A Quantitative Evaluation of the Geometric Aspects of Highways

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This study is an investigation of a quantitative measure of the resistance to the flow of traffic as offered by geometric highway features. Under consideration is a mechanistic model resulting from the postulate that traffic reacts to a motivating pressure potential which in turn reflects the behavior of the traffic traversing a particular section of highway. When solved, the governing differential equation yields a parameter called the modulus of geometric aspects. This parameter is a measure of the ease with which traffic traverses the given roadway section.

To evaluate the developed model and determine the reasonableness of the modulus of geometric aspects, a detailed study was undertaken of vehicle speeds on an actual highway curve. A procedure was developed whereby the spot speeds could be calculated from observations recorded by photographic means. Statistical methods were used to analyze the data and to determine the goodness of fit of the theoretical and observed speed distributions.

The success of the results obtained from the study of the first highway curve indicated the advisability of extending the study to additional geometric highway features (other curves, merging conditions, etc.). Additional field experiments were conducted to provide a more generalized basis of evaluating the reliability of the developed modulus.

The results of the study reveal that the mechanistic model as developed conforms closely to the observed speeds in the vicinity of geometric features except for special highway features requiring extreme speed changes. Subject to the same condition, the modulus of geometric aspects provides a reproducible quantitative rating of the geometric highway feature.

•THE PRIMARY objective of this study is to evaluate a proposed method of rating geometric features of highways. The rating, named the "modulus of geometric aspects" and shown symbolically as F_0, is a measure of the ease (as determined by speed changes) with which a vehicle may traverse a section of highway with a particular geometric feature, that is, F_0 is the reciprocal of flow resistance offered to the vehicle by the feature. The theoretical development of this modulus follows.

NOMENCLATURE

p = motivating pressure potential,
x = a specific location along the roadway,
L = a particular length of roadway (ft),

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v = vehicle speed (mph),
t = time (sec),
ρ = vehicle density (vehicles per unit of effective area),
w = effective width of roadway,
N = number of vehicles,
c_1, c_2, k = coefficients of proportionality,
a^2 = $(c_1 + c_2)(k)^{-1}$ = constant,
P = passenger car,
T = truck, and
P.C. = point of curvature, that is the location where the vehicle first encounters the actual change in direction in a horizontal highway curve. The station of the P.C. in all cases was taken as 0 + 00. The stations of all points was taken as positive or negative relative to the P.C. For example, a point 243 ft on the curve would be denoted as Sta. 2 + 43, whereas a point 243 ft prior to the P.C. would be Sta. - 2 + 43.

THEORY

The development of the "modulus of geometric aspects" (1) is based upon three assumptions:

1. The behavior of a vehicle upon a roadway is the result of the driver's reaction to a motivating "pressure potential" under the prevailing ambient conditions. The pressure potential is defined so that

$$\frac{\partial p}{\partial x} = -kv \tag{1}$$

2. The change in vehicular density with respect to the pressure potential is proportional to the density:

$$\frac{\partial \rho}{\partial p} = c_1 \rho \tag{2}$$

3. The change in effective width of the roadway with respect to the pressure potential is proportional to the effective width:

$$\frac{\partial w}{\partial p} = c_2 w \tag{3}$$

Traffic flow may be considered as a conserved flow; that is, the change in the number of vehicles within a section, during a specified time interval, must equal the difference in the number of vehicles entering the section and the number of vehicles leaving the section (during the time interval). When expressed mathematically and reduced by the assumptions listed above, the controlling equation is of the form

$$\frac{\partial^2 p}{\partial x^2} = a^2 \frac{\partial p}{\partial t} \tag{4}$$

The unique solution of Eq. 4 depends upon the imposed boundary and initial conditions. For this purpose consider the section of roadway shown in Figure 1. At $x = 0$, the

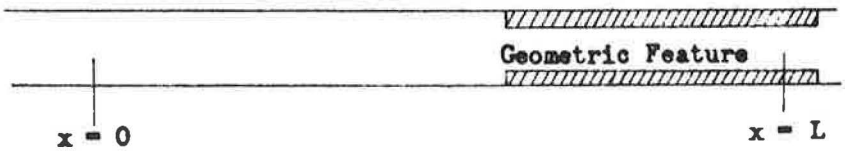


Figure 1. Schematic illustration of roadway section.

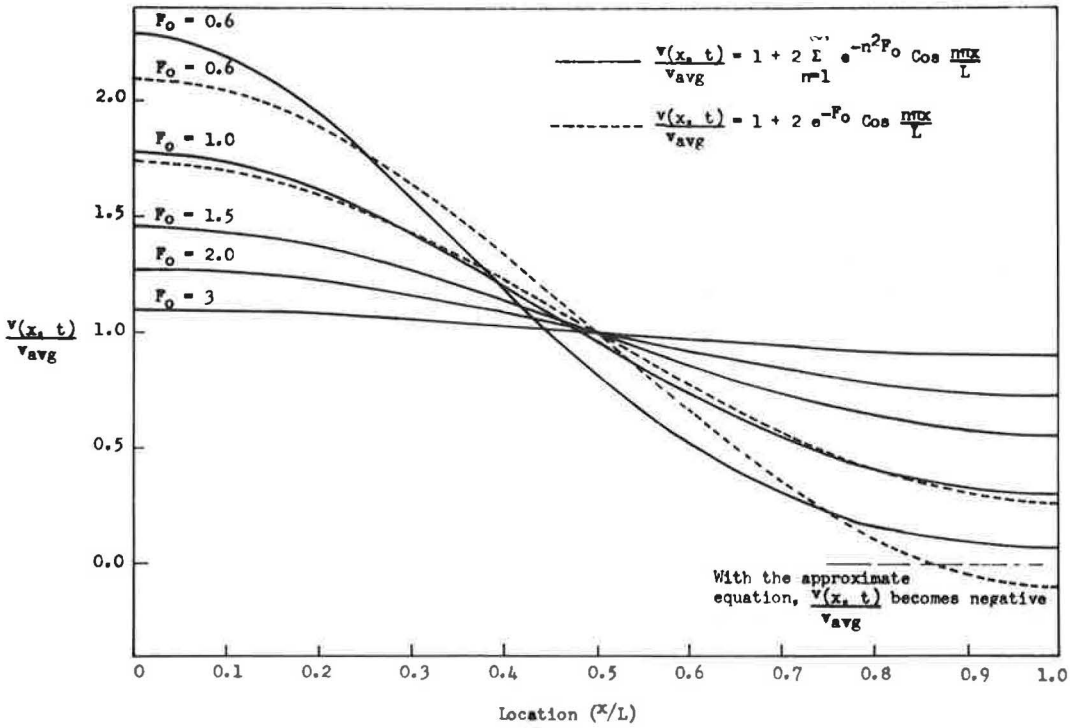


Figure 2. Theoretical speed ratio-distance relationships for selected values of F_o.

