Friction Factors for Hydraulic Design of Corrugated Metal Pipe

JOHN L. GRACE, JR., U. S. Army Engineer Waterways Experiment Station

Results of model tests of two types of corrugated metal pipe including friction factor-Reynolds number diagrams and mean flow formulas developed from velocity distribution data are reported. Calculated maximum values of the friction factor due to the corrugations and the bolt nuts on the crests of the structural plate corrugations for various sizes of each type of pipe are compared with those of similar prototypes as reported by other investigators. Recommended design values of the friction factor for annular corrugated pipes with corrugation depth-spacing ratios of 1:3 and 1:5.33 are related to diameter, and simple empirical equations describing the relations are developed.

•STRUCTURAL PLATE PIPE, widely used in drainage systems, is made of corrugated metal sections bolted together in the field. These sections permit erection of pipe 5 ft in diameter or larger (in increments of 0.5 ft). Structural plate corrugations have a depth of 2 in. and a pitch of 6 in. In standard corrugated metal pipe the depth of the corrugations is only ½ in. and the pitch or spacing of the corrugations is 2⅔ in., crest-to-crest.

Tests to determine friction factors for standard corrugated metal pipe were made on pipes 3, 5, and 7 ft in diameter at the U. S. Army Engineer Bonneville Hydraulic Laboratory which published the results in 1955 (1, 8). Roughness coefficients determined in these tests are used generally in culvert design. However, extrapolation of these roughness coefficients to values applicable to structural plate pipe, which has corrugations four times as deep and a depth-pitch ratio of 1:3 rather than 1:5.33 was considered unreliable. The HRB Committee on Surface Drainage of Highways has long recognized the need for field or laboratory determination of hydraulic design coefficients for this commonly used drainage material.

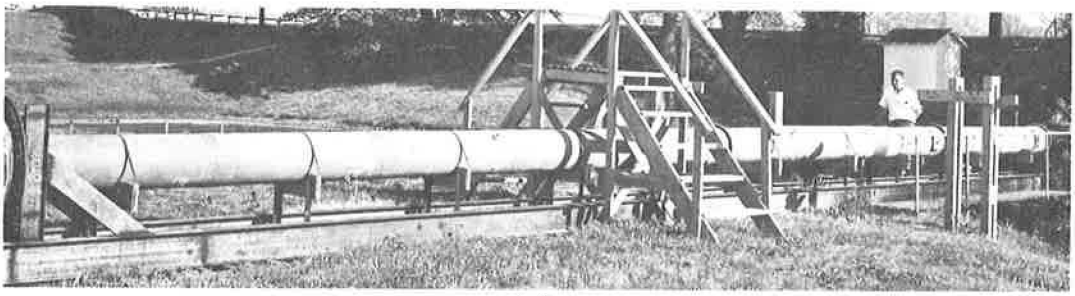
Anticipating that full-scale tests would have been costly, and that it would have been feasible to test only the smaller sizes of structural plate pipe, the Bureau of Public Roads and the Office, Chief of Engineers, initiated in 1958 a hydraulic model investigation at the U. S. Army Engineer Waterways Experiment Station (WES) for the purpose of determining friction factors for structural plate pipe. One model simulating a 5-ft diameter standard corrugated pipe was tested to permit comparison of model and prototype results and to check the applicability of simulating corrugated metal pipes with corrugated fiber glass conduits. The good agreement obtained between results of the WES model and the Bonneville Hydraulic Laboratory prototype tests of 5-ft-diameter standard corrugated metal pipe warranted the use of the fiber glass models.

MODELS AND TEST PROCEDURES

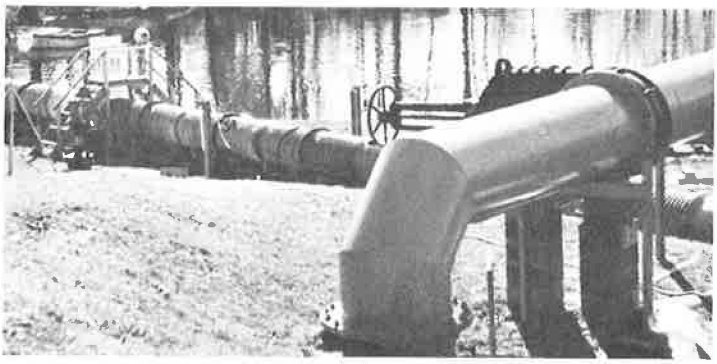
Four models were constructed: a 1:4-scale model of 5-ft-diameter standard corrugated pipe and three simulating structural plate pipes 5, 10, and 20 ft in diameter at scales of 1:2.2, 1:8, and 1:16, respectively. The diameter between crests of corrugations of all models was 15 in. with the exception of the model simulating 5-ft-diameter

Paper sponsored by Committee on Surface Drainage of Highways.

Figure 1. Sections of models representing (left to right) 5-, 10-, and 20-ft-diameter structural plate pipes.



(a)



(b)

Figure 2. Structural plate pipe models of 40-diameter lengths; (a) 1:8-scale model of 10-ft-diameter pipe, and (b) 1:2.2-scale model of 5-ft-diameter pipe.

Figure 3. Piezometers and velocity probes.

Figure 4. Velocity probes.

structural plate pipe which utilized a diameter of 27.27 in. The crests of corrugations referred to throughout the paper are those nearest the axis of the pipe and the diameters quoted are the actual minimum inside diameters except in the cases where results are related to nominal pipe diameter. This unusual diameter (27.27 in.) and model scale (1:2.2) was calculated to be necessary to obtain flows with Reynolds numbers representative of prototype conditions using an available pumping system with a rated capacity of 100 cfs under a 55-ft head. Fabricated sections of the models simulating structural plate pipes are shown in Figure 1. The sections were assembled and tested in lengths ranging from 22 to 100 times the respective pipe diameter. Models of 5- and 10-ft-diameter structural plate pipes are shown in Figure 2.

