Engineering of Location: The Selection and Evaluation of Trial Grade Lines by an Electronic Digital Computer

JOHN H. SUHRBIER and PAUL O. ROBERTS, respectively, Research Engineer, and Assistant Professor of Civil Engineering, Massachusetts Institute of Technology

One aspect of the highway route location problem, vertical profile selection and evaluation, has been chosen to show how digital computer hardware and software advances now under development can be used to improve the decision-making capabilities of the engineer.

The large number of variables involved complicates the total highway location and design problem. It is difficult to identify all pertinent variables for any particular problem; it is even more difficult to identify their interrelationships. However, these complications are somewhat fewer for the vertical profile phase of the location portion of this problem. In the usual problem, just three cost variables will primarily be a function of the choice of vertical profile. These are the earthwork portion of the initial capital investment and the user fuel and time portions of the continuing costs. The relationship between these variables is such that an increase in earthwork costs usually results in a decrease in user fuel and time costs. One possible criterion for profile selection is to select that profile for which the increase in earthwork cost just equals the decrease in user costs.

A system of computer programs has been formulated and tested which has as its basic input a digital model of the terrain, the horizontal alignment of the road, the typical roadway cross-section, the predicted traffic volumes, and a description of the types of vehicles that will use the road. A vertical profile suitable for project planning and preliminary engineering is automatically selected and evaluated.

The profile selection process is based on a method where a weighted average of the elevations of a range of terrain points, both in front of and behind a given point, is used to determine a trial elevation of the profile at that given point. This trial elevation is then adjusted to account for any control points within the range. The program marches forward from one end of the alignment continuously selecting profile elevations. At each point, the selected elevation is checked against grade, rate of change of grade, and control point restrictions. If these restrictions are not met, the program "backs up" and sequentially adjusts previously selected elevations until all restrictions are met and possible phase errors eliminated. A point elevation is accepted as final only when it is indexed out of the range of points then under consideration. Earthwork volume and user cost computations are performed as the profile elevation for each station is accepted as final.

The vertical alignment generated by the system can be controlled by adjusting either the "look ahead" and "look behind" distances or the shape of the weighting function. In this way, the profile can be varied until the difference in volumes cost just equals the difference in user costs.

Paper sponsored by Committee on Geometric Highway Design and presented at the 44th Annual Meeting.

The automatic selection and evaluation of vertical alignments represents one phase of a problem at a particular level in the hierarchy of the complete highway location and design problem. Testing has indicated that not only is a system of this nature feasible and valuable, but also that computer models at the higher levels of analysis are a requirement if an engineer is to perform his work in a professional and responsible manner.

•ONE aspect of the highway route location problem, that of vertical profile selection and evaluation, has been chosen to show how mathematical techniques, when used in conjunction with electronic digital computers, can be used to improve the decision-making capabilities of an engineer. A system of computer programs has been formulated and tested which has as its basic input a digital model of the terrain, the horizontal alignment of the road, the typical roadway cross-section, the predicted traffic volumes, and a description of the types of vehicles that will use the road. With this information, along with certain cost and economic analysis measures, a vertical profile suitable for project planning and preliminary engineering is automatically selected and evaluated.

This paper presents a general discussion of the research which has been performed and of the resulting system of computer programs. Major emphasis is placed on a definition of the profile selection problem, a description of some of the problems which have been encountered in previous attempts to machine-select grade lines, a description of the basic system methodology, a summary of the various computer hardware configurations which might be used to implement the existing system of computer routines, and a brief description of the field tests which have been performed. The paper does not describe the detailed logic of each of the computer routines, nor does it give precise using and operating instructions. It is also not now possible to give a complete discussion of how the system might be used in the solution of a highway location and design problem. Obviously, any research concerned with "computer aided engineering design" has many deep and complicated effects on the way in which engineering will be performed in the future. The full implications of this work cannot be fully known until extensive field utilization of the system has been achieved. The possible uses of these programs will, therefore, be described only in sufficient enough detail to give an idea of the sort of analyses which might be performed.

STATEMENT OF THE PROBLEM

The Highway Engineering Process

In developing a definition of the profile selection problem, it is useful to examine briefly the total highway engineering process in order to place the problem into a real world context and to develop a motivation for its actual solution.

The process of highway engineering is essentially one of decision-making, involving operations upon information and culminating in a selected course of action. These operations include identifying goals and decision criteria, searching for a set of alternatives, predicting the physical consequences of each alternative, evaluating these consequences according to a value scheme, and deciding by means of some decision-making scheme to either accept an existing alternative or to continue the search process (Fig. 1).

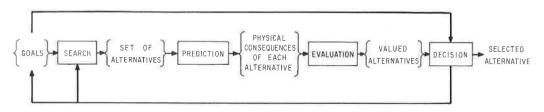


Figure 1. Schematic representation of the engineering process.

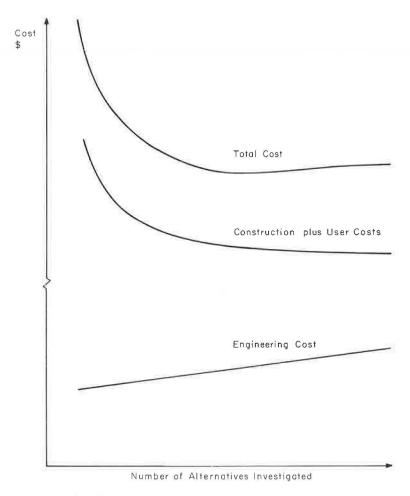


Figure 2. The cost and benefits of improved engineering.

Further examination of the highway location and design problem shows that it is hierarchical in structure. Typical of the higher levels of analysis are decisions such as choice of mode and geographical areas to serve. At somewhat lower levels, there are the questions of optimum network configuration involving the selection of bands of interest between individual nodes of the network. Still lower decision levels are concerned with the planning of specific projects and with the preliminary location of links within these bands of interest. The lower levels of this hierarchy are represented by preliminary engineering design and by the preparation of design plans and specifications. A characteristic of each level in this hierarchy is that it can be broken down into the five engineering phases. Each level is concerned with goals, search, prediction of consequences, evaluation, and decision.

An efficient and economical transport system is achievable only by good engineering at each level in this hierarchy. Highway engineers must employ engineering methods which produce designs which are consistent between the various levels and whose results at any one level can be directly and usefully presented to other levels in the design sequence. Suboptimization is meaningless unless there is feedback, involving both review and re-evaluation, at each level of the design process.

There are, at each stage in the hierarchy, an extremely large number of alternative courses of action open to an engineer. There is virtually no way to investigate them all, nor do they all necessarily need to be investigated. Figure 2 shows an idealized

form of the relationship between engineering, construction, and user costs. Although the plot is hypothetical, it indicates what is felt, and hoped, to exist in the engineering world. As more alternative designs are investigated, higher engineering costs are incurred, but the payoff for these higher engineering costs is a lower "total cost" solution. One of the difficulties with current engineering methods is the impossibility of predicting with any certainty the quality of an alternative which is produced by additional search activity. This quality, or "cost," is a function of the particular problem under investigation, the amount of search activity already completed, the skill of the engineer, and the methods by which these additional alternatives are being generated. The development of new means for rapidly and selectively generating, evaluating, and improving alternative alignments, particularly at the higher analysis levels, would increase the productivity of an engineer and would increase his ability to examine systematically more of the literally millions of possibilities which do exist (11).

The highway design phases have received the majority of recent research efforts. Systems of digital computer programs now exist which permit the detailed analysis of earthwork quantities and which facilitate computation of right-of-way, interchange, and superelevation geometrics. On-line plotters are beginning to be used for the automatic preparation of plans. Vehicle operating costs and travel times can be examined by means of either tables or charts or by simulation programs. However, the analysis levels concerned with project location and planning and also with overall system design have received somewhat less attention. Simple cost per mile figures are often used to evaluate construction cost. Vehicle operating and travel time costs are still only rarely considered. Very few alternatives are typically investigated because of the limitations imposed by hand methods. Since the costs of a project are almost completely determined at these higher analysis levels, it is these levels that should receive future research interest and to which attention is turned in this paper.

