A Comparative Evaluation of Trip Distribution Procedures

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The results of a research project designed to evaluate on a common basis the Fratar, gravity, intervening opportunities and competing opportunities trip distribution procedures are reported. Each of the procedures was calibrated using the 1948 Washington, D.C., O-D survey travel data as a base. Projections were made to 1955 using the procedures recommended by the principal developers of the techniques. The 1955 projections were then comprehensively tested against the 1955 Washington, D.C., O-D survey travel data.

Each procedure is evaluated for travel pattern simulation ability as well as the forecasting stability of the parameters. Various methods evaluate the accuracy of the models including trip length frequency duplication, screenline checks, specific movement checks and overall statistical evaluations of the estimated movements. These tests are performed for each technique and comparisons of the relative accuracies are also made. Appropriate changes in the calibration procedures are recommended.

•THE RAPID evolution of computer-oriented trip distribution techniques coupled with the pressing deadlines of the major urban transportation studies has made it difficult for the studies themselves to mount a comprehensive program for testing and evaluating the most widely used trip distribution techniques. Individual applications of trip distribution models have often involved a certain amount of research, and as a by-product of these applications, revisions and improvements in each of the techniques have been made. In the last 2 years, however, the rate of evolutionary development has slackened to the extent that most of the techniques are now considered to have reached a somewhat mature status.

This paper reports on the results of a research project conducted by the Urban Planning Division of the U.S. Bureau of Public Roads to test, evaluate, and compare four major trip distribution techniques: (a) the Fratar growth factor procedures as developed by Thomas J. Fratar and utilized by many transportation studies (1); (b) the so-called "gravity model," currently the most widely used of the mathematical travel formulas (2); (c) the intervening opportunities model developed by Morton Schneider of the Chicago Area Transportation Study (CATS) and since utilized by several other major studies (3); and (d) the competing opportunities model suggested by Anthony Tomazinis of the Penn-Jersey Transportation Study (PJ) but not yet utilized in an operational study (4, 5).

The mathematical model techniques present interesting contrasts in their approach to the trip distribution problem. These models can be classified into two categories: growth factor procedures and interarea travel formulas. The growth factor procedures utilize growth factors reflecting land-use changes in the zones to expand a known

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travel pattern to some future year. The interarea travel formulas simulate travel distributions by relating them to characteristics of the land-use pattern and of the transportation system. The interarea travel formulas require calibration—i. e., determining the effect of spatial separation on travel—before their actual application as forecasting tools.

STUDY PROCEDURES

An attempt was made to establish a standard set of test conditions for evaluating the four procedures. It was not possible to adhere to strictly comparable conditions in all instances, but each variation from a common base is fully discussed.

Basic data sources for the analysis were the 1948 and 1955 home interview travel surveys conducted in Washington, D. C. The 1948 survey covered 5 percent of the dwelling units in the metropolitan area. In 1955 a repeat survey was conducted. In the repeat survey, occupants of 3 percent of the dwelling units were interviewed within the District of Columbia. Elsewhere in the area, occupants of 10 percent of the dwelling units were interviewed. Figure 1 shows a map of the study area.

The boundaries of the 1948 and the 1955 study areas were not exactly matched. Every attempt was made, however, to make the 1948 and 1955 analysis zones compatible. This was not a critical problem with respect to the interarea travel formulas since the only variable projected directly is the effect of spatial separation on trip-making. This variable is independent of zone configuration. The Fratar procedure, however, requires compatible zones for base and projection years. For the Fratar analysis it was necessary to reduce the 400 zones utilized in the standard analysis to 362 more comparable units. In most instances this involved eliminating zones which were external to the 1948 study area but internal to the 1955 area and thus having zero trips ends in 1948. Certain irregularities in zonal boundaries still were present; however, their effect was not serious. Because of changes in the location of external cordon stations between 1948 and 1955, all trips crossing the cordon—i. e., external trips—were omitted from the analysis. The basic trips considered were the total person trips by all modes expanded from the home interview surveys. Trips recorded in the special truck and taxi surveys were not included.

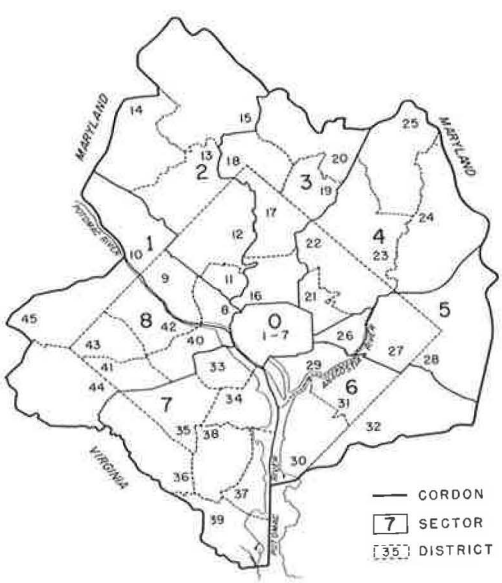

Figure 1. Study area, Washington, D. C., 1948 and 1955.

Although the test period covered by this analysis was only 7 years, the characteristics of the area experienced significant changes in this period of time. The total population increased 38 percent to almost 1.5 million. The number of person trips increased by over 42 percent. During the same interval, the number of passenger cars owned by residents almost doubled, increasing by 96 percent.

Probably the most significant change in the study area within the 7-year period was the decentralization of many activities. Residential, employment, and shopping activities were all relatively less oriented to the central business district (CBD) in 1955 than in 1948 (6). Total trips to the CBD likewise showed a relative decrease from 28 to 21 percent of the total person trips.

The study was designed so that the 1948 survey data would be used as the base year travel pattern for the Fratar procedure and as a calibration source for the interarea travel formulas. The 1955 travel survey data were used as a control against

which all forecasts were checked. The trip ends reflecting the 1955 characteristics were taken directly from the 1955 O-D survey trip ends to establish the Fratar growth factors. In addition, they were used directly as producing and attracting powers of the zones when calculating the synthetic distributions with the interarea travel formulas. The 1955 trip ends were used, rather than estimates developed in a land use-trip generation analysis, to restrict the possible sources of error to those inherent within each of the distribution procedures.

TRAVEL MODELS

Fratar

The Fratar procedure has been proven to be computationally the most efficient of the growth factor techniques (7). The basic premise of the Fratar procedure is that the distribution of trips from a zone is proportional to the present movements out of the zone modified by the growth factor of the zone to which the trips are attracted. The future volume of trips out of a zone is determined from the present trips out of the zone and the growth factors developed for the zone. Most earlier applications of the Fratar procedure considered only one general trip purpose. The Urban Planning Division of the U.S. Bureau of Public Roads in 1962 developed a Fratar procedure that considers up to 10 trip purposes. This program also allows the application of growth factors by mode, time of day, or separately for trips entering or leaving a zone. The basic formula for the directional purpose Fratar procedure is

$$T_{ij}^{(p,q)} = t_{ij}^{(p,q)} \cdot G_i^{(p)} \cdot G_j^{(q)} \cdot \frac{L_i^{(p)} + L_j^{(q)}}{2} \qquad (1)$$

where

$$L_i^{(p)} = \text{locational factor} = \frac{t_i^{(p)}}{\sum_{j=1}^{n} t_{ij}^{(p,q)} \; G_j^{(p)}};$$

$T_{ij}^{(p,q)}$ = future year trips from zone i to zone j with a purpose p at zone i and purpose q at zone j;

$t_i^{(p)}$ = base year trip ends at zone i for purpose p;

$t_{ij}^{(p,q)}$ = base year trips between zone i and zone j with a purpose p at zone i and a purpose q at zone j; and

$G_i^{(p)}$ - growth factor for zone i, purpose p.

The purpose Fratar allows the procedure to be sensitive to the type of land-use changes that are occurring in a given zone. For example, work trips can be expanded as a function of employment changes only. Before the development of the new computer program, all trips, irrespective of their purpose, were expanded by a measure of the overall growth of the zone.

Gravity Model

The gravity model is the most thoroughly documented of the trip distribution techniques (8-11). This approach, loosely paralleling Newton's gravitational law, is based on the assumption that all trips starting from a given zone are attracted by the various traffic generators and that this attraction is in direct proportion to the size of the generator and in inverse proportion to the spatial separation between the areas. This research study utilized the Public Roads computer program battery gravity model program. The basic gravity model formulation of this program is

$$T_{ij} = \frac{P_i A_j F_{ij} K_{ij}}{\sum_{j=1}^{n} A_j F_{ij} K_{ij}} \tag{2}$$

where

T_{ij} = trips produced in zone i and attracted to zone j;
P_i = trips produced in zone i;
A_j = trips attracted to zone j;
F_{ij} = empirically derived travel time factors (one factor for each 1-min increment of travel time) that are a function of the spatial separation between the zones and express the average areawide effect of spatial separation on trip interchange;
K_{ij} = specific zone-to-zone adjustment factor to allow for the incorporation of the effect on travel patterns of defined social or economic linkages not otherwise accounted for in the gravity model formulation.

The travel time factors F_{ij} are developed in an iterative procedure which is continued until the synthetic trips calculated for each trip length interval closely match the surveyed trips reported for the same intervals. Any convenient set of travel time factors may be used to start the iteration procedure.

Intervening Opportunities Model

The intervening opportunities model utilizes a probability concept which in essence requires that a trip remain as short as possible, lengthening only as it fails to find an acceptable destination at a lesser distance. An equal areawide probability of acceptance for any origin is defined for all destinations in a given category. All trip opportunities or destinations are considered in sequence by travel time from zone of origin. In operation, the first opportunity considered is the one closest to the origin and has the stated areawide probability of acceptance. The next opportunity has the same basic probability of acceptance; however, the actual probability is decreased by the fact that the trip being distributed has a chance of already having accepted the first opportunity. The procedure continues with each successive opportunity having, in effect, a decreased probability of being accepted.

Thus, spatial separation for the intervening opportunities model is measured, not in terms of the absolute travel time, cost, or distance between one zone and the other, but rather in terms of the number of intervening opportunities. These intervening opportunities (destinations) are determined by arraying the available destinations in all zones by travel time from the zone of origin. The formulation for the procedure is

$$T_{ij} = O_i \left[e^{-LD} - e^{-L(D + D_j)} \right] \tag{3}$$

where

T_{ij} = trips originating in zone i with destinations in zone j;
O_i = trip origins in zone i;
D = trip destinations considered before zone j;
D_j = trip destinations in zone j;
L = measure of probability that a random destination will satisfy the needs of a particular trip; it is an empirically derived function describing the rate of trip decay with increasing trip destinations and increasing length of trip; and
e = base of natural logarithms (2.71828).

This model is calibrated by varying the probability values until the simulated trip distribution reproduces the person-hours of travel and percent intrazonal trips of the surveyed trip distribution.

Competing Opportunity Model

Essentially, the basic concept of the competing opportunity model is that opportunities or destinations compete for trips within equal travel time, travel distance, or travel cost bands as measured from the zone of origin. Within a given band, every opportunity has an equal probability of acceptance. The probability that trips will distribute to a certain zone is the product of two independent probabilities. The first, called the "probability of satisfaction," reflects the chances that a trip will be of a particular length and is a function of the opportunities at a greater distance than the time band under consideration. The determination of the specific destination within this trip length is quantified by a "probability of attraction" related to the available opportunities which fall within the area up to and including the time band considered.

The mathematical formulation for this procedure is

$$T_{ij} = O_i \rho_{a_j} \rho_{s_j} \tag{4}$$

where

T_{ij} = trips produced in zone i and attracted to zone j;
O_i = trip origins in zone i;
ρ_{a_j} = probability of attraction = $\dfrac{\text{destination available in zone j}}{\text{sum of dest. avail. in time bands up to and incl. band m}}$ = $\dfrac{D_j}{\sum_{k=0}^{m} D_k}$
ρ_{s_j} = probability of satisfaction =

$$1 - \frac{\text{Sum of dest. avail. in time bands up to and incl. band m}}{\text{sum of total destinations in study area}} = 1 - \frac{\sum_{k=0}^{m} D_k}{\sum_{k=0}^{n} D_k};$$

k = any time band;
m = time band into which zone j falls;
D_k = destinations available in time band k;
n = last time band as measured from origin zone i; and
D_j = destinations available in zone j.