vehicle is unhampered by the geometric feature and is operating at a potential p_0. At $x = L$, a location influenced by the geometric feature, the potential has changed to p_1 where $p_1 = p_0 - \Delta p$. At time $t = 0$, the drivers of the vehicles are unaware of the necessity of a change in potential on the roadway, thus $p(x, 0) = p_0$.

A Fourier series solution to the controlling equation may be obtained with the listed conditions. The result is

$$\frac{v(x,t)}{v_{avg}} = 1 - 2\sum_{n=1}^{\infty}(-1)^{n-1} e^{-n^2 F_0} \cos \frac{n\pi x}{L} \tag{5}$$

where

$F_0 = \pi^2 t/a^2 L^2$.

The speed relationship is symmetrical about the location $x = L$ and repeats in intervals of 2L. The ratio increases as x goes from 0 to L and decreases as x goes from L to 2L. In the evaluation of specific geometric aspects vehicle speeds normally decrease

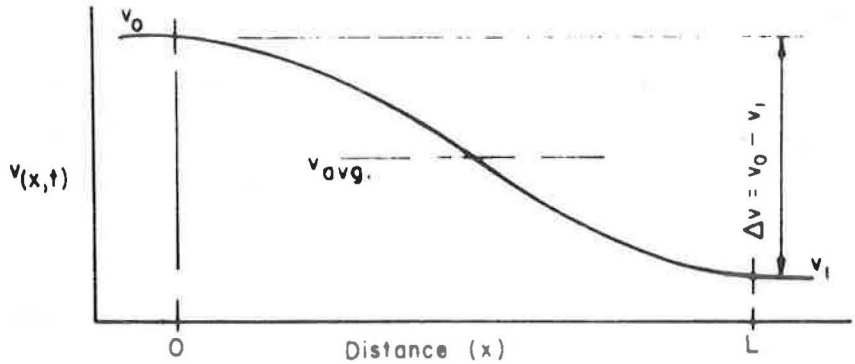


Figure 3. Theoretical speed-distance relationship.

as the feature is approached because traffic flow is impeded. For these features the theoretical curve from x = L to x = 2L (area where the ratio is decreasing) would be considered. Rewriting the speed ratio equation so that the origin is moved L units to the right (replacing x by x + L) and simplfying produces:

$$\frac{v(x,t)}{v_{avg}} = 1 + 2\sum_{n=1}^{\infty} e^{-n^2 F_0} \cos\frac{n\pi x}{L} \tag{6}$$

Figure 2 shows the speed ratio-distance curves obtained by introducing various values of F_0 into the theoretical equation. The dashed line curves are secured by using only the first term of the Fourier series. When F_0 is above 1.0, the first term approximation and the complete series curves are indistinguishable. Thus the first term may be used without significant error when F_0 is equal to or greater than 1.0.

$$\frac{v(x,t)}{v_{avg}} = 1 + 2e^{-F_0} \cos\frac{\pi x}{L} \tag{7}$$

The general shape of speed ratio-distance curve is shown by Figure 3. At x = 0, the vehicle speed is v_0 and $v_0 = v_{avg}\left(1 + 2e^{-F_0}\right)$. At v = L, vehicle speed is v_1 ($v_1 = v_0 - \Delta v$) and $v_1 = v_{avg}\left(1 - 2e^{-F_0}\right)$. Solving for e^{-F_0} results in

$$e^{-F_0} = (v_0 - v_1)/2(v_0 + v_1) = \Delta v/(4v_0 - 2\Delta v) \tag{8}$$

If desired, F_0 may be obtained as a natural logarithm function of speed and speed change:

$$F_0 = \ln(4v_0 - 2\Delta v) - \ln \Delta v \tag{9}$$

DATA COLLECTION AND ANALYSIS

To obtain complete information of vehicle behavior on a section of roadway, a photographic method of data collection was used. A 16-mm motion picture camera, equipped with telephoto lens, was mounted at a vantage point some 1,500 to 2,000 ft from the geometric feature under consideration. From this distance, at an approximate right angle to the feature, a total roadway distance of 1,200 to 1,500 ft could be covered without difficulty. Fifty-foot intervals were measured along the approach and through the feature. These measurements were projected radially to the camera site and marked by white stakes located in the fence line along the edge of the right-of-way. From the camera location, a distance along the roadway could be determined by observing the vehicle passing the radial stakes. The motion picture camera was operated at a selected speed (generally 30 frames per second) and checked periodically by recording a stop watch on film for several seconds.

The developed film was viewed by a time-motion study projector. The viewer could estimate to the nearest one-tenth of a frame the arrival of a vehicle at any particular marker. Thence the time period required for the vehicle to pass from one marker to the next could be estimated to the nearest one three-hundredth of a second. With a known distance and a known elapsed time interval, "spot" speed in miles per hour could be calculated for the roadway section under study.

After the data had been tabulated they were separated into observations of passenger cars and observations of trucks because the two have different operating characteristics. Next the recorded data were reduced to vehicle speeds, summed and averaged for each section of roadway. The average speed was plotted versus location (Figs. 4-10).

