating causes and effects, of being suitable for intelligent use, and providing sound and trustworthy design. It is not entirely "scientific"—but then very few, if any, engineering design methods are.

The elastic layered theory, however, has lagged far behind. Its advance and development have been slow, hampered by the tremendous mathematical complexity of its equations in analytical form (6). Nevertheless, many research engineers have retained hope in this theory as the best scientific approach to the design problem. A good advance was made by several authors in the numerical computation of influence coefficients for stresses and deflections by the layered theory. The calculation of most of these coefficients required the use of modern electronic computers. The first numerical coefficients for deflections for a two-layer system were computed by Burmister himself in 1943 (4). Coefficients for two-layer stresses were published by Hank and Scrivner (7) and Fox (8) five years later. However, the two-layer elastic system is a grossly oversimplified model of the flexible pavement. Stress coefficients for the three-layer system were published in 1951 by Acum and Fox (9), and were extended to a wider range of parameters by Jones in 1962 (10). Coefficients for three-layer deflections are still unavailable, except for a particular case computed by Shiffman (11), and an approximate process by Jeuffroy and Bachelez (32). The three-layer elastic system is still a simplified model of the multilayer pavement, but a more complex model would hardly be susceptable to calculation.

The numerical computation of influence coefficients made possible the experimental testing of the elastic layered theory. With the works of Sowers and Vesic (13) and other researchers, the layered theory was put through a severe trial. The available results show a reasonable agreement between measured stresses and deflections and theoretic computed values, but only for pavements possessing some tensile strength, such as soil-cement bases. These results tend somewhat to confirm the theory. Unfortunately, for flexible pavements which do not have tensile strength, that is, for truly flexible pavements, the measured and computed values show a disagreement great enough to overrule the physical analogy between the theoretical elastic model and the real pavement.

It is not likely that the application of rheologic principles could bring much improvement to the present concept of the layered theory. The rheological analysis must include elastic components in the complex rheological model. The influence of the elastic components would still be solely governed by the elastic layered theory—and this theory has proven to be not entirely correct.

The need is apparent for a better theoretical analysis than the present layered theory. It is hoped that such improved theoretical analysis can evolve in time to meet halfway the steadily improving empirically originated CBR analysis. Both systems of knowledge would then be encompassed by one larger branch of applied science, pavement mechanics. This seems to be the discernible trend.

This paper reviews the logical foundations of the elastic layered theory, discusses some of its inconsistencies, and points out possible ways of improvement.

THEORY OF ELASTICITY APPLIED TO FLEXIBLE PAVEMENTS

The first question that arises in trying to apply the elastic theory to pavement structures is whether a flexible pavement is an elastic body, at least approximately following Hooke's law. Considerable research and theoretical studies have been done on this point by several authors. One of the best approaches to the problem is the one proposed by Ruiz (14) and Baker and Papazian (29). The flexible pavement structural behavior normally falls in one of the two following patterns:

1. Case A.—Under a finite number of repetitions of loads of short duration (of the same order of duration as moving wheel loads) with intensity of loading well below breaking strength, the behavior of pavements of adequate design and construction is predominantly elastic, especially at low temperatures. The AASHO Road Test (15) for instance, determined that "deflections of flexible pavement surfacing increased almost linearly with load . . . ." The pavement in this case can be adequately represented by an elastic model whose most important characteristic is its modulus of elasticity. The elastic modulus of pavements in Case A should be determined under conditions typical of this case.
2. Case B.—For slow-moving or static loads, close to the breaking strength, especially at high temperature and for new pavements, the behavior is predominantly viscous or plastic. The pavement in this case is best represented by a rheologic viscoelastic model, such as the Voigt model. Peltier (12) suggests that the behavior of pavements in Case B is elastic delayed. The final deflection, given time, depends on the final modulus of elasticity but not on the viscosity of the pavement. The viscous components of the pavement structure affect the immediate deformation but not the final deflection if enough time is allowed for the time-dependent part of the deflection to take place. The elastic model would also apply, within certain limitations, to Case B if the final modulus of elasticity is chosen as the working modulus.

These considerations justify the conclusion that an elastic model is adequate to represent the pavement in Case A, and to a lesser extent in Case B. A rheologic model would better represent the pavement in Case B. However, the rheologic model must include elastic components. The influence of the elastic components would necessarily follow the elastic theory. Therefore, the theory of elasticity is the governing law in Case A, and also plays an important role in Case B.

Elastic Model of Pavement--Elastic Constants

The term "pavement" here refers to the whole structure built above the subgrade, including all existing layers of subbase, base, binder course and surface course. The term "subgrade" means the soil mass resulting from earthmoving operations and is located below the pavement.

The simplest elastic model of a pavement is the two-layer elastic system depicted in Figure 1. The subgrade soil and the pavement materials are supposed to be homogeneous, isotropic and perfectly elastic. The subgrade is considered to be a layer of infinite depth, limited in its upper boundary by the subgrade surface and unlimited in the vertical downward direction and the horizontal direction. The pavement is considered to be a layer of finite thickness, h, unlimited horizontally, lying over the subgrade. The dividing surface between the two layers is the interface. The wheel load, Q, is represented by a contact pressure, p, uniformly distributed over a circular contact area of radius, r. The wheel load is initially considered a vertical static load. The effects of dynamic impact, horizontal forces and repetition of loads are later incorporated into the analysis, usually as corrective coefficients. The dead load due to the weight of the materials is usually considered to be negligible as compared to the live loads.

It is known from the theory of elasticity that a homogeneous, isotropic and elastic material is characterized by two independent elastic constants. These constants can

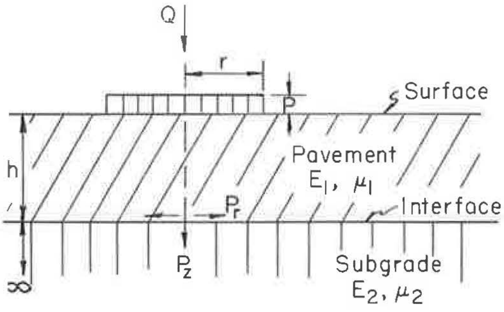

Figure 1. Two-layer system.

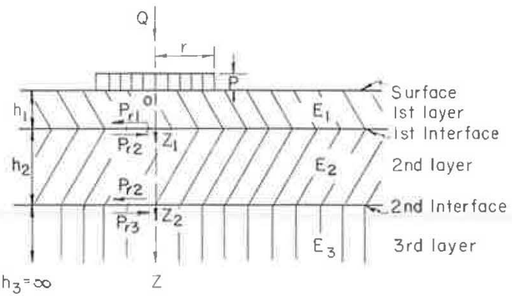

Figure 2. Three-layer system.

be taken as the modulus of elasticity or Young's modulus, E, and the Poisson's ratio, μ. A nonisotropic material has up to 15 independent elastic constants, according to the degree of anisotropy. A nonhomogeneous material has variable physical characteristics. A nonelastic material obviously has no elastic constants. In the two-layer elastic model, there are then four constants, namely E_1, μ_1 for the upper layer and E_2, μ_2 for the lower layer.

A further improvement of the elastic model is the three-layer elastic system indicated in Figure 2. In this system are six elastic constants, two for each layer. The remaining assumptions apply as for the two-layer system. A still better analogy to the real pavement would be a multilayer elastic system. However, the solution of an elastic model with more than three layers would present an exceedingly complex mathematical problem.

Of all simplifying assumptions made in the construction of the elastic model of the pavement, the hypothesis of perfect elasticity is the one which most departs from reality. The stress-strain relationship of real materials is not exactly linear; moreover, it depends on the material conditions. In other words, the elastic "constants" of real materials are not really constants, but depend on factors such as load, state of stress, rate of loading, temperature, compaction, and moisture. However, if we choose adequate average values for the elastic constants, valid for the particular conditions of the problem under study, the elastic theory should provide a reasonably accurate framework for the solution of many problems.

