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54 Traffic Flow
53 Traffic Control and Operations
55 Traffic Measurements

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Foreword

Record 118 contains material devoted to statistical and mathematical considerations of traffic movement. The ever-increasing complexities of urban traffic have fostered research designed to employ the use of mathematics and statistics in an effort to unfold some of the relationships that exist.

The five papers and one abridgment presented are concerned with theoretical aspects of ramp capacity, better movement of traffic through traffic signals, driver performance at stop-controlled intersections, fundamentals of driver decisions at intersections, use of short-term traffic counts to arrive at long-term traffic figures, and use of a computer to simulate driver behavior at intersections.

Robert Dawson and Harold Michael in their paper analyze capacities of three different freeway on-ramp designs by employment of a deterministic queuing model for the prediction of possible capacity, development of a Monte Carlo simulation model for ramp traffic flow study, evaluation of delay and queue length incurred by on-ramp vehicles, and evaluation of the possible and practical capacities for the three different ramp designs.

John Little, Brian Martin, and John T. Morgan have collaborated in producing the the next paper on synchronizing traffic signals for maximal band width (or green time at signalized intersections). By use of an IBM 1620 computer, a mathematical method for calculating maximum band widths, and consequently better traffic movement through signalized intersections, has been set forth.

Per Solberg and Joseph Oppenlander have investigated lag and gap acceptance characteristics for vehicles entering a major roadway from a stopped position. Three methods of analysis were used and tested and found to have general agreement. Relationships expressed in mathematical terms were formed and described.

Frederick A. Wagner, Jr. has evaluated fundamental driver decision and reaction parameters at a stop-controlled intersection. Among other results Wagner found that traffic factors most greatly affecting driver decisions were traffic pressure, direction of movement, and sequences of gap information.

Robert Drusch has evolved a procedure for estimating annual average daily traffic from short-term counts. Using a system of referral to traffic counts made in past years and a system of moving base averages, a feasible system was evolved for rural roads in Missouri.

Edwin A. Kidd and Kenneth R. Laughery have formulated a digital computer model for the highway intersection situation. Involving perceptual, decision-making, and response processes of the driver, the model presents a simulation of human behavior in a dynamic control task. Preliminary study indicated that the model is reasonable and realistic and would serve as a basis for further study of driving behavior.

This Record will be of chief interest to traffic researchers although some items will be of interest to those concerned with aspects of freeway and intersection design and capacity. Traffic engineers will find the intersection performance papers and the signalized intersection paper of interest.

Contents

Analysis of On-Ramp Capacities by Monte Carlo Simulation

ROBERT F. DAWSON and HAROLD L. MICHAEL
Respectively, Assistant Professor of Civil Engineering, University of Vermont; and Associate Director, Joint Highway Research Project, Purdue University

•IN recent years thousands of miles of freeway-type highways have been constructed to provide for the safe, convenient and efficient transportation of persons and goods. Access to these facilities is provided by on-ramps designed to merge ramp traffic into the high-speed, high-volume traffic stream. The efficiency of traffic movement on freeways, and the extent to which the potential capacity of freeways can be realized, depends in part on the adequacy of the access facilities. Improperly designed entrances limit the volume of traffic that can use an expressway and generate congestion that often extends back onto the local system.

PURPOSE AND SCOPE

The purposes of this study were the following:

1. To develop criteria for defining the practical capacity of freeway on-ramps;
2. To develop general models for the analysis of flow through ramp-freeway merge areas;
3. To evaluate vehicle delays and queue lengths incurred by on-ramp vehicles for various combinations of freeway and ramp volumes; and
4. To define the practical capacity of each of three different freeway on-ramp design-control situations.

Freeway on-ramp capacity is controlled at one or more of three locations along the typical ramp. These locations are: (a) the entrance to the ramp from the local system or another freeway, (b) the ramp proper, and/or (c) the merge area at the freeway terminal of the ramp. This study was devoted to an analysis of the merge area at the freeway terminal, as it is more commonly the restricting element of the ramp.

Only ramps with geometric configurations wherein on-ramp merge maneuvers are not compounded with off-ramp diverge maneuvers were considered. Thus the analysis is pertinent to the on-ramps of diamond interchanges and to the outer-loop connectors of cloverleaf interchanges. Typical ramp-terminal designs and controls used on existing freeways were analyzed and compared: no acceleration lane with stop-sign control, no acceleration lane with yield-sign control, and an acceleration lane with no sign control. The layouts assumed for these control situations are shown for no acceleration lane with stop- or yield-sign control (Fig. 1) and for an acceleration lane with no sign control (Fig. 2).

The conduct of a field study of adequate scope was impractical with respect to both time and cost, and data from numerous traffic studies of existing access facilities located throughout the country were already available. The existence of these data, plus the availability of a modern high-speed digital computer, suggested the development of simulation models for analyzing ramp capacity.

Paper sponsored by Committee on Highway Capacity.

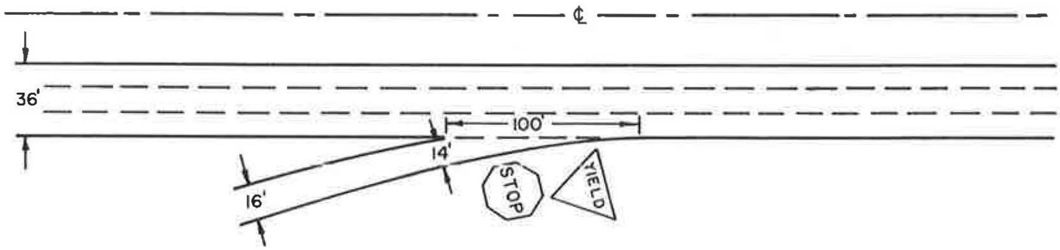

Figure 1. Typical on-ramp without acceleration lane.

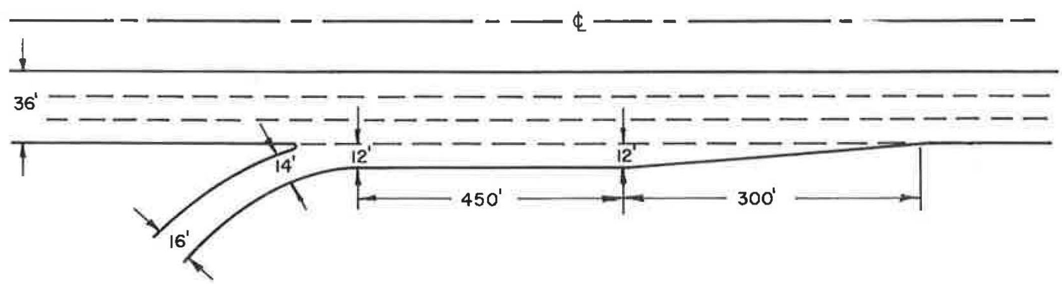

Figure 2. Typical on-ramp with acceleration lane.

CRITERION FOR ON-RAMP CAPACITY

The term capacity, as it is applied to highway traffic facilities, is not uniquely descriptive. In general, it pertains to the ability of a facility to accommodate traffic; but without some criterion indicative of the level of performance associated with the volume of flow, a statement of numerical capacity is incomplete.

Two variables that are indicative of level of performance, and that could be used in an analytical determination of capacity, are vehicle delay and queuing characteristics. Vehicle delay might be expressed in terms of the average delay incurred by a vehicle for various combinations of ramp and freeway volumes, or as the probability that delay exceeds some established level. Queuing characteristics could be defined by the mean queue length, or in terms of some percentile value such as the 85th percentile queue length.

In an attempt to establish a uniform capacity concept, the Highway Capacity Committee of the Highway Research Board (1) adopted definitions for highway and intersection capacity at a fairly free level of flow termed practical capacity. Since the area over which the on-ramp merges with the freeway is an at-grade intersection, the definition for the practical capacity of an intersection was adopted with two modifications. The original definition stated:

> The Practical Capacity of an intersection approach under signal control is the maximum volume that can enter the intersection from that approach during one hour with most of the drivers being able to clear the intersection without waiting for more than one complete signal cycle.

The qualitative index "with most of the drivers being able to clear the intersection" was replaced by the quantitative index "with 85 percent of the drivers being able to clear the intersection;" and since signals are not commonly used for traffic control on on-ramps, the time unit "one signal cycle" was replaced by an approximately equivalent time period of "60 seconds." The definition proposed for the practical capacity of a freeway on-ramp is as follows:

The practical capacity of a freeway on-ramp is the maximum volume of vehicles that can enter the freeway during one hour with 85 percent of the drivers being able to leave the ramp without being delayed for more than 60 seconds.

DESCRIPTORS OF RAMP SITUATION

The many variables involved in the operation of the ramp area traffic system can be given five classifications: roadway characteristics, vehicle characteristics, driver characteristics, traffic and environmental conditions, and rules of operations.

Roadway Characteristics

Geometric Layout. —The dimensions of the two on-ramp geometric layouts (Figs. 1 and 2) assumed for the no acceleration-lane design were based on a survey of plans of existing facilities. This design provided 108 ft for deceleration from ramp speed to a stop at the stop-line. It also included 92 ft of transition from the stop-line to the point of entry into the shoulder lane.

The dimensions for the acceleration-lane design were based on a survey of recommended ramp designs. With a 450-ft acceleration lane and a 300-ft taper, ramp vehicles had approximately 500 ft of acceleration distance available before encroaching on the shoulder lane. This distance was just adequate to provide for acceleration from a stop at the ramp nose to the maximum average shoulder-lane speed. Again the ramp geometry was adequate for the driver to decelerate from the ramp speed to a stop at the ramp nose with a comfortable rate of deceleration, when such a maneuver was deemed necessary.

Traffic Control. —Three separate traffic-control conditions were analyzed. Both stop-sign and yield-sign control devices were established on the no acceleration-lane layouts; no sign control was established on the acceleration-lane layout. Results reported from previous research indicated that a stop-sign control device on a ramp often functions as a yield sign, or as a composite of a stop sign and a yield sign, but for the purposes of this study each device was assumed to function in accordance with the Uniform Vehicle Code.

Vehicle Characteristics

All of the vehicles traversing the ramp system, whether on the ramp or on the shoulder lane, were assumed to have the geometric and operating characteristics of passenger cars. Overall length was established at 16.5 ft, the approximate average for all passenger cars, although this is considerably shorter than the AASHO defined P design vehicle (1).

In addition, each vehicle was assigned constant acceleration and deceleration potentials of 5 and 6 mi/hr/sec, respectively. In reality, acceleration and deceleration rates have distributions which are functions of the vehicle, the driver, the roadway, and the environment; but because of inadequate data and for simplicity, these variables were defined as constant vehicle characteristics.

Driver Characteristics

PIEV Time. —Although the driver is probably the most complex and certainly the dominant element in the ramp traffic system, he was modeled as a relatively simple machine with a capability for completing the PIEV process in 1.5 sec. Perception, intellection and volition time requirements vary among and within drivers, as well as among situations, but lack of information on this distribution led to the selection of the foregoing constant as a representative time for the average driver.

Minimum Time and Space Clearances. —The minimum time and space clearances a driver demands as a buffer between himself and a lead vehicle are undoubtedly closely related to his PIEV time. Various minimum clearances were established. A driver normally would not position his vehicle with less than 5 ft of clearance to a leading vehicle, and he would not move into a shoulder-lane gap behind a shoulder-lane vehicle with a time clearance of less than 0.5 sec. Minimum clearance time for ramp vehicles following a leading ramp vehicle through the system varies with the ramp design and the type of traffic control; the minimum was established at 2.0 sec with no acceleration lane and yield-sign control, whereas it was set at 1.8 sec with an acceleration lane and no control. In the latter case sudden, abrupt stops are less likely to occur. No limits were established for the stop-sign condition as the minimum clearance to a leading ramp vehicle never controls. Minimum headway spacings in the moving ramp and shoulder-lane streams were also defined.

Gap Acceptance. —Gap acceptance was the final driver characteristic to be modeled. From studies conducted in recent years (7, 8, 9, 10, 11, 16, 18, 19, 20), it was possible to develop two families of gap-acceptance models. In the case of on-ramps without acceleration lanes, and with either stop-sign or yield-sign control, the gap acceptance models were of the following form:

$$\text{Pr (Acpt)} = 1 - e^{-\left(\frac{t - t_{min}}{\bar{t} - t_{min}}\right)} \tag{1}$$

where

Pr (Acpt) = probability of accepting a gap of length, t;
t = any gap greater than t_{min};
t_{min} = minimum acceptable shoulder-lane gap; and
$\bar{t}$ = average acceptable shoulder-lane gap.

The gap-acceptance models for ramps with acceleration lanes and no sign control were of the general form:

$$\text{Pr (Acpt)} = \ln\left(\frac{t}{t_{min}}\right) * \frac{1}{\ln\left(\frac{t_{max}}{t_{min}}\right)} \tag{2}$$

where

t = any gap length between the limits of t_{min} and t_{max};
t_{min} = minimum acceptable gap; and
t_{max} = minimum gap length for which probability of acceptance is one.

In both cases distinction was made between gap acceptance by stopped, first-in-line vehicles and gap acceptance by vehicles moving as they passed the first-in-line position.

Traffic and Environmental Characteristics

Traffic and environmental characteristics are presented together as they are closely related. Changes in environmental conditions such as weather, lighting, and roadside development tend to modify traffic characteristics. For the purposes of this research environmental conditions were assumed ideal.

Traffic Distribution Between Lanes. —The number of vehicles that can enter the freeway from an on-ramp, in any time period, is directly affected by the volume of freeway traffic using the shoulder lane. To provide some flexibility for the applications of results, the simulation analyses were made with shoulder-lane volume as an independent variable. The highway designer using these results is free to select a technique for estimating the distribution of the freeway traffic between the through lanes. Most of the models (2, 7, 10, 11, 12, 16) available for this purpose describe lane distribution as a function of the number of lanes and the total one-direction flow. Two recent studies, by Moskowitz and Newman (13), and by Hess (4), reported that lane distribution is a more complex phenomenon involving variables such as the number of freeway lanes, total freeway volume, distance upstream to last off-ramp, traffic volume off at next off-ramp, and ramp traffic on the ramp under consideration.

Headway-Vehicle Generators. —Several probabilistic models are available as descriptors of headways in traffic streams. The more common ones are the negative-exponential distribution (3), the shifted-exponential distribution (3), the hyper-exponential distribution (6, 17), and a modified binomial distribution (3, 20). For the purposes of this study the shifted-exponential model was used to describe headways in the shoulder-lane stream, and the hyper-exponential model was used to describe ramp headways.

The shifted-exponential model is described by

$$P(h \geq t) = e^{-\left(\frac{t - D}{\bar{t} - D}\right)} \tag{3}$$

where

$P(h \geq t)$ = probability that a headway is equal to or greater than t;
t = any time;
$\bar{t}$ = average headway in stream,
= 3,600/hourly vol; and
D = minimum allowable headway in stream.

By trial-and-error, D values were defined for various shoulder-lane volumes to effect an apparent good fit to the headway curves for multilane traffic streams given in the Highway Capacity Manual (2).

The hyper-exponential headway distribution used to describe the ramp traffic was originally proposed by Schuhl (17), but the necessary statistical evaluation was performed by Kell (6). This distribution is based on the theory that a traffic stream is made up of two populations of moving vehicles, a restrained population and a free-moving population, each with its own headway distribution. The overall headway distribution is therefore defined by

$$P(h \geq t) = (1 - \alpha)\, e^{-\left(\frac{t - \Delta_1}{T_1 - \Delta_1}\right)} + \alpha\, e^{-\left(\frac{t - \Delta_2}{T_2 - \Delta_2}\right)} \tag{4}$$

where

α = proportion of traffic stream in restrained population;
$1 - \alpha$ = proportion of traffic stream in free-moving population;
T_1 = average headway of free-moving population;
T_2 = average headway of restrained population;
Δ_1 = minimum allowable headway of free-moving population; and
Δ_2 = minimum allowable headway of restrained population.

Kell evaluated the parameters of this model on a two-lane urban street on which there was negligible passing opportunity. Since the characteristics of a one-lane ramp are not unlike those of the directional channels of an urban street, Kell's model was accepted as an adequate descriptor of headways in a ramp stream.

Speed Models. —Although speed is known to follow an approximately normal distribution in freeway flow, this characteristic was described by much simpler models for the ramp and shoulder-lane streams. All ramp vehicles were assigned a speed of 30 mph, on generation into the system, on the assumption that ramp geometry governs speed regardless of traffic conditions. Shoulder-lane speeds were described by an equation developed from the results of research at the Midwest Research Institute (7):

$$SP = 52.0 - 0.008\, V_S \tag{5}$$

where

SP = shoulder-lane speed in ramp vicinity, and
V_s = shoulder-lane volume (vph)

Rules of Operation

The rules of operation include a queuing discipline and rules for the driver-vehicle under various traffic conditions.

Queuing Discipline. —The appropriate queuing discipline is established by the physical situation. The geometry of the ramp area provides service to ramp traffic on a first-come, first-served basis, i.e., no trailing vehicle can preempt service priority and pass a leading vehicle to accept a gap in the shoulder lane.

Vehicle and/or Driver Behavior. —A driver arriving at the entry point to the ramp system should immediately decide his course of action. If there is no acceleration lane and stop-sign control exists, the driver's decision should be to decelerate to a stop. Since there are 108 ft available between the point of entry into the ramp system and the stop-line in this study, this maneuver can be affected at a comfortable rate of deceleration. On leaving the ramp area all drivers are assumed to use the same acceleration rates and require the same minimum time and space clearances previously established.

In the cases of no acceleration lane with yield-sign control, and an acceleration lane with no sign control, the driver's decision process at the point of entry into the system is somewhat more complex. On passing this entry point he should evaluate both shoulder-lane and ramp traffic conditions. His decision may be to stop on, or before, reaching the stop-line; or his decision may be to proceed through the ramp area and into the shoulder lane. To arrive at the latter decision the driver has to project the positions and speeds of all other vehicles in the system, as well as his own, to the most critical point in both time and space. His decision to stop or proceed is based entirely on gap acceptance. He may determine the acceleration-deceleration pattern, within the capabilities established for his vehicle, that will maximize his probability of accepting a gap. It is probable, however, that a driver will not follow the speed pattern that maximizes the gap available to him (maximizing the gap maximizes the probability of acceptint the gap), but he undoubtedly considers the best situation he can create for himself before making a decision.

Some restrictions were necessary, however, to control this complex situation so that a model could be developed. It was assumed that the driver would stop at the stop-line if an alternate course of action would result in a speed downstream from the stop-line lower than the speed attained during acceleration from a stop at the stop-line.

MONTE CARLO ON-RAMP SIMULATORS

The ramp situation was described in micro detail. A general description of the macro framework within which the micro models were assembled as functional systems is now presented.

Structure of the Simulator

The on-ramp traffic simulators were programmed in FORTRAN IV and MAP coding (5) using both open and closed subroutines under the control of a monitor or master program. The main advantage of this type of structuring is the relative simplicity with which small segments of the overall model can be isolated, programmed, tested, and "debugged." In fact, by documenting each of the segmented programs with descriptive comments written in English, it was possible to prepare completely intelligible simulator programs without first preparing flow diagrams. The advantage of a program that can be readily digested by both the engineer and the computer is obvious.

Sampling the Simulated Traffic

Simulation runs were initiated with an empty system. That is, there were no vehicles in the simulation area when relative simulation time was zero. If the traffic characteristics of the first few simulated vehicles had been recorded and considered in the analysis of the level of performance, they would undoubtedly have biased the results. To guard against this bias the simulator was loaded before the actuation of the surveillance system. This pre-loading was effected by simulating the flow of 300 ramp vehicles through the ramp area; of course, the shoulder-lane flow was simulated simultaneously, but the number of shoulder-lane vehicles involved in the pre-loading operation was a function of the ratio of shoulder-lane volume to ramp volume. During this initial period no delay or queuing characteristics were recorded. The number of ramp vehicles that were simulated for pre-loading purposes was established arbitrarily; but it was assumed that 300 vehicles (an average of approximately one-half hour of real traffic flow) was adequate to establish equilibrium conditions in the ramp area.

Following the pre-loading operation the surveillance system was actuated and an additional 1,000 ramp vehicles were generated and observed. In this case the sample size was established by a dollars constraint rather than by statistical design. After estimates of running time had been prepared from the results of a pilot study, sample sizes were established to conform with the available project funds.

RESULTS AND DISCUSSION

The results obtained from the simulator runs did not directly describe on-ramp capacities. Each simulation run produced a record which described delay and queue characteristics at various combinations of shoulder-lane and ramp volumes. Subsequent statistical analyses of the delay characteristics provided the bases for descriptions of capacity. Related queuing characteristics were described by both graphical and mathematical models.

Simulation Results

Generated vs Requested Volumes.—The ramp and shoulder-lane traffic flows were generated by a simulated-sampling technique whereby theoretical headway distributions were sampled using random numbers. At the start of each simulation run parameters of the headway distributions were established for the particular volumes desired. The generated volumes varied slightly from the requested volumes due partly to sampling error and partly to small discrepancies inherent in the equations for predicting the volume related distribution parameters.

TABLE 1

RAMP VOLUMES GENERATED BY SIMULATOR COMPARED WITH RAMP VOLUMES REQUESTED

Volume (veh/hr)	
Requested	Generated
100	92
200	194
300	297
400	405
500	503
600	598
700	694
800	783
900	872
1,000	957
1,100	1,052
1,200	1,156

Table 1 compares requested and generated ramp volumes. Since each simulation run was continued until 1,300 ramp vehicles had been generated, and since the same sequence of random numbers was used for each run, identical ramp volumes were generated each time the same volume was requested. In contrast, the number of shoulder-lane vehicles generated at a given volume level was dependent on the ramp volume with the result that the simulated shoulder-lane volumes were different for almost every run. Table 2 compares the requested and the generated shoulder-lane volumes.