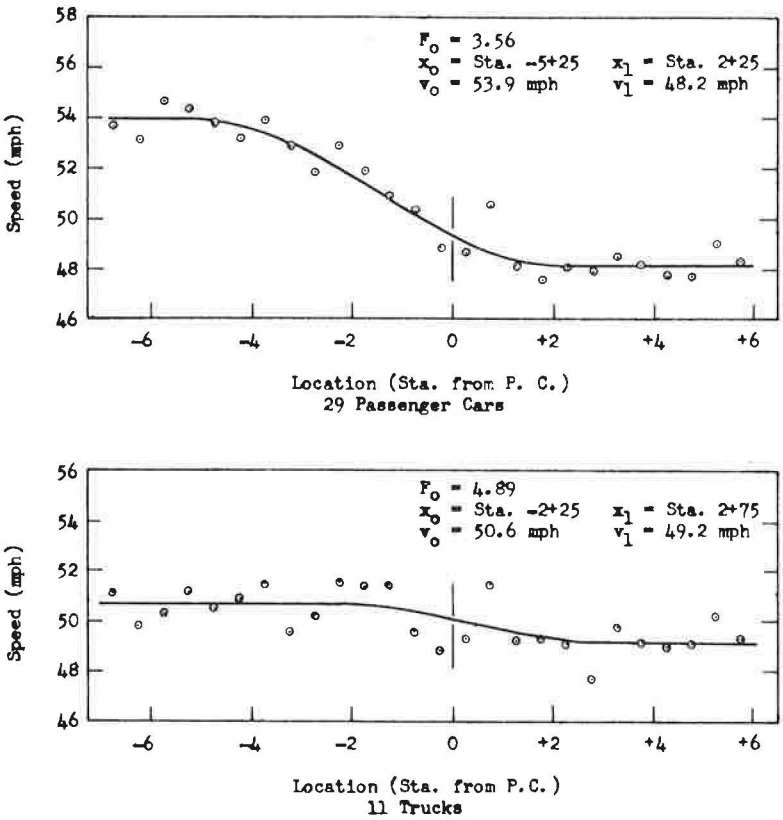
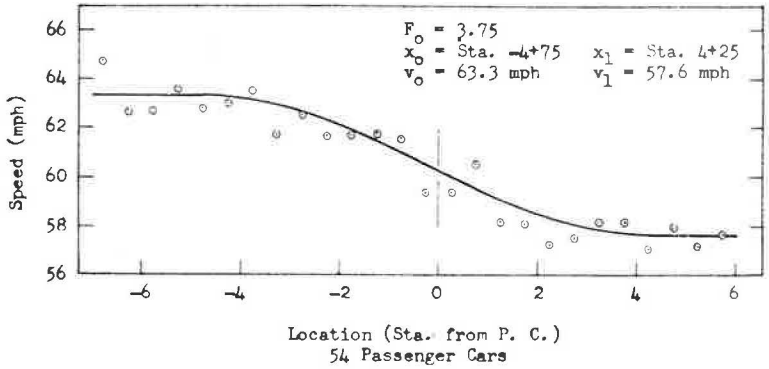


Figure 4. Speed-location graph for curve on US 24 near Reynolds, Ind.—Data Set No. 1.



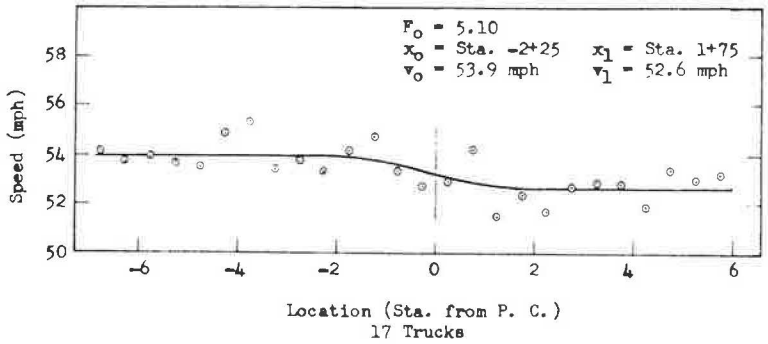
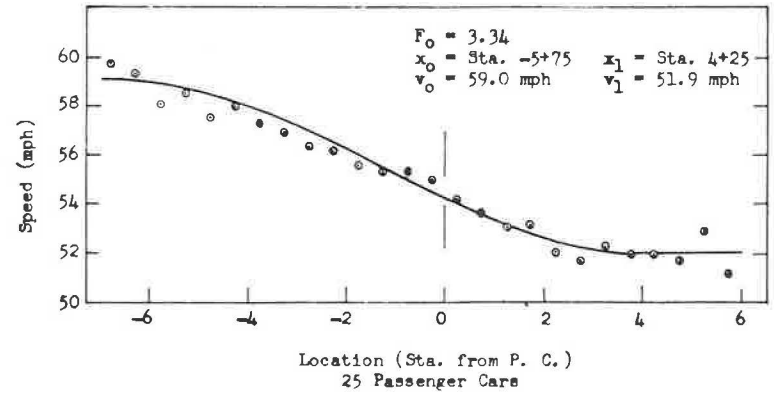


Figure 5. Speed-location graph for curve on US 24 near Reynolds, Ind.—Data Set No. 2.



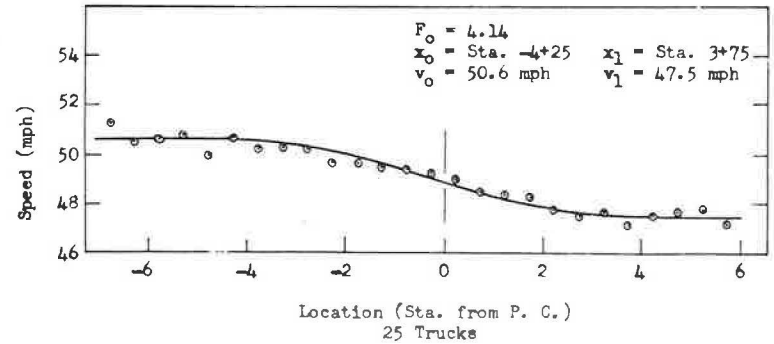
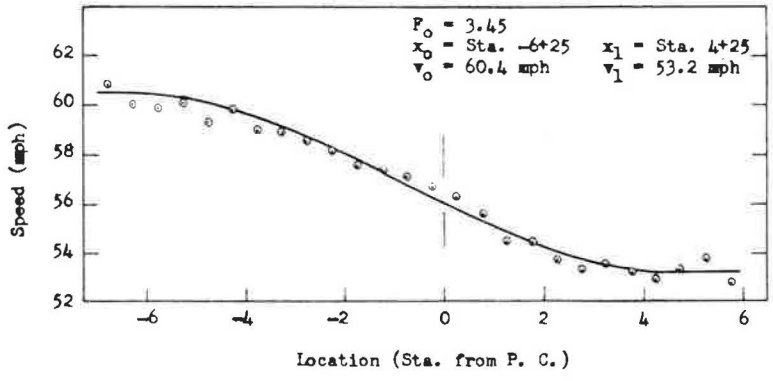
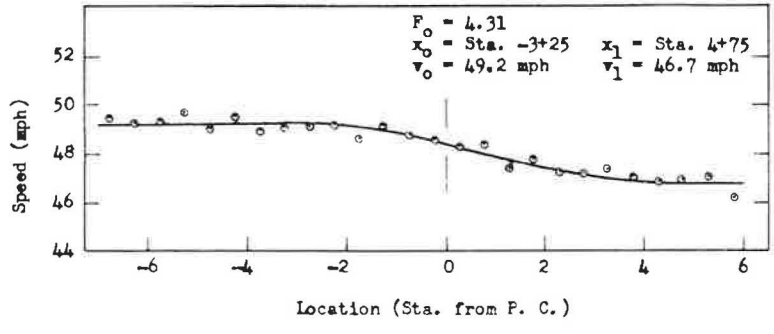