This model is calibrated by varying the width of the attracting bands until the trip length characteristics of the synthetic trips correspond to the trip length characteristics of the surveyed trips.

BASIC TESTS USED TO EVALUATE DISTRIBUTION MODELS

Four basic tests were employed to measure the ability of the various procedures to reproduce the total person trip movements of the known travel patterns: (a) ability to match the trip length frequency distribution from the O-D survey; (b) ability to produce river crossing volumes that match O-D survey volumes; (c) ability to match O-D survey trip movements by corridor to and from the CBD; and (d) accuracy of model as measured by statistical comparison of O-D survey and model of trips assigned to a "spider network."

No 1948 tests could be made with the Fratar procedure because its base is the survey data. However, in the case of the other travel formulas, some validation was accomplished against base conditions. Such validation is an essential part of calibrating the models before moving to projections. The accuracy of this base year simulation is typically the most important check in the calibration procedure. This check follows from an assumption that if the calibrated travel model will accurately simulate a base year travel pattern, the same model will also accurately simulate a future year travel pattern.

The trip length frequency comparisons were made by 1-min time intervals. A consideration of the trip length frequency curves and the mean trip lengths provides a measure of the accuracy of the person-hours of travel estimate for the total area as well as an indication of the accuracy of the trip distribution.

The river crossing tests were made on the basis of screenlines set up on both the Potomac and the Anacostia Rivers. Because of the trip definition, the base screenline values were the O-D survey person movements rather than actual vehicle counts.

The analysis of movements by corridor to and from the CBD was designed to detect any bias in the estimated travel patterns. The gravity model computer program provides for the use of adjustment factors to correct for bias. With the other techniques it is usually assumed that the procedure adequately distributes trips without need for adjustment.

The final test was the statistical analysis of trips assigned to a "spider network," a network consisting of airline distance connections between adjacent zone centroids. The resulting differences between the O-D and model assignments are arrayed by volume group and the root-mean-square error (RMSE) is calculated. This test provides a measure of the overall accuracy of the final trip distribution.

CALIBRATION OF INTERAREA TRAVEL FORMULAS

Gravity Model

Prior gravity model research with Washington data used the 1955 O-D data as a calibration base rather than the 1948 data (8, 9). The model parameters were, in effect, forecast backward from 1955 to 1948. For the subject research, the gravity model was recalibrated using the 1948 O-D data as a base and these 1948 model parameters were used to forecast 1955 travel patterns. The research showed that the same travel time factors held good for both 1948 and 1955 and that the K factor (socioeconomic adjustment factor) also maintained the same relationship with average family income by district for both periods. One somewhat questionable point was whether the river crossing time impedances, which varied from 5 and 3 min for work and nonwork trips, respectively, in 1948 to 6 and 5 min for these same trip categories in 1955, could have been properly forecast without the knowledge gained in the research. The 1955 river crossings were forecast from 1948 on the basis of the relative congestion levels for the 2 years (9, p. 93). For purposes of the present comparisons, however, it was assumed that the river barriers could be properly forecast. The travel time factors for each of the six trip purposes used for both 1948 and 1955 are given in Table 1.

TABLE 1

TRAVEL TIME FACTORS BY TRIP PURPOSE
WASHINGTON, D. C., 1948 AND 1955

Travel Time	Home-Based Trips					Nonhome-Based Trips
	Work	Shopping	Social-Rec.	School	Misc.	
1	1,000	8,700	2,000	4,200	2,600	1,600
2	1,000	8,700	2,000	4,200	2,600	1,600
3	1,000	8,700	2,000	4,200	2,600	1,600
4	1,000	8,700	2,000	4,200	2,600	1,600
5	1,000	8,700	2,000	4,200	2,600	1,600
6	1,000	8,700	2,000	4,200	2,600	1,600
7	1,000	8,700	2,000	4,200	2,600	1,600
8	1,000	8,700	2,000	4,200	2,600	1,600
9	680	5,400	1,475	2,800	1,700	1,100
10	500	3,600	1,100	2,000	1,200	780
11	400	2,300	820	1,475	875	580
12	320	1,600	640	1,075	650	440
13	270	1,120	500	800	500	340
14	235	800	400	625	390	265
15	205	580	320	480	300	215
16	180	420	260	370	235	170
17	160	310	220	280	190	140
18	145	235	180	215	150	110
19	130	180	152	165	125	92
20	120	140	130	135	105	78
21	110	105	110	110	87	65
22	100	95	95	90	72	54
23	93	70	82	70	60	46
24	87	58	72	57	51	40
25	82	45	64	47	43	33
26	77	38	56	40	38	29
27	70	32	49	32	32	25
28	63	26	42	26	28	22
29	58	21	38	22	24	20
30	53	17	34	18	21	17
31	49	13	30	15	18	15
32	44	10	27	12	15	13
33	40	8	24	10	13	12
34	37	6	21	9	12	10
35	34	5	19	7	10	9
36	29	4	17	6	8	8
37	27	3	15	5	7	6
38	24	2	13	4	6	5
39	22	2	11	4	5	4
40	19	1	10	3	4	3
41	17	–	8	3	3	2
42	15	–	7	2	3	1
43	13	–	6	2	2	–
44	11	–	5	2	1	–
45	9	–	4	1	1	–
46	7	–	3	1	–	–
47	6	–	3	1	–	–
48	5	–	2	1	–	–
49	4	–	1	–	–	–
50	3	–	1	–	–	–
51	3	–	–	–	–	–
52	2	–	–	–	–	–
53	2	–	–	–	–	–
54	1	–	–	–	–	–
55	1	–	–	–	–	–

Intervening Opportunities Model

Several methods of calibration of the intervening opportunities model were tried for the 1948 Washington area. The best procedures and the final calibration parameters were incorporated into this study. The several methods of calibration and the resulting findings are documented elsewhere (12). The method of calibration and forecasting

of the model examined here are very close to those used previously in Chicago and elsewhere, with the exception that procedures were developed to insure that the model would both send and attract approximately the correct number of trips for each zone in the study area. Without these adjustments only 84 percent of total trips were distributed and trips to the CBD were overestimated by 20 percent.

Trip ends were stratified into long residential, long nonresidential, and short. Both long and short L values were developed through an iterative process to insure that when the final L values were applied to the appropriate trip ends, a satisfactory average trip length, trip length frequency curve, and number of intrazonal trips would be obtained for the total trips (all three trip types combined).

River crossing time impedances were shown to be needed for the intervening opportunities model, in the same manner as for the gravity model. The additional bridge crossing time required for the 1948 intervening opportunities model calibration was 5 min. The use of procedures developed in the gravity model research to forecast the impedance for the intervening opportunity model estimated the impedance required in 1955 at 8 min. Although the use of this 8-min forecasted time penalty did materially improve model accuracy, estimated Potomac River crossings were still approximately 16 percent high. The differing forecasted values of the gravity model and the intervening opportunity model impedances were caused by the differing trip purpose categories which required different weighting of peak hour trips. The basic structure of the models also necessitated the use of differing 1948 impedances.

An increase in the total number of trip destinations or opportunities requires that the probability that any one of these destinations will be acceptable to any given origin be reduced. Therefore, because of the growth in total and intrazonal trips in the study area, the 1948 L (probability) value required reduction for use in 1955. The final 1948 long and short L values are 2.50×10^{-6} and 13.00×10^{-6}, respectively. They were reduced to 1.65×10^{-6} and 10.80×10^{-6} for the 1955 forecasts. These adjustments were made on the basis of the growth in total destinations between 1948 and 1955 (12).

Competing Opportunity Model

This model proved to be very difficult to calibrate. Because no systematic calibration procedures were available, it was necessary to try many alternate approaches for obtaining a simulated trip distribution with the same trip length characteristics as the 1948 Washington survey data. Initially, equal time bands were tried for work trips with little success (Fig. 2). Next, varying width time bands were utilized and the results became more meaningful. It appears that the best simulation for work trips was obtained when the first time band incorporated the majority of the opportunities in the study area. This broad band was followed by equal 2-min bands. Even with this approach, however, it was not possible to obtain a trip length frequency distribution approaching the O-D trip length frequency. As shown in Figure 3, the curve A peaks are much too high, whereas curve B, similar in shape to the O-D curve, is offset approximately 4 min to the right. No grouping of time bands was found that would fit the O-D curve.

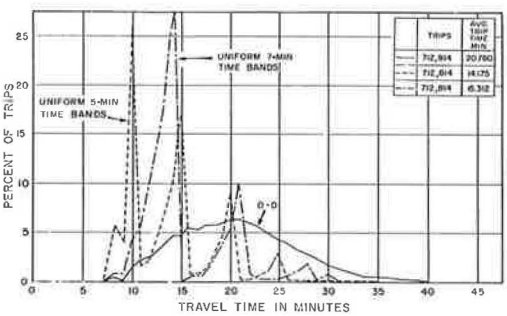

Figure 2. Comparison of trip length distribution (O-D vs competing opportunities model, uniform time bands), work trips, Washington, D. C., 1948.

The calibration of this model in the PJ area involved a district rather than zonal analysis. This, in effect, restructured the grouping of opportunities by greatly increasing the number of intrazonal trips. To date a calibration at the zonal level has not been attempted at PJ. For purposes of the subject research it was felt that the model would have to prove operational at the zonal level to be of universal

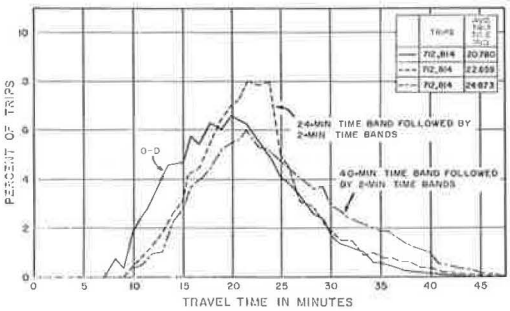

Figure 3. Comparison of trip length distribution (O-D vs competing opportunities model, variable time bands), work trips, Washington, D. C., 1948.

value. District analysis was not attempted as a part of the subject research. The only other difference from the PJ application involved the measure of spatial separation.

Because of the grossness of the measure, particularly with respect to the first opportunity band, where all trips in a ±20-min time band would be treated equally, the use of travel time rather than travel costs as the measure of spatial separation appears justified.

ANALYTICAL TESTS

The analytical tests, when viewed as a group, show not only measures of accuracy of the various procedures but also yield insight into the theoretical differences underlying the techniques. Do urban residents maintain a continuum of travel patterns over time modified only by the growth of the area as reflected in the Fratar procedure? Or, when considering making a trip, do they follow gravitational concepts weighting all attractors in direct proportion to the size of the attractors and in inverse proportion to the spatial separation as measured by the travel time between the zones? Or can travel patterns be best explained by opportunity concepts in the intervening opportunity model which assumes that people do not consider time directly, but rather consider opportunities in sequence by travel time and proceed on to any specific opportunity only after having considered and rejected all closer opportunities. Or does a person consider all opportunities in rather broad time or cost bands with all opportunities in a given band having an equal probability of acceptance as in the competing opportunity model.

One can be sure that people as social beings do not order their lives according to strict physical or mathematical laws and that no single model could ever be expected perfectly to match reality. However, one should expect that certain "theories" will be more explanative than others. With this in mind, the following tests should then be viewed in several lights. Is the particular procedure rational? Is the application simple enough that the procedure may be applied by urban planning studies lacking the experience in the procedure gained by research or earlier applications? Does the specific procedure fit the urban area to be studied; for example, are there local conditions such as relatively slow or rapid growth, inherent socioeconomic trip linkages, and large analysis units that might make one or more of the procedures more applicable?

Certain underlying differences in the procedures might best be described at this time. One of the most relevant differences is the weight placed on the role of travel time as an influence on trip distribution.

The Fratar procedures expand the existing travel patterns by considering growth in each portion of the study area without any specific consideration of the transportation network. If changes in the travel time between zones are sufficient to bring about change in travel patterns in the forecast year, the Fratar or any other growth factor technique would not reflect this.