The main difficulty is then of an experimental nature: the determination of adequate values for the elastic constants. The determination of elastic modulus of soils and pavement materials can be made by experimental methods such as triaxial tests, load bearing tests, drop impact, and pulse velocity. Each of these methods has its merits and shortcomings, its limitations and range of application. This subject has been widely studied by several authors. For our purposes it is enough to recognize that the elastic modulus is a physical characteristic of materials that can be measured experimentally for a given set of conditions. The experimental methods of measurement will not be further discussed in this paper, which is directed to the discussion of the conceptual or philosophical foundations of the theories.

The determination of Poisson's ratio for soils and flexible pavement materials poses a different problem, generally overlooked by most authors. For soils, Poisson's ratio is a very tenuous property which has never been satisfactorily determined. It is usually supposed to be near 0.5, the value for an incompressible engineering material (16). Because saturated nondraining soils can be assumed to be incompressible within the range of practical loads, this value appears to be reasonable. It has the additional advantage of simplifying theoretical equations, in which the term 2 μ is frequently found. There is no reliable test to measure Poisson's ratio for soils and pavement materials, and no sound criterion to chose a value different from 0.5. The author suggests that the difficulty of measuring Poisson's ratio may go deeper than testing complexities. It is possible that the Poisson's ratio as such—a relation of radial and axial unit elongations—may have no physical significance for soils and pavement materials. The concept of Poisson's ratio, according to the elastic theory, is based on the assumption that the deformations are small and the corresponding small displacements do not substantially affect the action of the external forces. These conditions are not generally met for soils. Consequently, the elastic principle of superposition of effects is not exactly valid for the stresses and strains acting on the elemental volume within the soil mass. The principle of superposition may eventually give acceptable results for the effects of external macro loads, as a sort of gross average blanketing the interrelated effects of several micro phenomena, but it is not strictly valid for the point stresses and strains. Therefore, the elastic Poisson's ratio becomes meaningless—and it is small wonder that it could never be satisfactorily determined. But according to theory, a second elastic constant, in addition to the modulus of elasticity, is necessary to define the elastic material. For practical applications, a coefficient related to the volumetric change, such as the elastic modulus of compression, would be more reliable as the second elastic constant. The condition most likely to prevail in actual subgrades is the permanence of volume, corresponding to a coefficient of volumetric change equal to zero. It can be demonstrated that this zero coefficient corresponds to a theoretical value of Poisson's ratio equal to 0.5. Therefore, this value of $\mu = 0.5$ seems to be well justified from a theoretical standpoint.

In the Vicksburg tests (17), the value of $\mu = 0.5$ was found to be adequate for the clayey-silt soil test section. For the air-dry pure sand test section (18), the most probable value was considered to be 0.3. However, the result for the pure sand is not as convincing as the one for the clayey-silt soil. Closely examining the sand section data presented (18, Plate 96), one can see that the agreement between measured and computed horizontal stresses is very poor, both for 0.3 and 0.5 values of μ, the former being only a little better than the latter. The pure sand section test results deviate in several respects from the elastic theory, much more than the clayey-silt soil section. There are several reasons to believe that most actual subgrades are closer to the clayey-silt soil than to the pure sand and, therefore, closer to $\mu = 0.5$.

In the absence of more experimental work, we have to rely on the theoretical analysis. It is fortunate that the theoretical influence of Poisson's ratio on the vertical stresses and deflections is small, for both homogeneous and layered systems. Shiffman (11) computed the vertical stresses and deflections for a three-layer system with Poisson's ratio of 0.4, 0.2 and 0.4, respectively. The author compared the vertical stresses from Shiffman with the values for Poisson's ratio of 0.5 interpolated from the tables of Jones (10) and found differences less than 5 percent, below experimental accuracy. The author also compared the vertical deflections from Shiffman to the deflections of a corresponding two-layer system with Poisson's ratio of 0.5 given by the Burmister graph (4). This comparison is valid because the two upper layers of the Shiffman case have the same modulus of elasticity, being different only in Poisson's ratios. The two upper layers of the Shiffman three-layer case correspond to the upper layer of the Burmister two-layer case. The comparison shows that the influence of Poisson's ratios of the upper layers is entirely negligible. The influence of the lower layer is sensible but less than 15 percent. This difference is of little significance because the lower layer of real pavements, that is the moist subgrade, is most likely to have a Poisson's ratio of 0.5.

Available experimental and theoretical evidence indicates that a Poisson's ratio of 0.5 seems to be the most adequate for general use. The discussion of alternative values for Poisson's ratio belongs to the realm of theoretical hypothesis. Practical graphs or formulas for deflection analysis including values of Poisson's ratio other than 0.5 are utterly unwarranted. This inclusion is misleading because it presupposes the wrong notion that Poisson's ratio is a parameter that can be varied to suit particular project conditions. Actually there is no such thing as the selection of a μ value in practical problems of deflection analysis, at least in the light of present knowledge.

This adoption of a fixed value for one of the two independent elastic constants somewhat simplifed the elastic model. We have now just one elastic constant, the modulus of elasticity, for each layer. As stated before, it is most important for the validity of the elastic model that the actual values of the modulus be measured under conditions similar to the service conditions of the real pavement. The elastic modulus is the physical parameter that binds the theory to the ground—and so to say, it is the clay foot of many theoretic giants.

The elastic model of the pavement will be completely defined by the following conditions and parameters (6):

1. Elasticity condition.—The subgrade soil and pavement materials are supposed to be homogeneous, isotropic and perfectly elastic, obeying Hooke's law. The modulus of elasticity is supposed constant, and usually assumed to be the same for tension and compression.
2. Geometric parameters.—These are radius, r, and layer thicknesses, h_1, h_2, h_3, ... It is convenient to express all thicknesses as nondimensional multiples of the radius, which is the same as to consider $r = 1$. The radius is thus eliminated as an independent parameter. There is one independent geometric parameter for each layer: h_1/r, h_2/r, h_3/r, ...
3. Loading parameter.—The applied contact pressure, p, is usually assumed to be normal to the surface, that is, of vertical direction. It is convenient to adopt the value, $p = 100$ percent and express all induced stresses in the model as a percentage of p. The contact pressure is then also eliminated as an independent parameter. The dead weight of the layers is usually neglected.
4. Physical parameters.—There is a modulus of elasticity for each layer, E_1, E_2, E_3, ... It is convenient to adopt as independent parameters the nondimensional ratios between successive modulus, plus the lowest modulus: E_1/E_2, E_2/E_3 and E_3. Sometimes it is useful to express the combined effects of all different moduli as an equivalent modulus E_e.
5. Boundary conditions.—The top surface of the layer is assumed to be free of any stresses outside the contact area and of shearing stresses inside the contact area. The lower boundary at infinite depth is supposed to be free of any stresses and strains.
6. Continuity conditions.—The interfaces between the layers are assumed to fall within one of the two limiting cases: Case 1 (Rough interfaces)—with perfect continuity and the layers in continuous contact, acting together with no slippage at the interfaces; or Case 2 (Smooth interfaces)—with no friction and the layers in continuous contact but perfectly free to move horizontally relative to each other. In reality, the actual conditions at the interface are intermediate between these two extremes, but probably closer to the first case than to the second.

The mathematical problem posed by the elastic model consists in expressing all induced stresses and strains within the model as functions of the independent parameters, for the given conditions.

Relative Rigidity of Flexible Pavement

Because the pavement modulus is greater than the subgrade modulus, there is a relative rigidity of the flexible pavement with respect to the subgrade. The relative rigidity improves the pavement performance in two ways: (a) it reduces the vertical downward pressure transmitted to the subgrade beneath the loaded area, and (b) it increases the ultimate shearing strength opposing the upward movement of the subgrade soil around the loaded area. According to Hveem (19), the second effect is more important than the first one. Certainly, the most important effect of the rigidity is a reduction of the total deflection for a given load. A convenient form of indicating the effect of the pavement rigidity is to compare the total deflection of the layered system with the corresponding deflection of a uniform medium of the same modulus of elasticity as the subgrade modulus. The ratio between the two deflections is the deflection factor. The most important problem to be solved in the elastic model is the calculation of the deflection factor.