Water used in the operation of the models was supplied by centrifugal pumps and measured by means of either a calibrated venturi meter or traverses of velocity across the pipes. Piezometers located on the crests of the corrugations (Fig. 3) were used to observe the hydraulic gradients. Velocity probes and traversing mechanisms (Fig. 4) were equipped with total pressure and static pressure tubes to obtain velocity and static pressure distribution data.

Before beginning a test, a discharge sufficient to remove air entrapped in the corrugations at the top of the pipe was set and instruments used to measure discharge, pressure, and velocity were primed. The test discharge was established, and all data desired at that discharge were obtained without interruption or modification of flow. The hydraulic gradient was observed. Traverses of total and static pressures across the pipe normal to the crest of a corrugation were obtained. The temperature of the water was measured during each test. Flow with a Reynolds number of 5×10^6 in the model simulating 5-ft-diameter structural plate pipe is illustrated in Figure 5.

Figure 5. Flow in model of 5-ft-diameter pipe, $\Re + 5 \times 10^6$.

In determining the slope of the hydraulic gradient, pressure readings near the entrance and exit of the test sections were neglected to eliminate the respective effects of boundary layer development and acceleration of flow. The average velocities, V, the slopes determined from the hydraulic gradients, S, and the actual diameter between crests of the corrugations, D, were used to determine values of the friction factor, f, by means of the Darcy-Weisbach equation. Values of the shear velocity, v, computed by means of the basic relation, $v^* = \sqrt{\frac{D}{4} Sg}$, were used to determine values of a parameter termed wall Reynolds number, $R_W = (v^*k/\nu)$. The symbols k and ν represent depth of corrugation in feet and kinematic viscosity, respectively.

STANDARD CORRUGATED PIPE

Although the relative roughness, K/D, of the model of 5-ft-diameter standard corrugated pipe was 0.00936 rather than the expected value of 0.0083, the resistance coefficient curve, f, versus wall Reynolds number, of the model was similar in shape to that of the prototype reported by Webster and Metcalf (8) for wall Reynolds numbers up to 1600 (Fig. 6). The maximum value of the resistance coefficient agreed most favorably with that interpolated based on the results of the 3- and 5-ft-diameter standard corrugated pipes. Thus, it was concluded that the material effect of fiber glass on the resistance coefficient was essentially the same as that of metal and that geometrically similar fiber glass models would adequately simulate corrugated metal pipes.

Analysis of the Bonneville Hydraulic Laboratory prototype test data indicated that the maximum value of the resistance coefficient of standard corrugated pipes occurs at flows with a common wall Reynolds number of 1300 (see Fig. 6). Therefore, appro-

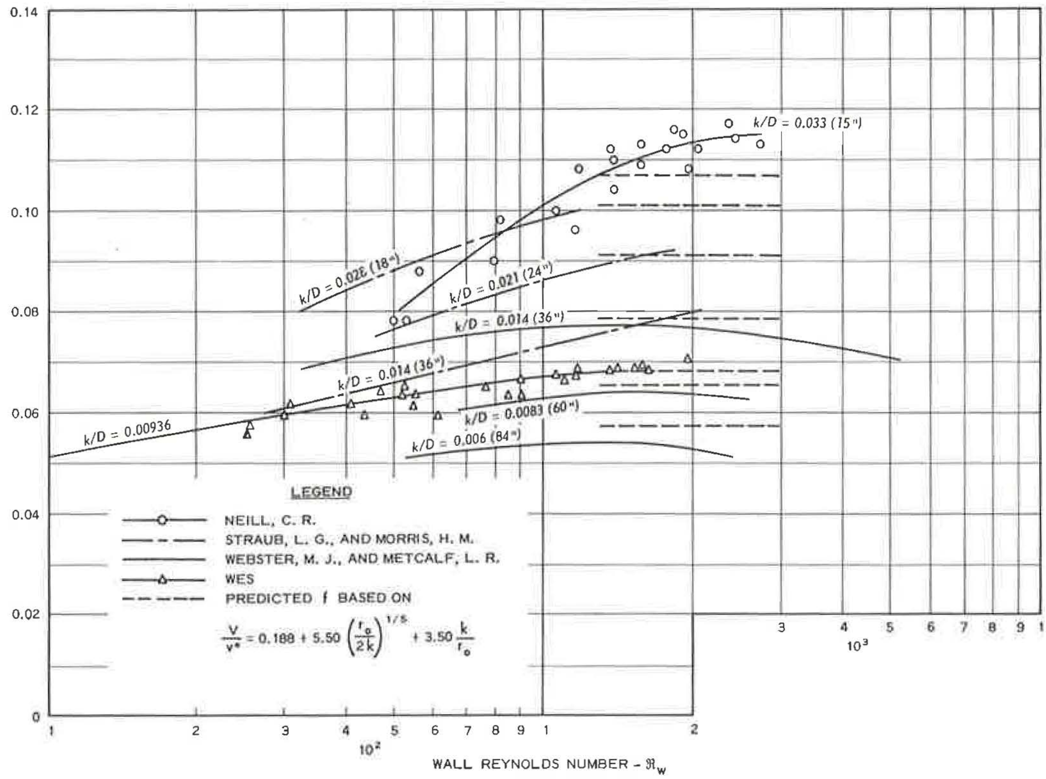


Figure 6. Resistance coefficients vs wall Reynolds number—standard corrugated metal pipe, full pipe flow.