Traffic Performance Characteristics.—Traffic performance on each of the three ramp designs was described by six characteristics for each combination of ramp and shoulder-lane volumes. These six characteristics were the average length of queue, and the 85th, 90th, and 95th percentile queue lengths found on the ramp,

TABLE 2

SHOULDER-LANE VOLUMES GENERATED BY SIMULATOR AT VARIOUS REQUESTED COMBINATIONS OF RAMP AND SHOULDER-LANE TRAFFIC VOLUMES

Shoulder-Lane Volume Requested (veh/hr)	Ramp Volume Requested (veh/hr)											
	100	200	300	400	500	600	700	800	900	1,000	1,100	1,200
100	100	97	93	91	89	89	90	91	85	82	80	77
200	197	200	194	194	185	186	182	180	180	178	180	178
300	300	297	302	294	291	291	282	280	277	277	271	267
400	395	396	394	402	393	387	391	390	381	373	373	371
500	498	494	494	497	503	493	493	482	489	487	481	471
600	598	598	595	594	598	603	587	590	584	576	587	585
700	694	701	694	693	694	693	704	690	687	689	677	672
800	792	792	791	791	788	789	799	804	795	790	784	788
900	891	890	897	887	893	893	891	900	900	903	895	
1,000	992	993	1,003	990	991	989	994	990	993	994		
1,100	1,097	1,098	1,099	1,098	1,086	1,095	1,084	1,092	1,089			
1,200	1,199	1,198	1,190	1,200	1,188	1,191	1,192	1,193				
1,300	1,298	1,296	1,287	1,300	1,282	1,283						
1,400	1,393	1,390	1,386	1,402	1,396							
1,500	1,495	1,490	1,494	1,498	1,502							
1,600	1,596	1,587	1,593	1,585								
1,700	1,695	1,685	1,698									
1,800	1,796	1,783										

TABLE 3

PREDICTION EQUATIONS
No Acceleration Lane—Stop-Sign Control[a]

Ramp Vol (veh/hr)	Regression Coefficients			Statistical Characteristics			
				Range of Analysis			
	a	b	c	Low x	High x	R^2	No. of Observ.
			Average Queue Lengths				
100	-1.9975	+0.0000625	+0.0000012797	100	1,800	0.963	18
200	-1.6397	+0.0018591	+0.0000009418	100	1,200	0.971	12
300	-1.5500	+0.0043815	+0.0000002995	100	800	0.984	8
400	-0.9395	+0.0032663	+0.0000071594	100	600	0.996	6
500	-0.7899	+0.0002321	+0.0000079973	100	400	0.999	7
600	+0.3085	-0.0010991	+0.0000643069	50	225	0.998	8
700	-2.0566	+0.0751588	-0.0002365566	50	150	0.972	5
			Average Delay (sec)				
100	+2.9952	-0.0013762	+0.0000016831	100	1,800	0.938	18
200	+2.7652	-0.0004893	+0.0000019405	100	1,200	0.967	12
300	+3.1612	-0.0034973	+0.0000077949	100	900	0.947	9
400	+2.8651	-0.0019709	+0.0000116908	100	600	0.998	6
500	+2.6811	-0.0004187	+0.0000189636	100	400	0.999	7
600	+3.2122	-0.0072780	+0.0000717930	50	225	0.996	8
700	+1.0967	+0.0513798	-0.0001375146	50	150	0.979	5
			Probability that Delay Exceeds 60 Sec				
100	-11.7913	+0.0123782	-0.0000032853	500	1,800	0.985	14
200	-9.3662	+0.0126359	-0.0000042133	200	1,200	0.936	11
300	-9.5943	+0.0205038	-0.0000111879	300	800	0.969	6
400	-8.0900	+0.0249755	-0.0000193204	200	600	0.996	5
500	-9.4459	+0.0493143	-0.0000651674	100	400	0.995	7
600	-13.3415	+0.1252529	-0.0003035478	100	225	0.985	7
700	-6.6710	+0.1034048	-0.0004176055	50	120	0.941	5

[a]Prediction model: $y = e^{(a + bx + cx^2)}$ where x is shoulder-lane volume expressed in vehicles per hour.

the average delay incurred by ramp vehicles, and the probability that the delay incurred exceeds 60 sec. Empirical least-square prediction models were constructed to describe average queue length, average delay, and the probability that delay exceeds 60 sec as functions of shoulder-lane volume, with ramp volume held constant at several different levels. The results of these model analyses are summarized in Tables 3 through 5. The various estimates obtained are plotted as functions of shoulder-lane volume in Figures 3 through 11.

Practical Capacity Analysis

Numerical Limits for Practical Capacity. —The empirical models describing the probability that a vehicle will incur delay in excess of 60 sec were used to define the practical capacities of the three ramp designs. These models were solved at each level of ramp volume to establish the shoulder-lane volume at which the probability of delay

TABLE 4

PREDICTION EQUATIONS
No Acceleration Lane—Yield-Sign Control[a]

Ramp Vol (veh/hr)	Regression Coefficients			Statistical Characteristics			
	a	b	c	Range of Analysis: Low x	Range of Analysis: High x	R^2	No. of Observ.
Average Queue Lengths							
100	-4.2729	+0.0033557	-0.0000000456	100	1,800	0.977	18
200	-3.2649	+0.0035794	-0.0000001103	100	1,400	0.964	14
300	-3.0580	+0.0048794	-0.0000002977	100	1,100	0.973	11
400	-2.5712	+0.0032686	-0.0000023291	100	1,000	0.982	10
500	-2.4593	+0.0045387	-0.0000025817	100	900	0.972	9
600	-1.6940	+0.0021831	-0.0000065882	100	800	0.988	8
700	-1.5052	+0.0026259	-0.0000078505	100	700	0.987	7
800	-1.4855	+0.0039350	-0.0000076947	100	700	0.996	7
900	-1.2510	+0.0042842	-0.0000097527	100	600	0.994	6
1,000	-1.1467	+0.0055615	-0.0000116985	100	500	0.991	5
1,100	-1.4023	+0.0081140	-0.0000135458	100	400	0.999	4
1,200	-2.2924	+0.0261991	-0.0000222119	100	450	0.994	8
Average Delay (sec)							
100	-0.0969	+0.0022760	+0.0000004194	100	1,800	0.984	18
200	+0.0931	+0.0025321	+0.0000006884	100	1,400	0.973	14
300	-0.1598	+0.0036479	+0.0000005528	100	1,100	0.984	11
400	+0.0532	+0.0024242	+0.0000027514	100	1,000	0.981	10
500	+0.1778	+0.0024766	+0.0000041782	100	900	0.975	9
600	+0.5665	+0.0009040	+0.0000076655	100	800	0.988	8
700	+0.6343	+0.0010002	+0.0000094279	100	700	0.986	7
800	+0.5380	+0.0019045	+0.0000101179	100	600	0.990	6
900	+0.6158	+0.0028395	+0.0000113563	100	600	0.996	6
1,000	+0.5615	+0.0041011	+0.0000137510	100	500	0.994	5
1,100	+0.4076	+0.0056497	+0.0000167166	100	400	0.999	4
1,200	-0.2721	+0.0189543	-0.0000071903	100	400	0.994	7
Probability that Delay Exceeds 60 Sec							
100	-16.1550	+0.0170741	-0.0000045908	700	1,800	0.989	12
200	-15.9561	+0.0191254	-0.0000056410	600	1,400	0.988	9
300	-17.5748	+0.0271073	-0.0000105851	500	1,100	0.988	7
400	-21.3262	+0.0370901	+0.0000156363	500	1,000	0.987	6
500	-19.3923	+0.0393628	+0.0000194069	400	850	0.998	6
600	-29.0753	+0.0781986	+0.0000528842	400	750	0.986	5
700	-47.3614	+0.1526573	-0.0001346453	400	650	0.981	6
800	-21.6138	+0.0675914	+0.0000529782	300	625	0.997	5
900	-28.8463	+0.1063333	-0.0000980491	300	550	0.999	6
1,000	-14.7238	+0.0494355	+0.0000386172	200	500	0.966	4
1,100	-21.5913	+0.1035501	-0.0001235440	200	400	0.999	5
1,200	-26.5800	+0.1917022	-0.0003487793	150	300	0.999	4

[a]Prediction model: $y = e^{(a + bx + cx^2)}$ where x is shoulder-lane volume expressed in vehicles per hour.

TABLE 5

PREDICTION EQUATIONS
Acceleration Lane—No Sign Control[a]

Ramp Vol (veh/hr)	Regression Coefficients			Statistical Characteristics			
				Range of Analysis		R^2	No. of Observ.
	a	b	c	Low x	High x		
Average Queue Lengths							
100	-5.0258	+0.0016410	+0.0000005153	200	1,800	0.978	17
200	-4.6655	+0.0022476	+0.0000005141	200	1,800	0.978	17
300	-4.4841	+0.0028177	+0.0000005433	100	1,700	0.988	17
400	-4.3162	+0.0019374	+0.0000018261	100	1,600	0.974	16
500	-4.2762	+0.0027761	+0.0000017289	100	1,500	0.981	15
600	-4.4491	+0.0043213	+0.0000010611	100	1,300	0.983	13
700	-3.8462	+0.0027600	+0.0000027415	100	1,200	0.989	12
800	-3.0714	+0.0004599	+0.0000050207	100	1,200	0.994	12
900	-4.1394	+0.0047775	+0.0000027603	100	1,100	0.988	11
1,000	-3.5794	+0.0053980	+0.0000024726	100	1,000	0.988	10
1,100	-3.1336	+0.0051631	+0.0000031242	100	900	0.991	9
1,200	-3.0124	+0.0065285	+0.0000026169	100	800	0.984	8
Average Delay (sec)							
100	-2.1045	+0.0029398	-0.0000000386	100	1,800	0.985	18
200	-1.6750	+0.0023434	+0.0000004250	100	1,800	0.990	18
300	-1.6785	+0.0023707	+0.0000007087	100	1,700	0.989	17
400	-1.4401	+0.0016767	+0.0000014566	100	1,500	0.987	15
500	-1.5498	+0.0021741	+0.0000015970	100	1,400	0.987	14
600	-1.4659	+0.0019993	+0.0000021832	100	1,300	0.985	13
700	-1.1070	+0.0005302	+0.0000039077	100	1,200	0.991	12
800	-0.9499	+0.0003271	+0.0000044638	100	1,100	0.995	11
900	-1.4987	+0.0027609	+0.0000032714	100	1,000	0.989	10
1,000	-1.0421	+0.0018696	+0.0000050159	100	1,000	0.992	10
1,100	-0.9161	+0.0021628	+0.0000054528	100	900	0.993	9
1,200	-0.7589	+0.0024525	+0.0000063174	100	800	0.991	8
Probability that Delay Exceeds 60 Sec							
100	-12.7287	+0.0026419	+0.0000019410	1,200	1,800	0.927	7
200	-32.2409	+0.0318443	-0.0000080515	1,100	1,800	0.997	8
300	-27.9335	+0.0298356	-0.0000081325	1,000	1,700	0.937	8
400	-38.0136	+0.0454645	-0.0000137083	1,000	1,500	0.953	6
500	-19.3308	+0.0201150	-0.0000046270	800	1,400	0.944	7
600	-31.4166	+0.0417297	-0.0000136855	800	1,200	0.995	5
700	-56.4630	+0.0927711	-0.0000381728	800	1,200	0.989	5
800	-59.8336	+0.1030377	-0.0000444457]	800	1,100	0.991	4
900	-42.7534	+0.0738793	-0.0000315796	700	1,000	0.999	4
1,000	-39.8221	+0.0778939	-0.0000361358	600	850	0.995	5
1,100	-21.1578	+0.0339636	-0.0000090505	500	800	0.999	5
1,200	-41.9321	+0.1061389	-0.0000674651	500	750	0.999	6

[a]Prediction model: $y = e^{(a + bx + cx^2)}$ where x is shoulder-lane volume expressed in vehicles per hour.

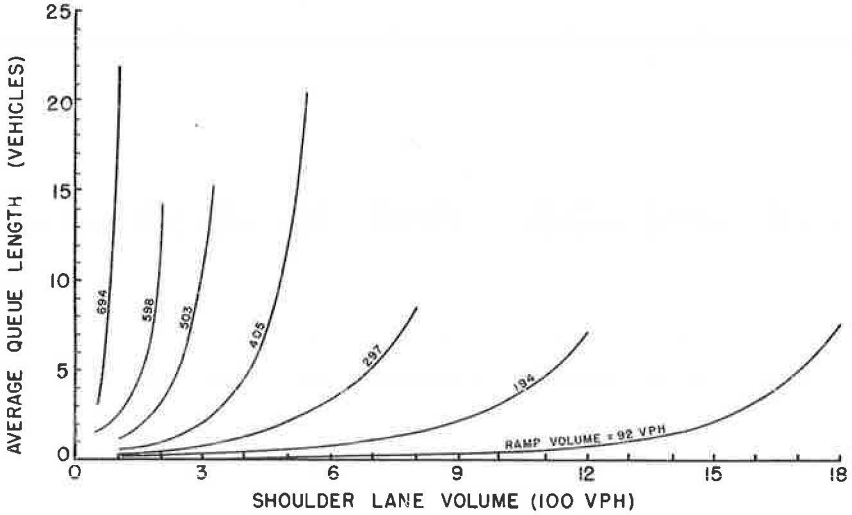

Figure 3. Average queue length on ramp with stop-sign control.

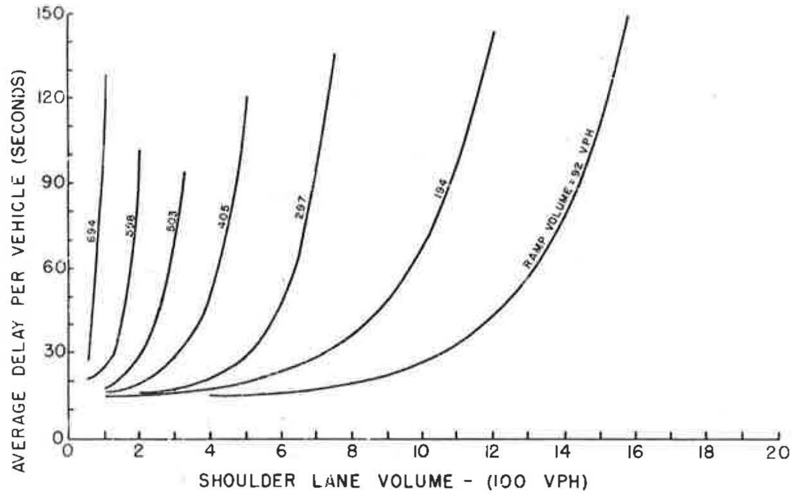

Figure 4. Average delay to ramp vehicles with stop-sign control.

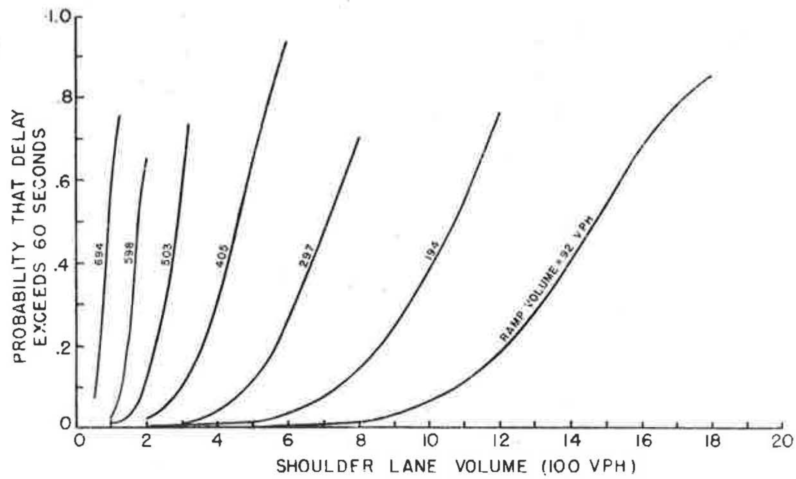

Figure 5. Probability that delay exceeds 60 sec with stop-sign control.

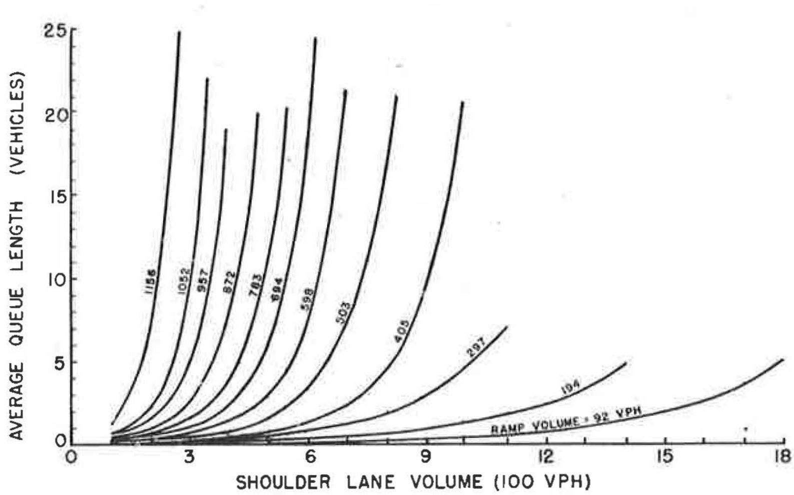

Figure 6. Average queue length on ramp with yield-sign control.

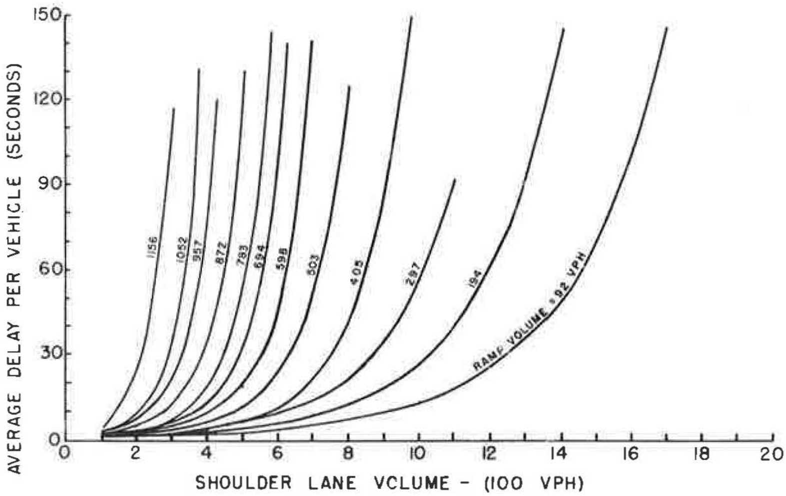

Figure 7. Average delay to ramp vehicles with yield-sign control.

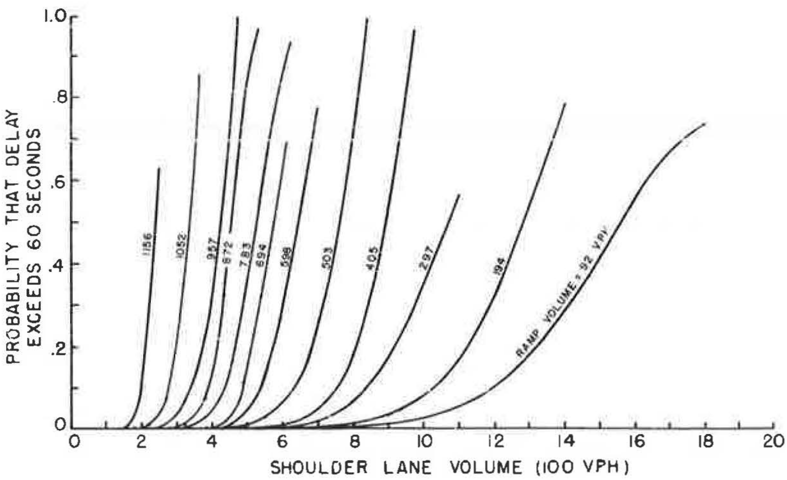

Figure 8. Probability that delay exceeds 60 sec with yield-sign control.

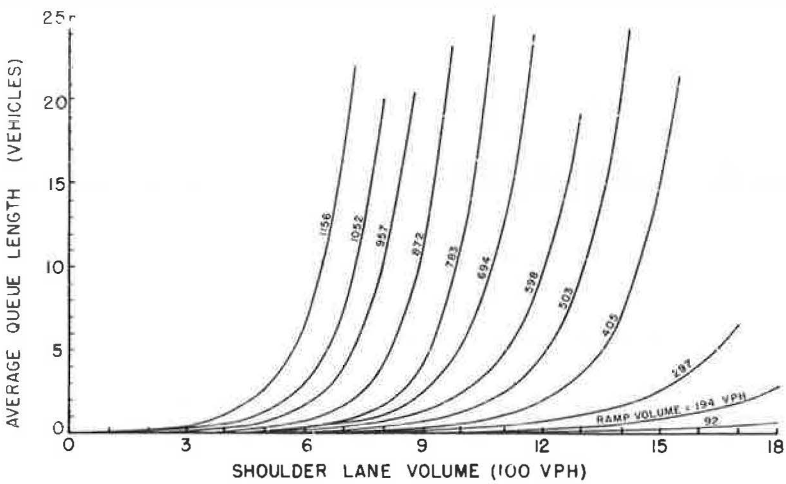

Figure 9. Average queue length on ramp with acceleration lane.

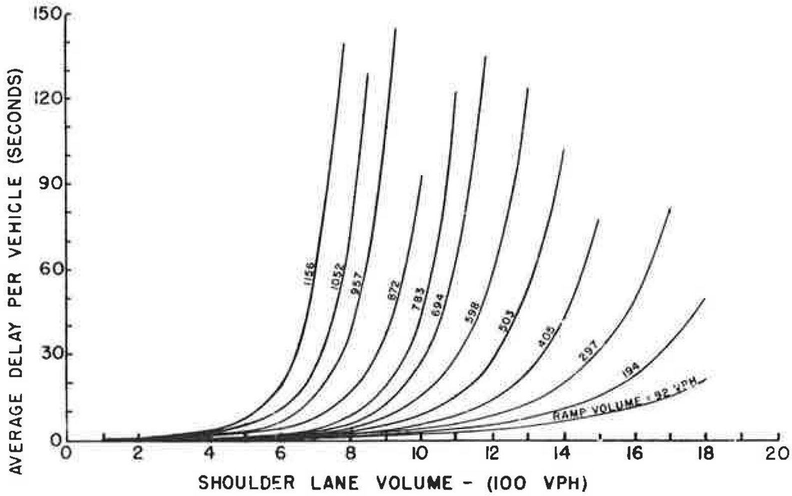

Figure 10. Average delay to ramp vehicles with acceleration lane.

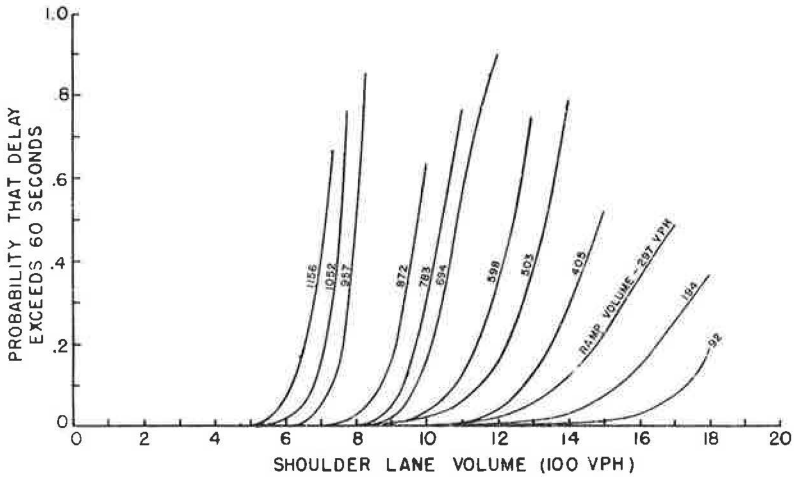

Figure 11. Probability that delay exceeds 60 sec with acceleration lane.

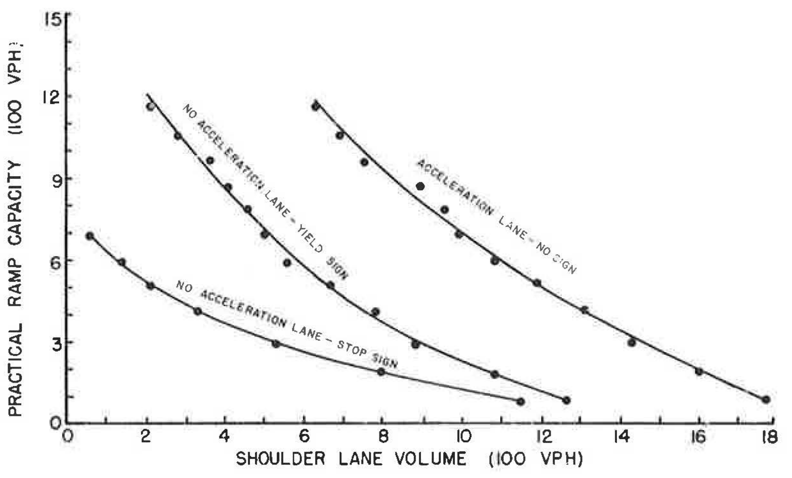

Figure 12. Practical capacities of freeway on-ramps.

in excess of 60 sec was 0.15. The resulting ramp volume, shoulder-lane volume data sets described the relationship between practical ramp capacity and shoulder-lane volume. These data sets are plotted for each of the three ramp designs in Figure 12. The curves drawn through these points are least-square fits to a model of the form

$$y = e^{(a + bx + cx^2)} \tag{6}$$

The complete analyses are summarized in Table 6.