PREVIOUS STUDIES

Almost all mathematical solutions of the layered system are based on the classical Boussinesq-Love solution for the semi-infinite uniform medium presented by Love (20). There were a few partial solutions of the layered system problem before the Burmister

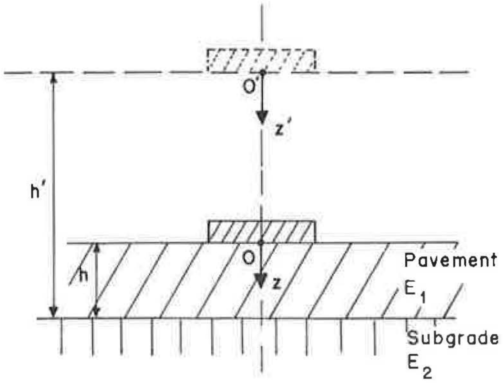

Figure 3. Equivalent thickness.

analysis. One of the most widely known is the solution presented by Hogg (21) of the problem of a thin rigid slab supported by a semi-infinite elastic foundation. Hogg's solution introduces an additional condition in the elastic model previously defined. It supposes that the thin slab deflects according to Navier hypothesis; that is, top and bottom faces and median plane deflect along parallel curves. This hypothesis causes a considerable simplification in the calculus, but it covers only approximately Case 2 of the continuity condition. Hogg's solution was numerically computed and presented in graphical form by Odemark (22). An extension of Hogg's solution to the three-layer problem was recently presented by Jeuffroy and Bachelez (32).

For the two-layer system, the deflection factor and vertical stress computed by Hogg and Odemark are practically equal to the corresponding values given by the Burmister analysis. The radial and shear stresses in the top layer are slightly different in the two solutions, but the radial and shear stresses in the lower layer cannot be correctly calculated with Hogg's solution. For the three-layer system the agreement is not as good.

Semi-Empricial Solutions

Palmer and Barber (23) presented an approximate process for calculation of deflections of the two-layer system which gave results close to the Burmister analysis. Palmer and Barber start from the hypothesis that, for deflection computation, the pavement thickness, h, could be replaced by an equivalent thickness, h', of subgrade soil (Fig. 3), satisfying the condition:

$$\frac{h'}{h} = \left(\frac{E_1}{E_2}\right)^{1/3} \qquad \left(\mu_1 = \mu_2 = \frac{1}{2}\right) \tag{1}$$

The factor $(E_1/E_2)^{1/3}$ had been previously proposed by Marguerre as the relative rigidity factor for slabs. It should be noted that the replacement of h by h' corresponds to a transformation of coordinates, displacing the origin 0 to 0' but keeping the interface unchanged.

By combining the equivalent thickness hypothesis of Palmer and Barber with the Boussinesq-Love analysis of the uniform medium and neglecting the deformations within the pavement itself, the subgrade deflection may be substituted for the total deflection, and expressed by:

$$W = \frac{1.5\ pr^2}{E_2\sqrt{r^2 + h^2\left(\frac{E_1}{E_2}\right)^{2/3}}} \tag{2}$$

This equation may be rewritten in another form by calling W_o the total deflection of the uniform medium and F' the Palmer and Barber deflection factor:

$$W_o = \frac{1.5\ pr}{E_2} \tag{3}$$

$$F' = \frac{1}{\sqrt{1 + \left(\frac{h}{r}\right)^2 \left(\frac{E_1}{E_2}\right)^{2/3}}} \tag{4}$$

therefore

$$W = W_o\ F' \tag{5}$$

Eqs. 2 or 5 will give the total deflection if the pavement deflection is neglected. If the ratio E_1/E_2 is greater than 100, the pavement deflection is much smaller than the subgrade deflection and can be safely neglected. But if the modular ratio is smaller than 100, this cannot be done. In actual practice, the modular ratio of flexible pavement is seldom greater than 100 and the pavement deflection should be taken into consideration. By an extension of the Palmer-Barber analysis (4, Discussion), it can be demonstrated that the total deflection may be expressed by the following equation:

$$W_t = W_o \left[F' \left(1 - \frac{E_2}{E_1}\right) + \frac{E_2}{E_1} \right] \tag{6}$$

By making:

$$F = F' \left(1 - \frac{E_2}{E_1}\right) + \frac{E_2}{E_1} \tag{7}$$

we have:

$$W_t = W_o F \tag{8}$$

Eq. 8 will give values for the total deflection practically equal to the values given by the Burmister analysis. This final agreement justifies the initial hypothesis of substituting h′ for h, according to Eq. 1.

The vertical stress in any point of the vertical axis passing through the center of the contact area, within the pavement or the subgrade, can be calculated by the equivalent thickness hypothesis combined with the Boussinesq-Love equation. However, the Palmer-Barber analysis is less correct for stresses than for deflections. The vertical stress values given by the Palmer-Barber analysis are smaller than the corresponding values computed by the Burmister analysis. Moreover, the radial and shear stresses cannot be calculated by the Palmer-Barber analysis.

From Eq. 2 comes the well-known semi-empirical Kansas formula (25):

$$h = \sqrt{\left(\frac{1.5\ Q}{\pi E_2 W_a}\right)^2 - r^2}\ \sqrt[3]{\frac{E_2}{E_1}} \tag{9}$$

in which

h = required thickness of pavement (in.),
Q = design wheel load (lb),
r = radius of contact area (in.),
E_1 = elastic modulus of pavement (psi),
E_2 = elastic modulus of subgrade (psi), and
W_a = allowable deflection of subgrade (usually assumed to be 0.1 in.).

For practical use, the wheel load, Q, is affected by two empirical factors related to traffic and rainfall, as follows:

$$Q = Pmn \tag{10}$$

in which

P = maximum wheel load, usually 9,000 lb;
m = traffic coefficient, between 0.5 and 1; and
n = rainfall coefficient, between 0.6 and 1.

The subgrade modulus E_2 is determined by the triaxial test, with the use of a graphical process that takes into consideration the variation of the vertical and radial stresses with depth. The pavement modulus, E_1, can be determined by the triaxial test, or a value for E_1 is assumed from previous experience. The thickness of each component layer of the pavement is also calculated with the help of the rigidity factor; that is, the thickness is inversely proportional to the cube root of the modulus.

The Kansas method, however useful in actual practice, has several shortcomings when viewed from a theoretical standpoint. To begin with, it neglects the pavement deflection. Actual tests, such as the AASHO Road Test (15), show that the pavement deflection often amounts to 30 percent or more of the total deflection. Furthermore, the determination of the elastic modulus by the triaxial test is, at best, a delicate operation and does not take into consideration the rate of loading in the two patterns of structural behavior previously discussed. But the weakest point of the method is the assumption of a fixed and arbitrary value for the allowable deflection, regardless of the pavement type. The assumed allowable deflection is very critical because it has a great influence on the resulting thickness. The usual value of 0.1 in. is rather large. Several performance studies, including the AASHO Road Test, indicate that 0.04 in. would be a more realistic value to prevent the early deterioration of the pavement. However, if this is introduced in the Kansas formula, unrealistically high thicknesses will result. Many authors have shown the difficulty of designing for an arbitrary allowable deflection. McLeod (26) points out that a given deflection under a specified wheel load does not indicate the same ability to carry traffic if the strengths of the underlying subgrades are different. In spite of its theoretical inconsistencies, the Kansas design method has been reportedly used with success (25). This might be explained by the experience and "engineering judgment" of Kansas engineers in supplementing their method.