Figure 6. Speed-location graph for curve on US 24 near Reynolds, Ind.—Data Set No. 3.

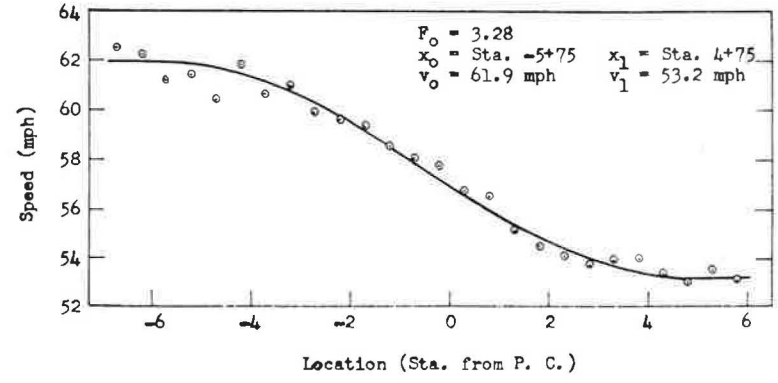


35 Passenger Cars

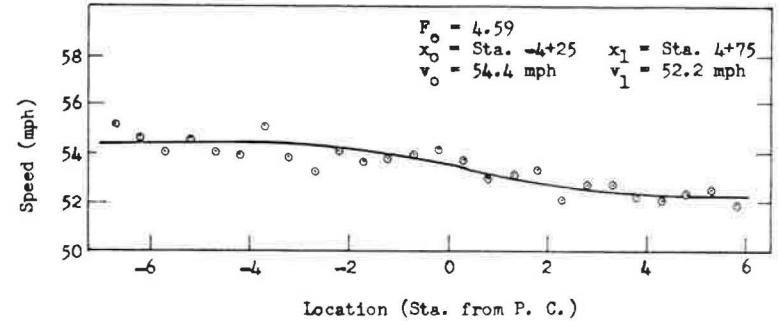


15 Trucks

Figure 7. Speed-location graph for curve on US 24 near Reynolds, Ind.—Data Set No. 4.

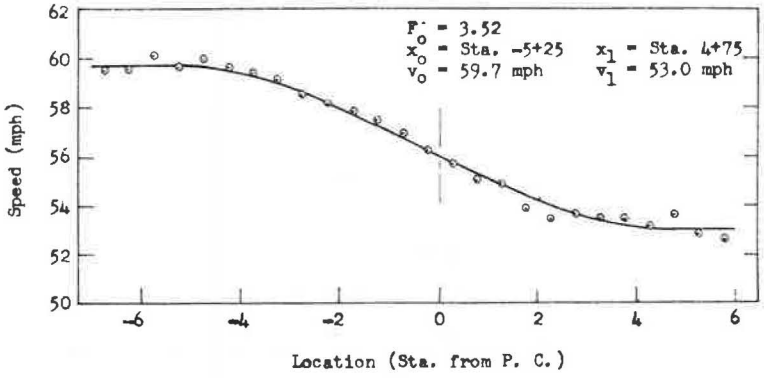


37 Passenger Cars

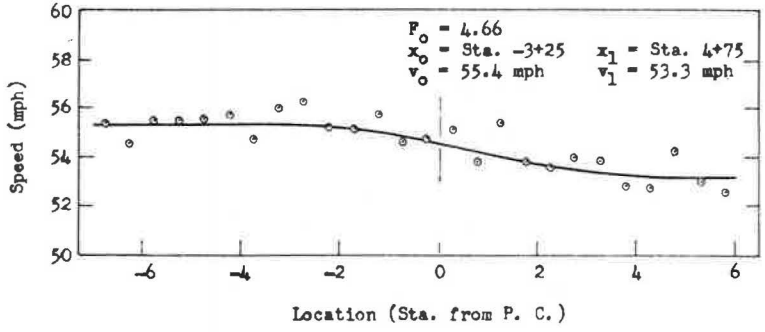


13 Trucks

Figure 8. Speed-location graph for curve on US 24 near Reynolds, Ind.—Data Set No. 5.