The Profile Selection Problem

A highway can be described as a curve in three dimensional space. To describe this curve fully, it is necessary to specify, either explicitly or implicitly, three coordinates for every point on this curve. In usual highway engineering practice, this is done by identifying a series of horizontal lines and curves defining a vertical curved surface on which a set of vertical lines and curves can then be defined. These two sets of lines and curves are known as the horizontal and vertical alignments, respectively.

The selection of a spatial location for this three dimensional curve, along with the decision as to the volume level of traffic to serve, determines the costs associated with transportation along this highway. These costs can be separated into capital, or first, costs and into operating, or continuing, costs. Capital costs include the initial expenditures for land, structures, earthwork, pavement, drainage, and interchanges; continuing costs consist of the expenses to the highway user and the expenses required to operate the facility. In order to use a road, a user incurs time, fuel, tire, oil, as well as vehicle maintenance and depreciation costs. Continuing costs associated with the highway itself include maintenance, snow removal, police patrol, and administration.

The relationships of these cost variables to the horizontal and vertical locations and to traffic volume, while not being completely defined, are understood well enough to permit some general statements to be made. The decision as to the level of traffic to serve specifies pavement width and number of lanes. Right-of-way and pavement costs are primarily a function of horizontal location only. In addition, such cost variables as structures, drainage, and road maintenance, although affected by vertical alignment, are largely determined by the horizontal location. This leaves earthwork, vehicle fuel, and user time costs as the cost variables that have a large dependence upon vertical alignment. These same three cost variables generally constitute the largest percentage of the total "cost" of a highway project. A summary of these cost relationships is given in Table 1.

Vehicle fuel and user time costs can be combined into a single term called "user cost." The implication of the preceding paragraph is that if traffic volumes are as-

TABLE 1
COST VARIABLE RELATIONSHIPS

Cost Variable	Function of Horizontal Alignment	Function of Vertical Alignment
Earthwork	X	X
Structures	X	CP^1
Pavement	X	
Drainage	X	?2
Interchange	X	CP
Relocation	X	
Right-of-way	X	
Fuel	X	X
Tire	X	
Oil	X	?
Vehicle mainte-		
nance	X	
Vehicle depre-		
ciation	X	
User time	X	X
Road mainte-		
nance	X	?
Accident	X	?
Political	X	
Social	X	
Aesthetic	X	

¹ Cost variable is usually accounted for through use of a control point.

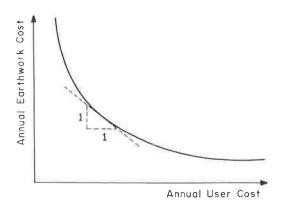


Figure 3. The relationship of earthwork to user time and fuel cost for a horizontal alignment alternative.

sumed fixed and once a single horizontal alignment is selected, the various vertical alignments which might be designed over this alignment may be ranked by considering only these two major variables, earthwork cost and user cost. The vertical alignment for which the sum of earthwork and user costs is minimum might be considered to be the "optimum" profile. At least the grade line is balanced in the sense that increased money spent on earthwork will not produce a corresponding savings in user cost.

This decision criterion can be demonstrated in a slightly different fashion. A theoretical plot of annual earthwork cost vs annual user cost for a range of various vertical alignment possibilities is shown in Figure 3. Points with high user costs and low earthwork costs correspond to highway profiles that follow the ground quite closely, while points having low user costs and high earthwork costs correspond to relatively smooth or flat grade lines that tend to deviate rather markedly from the existing ground profile. This plot indicates that for any one horizontal alignment, there is a point for which the decrease in user cost exactly equals the increase in earthwork cost. In theory, the vertical alignment selected by the engineer should correspond to this point, or at least be very close to it.

The highway profile must generally meet certain engineering restrictions. These restrictions are commonly associated with grades, rate of change of grade, and control points. Grade restrictions determine the maximum positive and negative grades that can exist on a road. The purpose of these restrictions is, ordinarily, to account for the relationships shown in Figure 3; that is, to prevent grades which result in excessively high operating costs caused by vehicle slow downs or hazardous operating conditions. Rate of change of grade restrictions provide adequate passing and stopping sight distances. Bridge elevations, railroad crossings, and major river crossings are typical examples of points where vertical control might be imposed. Control points then represent locations along the profile where the roadway should be at a specified elevation. The vertical alignment sought is that which results in the best balance of earthwork and user costs, yet meets the restrictions imposed by grade, sight distance, and control points.

²Effect of vertical alignment is considerably less than that of the horizontal alignment.

The highway location and design problem has been described as being hierarchical in structure, and the earthwork and user costs have been shown to be critically related to the vertical profile. These two ideas can now be examined together to see what the role of a profile selection capability might be. Profile selection becomes important at that level where the actual link locations are being considered. At higher levels of analysis, decisions are more concerned with questions such as modal split and system planning. The vertical profile is ordinarily unimportant here. At the project planning or project location levels, link locations are considered in more detail. Within the band of interest of an individual link, there typically will be a large number of possible horizontal alignments, each having a large number of feasible vertical alignments. With present methods, an engineer either uses a simple cost per mile estimate which ignores the vertical profile altogether or he is forced to use basically the same method as at the final design level. A horizontal alignment is specified by hand and input to a computer program. The machine then calculates the horizontal geometry and plots the ground profile. After a vertical profile is chosen by hand, additional machine passes are required to complete the evaluation. Thus, almost the same amount of engineering effort is required to investigate an alternative alignment at the project planning and location phases as at the preliminary or final design phases.

The method with which an engineer selects a vertical profile is quite complicated and, at present, not completely understood. For this reason, it may be best to continue to hand-select the final vertical profile. However, at the higher levels of analysis where accuracy requirements are not so tight, this essentially two-pass selection-evaluation procedure seems inefficient. It would be desirable to have a method of analysis which would permit the engineer, by merely specifying the horizontal alignment, to obtain a rapid and reasonably accurate evaluation of this alternative.

Previous Work on the Profile Selection Problem

The desire to select automatically a highway profile is not new. It has existed, in fact, since the introduction of electronic digital computers to the highway engineering field. A number of the past efforts in this area were examined to gain an idea of the various approaches that had been taken towards this problem and to learn and understand the major problems that had been encountered.

Most of the early work was directed toward the area commonly referred to as reconnaissance or preliminary engineering. The majority of the approaches examined, in essence, made a least squares fit of a series of consecutive and continuous polynomial curves (first, second, or third degree and combinations thereof) to a set of terrain profile points subject to grade and control point restrictions. This does not imply that these approaches are all similar in detail or even in basic method. They differ as to the objective function employed (if, indeed, one is used at all), the manner in which control points are handled, the method by which grade restrictions are satisfied, and the computer configurations required. Other studies have been performed in the area of automatic profile selection, the most notable of which is probably the application of the calculus of variations. However, these studies have not yet had completely acceptable results, nor have they proven to be economically feasible for use by public highway departments.

Basically, these procedures start from an origin point which has specified initial elevation and slope conditions. A least squares fit, taking account of control points, of a curve is made to the terrain points for a specified range of fit. This range of fit can extend either in front of and behind the origin point, or just in front of the origin point. The resulting curve is used to compute the highway grade at the station of the next terrain profile point ahead. If the slope does not exceed the designated limits, the curve is accepted and the elevation of the highway at this next station is computed. If the slope restrictions are not satisfied, the appropriate limiting slope is used and the curve corrected before the highway elevation is computed. The highway elevation and slope at the new station are then taken as the new initial conditions and the curve fitting process repeated at the new point. The computations systematically step ahead, yielding a continuous highway grade line.