priate velocity distribution data of both the WES model and the 5-ft-diameter prototype were used to develop the following mean flow formula which can be used to compute the maximum value of the resistance coefficient of any size of standard corrugated pipe.

$$\frac{V}{v*} = \sqrt{\frac{8}{f}} = 0.188 + 5.50 \left(\frac{r_o}{2k}\right)^{1/5} + 3.50 \frac{k}{r_o}$$

Resistance coefficients computed by means of the mean flow formula agree most favorably with the maximum values reported by the Bonneville (1) and the Saint Anthony Falls (2) Hydraulic Laboratories but are approximately seven percent less than the maximum value reported by Neill (5) for 15-in.-diameter pipe and that reported by Garde (4) and Chamberlain (3) for 12-in.-diameter standard corrugated pipe. Admittedly, the mean flow formula for standard corrugated pipe was developed from limited velocity distribution data (especially in the region of threshold velocities) due to practical considera-

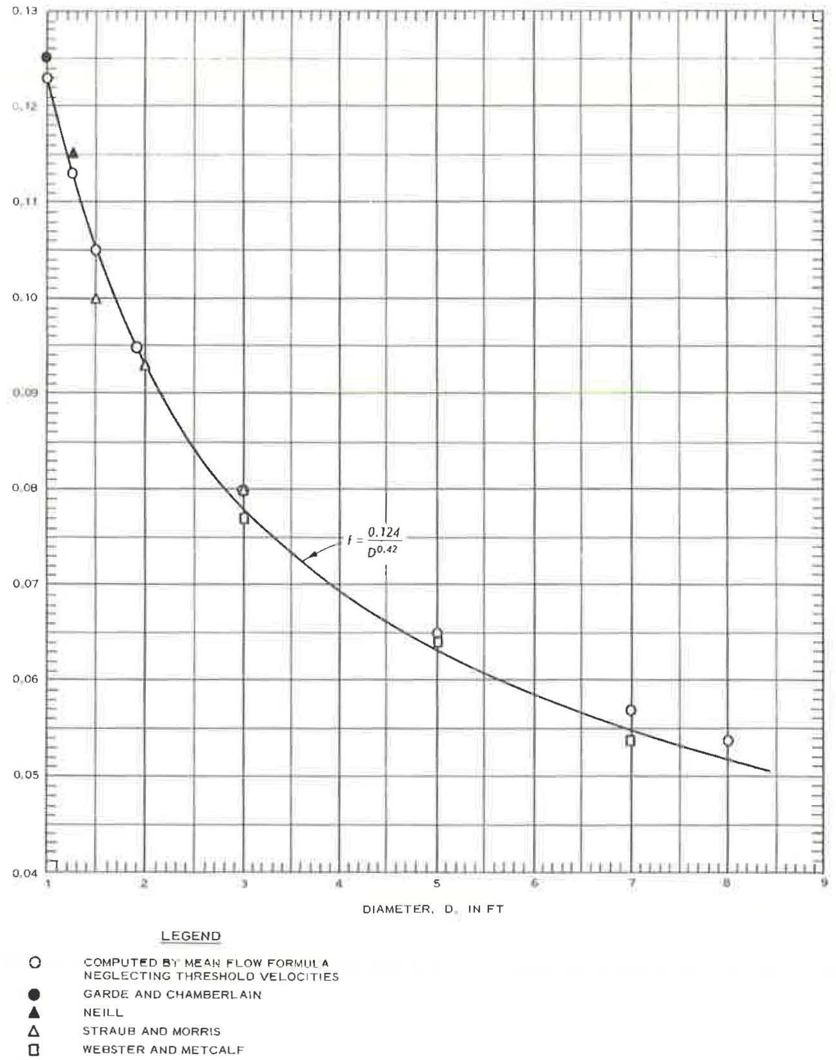


Figure 7. f vs pipe diameter—standard corrugated pipe.