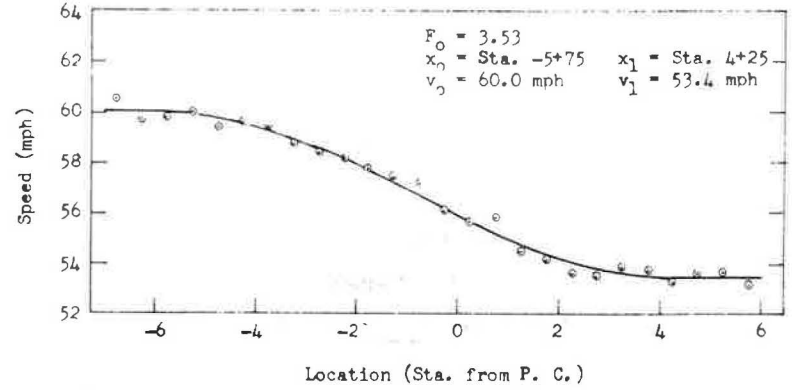


47 Passenger Cars

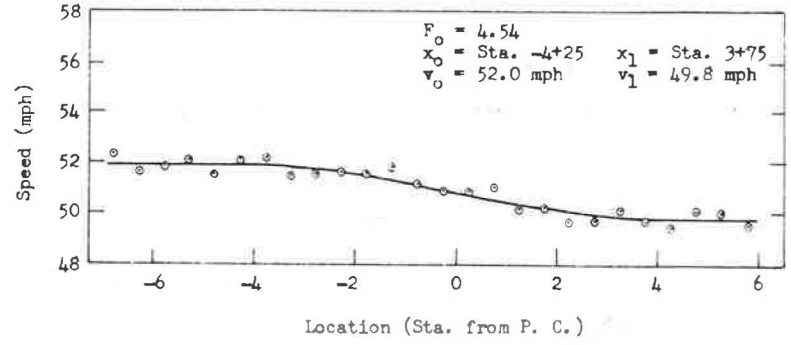


7 Trucks

Figure 9. Speed-location graph for curve on US 24 near Reynolds, Ind.—Data Set No. 6.



208 Passenger Cars



88 Trucks

Figure 10. Speed-location graph for curve on US 24 near Reynolds, Ind.—all data sets.

PILOT STUDY

The first feature studied was a section of US 24 approximately one mile west of Reynolds, Indiana. The highway exhibits a $4\frac{1}{2}$-deg curve extending through a central angle of 45 deg. The curve is level throughout, exhibits no degree of superelevation and has no apparent change in road cross-section. The approach to the curve is a mile long straightaway with no perceivable grade; thus visibility is not restricted for the approaching driver. The section contains no intersecting roads or driveways except for farm entrances to the adjacent fields. No vehicle was seen entering, leaving, or stopping in or near the study section.

Data from this feature were collected at six different times to secure variations in vehicle performance under varying ambient conditions. The first set of data was taken when the roadway was partially covered with ice and snow; the second set, on a clear cold day with the road surface clear but snow on the adjoining fields; the third set, on a clear warm spring day at approximately noon; the fourth set, during late afternoon of the same day; the fifth set, during early morning hours of a weekday (drivers headed directly into the sun as they negotiated the curve); and the sixth set recorded the response of Sunday afternoon drivers. Figures 4 through 10 show the speed-location curves obtained from these data.

Visual inspection of the shape of the measured speed location curves indicated a striking similarity to the theoretical curve of Figure 3. Quantitative curve fitting of the observations to the theoretical curve (and hence an estimate of the modulus of geometric aspects, F_0) was obtained by an application of the principles of least squares. Various locations were assigned for $x = 0$ and $x = L$, the theoretical curve was calculated between these end points and the squares of the difference between the theoretical curve and the observed data were summed.

The theoretical curve which "bestfits" (Figs. 4-10) the observations was determined by selecting the minimum value of the sum of squared differences.

RESULTS

Fit of Theoretical Curve

The speed variability removed by the pressure potential theory is extremely high. The lowest statistical F-value (Table 1) for any of the data sets for the US 24 highway section is 8.9 (trucks of data set number 1) which is significant at the 0.005 level of probability; that is, only five times in a thousand would this amount of variability be removed by pure chance. The other data sets, for both passenger cars and trucks, indicate an even better fit of the theoretical curve to the observed data. Therefore, it must be concluded that all the data exhibit definite agreement with the advocated theory.

TABLE 1

STATISTICAL F VALUES FOR THE SIGNIFICANCE OF THE PRESSURE POTENTIAL THEORY

Data	Passenger Cars		Trucks	
	Stat. F	$(R^2)^*$	Stat. F	$(R^2)^*$
Set 1	68.5	(85.6)	8.9	(43.6)
Set 2	137.0	(92.2)	10.7	(48.2)
Set 3	245.0	(96.4)	214.4	(95.0)
Set 4	573.1	(98.0)	157.5	(93.0)
Set 5	622.0	(98.2)	41.8	(78.6)
Set 6	1016.0	(98.9)	28.6	(71.4)
All	949.1	(98.8)	131.6	(91.9)

*R^2 provides an estimate of the percentage of the variability removed by the pressure potential theory.

Speed Changes (v_0 - v_1)

The speed changes occurring on the approach to and within the feature produced a definite pattern. First, the variance within data sets by type of vehicle was statistically the same for all data sets. Second, the mean speed change for a vehicle type was statistically the same under all observed ambient conditions. Third, these changes followed a normal distribution. The interpretation of these observations indicates that a particular type of vehicle may tend to reduce speed the same amount at a specific geometric feature regardless of roadway and other conditions.

Modulus of Geometric Aspects (F_0)

Because F_0, as obtained from individual vehicles, is given as a function of the natural logarithm of the speed change and the speed change distribution was found to be normal, the distribution of F_0 for individual vehicles cannot be normal. Thus the study of speed changes reflects a study of the individual F_0 values under a required transformation to obtain a normal distribution. Therefore the individual F_0 distribution was not analyzed.

Although F_0 values as calculated from speeds and speed changes of individual vehicles do not represent the flow restriction, the F_0 as calculated from the average of several vehicle responses does provide a measure of flow restriction (as evidenced by the fit of the observed data to the theoretical curve). Based upon the fit of the theoretical curve to the average of all observations, the highway curve on US 24 has an F_0 of 3.53 for passenger cars and 4.54 for trucks.

To secure an estimate of the number of observations required to estimate F_0, it is necessary to consider the logarithmic equation, $F_0 = \ln (4v_0 - 2\Delta v) - \ln v$. With the conditions indicated for the geometric feature on US 24 (that is, average v_0 of 60 mph, average speed change of 6.36 mph, and a standard deviation for the speed change of 4.36 mph) 25 passenger cars will provide an estimate of F_0 within $\pm$ 0.6 at a 95 percent confidence level. For trucks (average v_0 of 52 mph, average speed change of 2.26 mph with a standard deviation of 3.04), 16 observations would produce an estimate within $\pm$ 0.65 at a 90 percent confidence level. (These limits for cars and trucks are not strictly symmetrical because the logarithm varies more with a unit change in Δv when Δv is small than when Δv is large.) It is recommended that a minimum of 25 passenger cars and 16 trucks be used to evaluate F_0 from the average observed values.