Queuing Conditions at Practical Capacity. —The average queue-length models were solved and percentile queue data were evaluated at practical-capacity volume conditions. The various queue length estimates obtained are plotted as functions of shoulder-lane

TABLE 6

SUMMARY OF LEAST-SQUARE EQUATIONS FOR PREDICTING PRACTICAL CAPACITIES OF FREEWAY ON-RAMPS[a]

Regression Coefficients			Statistical Characteristics			
			Limits of Analysis—x			
a	b	c	Low	High	R^2	No. of Observ.
No Acceleration Lane and Stop-Sign Control						
+6.5781	-0.0014546	-0.0000002356	63	1,150	0.986	7
No Acceleration Lane and Yield-Sign Control						
+7.2901	-0.0009507	-0.0000009388	203	1,267	0.996	12
Acceleration Lane and No Sign Control						
+6.9814	+0.0008034	-0.0000012041	633	1,783	0.996	12

[a]Prediction model: $y = e^{(a + bx = cx^2)}$ where y = practical capacity (veh/hr) and x = shoulder-lane volume (vph).

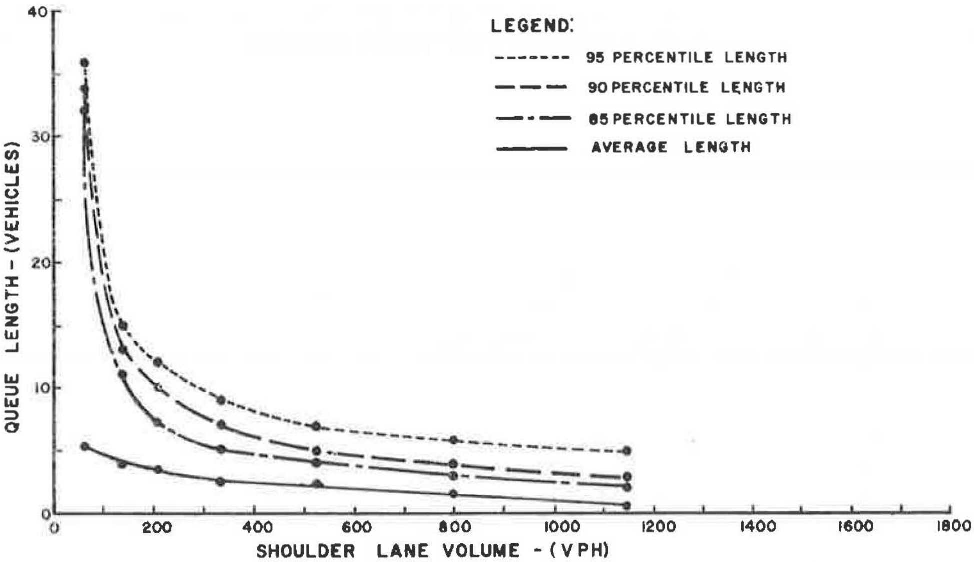

Figure 13. Queue lengths at practical capacity with stop-sign control.

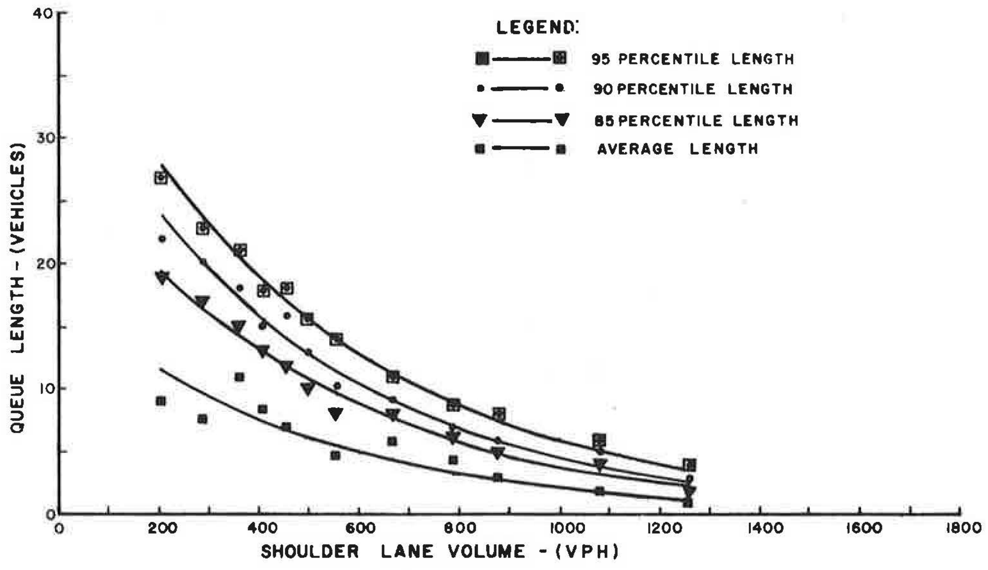

Figure 14. Queue lengths at practical capacity with yield-sign control.

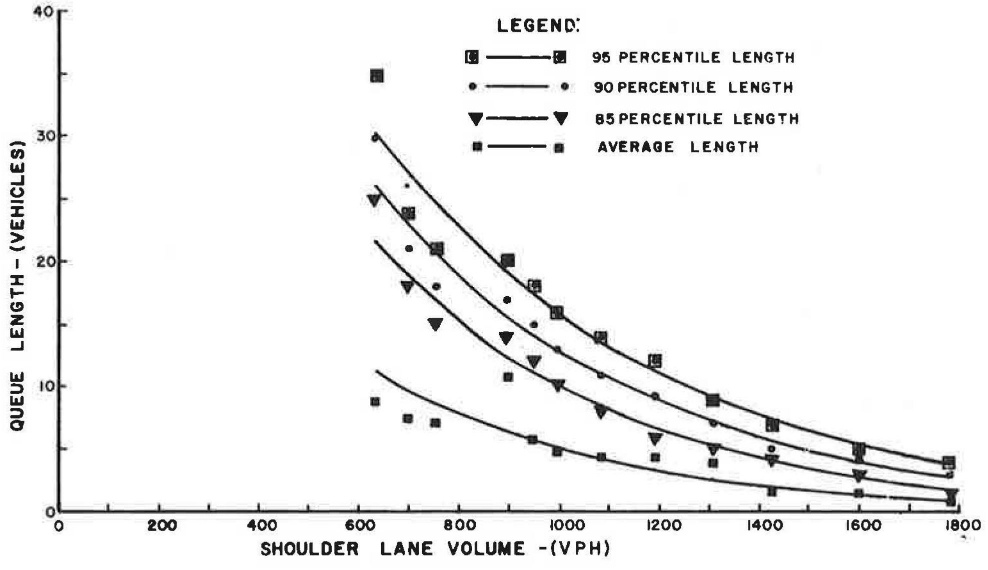

Figure 15. Queue lengths at practical capacity with acceleration lane.

volume in Figures 13, 14, and 15, respectively, for ramps with no acceleration lane and stop-sign control, no acceleration lane and yield-sign control, and an acceleration lane with no sign control.

Although there was relatively little scatter in the data describing the queuing conditions on the stop-sign controlled ramp, statistical analyses were performed for the purpose of driving prediction models. An empirical equation

$$y = e^{(a + bx)} \qquad (7)$$

was fitted to the data describing average queue lengths using the method of least-squares. Equations of the form

$$y = e^{\frac{1}{a + bx + cx^2}} \tag{8}$$

were derived for prediction of 85th, 90th and 95th percentile queue lengths. The results of the statistical analyses are given in Table 7, where the multiple R^2's (r^2 in the case of the average queue-length model) for the transformed equations were all equal to or greater than 0.967.

The various practical-capacity queuing characteristics for the ramp with yield-sign control and the ramp with an acceleration lane were described by empirical, least-square equations of the form

TABLE 7

SUMMARY OF LEAST-SQUARE EQUATIONS FOR PREDICTING QUEUE CHARACTERISTICS UNDER PRACTICAL CAPACITY CONDITIONS ON ON-RAMPS WITH NO ACCELERATION LANE AND STOP-SIGN CONTROL[a]

Variable Predicted (queue)	Regression Coefficients			Statistical Characteristics			
	a	b	c	Limits of Analysis—x Low	Limits of Analysis—x High	R^2	No. of Observ.
Average	+1.7006	-0.0017448		63	1,150	0.967 (r^2)	7
85 percent	+0.3214	+0.0006068	+0.0000002942	63	1,150	0.976	7
90 percent	+0.2714	+0.0007434	-0.0000001732	63	1,150	0.988	7
95 percent	+0.2738	+0.0005952	-0.0000002623	63	1,150	0.972	7

[a]Average queue prediction model, $y = e^{(a + bx)}$
Percentile queue prediction model, $y = e^{(1/(a + bx + cx^2))}$; where y = queue variable predicted (veh) and x = shoulder-lane volume (veh/hr) for both models.

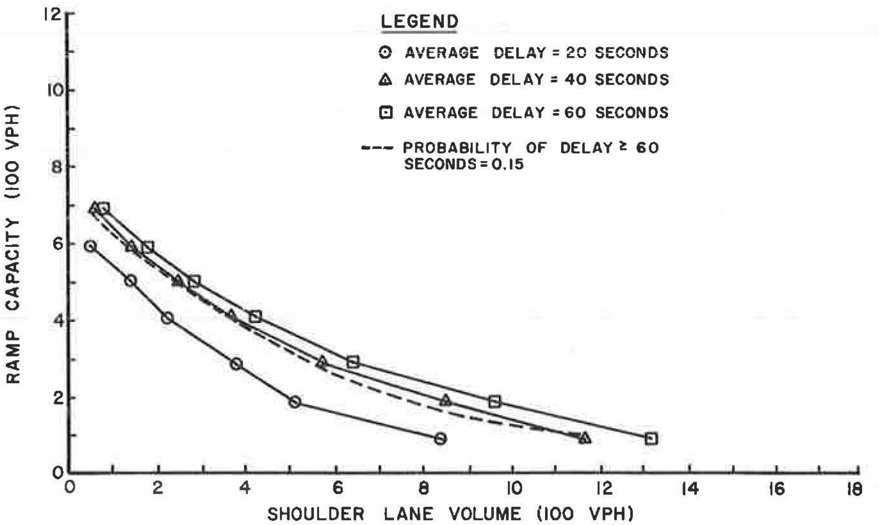

Figure 16. Comparison of capacities defined by several delay criteria for on-ramps with no acceleration lane and stop-sign control.

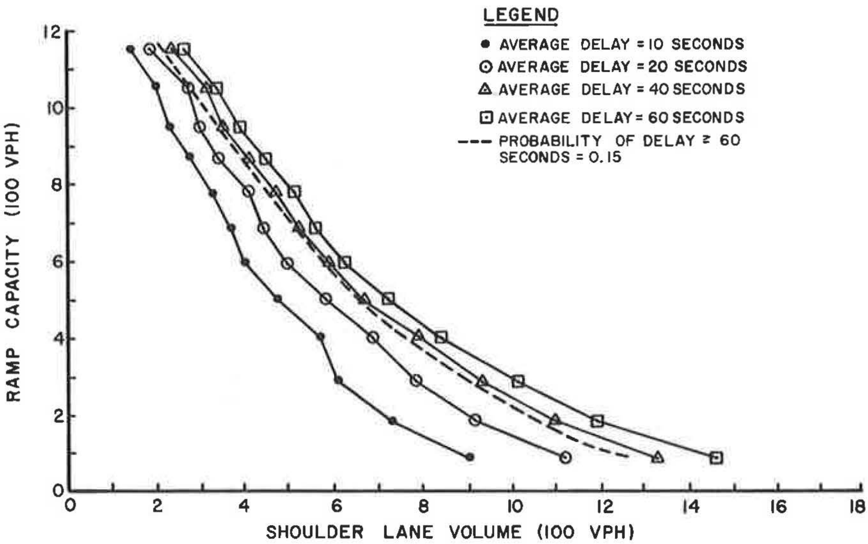

Figure 17. Comparison of capacities defined by several delay criteria for on-ramps with no acceleration lane and yield-sign control.

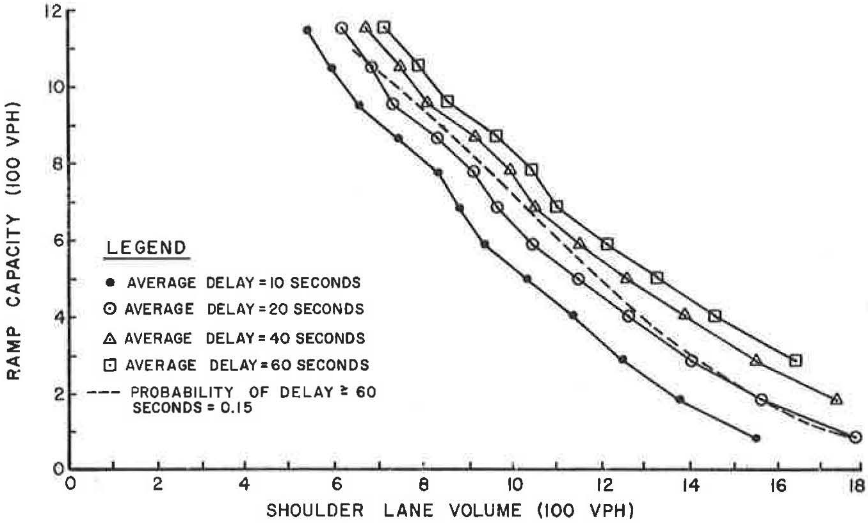

Figure 18. Comparison of capacities defined by several delay criteria for on-ramps with an acceleration lane and no sign control.

$$y = e^{(a + bx)} \tag{9}$$

and the results are given in Table 8. Because of scatter in the average queue-length data, the r^2 values were only 0.916 and 0.883 for the yield-sign control condition and the acceleration-lane condition, respectively.

Practical Capacities by Other Criteria. —In Figures 16, 17 and 18 practical-capacity relationships based on the proposed definition of practical capacity are compared to several other capacity relationships described by volume combinations that generate

TABLE 8

SUMMARY OF LEAST-SQUARE EQUATIONS FOR PREDICTING QUEUE CHARACTERISTICS UNDER PRACTICAL CAPACITY CONDITIONS ON ON-RAMPS WITH NO ACCELERATION AND YIELD-SIGN CONTROL AND ON ON-RAMPS WITH AN ACCELERATION LANE AND NO SIGN CONTROL[a]

Variable Predicted (queue)	Regression Coeff.		Statistical Characteristics			
			Limits of Analysis—x			
	a	b	Low	High	r^2	No. of Observ.
No Acceleration Lane and Yield-Sign Control						
Average	+2.8634	-0.0021425	203	1,267	0.916	12
85 percent	+3.3754	-0.0020187	203	1,267	0.980	12
90 percent	+3.6052	-0.0021090	203	1,267	0.964	12
95 percent	+3.7252	-0.0019438	203	1,267	0.984	12
Acceleration Lane and No Sign Control						
Average	+3.7821	-0.0021669	633	1,783	0.883	12
85 percent	+4.4148	-0.0021100	633	1,783	0.988	12
90 percent	+4.4861	-0.0019356	633	1,783	0.986	12
95 percent	+4.5476	-0.0017941	633	1,783	0.983	12

[a]Prediction model: $y = e^{(a + bx)}$ where y = queue variable predicted (veh) and x = shoulder-lane volume (veh/hr).

various levels of average delay. In general, practical capacities based on the proposed definition are similar to the capacities that could be realized with average delays in the range of 30 to 40 sec.

SUMMARY AND CONCLUSIONS

1. A quantitative criterion was established for measurement of the practical capacity of freeway on-ramps. This criterion was presented in the following definition: "The practical capacity of a freeway on-ramp is the maximum number of vehicles that can enter the freeway during one hour with 85 percent of the drivers being able to leave the ramp without being delayed more than 60 seconds."

2. The micro aspects of freeway on-ramp areas and their traffic were modeled in the mathematical mode, within the present understanding of traffic flow theory. In some cases, empirical estimates were substituted for presently undefined functional relationships.

3. Rules of operation were established for the on-ramp area that provided a framework within which the models describing micro aspects were assembled as functional systems. These rules were designed and implemented as control mechanisms in a computer-oriented ramp simulator. A wide range of ramp and shoulder-lane volume combinations were realistically generated by this model. Traffic monitors constructed as integral parts of the simulator measured and recorded several indexes of traffic performance. The most important of these were average and percentile queue lengths, average delay, and the probability that delay exceeds 60 sec.

4. Statistical models were derived to define the various indexes of performance as functions of ramp and shoulder-lane volume conditions for each type of ramp design considered.

5. Practical capacities were defined by obtaining solutions to the empirically derived models describing the probability of delay in excess of 60 sec. Ramp and shoulder-lane volume combinations that generated a probability of 0.15 constituted a practical capacity situation.

6. The average-queue models were solved and percentile-queue data were evaluated at practical-capacity volume conditions to obtain ramp storage requirements for ramps operating at practical capacity.

7. The results obtained from the simulation analyses can be extremely useful in the design of new on-ramp facilities and in evaluating the adequacy of existing facilities. The procedure for applying these results to a particular ramp situation involves two steps: (a) Obtain an estimate of the amount of traffic that is using, or is expected to use, the shoulder lane. This may be done by actual field study or by using Hess's lane-distribution models (4). (b) Obtain the practical capacity and performance characteristics associated with practical capacity from the appropriate models derived in this study.

8. Monte Carlo simulation is a useful, practical and efficient technique for studying freeway on-ramp operations. The proposed simulator required approximately two minutes for each combination of ramp and shoulder-lane volumes that was simulated. Although constant sample sizes of 1,000 ramp vehicles were observed on each run, variations in the ramp flow-rates resulted in variation of the real time/computer time ratio. These ratios ranged from 360/1 to 30/1. Approximately one-half of the computer time was spent pre-loading the ramp system, preparing statistical summaries of the results, and writing the simulation reports.

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Synchronizing Traffic Signals for Maximal Bandwidth

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Traffic signals can be synchronized so that a car, starting at one end of a street and traveling at preassigned speeds, can go to the other end without stopping for a red light. The portion of a signal cycle for which this is possible is called the bandwidth for that direction. Ordinarily the bandwidth in each direction is single, i. e., is not split into two or more intervals within a cycle. Two problems are solved for this case: (a) given an arbitrary number of signals along a street, a common cycle length, the green and red times for each signal, and specified vehicle speeds in each direction between adjacent signals, synchronize the signals to produce bandwidths that are equal in each direction and as large as possible; and (b) adjust the synchronization to increase one bandwidth to some specified, feasible value and maintain the other as large as is then possible. The method of calculation has been programmed for a 20K IBM 1620.

•TRAFFIC signals prevent chaos at busy intersections, but nobody likes the frequent stops that often occur on streets with many signals. The number of stops can be reduced by proper synchronization of the signals.

Consider a street with a sequence of signals all of which have the same cycle length. The bandwidth along the street will be defined as that portion of a cycle during which a car could start at one end of the street and, by traveling at preassigned speeds (not necessarily all the same), go to the other end without stopping for a red light. Each direction has its own bandwidth. For example, it is an easy matter to synchronize the signals so that, for one direction, a car that passes the first signal just as it turns green passes all others in the same way. We shall call this a complete one-way synchronization. The bandwidth for that direction is as large as possible and equals the shortest of the green times of the signals on the street. Bandwidth in the other direction, however, is likely to be small or zero, unless the distances between signals are particularly fortuitous. Signals synchronized to create a substantial bandwidth are called progression systems.

It is possible to construct examples where the bandwidth in a single cycle in a single direction is split up into two or more intervals separated by very short reds. Since it seems rather unlikely that split bandwidths would often occur in practice, and since the extension of results to cover these cases appears rather cumbersome, we restrict ourselves unless otherwise stated to problems for which the maximal bandwidths are unsplit.

Procedures are given for solving the following two problems. Problem 1: given a common cycle length, green splits for each signal, and specified speeds in each direction between adjacent signals, determine offsets for the signals, so as to produce bandwidths which are equal in each direction and as large as possible. Problem 2: adjust

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the offsets to favor one direction with a larger bandwidth, if feasible, and give the other direction the largest bandwidth then possible.

The paper is divided into three parts. After an introduction, the first section discusses the background of the problem and describes briefly a computer program that calculates the desired offsets. The second section develops the mathematical theory underlying the solution. The third section describes the computer program and its operation in detail.

BACKGROUND AND COMPUTER PROGRAM

Background

The objective of maximizing bandwidth has an intuitive appeal and is widely used. A more obvious criterion might be trip delay, but almost any kind of synchronization that treats the street as a whole leads to the concept of a planned speed. Once specified, the planned speed tends to determine trip delay (1), unless input flow exceeds street capacity, in which case delay is determined mostly by the amount and duration of the overload. Changes in synchronization tend to produce changes in trip delay which in terms of percentage are small. The stops themselves may be more irritating than the delay. However, the driver's trade-off between stops and delay does not seem to have been much investigated.

In any case, increases in bandwidth usually tend to decrease both stops and delay. For example, in von Stein's (2, 3) approach to traffic control, drivers are encouraged by various signaling devices to form compact platoons which travel nonstop through the system at a preset speed. Insofar as this is successful, trip delay is fixed by the speed. The bandwidth determines the maximal platoon size for which stops can be avoided. A stop forces a driver back into the following platoon with a delay of some fraction of a period. Therefore, the objective studied here is that of maximizing main street bandwidths subject to the constraints imposed by service for the cross streets, pedestrian crossings, etc. For further discussion of signal synchronization and for other approaches to the problem, see Newell (12, 13) and Grace and Potts (14).

The literature on bandwidth contains a number of methods, mostly graphical, for solving special cases of Problem 1. Matson, Smith and Hurd (4) consider primarily signals with constant spacing. Bruening (5) and Petterman (6) approach the problem by trial and error. Raus (7) treats a limited class of problems algebraically.

Bowers (8) gives a graphical method for maximizing bandwidth when the green times are all the same and speed is a constant. His standard procedure involves solving the problem for a range of (speed) × (period) and identifying those values which yield the largest bandwidth as a percentage of period. Evans (9) presents Bowers' method. Davidson (10) also uses this method, but redefines the problem slightly by taking the bandwidth for the main street as given and seeking to maximize the smallest percentage of green assigned to any cross street. This criterion determines green splits for a few critical signals with the rest given the largest cross street green consistent with the specified main street bandwidth. The resulting synchronization is the same as that of Bowers' method.

Our method solves the foregoing cases and handles two generalizations which have not, to our knowledge, been handled previously in any formal way: (a) arbitrary planned speeds are permitted in either direction between any two adjacent signals; and (b) a device is given for apportioning bandwidth between directions on the basis of platoon size. In addition, the method is designed for machine computation and has been programmed for an IBM 1620.

Computer Program

The calculation of offsets to give maximal bandwidth is an easy job, thanks to computers. A program, called TSS3, has been written for a 20K IBM 1620. The machine language object deck* will run on any basic 1620 installation. If the installation has a

*Obtainable from Director, Civil Engineering Systems Laboratory, M.I.T., Cambridge, Massachusetts, 02139.

Cal-Comp Digital Plotter available for use on a line, a further program, TSS4, will take the output of TSS3 and plot a space-time diagram for the final signal settings.

The data required to operate the program are as follows: number the signals 1, 2, 3,... in the direction of increasing distance from an origin at one end of the street. This direction will be called outbound, the opposite direction inbound. To find the maximal equal bandwidths, the program requires as input: (a) number of signals, (b) cycle length (sec), (c) distance of each signal from chosen origin (ft), (d) red phase of each signal (sec), and (e) vehicle speed in each direction between each pair of adjacent signals (mph). For the case of unequal volumes in the two directions, the program will adjust the bandwidths to favor the heavy volume direction. For this purpose, the program requires: (a) inbound volume (veh/hr), (b) outbound volume (veh/hr), and (c) headway between vehicle (sec).