However, each of the interarea travel formulas considered (gravity, intervening opportunity, and competing opportunity) uses time separation as a key variable. Thus, changes in the transportation system and the concomitant changes in accessibility between certain portions of the study area are directly reflected in the models.

The gravity model uses a travel time factor for each 1-min increment and, therefore, makes the most explicit use of absolute travel time of any of the procedures. These travel time factors are adjusted in the calibration process until there is close agreement between the estimated trip length frequency curve and the actual curve at

all increments of travel time. These factors, or relative weights of making trips of certain lengths, are then assumed to remain constant over the forecast period.

In contrast to the gravity model, the intervening opportunities model does not make such explicit use of absolute travel time. Travel time is used instead to rank all possible destination zones from a particular origin zone. This ranking then is used to determine the number of intervening opportunities, i.e., the number of destinations already considered before a particular destination zone is considered. Changes in the transportation system and accessibility between zones over the forecast period are thus reflected in the forecasting model. Two probability factors generally described as the long and short L values are used in conjunction with the intervening opportunities model to determine trip interchanges between zone pairs.

The procedure of ranking used in the intervening opportunity model does bring about certain situations unique to this model. Consider a small community on the fringe of the study area 5-min distance from the nearest developed area. From zones in the center of the study area, the intervening opportunity model would consider all opportunities in this fringe community immediately after considering the opportunities in the nearest developed area. In effect, the 5-min separation would be ignored. The gravity model would have considered the 5 min and thus decreased the possibility of a trip crossing the gap.

The competing opportunity model is somewhat unique, approaching the gravity model if small time bands are used and tending to ignore spatial separation when large time bands are used.

In evaluating and comparing the results of the following tests, consideration should be given to the formulation and parameter makeup of each of the procedures. The amount of the actual O-D data used for the base calibration and the number of parameters requiring forecasting are important in weighing the results of one model against others. The Fratar procedure uses all of the base year travel data from the home interview survey. The travel models, however, all require less O-D data than the Fratar. However, the amount of data used and the number of parameters used to represent these data vary to a considerable degree between the travel models tested.

Trip Length Frequency Comparison

Base Year.—Comparisons of the final calibrated model trip length frequency curves to actual trip length frequency curves for the gravity model, the intervening opportunities model, and the competing opportunities model are given in Figures 4 through 6. Each of these plots is shown on a slightly different basis due to the manner in which the research was carried out. However, each is compatible with the survey data with which it is compared.

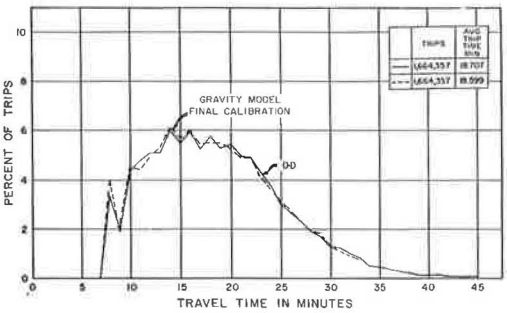

Figure 4. Comparison of trip length distribution (O-D vs gravity model, final calibration), total purpose trips, Washington, D. C., 1948.

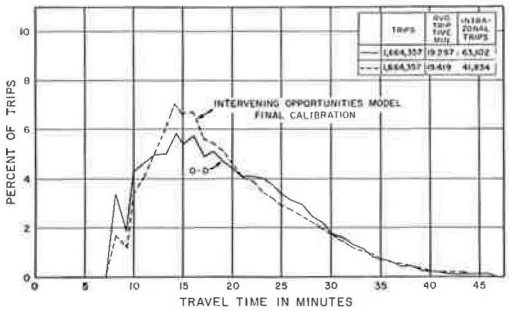

Figure 5. Comparison of trip length distribution (O-D vs intervening opportunities model, final calibration), total purpose trips, Washington, D. C., 1948.

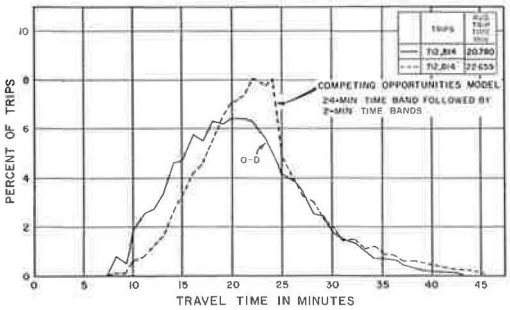

Figure 6. Comparison of trip length distribution (O-D vs competing opportunities model, best calibration), work trips, Washington, D. C., 1948.

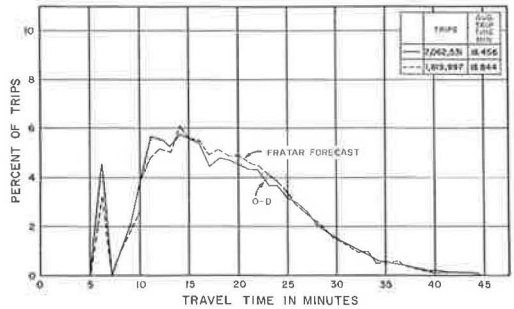

Figure 7. Comparison of trip length distribution (O-D vs Fratar forecast, total trips), Washington, D. C., 1955.

The curves in Figure 6 for the competing opportunities model are for work trips only. Due to calibration problems, a full analysis of this procedure could not be made. The information in Figure 6 was selected as the best calibration achieved with this procedure.

As expected, due to the refined degree of adjustment during the calibration phase, the gravity model shows the best agreement through most portions of the trip length frequency curves. Both the gravity and intervening opportunities models show good duplication of the total hours of travel in that both models agree with the appropriate average trip length.

Even though the two curves in Figure 6 for the competing opportunity model show some agreement, no rational method could be found to adjust toward a more satisfactory model.

Forecast Year. —The trip length frequency curves from the travel patterns as estimated by each of the procedures are compared to the appropriate O-D information in Figures 7 through 9. No forecast was made for the competing opportunities model and, therefore, no information is included for this model.

The Fratar procedures provided a good duplication of average trip length for 1955 as shown in Figure 7, even though approximately 195,000 trips out of the total available of 2,012,947 trips were not distributed because of zero trip ends for certain purposes in particular zones in 1948. The average trip length of the expanded patterns for 1955 of 18.8 min compares favorably with that of 18.5 min from the surveyed information.

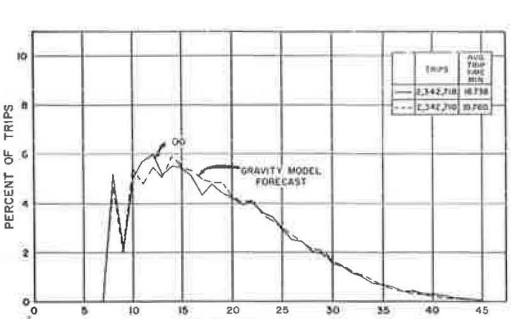

Figure 8. Comparison of trip length distribution (O-D vs gravity model forecast), total purpose trips, Washington, D. C., 1955.

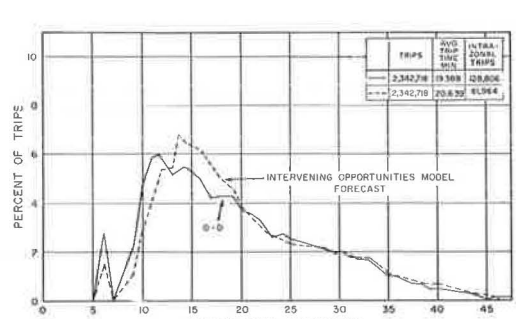

Figure 9. Comparison of trip length distribution (O-D vs intervening opportunities model forecast), total purpose trips, Washington, D. C., 1955.

Travel patterns forecast with the gravity model also provide an extremely good duplication of the average trip length as well as close agreement with the trip length frequency curve as shown in Figure 8. The average travel time for the forecast gravity model results of 18.8 min compares quite well with 18.7 min for the surveyed data.

The intervening opportunities model forecast is shown in Figure 9. The average travel time (driving time plus terminal times) of 20.6 compares with the actual of 19.4. These figures include the use of a river impedance.

River Crossings

The tests of estimated river crossings made on the various model results were developed because definite bias in the simulated trip distributions of two of the models became apparent during the calibration of the models. Both the gravity model and the intervening opportunities model required the use of time penalties on the Potomac River in the base year and in the forecast year. Different impedances were required for the two models. The gravity model research was completed first and procedures to forecast these time penalties were developed at that time. These procedures, when applied during the intervening opportunities research, reduced the error substantially in the forecast year, but not completely. The penalties required in the gravity and intervening opportunities models were different and the fact that different methods were required to forecast the time penalties is likely related to the different manner in which time is used by each. Of course, the effect of the impedance to free travel in the form of the Potomac River bridges was present in the 1948 surveyed trip crossings which were expanded to 1955 by the Fratar procedures. Table 2 gives the relative accuracies of river crossing estimates for the Potomac and Anacostia Rivers for each of the models for both the calibration and forecasting phases. The effect of the use of time impedances for the gravity and intervening opportunities model is included.

Movements by Corridor to and from CBD

This test was developed to isolate any geographical bias present in model results. The incorporation and need for adjustment for geographical bias has been shown for the gravity model through the use of K factors. No such adjustments were found to be necessary in the Fratar or intervening opportunity procedures. Tables 3 and 4 give information relating the estimated patterns to and from the CBD by corridor to the actual patterns from the O-D survey for 1948 and 1955, respectively. Factors to adjust for geographical bias have been used for the work trips to the CBD in the gravity model.

TABLE 2

COMPARISON OF TOTAL TRIPS CROSSING POTOMAC AND ANACOSTIA RIVERS (SURVEY VS MODEL) FOR THE VARIOUS DISTRIBUTION PROCEDURES, WASHINGTON, D. C.

Year	Data Source	Potomac River		Anacostia River	
		Total Trips[a]	Diff.[b](%)	Total Trips[a]	Diff.[b](%)
1948	O-D survey	196,255	N.A.	183,696	N.A.
	Gravity model	202,237	+ 3.05	184,188	+ 0.27
	Intervening opportunities	188,134	- 4.14	193,398	+ 5.28
1955	O-D survey	246,268	N.A.	287,452	N.A.
	Fratar[c]	279,055	+13.31	281,881	- 1.94
	Gravity model	230,949	- 6.22	296,830	+ 3.26
	Intervening opportunities	287,447	+16.72	318,269	+10.72

[a]In thousands.
[b]Survey data used as base.
[c]Adjusted to common O-D survey base.

TABLE 3

COMPARATIVE ANALYSIS OF CALIBRATION ACCURACY OF VARIOUS MATHEMATICAL MODELS IN DUPLICATING HOME INTERVIEW DATA, WASHINGTON, D. C., 1948

Movements Between Zero Sector and Sector No.	Survey Trips	Gravity Model[a]		Intervening Opportunities[b]	
		Trips	Diff. (%)	Trips	Diff. (%)
0	134,951	141,105	+4.56	142,595	+ 5.66
1	44,771	46,110	+2.99	45,407	+ 1.42
2	72,206	66,494	-7.91	59,710	-17.31
3	195,114	181,860	-6.79	184,815	- 5.28
4	93,542	92,027	1.62	94,923	+ 1.48
5	62,484	58,550	-6.30	64,999	+ 4.02
6	80,275	83,684	+4.25	91,174	+13.58
7	67,835	68,898	+1.57	58,299	-14.06
8	42,833	43,505	+1.57	36,297	-15.26
Total	794,011	782,233	-1.48	778,219	- 1.99

[a]Includes K factors.
[b]Does not include K factors.