Summary and Recommendations

The modulus of geometric aspects provided a quantitative rating for the geometric feature of the pilot study when calculated from the average speeds of at least 25 passenger cars or at least 16 trucks. When working with individual vehicles, the speed change attributed to the geometric feature was a better representation of driver response to flow restriction.

On the basis of these findings, the study was extended to other geometric features in order to determine whether or not the proposed model—and its accompanying modulus of geometric aspects—may be applied in general to the geometric features of highways.

EXTENSION OF STUDY

Sites

On the basis of the findings of the pilot study, additional geometric aspects were selected for the extension of the study:

1. A 5¼-deg horizontal curve on US 41 (and 52) located approximately one mile north of Earl Park, Ind.
2. A right angle turn on Ind. 26 located approximately one mile east of Pine Village, Ind.
3. A transition section from 4-lanes to 2-lanes on US 52 north of Templeton, Ind.
4. A merging lane on the North River Road entrance to the William Henry Harrison Bridge in West Lafayette, Ind.
5. A narrow bridge on Ind. 43, located approximately two miles north of Chalmers, Ind.

Data Collection

From these sites data were recorded on days of comparatively good weather; that is, for all observations the road surface was clear and dry. Only one period of observation was used for each of the selected sites as it was believed that broader coverage should be obtained at the sacrifice of information for fewer sites under varying ambient conditions. In all cases, each vehicle for which data were recorded could choose its own speed—there was no vehicle immediately (10 seconds or less) preceding it in its particular traffic lane. However, where possible, the data were analyzed on the basis of encountering (or not encountering) vehicles traveling in the opposite lane. If an oncoming vehicle was met within the critical part of the feature, the data were classified as the opposing lane occupied. Otherwise the opposing lane was considered free of oncoming traffic.

Curve on US 41 (and 52)

Description.—The curve is on a heavily traveled section of US 41 and is the location of frequent accidents—some of which are severe. Both of the approaches are downgrade to the curve and visibility is not restricted. The pavement has been widened for the inside (northbound) lane. There is a minimum amount of superelevation through the facility. The approaches to the feature are marked by flashing amber caution lights and large signs proclaiming a dangerous curve ahead. Within the curve there is a minor county road intersecting the highway on the outside edge. During the period of observation no vehicle was seen using this minor facility. Data were recorded for vehicles traveling south (the outside lane).

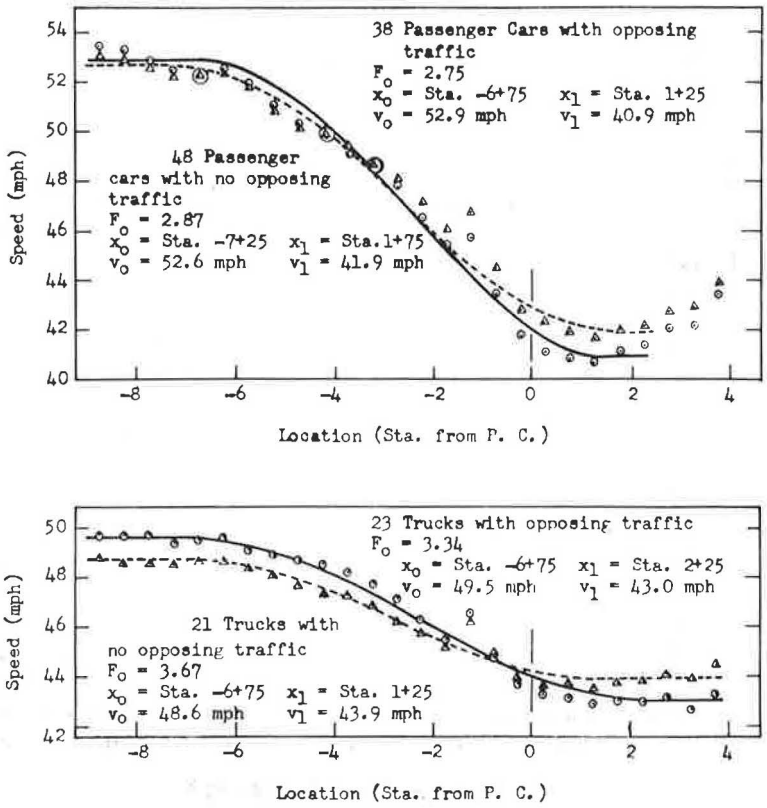


Figure 11. Speed-location graph for curve on US 41 (and 52) near Earl Park, Ind.

Results.—The difference between the two best-fit theoretical curves (Fig. 11) is not significant. Therefore, there is no evidence of a difference in driver response resulting from opposing traffic and all data may be considered as part of the population; that is, the pressure of oncoming cars produces no significant change in driver response to the highway feature.

The speed variability removed by the advocated theory is again extremely high, indicating an excellent fit between observation and theory. The distribution of speed changes plots as a normal distribution.

The best estimate of the modulus of geometric aspects for this highway curve from all observations is 2.78 for passenger cars and 3.42 for trucks.