The output of the program consists of: (a) offsets for each signal with respect to a reference signal, (b) number of reference signal, (c) inbound and outbound bandwidths, and (d) largest volumes that will fit unimpeded through the inbound and outbound green bands. The program also produces certain other information useful to the plotting program.

The treatment of volumes is based on the idea of platoons. A given volume and cycle length together imply some number of vehicles per cycle through each signal. Under suitable conditions these vehicles move as a fairly compact platoon through the system. The average headway between vehicles determines the time-length of the platoon. The computer program tries to arrange bandwidths so that both inbound and outbound platoons fit into their green bands. However, a number of special cases come up. Whenever the two platoons are equal, equal bandwidths are given each direction. If the sum of the two bandwidths is greater than the sum of the two platoon lengths, the individual bandwidths are made proportional to platoon lengths, as far as possible. If the sum of the bandwidths is less than the sum of the platoon lengths, the larger platoon is accommodated, if possible, and, thereafter, as much bandwidth as can be arranged is given to the direction with the smaller platoon. The final results are summarized by printing out the inbound and outbound volumes that would be obtained by putting through the largest platoons that fit unimpeded into the green bands.

The time to solve a 10-signal problem is only about a minute. Thus it is a reasonable task to explore a range of cycle lengths to look for particularly large bandwidths or to make sensitivity tests on other constants of the system.

Applications of the method have been made in Cleveland and, more recently, by Hesketh (15) outside Providence.

THEORY

Definitions and Notation

Consider a two-way street having n traffic signals. Directions on the street will be identified as outbound and inbound. The signals will be denoted S_1, S_2, ..., S_n with the subscript increasing in the outbound direction. Let

C = cycle length of the signals (sec);
r_i = red time of S_i on street under study (cycles);
b ($\bar{b}$) = outbound (inbound) bandwidth (cycles);
t_{ij} ($\bar{t}_{ij}$) = travel time from S_i to S_j in the outbound (inbound) direction (cycles); and
θ_{ij} = relative phase, or offset, of S_i and S_j, measured as time from center of a red of S_i to next center of red of S_j (cycles); by convention $0 \leq \theta_{ij} < 1$. (See Fig. 1.)

Any time quantity can be expressed in cycles by dividing by C. "Red time" is used as shorthand for "unusable time." A set of θ_{ij}, $j = 1, \ldots, n$ for any i will be called a synchronization of the signals.

Travel times between adjacent signals are presumed known and fixed. Then all t_{ij} may be calculated from the following:

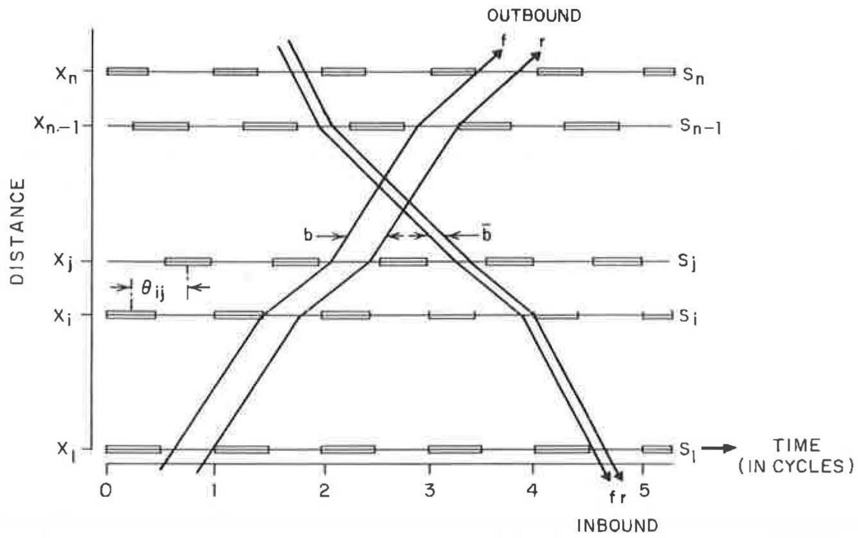


Figure 1. Space-time diagram showing outbound and inbound green bands; signals S_1 and S_i are critical signals.

$$
t_{ij} = \begin{cases} \sum_{k=i}^{j-1} t_{k,\,k+1} & j > i \\ 0 & j = i \\ -\sum_{k=j}^{i-1} t_{k,\,k+1} & j < i \end{cases}
$$

and all $\bar{t}_{ij}$ from corresponding expressions with each t replaced by $\bar{t}$. Although t_{ij} and $\bar{t}_{ij}$ are the basic inputs to the calculation, it is frequently more convenient to think in terms of speeds and distances. Let

x_i = position of S_i on the street (ft), and
v_i ($\bar{v}_i$) = outbound (inbound) speed between S_i and S_{i+1} (ft/sec).

Then

$$
t_{i,\,i+1} = \frac{x_{i+1} - x_i}{v_i C}
$$

$$
\bar{t}_{i,\,i+1} = \frac{x_i - x_{i+1}}{\bar{v}_i C} \qquad (1)
$$

Most previous work has assumed $v_i = \bar{v}_i = v$, in which case $t_{ij} = -\bar{t}_{ij} = (x_j - x_i)/vC$, but this work is not so restricted.

Figure 1 shows a space-time diagram for travel on the street. Heavy horizontal lines indicate when the signals are red. The zig-zag lines represent trajectories of cars passing unimpeded along the street in the directions indicated. Changes in slope correspond to changes in speed. The set of possible unimpeded trajectories in a given direction forms a green band whose horizontal width is the bandwidth for that direction.

The trajectory forming the front edge (earlier in time) of a band and the one forming the rear edge (later in time) have been marked f and r, respectively. Although the green bands are only drawn once, they appear once per cycle in parallel bands across the diagram.

Basis for Method

The basis for the method is developed in a sequence of lemmas and theorems. Before starting, let us examine the objectives. We want to maximize bandwidths but there are bandwidths in each direction, b and $\bar{b}$. We could maximize $b + \bar{b}$, but possibly this would produce an undesirable division of the total between b and $\bar{b}$; for example, one of them might be zero. To unravel the situation, consider the following three problems:

(1) Max $(b + \bar{b})$.
(2) Max $(b + \bar{b})$ subject to $b = \bar{b}$.
(3) Max $(b + \bar{b})$ subject to $b > 0$ and $\bar{b} > 0$.

This work shows that there is usually a whole class of synchronizations which solve (3) and, of these, at least one solves (2). Moreover, the max $(b + \bar{b})$ found in (2) and (3) is a constant which can, within certain limits, be divided arbitrarily between b and $\bar{b}$. However, in some cases, the constant will be less than the $(b + \bar{b})$ found in (1). The reason is fairly simple. Under sufficiently awkward red times and signal spacings the max $(b + \bar{b})$ of (2) and (3) can become quite small, even zero. On the other hand, no matter how awkward the spacing, we can always set up a complete one-way synchronization and obtain a $(b + \bar{b})$ at least as large as the smallest green time.

In any case, this work solves all three problems. The central problem is (2), which will be called the problem of finding maximal equal bandwidths and is solved by theorem 3. Theorem 4 expresses the solution of all three problems in what seems to be an operationally useful way.

Definition. —A signal S_j is said to be a critical signal if one side of S_j's red touches the green band in one direction and the other side touches the green band in the other direction. Thus, in Figure 1, signals S_1 and S_j are critical, but no others are.

Lemma 1. —If a synchronization maximizes $(b + \bar{b})$ subject to $b > 0$ and $\bar{b} > 0$, then:

(a) There exists at least one critical signal.

(b) The red time of any critical signal will touch the front edge of one green band and the rear edge of the other.

(c) All critical signals can be divided into two groups: Group 1 consists of signals whose reds touch the front of outbound and the rear of inbound and Group 2 of signals whose reds touch the front of inbound and the rear of outbound.

Proof. —Consider the set of signals whose reds touch a given side of the green band in one direction. Part (a) must be true or else all these signals could be shifted to in-

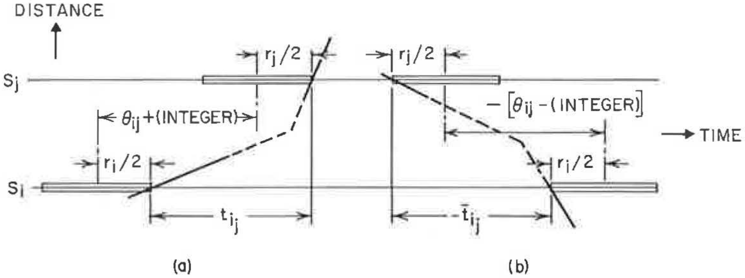


Figure 2. Geometry when two Group 1 signals limit the green band.

crease bandwidth in the one direction without reducing it in the other. Part (b) is a consequence of the definition of critical signal: since the right side of red can only touch front edges and the left side only rear edges, a critical signal must touch (at least) one of each.

Part (c) follows immediately from (b) since there are only two choices. Possibly a signal fits into both groups, in which case it will be considered to be in both. Possibly there is only one critical signal, but then it fits into both groups. This completes the proof.

Suppose two signals, S_i and S_j, are in the same group, for instance, 1. For each signal, the right-hand side of red touches the front of the outbound band and left-hand side touches the rear of the inbound band. Figure 2 shows the geometry for this situation. The quantities are presented in such a way that, if $j > i$, all the lengths shown are positive. The notation "integer" is used to indicate that some integer is to be added to an expression to make it valid.

From Figure 2a:

$$\frac{1}{2} r_i + t_{ij} = \frac{1}{2} r_j + \theta_{ij} + (\text{integer})$$

From Figure 2b:

$$\frac{1}{2} r_i - \bar{t}_{ij} = \frac{1}{2} r_j - \theta_{ij} + (\text{integer})$$

Consequently:

$$\theta_{ij} = \frac{1}{2} (t_{ij} + \bar{t}_{ij}) + \frac{1}{2} (\text{integer}) \tag{2}$$

Corresponding arguments lead to the same equation for Group 2. By convention, $0 \leq \theta_{ij} < 1$. Therefore, it may be seen that (2) has two solutions for θ_{ij}, to be found by adding whatever half integers will bring $(\frac{1}{2})(t_{ij} + \bar{t}_{ij})$ into the required range.

A more explicit expression for the two possible values of θ_{ij} can be developed. Let

$$\delta_{ij} = 0 \text{ or } \frac{1}{2}$$

man z = mantissa of z, as obtained by removing the integral part of z and, if the result is negative, adding unity

Thus, man (5.2) = 0.2, man (-0.2) = 0.8, and in general $0 \leq \text{man } z < 1$. Now (2) becomes

$$\theta_{ij} = \text{man} \left[\frac{1}{2} (t_{ij} + \bar{t}_{ij}) + \delta_{ij}\right] \tag{3}$$

The phasing represented by (3) will be called half-integer sychronization. The term can be consistently applied to a collection of signals. In other words, given a set $\delta_{i1}, \delta_{i2}, \ldots, \delta_{in}$, the resulting $\{\theta_{ij}\}$ have the property that $\theta_{ik} = \text{man}(\theta_{ij} + \theta_{jk})$. Furthermore, the same θ_{ik} is obtained by setting $\delta_{ik} = \text{man}(\delta_{ij} + \delta_{jk})$ in (3). The above summarizes into:

Lemma 2.—Under the conditions of lemma 1, each group of signals has half-integer synchronization.

The operational meaning of half-integer synchronization is easiest understood in the special case $t_{ij} = -\bar{t}_{ij}$, which occurs, for example, when speeds are the same in each direction. Then (3) gives $\theta_{ij} = 0$ or $\frac{1}{2}$ so that any two signals in the same group have the centers of their reds exactly in phase or exactly out of phase.

Theorem 1.—There is a half-integer synchronization which gives maximal equal bandwidths.

Proof (by construction). —Suppose we have a set of phases such that $(b + \bar{b})$ is maximal subject to $b > 0$ and $\bar{b} > 0$. (If none exists, the theorem is trivially true.) Divide the critical signals into Groups 1 and 2. Extend the reds of all other signals until they are critical, too, but not so far as to reduce bandwidth. The old reds lie wholly within the new. Move the center of the old red to the center of the new—this cannot extend the new red or change bandwidth. Classify the new critical signals into Groups 1 and 2. Change the pases of all Group 1 signals by an equal amount in the direction that will decrease the larger of $\bar{b}$ and b. The loss to the larger is just equaled by a gain to the smaller so that $(b + \bar{b})$ stays constant. Choose the amount of change so that $b = \bar{b}$.

Within each group there is half-integer synchronization. It remains to show that there is now half-integer synchronization between signals from different groups. Let S_i be from Group 1 and S_j from Group 2. Figure 3 shows that

$$\frac{1}{2} r_i + b + t_{ij} + \frac{1}{2} r_j = \theta_{ij} + (\text{integer}) \tag{4}$$

$$\frac{1}{2} r_i + \bar{b} + \bar{t}_{ij} + \frac{1}{2} r_j = -\left[\theta_{ij} - (\text{integer})\right] \tag{5}$$

But $b = \bar{b}$, whence

$$\theta_{ij} = \frac{1}{2}(t_{ij} + \bar{t}_{ij}) + \frac{1}{2}(\text{integer})$$

which is (2) again and so implies that S_i and S_j have half-integer synchronization.

In the foregoing we have also proved the following:

Corollary 1.—If the maximal equal bandwidths are greater than zero, max $(b + \bar{b})$ subject to $b > 0$ and $\bar{b} > 0$ equals max $(b + \bar{b})$ subject to $b = \bar{b}$.

Theorem 2. —Under any half-integer synchronization, $b = \bar{b}$.

Proof. —It suffices to consider critical signals. Let S_i be from Group 1 and S_j in Group 2. Figure 3 applies as do (4) and (5). Subtract (5) from (4) and substitute (2). It will be seen that $b = \bar{b}$.

Two special cases of Theorem 1 deserve separate mention. From (1) comes:

Corollary 2. —If speeds are the same in each direction at each point of the street, maximal equal bandwidths are achieved by a synchronization in which each θ_{ij} is either 0 or ½.

Explicit results are possible in the two signal case:

Corollary 3. —If there are only two signals, S_i and S_j, and speeds are the same in each direction, maximal equal bandwidths are achieved by

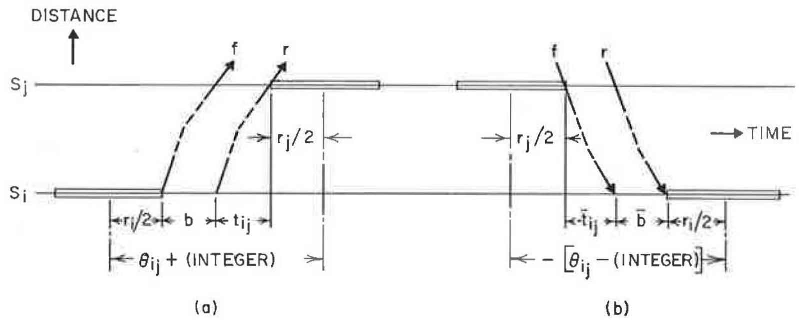


Figure 3. Geometry when a Group 1 and a Group 2 signal limit the green bands.

$$\theta_{ij} = \begin{cases} 0 & 0 \leq \text{man}\left[\frac{x_j - x_i}{vC}\right] < \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} \leq \text{man}\left[\frac{x_j - x_i}{vC}\right] < \frac{3}{4} \\ 0 & \frac{3}{4} \leq \text{man}\left[\frac{x_j - x_i}{vC}\right] < 1 \end{cases}$$

Notice that θ_{ij} does not depend on the green splits of the two signals. This is not always true with more than two signals. Corollary 3 may be proved by constructing a space-time diagram with S_i at $x = 0$ and the start of S_i's green at $t = 0$. S_j's position may be varied along an x-axis where $x = (x_j - x_i)/vC$. At each x there are, by Corollary 2, only two possibilities for placing the center of S_j's red and the best one is fairly obvious.

Synchronization for Maximal Equal Bandwidths

The significance of the results of the previous section is that a synchronization for maximal equal bandwidths can be found by searching through a relatively few cases. By Theorem 1, it suffices to examine half-integer synchronizations. By Theorem 2, it suffices to examine only the outbound direction.

b_i = greatest outbound bandwidth under half-integer synchronization if S_i's red touches the front of the outbound band; and
B = the value of one of the maximal equal bandwidths.

It will be helpful in computations to permit b_i and B to be negative at times; the operational interpretation as a zero bandwidth is clear.

If S_i's red touches the front of the outbound green band, the situation is as shown in Figure 3a. Take as an origin for measurements the right side of S_i's red. The trajectory (not shown) that touches the right side of S_j's red passes S_i at a time which will be denoted u_{ij}. Figure 3a shows that

$$\text{man}\left[\theta_{ij} + \frac{r_j}{2} - \frac{r_i}{2} - t_{ij}\right]$$

except that, when this expression is zero, we shall want $u_{ij} = 1$. This may be accomplished by writing

$$u_{ij} = 1 - \text{man}\left[-\theta_{ij} - \frac{r_j}{2} + \frac{r_i}{2} + t_{ij}\right]$$

Substituting (3) and making the dependence of δ_{ij} explicit:

$$u_{ij}(\delta_{ij}) = 1 - \text{man}\left[\frac{1}{2}(r_i - r_j) + \frac{1}{2}(t_{ij} - \bar{t}_{ij}) - \delta_{ij}\right]$$

The trajectory that touches the left side of S_j's red passes S_i at $u_{ij} - r_j$. Therefore, since δ_{ij} is to take on either the value 0 or ½ and since S_i's red is to touch the front, the best δ_{ij} is identified by

$$\max\left[u_{ij}(0) - r_j,\ u_{ij}\left(\frac{1}{2}\right) - r_j\right]$$

Therefore

$$b_i = \min_{j} \max_{\delta = 0, \frac{1}{2}} \left[u_{ij}(\delta) - r_j\right]$$

and, finally,

$$B = \max_{i} b_i$$

Summarizing, we have the following:

Theorem 3. —The maximal equal bandwidth is max (0, B) where

$$B = \max_{i} \min_{j} \max_{\delta = 0, \frac{1}{2}} \left[u_{ij}(\delta) - r_j\right]$$

Let i = c be a maximizing i and $\delta_{c1}, \ldots, \delta_{cn}$ be the corresponding maximizing δ's. Then, a synchronization for maximal equal bandwidths is $\theta_{c1}, \ldots, \theta_{cn}$ obtained by substituting the δ_{cj} into (3).

Maximal Unequal Bandwidths

Average platoon lengths usually differ between the inbound and outbound directions. If the length exceeds bandwidth in one direction and not the other, it may be possible to shift bandwidth from one direction to the other and pass both platoons. We first show how to shift bandwidth and then suggest a method for dividing total bandwidth between directions on the basis of platoon size.

Let $\theta_{c1}, \ldots, \theta_{cn}$ be a maximal equal bandwidth synchronization with S_c a critical signal whose red touches the front of the outbound green band. The corresponding $u_{c1}, \ldots, u_{cn}$ are presumed known as is the maximal equal bandwidth, B. Let

α_j = a phase shift for S_j (cycles),
θ'_{cj} = man $(\theta_{cj} - \alpha_j)$ = adjusted phase for S_j (cycles), and
$g = \min_{i} (1 - r_i)$ = smallest green time (cycles)

The shifting procedure is described in Theorem 4.

Theorem 4. —The outbound bandwidth, b, can be assigned any value, max [0, B] $\leq$ b $\leq$ g, by making a phase shift.

$$\alpha_j = \max\left[u_{cj} - 1 + b - B,\ 0\right]$$

Then $\bar{b}$ = max [2B - b, 0], and $\bar{b}$ is as large as possible for the given b.

Alternatively, the inbound bandwidth, $\bar{b}$, can be assigned any value, max [0, B] $\leq$ $\bar{b} \leq$ g, by making a phase shift:

$$\alpha_j = \max\left[\bar{b} + r_j - u_{cj},\ 0\right]$$

Then b = max [2B - $\bar{b}$, 0], and b is as large as possible for the given $\bar{b}$.

The shifting procedure may be developed as follows: suppose it is desired to increase outbound bandwidth to b > B, or, if B is negative, to b > 0. The trajectory at the front edge of the outbound band is moved to the left, pushing before it any reds that start to touch it. (See Fig. 4, or the change from Fig. 5 to 6.) During the movement, the critical signals will cut down $\bar{b}$ just as much as b is increased, except that, if $\bar{b}$ reaches zero, no further decrease can occur. Thus, by Corollary 1, $\bar{b}$ is as large as possible for the given b. There is a limit to the increase that can be made in b because eventually the pushing of a red to the left will bring the next red of that signal in from the right to cut into the rear of the outbound band. Then that signal limits both front and rear of the band. The signal must be one with the smallest value of green time; therefore b = g. From this argument we conclude that b can be increased from max [0, B] to any value less than or equal to g and that $\bar{b}$ is then max [B - (b - B), 0]. Analogous remarks apply to increasing $\bar{b}$.

The algebra of the shift may be worked out from Figure 4. Define a u-axis which measures right and left from the front edge of the outbound green band under the given maximal equal bandwidth synchronization. The rear of the band is then at u = B and the right side of a red of S_j is at $u = u_{cj}$.

Consider first the shift to obtain b. The front of the outbound band is pushed left to the position u = B - b. (See the dashed line in the outbound portion of Fig. 4.) This will

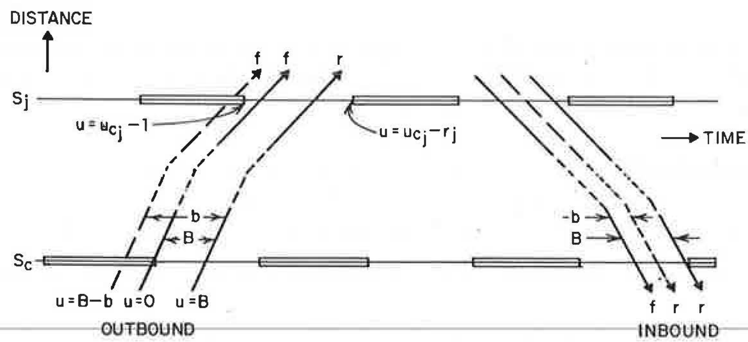


Figure 4. Widening the outbound green band from B to b.

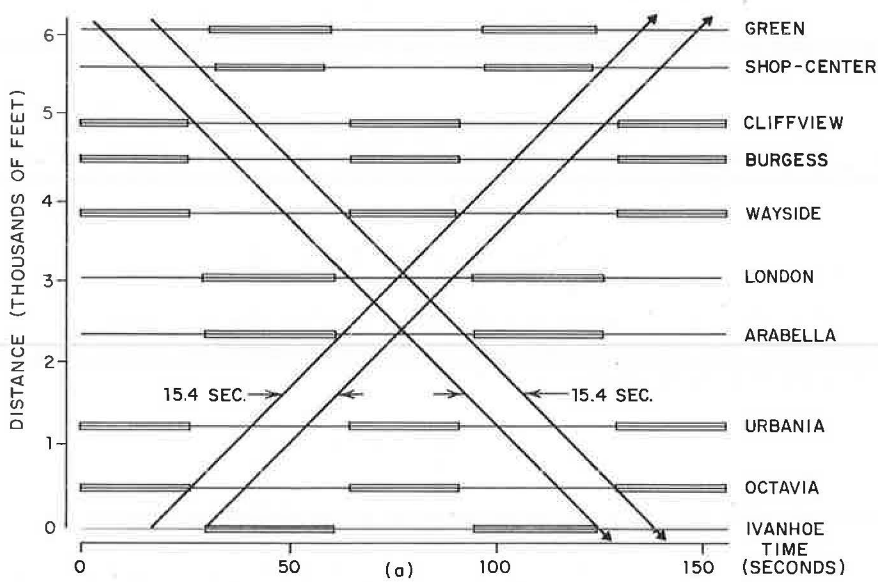


Figure 5. Space-time diagram for part of Euclid Avenue in Cleveland. C = 65 sec, $v = \bar{v} = 50$ ft/sec, $P = \bar{P}$.