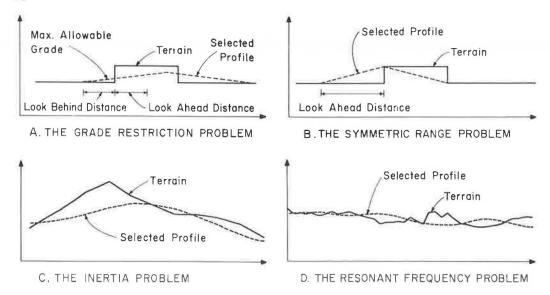


Figure 4. Summary of major problem areas in the automatic selection of highway profiles.

The following is a summary of the four major problems that have been encountered in the curve fitting type of profile selection techniques:

- 1. The grade restriction problem. Relatively good results can be obtained in gently rolling terrain. However, as the roughness of the terrain increases and grade restrictions are imposed, the resulting highway profile is skewed or biased to the right. This bias occurs for several reasons. The hills and valleys are often "seen" too late by the range of fit ahead to permit the highway to alter its course in time. This problem is compounded when grade restrictions set in, because the highway can only rise or fall at a specified maximum slope. If the terrain condition is seen too late and the grade restrictions are such that the terrain is falling or rising at a slope which the highway profile cannot match, the selected highway profile is bound to be skewed to the right (Fig. 4A).
- 2. The symmetric range problem. A second type of bias or skew is introduced when only terrain points aheadare taken into account in determining the highway grades and profiles. However, this will tend to skew the resulting profile to the left instead of to the right. This can be easily seen in the case of the "rectangular mountain" (Fig. 4B). By the time the roadway reaches the mountain, it will have already reached the highest elevation, since the only terrain points under consideration at this time will be those having elevations equal to the highest elevation. Similarly, the roadway elevation will have already returned to original ground level by the time the end of the mountain is reached.
- 3. The inertia problem. Whenever previous roadway elevations and grades are taken into account in determining the elevation and grade of other stations, there is a tendency for the selected profile to be skewed to the right (Figure 4C).
- 4. The resonant frequency problem. A "resonant frequency" type of problem can occur under special conditions. When the predominant wave length of the terrain profile is approximately equal in length to the total range of terrain points under consideration, an oscillation develops in the selected profile (Figure 4D).

METHODOLOGY OF AN AUTOMATIC PROFILE SELECTION-EVALUATION SYSTEM

The System

The logic of an integrated profile selection-evaluation system is shown in Figure 5. The system calculates the horizontal geometry of an alignment, generates and plots the

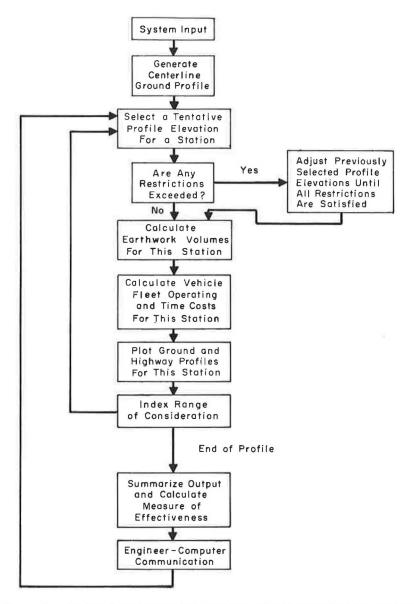


Figure 5. Logic of an integrated profile selection-evaluation system.

centerline ground profile, selects a tentative highway profile by means of a mathematical smoothing technique and plots it over the ground profile, calculates the earthwork volumes associated with this profile, simulates the operation of vehicles over this alignment to determine operating and time costs, summarizes the results in the form of total and annual costs, and finally asks the engineer whether he would like to try to improve the selected profile in order to obtain a better balance of earthwork and user costs. Calculations are based on the theory of the digital terrain model (DTM) and employ the routines and programs presently existing in the DTM location system.

The profile selection process is based on a method whereby a weighted average of the elevations of a range of terrain points, both in front of and behind a given point, is used to determine a trial elevation of the profile at that given point. This trial elevation is then adjusted to account for all control points within the range. The program

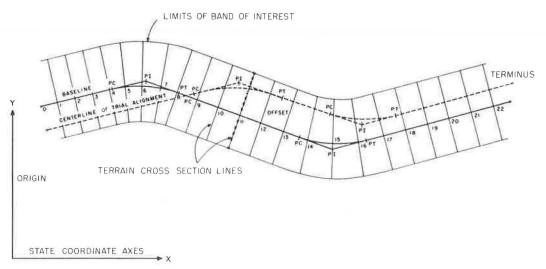


Figure 6. Digital terrain model showing baseline and terrain cross-section lines.

marches forward from one end of the alignment continuously selecting profile elevations. At each point, the selected elevation is checked against grade and control point restrictions. If these restrictions are not met, the program "backs up" and sequentially adjusts previously selected elevations until all restrictions are met and possible phase errors eliminated. A point elevation is accepted as final only when it is indexed out of the range of points then under consideration (see Fig. 10). Earthwork volume and user cost computations are performed as the profile elevation for each station is accepted as final.

The following sections describe each of the major computational blocks of Figure 5 and indicate how each is related to the overall goal of profile selection and evaluation.

Digital Terrain Model

Fundamental to an understanding of the complete profile selection-evaluation system is an understanding of how the terrain model is formed. The DTM is simply a sampling of the continuous surface of the ground by a number of selected terrain points with known X, Y, and Z coordinates in an arbitrary coordinate system. Terrain data are defined relative to a baseline (X-axis) and are taken along lines normal to this baseline called cross-section or scan lines (Y-axis) (Fig. 6). The reason for relating terrain data to a fixed baseline rather than to a centerline is that, for a new trial horizontal alignment, it is relatively easy to re-establish the relationship of the new centerline to the baseline. This enables the model to be used repeatedly for the fast evaluation of many different trial lines during planning and location studies without the necessity of retaking data.

Before the terrain data are taken, a band of interest is selected by the engineer. The width of this band varies with the amount of latitude that the location affords. This band of interest is then digitized for use by the computer.

The first step in the digitization process is the definition of a baseline within the band of interest. This serves as the X-axis of the data coordinate system and can be composed of either straight line segments or straight line segments connected by curves, depending upon the shape of the band of interest. Next, the surface of the ground is represented by a series of points (Fig. 7). Sample points are taken left to right across each cross-section and are referenced by giving the baseline station number (X-coordinate) of the cross-section line, their offset distance from the baseline (Y-coordinate), and their elevation (Z-coordinate). Points to the left of the baseline are recorded with a negative sign, while points to the right are considered to have a positive offset.

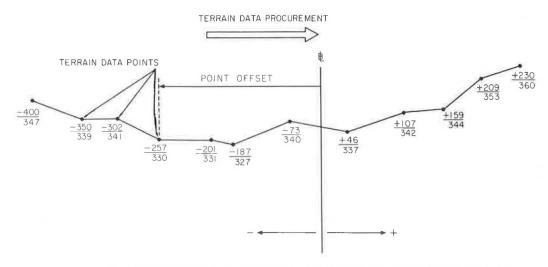
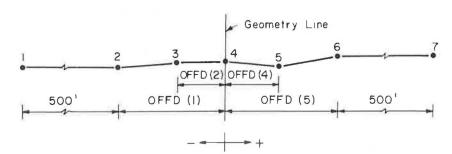


Figure 7. Typical cross-section showing definition of model by terrain points.



Note: OFFD (3) IS DEFINED AS HAVING AN OFFSET OF O.O.

Figure 8. Definition of approximated terrain section.

The spacing of the scan lines and the sample density of terrain points along a scan line depend upon the accuracy required, the maps available, and the nature of the terrain.

Terrain data for the DTM can be taken directly from field notes, from topographic maps either by hand or by the use of special instrumentation, or directly from the stereo model using automatic take-off equipment. Terrain data are taken only once and can be used repeatedly for the evaluation of many lines.

Location Alignment Program

The location alignment program calculates the geometry of the highway centerline alignment and determines its relationship with the baseline to which the terrain data are referenced. Once this relation has been established at each cross-section, an approximated terrain model for use in the actual profile selection-evaluation program can be interpolated from the real terrain model (Fig. 8).