Right Angle Corner on Ind. 26

Description.—This feature is on a lightly traveled section of Ind. 26; however, it is the location of frequent accidents, most of which are limited to minor damage. The

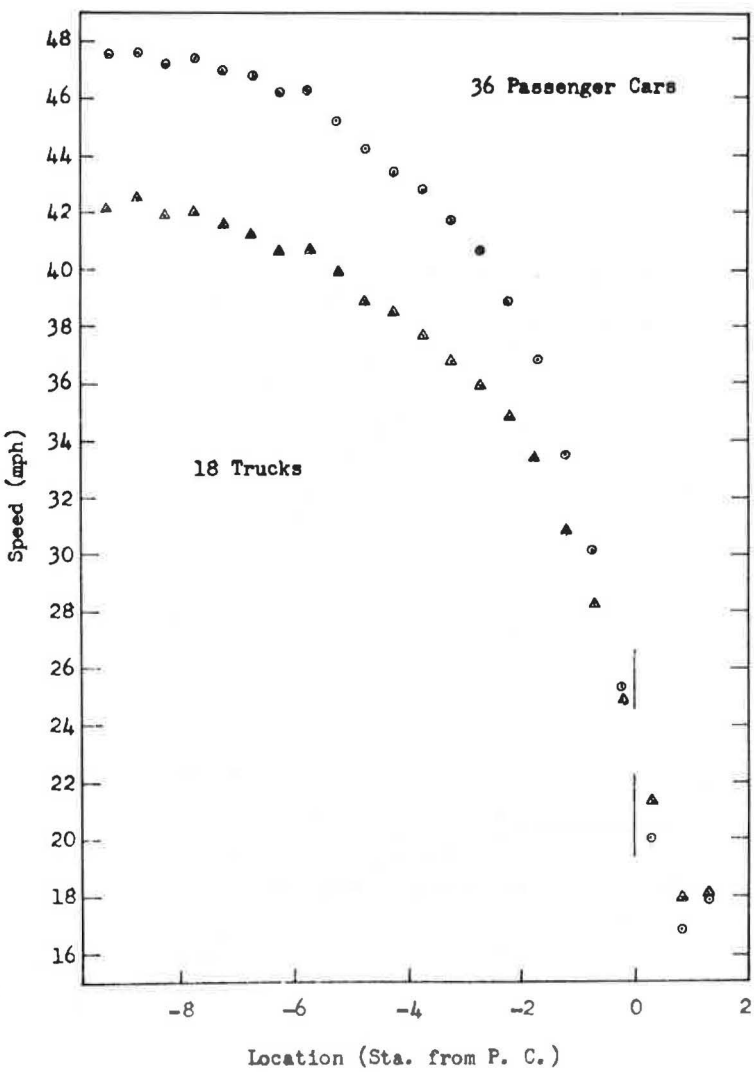


Figure 12. Speed-location graphs for corner on Ind. 26 near Pine Village, Ind.

approach to the turn is slightly downgrade and visibility is not restricted. There is considerable banking of the asphalt surface in the turn. Approaching drivers are informed of the conditions by means of a sharp turn sign only.

The roadway makes a complete 90-deg bend with an estimated radius of 42 ft for the westbound travel lane for which observations were recorded; thus, this condition is an example of an extreme rural highway curve.

Results.—The speed-location curve (Fig. 12) is not similar to the ones previously encountered in this study. Approaching vehicles begin to reduce speed approximately 1,000 to 1,200 ft prior to the turn—as in the other curves studied. However, in this case vehicles continue their deceleration at a more progressive rate as the feature is approached. When the minimum speed is reached (at approximately the center of the turn) the vehicles immediately undertake an acceleration. This type of response to the feature differs from the pressure potential theory previously discussed.

Matson et al. (2) show a comparable speed-location relationship which has been computed for the approaches to stop signs from a study by Beaky (3). The plotted observations for the subject curve on Ind. 26 resemble this condition more closely than the previous features in this study. Consequently this curve would be classified as a severe condition; that is, it requires an extremely large change of speed to permit vehicles to traverse it safely.

The shape of the speed-location curve may be attributed to this severity of operation. As the driver approaches the turn, he becomes aware of the increased resistance and subsequently allows his vehicle to decrease speed. As the turn becomes closer, the potential changes, and the driver must apply the vehicle's brakes to reduce speed more rapidly. Thus, at least two moduli of geometric aspects are required to be able to describe the response. An indication of this may be viewed in the similarity of the

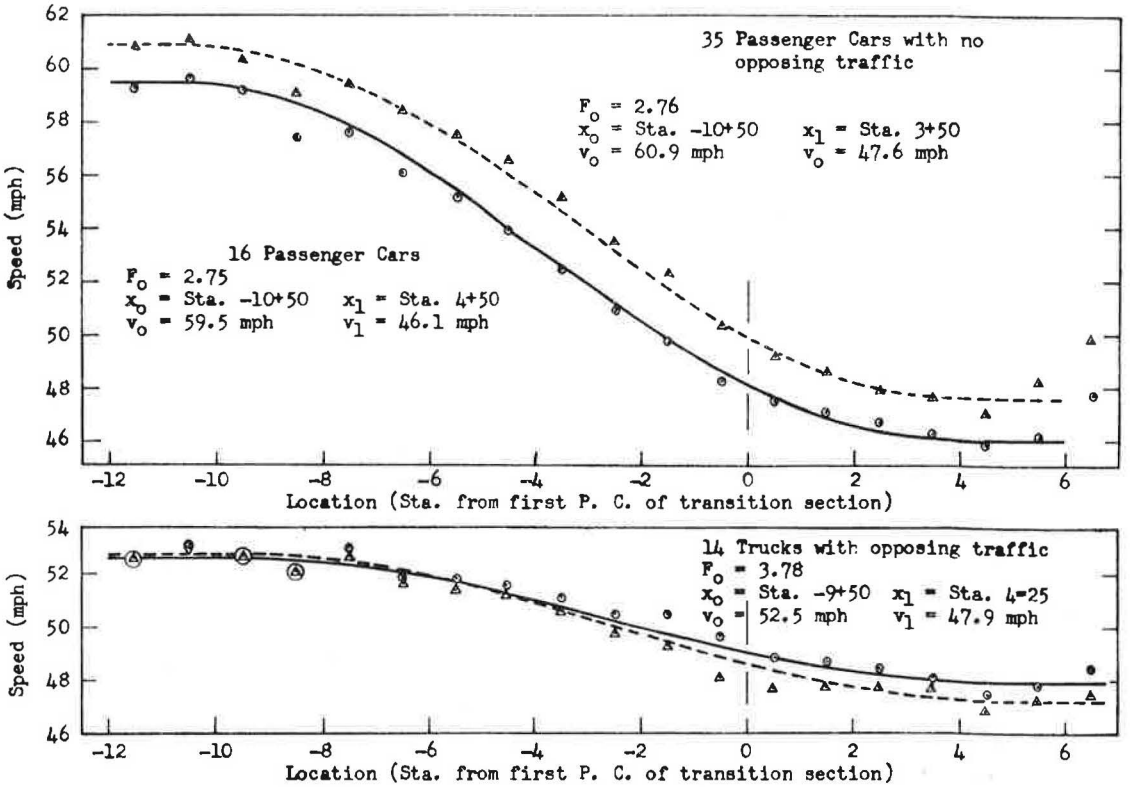
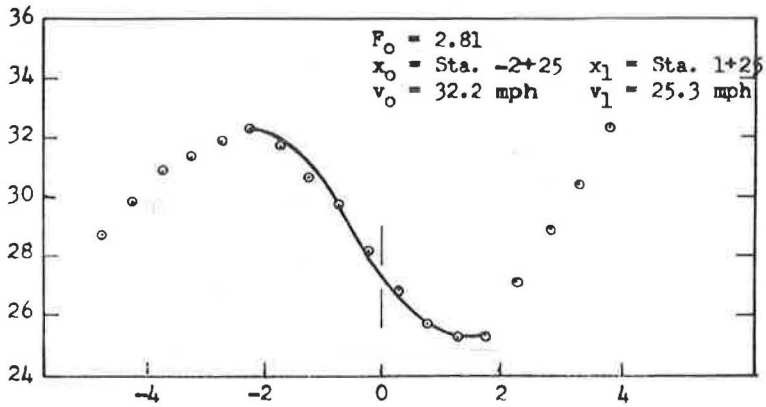
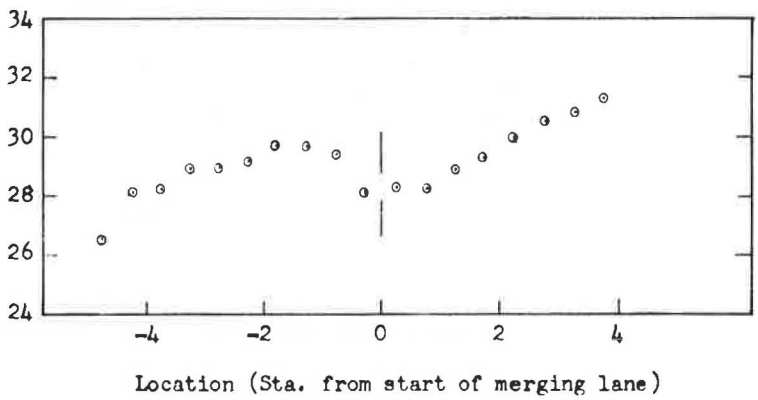