Another semi-empirical method employing the triaxial test in a modified form is the Texas method (27) which uses a correlation-based classification chart rather than a design formula. The subgrade soil and pavement materials are classified by the position of their Mohr's envelopes in the classification chart, and the required thickness is taken from a design chart using the classification of materials and the design wheel load. The Texas method employs the triaxial test in a rather dependable manner. Instead of using just one numerical value for the modulus of elasticity of each material, it relies on several points of the shear envelope, each point being determined by stressing the material until failure at various lateral pressures. The theoretical justification of the method is based on the Burmister analysis, making use of the Hank-Scrivner solution (7). The Texas method represents definite progress towards the development of a scientific design method, but because it does not properly present a new solution for the mathematical problem of the elastic model, it will not be further discussed here.

Ivanov (28) presented a semi-empricial thickness design method that is said to have been the official Russian method for many years. The Ivanov method has some resemblance to the Palmer-Barber analysis. It is essentially based on the assumption of an arbitrary allowable deflection and the calculation of stresses and deflection with approximate formulas. The rigidity factor of the pavement is assumed to be the following:

$$n = \frac{h'}{h} = \left(\frac{E_1}{E_2}\right)^{1/2.5} \tag{11}$$

Comparing Eqs. 1 and 11, we can see that Ivanov attributes greater rigidity to the pavement than do Palmer and Barber. On the other hand, Ivanov permits smaller allowable deflection, of the order of 0.04 in.

Ivanov proposes an approximate formula for the vertical stresses but gives no formulas for the radial and shear stresses. The vertical stresses computed by the Ivanov formula for the uniform medium are very close to the values given by the classical Boussinesq-Love equation. For the two-layer system, Ivanov's formula gives vertical stresses well below the values given by the Burmister analysis. For the deflections, the two theories present a reasonable agreement.

The final deflection equation of Ivanov is:

$$W = \frac{2\,pr}{E_2} \quad \frac{\pi}{2\sqrt{a}} \left[1 - \frac{2}{\pi}\left(1 - \frac{1}{n^{3.5}}\right) \text{arc tan} \frac{nh\sqrt{a}}{2\,r}\right] \tag{12}$$

in which

p = contact pressure,
r = radius of contact area,
a = empirical constant,
E_2 = subgrade modulus of elasticity,
n = rigidity factor, and
h = pavement thickness.

The elastic modulus is determined by load bearing tests or dynamic tests. The proposed values for the constant a are as follows: for a uniform medium $a = 2.5$; for a two-layer system $a = 2$; and for a three-layer or greater system, $a = 1$.

Implied in the analytical derivation of Ivanov's equation, though not clearly stated, is the assumption of $\mu = 0$ because the effect of the radial stress on the deflection is neglected. Ivanov's deflection equation for a uniform medium is:

$$W_o = \frac{2\,pr}{E} \tag{13}$$

The same expression would be given by the Boussinesq-Love analysis for $\mu = 0$. This condition should be borne in mind when using Eqs. 12 and 13. Of course, if the elasticity modulus is determined with load bearing tests by Eq. 13 and the resulting value is introduced in Eq. 12, the difference is canceled out. But if the modulus is determined in the usual way, for $\mu = 0.5$, it should be multiplied by a factor of 1.33 before entering it into Eqs. 12 and 13. Eq. 12 may be somewhat simplified for the two-layer system by using the values $a = 2$, $n = (E_1/E_2)^{1/2.5}$, and $W_o = 2\,pr/E_2$:

$$F = \frac{W}{W_o} = 0.7\left[\text{arc tan} \frac{1}{\left(0.7\,n\,\frac{h}{r}\right)} + \frac{1}{n^{3.5}}\ \text{arc tan}\left(0.7\,n\,\frac{h}{r}\right)\right] \tag{14}$$

and, therefore,

$$W = W_o F \tag{15}$$

From Eq. 14 Ivanov defines a uniform medium of equivalent modulus, E_e, which would have the same deflection of the layered system. It is possible to calculate the equivalent modulus of a multilayered system, considering the layers two by two, from the lowest to the top. This concept of an equivalent modulus, based on equality of deflections, is the most outstanding feature of Ivanov's method. From Eqs. 13 and 14, the equivalent modulus is:

$$E_e = \frac{E_2}{F} \tag{16}$$

Burmister Analysis

Burmister presented in 1943 the first rigorous and complete theoretical solution of the problem of stresses and strains in the elastic model of the pavement (4, 6). The Burmister solution follows exactly all conditions of the elastic model previously defined. In his first paper (4), Burmister extended the Boussinesq-Love equations to the two-layer system, determining the parameters by the boundary and continuity conditions, and checking the compatibility of the equations. The Burmister solution, with the use of Bessel auxiliary functions, gives the stresses and displacements in any point of the two-layer system for the two cases of interfaces. In his second paper (6), Burmister extended his analysis to the three-layer system with rough interfaces, but derived only the equation for total deflection at the surface. The Burmister equations in analytical form are exceedingly complex and not suitable for immediate application. For practical use, they require the computation of numerical influence coefficients for stresses and deflections.

The first numerical coefficients were computed by Burmister (4) for the total deflection at the surface beneath the center of the contact area of a two-layer system with rough interface and with 0.5 Poisson's ratio in both layers. The Burmister equation for deflection is:

$$W = \frac{1.5\ pr}{E_2} \quad F \tag{17}$$

$$F = f\ (E_1/E_2,\ h/r)$$

in which

W = total deflection,
p = contact pressure,
r = radius of contact area,
h = pavement thickness,
E_1 = pavement modulus of elasticity,
E_2 = subgrade modulus of elasticity, and
F = deflection factor, function of the ratios E_1/E_2 and h/r.

In the Boussinesq-Love analysis, the total deflection of a uniform medium of modulus, E_2, loaded with a uniform pressure is:

$$W_o = \frac{1.5\ pr}{E_2} \tag{18}$$

Therefore,

$$W = W_o F \tag{19}$$

for $F = 1$ and $W = W_o$.

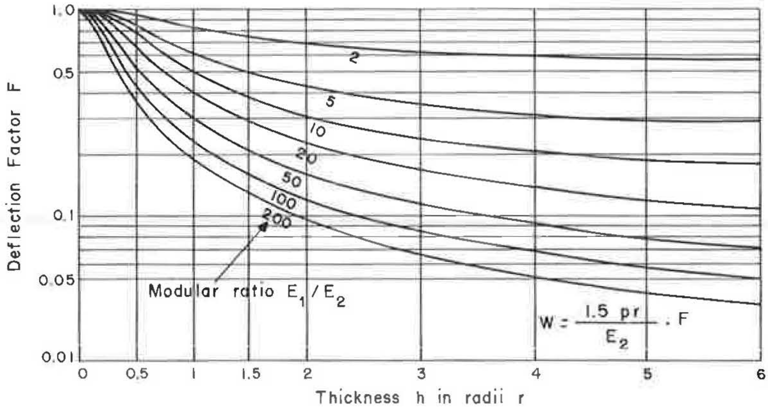

Figure 4. Two-layer deflection factor.

The total deflection of a uniform medium loaded with a rigid plate is:

$$W_O = \frac{1.18\ pr}{E_2} \tag{20}$$

Eq. 19 is valid with the substitution of the value given by Eq. 20 for W_O. The theoretical analysis shows the influence of the two parameters E_1/E_2 and h/r on the deflection. Burmister computed numerically the deflection factor, F, for usual values of the parameters, and presented the results in graphical form (Fig. 4).

Burmister Design Method

Burmister also suggested (4) a semi-empirical thickness design method based on his analysis. It consists essentially of fixing an arbitrary value for the allowable deflection and calculating the required thickness by the deflection graph. The subgrade modulus is determined by load bearing tests, and the pavement modulus is assumed from experience. The total deflection is then checked in a trial section of the pavement. This method has the same weakness already discussed in the Kansas method, namely the difficulty and inconsistency of assuming an arbitrary value for the allowable deflection. The required thickness is very sensitive to the chosen value of the allowable deflection. Burmister's design method has not encountered the same acceptance as has his theoretical analysis of stresses and deflections.