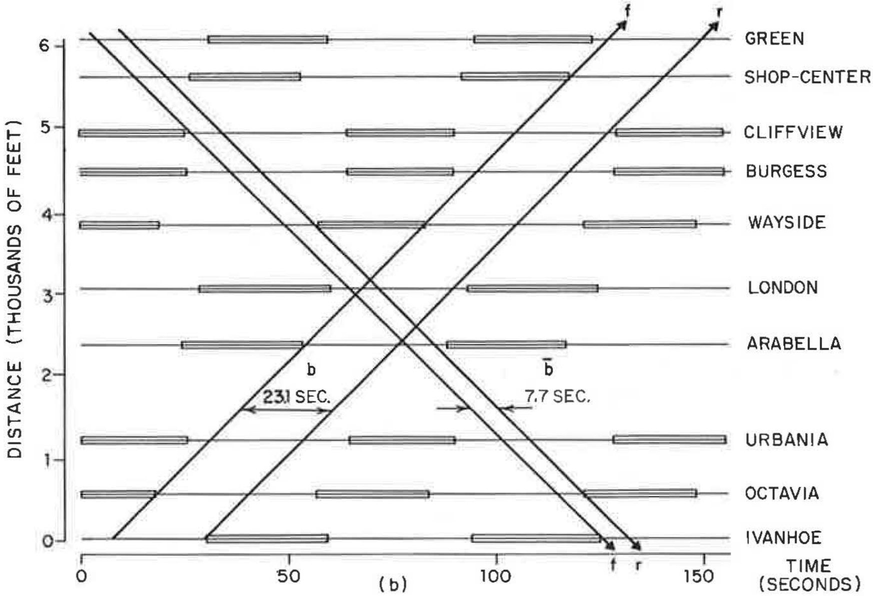


Figure 6. Space-time diagram for part of Euclid Avenue in Cleveland. $C = 65$ sec, $v = \bar{v} = 50$ ft/sec, $P = 0.3$ cycles, $\bar{P} = 0.1$ cycles.

require moving some reds but no more will be moved than necessary and those moved will be moved as little as possible. The next S_j red to the left of the old front edge is met at $u = u_{cj} - 1$. Therefore, the appropriate phase shift for S_j is to the left by an amount:

$$\alpha_j = \max \left[(u_{cj} - 1) - (B - b),\ 0\right]$$

For the case of shifting to obtain $\bar{b}$, it is first observed that under the given synchronization, as under any half-integer synchronization, the distance from the front of the inbound green band to the next S_j red on the left is the same as the distance from the rear of the outbound green band to the next S_j red on the right. (Otherwise we could contradict Theorem 2 by enlarging r_j until S_j started to reduce one green band and not the other.) Consequently we can calculate how much to shift S_j by seeing what is required to move the rear of the outbound band to the right and make it $\bar{b}$ wide. From Figure 4, we find that the magnitude of the shift should be

$$\alpha_j = \max \left[\bar{b} - (u_{cj} - r_j),\ 0\right]$$

To move the front of inbound to the left, these shifts are made to the left. This concludes the proof of Theorem 4. For completeness, if $g > 2B$, $\max (b + \bar{b}) = g$; otherwise $\max (b + \bar{b})$, $\max (b + \bar{b})$ subject to $b = \bar{b}$, and $\max (b + \bar{b})$ subject to $b > 0$ all equal $2B$.

Finally, we give a way to apportion total bandwidth between directions on the basis of the length (in time) of the platoons. Let

P ($\bar{P}$) = platoon length in the outbound (inbound) direction (cycles).

Whenever $P = \bar{P}$ maximal equal bandwidths are proposed. Otherwise, we proceed as follows: If $P + \bar{P} \leq 2B$, there may be enough bandwidth to accommodate both platoons. The bandwidth is made proportional to platoon length if possible. Thus, if $P > \bar{P}$:

$$\begin{aligned} b &= \min[2BP/(P+\bar{P}),\ g] \\ \bar{b} &= \max[2B - b,\ 0] \end{aligned}$$

If $P + \bar{P} > 2B$, the larger platoon is accommodated, if possible, and the remainder, if any, is given to the smaller. Thus, if $P > \bar{P}$,

$$\begin{aligned} b &= \min(P,\ g) \\ \bar{b} &= \max(2B - b,\ 0) \end{aligned}$$

except that if $\bar{b} = 0$, b is set to g. Appropriate interchanges apply if $\bar{P} > P$.

Summary of Method

To synchronize signals for maximal equal bandwidths, first number of signals in order of distance along the street, say, $i = 1, 2, \ldots, n$. The direction of increasing i will be called outbound. Next specify the following data: the signal period, C, in seconds; the red times, $r_1, \ldots, r_n$, in fractions of a cycle; the signal positions, $x_1, \ldots, x_n$, in ft; the outbound speeds between signals, $v_1, v_2, \ldots, v_{n-1}$, in ft/sec; and the inbound speeds between signals $\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_{n-1}$ in ft/sec.

The computation proceeds in the following steps:

1. Calculate $y_1, \ldots, y_n$ from

$$\begin{aligned} y_1 &= 0 \\ y_i &= y_{i-1} - \frac{1}{2}(r_i - r_{i-1}) + (x_i - x_{i-1}) \frac{1}{2C} \left[\frac{1}{v_{i-1}} + \frac{1}{\bar{v}_{i-1}} \right] \end{aligned}$$

2. Calculate $z_1, \ldots, z_n$ from

$$\begin{aligned} z_1 &= 0 \\ z_i &= z_{i-1} + (x_i - x_{i-1}) \frac{1}{2C} \left[\frac{1}{v_{i-1}} - \frac{1}{\bar{v}_{i-1}} \right] \end{aligned}$$

3. Calculate

$$B = \max_{i} \ \min_{j} \ \max_{\delta = 0,\ \frac{1}{2}} \left[u_{ij}(\delta) - r_j \right]$$

where

$$u_{ij}(\delta) = 1 - \text{man}\,(y_j - y_i - \delta)$$

and the operation "man" is as defined earlier. Consider a specific i. As the max over δ is performed, the maximizing value (one for each j) may be recorded in a temporary table, $\delta_{i1}, \ldots, \delta_{in}$. As the max over i is performed, the maximizing i, say $i = c$, identifies the best set, $\delta_{c1}, \ldots, \delta_{cn}$, which is saved. For the following computations, it is necessary to save the set, $u_{c1}, \ldots, u_{cn}$, corresponding to the $\delta_{c1}, \ldots, \delta_{cn}$. This means saving the value of u_{ij} whenever a value of δ_{ij} is saved.

4. A synchronization, $\theta_{c1}, \ldots, \theta_{cn}$, for maximal equal bandwidths is calculated from

$$\theta_{cj} = \text{man}\,[z_j - z_c + \delta_{cj}]$$

The bandwidth in each direction is max (0, B).

To adjust the synchronization for platoon lengths of P, outbound, and $\bar{P}$, inbound, specify P and $\bar{P}$, perform the foregoing calculations and continue as follows.

5. Calculate $g = \min(1 - r_i)$.
6. If $P = \bar{P}$ accept equal bandwidth solution.
7. If $\bar{P} > P$, go to Step 11, otherwise continue.

8. If $P + \bar{P} \leq 2B$, set $b = \min [g, 2BP/(P + \bar{P})]$. Otherwise, set $b = \min (P, g)$; unless $P \geq 2B$, in which case, set $b = g$.
9. Calculate $\alpha_1, \ldots, \alpha_n$ from $\alpha_j = \max (u_{cj} - 1 + b - B, 0)$.
10. Calculate $\bar{b} = \max (2B - b, 0)$. Go to Step 14.
11. If $P + \bar{P} \leq 2B$, set $\bar{b} = \min [g, 2B\bar{P}/(P + \bar{P})]$. Otherwise, set $\bar{b} = \min (\bar{P}, g)$, unless $\bar{P} \geq 2B$, in which case, set $\bar{b} = g$.
12. Calculate $\alpha_1, \ldots, \alpha_n$ from $\alpha_j = \max (\bar{b} + r_j - u_{cj}, 0)$.
13. Calculate $b = \max (2B - \bar{b}, 0)$
14. The adjusted synchronization, $\theta'_{c1}, \ldots, \theta'_{cn}$, is calculated from

$$\theta'_{cj} = \text{man} (\theta_{cj} - \alpha_j)$$

and the bandwidths are b, outbound and $\bar{b}$, inbound as previously determined.

For plotting space-time diagrams it is helpful to know where the edges of the green bands are. Take as a reference point the center of a red of S_c. The left side of an outbound band is at $r_c/2$, the right side at $(r_c/2) + b$. An inbound band has its left side at $1 - (r_c/2) - \bar{b}$ and its right side at $1 - (r_c/2)$. The edges of the same outbound band at S_j are found by adding t_{cj} to the edges at S_c; for inbound, add $\bar{t}_{cj}$.

Examples

The method has been used to synchronize the signals on a stretch of Euclid Avenue in Cleveland under off rush-hour conditions. (During rush hours a complete one-way synchronization is used.) Signals are at 0, 550, 1250, 2350, 3050, 3850, 4500, 4900, 5600, 6050 ft. Corresponding red times are 0.47, 0.40, 0.40, 0.47, 0.48, 0.42, 0.40, 0.40, 0.40, 0.42 cycles. C = 65 sec, v = 50 ft/sec in both directions. Figure 5 shows the space-time diagram for maximal equal bandwidths. B = 0.237 or 15.4 sec. Figure 6 shows the case: $P = 0.30$ cycles, $\bar{P} = 0.10$ cycles.

Discussion

The ability to handle different platoon speeds between different signals make it possible to adjust the synchronization for the presence of a queue waiting at a signal. Such a queue might arise because turning traffic is entering the main street at the previous intersection or might be the tail of a platoon that does not fit through the green band. Let τ be the time length of the queue waiting when the next platoon tries to come through. Unless the queue is released early, the arriving platoon will have to stop or slow down. Let v be the normal platoon speed and x the distance from the preceding signal. If the speed, $v' = v/[1 - (\tau/x) v]$, is used in the computation, the synchronization will permit the platoon to travel at v and not stop (but note that the departing platoon is longer than the arriving platoon). Similarly, an allowance can be made for cars leaving the platoon, although, unless the cars always leave from the head, it may be more reasonable to expect (or guide) the lead car to maintain a planned speed and encourage subsequent cars to adjust speed to close the gaps. A negative value of v' is permitted by our calculation; this would imply a backward movement of the green wave.

Although we have ruled out problems for which the maximal bandwidth is split in one or both directions, our results extend to one aspect of these cases. Denote the largest unsplit segment of a bandwidth as the primary bandwidth and the corresponding green band as the primary green band. The substitution of these terms for bandwidth and green band throughout the analysis will make the results hold for any problem. In most problems our method is likely to maximize total bandwidth even if split, but it is possible to construct examples where this does not happen.

The method may be useful in connection with traffic control by on-line computer. The α_j adjustment could be made to follow actual flow. Another possibility is to vary C. Suppose that the green split restrictions are expressed in cycles and remain valid for a range of C. From (1) it may be seen that, if each speed is multiplied by a con-

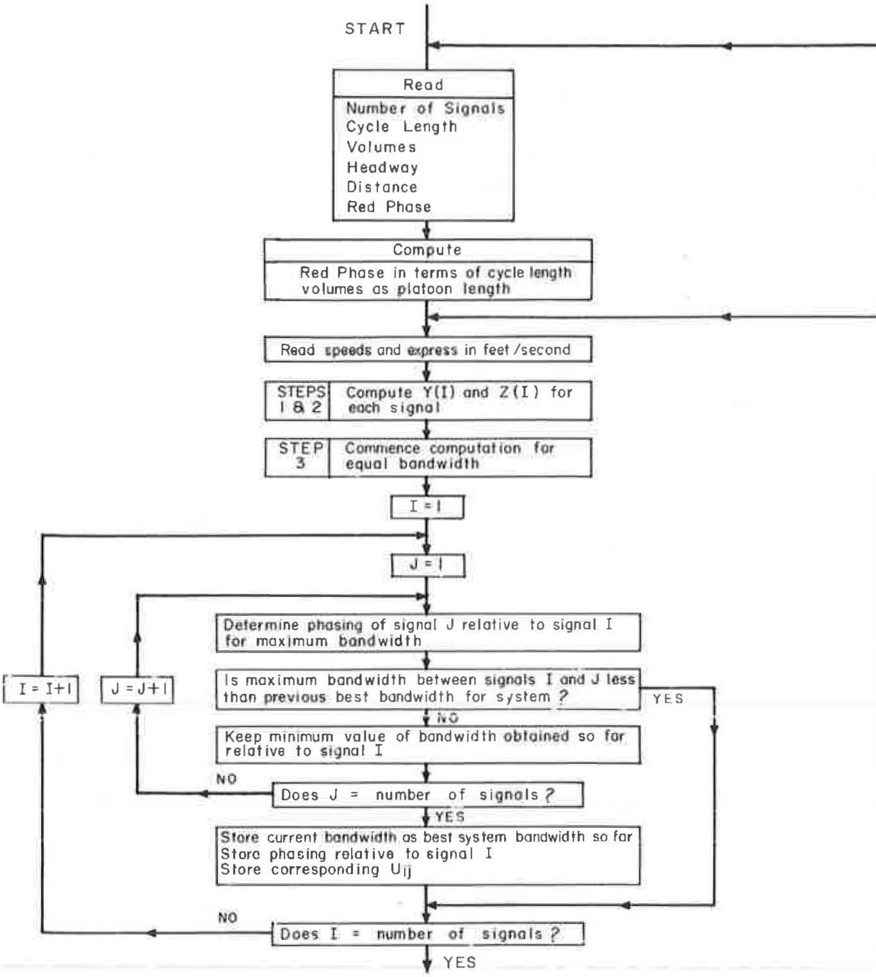


Figure 7. Flow chart of Program TSS3.

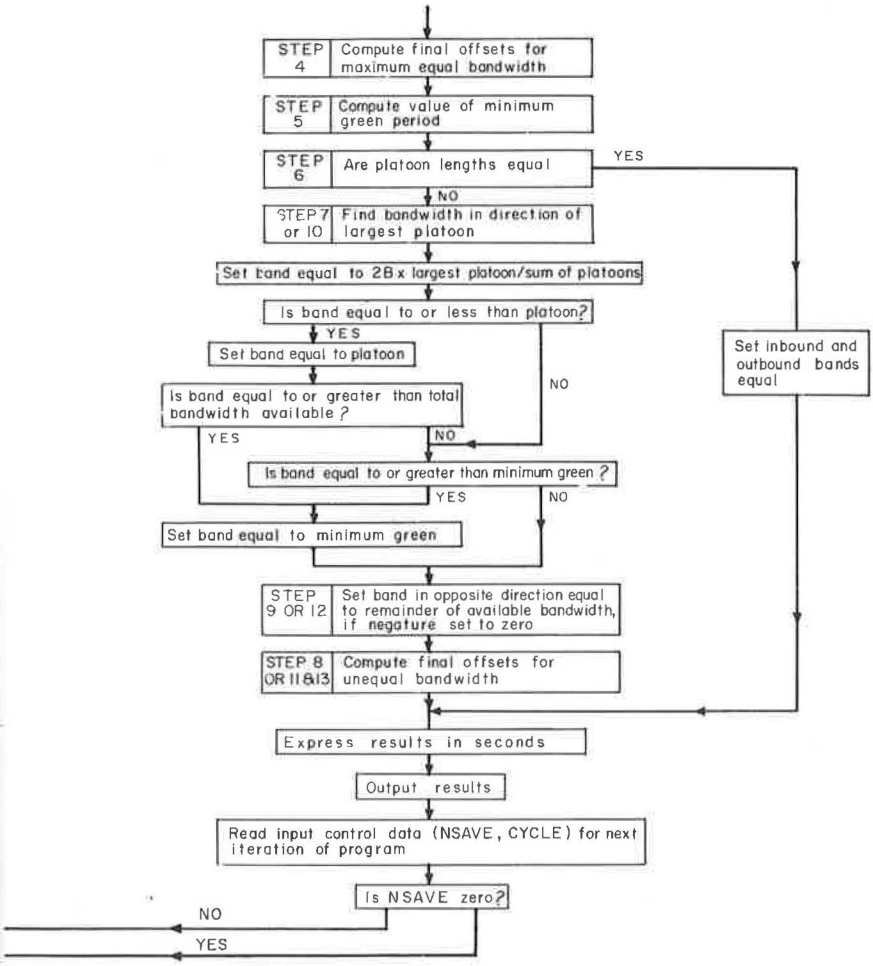
stant and C is divided by that constant, the travel times (expressed in cycles) between signals are unchanged. Therefore, the synchronization for maximum bandwidth is unchanged. Suppose, then, that in real operations the traffic speed temporarily decreases from the planned speed because of weather, increased vehicle density, or the like. Normally, the synchronization becomes invalid, but this will not happen if the signal period is correspondingly lengthened, as could easily be done in real time control. Such lengthening would probably increase capacity slightly.

COMPUTER PROGRAMS

Description of Programs

Program TSS3. —Program TSS3* performs all the necessary computation to determine the offsets of the signals and the green bandwidths. The program is written in LOAD

*A listing of the source program and a storage map are available (at cost of xerox reproduction and handling) for both Program TSS3 and Program TSS4—Supplement XS-7 (Highway Research Record 118), 12 pages.



Note: Steps refer to summary in text

and GO FORTRAN, a system developed by the Civil Engineering Systems Laboratory for the IBM 1620 computer at the Massachusetts Institute of Technology. The program has been limited to 20K storage so that the smallest IBM 1620 installation can use it. The machine language object deck, which is available from MIT, will run on any basic 1620 installation. A flow chart of the program is shown in Figure 7.

Program TSS4. —Program TSS4* takes the output of program TSS3 and plots a space-time diagram, using the Cal-Comp Digital plotter on line. The program is also written in LOAD and GO FORTRAN; it does, however, use two subroutines, PLT, XNM, that are not generally available. These routines are in the object deck which can be used on any 20K installation having a digital plotter on line.

The flow chart (Fig. 8) shows the structure of the program.

*See footnote on preceding page.

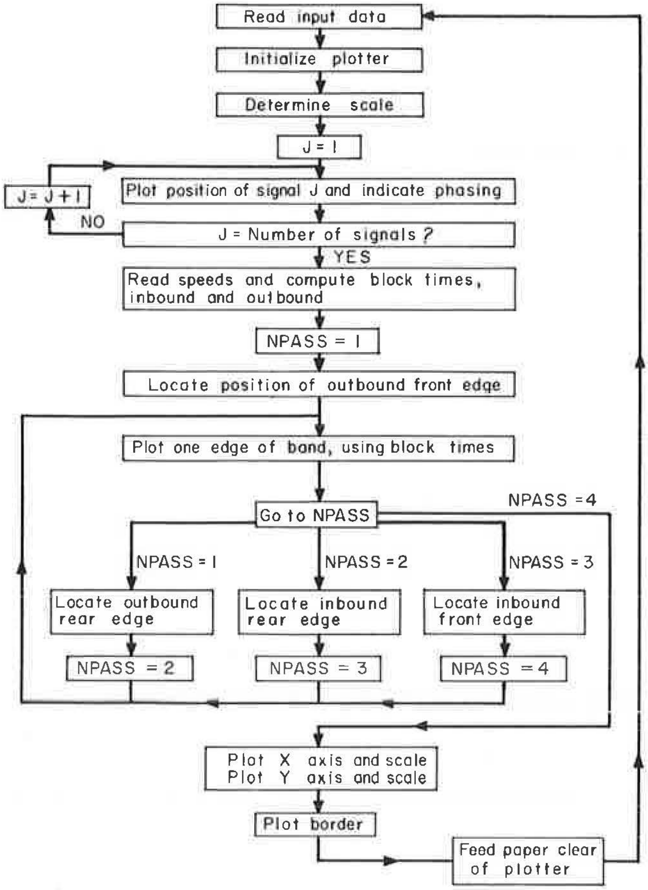


Figure 8. Flow chart of Program TSS4.

Operating Details

Program TSS3

INPUT. All of the data are input from punched cards. There are four types of cards used as follows:

Card type 1, system parameters;
Card type 2, signal characteristics;
Card type 3, speeds; and
Card type 4, control card.

The data included on each card are summarized in Table 1. In the LOAD and GO system there is no format; data must, however, be separated by blank columns. Blank columns are not interpreted as zero in the LOAD and GO system.

Card Type 1. Card contains the data indicating the number of signals in the system, the cycle length, the inbound and outbound volumes and the vehicle headway. The vehicle headway is used to convert the hourly volumes into platoon lengths (sec), as follows:

$$\text{Platoon length} = \frac{\text{Hourly volume} \times \text{vehicle headway}}{3,600}$$

TABLE 1

INPUT DATA TO PROGRAM TSS3

Entry	Description of Data Item	Variable Name	Units	Form
Card type 1				
1	Number of Signals	NSIG	Integer	Fixed Point
2	Cycle Length	CYCLE	Seconds	Floating Point
3	Inbound Volume	VOLIN	Veh/Hr	Floating Point
4	Outbound Volume	VOLOT	Veh/Hr	Floating Point
5	Vehicle Headway	HEDWY	Seconds	Floating Point
	ONE CARD PER COMPUTATION			
Card type 2				
1	Distance of Signal from origin	X (I)	Feet	Floating Point
2	Red Phase of Signals	RED (I)	Seconds	Floating Point
	NUMBER OF CARDS=NUMBER OF SIGNALS			
	CARDS IN ORDER OF INCREASING X			
Card type 3				
1	Inbound block speed	SPEDI(I)	MPH	Floating Point
2	Outbound block speed	SPEDO(I)	MPH	Floating Point
	NUMBER OF CARDS=NUMBER OF SIGNALS-1			
	CARDS IN ORDER OF INCREASING X			
Card type 4				
1	Constant	NSAVE	Integer	Fixed Point
2	Cycle length for next Iteration	CYCLE	Seconds	Floating Point

For streets with several lanes the engineer may use either hourly lane volumes or adjust headways accordingly.

Card Type 2. Each card contains two entries, the first being the x value; that is, the distance of the signal in feet from the origin, taken as the first signal. The second entry indicates the length of the red phase for the signal in seconds. Increasing x is defined as the outbound direction: type 2 cards must be in order of increasing x.

Card Type 3. Each card contains two entries, the first giving the block speed inbound and the second the block speed outbound. The cards must be in order of increasing x, i. e., outbound.

Card Type 4. Card is used to direct the program operations for the following problem, if any. If a completely new problem is to be run, or if the volumes on the previous problem are to be changed, set: NSAVE = positive integer, and CYCLE = any value. The program will complete the current problem and return to start a new problem by reading card type 1. Note that a value for CYCLE must be given; if it is left blank the program will take the first data item on the following card. The program can be terminated by trying to read a card type 1 or a card type 4.

If it is desired to rerun the same physical problem, i.e., distances, red and volumes, but changes must be made in cycle length and/or speeds, set: MSAVE = 0, CYCLE = value wanted. The program will complete the current problem and return to read card type 3 and another card type 4.

PROGRAM OPERATION. The program does not require any special sense switch settings. Running times depend on the particular problem. However, Table 2 indicates running times obtained with the program on problems with 10, 20, 30, 40 and 50 signals, and these may be considered as generally representative.

Card type 4 can be used to make several runs with the same set of signals and volumes but with different cycle lengths or speeds. If the volumes are to be changed or a different problem is to be run the NSAVE entry on card type 4 should be positive and CYCLE may be set to any value, but some value must be given.