TABLE 4

COMPARATIVE ANALYSIS OF FORECASTING ACCURACY OF VARIOUS MATHEMATICAL MODELS IN DUPLICATING HOME INTERVIEW DATA, WASHINGTON, D. C., 1955

Movements Between Zero Sector and Sector No.	Survey Trips	Gravity Model[a]		Intervening Opportunities[b]		Fratar		
		Trips	Diff. (%)	Trips	Diff. (%)	O-D Survey[c]	Trips	Diff. (%)
0	112,471	123,243	+ 9.58	119,613	+ 6.35	112,007	113,972	+ 1.75
1	52,391	53,830	+ 2.75	53,680	+ 2.46	52,213	47,485	- 9.06
2	100,710	87,896	-12.72	82,498	-18.08	88,865	79,388	-10.66
3	197,167	182,558	- 7.41	187,026	- 5.14	191,362	181,933	- 4.93
4	102,384	105,943	+ 3.48	108,668	+ 6.14	97,906	98,860	+ 0.97
5	64,788	62,019	- 4.27	70,485	+ 8.79	64,623	63,348	- 1.97
6	95,461	100,579	+ 5.36	107,037	+12.13	92,087	84,960	- 7.74
7	69,221	64,911	- 6.23	66,541	- 3.87	62,125	62,161	+ 0.06
8	57,847	54,652	- 5.52	53,258	- 7.93	51,154	49,653	- 2.93
Total	852,440	835,631	- 1.97	848,806	- 0.43	812,342	781,760	- 3.76

[a]Includes K factors.
[b]Does not include K factors.
[c]Contains information from 362 zones only as used in Fratar analysis.

Statistical Analysis of Assigned Trips

As a common measure of the accuracy of each of the model distributions, the total person trip output for the calibration and forecast runs of each model were assigned to a spider network and compared by link with the O-D survey assigned to the same network. All trips are defined as going from origin zone to destination zone. To achieve uniformity, the gravity model trips had to be redefined. Standard gravity model procedures were used to adjust the production to attraction trip tables to true origin to destination trip tables for directional assignments. To do this, a 50/50 split was assumed of all production to attraction zone-to-zone transfers to get back to true origin to destination tables. For example, in determining the number of trip productions and trip attractions in any zone, the home end of any home-based trip is always called the production and the nonhome end the attraction. All trips with the general purpose "work" would be considered as going from home to work, the work to home

portions, in effect, being reversed. After the model simulates trips by this definition, again assuming work trips, all home-based trips are then converted to directional volumes by assuming 50 percent are from work to home trips and 50 percent are the reverse.

The comparisons are of directional link volumes assigned to a spider network, with differences recorded by volume group. Statistical analyses were made of these comparisons with the RMSE calculated for each model for each O-D volume group. The results of these analyses for the calibration or base year gravity and intervening opportunities models are shown in Figure 10. This figure illustrates the accuracy attained in the final research 1948 calibrations. Each model output includes the river time penalties. The gravity model used K factors to adjust the work trips to the CBD.

Next, shown on Figure 11, are the comparisons of RMSE by volume group for the forecasted travel patterns for each of the models. The Fratar output is compared with O-D from only the 362 zones where compatability for 1948 and 1955 could be achieved.

The results indicate that the gravity model forecasts compare best with the O-D assignments in most volume groups up to 1,500 trips with the Fratar procedure and the intervening opportunity model showing slightly better accuracy in the very highest volume groups. River impedances were used with both the gravity and intervening opportunities models. However, the opportunities model could not be adjusted as closely as the gravity model to conform with actual river crossings.

The results of the Fratar as assigned and compared are biased in that there are 195,000 trips which the Fratar, through one cause or another, could not expand. It might be expected that the Fratar procedures would produce results which have increasing error as the forecast period lengthens and land-use changes increase in significance. But, even over such a relatively short time period as 7 years, the Fratar results are not significantly better than the model results.

SUMMARY AND ANALYSIS

An attempt was made to test on a common basis the four available procedures to distribute and forecast urban travel patterns. When dealing with large masses of data with a series of formulations requiring different definitions and calibration procedures, variations in the base conditions are bound to occur. These variations in the test conditions did not seriously detract from the analysis of the relative merits and weakness of each of the procedures.

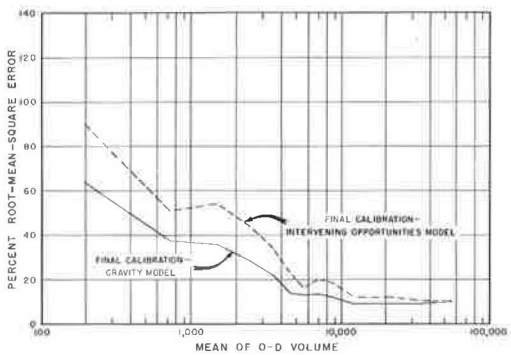

Figure 10. Comparison of RMSE (O-D vs model) by volume group, directionally assigned volumes on spider network, total purpose trips, Washington, D. C., 1948.

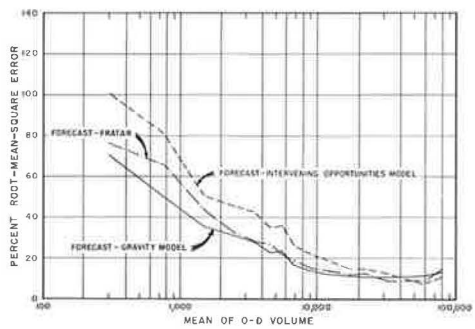

Figure 11. Comparison of RMSE (O-D vs model) by volume group, directionally assigned volumes on spider network, total purpose trips, Washington, D. C., 1955.

Fratar

This procedure, requiring no calibration, performed essentially as expected. Six trip purposes—home, work, shop, social-recreation, school, and miscellaneous—were utilized. Over the 7-year period, the Fratar procedure demonstrated a high level of accuracy in all analytical tests. It was not, however, tested specifically in one most critical area—the correct expansion of trips from zones changing from essentially undeveloped rural land uses to full urban development. Most zones in this class had to be eliminated from the analysis because of incompatibility of 1948 and 1955 zone boundaries. The model by its nature does not require any type of adjustment due to the already built-in socioeconomic trip linkages in the travel patterns expanded. It was surprising, therefore, that the Fratar procedure was only moderately better in estimating trips to and from each of the eight sectors to the CBD than the gravity and intervening opportunity models. This particular test is the most sensitive indicator of socioeconomic bias.

The multipurpose Fratar, although having distinct advantages in the proper expansion of trips by purpose, also has certain drawbacks when compared with a single-purpose Fratar. By expanding the number of trip categories to six, the possibility of zero volumes in the trip tables increases as the square of the number of trip categories. In the Washington area, 242,000 trips were "lost" in the expansion because for certain zones and trip categories no trips were made in 1948, but in 1955 in the same zones and for the same trip purposes 242,000 trips were made. This amounted to over 10 percent of the 1955 trips. Had it been possible to include all fringe area zones in the analysis, the problem would have been much more serious. Again, the most serious problems are in the urban fringe areas where, for example, shopping centers and golf courses are developed on farm or vacant land. Correct trip distributions cannot be achieved in these instances unless base year trips are first synthesized for these areas with an interarea travel formula and then artificially superimposed on the base year travel pattern before the Fratar expansion.

Gravity Model

This model proved adequate in most respects. It is particularly strong in the calibration phases, that is, in having an orderly procedure allowing for fine adjustments in the travel time factors and the direct adjustment for socioeconomic or geographic bias. The travel time factors have been shown to be stable over the 7-year period.

One problem inherent in the procedure is the necessity found for socioeconomic adjustment factors. Thirty-four factors ranging from 2.23 to 0.29 were utilized. Developing relationships between these factors and characteristics of the districts of residence or attraction can present problems in forecasting these characteristics. In Washington, the factors used to adjust work trips to the CBD were highly correlated with the average incomes of the residence zones. Another problem is the forecast of "river impedances." These topographical impedances most likely related to historical deficiencies of capacity, including the complete lack of facilities, can be projected on the basis of present and projected volume-capacity ratios. River barriers are a problem in that they require a detailed, though not complex, analysis and because they relate to such a critical area in terms of the analysis of future transportation system needs.

Intervening Opportunity Model

This model, although not previously utilized operationally by the researchers, performed very well. Several methods of calibration were tried and after selection of the best procedures, the model was calibrated with little difficulty. No socioeconomic adjustment factors were necessary for Washington, D.C.—a very strong point in this model's favor.

The trip purposes are defined in such a manner that directional trips are maintained at all times. Fairly high river impedances were required and, as with the gravity

model, their projection, although requiring detailed analysis, is straightforward. Even with the projected river impedance of 8 min, the 1955 model overestimated Potomac River crossings to a considerable extent. Examination of the results and the skim trees indicated that very little further improvement could have been made even with a higher impedance value.

One drawback to this model is the fact that the L values change with time. In this analysis, the change in L value was forecast as a function of the change in the number of trips. Refinement in methods of forecasting these L values will require refinements in methods to project future trip length. Such a projection was not attempted in the application reported here. Although considerable research is currently under way on trip length trends, they are presently the subject of much discussion. For example the Institute for Urban Studies at the University of Pennsylvania, in cooperation with the Urban Planning Division of the U.S. Bureau of Public Roads, has recently completed one research project and is currently undertaking a second on these trends.

An additional point for consideration is the fact that in the calibration phase, the intervening opportunity models for the individual trip purposes do not necessarily reproduce the trip length frequency characteristics of the corresponding O-D trips. When the individual purposes are summed to a total purpose, the trip length frequency characteristics are good because of compensating deviations in the individual trip purposes.

The explanation given for this situation is that the opportunity model does not consider trip purposes per se, but rather uses the survey trip purposes as a convenient way of grouping trip ends to apply individual L values. There may be problems when desiring to distribute trips by purpose, for example, when performing a modal split analysis. The L value derived to treat a single purpose would differ from the L value used if the trip purpose were to be combined with others to form a total trip distribution. In essence, the trip distributions by purpose are only meaningful when summed to a total trip purpose distribution.

Competing Opportunity Model

It was disappointing that this model could not be calibrated with the Washington, D.C., O-D data and on a zonal basis. Time bands of uniform width were not at all applicable, and no simple procedure could be derived for selecting nonuniform time bands. Many different combinations of time bands were tried before a set was obtained which even approached giving correct trip length characteristics.

When the various trial-and-error approaches of arriving at appropriate time bands proved futile, a theoretical approach to the problem was attempted. The required type of probability curve for selected Washington, D.C., zones was derived as a plot of the percent of trips remaining to be distributed vs the accumulated available opportunities. Working within the framework of the model, it was not possible to duplicate this probability curve derived from the selected zonal O-D data. Figure 12 illustrates the degree to which two different time band groupings approach the actual O-D probability groupings.

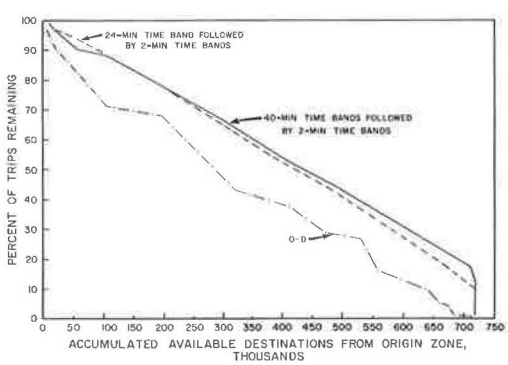

Figure 12. Distribution of 10,687 home-based work trips from zone 48, Washington, D.C., 1948, competing opportunities model.

SUMMARY

The overall accuracy of the gravity model proved to be slightly better than the accuracy of the intervening opportunity model in base year simulation and in forecasting ability. This fact must, however, be considered in light of the need for and use of socioeconomic adjustment factors in the gravity model for the work trip

calibration. In effect, more parameters were used in the gravity model calibration.

With the use of these adjustment factors, the gravity model exhibited less error than the intervening opportunities model when trips by sector to the CBD were examined. However, the opportunity model was better than the unadjusted gravity model. It is not clear whether this is due to the conceptual basis of the models or to the trip purpose stratifications used.

Due to the fewer parameters used, the intervening opportunities model proved slightly less difficult to calibrate. However, adjustments necessary in future L values reduce this advantage in making the forecasts. Considering all factors, the gravity and intervening opportunity models proved of about equal reliability and utility.