TABLE 1

INFLUENCE COEFFICIENTS FOR TWO-LAYER STRESSES[a]

E_1/E_2	h/r	Vertical Stress (Compression), p_z/p (%)	Radial Stress[b] p_r/p (%)	Shear Stress, p_s/p (%)
		(a) Rough Interface		
100	1	8.1	352	180
	2	2.4	122	62
	5	0.4	23	12
	10	0.1	6.0	3.0
20	1	20.5	218	119
	2	6.6	84	45
	5	1.2	16	8.7
	10	0.3	4.0	2.1
10	1	29.2	158	94
	2	10.1	66	38
	5	1.8	13	7.4
	10	0.5	3.3	1.9
2	1	54.7	30	42
	2	22.4	18	20
	5	4.4	3.9	4.2
	10	1.1	1.0	1.1
		(b) Smooth Interface		
10	1	30.5	106	108
	2	10.5	76	43
	5	1.9	15	8.5
	10	0.5	3.8	2.3
2	1	59.8	81	70
	2	24.0	41	33
	5	4.7	8.7	6.7
	10	1.2	2.2	1.7

[a] At interface on vertical axis, as percentage of contact pressure (7).
[b] In top layer (tension).

Extensions of the Burmister Analysis

Hank and Scrivner (7) published numerical coefficients for two-layer stresses and some three-layer stresses at the interfaces, in the vertical axis for rough and smooth interfaces. Some of these values are given in Table 1. The vertical stresses are very similar in both types of interfaces, but the radial and shear stresses at the base of first layer are greater for the smooth interface, particularly for the lower modular ratios. The radial stress at the top of the second layer (not shown in Table 1) is always smaller than the radial stress at the base of first layer. There is always a discontinuity of radial stresses, even in the case of rough interfaces, induced by an equal strain under different moduli of elasticity.

Fox (8) presented numerical values for the two-layer stresses in the vertical axis and also in points off-set from the axis, computed with the help of the relaxation method of Southwell. The Road Research Laboratory (24) published some of Fox results in graphical form, comparing the two-layer stresses with the corresponding stresses for a uniform medium (Fig. 5). This comparison indicates that the two-layer vertical stresses are considerably lower than the uniform medium stresses. For example, at a depth $z/r = 1$, the uniform medium vertical stress is 65 percent of p, whereas in the two-layer system it is reduced to 29 percent of p. This result indicates that the pavement rigidity reduced the vertical stress at that particular point to less than half. However, at greater depths, i.e., for z/r greater than 3, the stresses in the two systems are very similar.

The reduction of the vertical stresses in the layered system is accompanied by a considerable increase in the radial and shear stresses and the appearance of tensile stresses in the top layer. The greatest values of the layered system tensile stress are located beneath the center, in the lower face of the top layer. The tensile stress depends on the same parameters E_1/E_2 and h/r. The uniform medium ($E_1/E_2 = 1$) with $\mu = 0.5$ has no tensile stress in the vertical axis. When the modular ratio exceeds 1.5,

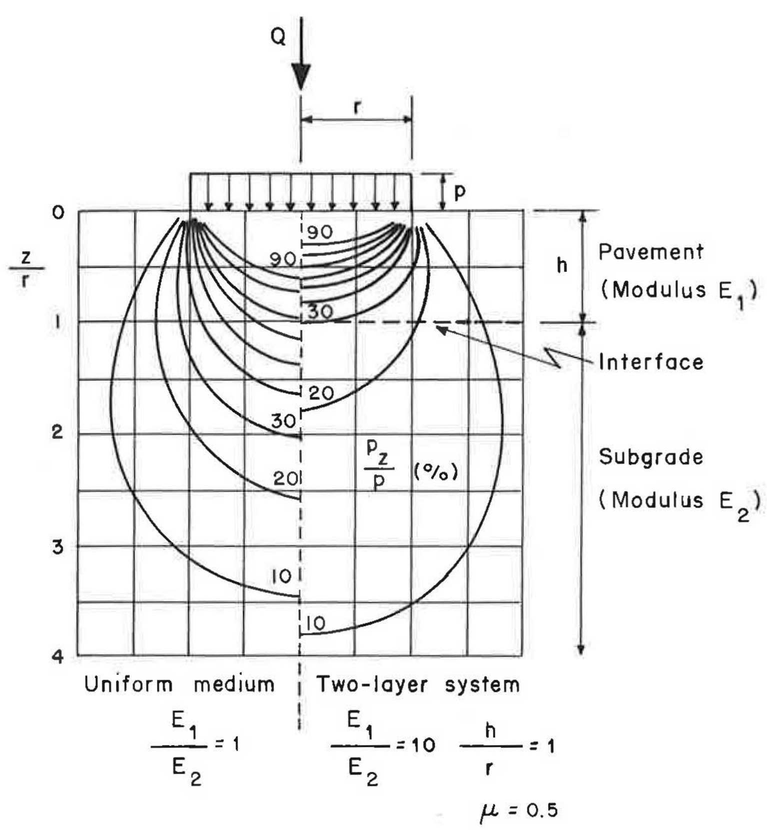

Figure 5. Pressure bulb for the two-layer system (Burmister) compared to a uniform medium (Boussinesq-Love).

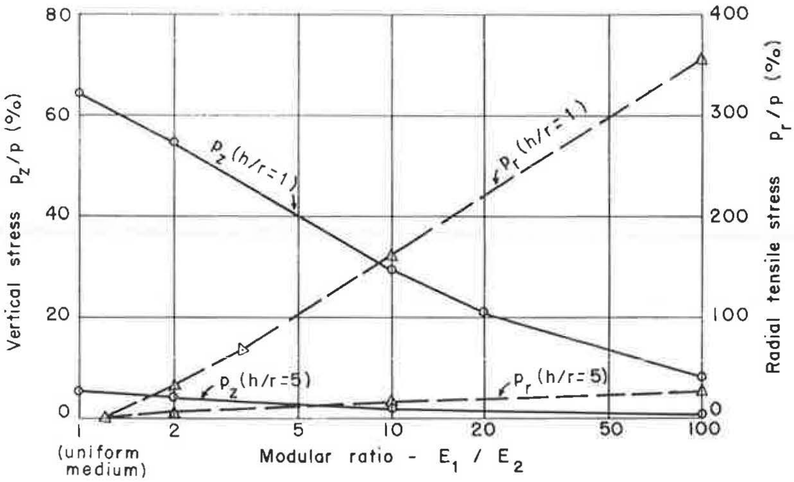

Figure 6. Variation of vertical stress and radial tensile stress in a two-layer system with h/r = 1 and h/r = 5, as functions of the modular ratio, E_1/E_2.

tensile stresses begin to appear. Figure 6, based on Hank-Scrivner values, indicates the reduction of vertical stresses and the increase of radial stresses in a two-layer system, with the increase of the modular ratio. For $h/r = 1$ and $E_1/E_2 = 2$, the tensile stress already reaches one-third of the contact pressure. For $E_1/E_2 = 100$, which is the greatest modular ratio normally occurring in flexible pavements, the tensile stress exceeds three times the contact pressure. In the last case, the flexible pavement has a stress distribution similar to a rigid slab in flexure, with top and lower face stresses almost equal in absolute value but opposite in signal, and middle plane stresses almost zero. Baker and Papazian (29) also noted this basic similarity in the structural design of rigid and flexible pavements. They computed tensile stresses of flexible pavements by the Burmister analysis, and the correspondent rigid slab stresses by the Westergaard analysis, and found comparable stresses, particularly for the thicker pavements. As for the influence of thickness, the tensile stress reaches a maximum for a certain critical thickness (of the order of half the radius) and then decreases with an increase in the thickness of the top layer. This theoretical finding explains the recognized fact that thin surfaces are more prone to tensile cracking and, therefore, should be more flexible.