OUTPUT. All output is by punched cards, there being five types of cards. Table 3 summarizes the arrangement of the output cards.

Card Type 1. The entries correspond to the input entries for number of signals, cycle length and vehicle headway.

Card Type 2. The first two entries correspond to the input values of inbound and outbound volumes. The last two entries indicate the inbound and outbound volumes possible through the computed green bandwidths for the input vehicle headway.

Card Type 3. The first entry indicates the inbound bandwidth in seconds; the second entry is the outbound bandwidth in seconds. The final entry gives the number of a critical signal, where the signals are numbered in the outbound direction.

Card Type 4. Entries 1 and 2 correspond to the entries on input card type 2. Entry 3 indicates the time in seconds from the origin to the right hand side of red for each signal, where the origin is taken as the center of red for the critical light (final entry on card type 3). This information is useful for plotting the space-time diagram by hand. The final entry indicates the offset of the signal with respect to the critical signal, given in terms of cycle length.

Card Type 5. Card is principally of use to program TSS4. The first entry is the position of the front edge of the outbound band and the second entry is the position of the rear edge of the inbound band, at the first signal. The third and fourth entries are the total system travel time, inbound and outbound, respectively.

Program TSS4

INPUT. All of the data are input from punched cards. There are two different sources for the input data, the first being the output from program TSS3 and the second the speed data as input to program TSS3 on cards type 3. Table 4 summarizes the input data to program TSS4.

TABLE 2

RUNNING TIMES WITH PROGRAM TSS3

(All entries in seconds)

Number of Signals	Read Object Deck	Read Data	Computations	Total
10	30	10	20	60
20	30	20	25	75
30	30	30	75	135
40	30	40	110	180
50	30	50	200	280

TABLE 3

OUTPUT DATA FROM PROGRAM TSS3

Entry	Description of Data Item	Units
Card type 1		
1	Number of Signals	Integer
2	Cycle Length	Seconds
3	Vehicle Headway	Seconds
	ONE CARD PER PROBLEM	
Card type 2		
1	Inbound Volume (input)	Veh/Hour
2	Outbound Volume (input)	Veh/Hour
3	Possible Inbound Volume through band	Veh/Hour
4	Possible Outbound Volume through band	Veh/Hour
	ONE CARD PER PROBLEM	
Card type 3		
1	Inbound bandwidth	Seconds
2	Outbound bandwidth	Seconds
3	Number of Restricting Signal	Integer
	ONE CARD PER PROBLEM	
Card type 4		
1	Distance of Signal from Origin	Feet
2	Red Phase of Signals	Seconds
3	RHS of red phase	Seconds
4	Offset relative to restricting signal	Fraction of Cycle Length
	NUMBER OF CARDS = NUMBER OF SIGNALS	
Card type 5		
1	Front edge of outbound band	Fraction of
2	Rear edge of inbound band	Cycle Length
3	Total system travel time inbound	Fraction of
4	Total system travel time outbound	Cycle Length
	ONE CARD PER PROBLEM	

PROGRAM OPERATION. The program does not require any special sense switch settings. The width of the plot has been scaled so that it will always be 8 in. wide, the convenient plotting width of the 12-in. Cal-Comp plotter. The horizontal scale of the plot will always be 20 sec/in. Running times for the program depend on the problem and exact hardware configuration. However, plots will normally take between 5 and 10 min.

OUTPUT. The only output is the plot, examples of which are shown.

TABLE 4

INPUT DATA TO PROGRAM TSS4

Entry	Description of Data Item	Units
1	Data cards exactly as output from program TSS3	
2	Block speed inbound Block speed outbound (Exactly as input card type 3) NUMBER OF CARDS = NUMBER OF SIGNALS-1	MPH MPH

TABLE 5

SAMPLE PROBLEM

Signal Number	X Value Feet	Red Phase Seconds	Block Speed MPH Inbound	Block Speed MPH Outbound
1	000	30.5	30.0	30.0
2	550	26.0	30.0	30.0
3	1250	26.0	30.0	30.0
4	2350	30.5	50.0	50.0
5	3050	31.0	50.0	50.0
6	3850	27.0	50.0	50.0
7	4500	26.0	40.0	40.0
8	4900	26.0	40.0	40.0
9	5600	26.0	40.0	40.0
10	6050	27.0		

Restrictions

Program TSS3 is limited to problems with 50 signals or less, by the 20K storage restriction imposed on the program. Block speeds may be either positive or negative but not zero. All other variables should be positive.

Program TSS4 is limited to problems with 20 signals or less, if the smaller Cal-Comp plotter is being used. For the larger Cal-Comp plotter 50 signals could be accommodated in the 20K storage, although plots are likely to be very long and consequently slower. Program TSS4 will accept any output from Program TSS3. Zero bandwidth is plotted as one straight line.

TABLE 6 SAMPLE PROBLEM INPUT TO PROGRAM TSS3

10	65.0	400.	400.	2.
0.	30.5			
550.	26.0			
1250.	26.0			
2350.	30.5			
3050.	31.0			
3850.	27.0			
4500.	26.0			
4900.	26.0			
5600.	26.0			
6050.	27.0			
30.0	30.0			
30.0	30.0			
30.0	30.0			
50.0	50.0			
50.0	50.0			
50.0	50.0			
40.0	40.0			
40.0	40.0			
40.0	40.0			
1	1.			
10	65.0	850.	0.	2.
0.	30.5			
550.	26.0			
1250.	26.0			
2350.	30.5			
3050.	31.0			
3850.	27.0			
4500.	26.0			
4900.	26.0			
5600.	26.0			
6050.	27.0			
30.0	30.0			
30.0	30.0			
30.0	30.0			
50.0	50.0			
50.0	50.0			
50.0	50.0			
40.0	40.0			
40.0	40.0			
40.0	40.0			
1	1.			
10	65.0	600.	200.	2.
0.	30.5			
550.	26.0			
1250.	26.0			
2350.	30.5			
3050.	31.0			
3850.	27.0			
4500.	26.0			
4900.	26.0			
5600.	26.0			
6050.	27.0			
30.0	30.0			
30.0	30.0			
30.0	30.0			
50.0	50.0			
50.0	50.0			
50.0	50.0			
40.0	40.0			
40.0	40.0			
40.0	40.0			
1	1.			

TABLE 7 SAMPLE PROBLEM OUTPUT FROM PROGRAM TSS3

10	65.000000	2.0000000		001
400.00000	400.00000	324.75528	324.75528	002
11.727274	11.727274	7		003
.00000000	30.499999	47.750000	.50000000	004
550.00000	26.000000	45.500000	.50000000	005
1250.0000	26.000000	13.000000	.00000000	006
2350.0000	30.499999	47.750000	.50000000	007
3050.0000	31.000000	48.000000	.50000000	008
3850.0000	27.000000	46.000000	.50000000	009
4500.0000	26.000000	13.000000	.00000000	010
4900.0000	26.000000	13.000000	.00000000	011
5600.0000	26.000000	13.000000	.00000000	012
6050.0000	27.000000	13.500000	.00000000	013
.92727280	2.0727272	1.6791956	1.6791956	014
10	65.000000	2.0000000		015
600.00000	200.00000	600.00000	49.510566	016
21.666666	1.7878816	7		017
.00000000	30.499999	47.750000	.50000000	018
550.00000	26.000000	35.560608	.34708627	019
1250.0000	26.000000	13.000000	.00000000	020
2350.0000	30.499999	47.750000	.50000000	021
3050.0000	31.000000	48.000000	.50000000	022
3850.0000	27.000000	39.196968	.39533796	023
4500.0000	26.000000	13.000000	.00000000	024
4900.0000	26.000000	13.000000	.00000000	025
5600.0000	26.000000	11.583338	.97820520	026
6050.0000	27.000000	3.9128900	.85250600	027
.92727280	2.0727272	1.6791956	1.6791956	028
10	65.000000	2.0000000		029
850.00000	.00000000	941.53860	.00000000	030
34.000005	.00000000	7		031
.00000000	30.499999	35.727264	.31503484	032
550.00000	26.000000	23.227269	.15734260	033
1250.0000	26.000000	7.3181750	.91258730	034
2350.0000	30.499999	47.318175	.49335654	035
3050.0000	31.000000	37.772725	.34265730	036
3850.0000	27.000000	26.863629	.20559429	037
4500.0000	26.000000	13.000000	.00000000	038
4900.0000	26.000000	11.181814	.97202790	039
5600.0000	26.000000	64.249998	.78846150	040
6050.0000	27.000000	56.579550	.66276230	041
.92727280	2.0727272	1.6791956	1.6791956	042

Program Availability

Machine language object decks for both programs TSS3 and TSS4 can be obtained from the Department of Civil Engineering at MIT. Program TSS3 can be punched from the source listing given in this report and compiled with FORTRAN compilers without format. However, on 20K machines memory overflows may occur if trace instructions are also included at compilation time. Program TSS4 contains plotting routines not generally available. The source listing is included for users who may wish to use their own routines. In program TSS4 the plotting routines are as follows: PLT for drawing lines and moving pen; and XNM for plotting numbers.

Sample Problem

The sample problem consists of a system of 10 signals with the characteristics given in Table 5. The input cards to program TSS3 are given in Table 6; the output from this program is given in Table 7. The input to program TSS4 is listed in Table 8. The plotted output from program TSS4 is shown in Figures 9, 10 and 11. The system parameters are as follows: cycle length, 65 sec; headway, 2 sec.

TABLE 8 SAMPLE PROBLEM INPUT TO PROGRAM TSS4

10	65.000000	2.0000000		001
400.00000	400.00000	324.75528	324.75528	002
11.727274	11.727274	7		003
.00000000	30.499999	47.750000	.50000000	004
550.00000	26.000000	45.500000	.50000000	005
1250.0000	26.000000	13.000000	.00000000	006
2350.0000	30.499999	47.750000	.50000000	007
3050.0000	31.000000	48.000000	.50000000	008
3850.0000	27.000000	46.000000	.50000000	009
4500.0000	26.000000	13.000000	.00000000	010
4900.0000	26.000000	13.000000	.00000000	011
5600.0000	26.000000	13.000000	.00000000	012
6050.0000	27.000000	13.500000	.00000000	013
.92727280	2.0727272	1.6791956	1.6791956	014
30.0	30.0			
30.0	30.0			
30.0	30.0			
50.0	50.0			
50.0	50.0			
50.0	50.0			
40.0	40.0			
40.0	40.0			
40.0	40.0			
10	65.000000	2.0000000		015
600.00000	200.00000	600.00000	49.510566	016
21.666666	1.7878816	7		017
.00000000	30.499999	47.750000	.50000000	018
550.00000	26.000000	35.560608	.34708627	019
1250.0000	26.000000	13.000000	.00000000	020
2350.0000	30.499999	47.750000	.50000000	021
3050.0000	31.000000	48.000000	.50000000	022
3850.0000	27.000000	39.196968	.39533796	023
4500.0000	26.000000	13.000000	.00000000	024
4900.0000	26.000000	13.000000	.00000000	025
5600.0000	26.000000	11.583338	.97820520	026
6050.0000	27.000000	3.9128900	.85250600	027
.92727280	2.0727272	1.6791956	1.6791956	028
30.0	30.0			
30.0	30.0			
30.0	30.0			
50.0	50.0			
50.0	50.0			
50.0	50.0			
40.0	40.0			
40.0	40.0			
40.0	40.0			
10	65.000000	2.0000000		029
850.00000	.00000000	941.53860	.00000000	030
34.000005	.00000000	7		031
.00000000	30.499999	35.727264	.31503484	032
550.00000	26.000000	23.227269	.15734260	033
1250.0000	26.000000	7.3181750	.91258730	034
2350.0000	30.499999	47.318175	.49335654	035
3050.0000	31.000000	37.772725	.34265730	036
3850.0000	27.000000	26.863629	.20559429	037
4500.0000	26.000000	13.000000	.00000000	038
4900.0000	26.000000	11.181814	.97202790	039
5600.0000	26.000000	64.249998	.78846150	040
6050.0000	27.000000	56.579550	.66276230	041
.92727280	2.0727272	1.6791956	1.6791956	042
30.0	30.0			
30.0	30.0			
30.0	30.0			
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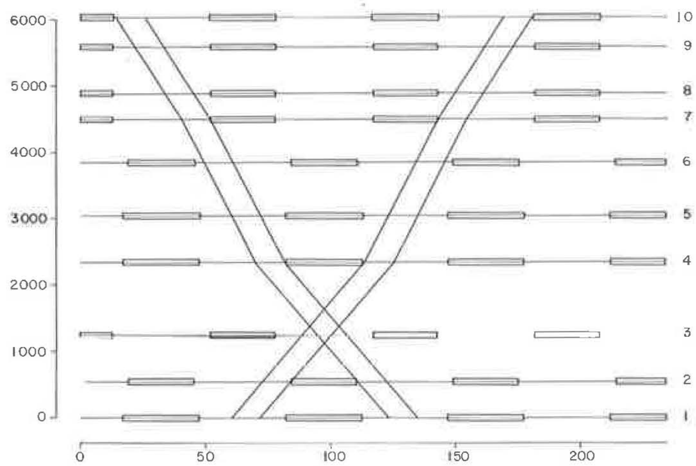

Figure 9. Input volumes: inbound, 40 veh/hr; outbound, 400 veh/hr.

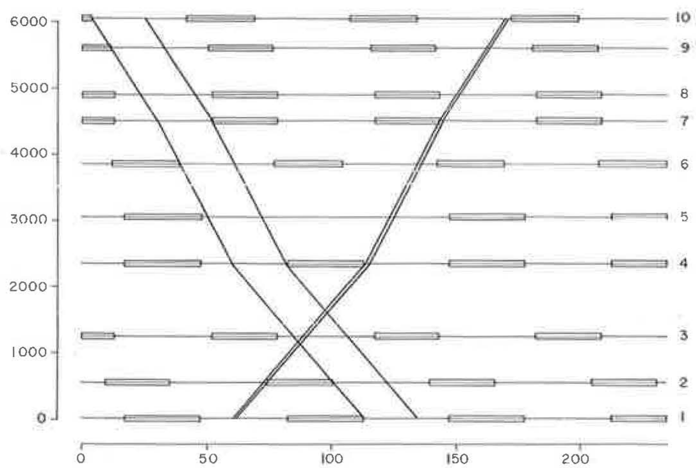

Figure 10. Input volumes: inbound, 600 veh/hr; outbound, 200 veh/hr.

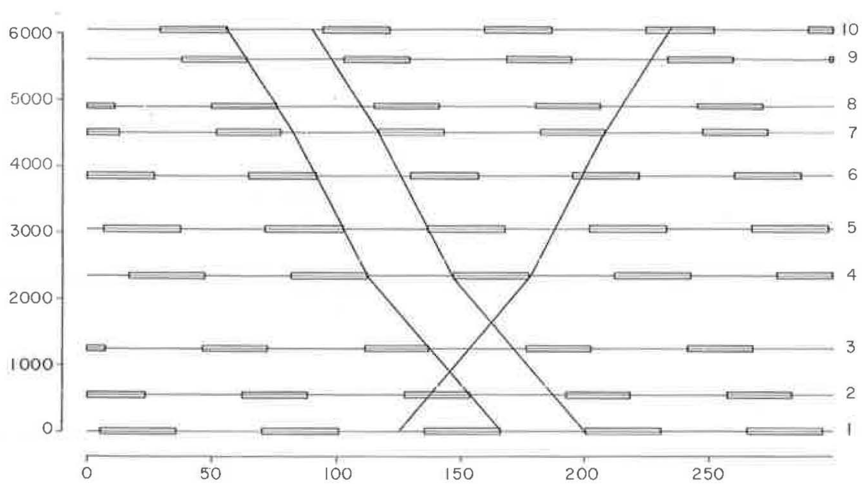

Figure 11. Input volumes: inbound, 850 veh/hr; outbound, 0 veh/hr.

Computations are to be made in volume combinations as follows:

Inbound Volume	Outbound Volume
400 (veh/hr)	400 (veh/hr)
600 (veh/hr)	200 (veh/hr)
850 (veh/hr)	0 (veh/hr)

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Appendix

DICTIONARY OF VARIABLE NAMES USED IN COMPUTER PROGRAMS

PROGRAM TSS3

Program Variable	Equivalent Variable Used in Formulation of Technique	Description
Variables as first defined		
ALHS	None	See definition in program listing
BAND	B	Bandwidth in one direction
BIN	$\bar{b}$	Inbound bandwidth
BOUT	b	Outbound bandwidth
BRHS	None	See definition in program listing
CYCLE	C	Cycle length of system
HEDWY	None	Vehicle Headway
LTBST	C	Critical light
NSIG	n	Number of Signals
PHASE (I)	δ_{cj}	Relative Offset of Light
PLATI	$\bar{P}$	Inbound platoon length
PLATO	P	Outbound platoon length
RED (I)	r_i	Red phase of Signal
SPEDI (I)	$\bar{v}_i$	Block speed inbound
SPEDO (I)	v_i	Block speed outbound
TIME (I)	None	Used to find time to signal from origin
VOLIN	None	Inbound Volume
VOLOT	None	Outbound Volume
W (I)	u_{ij}	See definition in summary step 3
X (I)	x_i	Distance of signal from first signal
Y (I)	y_i	See definition in summary step 1
Z (I)	z_i	See definition in summary step 2

NOTES

Variables A, B, I, J, K, M, NSAVE, SAVE and WMIN used as indices and temporary storage.
Variables SPEDI (I), SPEDO (I), PHASE (I) are redefined during the computation as follows:

SPEDI (I) used to save values of PHASE (I)
SPEDO (I) used to save values of W (I)
PHASE (I) used to save values of final offsets for equal and unequal bandwidth cases.

PROGRAM TSS4

Variables given where different to meaning in TSS3

Program Variable	Description
IXY (1)	X co-ordinate at start of line
IXY (2)	Y co-ordinate at start of line
IXY (3)	X co-ordinate at end of line
IXY (4)	Y co-ordinate at end of line
RHS	Length of plot in X(time) direction
SCHR	Horizontal scale factor
SCVT	Vertical scale factor
X1, X2, X3	as IXY (1 or 3)
Y1, Y2, Y3	as IXY (2 or 4)
Y (I)	Distance of a signal from first signal

NOTES

Variables I, B, DUMB, YLS, ALS, NSAVE, M, SAVE and NPASS used as indices and temporary storage.

Lag and Gap Acceptances at Stop-Controlled Intersections

PER SOLBERG and J. C. OPPENLANDER
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The purpose of this research study was to investigate the lag and gap acceptances for drivers entering and crossing a major roadway from a stopped position. This driver-behavior evaluation included a determination of a lag-and-gap acceptance distribution for the sidestreet drivers, consideration of community influence on this distribution, and comparisons of time-interval acceptances by drivers making through, left-turn, and right-turn movements.

The study was performed at right-angle intersections formed by two-way, two-lane, urban streets. Four sites, selected in Lafayette and Indianapolis, Ind., were as identical as possible regarding geometry and adjacent land use. The data were collected at these sites by means of a motion picture camera. The technique of probit analysis was employed in the statistical treatment of the observations. In addition, two other methods, one developed by Raff and the other by Bissell, were considered in this evaluation of driver behavior at stop-controlled intersections.

The acceptance distributions were well described by a linear relationship between the probit of acceptance and the logarithm of acceptance time. There were no significant difference between the median lag-acceptance and the median gap-acceptance times at the four intersections. However, significant variations were found between right- and left-turning drivers and between drivers proceeding through the intersection and those making left turns. Right-turning drivers and those crossing the intersection had statistically equal median acceptance times. Community size apparently has some influence on driver performance at intersection approaches controlled by stop signs. A general agreement existed among the three methods of analysis investigated.

•THE INTERSECTION of streets at grade in urban areas is a primary location of traffic accidents and a point of considerable congestion and delay. One-half of all urban traffic accidents and more than three-fourths of all vehicular delays experienced in urban areas occur at these locations (6). The intersection is a critical element because vehicles arriving from different directions converge on this small area. The efficiency and capacity of the entire street system is generally dependent on the characteristics of the intersections in the system. Also, the safety of the individual driver is related to the intersectional characteristics of the street system. The type of traffic control used at intersections influences the frequency and severity of traffic accidents.

Paper sponsored by Committee on Characteristics of Traffic Flow.

The principle that a majority takes precedence applies in the field of traffic engineering when two traffic streams of unequal volumes come into conflict. The movement with the greater volume is usually less likely to respect the rights of the minor flow. The traffic engineer recognizes this principle when he finds it necessary to stop the minor stream by placing stop signs at the intersection. Whenever a gap in the major flow is equal to or greater than some acceptable value, one or more vehicles in the minor flow merge with or cross the major stream. In selecting acceptable gaps, attention must be focused on the distribution of large openings in the primary traffic stream.

The purpose of this study was to investigate the gap and lag acceptances for drivers entering and crossing a major roadway from a stopped position. A gap is defined as the time interval between the passing of the path of the side-street vehicle by two successive vehicles in a lane of traffic flow on the main street. Gaps are normally measured from front-to-front of the successive vehicles and, thus, include the length of the lead vehicle. On the other hand, a lag is the time interval measured from the arrival of a side-street vehicle at the stop bar of the intersection approach to the crossing of the path of this vehicle by the first main-street vehicle. Lag intervals are measured between the times when the fronts of the vehicles arrive at or cross their respective determination points. This driver-behavior evaluation was subdivided into the following main categories:

1. Determination of lag-and-gap acceptance distribution for side-street traffic regulated by a stop sign;
2. Consideration of community influence on these distributions; and
3. Comparison of driver time-interval acceptance for through, left-turn, and right-turn movements.

For each of these items various statistical tests were employed to evaluate the significance of the findings.

Simulation methods are presently being developed to analyze traffic flow and its characteristics at intersections and at ramps on freeways. However, simulation techniques are dependent on field investigations of traffic-flow performance. The results of driver-behavior studies are required to construct realistic mathematical models which can be used to simulate traffic situations in computer analyses. In addition, time-acceptance distributions provide fundamental information for the development of warrants for traffic-control devices and for the determination of intersection capacities.

PREVIOUS INVESTIGATIONS

Several research projects have been conducted to study the traffic characteristics of at-grade intersections. In these investigations various techniques were used to analyze intersectional flow patterns under different roadway and traffic conditions. In 1944, B. D. Greenshields employed time-motion pictures to study the time intervals accepted by drivers when crossing another traffic stream. Both controlled and uncontrolled intersections were studied, and, in particular, stop sign controlled intersections were included in these investigations. The average minimum acceptable time gap was defined as that value which is accepted by 50 percent of the drivers (3).

A few years later a similar study was made with a 20-pen graphic recorder by M. S. Raff; the concept of a time lag was introduced and evaluated. Instead of Greenshields' definition of an average minimum time gap, Raff developed the critical lag, which is defined as the median time lag; that is, the number of accepted lags shorter than the critical time lag is equal to the number of rejected lags longer than this specific value. In this study the critical lags were not constant but varied from intersection to intersection. Critical lags were influenced by sight obstructions, main-street speeds, main-street width, and the patterns of traffic flow on the side street. However, traffic volumes on the main street did not significantly modify the critical-lag value. Turning movements, which probably affect the amount of delay to the side-street vehicles, received little attention in that study. In comparing the critical lag with the time gap, Raff noted that this gap averaged about 0.2 sec greater than the critical lag (4).

Although most projects were limited to the consideration of vehicular delay and speed-change performance, H. H. Bissell considered vehicular movements through the intersection as through, left turn, and right turn. A 20-pen graphic recorder was used to obtain the necessary data for two intersections within similar urban areas. In the analysis of the data it was determined that the acceptance of lags was not significantly different from the acceptance of gaps. This homogeneity of lags and gaps was demonstrated by the overlapping of the confidence intervals determined for a confidence coefficient of 80 percent. A mathematical formula of the accumulative logarithmic normal distribution for pooled lags and gaps was devised to describe the human judgment for accepting or rejecting the main-street traffic gaps offered to drivers stopped on the side street. Although the lane position (near or far) of the main-street traffic did not influence the gap acceptance for the traffic entering from the side street, the type of entering maneuver produced different gap-acceptance distributions (1).