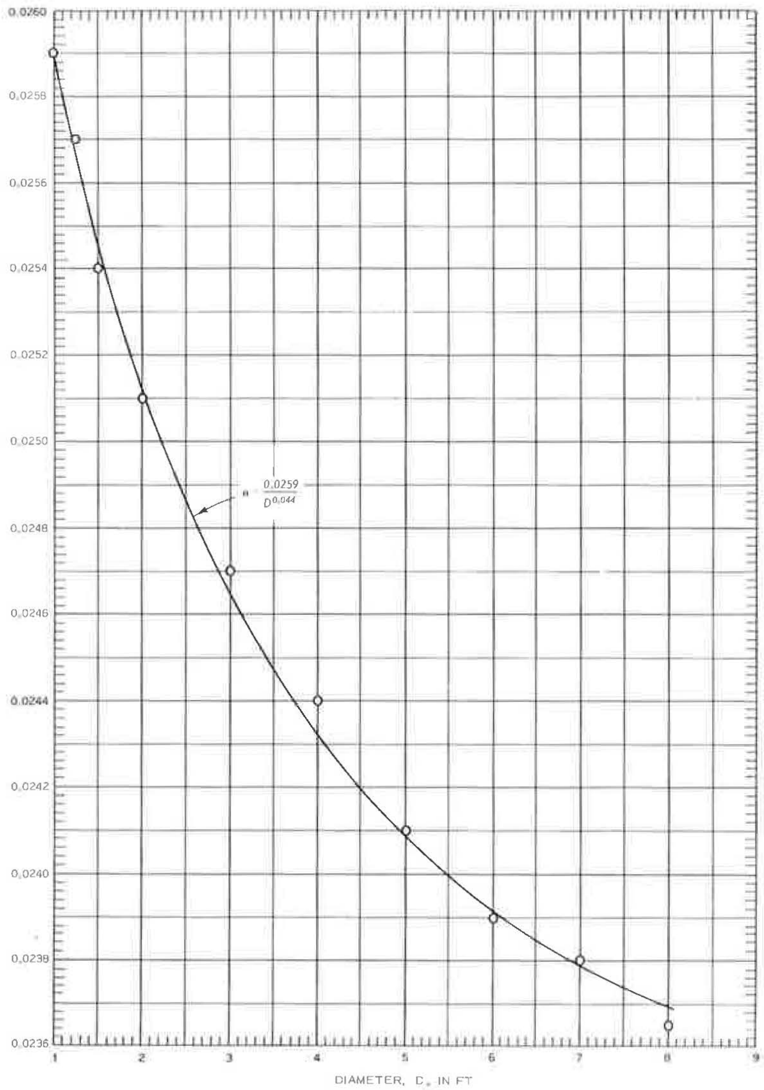


Figure 8. Manning's n vs pipe diameter—standard corrugated pipe.

tions and, therefore, the mathematical expression derived for the threshold velocities is questionable. If the term 3.50 (k/r_o) is neglected, the modified mean flow formula predicts maximum values of the resistance coefficients that agree favorably with those reported by other investigators for standard corrugated metal pipes ranging from 1 to 7 ft in diameter as shown in Figure 7. This does not imply that the threshold velocities do not exist in standard corrugated metal pipe but merely that the expression derived is not adequate and this was expected in view of the lack of appropriate data near the boundary of this type of pipe. Figure 7 indicates that the maximum or design value of the resistance coefficient of any size of standard corrugated pipe may be calculated by means of the empirical equation, $f = 0.124/D^{0.42}$, where D is pipe diameter in feet. Values of f were converted to Manning's n by means of basic relations and the relation between n and pipe diameter (Fig. 8) is satisfied by the empirical equation, $n = 0.0259/D^{0.044}$. These values of the resistance coefficient can be expected at flows with wall Reynolds numbers near 1300 and are considered applicable for design since values of R_W, encountered in field installations of 12-, 60-, and 96-in.-diameter standard corrugated pipes flowing full with friction slopes of 0.5 to 8.0 percent and water temperatures ranging from 45 to 75 F, range from 550 to 3400, 1250 to 7550, and 1550 to 9550, respectively. There may be objections to the recommendation that the maximum values of the resistance coefficients observed in standard corrugated pipes be used as a basis for selection of design values for all conditions since prototype tests (1, 3, and 4) indicate that the resistance coefficients decrease with increasing wall Reynolds numbers greater than 1300. Certainly this appears to be merited for the cases where the R_W of flow in standard corrugated pipes is expected to be well above the value of 1300 (the range of R_W where a maximum value of the resistance coefficient is indicated). In such cases, it is recommended that the results of the Bonneville Hydraulic Laboratory prototype tests (1) as shown in Figure 6 be used in extrapolating the design values of the resistance coefficient.

STRUCTURAL PLATE CORRUGATED PIPE

The resistance coefficient curve determined from tests of the model simulating 5-ft-diameter structural plate pipe (Fig. 9) revealed that the resistance coefficient attained a maximum value of 0.111 at a R_W of about 8000 and that f remained constant for R_W up to 22,000. Values of wall Reynolds numbers, expected in field installations of 5-, 10-, and 20-ft-diameter structural plate pipes flowing full with friction slopes of 0.5 to 8.0 percent and water temperatures ranging from 45 to 75 F, range from 5,000 to 30,000, 7,000 to 43,000, and 10,000 to 60,000, respectively. Thus, the conditions investigated with the model of 5-ft-diameter structural plate pipe simulate anticipated field flow conditions adequately. Unfortunately, the limiting value of R_W (8000) was

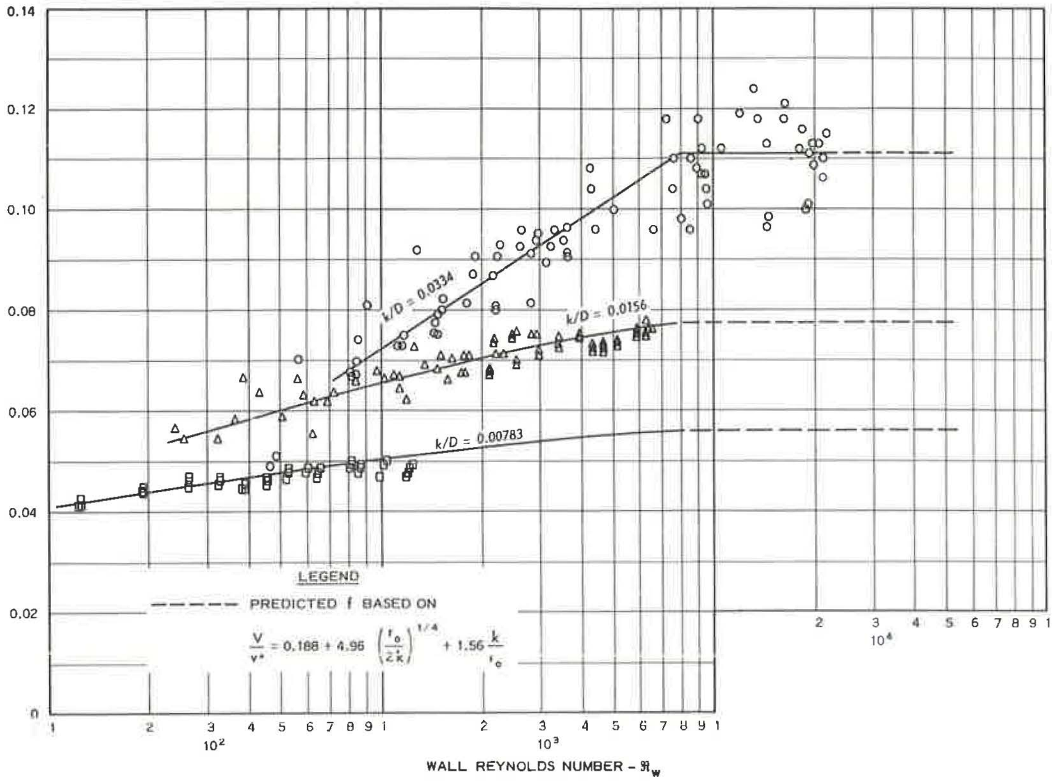


Figure 9. Resistance coefficient vs wall Reynolds number—structural plate pipe, full pipe flow.