The three-layer stress coefficients were computed by Acum and Fox (9) for points at the interfaces in the vertical axis. The relaxation method was found to be unsuitable for off-set points in the three-layer system. Jones (10) extended the Acum-Fox computation to a wide range of the parameters for the three-layer system with rough interfaces and Poisson's ratio of 0.5. The Jones tables were computed by the electronic computer at Shell Laboratorium, Amsterdam. Table 2 gives some of the Jones coefficients, expressed as a percentage of contact pressure. Peattie (30) organized a series of charts based on the Jones tables giving the stress and strain factors in graphical form for convenience of interpolation. The Jones-Peattie works are the most extensive set of numerical data now available on the elastic layered system. They give the stress and strain coefficients (but not the deflection factor) for the three-layer system for any combination of parameters within the following range (Fig. 2):

$$E_1/E_2 = K_1 = 0.2 \text{ to } 200$$
$$E_2/E_3 = K_2 = 0.2 \text{ to } 200$$
$$h_1/h_2 = H = 0.125 \text{ to } 8$$
$$r/h_2 = A = 0.1 \text{ to } 3.2$$

TABLE 2

INFLUENCE COEFFICIENTS FOR THREE-LAYER STRESSES[a]

$E_1/E_2 = K_1$	$E_2/E_3 = K_2$	$h_1/h_2 = H$	$h_2/r = \frac{1}{A}$	A	p_{z2}/p (%)	p_{r1}/p (%)	p_{s1}/p (%)
20	20	0.25	1.25	0.8	6.7	580.1	324.5
			2.5	0.4	1.9	378.3	207.0
			5	0.2	0.5	177.0	95.3
20	20	1	1.25	0.8	1.5	251.7	129.8
			2.5	0.4	0.4	79.7	41.2
			5	0.2	0.1	21.6	11.2
20	2	0.25	1.25	0.8	26.3	474.9	275.4
			2.5	0.4	7.9	348.9	193.4
			5	0.2	2.1	169.5	91.9
20	2	1	1.25	0.8	5.9	185.2	98.7
			2.5	0.4	1.6	62.9	33.3
			5	0.2	0.4	17.5	9.2
2	20	0.25	1.25	0.8	9.3	-70.8	11.0
			2.5	0.4	2.7	7.9	42.2
			5	0.2	0.7	24.6	33.5
2	20	1	1.25	0.8	3.6	43.0	38.1
			2.5	0.4	0.9	16.8	14.8
			5	0.2	0.2	4.9	4.3

[a]At interfaces (rough interfaces) on the vertical axis as percentage of contact pressure; symbols used are as follows:

p_{z2} = vertical stress at 2nd interface (pressure on subgrade),

p_{r1} = radial stress at 1st interface (tension at base of top layer), and

p_{s1} = shear stress at 1st interface (at base of top layer).

(Interpolation between given values of K_1 and K_2 can be performed graphically on log-log paper.)

The last parameter is the reciprocal of the usual one, that is:

$$h_2/r = 0.32 \text{ to } 10$$

Shiffman (11) computed the stresses and also the deflection factor for a particular case of a three-layer system having $E_1/E_2 = 1$ and $E_2/E_3 = 10$; that is, the same modulus in the two top layers, but different Poisson's ratios for each layer: $\mu_1 = 0.4$, $\mu_2 = 0.2$ and $\mu_3 = 0.4$. Except for this specific case, which is of little practical interest, there are no published values for the three-layer deflection factor determined by the rigorous Burmister analysis.

Shiffman has also presented (31) a general analysis of stresses and displacements in layered systems, developing analytical procedures for the consideration of normal and tangential loads and axisymmetric and inclined plates. Numerical coefficients were not presented.

Jeuffroy and Bachelez (32) proposed an approximate solution for the three-layer system to which we have already referred. The Jeuffroy-Bachelez solution is an extension of Hogg's solution. The top layer is supposed to follow Navier's hypothesis. The first interface is assumed to be smooth and free of shear stresses. The second interface is assumed to be rough, but having all stresses and strains equal in both layers. This is a further simplification because, according to the rigorous Burmister analysis, the radial stresses at the interfaces are different in both layers, even for the rough interfaces. Jeuffroy and Bachelez presented a series of charts giving the stresses and deflection factor for the three-layer system, for single and dual wheel loads. The stresses computed by Jeuffroy and Bachelez are higher than the correspondent values in the Jones tables. The radial and shear stresses in the two lower layers cannot be computed by the Jeuffroy-Bachelez solution.

EXAMPLES OF APPLICATION

A few simple practical applications of the theories discussed are in order, to try the engineering "feeling" of so many tables, graphs and formulas. For example, if the wheel load is Q = 7,800 lb, p = 100 psi, and r = 5 in. and we assume a uniform medium of modulus E = 1,000 psi, the deflection at surface is:

$$W_o = \frac{1.5 \times 100 \times 5}{1,000} = 0.75 \text{ in.}$$

Calculation of vertical and radial stresses at an arbitrary depth of 10 in., according to the Boussinesq-Love analysis (35) yields p_z = 28.4% × 100 = 28.4 psi and p_r = 1.6% × 100 = 1.6 psi (compression) because z/r = 10/5 = 2. If over this subgrade we build a 10 in. base of modulus 20,000 psi, we have a two-layer system, whose parameters are h/r = 10/5 = 2 and E_1/E_2 = 20,000/1,000 = 20. The two-layer deflection, by the Burmister graph (Fig. 4), is F = 0.22 and W = 0.75 × 0.22 = 0.17 in. If the two-layer deflection is calculated by some of the other referred methods, the following results will be found:

1. By the Hogg-Odemark graph, W = 0.75 × 0.23 = 0.17 in.
2. By the Palmer-Barber approximated formula

$$F' = \frac{1}{\sqrt{1 + 2^2 \times 20^{2/3}}} = 0.182$$

and W = 0.75 × 0.18 = 0.14 in.

3. By the Palmer-Barber corrected formula, F = 0.182 (1 - 1/20) + 1/20 = 0.223 and W = 0.75 × 0.22 = 0.17 in.

4. By the Ivanov formula, n = $20^{1/2.5}$ = 3.31, h/r = 2, $1/n^{1/3.5}$ = $1/3.31^{1/3.5}$ = 0.015, 0.7 n h/r = 4.65 which yield F = 0.7 (arc tan 1/4.65 + 0.015 arc tan 4.65) = 0.164 and W = 0.75 × 0.16 = 0.12 in.

In calculating the stresses at the interface by the Hank-Scrivner table, because p = 100 psi, the percentages of p correspond to psi. The vertical and radial stresses are p_z = 6.6 psi and p_r = 84.4 psi (tension). Comparison of the two-layer system with the uniform medium indicates that the deflection was reduced from 0.75 to 0.17 in., and the vertical pressure on the subgrade from 28.4 to 6.6 psi, but at the same time the radial stress passed from a compression of 1.6 psi to a tension of 84.4 psi. The shear stress, not computed in this example, also increased. It should be noted that the computed values of radial and shear stresses are very high.

If we cap this base with a 2½-in. asphaltic surface course of modulus 400,000 psi, we have a three-layer system with the following parameters:

$$\frac{h_1}{h_2} = H = \frac{2.5}{10} = 0.25$$

$$\frac{h_2}{r} = \frac{1}{A} = \frac{10}{5} = 2 \qquad (A = 0.5)$$

$$\frac{E_1}{E_2} = \frac{400,000}{20,000} = 20$$

$$\frac{E_2}{E_3} = \frac{20,000}{1,000} = 20$$

The three-layer deflection, by the Jeuffroy-Bachelez method, is W = 0.75 × 0.18 = 0.14 in. This value is very close to the two-layer deflection.

The stresses at the interfaces, by the Jones tables, are as follows:

1. Radial stress at the base of first layer, p_{r1} = 428 psi (tension);
2. Radial stress at the base of second layer; p_{r2} = 43 psi (tension); and
3. Vertical stress at second interface, p_{z2} = 3.1 psi.

The vertical pressure on the subgrade was further reduced from 6.6 to 3.1 psi, and the radial tensile stress at the second interface was also reduced from 84 to 43 psi with respect to the two-layer system, but for this the surface course should resist a very high tensile stress of 428 psi. The shear stress would also be very high.