Both the baseline and centerline are defined by the state plane coordinates of each P.I. and one curve defining parameter given at each P.I. The location alignment program computes the geometry of the defined baselines and of corresponding centerline alignments. This geometry information is then used in computing the offset distances from the baseline to the centerline. The approximated terrain model is generated by interpolating terrain data on each cross-section to find the ground elevation along the centerline of the roadway and along four parallel alignments whose offset distances from the centerline have been specified by the engineer.

A more detailed description of the operations of the various program phases is given in the program manual of the DTM location system (19).

Profile Selection

The selection of a trial highway profile is done by a heuristic which may be thought of as attempting to model or simulate the actions of an engineer by determining a weighted elevation of terrain points within a range of interest and then adjusting this elevation to account for the control points. The mechanics of the profile selection and grade adjustment procedure are indicated in Figure 9 and will be described as a series of iterative steps. The range of terrain points and associated terminology are shown in Figure 10. The range has been indexed so that point 0 is the middle point; it is referred to as the origin point.

- 1. The search point is initially set equal to the origin point and a trial profile elevation for the search point station is determined by calculating a weighted elevation of all terrain points within the current range of interest and then adjusting this elevation to account for each of the control points within the current range.
- 2. Both the grade and the rate of change of grade of the highway between the search point station and the station of the immediately preceding point are calculated. If both are acceptable and the current search point is point 0, go to step 3. If both are acceptable and the search point is somewhere between point -n and point -1, go to step 4. If either is unacceptable and the search point is any point except point -n, go to step 5. Finally, if either does not satisfy the appropriate restriction and the search point is point -n, go to step 6.
- 3. Since all grades and all rates of change of grade are acceptable, the profile elevation of point -n is accepted as final and the total range of terrain points is indexed so that point -n passes out of the range of terrain points under consideration and point 1 becomes the new point 0. Calculations are returned to step 1.
- 4. All grades and rates of change of grade on the selected profile from the current search point station to the origin point station are checked. If all are acceptable, go to step 3. If any grade or rate of change of grade does not satisfy the restrictions and the search point is not point -n, go to step 5. If some grade or rate of change of grade is determined to be unacceptable and the current search point is point -n, go to step 6.
- 5. If a particular trial grade or rate of change of grade does not satisfy the given restrictions, this is an indication that a terrain configuration in front of the search point was "seen" too late to permit the use of an acceptable design standard. To correct this, the search point is moved back one point (from point 0 to point -1 or from point -j to point -(j + 1) and step 1 repeated. In effect, this throws out the previously accepted profile elevation for this station and a new adjusted elevation is determined.
- 6. The search point has reached the back of the range and additional "backing up" is not possible. Therefore, any grade or any rate of change of grade between the search point station and the origin point station that exceeds the design restrictions must be set equal to the appropriate maximum positive or negative value. When all

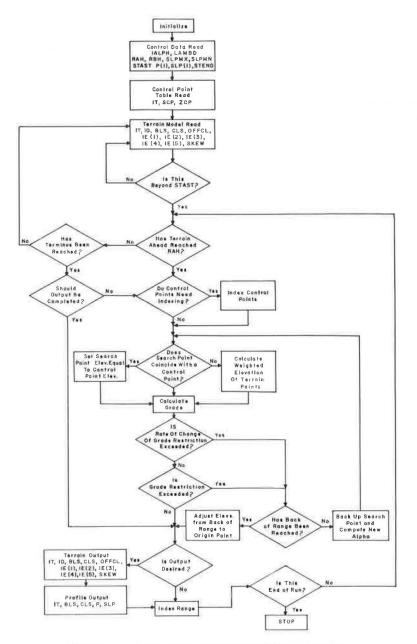


Figure 9. Macro flow diagram profile selection.

unacceptable conditions have been corrected, the range of terrain points is indexed so that point 1 becomes point 0. The search point is again set equal to point 0 and a new iteration is begun with step 1.

A profile selection procedure of the above nature must specify starting and stopping conditions. The manner in which these are handled is indicated in Figure 11.

<u>Case 1</u>: The information for the range of terrain points ahead of the initial origin point is read in and stored in memory before any profile selection calculations are made. The lengths of the range ahead and the range behind are specified by the engineer.

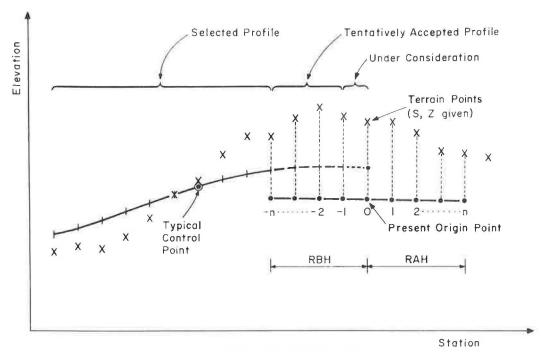


Figure 10. Range indexing procedure.

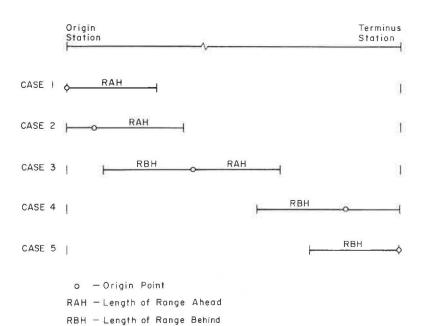


Figure 11. Starting and stopping procedures.

Case 2: As the origin point moves away from the initial or origin station, the range of terrain points behind the origin point is slowly built up until it reaches its full length.

Case 3: This is the normal operating procedure and is the case assumed in the description of the profile selection procedure.

Case 4: As the front of the range ahead reaches the terminus point, additional terrain information is not available to be read into storage. The origin point is indexed ahead in the normal manner, except the front of the range ahead stays fixed at the terminus point.

Case 5: When the origin point reaches the terminus point, the algorithm has reached an end. The elevations and stations for the selected profile between the back of the range behind and the origin point are output.

The initial selection of a tentative profile elevation is done by calculating a weighted elevation of the terrain points within the present range of interest. This initial elevation is then adjusted to satisfy the control point elevation constraints. A weight for each of the terrain point elevations is determined by the equation:

$$W = \frac{1}{\beta (\alpha + 1, \lambda + 1)} x^{\alpha} (1 - x)^{\lambda} (\alpha, \lambda > -1; 0 \le x \le 1)$$

where

$$\beta(\alpha+1,\lambda+1) = \frac{\Gamma(\alpha+1) \Gamma(\lambda+1)}{\Gamma(\alpha+\lambda+2)}$$
, and

 $\Gamma(n)$ denotes the generalized form of the gamma function

$$\Gamma(n) = \int_{0}^{\infty} x^{n-1} e^{-x} dx \quad (n > 0)$$

The variable x varies from 0 to 1. The back of the range behind the origin point is denoted as 0 and the front of the range ahead is denoted as 1. Stations between these two points have a fractional x value. The weighted elevations for the terrain points are added and their sum divided by the sum of the terrain point weights to determine a trial profile elevation.

The equation used to determine the terrain point weights is identical in form to the probability density function of the beta distribution. The effects of the parameters on the shape of the terrain point weighting function curve are shown in Figure 12. The special case $\alpha, \lambda = 0$ yields the rectangular distribution, while if one parameter is 0 and the other 1, the triangular distribution is obtained. The curve is U-shaped if both parameters are negative, J-shaped if only one is negative, and unimodal with the mode at $x = \alpha/(\alpha + \lambda)$ if both are positive. For the latter case, the curve is symmetrical if $\alpha = \lambda$, skewed to the left if $\alpha < \lambda$, and skewed to the right if $\alpha > \lambda$. One of the major reasons for selecting this form of the terrain point weighting function was the large flexibility available in the shape of the curve. The effects of different shapes could be determined and the skew of the curve could be controlled so that the search point elevation always receives the largest weight.