greater than that anticipated initially; and consequently, flows with wall Reynolds numbers equal to or greater than 8000 were not possible with the selected models of 10- and 20-ft-diameter structural plate pipes and the available water supply systems. However, results obtained with the model of 10-ft-diameter structural plate pipe and wall Reynolds numbers just below this limit agreed most favorably with that of the model of 5-ft-diameter structural plate pipe, and it was concluded that the resistance coefficient of any size of this type of pipe approaches a maximum value and remains constant for flows with R_w equal to or greater than 8000. Since an analysis of the results of Webster and Metcalf (8) indicate that the maximum value of the resistance coefficient of standard corrugated pipes (3, 5, and 7 ft in diameter) occurred at flows with a common wall Reynolds number, it seems quite reasonable that a similar relation would exist for structural plate pipes.

Velocity distribution data of the model simulating 5-ft-diameter structural plate pipe in the range of wall Reynolds numbers, where the resistance coefficient was at its maximum and constant value, were used to develop the following mean flow formula.

$$\frac{V}{v^*} = \sqrt{\frac{8}{f}} = 0.188 + 4.96\left(\frac{r_o}{2k}\right)^{1/4} + 1.56\frac{k}{r_o}$$

Velocity distribution data of the model simulating 10-ft-diameter structural plate pipe within the range of R_w near 8000 are satisfied by the mean flow formula also. Thus, it is concluded that the mean flow formula can be used to compute the maximum value of the resistance coefficient due to the corrugations of any size of structural plate pipe.

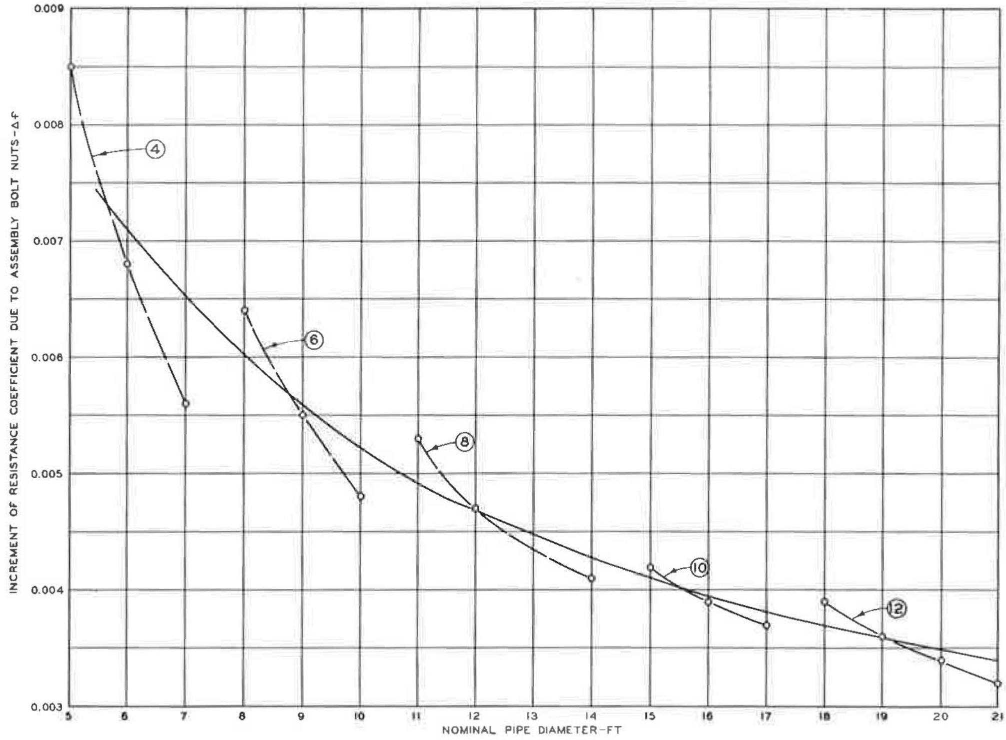


Figure 10. Resistance factor, Δf, attributable to assembly bolt nuts—structural plate pipe.