The Fratar growth factor procedure demonstrated a good ability to expand trips correctly for stable areas but showed significant weaknesses in areas undergoing land-use changes. Even by eliminating zones of completely new growth from the O-D test data, approximately 10 percent of the total 1955 trips were lost through the expansion. This 10 percent amounted to a much more significant portion of the increase in trips between 1948 and 1955. The concentration of error in areas experiencing growth in trips points up the need for supplemental procedures to provide a base year synthesized trip pattern in such areas. The magnitude of this problem, when examined in the light of the favorable results attained with the gravity and intervening opportunity models, indicates that the use of a travel model provides a more direct and efficient approach to trip distribution for growing urban areas.

CONCLUSIONS

1. The gravity model and the intervening opportunity model proved of about equal reliability and utility in simulating the 1948 and 1955 trip distribution for Washington, D. C.
2. The Fratar growth factor procedure demonstrated a good ability to expand trips correctly for stable areas but showed significant weaknesses in areas undergoing land-use changes.
3. It was not possible to calibrate adequately the competing opportunities model for use in determining trip distributions between areas as small as the traffic zones used in Washington, D. C. Its use in exploratory work in the PJ study at the district level (groupings of zones) offered promise which this particular research study has not been able to reproduce in Washington, D. C.

FUTURE RESEARCH

Several areas for future research were uncovered when the models were analyzed on a common basis. The use of different trip purpose categories as input to the gravity model trip distribution procedure should be explored as a means of eliminating the socioeconomic adjustment factors. As a first attempt, a five-purpose true O-D purpose definition model consisting of (a) home to work trips, (b) work to home trips, (c) home to other trips, (d) other to home trips, and (e) nonhome-based trips should be tried.

Research is needed to develop more sophisticated procedures to adjust the base year L values to the future year for the intervening opportunities model. Certainly, better information on future trip length in terms of either miles or minutes would be very helpful in this regard. Also, some work is required to test the effect of the trip universe used on accuracy and the need to make adjustments to force all the trips to be sent. For instance, the inclusion or exclusion of the external trip ends creates a slightly different set of intervening opportunities for any given origin zone.

Additional research is also needed to examine the impedance effect of physical or topographical features on travel. More insight into basic causes of the impedance is essential to the development of comprehensive techniques for projecting the impedance.

The advantages of the purpose Fratar—that is, the more direct consideration of land-use changes—must be investigated in view of the resulting highly significant loss in expanded trips.

Finally, research is required to develop calibration procedures for the competing opportunities model.

REFERENCES

1. Fratar, T. J. Forecasting Distribution of Interzonal Vehicular Trips by Successive Approximations. Highway Research Board Proc., Vol. 33, pp. 376-384, 1954.
2. Voorhees, A. M. A General Theory of Traffic Movement. Proc. Inst. of Traffic Eng., pp. 45-56, 1955.
3. Final Report, Chicago Area Transportation Study, Vol. 2.
4. Tomazinis, Anthony R. A New Method of Trip Distribution in an Urban Area. Highway Research Board Bull. 347, pp. 77-99, 1962.
5. Tomazinis, Anthony R. The Development of the Penn-Jersey Transportation Study Trip Distribution Model. Presented to Highway Research Board, Jan. 1964.
6. Silver, J. Trends in Travel to the Central Business District by Residents of the Washington, D. C., Metropolitan Area, 1948 and 1955. Highway Research Board Bull. 224, pp. 1-40, 1959; Public Roads, Vol. 30, pp. 153-176, April 1959.
7. Brokke, G. E., and Mertz, W. L. Evaluating Trip Forecasting Methods with an Electronic Computer. Public Roads, Vol. 30, pp. 77-87, Oct. 1958.
8. Hansen, W. G. Evaluation of Gravity Model Trip Distribution Procedures. Highway Research Board Bull. 347, pp. 67-76, 1962.
9. Bouchard, R. J., and Pyers, C. E. Use of Gravity Model for Describing Urban Travel—An Analysis and Critique. Record No. 88, pp. 1-43, 1965.
10. Calibrating and Testing a Gravity Model for Any Size Urban Area. U.S. Bureau of Public Roads, Washington, D. C., 1963.
11. Calibrating and Testing a Gravity Model with a Small Computer. U.S. Bureau of Public Roads, Washington, D. C., Oct. 1963.
12. Pyers, C. E. Evaluation of Intervening Opportunities Trip Distribution Model. Highway Research Record 114, 1966.

Discussion

DONALD E. CLEVELAND, Department of Civil Engineering, University of Virginia—
The need for a comparative study of the relative and actual effectiveness of the principal techniques of trip distribution has been apparent for some time. Those who have had the opportunity to study this paper will generally agree that the topic is timely, the results are of interest, and the conclusions are justified from the study and useful to the profession.

Unfortunately, the authors were able to study only one city, but this city, Washington, D. C., experienced a growth not significantly different from that expected in the planning period in many cities. This makes their findings of particular interest.

Trip distribution models attempt to explain rationally the movement of persons from one place to another, a phenomenon that depends on sociological, psychological, and economic effects and interactions. It could be asserted that efforts to develop such a model are bound to be unsuccessful or lead to brittle formulations requiring complex manipulations to reproduce reasonable patterns. The pragmatic practitioner is interested in successful models. The techniques tested by the authors include those classed as successful. The detailed methods used are of interest to the skeptic.

What characteristics should a trip distribution model possess? It must first be remembered that trip distribution does not carry the entire burden in developing reasonable estimates of transportation network usage. However, trip distribution models should respond satisfactorily to the following types of changes characteristic of urban

areas: (a) varying transportation networks, (b) changing locations and magnitudes of activities within the citywide framework, and (c) changing determinants of individual trip-making. It is believed that an effective trip distribution model should have a simple structure supported by a data management capability adequate to estimate the necessary constants or parameters. The parameters of a trip distribution model should be developable with a minimum amount of data, forecastable in the sense that future estimates of their values follow naturally from the studies and forecasting processes of urban transportation planning, reponsible in producing satisfactory results for known conditions, and minimal in the sense that effective models do not usually require elaborate parameter sets.

The authors have clearly described in some detail their steps in "calibrating" the models. Others faced with the calibrating task will benefit from studying the art as described in the paper. The careful reader may have questions concerning some of the procedures used and results obtained. It would be helpful to many if comments can be made on the following few questions occurring to the reader.

1. Does the Washington, D.C., metropolitan area itself have any characteristics that make it unusually good or poor as a study city for the comparison of trip distribution techniques? Has the long usage of this area in trip distribution research possibly biased the network characteristics? Would the authors speculate on the general validity of their conclusion regarding the relative effectiveness of the gravity and intervening opportunity models?
2. The Fratar technique cannot respond to the unbalanced improvement of access resulting from most transportation improvement programs. What results were obtained for stable areas where accessibility was improved?
3. The introduction of network impedances at river crossings is unsatisfying. Do the authors feel that the assignment of 24-hr person trips to an off-peak automobile travel time network could have contributed to the need for this additional impedance? How should the need for such adjustments be determined and how should this activity be incorporated in the formulation of the general model?
4. Do the authors believe that the differences among the models tested would have influenced a transportation facility planning decision?
5. Unsatisfactory trip length distributions were obtained in the calibration of the competing opportunities model. Could these results have cancelled out in the total person trip distribution as they apparently did to a lesser extent in the application of the intervening opportunities model?

As a further comment, innovations have been and are being made in the application of each of the models tested. Each of these changes resulted from the necessity to cope with unsatisfactory behavior of the parent model or a desire to strengthen the basis for utilization of the model. The Fratar technique has had reasonable and reproducible techniques developed to improve predictions to and from new areas. The intervening opportunity model as used has undergone changes at the Chicago and Pittsburgh studies. There are indications that the gravity model may become more flexible as significant trip-making determinants are more completely understood. The competing opportunity model may now respond better to varying city and analysis zone sizes based on a recently developed calibration procedure.

Where does the profession stand in the development of trip distribution models? There have been several generations of observe-formulate-predict-test and this activity continues. We know that we are doing better than we were 5 years ago. We may even be doing well enough. I have seen no analysis that tells us this. Relations among trip generation, distribution, and assignment should be sought. Meanwhile, sharpening existing models proceeds and efforts should be devoted to seeking the elusive and simple law which will describe this aspect of traffic behavior.

ROBERT S. VOGT, Vogt, Ivers and Associates, Cincinnati, Ohio—Any discussion of existing trip distribution procedures should consider the direction of current research and technology, the limitations of present techniques, and the need for additional technical capability in the future. After slightly more than a decade of concerted effort in the science of urban transportation planning, we can take some pride in the results which have been accomplished. However, there seems to be an air of finality about the trip distribution techniques in current use. Such confidence should be avoided.

In the paper under discussion, four procedures are described. The gravity model procedure as currently used places heavy weight on the structure of the present community, yet we are well aware that the same model parameters do not apply with equal reliability in all communities. The Fratar forecasting procedure relies so heavily on the existing structure that future development patterns seemingly can never be predicted without special techniques. Although the intervening opportunity model appears to account well for the changing structure of the community without overweighting the effect of the existing structure, it is evident that this model, too, does not properly nor consistently distribute trips between subareas. This inconsistency is particularly evident for trip patterns between the main urban mass and separate satellite communities in the region.

In the use of the gravity model procedures, there is a strong tendency to object to the iterative procedure of calculating the travel time factors ($F_{t_{i-j}}$). Admittedly, it assists in improving the calibration but it tends to say that the community in many respects is homogeneous insofar as trip distribution is concerned. To modify any lack of homogeneity which seems apparent to the analyst, K factors are applied, travel time barriers are added, and terminal times are varied. Although this can be done with some finesse with experience, the procedure lacks the identification of community functions and structure which define different urban areas. Why is it logical to use the same travel time factors for 1964 and 1990 if it is not logical to use the same travel time factors and terminal times in Cleveland as in Baltimore for the same or different projection years?

Actually, the urban area is a changing and diverse organism, subject to subregional variations which can only be determined by subregional analysis. If we consider the difference between the small community (less than 50,000 population) and the large metropolitan areas, it is evident that the same travel time factors do not apply. Why do we consider it logical to hold the travel time factors in the analysis of the larger communities through time?

A more realistic view is to develop interarea travel formulas which are sensitive to changing social value factors and, therefore, to the changing structure of the urban area. Using this philosophy it is assumed that trip end generation is a function of the characteristics and affluence of the population and may be specifically calculated, given specific data concerning those characteristics. (To a large extent this is current practice). Distribution then is assumed to be a function of the characteristics of people where those characteristics are based on evaluation and analysis of existing travel patterns. Essentially, this view theorizes that the gravity model distribution technique or the intervening opportunity distribution technique are only mathematical procedures, either of which may provide a significant distribution process. The important aspects of trip distribution which would be recognized in this procedure include the following:

1. Some trip patterns can be more accurately predicted than others;
2. Trips once distributed reduce the trip end total at both the origin and destination so that the attraction function in the gravity model formula is constantly reduced until all trips are distributed; and
3. Trip patterns can be related to community characteristics so that changes in characteristics over time can be the basis for estimating future trip patterns.

The difference between these suggested criteria and current practices is the belief that trip patterns between some areas are much more stable than between others; therefore, they are easier to predict with reliability and should be distributed first.

An additional variation is that the suggested criteria assume that the travel time factors vary from zone to zone and from one time period to another for a similar zone. Finally, it is assumed that these change statements are predictable. These conclusions are based on considerable study of the results of interarea trip distribution in a number of communities of different size.

Often it seems that the gravity model distribution procedure raises nearly as many questions as it answers. Following are a few of the more apparent.

1. Why does it take travel time barriers at major river crossings to calibrate the model? Are they reasonable inclusions in trip distribution procedures?
2. How do we know whether a travel time barrier is more realistic than K factors in the calibration of certain trip patterns?
3. To what extent are terminal times a realistic function of trip distribution and to what extent are they used only as a means of calibration?