The beta function cannot be integrated formally from 0 to x unless α and λ are both integral multiples of $\frac{1}{2}$. The profile selection computational procedure assumes that the values input to the program meet this restriction. All adjustments of α , λ within the program are made so that each will continue to be a multiple of $\frac{1}{2}$.

Control point elevations are introduced by adjusting the weighted terrain point elevation once for each control point within the current range of interest. The equation used for this adjustment is:

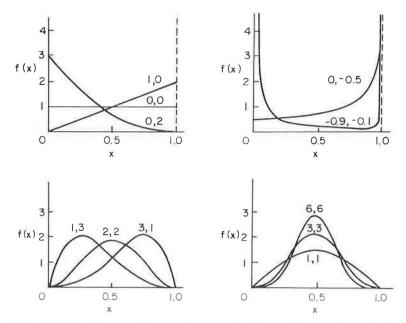


Figure 12. Beta distributions with various selections of parameters α , λ ; the symbol a, b means α = a, λ = b (From "Introduction to Probability and Random Variables" by Wadsworth & Bryan, Copyright 1960, McGraw-Hill Book Company, Inc.).

where FACTOR equals the absolute value of the difference between the control point and the search point stations divided by the difference between the search point station and the front of the range of interest. A control point is thus given linearly increasing importance as it nears the search point. As a control point enters the range of interest, its weight will be 0; as the search point gets progressively closer to a control point, the control point weight increases to 1.0; and when the stations of the control and search point are equal, the weighted terrain elevation will be ignored completely since the FACTOR value in this case will be 0. If more than one control point is within the range of interest, they are accounted for in inverse order as their distance from the search point station increases. Thus, the farthest control point is accounted for first and the closest is accounted for last in adjusting the weighted terrain point elevation.

This approach to the automatic selection of highway profiles eliminates three of the four major problems that have been encountered in previous work. The immediate history of the roadway is ignored in computing new profile elevations, thereby eliminating inertia effects and its associated right-hand skew. Grade and rate of change of grade restrictions are not satisfied by simply setting excesses to the appropriate maximum positive or negative value. When a restricted area is encountered, the selection algorithm backs up until all grades and rates of change of grade have been adjusted to acceptable levels. Restricted values are used only if this backing up continues until the back of the range of terrain points presently under consideration is reached. This back up procedure will tend to eliminate the right-hand bias resulting from seeing a terrain configuration too late. Terrain points ahead and behind the search point are used in determining the profile elevations. Problems created by using an unsymmetrical influence zone are, therefore, eliminated. Problems associated with resonant frequency can still occur. However, if the length of the range of terrain points being considered is greater than the wave length of all objectionable oscillations in the profile, resonant frequency should not be a problem. Oscillations may still occur, but since their wave lengths will be greater than or equal to the length of the influence zone, these oscillations have been implicitly approved of by the engineer.

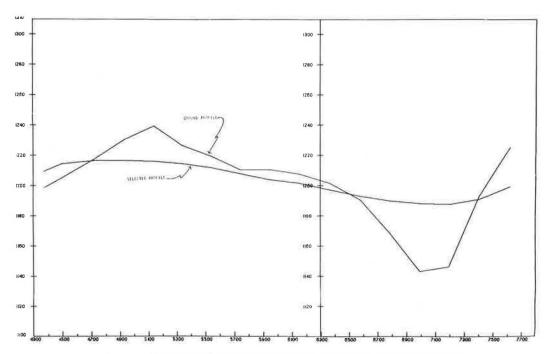


Figure 13. Example of an automatically generated highway profile.

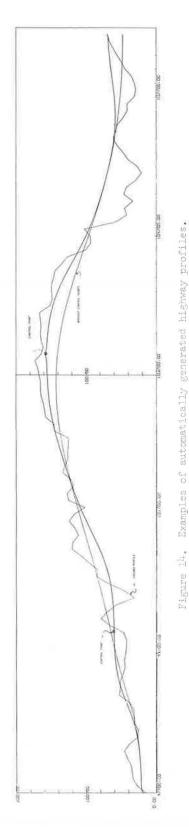
System criteria are satisfied by this selection procedure. If the selected profile is to be iteratively improved upon, it is necessary to have a way in which the smoothness or quality of the profile can be controlled. It is possible in this procedure to adjust both the lengths of the ranges ahead and behind the origin point and the parameters α and λ .

Examples of profiles generated by this routine are shown in Figures 13 and 14.

Roadway Design and Earthwork Volumes

As the selected profile elevation for each station is indexed out of the range of terrain points under consideration, roadway design and earthwork volume computations are made for this cross-section. Using the approximated terrain model generated in the location alignment program and the profile grade chosen by the profile selection procedure, the roadway template is constructed from the defined template links and parameters, the slopes are selected according to specified design criteria, and the slope intercept and earthwork volume calculations made. Output consists of the total accumulated cut and fill volumes for the entire job with the slope stake and volume information for any particular station available at the option of the engineer.

The template is composed of a series of links, each defined by dy and dz distances, and a series of low and high cut and fill slopes (Fig. 15). The five-point approximate terrain model from the location alignment program is used as the terrain description in the template construction and earthwork computations. This model contains five offsets and elevations for each cross-section. Normally, two of these are to the right of the centerline, two are to the left, and one is the centerline. An additional two points, which actually make this a seven-point terrain model, are defined internally by the routine. These are 500 ft outside each of the two exterior offset points and are at the same elevation as the corresponding exterior offset point. This approximate terrain model, rather than the actual DTM, is used for two reasons: first, it results in a large increase in running speed with only a slight reduction in accuracy over the more exact terrain model. This is important since this is only one routine of a much larger pro-



gram system. In addition, it is felt that this lower accuracy is acceptable during preliminary location work. Second, by internally defining the two extreme points, the engineer can select arbitrary offsets from the profile grade line without having to worry about whether or not the template slopes will intercept within the defined terrain.

Vehicle Fleet Operating and Time Costs

An engineer, in performing an analysis of a particular highway alternative, is interested in the operating and time costs incurred by the vehicle fleet for several reasons. In order to compare this alternative with others, the costs to the road user must be measured and compared with those of other alternatives. In the selection of a vertical alignment an engineer almost always, either implicitly or explicitly, makes an effort to balance construction and user costs. Lastly, if a highway is to serve as an efficient transport link, it must permit the individual vehicles using the facility to operate efficiently, i.e., vehicles, especially trucks, must be able to perform satisfactorily. While the output necessary for the profile selectionevaluation feedback loop is only the total annual user costs, it is also desirable to be able to obtain, at the option of the engineer, information concerning the operation and performance of various types of vehicles.

A considerable amount of work has been done in the field of vehicle performance prediction. These efforts fall into essentially three categories: (a) the tabulation of experimental data into tables and graphs, (b) the statistical analysis of data to construct regression equations, and (c) the development of systems of performance equations which can be solved by either hand or computer to give vehicle performance and operation.

In the usual table, cost per mile figures for fuel, oil, tires, maintenance, and depreciation are given as functions of profile grade, pavement condition, and vehicle class. These are generally empirical studies and give cost information only. Since these figures will not necessarily apply to vehicles manufactured outside the country in which the study was made and since an actual speed profile is not obtainable, this method of user cost pre-

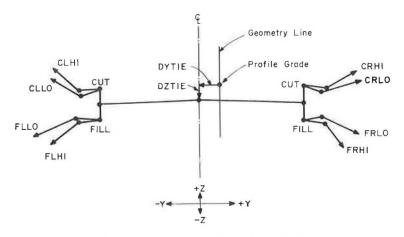


Figure 15. Definition of template.

diction is not considered to be completely satisfactory for a profile selection-evaluation system.

Because of the complexity of vehicle performance, statistical studies have necessarily been limited to rather special cases. For example, the cost per mile is given as a function of velocity for various vehicle classifications or fuel consumption for the acceleration of gasoline or diesel trucks is given as a function of magnitude and length of grade. To date these studies have not been general enough in nature to permit them to be used for the prediction of vehicle fleet performance over a specific alignment.