TABLE 1

BOLT-NUT RESISTANCE FACTOR, Δf, STRUCTURAL PLATE PIPE

$$\Delta f = \frac{C_D N a}{0.785 D^2} \left(\frac{v}{V}\right)^2$$

$C_D = 1.1$ $a = 0.0070$ sq ft

No. of Plates per Ring	Pipe Diameter		No. of Nuts per Diameter	$\frac{v}{V}$	Δf
	Nominal (in.)	Actual (ft)			
4	60	4.93	50	0.649	0.0085
	72	5.94	63	0.621	0.0068
	84	6.97	77	0.598	0.0056
6	96	7.98	123	0.580	0.0064
	108	9.00	143	0.564	0.0055
	120	10.02	164	0.549	0.0048
8	132	11.04	227	0.537	0.0053
	144	12.06	254	0.525	0.0047
	168	14.09	321	0.506	0.0041
10	180	15.11	398	0.498	0.0042
	192	16.13	434	0.490	0.0039
	204	17.15	470	0.483	0.0037
12	216	18.17	575	0.476	0.0039
	228	19.18	616	0.469	0.0036
	240	20.21	660	0.464	0.0034
	252	21.22	705	0.459	0.0032

Δf = increment of resistance coefficient attributable to bolt nuts.
C_D = coefficient of drag.
N = number of objects (bolt nuts) on crest of corrugations in a length of one pipe diameter.
a = projected area of object in a plane normal to direction of flow, sq ft.
D = actual diameter of pipe between crests of corrugations, ft.
v = local veolcity at midheight of object, fps.
V = mean velocity of flow, fps.

Assembly bolt nuts which are located on the crests of the corrugations in prototypes were not simulated in the models and, therefore, the mean flow formula does not reflect the added resistance they would entail. However, H. G. Bossy of the U. S. Bureau of Public Roads made a detailed review of literature concerned with the coefficient of drag of shapes similar to the bolt nuts and developed a method to determine the increment of resistance attributable to the assembly bolt nuts of structural plate pipe. The results of Bossy's analysis, presented in Table 1 and Figure 10, indicate that the increment of the resistance coefficient, Δf, which can be attributed to the bolt nuts varies with pipe diameter, and that a Δf of 0.0085 is reasonably applicable for the 5-ft-diameter structural plate pipe. Adding this increment to the f determined from the mean flow formula based on an actual diameter between corrugation crests of 59.1 in., that recommended by the manufacturer, gives an f of 0.12. The first and only reported prototype tests by Neill (5) of a 5-ft-diameter structural plate pipe (within this range of wall Reynolds numbers) indicate a maximum constant value of 0.13 for the resistance coefficient based on a diameter from crest to crest of corrugations of 59 in. (7). Thus, the maximum value of the resistance coefficient predicted from the WES model tests agrees favorably with that indicated by Neill's prototype tests. Additional friction-loss data of a small model of 5-ft-diameter structural plate pipe presented by Kellerhals (6) confirm the data of the WES model of 5-ft-diameter structural plate pipe in the lower range of central Reynolds number, VD/ν (2 to 5×10^5).

Resistance coefficients due to the corrugations of structural plate pipes with nominal diameters ranging from 5 to 20 ft were calculated by means of the mean flow formula and the actual inside diameters between crests of corrugations as given by the manufacturers. The increment of the resistance coefficient attributable to the assembly bolt nuts determined by Bossy (Fig. 10) was added to the value of the resistance coefficient due to the corrugations to determine the total resistance coefficient of each of the several sizes of structural plate pipe. The relation between total resistance coefficient and diameter of pipe (Fig. 11) is satisfied by the empirical equations, $f = 0.258/D^{0.482}$ and $f_n = 0.320/D_n^{0.576}$. It is noted that the equation based on nominal pipe diameter will yield a value of the resistance coefficient other than that determined by the equation based on actual pipe diameter. This is required in order that the head loss computed by the Darcy-Weisbach equation using the nominal diameter and a velocity based on the nominal diameter and design discharge will agree with that determined using actual diameter and velocity, i.e.,

$$h_l = f\frac{L}{D}\frac{V^2}{2g} = f_n\frac{L}{D_n}\frac{V_n^2}{2g} \text{ and } f_n = f\left(\frac{D_n}{D}\right)^5$$

The recommended design value of the total resistance coefficient obtained from the foregoing equations of Figure 11 is that expected to occur at flows with wall Reynolds numbers of 8000 or greater (the range of R_w in which f has attained a constant maximum value and also that to be expected in the field).

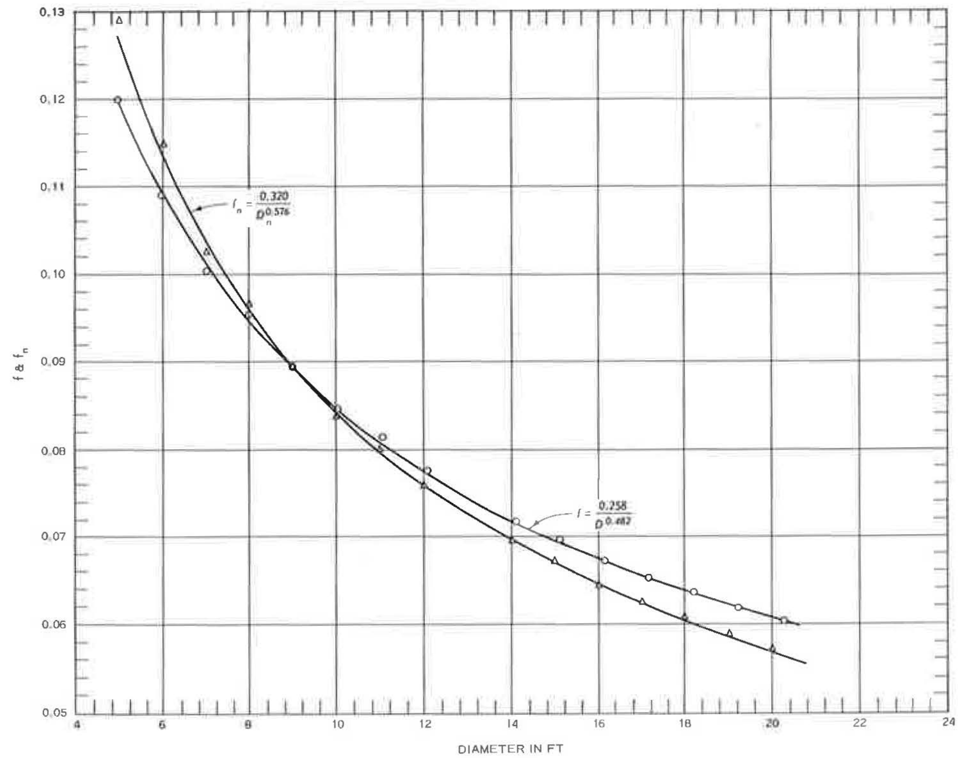


Figure 11. f and f_n vs pipe diameter—structural plate pipe.