Figure 13. Speed-location graph for transition from 4-lanes to 2-lanes on US 52 near Templeton, Ind.



42 Passenger Cars



5 Trucks

Figure 14. Speed-location graph for merging section of approach to Wm. H. Harrison Bridge in West Lafayette, Ind.

first 500 ft of deceleration shown by the observed data for all features. Although the initial portion of the observed relationship is comparatively uniform, the remaining section exhibits extreme differences.

The detailed study of how to handle this particular response was beyond the scope of this study and was left for future investigations into the pressure potential theory.

US 52, 4-Lane to 2-Lane

Description.—The feature under consideration is a moderately heavily traveled section of US 52. For the vehicles observed the feature consists of a straight, level approach on the divided 4-lane pavement, a curve to the left, a short straight section, and a curve to the right which brings traffic onto the 2-lane roadway. Yellow paint has been applied to the pavement on the approach to encourage traffic to merge into a single lane prior to the transition. Warning and directional signs (for the transition) are located approximately 1,000 ft before the actual feature. There is only a minor degree of superelevation evident on the curved portion.

Results.—Although the speed-distance charts (Fig. 13) indicated some difference between the speeds of vehicles encountering oncoming traffic and those not encountering such traffic, this difference was found to be statistically insignificant.

The speed variability removed by the theoretical curve is extremely high and an excellent fit is evident both visually and by statistical F tests. The speed changes for passenger cars were normal; however, the speed changes for trucks were found to be skewed towards zero.

The best estimate of F_0 for this feature at the time of test was 2.76 for passenger cars and 3.69 for trucks.

Approach to Wm. H. Harrison Bridge

Description.—An investigation of the suitability of the modulus of geometric aspects was undertaken for the merging section of the North River Road approach to the Wm. H. Harrison Bridge. Observations were recorded of vehicles on the straight ramp approach, through the merging area, and onto the bridge itself.

Results.—Although the entire speed-location graph (Fig. 14) is rather complex, it was easily broken into individual parts for study. The speed-location graph for trucks is shown, but no conclusions were drawn from these data because only 5 trucks were observed.

The theoretical curve again closely approximates the observed speeds for the merging section, thus indicating an excellent fit of the theoretical curve and the observed data. Here also the speed change distribution plots as a straight line on probability paper. The modulus of geometric aspects for the merging area was found to be 2.81 from the observed data.

Ind. 43 near Chalmers

Description.—The bridge is an open truss bridge with an interior width of 22 ft 4 in. which is the same as the width of pavement on the approach to the facility. The approach is level and straight. Visibility is not restricted. A sign warning of a narrow bridge is located about 600 ft before the bridge.

Results.—There was only a minor speed change observed on the approach to the narrow bridge. This small speed reduction was statistically insignificant and thus the other calculations would have little basis for meaning. The only estimate of F_0 would be a high value indicating no resistance offered by this highway feature.

CONCLUSIONS

The pressure potential theory provides an excellent theoretical model for explaining the variations observed in speeds on the approach to and through most geometric highway features. It does not apply to geometric features requiring very severe speed changes.

The modulus of geometric aspects provides a reproducible quantitative rating of the ease of traffic flow through all geometric features except those requiring very severe speed changes.

RECOMMENDATIONS

The success of the modulus of geometric aspects as a quantitative rating of the highway features considered in this study suggests that the following additional investigations be undertaken:

1. A systematic tabulation of the modulus of geometric aspects for all pertinent highway features should be initiated. It is hoped that a spectrum of these values would (a) allow design decisions to be made in a quantitative manner; and (b) provide a basis for simulation approaches to traffic flow.
2. The relationship between the modulus of geometric aspects for cars and that for trucks should be explored in an attempt to obtain an equivalency between these vehicle types.

3. The theory should be amplified to include the compound modulus of geometric aspects as revealed in highway features requiring severe speed changes.

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