The high values of the theoretically computed tensile stresses in the base, and particularly in the surface course, seem to be very unrealistic. It is unlikely that common flexible pavement materials would withstand such high stresses, unless at very low temperatures. And according to the Jones tables the theoretical tensile stresses in the three-layer system can go even higher than those in this example, surpassing six times the contact pressure, i.e., to over 600 psi. It is curious to note that few authors have shown much concern for this apparent unrealism of the layered theory.

The stress values included in Table 2 allow another interesting comparison. It is known that at low temperatures asphaltic surface courses have high elastic moduli and relatively high tensile strength. When the temperature rises, the surface modulus is reduced, but the base and subgrade modulus are practically unaffected. As the ratio E_1/E_2 decreases, the tensile stress is drastically reduced, while the vertical stress increases just a little. Consequently, the "slab effect" of the flexible pavement is considerably reduced and the structural behavior changes from elastic to plastic. We should investigate, for example, what would theoretically happen in a thin pavement when the modular ratio E_1/E_2 changes from 20 to 2, without alteration in the other modulus. For the parameters E_1/E_2 = 20, E_2/E_3 = 20, h_1/h_2 = 0.25, h_2/r = 1.25, and p = 100 psi, we have p_{z2} = 7 psi and p_{r1} = 580 psi (tension). For E_1/E_2 = 2 with the other parameters remaining the same, we now have p_{z2} = 9 psi and p_{r1} = 71 psi (compression). In this particular case, simply because of a temperature change, the radial stress changed from a tension of 580 psi to a compression of 71 psi, while the vertical stress was only slightly affected.

EXPERIMENTAL TESTING OF THE ELASTIC LAYERED THEORY

The numerical computation of influence coefficients made possible the experimental testing of the elastic layered theory. In his original paper, Burmister (4) proposed a graphical solution of his multiple equations system to calculate the deflection of the two-layered system. The graphical process consists in drawing trial curves of the deflection factor, F, on Burmister's graph (Fig. 4) and selecting the best fitting curve. Later Burmister (33) presented an evaluation of pavement systems of the WASHO Road Test by this method. Yet, under close examination the results seem to be somewhat disappointing. It is apparent that the sole criterion of the shape of the trial curves is not reliable enough to permit an evaluation of the acting modulus of elasticity.

There have been several experimental studies that have attempted to interpret measured stresses and deflections through the layered theory and to correlate measured and theoretic values. The prevailing trend in many technical reports has been to underestimate observed discrepancies, explaining the deviation between measured and computed values as inconsistencies of the measured data. However, a close check on the comparison often shows that the differences are not to be disregarded.

Sowers and Vesic (13) presented an experimental study which factually compared and reported measured and computed vertical stresses in the subgrade beneath statically loaded flexible pavements. In a recent paper (34), Vesic widened the range of his conclusions, showing on a graph, besides his own findings, data from other researchers such as Griffith, McMahon and Yoder, the Road Research Laboratory and the AASHO Road Test. The general conclusion of these studies indicates a reasonable agreement between measured and computed values only for pavements possessing some tensile strength, like soil-cement and tar-macadam bases. For flexible pavements devoid of tensile strength, the measured and computed values show a considerable disagreement. The vertical stresses beneath the latter type of pavement are much higher than the values computed by the layered theory, being closer to the values given by the Boussinesq-Love analysis for a uniform medium. The vertical stresses in a homogeneous soil measured by the U. S. Corps of Engineers in the Vicksburg tests (17) are close to the distribution predicted by the Boussinesq-Love analysis.

The mass of experience available from several sources is not yet large enough to warrant a final conclusion, but it strongly indicates a serious deficiency of the layered theory. At least there is not one experimental study showing a good and general agreement between measured and computed stresses. The consistency of results among different researchers utilizing different procedures rules out the possibility of a systematic error in the measuring system. The inadequacy of the layered theory appears to be so great that the discussion of the degree of physical analogy between the theoretic elastic model and the real pavement would be meaningless. The tentative conclusion would be that the theory simply does not apply to the non-tension-resisting pavements.

It cannot be said, however, that pavements without tensile strength follow the Boussinesq-Love analysis. This would be a mathematical absurdity because the Boussinesq-Love analysis starts from the assumption of a uniform medium with constant modulus of elasticity, and is not valid for a layered system. What can be said is that such pavements, for unknown reasons, present a stress distribution that happens to be similar to the Boussinesq-Love curve for a uniform medium.

INTERPRETATION OF EXPERIMENTAL RESULTS

Several factors could be pointed out as possible causes of the disagreement between theory and practice. The most important factor seems to be the lack of tensile strength of the pavement materials. The layered theory assumes equal moduli of elasticity for tension and compression, but most actual flexible pavement materials have practically no tensile strength, that is, a zero tensile modulus. A new layered theory for zero tensile modulus would be helpful to test this hypothesis, but it is not avialable. Another important factor is the nonlinearity of the stress-strain relationship. The compressive modulus depends on the load and the lateral pressure, hence on the depth. This latter factor should be less important because it is also present in the uniform medium and it does not affect the validity of the theory for that medium.

There are some other disturbing factors that the theoretical analysis shows to be of lesser importance: anisotropy of layered systems, adhesive restraint between the tire and the pavement, etc. Of course, the lack of homogeneity of the materials would also be of a disturbing factor, but its effect would be a scattering of the data and not a definite trend as shown by the test data.

If more experimental testing supports the tentative conclusions based on the data now available, the final conclusion will be warranted that the pavement rigidity has practically no effect on the stress distribution and on the reduction of the pressure transmitted to the subgrade. Semirigid pavements, like soil-cement, consitute an exception and should be designed by proper methods that take into account its tensile strength. But truly flexible pavement, devoid of tensile strength, does not contribute strength by the slab effect. It resists applied forces only by lateral distribution of stresses in depth, like a uniform medium. If this conclusion is proved true, as it seems to be, the Burmister analysis will have lost its usefulness. On the other hand, the usual CBR method that fixes the required total thickness based on subgrade strength, without much regard for quality of pavement materials, will be proved scientifically correct, at least in what concerns the pressure reduction. The shear strength of the system will be the determining factor of the pavement bearing capacity. Shear strength computations will be very much simplified by the hypothesis of Boussinesq-Love stress distribution.

The problem of deflection computation remains unsolved, however. The CBR method bypasses the deflection problem through the use of correlation curves, but this is a limited method. The correlation is strictly valid only for the conditions previously experienced. The deflection calculation is essential to any scientific design method because the maximum deflection has a large bearing on the fatigue resistance of pavement layers subject to repeated loadings. It is a recognized fact that the pavement rigidity has an important influence on the reduction of the maximum deflection of the layered system, as compared to a uniform medium. If the layered system, according to Sowers and Vesic, has a stress distribution similar to the uniform medium but the deflections are different, this difference in behavior could be interpreted as a difference in the respective elasticity moduli. The problem could be approached by the computation of an effective or equivalent modulus of the system, as a function of the layer modulus and thicknesses, so as to make the computed deflections agree with the experience. Presently there is no entirely valid theoretical analysis of this problem.

VESIC'S PROCESS OF DEFLECTION COMPUTATION

As a first solution to this problem, Vesic (34) proposed an approximate method of deflection computation based on the assumption that the layered system has a stress distribution similar to the Boussinesq-Love curve, but with a different modulus of elasticity for each layer. A slight mofidication of the Vesic process is shown in Figure 7. The modification consists in using the well-known deflection factor for a uniform medium computed by Foster and Ahlvin (35) instead of a new definition of the deflection factor as proposed by Vesic, and suppressing the curves for Poisson's ratios other than 0.5, for the reasons previously mentioned. Also a correction was made in the curve for rigid plate.