Simulation techniques have typically been of two types: those employing a digital computer to solve the system equations, and those employing a graphical or hand method of solution. Of necessity, the hand solutions have been significantly more limited in flexibility than the computer-oriented approaches.

The basis of any simulation is a determination and calculation of all forces acting on the vehicle (Fig. 16). The forces resisting the movement of the vehicle are air resistance, rolling resistance, and grade resistance. Rolling resistances are those forces inherent in the vehicle itself that tend to retard its motion. The grade resistance

Force Available to Accelerate Vehicle = Tractive Effort — Rolling Resistance — Air Resistance — Grade Resistance

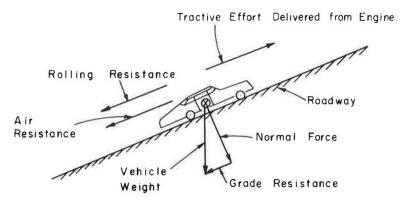


Figure 16. Simplified free body diagram of vehicle and roadway.

TABLE 2 FEATURES OF VEHICLE OPERATING COST ROUTINE

Cars and trucks	Yes
Torque converter	No
Shifting deceleration	Yes
Gear shift oscillations	Yes
Engine power curve	Torque vs rpm
Fuel prediction	Gen'l fuel map
Speed changes and stops	No
Chassis friction	% Efficiency
Turn around	Yes
Independent variable	Velocity
Effect of temperature	No
Horizontal curvature	No
Vertical alignment	Tangents only
Station equation	No
Summary information	Yes
Output comments	Yes
Data control cards	No
Program language	FORTRAN II
_ 0 0	

is equal to that component of the vehicle's weight which is acting in a direction parallel to the surface of the road. The force available to overcome these resistance forces and to move the vehicle forward is called the tractive effort force, which is equal to the force supplied by the engine minus certain internal friction losses. The difference between this motivating force and the total resisting force is the force available to accelerate the vehicle. All simulation techniques are similar in that they are concerned with the manipulation of these forces. The logic of the technique, whether computer oriented or not, drives any vehicle over any selected alignment. The vehicle is accelerated, decelerated, stopped, upshifted, downshifted, and, in general, operated in the same manner as if it were being driven by a normal driver. In this manner, the effect of such highway characteristics as vertical profile, superelevation, pavement type, and speed limits on vehicle road speed, engine rpm, vehicle tractive effort, and percent of full engine load can be determined. Since these vehicle performance

characteristics are in turn directly related to the user costs, it is possible to study the sensitivity of these costs to changes in the highway design.

In comparing existing methods of vehicle performance prediction to the criteria of the profile selection-evaluation system, no satisfactory solution is found to be currently available. The computer simulation techniques are the best; yet, even these are not completely satisfactory in their existing form. The vehicle simulation and operating cost system developed at M.I.T. is in effect too flexible and consequently too expensive to use. It was decided that a simulation method which combined some of the flexibility of the existing M.I.T. method with the logical advantages of some of the other vehicle simulation techniques presently available would produce an efficient predictor of user fuel and time costs suitable for all classes of vehicles in not only the United States, but also foreign countries. Such user cost items as tires, oil, depreciation, and vehicle maintenance are not sufficiently well understood to merit their inclusion into a simulation routine on anything more than a cost per mile as a function of pavement type basis.

The basic features of the proposed vehicle simulation routine are given in Table 2. The independent variable is velocity. Vehicle velocity is changed in 1-mph units during acceleration or deceleration. The model equations are recalculated only when the velocity changes or when the resisting forces change due to a change in profile grade.

Input data required for this routine, in addition to the fuel map, are essentially of three types:

- 1. Vehicle data include such things as vehicle weight, frontal area, tire size, number of cylinders, engine bore and stroke, gear shift speeds and ratios, and full-throttle torque curve:
- 2. Control data include the speeds at which the vehicle is to travel and the maximum acceleration and deceleration rates to be used by the vehicle; and
- 3. Highway profile data are required in the form of a series of straight line segments.

Output data consist of two types. The first is the cost information required for the feedback loop—the total annual user fuel and time costs for each of the vehicle types selected. The second type of output is optional and is under control of the engineer.

For any vehicle operating on a particular vertical profile, it would include such things as current vehicle speed, distance traveled en route, elapsed time, rate of fuel consumption, and total fuel consumed to date. Areas where the vehicle was not capable of maintaining the desired road speed would be so indicated.

The proposed vehicle fleet operating and time cost routine has not yet been programmed. The versions of the profile selection-evaluation system currently in use employ a table look-up procedure based on data published in the AASHO Red Book (1). Direct operating costs (fuel, tires, oil, maintenance and repairs, and depreciation) are obtained directly from the Red Book while time costs are calculated separately. Input data include the type or class of alignment, speed, user time cost, number of persons per car, truck factor, percentage trucks, present traffic volume, and projected traffic volume. Although no vehicle performance information can be obtained with this approach and although the basic cost data that are incorporated in the routine prevent it from being meaningful to areas outside the continental United States, this procedure is still thought to be acceptable for the purpose of testing the basic system and its operation.

Graphical Display of the Ground and Selected Highway Profiles

The manner in which information is displayed is critically important for any profile selection scheme. As a mass of numbers, a highway or ground profile is difficult to comprehend or to picture, but as a graphical plot it is clear and concise. Routines are incorporated in the system to simultaneously plot the ground and selected highway profiles. Depending on the machine configuration being used, these displays are in the form of either continuous line plots (Figures 13 and 14) or character plots and are generated via either on-line digital plotters (either a California Computer Products incremental Digital Plotter, Model 565, or a Gerber Scientific Instrument Company line plotter, Model VP 600) or printers.

Engineer-Computer Communication and Feedback Loop

The lower portion of Figure 5 indicates an engineer-computer feedback loop. This loop, by incorporating the concepts of a problem-oriented computer language, permits an engineer to use ordinary English and engineering terminology to change any of the initial input data and then to investigate the effects of these changes on the selected profile. He can modify such items as the roadway width, the grade restrictions, the sight distance or rate of change of grade restraints, the elevation of a particular control point, the predicted traffic volumes, or the expected percentage of trucks to determine how much a particular design standard or restriction is costing in terms of both construction and user costs.

The "smoothness" of the vertical alignment generated by the system can be controlled via this same engineer-computer feedback loop through the adjustment of the program parameters which control the length of the range of influence ahead and behind the origin point. In this way, the profile can be varied until the most economic balance of initial and continuing costs is determined. This iterative improvement of the highway profile is not presently being automatically done by the program because of the lack of specific knowledge concerning the exact relationship of user to construction costs. Past work in this area has been largely of a theoretical nature. Experimental data from a number of real projects will be necessary before this "learning type of behavior" can be confidently accomplished without engineer intervention.

COMPUTER HARDWARE CONFIGURATIONS

The engineering problems under discussion are basically ones in information acquisition, processing, storage, and display. The selected computer hardware should perform these information handling operations in as efficient a manner as is possible. In addition, the chosen computer configuration must result in an overall profile selection-evaluation system which is competitive with existing methods of profile selection and evaluation. In order for a method such as the one being proposed either to replace or augment present hand techniques, it must be approximately equal in cost. If not, it

must provide enough additional information in a short enough time period to merit the additional expense. Thus, the proper choice of computer hardware is an extremely important part of the development work of this system.

Existing Equipment Requirements

Three separate versions of the system have been developed. These have been for a small-scale computer (either a 20,000 or a 60,000 digit memory IBM 1620 data processing system), a medium size computer (a 32,000 word memory IBM 7040 data processing system with 6 magnetic tape drives and on-line card reader, card punch, and printer); and a large-scale computer (a time-shared IBM 7094). The 1620 and 7040 versions have been used primarily for program development and field testing, while the time-shared 7094 version has been used for experiments in man-machine interaction and in the development of large and more sophisticated highway oriented programming systems. In the time-shared version, the engineer-user has essentially complete on-line control of a large-scale computer (currently an IBM 7094 under control of the M.I.T. compatible time-sharing system). The response time between input query and output result is such that entirely new and different approaches to engineering utilization of computers are possible.