As is well known, K factors reduce the attraction of the destination zones so that the trips from all zones where K factors are applied are reduced or increased by a factor equal to the K factor. The application of travel time barriers between the same zones has a different effect. Since the travel time barrier is uniformly applied to every trip transfer which crosses the barrier, the effect on trip distribution is related to the length of the trip and the travel time factors ($F_{t_{i-j}}$) which are applied. The resulting effect is to impede the shorter trips more drastically than the longer trips. Increases in terminal times are similar to the travel time barrier in that the shorter trips are impeded to a greater extent than the longer trips. Using the curves shown in Figure 2 of the paper, it is interesting to consider the changes in trip values which occur under certain logical changes in travel time barrier times, terminal times and K factors. The original data between zone pairs can be assumed to be the following (using the travel time factors for nonhome-based trips):

Zone$_{1-2}$—travel time 20 min, original transfer 100 trips;
Zone$_{1-3}$—travel time 10 min, original transfer 100 trips;
Using K-5, the transfers become T_{1-2} = 50 trips, T_{1-3} = 50 trips;
Using travel time barrier of 5 min, $T_{1-2} = 100 \times 0.9/2.0 = 45.0$ trips, $T_{1-3} = 100 \times 6.0/17.5 = 34.4$ trips; and
Using travel time barrier of 10 min, $T_{1-2} = 100 \times 0.48/2.0 = 24.0$ trips, $T_{1-3} = 100 \times 2.0/17.5 = 11.4$ trips.

The number of trips computed is not important since that computation depends on the number of transfers so affected. The important consideration is the relative proportion of trips distributed in each case. Thus, in the gravity model development, numerous modifiers have varying effects on the final distribution. Until we clearly define how these modifiers should be applied, we do not have a "mature" procedure. Can we really reason with assurance that the adjustments are consistent? Can we justify their application to large blocks of zonal interchanges, as is current practice?

If this discussion has seemed critical, it is not meant to be. We have been consistent users of the gravity model and have used the results as the basis for many design recommendations. Although our concerns are based on the previous discussion, our confidence in the current procedure also has some factual basis. In Dayton, in 1957, a postcard O-D survey was synthesized by first accepting the trip ends from the survey and then distributing the trips between zone pairs using the gravity model. Purposes were established as follows:

Purpose 1—all trips with origin or destination in the CBD;
Purpose 2—all trips with an origin or destination in home zone; and
Purpose 3—all trips with neither origin nor destination in home zone.

Even with these minimum purpose descriptions and an exponential function ($x = 0.6$ for purpose 1, $x = 2.0$ for purpose 2, and $x = 2.2$ for purpose 3), a comparison of assignments gave the following results by volume groups:

Volume (2-999)—mean O-D volume (430), RMS (310);
Volume (1,000-1,999)—mean O-D volume (1,460), RMS (615);
Volume (2,000-3,999)—mean O-D volume (2,900), RMS (700);
Volume (4,000-5,999)—mean O-D volume (4,840), RMS (1,000); and
Volume (6,000-7,999)—mean O-D volume (6,880), RMS (1,250).

Since this model fit (Fig. 13) is at least equal to that shown in Figure 11 of the paper with considerably fewer purpose categories, one is inclined to question the need for the greater detail.

In Muncie, Ind., a basic external cordon survey with screenline interviewing (1957) was synthesized except that trip production by purpose (four purposes developed) was computed as the input to the gravity model. Purposes were established as follows on an O-D rather than production-attraction basis:

Purpose 1—total home-based auto trip ends with "work" as a purpose;
Purpose 2—total of all other home-based auto trip ends;
Purpose 3—total of all other nonhome-based auto trip ends; and
Purpose 4—total of commercial trip ends.

Again, with these minimum purpose descriptions and an exponential function (x = 2.5 purpose 1, x = 2.5 for purpose 2, x = 1.8 for purpose 3, and x = 2.0 for purpose 4), a comparison of assignments gave the following results by volume groups:

Volume (2-199)—mean O-D volume (88), RMS (88);
Volume (200-399)—mean O-D volume (282), RMS (142);
Volume (400-599)—mean O-D volume (500), RMS (186);
Volume (600-799)—mean O-D volume (707), RMS (155);
Volume (800-999)—mean O-D volume (908), RMS (265);

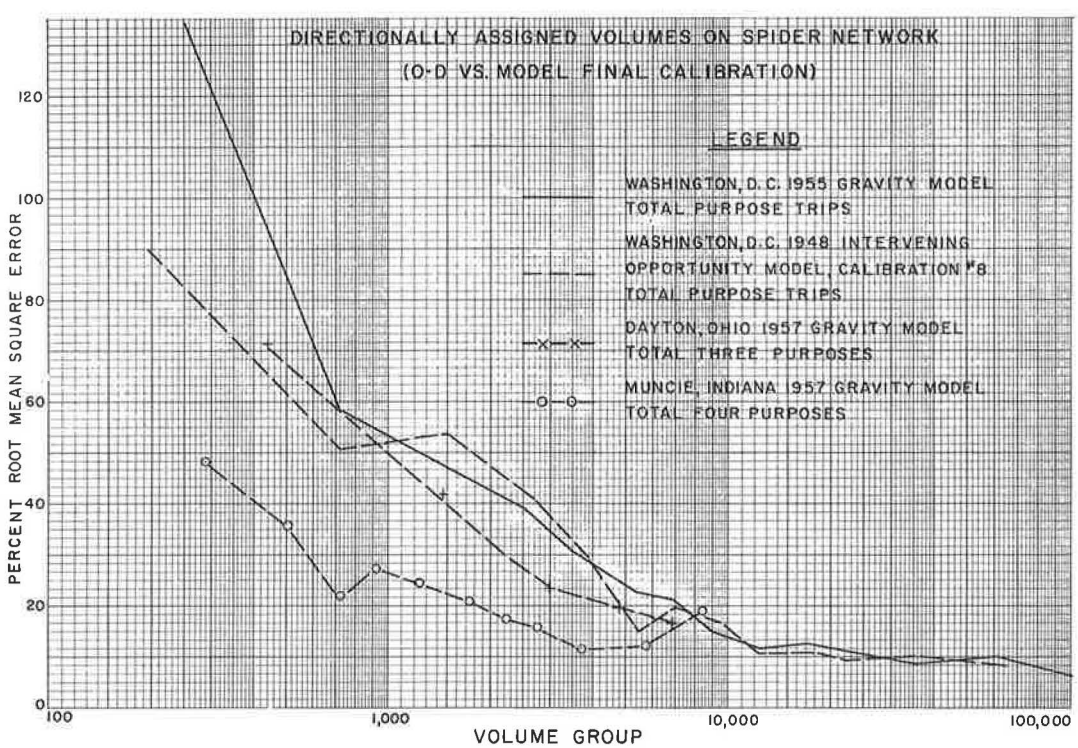

Figure 13. Comparison of root-mean-square error by volume groups.

Volume (1,000-1,499)—mean O-D volume (1,223), RMS (325);
Volume (1,500-1,999)—mean O-D volume (1,753), RMS (363);
Volume (2,000-2,499)—mean O-D volume (2,241), RMS (376);
Volume (2,500-2,999)—mean O-D volume (2,745), RMS (450);
Volume (3,000-4,999)—mean O-D volume (3,712), RMS (460);
Volume (5,000-6,999)—mean O-D volume (5,768), RMS (765); and
Volume (7,000-8,999)—mean O-D volume (8,171), RMS (1,560).

The results are again considerably better than those reflected in Figure 11, although in this case it must be noted that a small percentage (20 percent) of the total trips in the O-D values were synthesized to provide a complete trip matrix. The O-D survey procedure did not provide this information directly from the survey data.

To carry this discussion one step further, the same O-D survey was synthesized using a one-purpose model which was the sum of all purposes previously described. An exponential function (x = 2.5) was used. A comparison of assignments gave the following results by volume groups:

Volume (2-199)—mean O-D volume (88), RMS (78);
Volume (200-399)—mean O-D volume (282), RMS (118);
Volume (400-599)—mean O-D volume (500), RMS (169);
Volume (600-799)—mean O-D volume (707), RMS (148);
Volume (800-999)—mean O-D volume (908), RMS (242);
Volume (1,000-1,499)—mean O-D volume (1,223), RMS (305);
Volume (1,500-1,999)—mean O-D volume (1,753), RMS (354);
Volume (2,000-2,499)—mean O-D volume (2,241), RMS (354);
Volume (2,500-2,999)—mean O-D volume (2,745), RMS (422);
Volume (3,000-4,999)—mean O-D volume (3,712), RMS (438);
Volume (5,000-6,999)—mean O-D volume (5,768), RMS (874); and
Volume (7,000-8,999)—mean O-D volume (8,171), RMS (1,770).

Although these results are not quite as good as with the four-purpose model, the difference is so slight as to raise questions concerning the need for the additional detail.

The purpose of interarea travel formulas is to provide procedures which predict future travel. The fact that these methods will reproduce an existing O-D survey is only a first step in the process. Since our ability to reproduce the present is only fair and to produce the future is worse, more study of the many varied aspects of trip distribution is necessary.

G. E. BROKKE, Research Assistant, Urban Planning Division, U.S. Bureau of Public Roads—The task accomplished by the authors is one of considerable magnitude. Although it may appear that the data were fed into a computer and the results poured forth, there were, in reality, several dozen programs involved. Each of them has the possibility of introducing spurious results, and the constant checking and evaluations to guard against this eventually might have discouraged less tenacious and understanding authors.

The tests of the models are certainly objective and, in my opinion, the authors are equally objective. Yet there remain various acts of loving kindness and tender care that are perhaps somewhat unequally divided. For example, considerable experimentation was conducted to select appropriate "river barrier" factors and a set of 34 K factors ranging in value from 2.23 to 0.23 for the gravity model. Similar techniques were not tried with the Fratar or Chicago model, although in the case of the Fratar, it has been shown that the majority of the "lost" trips can be accommodated by aggregating zones into districts to obtain the interchange potential. To some extent this uneven care is probably due to the ability of the gravity model to accommodate hindsight and perhaps also to the deep understanding of the authors in the use of the gravity model.

The paper correctly states that the Fratar procedure is consistently low in the accumulation of trips on the spiderweb network. However, the discrepancy is not as large as might be supposed, as shown in Figure 14. The lower dashed line indicates the average error in each of the several traffic volume groups for the Fratar method, and the solid line indicates the average error for the gravity model. As a matter of fact, the consistency of the Fratar in underestimating is very close to the 10 percent mentioned in the paper. If the "lost" trips had been apportioned to the network in accordance with the assigned volumes, the results of the Fratar would have been measurably improved.

On the same graph, the average error in assigning present trips to the present system is shown for each of the several volume groups in Salt Lake City, Utah. It would have been preferable to show the data from Washington, D. C., but the necessary count and capacity information were not available. Coincidently or otherwise, it happens that the number of directional links in the spiderweb for Washington, D. C., is very nearly equal to the number of two-way highway links in the Salt Lake City network for all traffic volume groups up to about 17,500 veh/day. Above this volume there are more links in the Washington, D. C., network.

Because it will be significant at a later stage, it should be noted that both the gravity model and the assignment process are high, up to about 15,000 veh/day, whereas the Fratar method is low over this entire range. In addition, the assignment and both distribution procedures are significantly low in the 20,000 to 25,000 range.

It seems worthwhile to inquire into the relative accuracy of the assignment and distribution processes and, inasmuch as these are independent occurrences, combine the error of the two events. Figure 15 shows the error in the various procedures. In general, it shows that the error in using either the gravity model or the Fratar method is roughly half the error of assignment.

In addition, the figure shows that the addition of the error in the forecasting distribution by either method is hardly noticeable except at the higher traffic volume ranges.