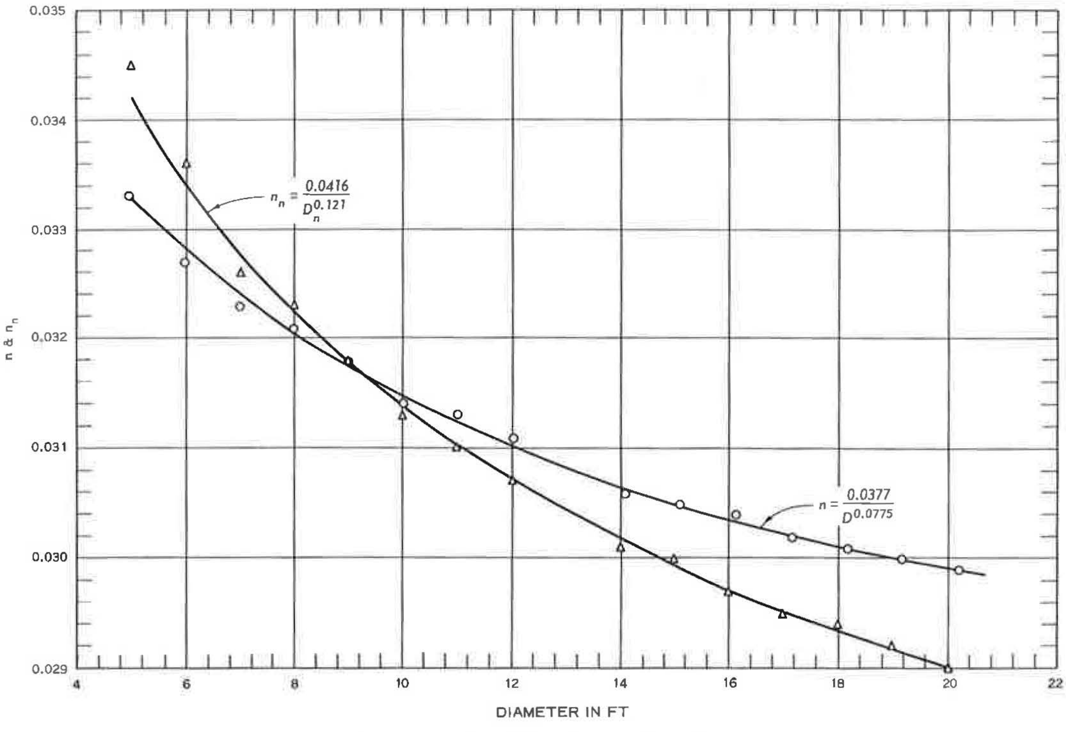


Figure 12. n and n_n vs pipe diameter—structural plate pipe.

Values of f were converted to Manning's n by means of basic relations. The relation of recommended design values of Manning's n to pipe diameter (Fig. 12) is satisfied by the empirical equations, $n = 0.037/D^{0.0775}$ and $n_n = 0.0416/D_n^{0.121}$

OTHER CORRUGATED PIPE

Since the depth-to-pitch ratio of corrugations 1 in. by 3 in. is the same as that of structural plate corrugations, 2 in. by 6 in., the mean flow formula for structural

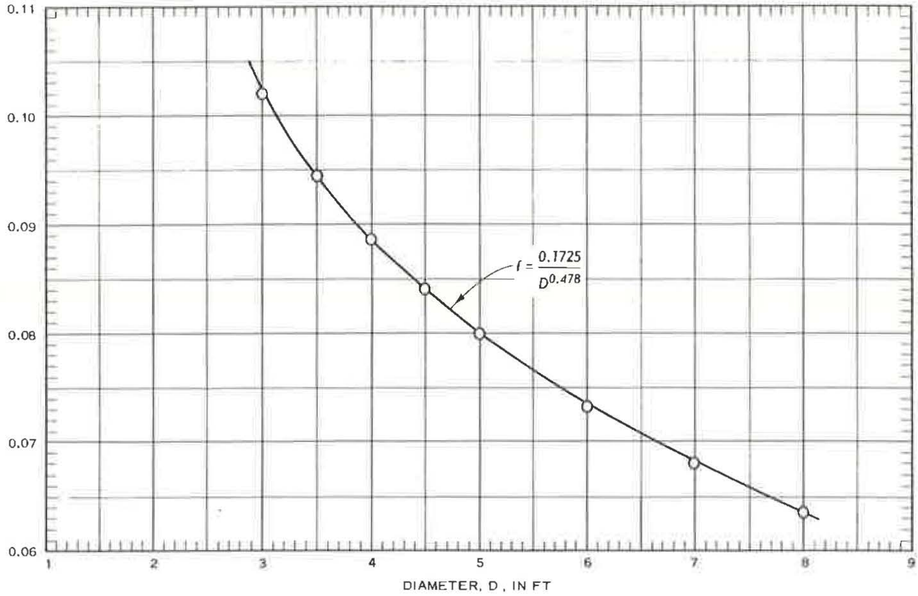


Figure 13. f vs pipe diameter—1- × 3-in. corrugations.