Input-Output and Information Display Developments

A number of capabilities have recently been introduced which permit the performance of operations upon information in a more efficient manner than previously possible. One of these is the use of small-scale "satellite" computers to communicate with and to act as remote input/output consoles to a large computer. As a prototype development, the M.I.T. Department of Civil Engineering's IBM 1620 computer is now connected via a dataphone to an IBM 7094 computer. In this mode of operation, the 1620 can be used as a rather complete remote console in the compatible time-sharing system. The terrain vertical profile and the selected highway vertical profile data can be transmitted directly from the 7094 into the 1620 core or disc memory. A plotting program stored in the 1620 could then plot both the ground and highway profiles on a plotter attached on-line to the 1620. Because of the length and size of plots required, this means of plotting would most likely be preferred to either a scope attached directly to the 7094 or the teletype remote console produced character plots.

SYSTEM TESTING

Profile Selection

An important phase in the development of any new engineering technique is the testing to which this technique is subjected before it is actually placed into full production use. In addition to the tests being performed by the M.I.T. Department of Civil Engineering in cooperation with the projects sponsor, the Massachusetts Department of Public Works, a series of field experiments are being conducted by the Maine State Highway Commission. Both of these series are of a continuing nature and have been under active study by one or the other of these state highway departments. They differ principally in that the M.I.T.-Massachusetts series is being conducted primarily at M.I.T. while the Maine tests are being performed and evaluated by actual state highway department personnel. In both test series, the conclusion has been that the profiles generated by the program are generally satisfactory for the purpose of preliminary engineering location and they are at least equal to the first or second trial profiles chosen by engineers. Although it is not possible to describe all of the details and all of the results of the various tests which have been performed, the general conclusions can be summarized.

The three bias problems which existed in some of the earlier attempts at automatic profile selection appear to have been eliminated. Testing in both smooth and rough terrain has produced a profile that is "in phase" with the terrain; that is, the peaks and valleys of the profile occur at the same points as those for the terrain. Operation in grade restricted areas has been examined and the results indicate that the backing up procedure generally results in grades below the maximum allowable grade and that the

problems associated with seeing a major terrain configuration too late have been eliminated or considerably reduced. If the engineer selects a look ahead and look behind range that is too short for the existing terrain, resonant frequency can still become a major problem. However, it is felt that this is more the result of poor input data than of the profile selection technique itself.

One of the criteria specified for the selection technique was an ability to change the quality of smoothness of the selected profile. If improvements are to be iteratively made in the profile selected for any one alignment, it must be possible to alter the profile in some manner. This is currently done either by changing the length of the look ahead and look behind ranges or by adjusting the α , λ parameters in the terrain point elevation weighting function. Testing has shown that as the length of the range is increased, the selected profile becomes significantly smoother. The majority of the testing has been conducted with both α and $\lambda = 3$. However, in tests where these parameters were decreased toward 0, the profile tended to become somewhat flatter. Although for α , $\lambda = 0$ (equivalent to a rectangular weighting function), the profile had many small dips and rises superimposed on the selected profile. A value of 3 for α and λ seems to give the best results.

A number of variations of the basic control point handling technique have been investigated. No single method has yet been found to be completely satisfactory for all cases. When the elevation of a control point is near the elevation of the desired profile, the

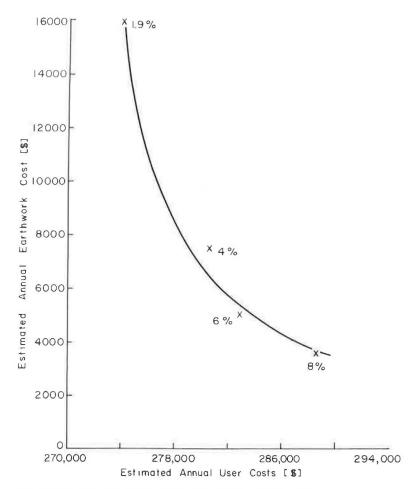


Figure 17. Relationship of earthwork to user costs for a hypothetical rectangular mountain.

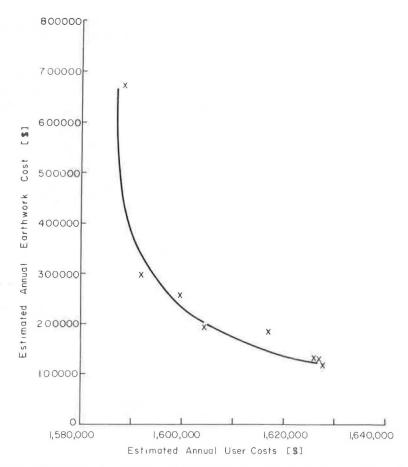


Figure 18. Relationship of earthwork and user costs for a 7-mile section of Interstate 495.

transition is both smooth and satisfactory. However, when the control point elevation is considerably different from the normal desired profile elevation, the selection technique does not account for the control point in a completely satisfactory manner. The conclusion of these tests is that the method of handling control points needs additional investigation.

The final and in the long run the only test of this technique is whether it produces a profile acceptable to the engineer. Present testing indicates that this goal can be reached.

Earthwork and User Cost Relationship

An extremely critical part of the selection-evaluation scheme is the curve (Fig. 3) showing the theoretical relationship of earthwork to user cost. Whether the proposed feedback loop is to be automatically included in the computer complex or is to be hand controlled by the engineer, real data must be collected to confirm the theorized shape of this cost relationship. If it turns out that these cost variables are not related or if their relationship is different than presently thought, it is doubtful that a profile selection-evaluation technique which attempts to balance earthwork and user costs will be either feasible or valuable.

A preliminary series of experiments that examined two separate test conditions has been performed to determine the relationship between these two cost variables. The

first is the case of the so-called rectangular mountain. It is easy in this relatively simple example to define a series of progressively smoother vertical alignments and to evaluate both earthwork and user costs by standard hand methods. The results (Fig. 17) indicate that, at least for this very simple case, the theoretical relationship holds.

The second test series was performed on a seven-mile portion of highway near Franklin, Mass. The actual alignment chosen was one of those considered in the location of Interstate 495 and passed through two rather large hills. Eight separate vertical alignments were investigated; the "roughest" of which followed the terrain quite closely, while the smoothest was a constant 0.2 percent grade from beginning to end. Geometry, roadway design, and earthwork volume computations were made with the DTM location system, while user operating and time costs were estimated with the M.I.T. vehicle simulation and operating cost system. The traffic volumes predicted by the Massachusetts Department of Public Works were used as a means of obtaining equivalent annual user costs. The resulting plot of earthwork cost vs total user cost is shown in Figure 18. Although the hypothesized relationship appears to hold, at no point is the decrease in user costs large enough to merit the corresponding increase in construction cost. This is an indication that because of the low traffic volumes being used in this particular example, the chosen profiles represent points only at one extreme of the hypothetical curve.

These tests have been of a preliminary nature and their results do not constitute enough evidence to say that this relationship is either valid or not valid. A much more extensive test series is needed to definitely confirm this cost variable relationship.

CONCLUSIONS

Basic System Approach

In mathematical and operations research terminology, this approach to the solution of the profile selection problem would be classed as a heuristic and opposed to a mathematically rigorous optimization procedure. It is a strategy, a simplification, or a rule which attempts to produce a solution which is "good enough most of the time;" it is not necessarily producing an "optimal" solution. The procedure is trying to select a profile which would be classed as acceptable, satisfactory, or intelligent if the same profile had been chosen by a human engineer the first or second time he had examined a problem. In brief, it is simply a device "which drastically limits search for solutions in large problem spaces" (6).

An example of a more mathematically rigorous approach to this problem is the dynamic programming solution, which could produce a solution which would guarantee an optimum balance of earthwork and user costs. Although this approach has not yet been fully formulated by the authors, two general comments can still be made. First, the solution appears to break down to an exhaustive search of the decision space. This type of search with its corresponding high number of required evaluations of the objective function would probably require a class of computer which is not yet available to the typical state highway department. Second, the selection of a highway vertical profile is not a neat, tidy, and clean problem which directly lends itself to an optimum solution. Any profile selection technique can at best make suggestions to the location engineer.