The studies by Greenshields and Raff were both conducted in New Haven, Conn., and Bissell investigated one intersection in Richmond and another in Oakland, Calif. As a general comparison of the three studies, Greenshields, Raff, and Bissell reported, respectively, a mean gap acceptance of 6.1 sec, a mean lag acceptance of 5.9 sec, and a mean lag-and-gap acceptance of 5.8 sec.

PROCEDURE

To establish the acceptance distributions for lags and gaps, it was necessary to observe driver behavior at selected intersection locations. Statistical estimations and various tests of hypothesis were used, respectively, to develop functional relationships and to appraise the significance of the findings.

Site Selection

The selection of suitable study sites involved the consideration of several factors. To obtain a representative sample of drivers, two at-grade intersections were chosen in each of two cities. Lafayette and Indianapolis, Ind., were selected as typical of small- and medium-sized standard metropolitan areas. These communities permitted a comparison of driving habits as related to city size.

The following limitations were imposed on the selection of study locations to control several roadway and traffic variables which could influence the study results:

1. The four intersections were located in residential sections of an urban area.
2. Commercial roadside development near the intersection, such as service stations, laundries, and ice-cream stands, were not considered objectionable if the rest of the immediate area was residential.
3. To obtain a random sample of gaps in the main traffic stream, the intersections were located at least 0.25 mi from any traffic-control device on the main street.
4. Traffic volumes on the main and side streets were in excess of 250 and 60 vph, respectively. These limits were established to provide for the collection of data within a reasonable period of time. Also, the range of gaps presented to the side-street drivers is a function of the volume on the main street. A wide range of gap and lag sizes was desired in this field investigation.
5. The intersections studied were very similar with regard to their geometry, consisting of two, two-way streets crossing each other at right angles. Sight distance conditions were about equal on all approaches, and the main-street width was approximately the same at all intersections.
6. Posted speed limits on the main and side streets were 30 mph, except for one side street which was posted with a speed limit of 25 mph.

A brief description of each intersection location is given in Table 1.

TABLE I

SUMMARY OF STUDY LOCATIONS

CITY	INTER-SECTION	MAJOR STREET	COMMERCIAL DEVELOPMENT AT INTERSECTION	GRADE AT INTER-SECTION	DEVELOPMENT ALONG STREETS	SIGHT CONDITIONS	POSTED SPEED LIMIT	AVERAGE MAJOR ST. VOLUME	AVERAGE MINOR ST. VOLUME
LAFAYETTE	A	N. 14 ST.	SOME	LEVEL	RESIDENTIAL	ADEQUATE	25 MPH	420 VPH	65 VPH
	B	KOSSUTH ST.	NONE	MOSTLY LEVEL	RESIDENTIAL	ADEQUATE	30 MPH	330 VPH	65 VPH
INDIANA-POLIS	C	N. ILLINOIS ST.	NONE	MOSTLY LEVEL	RESIDENTIAL	ADEQUATE	30 MPH	460 VPH	65 VPH
	D	N. COLLEGE ST.	SOME	LEVEL	RESIDENTIAL	ADEQUATE	30 MPH	590 VPH	65 VPH

Equipment

Time-motion pictures were chosen in this investigation as the best means of securing the necessary data. The camera used was a 16-mm Eastman Cine Kodak Special with a wide-angle lens. A spring motor drove the camera at the rate of 8 frames per sec. Therefore, elapsed time intervals were measured to the nearest 0.125 sec. This degree of precision was considered sufficient to measure lag and gap times. If a vehicle is traveling at 30 mph, approximately 1.0 sec is required for it to pass through an average intersection. About 8 pictures of this vehicle are recorded on the movie film.

Data Collection

Data collection was performed with the same procedure at all study sites. At each intersection the camera was mounted on a tripod at some vantage point located near the side-street approach. The camera was positioned about 30 ft from the main street to view the entire intersection area, and it was relatively inconspicuous to the passing traffic. A typical field installation is shown in Figure 1.

Data were collected on Monday, Wednesday, and Friday in the morning and afternoon off-peak periods. Approximately 5 days were spent at each site to obtain a wide range of traffic-volume levels. Field studies were performed only when the weather was clear and the pavements were dry. The speed of the camera was frequently calibrated with a stopwatch.

The camera was started whenever a side-street vehicle approached the intersection and stopped for the stop sign. After the side-street driver had accepted a time gap, the camera was stopped. The maximum time gap considered in this investigation was 15 sec, and the camera was stopped if the time interval accepted was longer than this limiting value. Only passenger cars and light commercial vehicles with passenger-car operating characteristics were considered in this field investigation.

The developed film was viewed by a time-motion study projector. The projector has a frame counter, and the film can be advanced or reversed one frame at a time. The pictures were projected on a screen with grid lines drawn to define the collision points. The locations of the possible collision points are shown in Figure 2. A stopped vehicle either proceeded straight through the intersection, turned right, or turned left. If a driver went straight through, the path of movement intersected that of vehicles from both the right and the left. When a right turn was made, the movement merged with traffic coming from the left and did not conflict with traffic from the right. On a left turn the path of a main-street vehicle approaching from the left was crossed, and the maneuver merged with the major stream coming from the right.

The frame number in which the vehicle stopped at or crossed the property line of the intersection approach (Fig. 1) was recorded. When the next opposing vehicle crossed the collision point, the frame number was again noted. The difference between these two frame numbers was divided by the camera speed of 8 frames per sec to produce

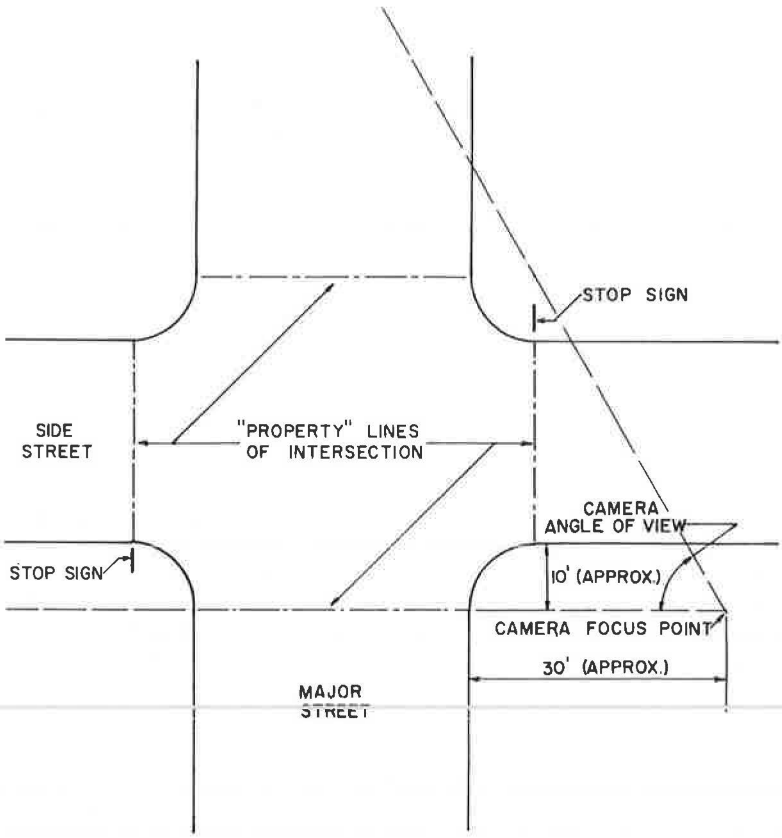

Figure 1. Typical field setup.

the available time lag in seconds. If a driver on the stop-signed street proceeded across the intersection in front of the crossing vehicle, the time interval was considered as accepted. Otherwise, the time opportunity was rejected. A time-gap interval was recorded as the difference in frame numbers, between two successive main-street vehicles passing the collision point, divided by 8 frames per sec.

Data Analysis

The statistical analysis was designed to investigate the significance of the differences in median acceptance times for the following categories:

1. Lag-acceptance time and gap-acceptance time;
2. Acceptance times for right turns, left turns, and through movements; and
3. Acceptance times in one community as compared with those in the other community.

A technique called probit analysis was applied to test these differences statistically. This method is especially applicable in research dealing with "all-or-nothing" responses (2).

The acceptance or rejection of a time gap is an all-or-nothing, or binomial, response, dependent on the size of the gap. The minimum time gap a driver accepts is defined as the tolerance level. The driver is assumed to reject all smaller time gaps and to accept all larger time gaps. This tolerance may be a fixed quantity for a subject, or it may vary with time.

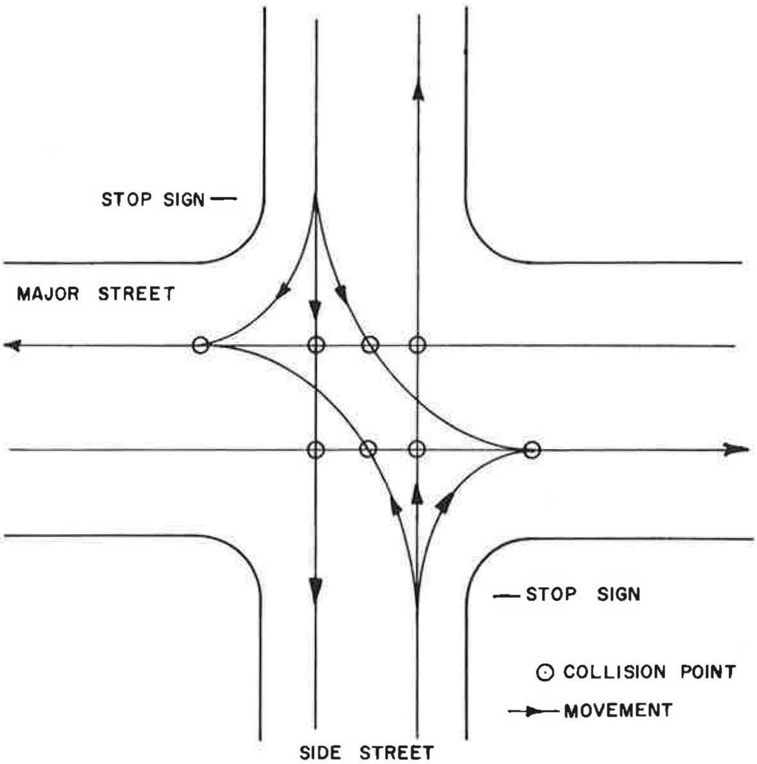

Figure 2. Typical collision points considered for side-street traffic.

A variation in the tolerance value exists from one member to another of the population. Thus, it was necessary to consider the distribution of tolerances over the population studied. The assumption of a normal distribution for the common logarithm of the tolerances suggested the application of the probit transformation. This transformation from percentages or proportions to probits forces the normal sigmoid curve of the untransformed data into a linear relationship.

The probit of the proportion (P) is defined as the abscissa which corresponds to a probability of P in a normal distribution having a mean of 5.0 and a variance of 1.0. A normalizing transformation for the time gap is required so that the transformed measure (x) of the time (t) is normally distributed. The normalizing function was provided by a logarithmic transformation in this investigation of driver acceptance times. The probit of the expected proportion accepting a time gap is related to the time gap by the following linear equation:

$$Y = 5.0 + \frac{1}{\sigma}(X - u)$$

where

Y = probit of the proportion accepting time gap,
X = logarithm of time gap,
u = mean of tolerance distribution, and
σ = standard deviation of tolerance distribution.

By means of the probit transformation the study data were used to obtain an estimate of this equation. The mean and standard deviation of the tolerance distribution were also determined. In particular, median gap- and lag-acceptance times were estimated as the antilogarithm of X when Y = 5.0.

Initially the data were tabulated into groups of 1-sec intervals. These observed data are binomial in nature, and within each time interval driver responses have a binomial distribution. If a driver, selected at random from a population, is exposed to a time interval of t sec, the probability of acceptance is P, and the probability of rejection is Q = 1 - P. The purpose of observing a group of drivers in each interval of the time series was to obtain an estimate of the proportion of drivers accepting this interval.

When experimental data on this relationship between time and acceptance have been obtained, either a graphic or an arithmetic procedure can be used to estimate the slope (b) of the regression line, which is an estimate of the reciprocal of the standard deviation, and the logarithm of the median acceptance time (m) at which Y = 5.0. The arithmetic analysis is necessary when an accurate assessment of the precision of the estimates is desired.

To conduct either type of analysis, the percentage of acceptance observed for each time gap was first calculated and converted to a probit. These probits were then plotted as a function of the logarithm of the time gap, and a straight line was visually fitted to these points. Only the vertical deviations of these points were considered in drawing the line. Very extreme probits outside the range of 2.5 to 7.5 are relatively unimportant and can usually be disregarded. However, these extreme values should be included in the analysis when more drivers are observed in these ranges than in the groups giving intermediate probit values. This regression line is an approximation of the functional relationship between the gap-acceptance probit and the logarithm of gap time. This relation was used to initiate the arithmetic process of estimating a better-fitting regression line. The mathematical basis for the method of estimating the probit regression equation by a process of successive approximations is given by Finney (2).

The statistical comparison of acceptance times is based on the assumption that the variances for the tolerance distributions are equal. This relationship is demonstrated in the probit analysis by the parallelism of the regression lines. If two series of data yield parallel probit regression lines, then a constant difference exists between the time gaps for all corresponding proportions of responding subjects. This constant time difference is determined by computing the antilogarithm of the difference between the common logarithms of the median acceptance times. The various steps followed in estimating the probit regression line are outlined by Finney (2). A test of parallelism for two or more regression lines was performed by comparing the sum of the individual chi-square values for the series with that for the total sums of squares and products.

The methods employed by Raff and Bissell in their analyses were applied to the original data collected in this study to make comparisons with the results obtained by the probit method. Raff determined the critical lag by plotting two cumulative distributions on the same graph. One curve describes the accepted number of lags shorter than a time interval, and the other shows the rejected number of lags longer than this interval. The value of the critical lag was determined as the time at which the two curves intersect (Fig. 3).

Bissell acknowledged the binomial character of the gap-acceptance distribution. The data were plotted on log-probability paper, and a straight line was visually fitted to these points. The lines representing lags and gaps were drawn with equal slope for right turns, left turns, and through movements in each comparison of varying conditions. However, the slopes were different for the various comparisons. A sample graph is shown in Figure 4. The standard deviation was determined directly from this plot by assuming that the mean time gap is the median value of the acceptance time. The standard deviation was then estimated as the difference between the median acceptance value and the time corresponding to an acceptance of 15.9 percent.

RESULTS

Various methods have been developed to determine the time interval an average driver accepts in crossing or merging with a traffic stream from a stopped position.

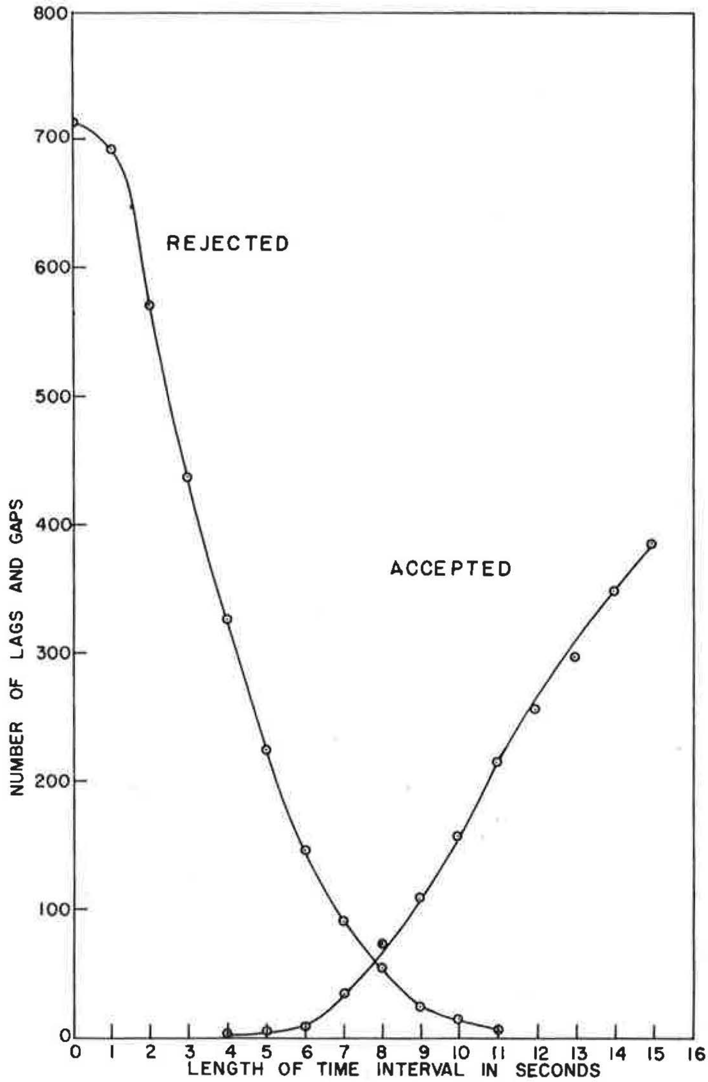

Figure 3. Distribution of accepted and rejected lags and gaps at intersection (A and B) left turns.

Drivers were observed at four different intersections, and the time interval required by each driver to enter or cross the major traffic stream was recorded. The technique of probit analysis was employed in the statistical treatment of these observations. In addition to probit analysis, two other methods, one developed by Raff and the other by Bissell, were considered in this study of driver behavior.

Probit Method

Probit analysis is based on the assumption that a particular transformation of an all-or-nothing response is normally distributed. In the problem of determining lag- and gap-acceptance times, previous studies have indicated that the logarithms of acceptance times are normally distributed. Thus, when the percentages of drivers accepting particular time intervals are converted to probits, a linear relationship exists between the probit of the percent acceptance and the logarithm of acceptance time.

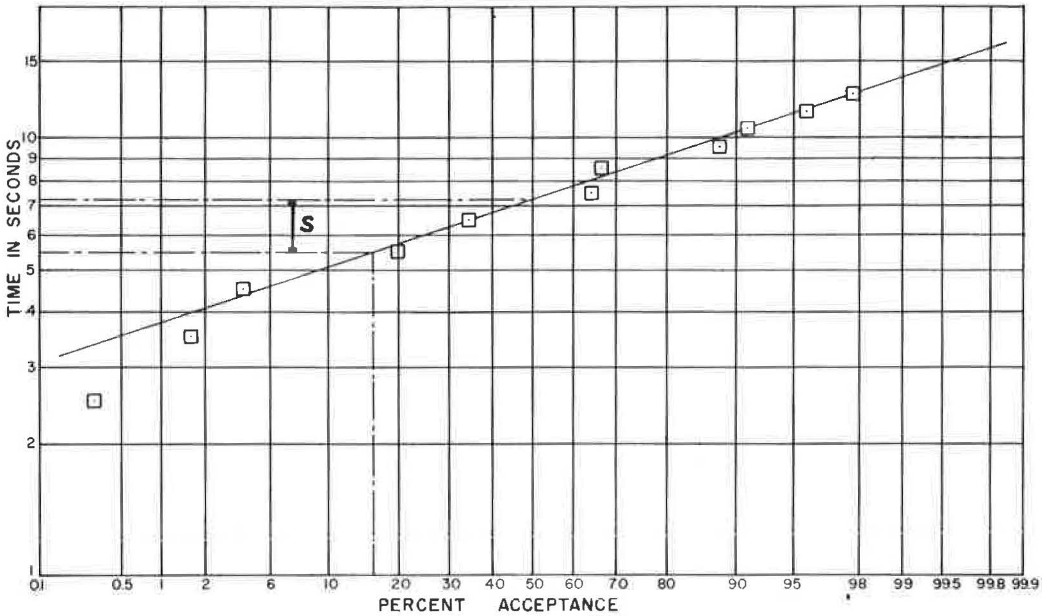

Figure 4. Lag-and-gap distribution for through movements in Lafayette and Indianapolis.

The relationships between lag acceptance and time and between gap acceptance and time are shown in Figures 5 and 6. Similar relations between lag-and-gap acceptance and time intervals for different traffic movements and at the various intersection locations are shown in Figures 7 to 10. Each linear regression represents the best fit of a straight line to the observed data and was used to estimate the median acceptance time. For a 5 percent level of significance the difference in acceptance times was considered as non-significant if the relative acceptance time (R) was equal to or less than 1.10 (2).

In previous studies the precision of the findings was not clearly stated, and no tests were performed to investigate the significance of the results. However, confidence limits for median acceptance time, as well as those for the differences between acceptance times, may be calculated with the probit technique. A test for the goodness of fit of the regression line to the data points measures the precision of the time-value estimates (2).

The differences in acceptance times between lags and gaps were first analyzed in this investigation of driver behavior. By pooling the data from the two intersections in Lafayette, the relative acceptance time was contained in the interval of 1.00 to 1.08 for a confidence coefficient of 95 percent. That is, the median gap-acceptance time is not expected to exceed 1.08 times the median lag-acceptance time for a level of significance of 5 percent. Because the test statistic of 1.08 is less than the critical value of 1.10, the difference between lag acceptances and gap acceptances was not considered significant. The median acceptance times for lags and gaps were, respectively, 7.48 and 7.71 sec. The respective standard errors of estimate were 0.13 and 0.16 sec. The findings for this comparison of lags and gaps are given in Table 2.

For the two intersections in Indianapolis, the relative acceptance time was 1.01 with 95 percent confidence limits of 1.00 and 1.06. With a gap acceptance that was only 1.01 times greater than the lag acceptance, this difference was not large enough to be considered significant. Median acceptance times for lags and gaps together with standard errors and confidence limits are given in Table 3. Because of the small differences that existed between lag acceptances and gap acceptances in both Lafayette and Indianapolis, it was assumed that these lags and gaps came from the same populations in the respective cities.

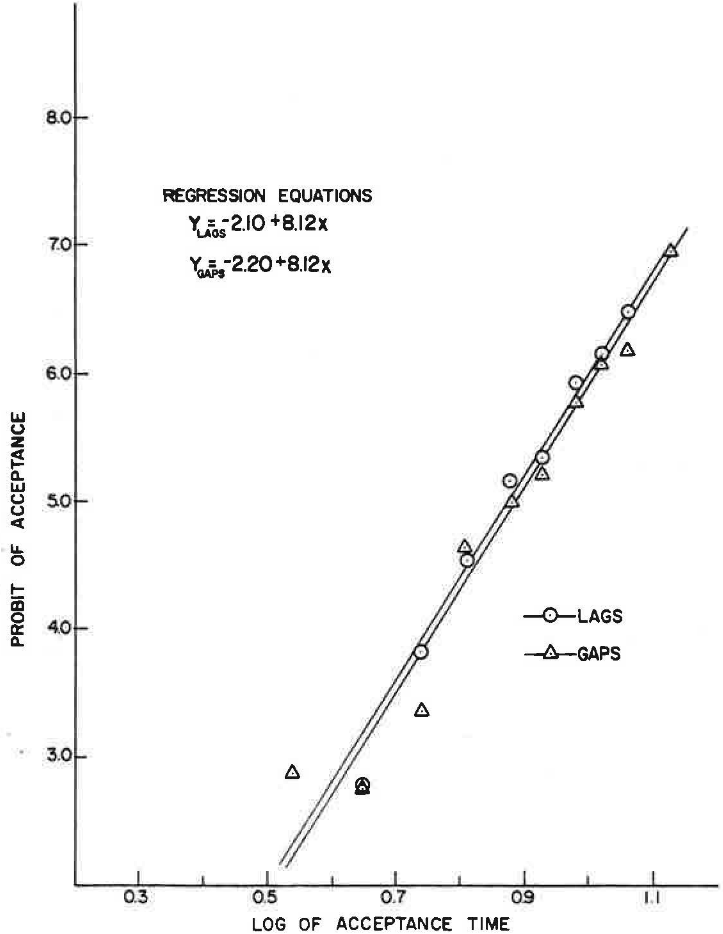

Figure 5. Probit regression lines for estimation of difference of lag acceptance time and gap acceptance time in Lafayette.