The deflection for a uniform medium subjected to a flexible load is (23, 25):

$$W = \frac{pr}{E} \quad F \tag{21}$$

$$F = \frac{1.5}{\sqrt{1 + Z^2}} \qquad (Z = z/r,\ \mu = 0.5) \tag{22}$$

For $z = 0$ and $F_0 = 1.5$, in which

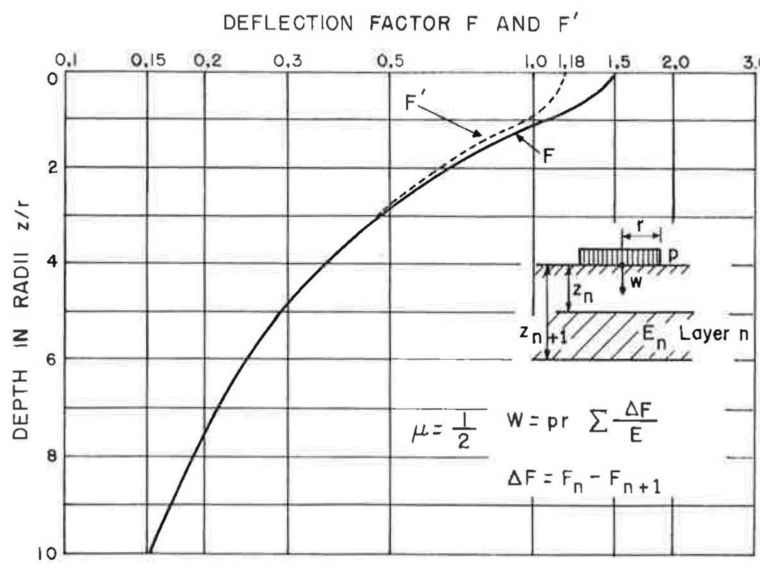

Figure 7. Vertical deflection factor for uniform load, F, and for rigid plate, F′.

W = deflection for flexible load,
p = contact pressure,
z = depth,
E = modulus of elasticity,
F = deflection factor,
r = radius of contact area,
Z = depth in radius, and
μ = Poisson's ratio.

The deflection equation for a uniform medium loaded with a rigid plate, not generally found in the technical literature, is the following:

$$W' = \frac{pr}{E} F' \tag{23}$$

$$F' = \frac{1.5}{2}\left[\frac{Z}{1 + Z^2} + \arc\tan\frac{1}{Z}\right] \tag{24}$$

For $z = 0$ and $F'_0 = 1.18$, in which

W' = deflection for rigid plate, and
F' = deflection factor.

For Z equal to or greater than 0.5, F' may be computed by the following approximate formula:

$$F' = \frac{1.5}{\sqrt{1.5 + Z^2}} \tag{25}$$

For Z greater than 3, F' is practically equal to F. This is a predictable conclusion because at greater depths the difference between the two cases of loading becomes less important. This fact is apparent in Figure 7. For Z greater than 6, both F and F' may be computed by the following approximate formula:

$$F = F' = \frac{1.5}{Z} \tag{26}$$

When Z is infinite, $F = F' = 0$. Factors F and F' are shown in Figure 7.

Starting from the premises assumed by Vesic, and from the differential deflection equation:

$$dW = \left(p_z - p_r\right) dz \qquad (\mu = 0.5) \tag{27}$$

it can be demonstrated that the total deflection of the layered system is given by the following equation (Fig. 7):

$$W = pr \sum \frac{\Delta F}{E} \tag{28}$$

$$\Delta F = F_n - F_{n+1}$$

For the rigid plate F' is substituted for F, and 1.18 for 1.50.

For the two-layer system, Eq. 28 becomes:

$$W = pr\left(\frac{1.5 - F_1}{E_1} + \frac{F_1}{E_2}\right) \qquad (F_2 = 0) \tag{29}$$

and for the three-layer system the deflection is:

$$W = pr\left(\frac{1.5 - F_1}{E_1} + \frac{F_1 - F_2}{E_2} + \frac{F_2}{E_3}\right) \qquad (F_3 = 0) \tag{30}$$

and so on.

Vesic presented an evaluation of the deflection of the WASHO and AASHO Road Tests by his process. Deflections computed by the Vesic process are much higher than those given by the Burmister analysis. Deflections of the two- and three-layer systems of the previous example determined by the Vesic process are as follows:

1. For the two-layer system ($h/r = 2$ and $F_1 = 0.67$):

$$W = 100 \times 5\left[\frac{1.5 - 0.67}{20,000} + \frac{0.67}{1,000}\right] = 0.35 \text{ in.}$$

The deflection found by the Burmister analysis was 0.17 in.

2. For the three-layer system:

$$W = 100 \times 5\left[\frac{1.5 - 1.34}{400,000} + \frac{1.34 - 0.56}{20,000} + \frac{0.56}{1,000}\right] = 0.30 \text{ in.}$$

The deflection computed by the Jeuffroy-Bachelez method was 0.14 in.

The equivalent modulus of a layered system by Vesic's process is given by the equation:

$$\frac{1.5}{E_2} = \frac{1.5 - F_1}{E_1} + \frac{F_1 - F_2}{E_2} + \frac{F_3 - F_4}{E_3} + \ldots \tag{31}$$

$$W = \frac{1.5\,pr}{E_e} \tag{32}$$

SUMMARY AND CONCLUSIONS

The structural behavior of the flexible pavement may be described as elastic or viscoelastic. An elastic model is adequate to represent the flexible pavement in the first case, and also to a lesser extent in the second case if the final modulus of elasticity is adopted as the effective modulus. The theory of elasticity is the governing law in the first case, and also plays an important role in the second.

The layered system model of the flexible pavements, with the usual assumptions of perfect elasticity, boundary and continuity conditions, is adequate for the development of valid theoretical analysis. However, this model should be expanded to include the condition of zero tensile modulus of the component materials. The elastic model is characterized by two elastic constants for each layer, the modulus of elasticity and Poisson's ratio.

The 0.5 value for Poisson's ratio seems to be the most adequate, in the light of the available experimental and theoretical evidence. Practical graphs of formulas for deflection analysis should not include other values for Poisson's ratio, for this inclusion would be unwarranted and misleading. The discussion of alternative values for Poisson's ratio belongs to the realm of theoretical hypothesis. A coefficient related to the volumetric change would be more reliable as the second elastic constant, besides the elasticity modulus.

Starting from the elastic model and known values of the model parameters and chosing adequate average values for the elastic constants, valid for the particular conditions of the problem under study, the elastic layered theory should provide a reasonably accurate framework for the solution of many problems. One of the main problems of the layered system is the calculation of the deflection factor.

The most important layered system analyses and theories were reviewed and discussed. Examples of application were computed. The available experimental data from many sources were used to test the validity of the layered system analysis. It is apparent from the available testing data that the present layered system theory is inadequate to explain the structural behavior of flexible pavements lacking tensile strength. However, more research is needed to confirm this statement.

It has been suggested that the most important cause of disagreement between theory and practice is the lack of tensile strength of common flexible pavement materials. The nonlinearity of stress-strain relationship is a secondary cause. Other disturbing factors exist but are of lesser importance.

The need is apparent for a better theoretical analysis than the present layered system theory. It is suggested that the layered theory should be corrected to apply to zero tensile modulus materials.

The deflection computation of layered systems could be profitably approached by the theoretical calculation of an equivalent modulus of the system as a function of moduli and thicknesses of the layers, so as to make the computed deflections agree with experience.

The CBR in its actual improved form is considered to be one of the most dependable methods of thickness design now available.

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In a paper of this kind, the author owes almost everthing to writers of original papers in the field. His personal task was to review, organize and discuss, and to a certain extent to evaluate, other authors' works. The author has endeavored to keep track of the sources of the most important theories and studies, which are acknowledged in the list of references. However, in the process of reading and utilizing a large number of books, papers and articles, comprising almost all available technical literature on the subject, it is possible that the author involuntarily missed making some reference. This is almost unavoidable when one tries to present a comprehensive and integrated picture of the subject, and not a mere catalog of papers.

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