Possible System Applications

An important question that should be investigated in the early stages of development of any system dealing with "computer-aided engineering design" is, "What is the demand by the engineering community for a system of this nature?" This is extremely difficult to determine; however, it is felt that a profile selection and evaluation system as described has a wide variety of uses.

To the student, it could serve as a valuable educational tool for studying the engineering of location. Certain real life subtleties, which are extremely difficult to teach in the classroom, would become immediately obvious after a few minutes of investigation into an actual problem.

To the researcher, such a system represents a powerful tool for studying and

analyzing the engineering process. Certain insights may be gained into intricate cost variable relationships.

To the highway engineer of a developing or emerging nation, it could represent a means of greatly increasing engineering capabilities. In these areas, the flexibility of locating a transport system link is generally much greater than in the United States, and hence the decision-making problems associated with location are compounded.

To a highway engineer in the United States, a profile selection and evaluation system could have a number of direct applications. It could help to provide a uniformity of vertical design standards. Unbiased vertical design would allow different horizontal alignments to be more directly and fairly compared since the same set of restrictions and the same design procedure had been used on each. It could provide a means of speeding up the processing of horizontal alignments. One of the current restrictions on the number of horizontal alignments which can feasibly be investigated by an engineering team is caused by the number of computer passes required for even a preliminary evaluation. There is presently a human selection of the horizontal alignment which is followed by one or more machine passes to produce a centerline ground profile plot. After human review and human selection of a vertical alignment, additional computer passes are required to obtain earthwork computations, slope limit plots, and, if desired, vehicle performance and user cost data. The automatic selection of a vertical profile would allow the entire computer sequence of operations to occur without interruption. If the profile selection-evaluation system is iteratively solved for each of a number of horizontal alignments, with each iteration on a particular alignment using a different combination of the smoothing parameters and projected traffic volumes, it becomes possible to generate a complete set of transportation production curves. These curves could then be used to provide the necessary construction and user cost input data for a regional transportation network analysis.

Long-Range Research Objectives

The ultimate objective of this work is an integrated system of computer programs, computer and computer related hardware, and engineering procedures which can be used to solve highway location and design problems. The automatic selection and evaluation of vertical alignments represents one phase of this larger problem, at one particular level in the design hierarchy. Although much computer-oriented work has already been directed towards the final design aspects of this overall system, investigations are only beginning in such areas as regional network analysis, horizontal alignment selection, drainage, interchange design, and guidance or management of the total engineering process. Many more areas exist where mathematical selection, evaluation, and analysis models need to be built. The formulation, design, construction, and implementation of the individual models required to fill in these "gap" areas and of the ultimate man-machine system represents a fertile area for future research.

ACKNOWLEDGMENTS

The authors wish to express their appreciation to the Massachusetts Department of Public Works and the U.S. Bureau of Public Roads who have sponsored this research. We would also like to express special appreciation to Mr. Walter Verrill and Mr. Roger Mallar of the Maine State Highway Commission for their frequent assistance, advice, and encouragement during all phases of research; and to Mrs. Raymond Igou for her secretarial assistance.

These activities have been done in part in the Civil Engineering Systems Laboratory, C. L. Miller, Director, a research facility of the Department of Civil Engineering, Massachusetts Institute of Technology; and in part in Project MAC, an M.I.T. research program sponsored by the Advanced Research Projects Agency, Department of Defense, under the office of Naval Research, Contract Number Nonr - 4102(01). Reproduction in whole or in part is permitted for any purpose of the U.S. Government.

REFERENCES

- 1. American Association of State Highway Officials, Committee on Planning and Design Policies. Information Report on Road User Benefit Analyses for Highway Improvements. Washington, D. C., 1960.
- 2. American Association of State Highway Officials. A Policy on Geometric Design of Rural Highways. Association General Offices, Washington, D. C., 1955.
- 3. Bellman, R. Dynamic Programming. Princeton Univ. Press, 1957.
- 4. Corbato, F. J., et al. The Compatible Time-Sharing System, A Programmer's Guide. Cambridge, Mass. The M.I.T. Press, 1963.
- 5. Dawson, R. F. F. Vehicle Operating Costs in 1962. Traffic Engineering and Control. Jan. 1963.
- Feigenbaum, E. A., and Feldman, J. Computers and Thought. New York, N.Y., McGraw-Hill, 1963.
- 7. Firey, J. C., and Peterson, E. W. An Analysis of Speed Changes for Large Transport Trucks. Highway Research Board Bull. 334, pp. 1-26, 1962.
- 8. Gladding, D. G. Automatic Selection of Horizontal Alignments in Highway Location. M.I.T., unpublished SM thesis, June 1964.
- 9. Hewes, L. I., and Oglesby, C. H. Highway Engineering. New York, John Wiley and Sons, 1960.
- Hildebrand, F. B. Methods of Applied Mathematics. Englewood Cliffs, N. J., Prentice-Hall, 1960.
- Lang, A. S., Manheim, M. L., Roberts, P. O., and Wohl, M. Preliminary Notes on Advanced Highway Engineering. M.I.T., Depart. of Civil Eng., unpublished course notes, 1961.
- Lang, A. S., Robbins, D. H., and Roberts, P. O. Vehicle Simulation and Operating Cost System, Program Manual. M.I.T., Civil Eng. Res. Rept. R62-35, Sept. 1962.
- 13. Lang, A. S., and Robbins, D. H. A New Technique for the Prediction of Vehicle Operating Costs in Connection with Highway Design. M.I.T., Civil Eng. Systems Lab. Publ. 141, April 1961.
- 14. Manheim, M. L. Highway Route Location as a Hierarchically-Structured Sequential Decision Process: An Experiment in the Use of Bayesian Decision Theory for Guiding and Engineering Process, M.I.T., Civil Eng. Res. Rept. R64-15, May 1964.
- Miller, C. L. Man-Machine Communications in Civil Engineering. M.I.T., Civil Eng. Systems Lab. Publ. T63-3, June 1963.
- 16. Miller, C. L. Mathematical Theory of an Automatic Highway Profile Design Program. M.I.T., Civil Eng. Systems Lab. Publ. 118, August 1958.
- 17. Miller, C. L., Lang, A. S., and Robbins, D. H. Research Report on the Vehicle Simulation and Operating Cost System, M.I.T., Civil Eng. Systems Lab. Publ. 143, July 1961.
- 18. Roberts, P.O., and Suhrbier, J. H. Highway Location Analysis, An Example Problem. M.I.T., Civil Eng. Res. Rept. R63-40, Dec. 1962.
- 19. Roberts, P. O., and Suhrbier, J. H. DTM Location System, Program Manual. M.I.T., Civil Eng. Res. Rept. R62-8, April 1962.
- Roberts, P. O., and Currie, J. A. DTM Design System, 40K Program Manual. M.I.T., Civil Eng. Res. Rept. R62-7, Dec. 1961.
- 21. Robbins, D. H. Selection and Optimization of Vertical Highway Alignment. M.I.T., unpublished SM thesis, August 1958.
- 22. Sawhill, R. B., and Firey, J. C. Predicting Fuel Consumption and Travel Time of Motor Transport Vehicles. Highway Research Board Bull. 334, pp. 27-46, 1962.
- 23. Smith, G. L. Multigrade Vehicle Performance Simulation. Detroit, Mich., General Motors Corp. Truck and Coach Div., unpublished paper, 1963.
- 24. Taborek, J. J. Mechanics of Vehicles. Machine Design, May-Dec. 1957.
- Vajda, S. Mathematical Programming. Reading, Mass., Addison Wesley Publishing Co., 1961.
- 26. Wadsworth, G. P., and Bryan, J. G. Introduction to Probability and Random Variables. New York, N.Y., McGraw-Hill, 1960.