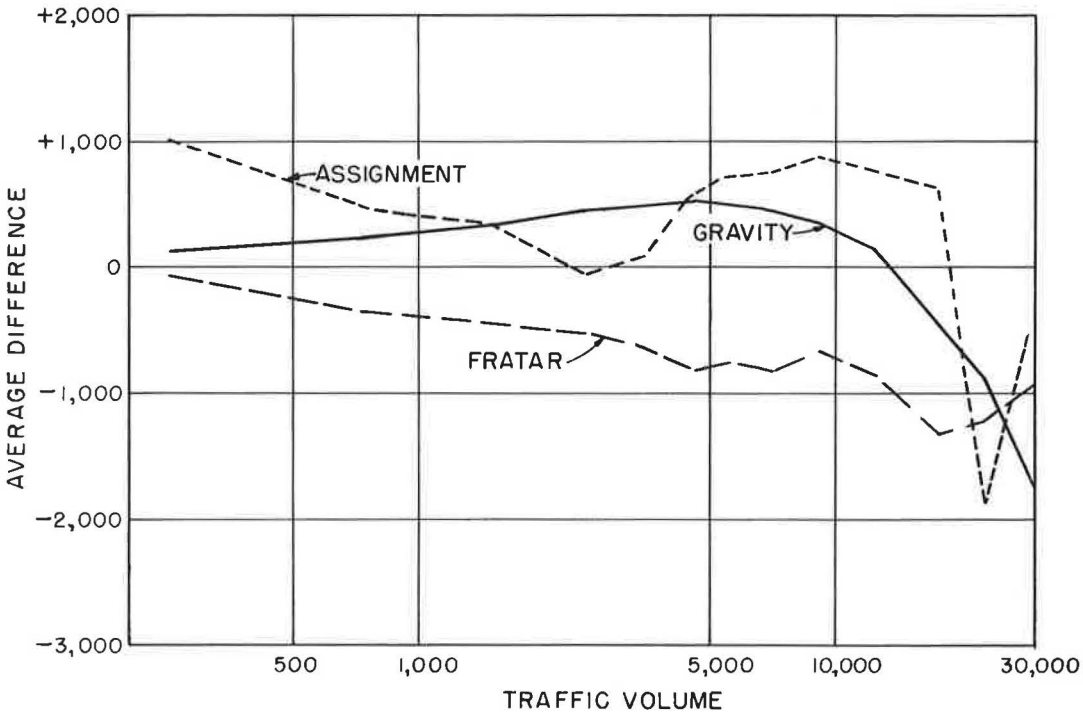

Figure 14. Average error in assignment and forecasting.

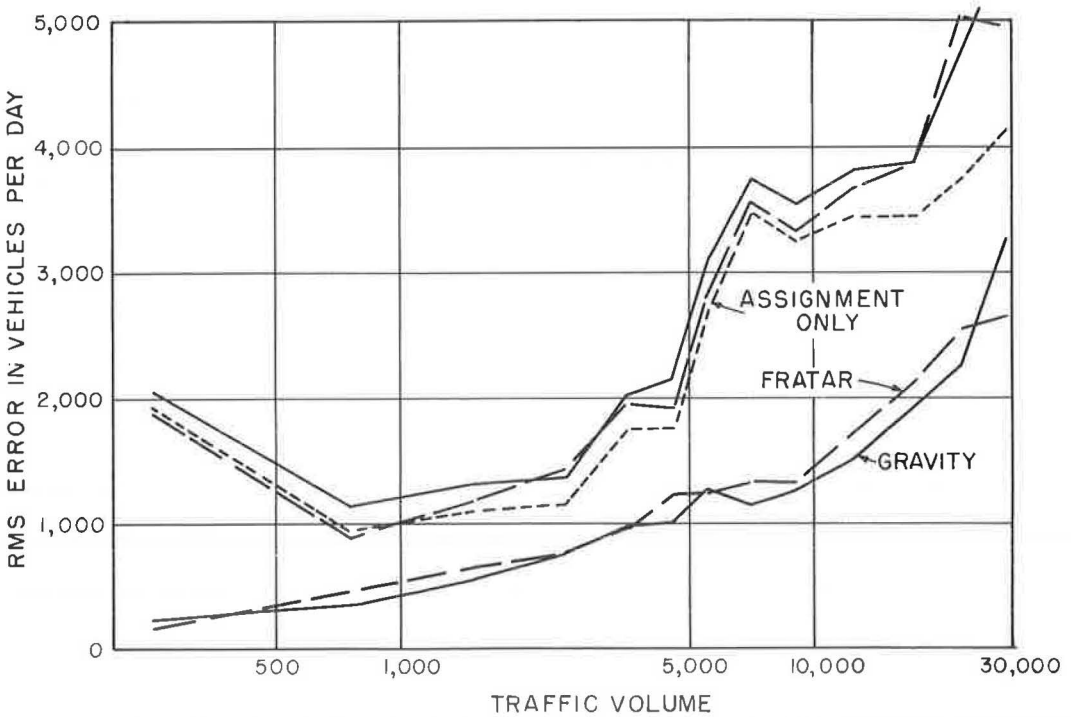

Figure 15. RMS errors in assignment and forecasting.

The procedure used in adding the errors for the two events of forecasting and assignment was to take the square root of the sum of the squares of the standard deviation and adjust this error to the average of the mean of the two events. As may be recalled, the average error for the Fratar method was consistently low for the entire range of traffic volumes. Both the gravity model and the assignment are high in the volume ranges up to about 15,000 veh/day, and all methods are consistently low in the volume ranges above 20,000 veh/day. This feature explains some of the anomalies of the graph. For example, the gravity model is slightly better than the Fratar method when viewed alone but slightly worse when viewed in combination with assignment. The consistently low mean volumes above 20,000 veh/day also explain, at least to some extent, the rather sharp rise in the RMS error above 20,000 veh/day.

As previously stated, these tests represent two cities of the more than 200 cities in the United States having populations over 50,000. The assignments were made to an existing network in a city of about ⅓ million population and the test of the distribution procedures to a spiderweb network in a city of about 1.8 million population. At this point in time, it is rather difficult to establish which characteristics of the procedures are inherent properties and which are accidental occurrences.

Yet I believe at least a tentative set of conclusions can be reached. It seems likely that the gravity model, the Fratar method, or the Chicago method could be used as a trip distribution procedure without inordinately affecting the error in the assignment procedures. I think we can also safely assume that, insofar as these two processes are concerned, improvement in the end product is primarily concerned with increasing the accuracy of the assignment procedure.

Is there any likelihood of this occurring? I think there is. At the moment the assignment procedures are capable of adding refinements but are "bogged down" by our inability to find in one place such prosaic items as a highly accurate O-D survey, a comprehensive transportation network, reliable traffic counts by direction and by peak hours on most of the network, and reliable capacity values on practically the entire arterial and freeway network.

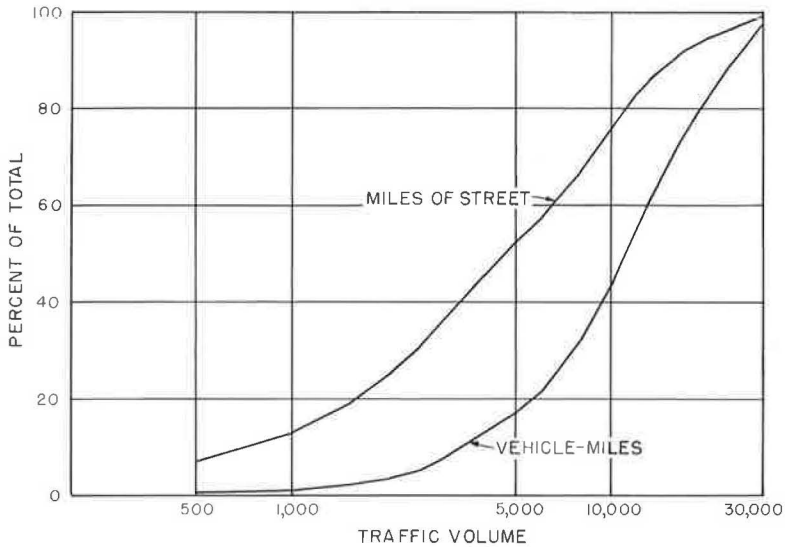

Figure 16. Distribution of street-miles and vehicle-miles in Salt Lake City, Utah.

With these errors such as they are, where does this leave us? Figure 16 indicates the composition of the street system in Salt Lake City. The top line indicates the cumulative percentage of miles of streets by increasing traffic volume groups. The lower line indicates the cumulative percentage of vehicle-miles on these same links. Thus, the highway administrator will have a tendency to feel that he has done an adequate job on a majority of the highway networks, but the public with its much greater probability of using the heavy volume links will have a tendency to be more critical of the mistakes on the heavily loaded links. For example, half the street-miles consist of links with less than 4,600 veh/day, but half the vehicle-miles are traveled on links with more than 11,000 veh/day.

It might be noted that between Figures 15 and 16 we have the elements we need to compute the probability of over- or underdesigning at any increments we might choose. This, however, is clearly beyond the scope of this discussion.

There are two major sources of error in forecasting traffic that have not yet been mentioned. One is the trip generation and attraction rates for various types of land uses and socioeconomic factors. This is receiving intensive study at the present time, and results should be available within a year. The second is the forecasting of the distribution of land use and the associated socioeconomic characteristics. The evaluation of this field is yet largely untouched. The principal problem is the lack of standardization of the factors that require forecasting. The view in this field is not particularly promising.

In summary, therefore, the distribution process by any of the models reported seems satisfactory. The error when combined with that of assignment is substantial, but the results are useful. It must be remembered that today's results are about twice as accurate as those of 3 years ago and perhaps more than 4 or 5 times as accurate as those of 10 years ago.

ROBERT T. HOWE, Associate Professor of Civil Engineering, University of Cincinnati—The authors have made a major contribution to the art of transportation planning by laying bare the limitations of several popular methods of trip forecasting. This commentator believes that certain further limitations of these methods are implicit in the report and wishes to bring these to the surface for discussion.

In the section on study procedures, the statement is made: "In addition, they [the 1965 characteristics] were used directly as producing and attracting powers of the zones when calculating the synthetic distributions with the interarea travel formulas." This evidently means that for work trips, for example, the number of trips originating in a zone was taken to be 20 times the auto and transit work trips found in the 1948 O-D survey and 33⅓ times those found in the 1955 survey without considering the actual number of workers living in that zone. Even more important, it would seem that no check was made to compare the expanded number of work trip destinations in a zone with the actual number of jobs available in that zone. Without such checks it is conceivable that the travel models actually produced more valid results than did the O-D surveys against which they were "calibrated."

The authors introduce a discussion of analytical tests with a series of speculations on how individual persons may decide on their trip patterns; they conclude that "people as social beings do not order their lives according to strict physical or mathematical laws.... However,... certain 'theories' will be more explanative than others." This commentator has previously pointed out that there are three elemental types of trips and each type satisfies certain conditions which the other two types do not satisfy: (a) the type in which a certain number of trips must originate in each zone at the same time a certain number must be destined for each zone, e.g., work trips; (b) the type in which a certain number of trips must originate in each zone but no exact number need be destined for any zone, e.g., shopping trips; and (c) the type in which no trips must originate in any particular zone and yet a definite number of trips do indeed end in a particular zone, e.g., recreation trips (13). It would seem that only those theories which take into account these three types of trip patterns can have any hope of being "more explanative than others."

The authors then pose the following criterion for a satisfactory theory: "Is the application simple enough that the procedure may be applied by urban planning studies lacking the experience in the procedure gained by research or earlier applications?" It would seem that the authors' applications of the four techniques tested in Washington, D.C., indicate that no one of the four can produce a positive answer to this question since the coefficients for the same city over a span of a mere 7 years had to be adjusted to give, in effect, post facto predictions. Another way of stating this would be to ask: how constant are the constants in the gravity model and the intervening opportunities model?

When giving the basic tests to evaluate distribution models, the authors state, "However, in the case of the other travel formulas, some validation was accomplished against base conditions. Such validation is an essential part of calibrating the models before moving to projections." If only some validation can be accomplished despite the fact that such validation is essential, there would appear to be a factor of safety of less than 1 in forecasting with these models.

Fratar Method

The authors' summary of the Fratar method—i.e., "The Fratar growth factor procedure demonstrated a good ability to expand trips correctly for stable areas but showed significant weaknesses in areas undergoing land-use changes"—should sound the death knell for this technique. A fundamental weakness of this method, over and above its inability to deal with formerly undeveloped areas, is the fact that movements between two zones may increase because of more direct means of transportation without any real change in the size of the "attraction."