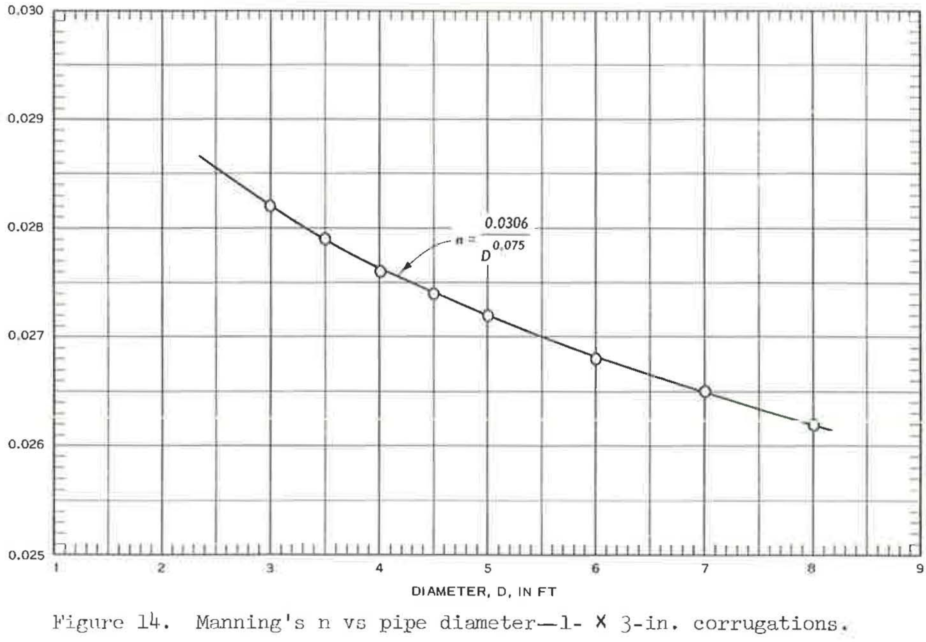


Figure 14. Manning's n vs pipe diameter—1- × 3-in. corrugations.

plate pipe is considered applicable to corrugated pipe with annular 1-in. by 3-in. corrugations. Values of f determined by means of the mean flow formula are related to pipe diameter in Figure 13 which indicates that the resistance coefficient of any size of this type of pipe can be calculated by the empirical equation, $f = 0.1725/D^{0.478}$. Manning's n may be computed directly by the equation, $n = 0.0306/D^{0.075}$ (see Fig. 14). Design values of Manning's n ranging from 0.0282 to 0.0262 are indicated for 3- to 8-ft-diameter pipes with annular 1-in. by 3-in. corrugations.

The results reported herein are believed to be most adequate for determining design values of the resistance coefficient for each type of corrugated pipe discussed. However, sufficient data are not available with which the effect of corrugation pitch or spacing, λ, can be determined. In addition, little is known of the effects of helical rather than annular corrugations on the resistance coefficient. It is believed that the need for tests to determine the resistance coefficient of various configurations, including both annular and helical, will arise in the near future and it is hoped that efforts will be directed to determine the importance of these geometric properties on velocity distribution in the range of maximum resistance, in order that a more complete understanding of the law of velocity distribution in corrugated pipe can be developed.

ACKNOWLEDGMENTS

The results reported herein are a summary of the research project sponsored jointly by the Bureau of Public Roads and the Office, Chief of Engineers and conducted at the U. S. Army Engineer Waterways Experiment Station for investigation of the hydraulic resistance of structural plate corrugated metal pipe. The author is indebted to K. S. Eff of the Office, Chief of Engineers, H. G. Bossy of the Bureau of Public Roads, T. E. Murphy of the Waterways Experiment Station, and G. H. Keulegan of the Bureau of Standards (now an employee of the Waterways Experiment Station) for many helpful discussions of the test results and review of the analytical methods employed. Special thanks are extended to Mr. Bossy for his analysis to determine the increase in the resistance coefficient attributable to the assembly bolt nuts of structural plate corrugated metal pipe and for invaluable review of the paper.

REFERENCES

1. Webster, M. J., and Metcalf, L. R. Friction Losses in Corrugated Metal Pipe. U. S. Army Engineer Bonneville Hydraulic Laboratory, Rept. 40-1, Bonneville, Oregon, July 1955.
2. Straub, L. G., and Morris, H. M. Hydraulic Data Comparison of Concrete and Corrugated Metal Culvert Pipes. St. Anthony Falls Hydraulic Laboratory, Univ. of Minnesota, Technical Paper 3B, Minneapolis, Minn., 1950.
3. Chamberlain, A. R. Effect of Boundary Form on Fine Sand Transport in Twelve-Inch Pipes. Colorado State Univ. Rept. CER No. 55, June 1955.
4. Garde, R. J. Sediment Transport Through Pipes. Colorado State Univ. Rept. CER No. 56, Oct. 1956.
5. Neill, C. R. Hydraulic Roughness of Corrugated Pipes. Proc., ASCE, Vol. 88, HY-3, New York, N. Y., pp. 23-44, May 1962.
6. Kellerhals, Rolf, and Bossy, H. G. Discussion of Hydraulic Roughness of Corrugated Pipes. Proc., ASCE, Vol. 89, HY-1, New York, N. Y., pp. 201-205, Jan. 1963.
7. Neill, C. R. Closure of Discussion of Hydraulic Roughness of Corrugated Pipes. Proc., ASCE, Vol. 89, HY-4, New York, N. Y., pp. 205-208, July 1963.
8. Webster, Morris J., and Metcalf, Lawrence R. Proc., ASCE, Hydraulics Division, Sept. 1959.