Gravity Model

This commentator has a great deal of difficulty in following the authors' use of the gravity model. In the first place, he cannot determine how many of the variables in the equation are "fixed" and how many are subject to "adjustment." The travel time, F_{ij}, from one zone to another would appear to be known for the base year and hypothesized for the forecast year, and yet this seeming "constant" was evidently juggled to account for "river impedance." Under analytical tests, it is stated that the gravity model "makes the most explicit use of absolute travel time of any of the procedures" but there is no indication of how this travel time is found for the future. At one point, K_{ij} is defined as a "specific zone-to-zone adjustment factor to allow for the incorporation of the effect on travel patterns of defined social or economic linkages not otherwise accounted for in the gravity model formulation." Nowhere can this commentator find how these social or economic linkages are defined. In the discussion of movement by corridor to and from the CBD, the statement is made that "The incorporation and need for adjustment for geographical bias has been shown for the gravity model through the use of K factors." It is evident that different values of K_{ij} for any given movement from i to j were needed in 1948 and in 1955, but there appears to be some intimation that a given K_{ij} may have been varied for the several "checks," i.e., river crossings, corridors, etc. If K is chosen on the basis of defined conditions, why must it be changed so much?

In the statistical analysis of assigned trips, the statement is made that "Standard gravity model procedures were used to adjust the production to attraction trip tables to true origin to destination trip tables for directional assignments." To one who is not familiar with these "standard procedures" it is not clear whether adjustments are made to the time factors, or to the K's, or to both. But again, if there are known travel times and defined social and economic conditions, how can these be "adjusted"? In the summary and analysis the authors acknowledge that "Developing relationships between these factors and characteristics... can present problems," but they offer no suggestions for resolving these problems in an area which has never before been "fitted" for a gravity model—and, indeed, they indicate that they could not "fit" the same model to the same city in two different years. In addition, it is startling to inspect Table 1 and find that as the travel time increases from 8 to 10 min, the travel time factors are cut in half!

Intervening Opportunities Model

This section of the paper should really be read together with Mr. Pyers' "Evaluation of The Intervening Opportunities Trip Distribution Model" (12), but the paper presently under discussion raises some serious questions about the method.

In the explanation of the terms of the basic equation, L is said to be "an empirically derived function describing the rate of trip decay with increasing trip destinations and increasing length of trip." The authors further state that "This model is calibrated by varying the probability values until the simulated trip distribution reproduces the person-hours of travel and percent intrazonal trips of the surveyed trip distribution." No indication is given, however, as to how "length of trip" is found. Does it include mass transit trips? Does it include walking to and waiting for mass transit? Do the intrazonal trips include walking trips? Are trips simulated by purpose, or all trips combined?

As with the gravity model above, the authors found it necessary to "adjust" the river impedance from a 1948 value to a 1955 value to improve the fit. How can such post facto adjustments be considered valid? If the impedance value for 1948 was 5 min and that for 1955 was 8 min, will the value for 1969 be a linear projection of this change with a value of $8 + 3 + 3 = 14$ min or a geometric projection with a value of $8 \times 1.6 \times 1.6 = 20.5$ min?

In the checks of the gravity model and intervening opportunity model shown in Figures 8 and 9, it is most interesting to note that the Washington area has so few trips of 9-min duration. It is even more interesting to note that the O-D curve in Figure 8 is quite different from that in Figure 9. Could one curve actually be from 1948 and the other from 1955?

In the summary and analysis, the statement is made that "In this analysis the change in L value was forecast as a function of the change in the number of trips." But if the purpose of travel pattern models is to predict changes in trip patterns, and L which "predicts" these trips while being dependent on them would seem to be intolerable!

Conclusions

In proposing future research, the authors indicate that the gravity model might better be calibrated by trip purpose. Since this paper reports on trip stratification used with the Fratar method, and since Mr. Pyers' companion paper mentions stratifying the intervening opportunities model, why was the gravity model not stratified herein if such a step might be expected to improve the results or to stabilize the K value?

Other calls for further research really appear to be admissions that the techniques used could not be juggled to give reasonable predictions even when using post facto "constants." As Colen Clark and G. H. Peters have said: "It may be said in conclusion that the principle of 'intervening opportunities' appears to be an important step forward in our knowledge relating to travel habits. At the very least it must further undermine our faith in the effects of distance, and it must surely force us to recast our thinking concerning the potential usefulness of gravity models" (14).

What is needed is research into a technique not tied to time or city, but only to land-use patterns and, perhaps, key points in a transportation system. This commentator believes that the electrostatic field theory (13, 15, 16, 17), which is tied only to land use and certain a priori assumptions now merits more thorough testing than it has yet been given.

References

13. Howe, R. T. A Field Theory of Land Use and the Movement of People. University Microfilms, Inc., Ann Arbor, MIC 59-4149. Ph.D. thesis, Purdue Univ., 1959.
14. Clark, Colin, and Peters, G. H. Intervening Opportunities Method of Traffic Analysis. Traffic Quarterly, Vol. 19, pp. 101-119, 1965.
15. Howe, R. T. A Theoretical Prediction of Work-Trip Patterns. Highway Research Board Bull. 253, pp. 155-165, 1960.
16. Howe, R. T. A Theoretical Prediction of Work Trips in the Minneapolis-St. Paul Area. Highway Research Board Bull. 347, pp. 156-181, 1962.
17. Howe, R. T. A Critical Analysis of an Origin-Destination Survey. Highway Research Record No. 41, pp. 79-98, 1963.

KEVIN E. HEANUE and CLYDE E. PYERS, Closure—Dr. Cleveland, in addition to placing this research in perspective, has raised several questions aimed at providing the reader with additional insight into travel model development. Comments directed at his specific questions are as follows:

1. Washington, D.C., was not selected because it was an "ideal" city on which to base this research but rather because of data availability. It does, however, have certain characteristics which make it somewhat appropriate, namely, significant population growth, large increases in car ownership, and decentralization of many retail and business functions. This last factor is particularly appropriate to the analysis of travel patterns over time.

No attempt is made to claim "general validity" for the study findings. We would not, however, hesitate to apply the general findings when selecting a travel model for use in other urban areas. At the same time, we urge other researchers to undertake similar analyses in other cities, hopefully using data for a longer period of time.

2. There were not sufficient data to relate the Fratar forecast accuracy to changes in accessibility. It would be most interesting to analyze the performance of each of the procedures in geographic locations where accessibility varied significantly between 1948 and 1955. Unfortunately, this would be difficult in that part of the study area

where actual trips had to be eliminated from the Fratar tests. Most of the zones eliminated were in the areas with the greatest changes in accessibility.

3. Network impedances, particularly their forecast, are among the weakest phases of urban trip distribution model work. Certainly the variance between 24 hr and peak hour travel time is a major contributing factor. A more difficult part of this so-called "impedance" is the portion due to historical bias against making a trip involving a river crossing. This type of bias has been noted in most all urban areas with river crossings. Improvement in accessibility modifies this factor, but not enough so that a bridge crossing link can be treated as any other link in the system. We offer no improvements to the present methodology based on a trial-and-error approach, but again state that the present methodology, though lacking computational elegance, does do the job in a straightforward manner.

4. The question of whether the authors believed that the differences among the models tested would have influenced a transportation facility planning decision is most difficult to answer. The Fratar procedure certainly has inherent weakness that could cause serious underestimates of trips in areas undergoing land-use changes. Most analysts undertake steps to overcome this weakness. These modifications or adjustments were not tested in the subject research.

Analysts should gain insight into tendencies for the gravity or intervening opportunities models to over- or underestimate trips in given portions of the study area during the calibration phase. Hopefully, such insight applied during the systems analysis would result in essentially the same system, irrespective of which of these two models were used.

5. The unsatisfactory trip length distributions obtained in the calibration of the competing opportunities model were too significant to have canceled out in the total person trip distribution. When using small time bands, the trip length curves were not even similar; when broader time bands were used, the trip length curves, though attaining the characteristic trip length frequency curve shape, were significantly offset from the comparable O-D survey data curves.

Mr. Vogt has contributed the background of one who has had practical experience in the use of gravity models. We share his uneasiness over the necessity to assume that travel time factors remain constant through time and to apply river impedances and socioeconomic adjustment factors. In spite of this uneasiness, the fact that such adjustments and impedances can be both logically and quantitatively derived and that the final model results can be quantitatively verified allows us to recommend the use of these procedures with a certain degree of confidence. Mr. Vogt has presented results from certain studies in which he was involved to demonstrate a reason for confidence in current procedures. These results were attained with significantly less than comprehensive data. With comprehensive survey trip data available, calibration results should be expected to attain a RMSE accuracy of less than 10 percent for the high volume groups, regardless of city size.

Mr. Brokke suggests that we may have treated the travel models with more "kindness" than we treated the Fratar procedure. He would have modified the basic purpose Fratar procedure by aggregating zones into districts to obtain interchange potential in areas where there is no base year travel pattern. The major difficulty with this approach is that there is no satisfactory procedure for bringing the analysis back to the zonal level.

A procedure that is more often recommended is to create a base year travel pattern in presently undeveloped areas through the use of a gravity or opportunity model and then to expand this synthetic pattern through the use of the Fratar procedure. This type of Fratar "adjustment" offers little appeal to the authors. This procedure accepts the validity of travel models and uses them as a crutch in determining a synthetic base year travel pattern in presently undeveloped areas. The synthetic pattern is fully reflective of land use and the transportation system. The procedure proceeds to ignore this inherent land-use transportation system-travel linkage in making projections. We suggest that it is far more logical to start with a travel model and to utilize it fully.

Mr. Brokke's point that "the consistency of the Fratar in underestimating is very close to the 10 percent mentioned in the paper" refers to a comparison between the Fratar results and base data where 38 zones in undeveloped areas were eliminated from the comparison. If this comparison had been made using the full universe of O-D trips, certainly the amount of error would have risen and be concentrated in the areas of significant land-use changes.

Mr. Brokke's quantitative relationships of the accuracy of trip distribution to the accuracy of traffic assignment is most interesting. In effect, he shows that had the final comparisons of travel models been made after a traffic assignment including capacity restraint, all the variation between the procedures would have disappeared. His findings should provide an impetus to the much needed improvements in traffic assignment techniques and to a quantitative evaluation of the several key technical phases of the transportation planning process. Such an evaluation will result in a much better appreciation of the accuracy of the total process as well as an indication of the sensitivity of the individual phases with respect to the final product.

Professor Howe's comments are difficult to handle. He raises several well-taken points about weaknesses in the use of these techniques with which the authors are quick to agree. Underlying these comments, however, the professor appears to be making a sales pitch for his own approach to trip distribution, namely, the electrostatic field theory. His model is essentially a gravity model which utilizes airline distance as the measure of spatial separation. No exponent is used to raise this distance measure to a higher power. The fact that he would use labor force as the measure of work trip generation, rather than the O-D survey results, is not pertinent to this discussion since our comments would apply should perfect trip generation data be available. What concerns us most is the use of airline distance as the sole measure of spatial separation. We cite the professor's own words, included in his comments on the Fratar procedure, to criticize his model. A fundamental weakness of this method is the fact that it fails to recognize that "movements between two zones may increase because of more direct means of transportation without any real change in the size of the attraction." Airline distance, totally insensitive to system and level of service, is a very weak attempt to overcome the Fratar's basic weakness.

In his admittedly skeptical reading of the portions of the paper dealing with the gravity and intervening opportunities models, Professor Howe has read between the lines and found all forms of "juggling." It may be helpful if we reduce the calibration and projection procedures to their essentials by summarizing changes in key parameters for the gravity model. The travel time factors were developed for 1948 and held constant to 1955. The river "impedances" were developed for 1948 and forecast to 1955 on the basis of the change in the level of service on the river crossings. The socioeconomic adjustment factors (K_{ij}) were developed for 1948 and related to 1948 district incomes. They were applied in 1955 on the basis of 1955 district incomes. The 1955 transportation system was used to determine the time inputs for the 1955 gravity model application. The travel time factors, river impedances and socioeconomic factors were developed and applied in that order. This involved no juggling, merely the same procedures used operationally by dozens of urban studies.

The point raised by Professor Howe regarding adjustments in L values for the forecast in the intervening opportunities model also deserves comment. He quotes from our paper: "In this analysis the change in L value was forecast as a function of the change in the number of trips." In his discussion he states, "But if the purpose of travel pattern models is to predict changes in trip patterns, an L which 'predicts' these trips while being dependent on them would seem to be intolerable!" We fail to see the problem. The total number of trips and the travel patterns in an area are two completely different things. The fact that one parameter in the distribution model is made a function of an areawide characteristic is appropriate rather than intolerable.