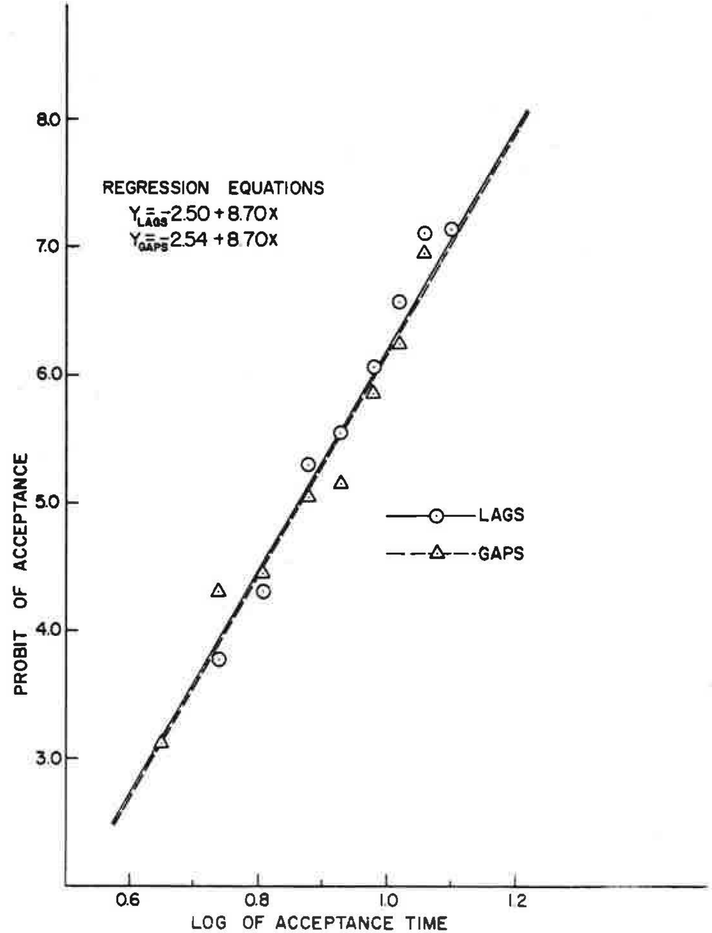

Figure 6. Probit regression lines for estimation of difference in lag acceptance time and gap acceptance time in Indianapolis.

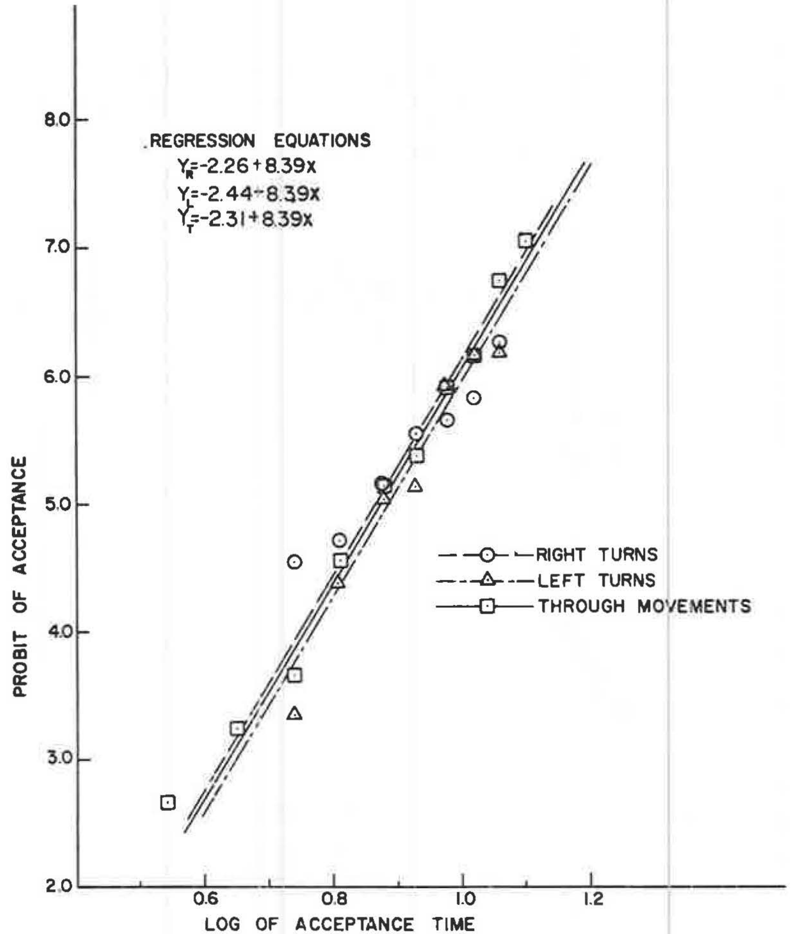

Figure 7. Probit regression lines for estimation of difference in acceptance time for various movements at intersections in Lafayette (lags and gaps pooled).

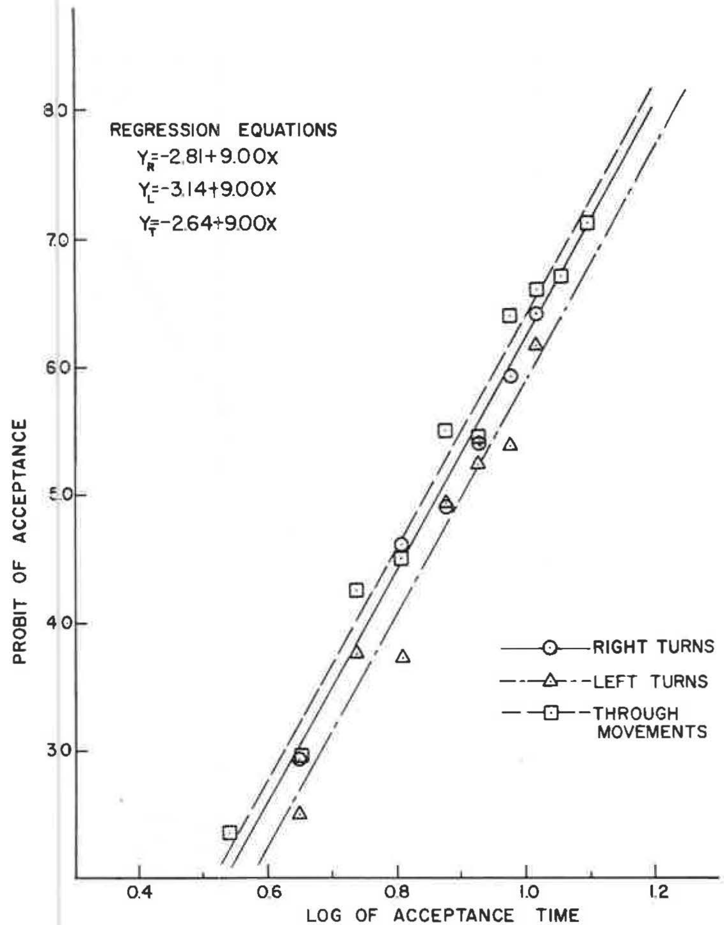

Figure 8. Probit regression lines for estimation of difference in acceptance time for various movements at intersections in Indianapolis (lags and gaps pooled).

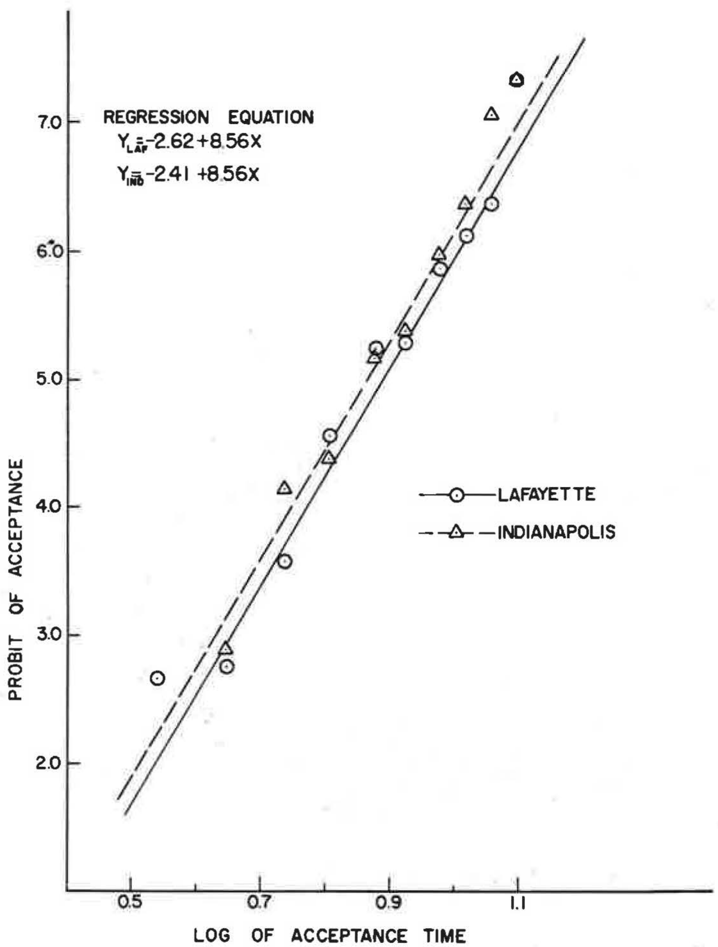

Figure 9. Probit regression lines for estimation of difference in acceptance time for intersections in Lafayette and Indianapolis (lags and gaps pooled, all movements pooled respectively).

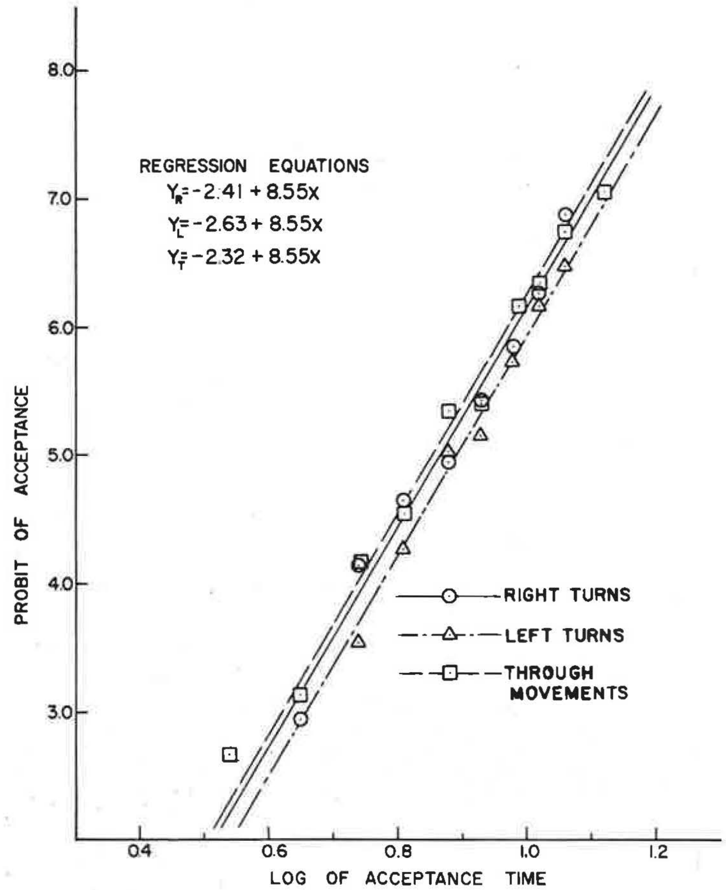

Figure 10. Probit regression lines for estimation of difference in acceptance time for various movements at intersections in Lafayette and Indianapolis (intersections pooled, lags and gaps pooled).

TABLE 2

DIFFERENCE BETWEEN MEDIAN LAG ACCEPTANCE AND MEDIAN GAP ACCEPTANCE AT TWO LAFAYETTE INTERSECTIONS[a]

Summary Statistics	Lags	Gaps
Log of mean acceptance time ($\bar{x}$)	0.893	0.897
Mean probit ($\bar{y}$)	5.15	5.08
Log of median acceptance time (m)	0.874	0.887
Median acceptance time (10^m sec)	7.48	7.71
Standard error of median acceptance time (sec)	0.13	0.16
95 percent confidence limits for median acceptance time (sec)	7.21; 7.75	7.38; 8.04

Test Statistics	Comparison Between Gaps and Lags
Difference in m values (M)	0.013
Relative acceptance time ($R = 10^M$)	1.03
Standard error of relative acceptance time	0.028
95 percent confidence limits for relative acceptance time	1.00; 1.08

[a]Summary of test results.

TABLE 3

DIFFERENCE BETWEEN MEDIAN LAG ACCEPTANCE AND MEDIAN GAP ACCEPTANCE AT TWO INDIANAPOLIS INTERSECTIONS[a]

Summary Statistics	Lags	Gaps
Log of mean acceptance time ($\bar{x}$)	0.898	0.866
Mean probit ($\bar{y}$)	5.31	4.99
Log of median acceptance time (m)	0.862	0.867
Median acceptance time (10^m sec)	7.28	7.36
Standard error of median acceptance time (sec)	0.13	0.13
95 percent confidence limits for median acceptance time (sec)	7.03; 7.53	7.11; 7.61

Test Statistics	Comparison Between Gaps and Lags
Difference in m values (M)	0.005
Relative acceptance time ($R = 10^M$)	1.01
Standard error of relative acceptance time	0.024
95 percent confidence limits for relative acceptance time	1.00; 1.06

[a]Summary of test results.

After the lags and gaps at the intersections in each city were combined, comparisons were performed among the through, left-turn, and right-turn traffic movements. The median acceptance times in Lafayette for right turns, left turns, and through movements were, respectively, 7.33, 7.71, and 7.43 sec. In the comparison between left-turning and right-turning drivers, the relative acceptance time was 1.05 times greater for left turns than for right turns. The 95 percent confidence limits for this relative acceptance time were 1.00 and 1.10.

The relative acceptance times for the comparisons between left turns and through movements and between through movements and right turns were 1.04 and 1.02, respectively. These values were contained in the intervals between 1.00 and 1.08 and between 1.00 and 1.07, respectively, for a 5 percent level of significance. These results are summarized in Table 4 for the various traffic-movement comparisons. According to the criterion that only relative acceptance times greater than 1.10 represent significant differences, the median acceptance times for the various intersectional movements were statistically equal in Lafayette.

Similar comparisons were performed for the data obtained at the two Indianapolis intersections. Significant differences were observed between the lag-and-gap-acceptance times for left turns and right turns and for left turns and through movements. However, the relative difference between right turns and through movements was not significant. The median acceptance times were 7.38, 8.02, and 7.06 sec for

TABLE 4

DIFFERENCE BETWEEN MEDIAN LAG-AND-GAP ACCEPTANCE FOR VARIOUS MOVEMENTS AT TWO LAFAYETTE INTERSECTIONS[a]

Summary Statistics	Side-Street Movements		
	Right	Left	Through
Log of mean acceptance time ($\bar{x}$)	0.905	0.904	0.892
Mean probit ($\bar{y}$)	5.33	5.14	5.17
Log of median acceptance time (m)	0.865	0.887	0.871
Median acceptance time (10^m sec)	7.33	7.71	7.43
Standard error of median acceptance time (sec)	0.22	0.14	0.15
95 percent confidence limits for median acceptance time (sec)	6.91 7.77	7.42; 8.00	7.13; 7.73
Test Statistics	**Comparison Between Movements**		
	Lt to Rt	Lt to Thru	Rt to Thru
Difference in m values (M)	0.022	0.016	0.010
Relative acceptance time ($R = 10^M$)	1.05	1.04	1.02
Standard error of relative acceptance time	0.024	0.028	0.025
95 percent confidence limits for relative acceptance time	1.00 1.10	1.00; 1.08	1.00; 1.07

[a]Summary of test results.

right turns, left turns, and through movements. The upper 95 percent confidence limits for the relative acceptance-time values were 1.18, 1.20, and 1.10, respectively, for the comparisons of left turns to right turns, left turns to through movements, and right turns to through movements (Table 5).

To evaluate the influence of community size on the observed lag-and-gap acceptances, the significance of the difference in the median acceptance values was tested for the combined traffic movements in the two study cities. The median acceptance times were 7.76 sec in Lafayette and 7.36 sec in Indianapolis. The 95 percent confidence limits for the acceptance times (Table 6) were 7.59 and 7.94 sec in Lafayette and 7.18 and 7.54 sec in Indianapolis. The upper 95 percent confidence limit for the relative acceptance time was 1.12. That is, the median lag-and-gap-acceptance time in Lafayette was significantly greater than in Indianapolis for a 5 percent level of significance. Drivers in small-sized cities apparently require larger openings to enter or cross a major traffic flow from a stopped position at an intersection than those operating vehicles in medium-sized communities.

Because the difference in median acceptance times was significant only to a slight degree, the lag-and-gap acceptances were combined for the intersections in Lafayette and Indianapolis. The resulting comparison of lag-and-gap-acceptance times performed between the various movements is given in Table 7. Left-turning drivers have

TABLE 5

DIFFERENCE BETWEEN MEDIAN LAG-AND-GAP ACCEPTANCE FOR VARIOUS MOVEMENTS AT TWO INDIANAPOLIS INTERSECTIONS[a]

Summary Statistics	Side-Street Movements		
	Right	Left	Through
Log of mean acceptance time ($\bar{x}$)	0.871	0.899	0.861
Mean probit ($\bar{y}$)	5.03	4.95	5.11
Log of median acceptance time (m)	0.868	0.904	0.849
Median acceptance time (10^m sec)	7.38	8.02	7.06
Standard error of median acceptance time (sec)	0.16	0.20	0.13
95 percent confidence limits for median acceptance time (sec)	7.06; 7.70	7.64; 8.40	6.82; 7.30
Test Statistics	**Comparison Between Movements**		
	Lt to Rt	Lt to Thru	Rt to Thru
Difference in m values (M)	0.036	0.055	0.019
Relative acceptance time ($R = 10^M$)	1.09	1.13	1.05
Standard error of relative acceptance time	0.039	0.036	0.028
95 percent confidence limits for relative acceptance time	1.00; 1.18	1.06; 1.20	1.00 1.10

[a]Summary of test results.

TABLE 6

MEDIAN LAG-AND-GAP ACCEPTANCE DIFFERENCE, COMBINED MOVEMENTS, BETWEEN LAFAYETTE AND INDIANAPOLIS[a]

Summary Statistics	Lafayette	Indianapolis
Log of mean acceptance time ($\bar{x}$)	0.891	0.876
Mean probit ($\bar{y}$)	5.02	5.09
Log of median acceptance time (m)	0.890	0.867
Median acceptance time (10^m sec)	7.76	7.36
Standard error of median acceptance time (sec)	0.09	0.09
95 percent confidence limits for median acceptance time (sec)	7.59 7.94	7.18 7.54

Test Statistics	Comparison Between Lafayette and Indianapolis
Difference in m values (M)	0.023
Relative acceptance time ($R = 10^M$)	1.05
Standard error of relative acceptance time	0.037
95 percent confidence limits for relative acceptance time	1.00; 1.12

[a]Summary of test results.

1.06 and 1.09 times greater median lag-and-gap-acceptance times, respectively, than those drivers turning right or proceeding straight through the intersection. Significant differences existed between these movements at the 5 percent significance level, because the upper confidence limits for the relative acceptance times were 1.12 for the first comparison and 1.14 for the second comparison. However, right-turning drivers required a median acceptance time that was only 1.03 times greater than that selected by drivers continuing straight through the intersection. The median acceptance times for these two traffic movements were considered statistically equal at the 5 percent level of significance.

Raff Method

The findings obtained by using the Raff method depend largely on the manner in which the curves are fitted to the data points. No test is presently available to check the precision of this visual fitting technique. The resultant values are relatively accurate if the curve closely follows the plotted points. The results of this method are given in Table 8.

In the investigation of median acceptance times for lags and gaps with the combined data for the two intersections in Lafayette, the median value for lags was 7.60 sec and that for gaps was 7.75 sec, or 0.15 sec longer. The median acceptance times for lags and for gaps were found to be equal to 7.35 sec for the two intersections in Indianapolis.

In Lafayette the median lag-and-gap-acceptance time for right turns was 7.55 sec, or 0.05 sec shorter than the corresponding value for through movements. The value for left turns was 7.80 sec, or 0.20 sec greater than for through movements. However, greater differences were evident in Indianapolis for certain traffic movements. The median acceptance times were 7.30 sec for right turns, 7.95 sec for left turns, and 7.10 sec for through movements (Table 8). Drivers moving straight through the intersection had the lowest median acceptance time, although this value was only 0.20

TABLE 7

DIFFERENCE BETWEEN MEDIAN LAG-AND-GAP ACCEPTANCE FOR VARIOUS MOVEMENTS AT FOUR INTERSECTIONS COMBINED IN LAFAYETTE AND INDIANAPOLIS[a]

Summary Statistics	Side-Street Movements		
	Right	Left	Through
Log of mean acceptance time ($\bar{x}$)	0.883	0.908	0.865
Mean probit ($\bar{y}$)	5.14	5.14	5.08
Log of median acceptance time (m)	0.867	0.893	0.856
Mean acceptance time (10^m sec)	7.36	7.82	7.18
Standard error of median acceptance time (sec)	0.14	0.11	0.09
95 percent confidence limits for median acceptance time (sec)	7.10; 7.64	7.60; 8.04	7.00; 7.36
Test Statistics	**Comparison Between Movements**		
	Lt to Rt	Lt to Thru	Rt to Thru
Difference in m values (M)	0.026	0.037	0.011
Relative acceptance time ($R = 10^M$)	1.06	1.09	1.03
Standard error of relative acceptance time	0.026	0.023	0.024
95 percent confidence limits for relative acceptance time	1.02; 1.12	1.05; 1.14	1.00; 1.07

[a]Summary of test results.

TABLE 8

MEDIAN ACCEPTANCE TIMES AT STUDY LOCATIONS—RAFF METHOD

Location	Combined Lags and Gaps (sec)			Combined Movements (sec)	
	Right Turns	Left Turns	Through Movements	Lags	Gaps
Lafayette	7.55	7.80	7.60	7.60	7.75
Indianapolis	7.30	7.95	7.10	7.35	7.35
Lafayette and Indianapolis	7.45	7.85	7.35		

sec shorter than that selected by drivers turning right. The left-turning drivers required a considerably longer median acceptance time.

When data in Lafayette and Indianapolis were grouped together, the median acceptance time for through movements was 7.35 sec, or only 0.10 sec lower than that for right turns. The value of the median lag-and-gap-acceptance time for the left-turning drivers was greater than that for drivers turning right or moving straight through the intersection.

Raff computed values varying from 4.6 to 6.0 sec for the median values of driver lag-acceptance time for the intersections studied in Connecticut (4). These median times are approximately 2.0 to 2.5 sec shorter than those measured in the present investigation. Raff found that 2.0 percent of the drivers accepted a time interval less than 1.0 sec and up to 7.0 percent were observed in the interval between 1.0 and 2.0 sec. This acceptance of extremely short time lags may account for his lower median acceptance times. Lags were measured with the near curb line as the reference point in the Raff study. However, in this study lags were referred to the collision points. The use of the longer approach path in the latter case may partially account for the differences between median acceptance times.

Bissell Method

The results obtained by the Bissell technique are predicated on the accuracy of fitting a straight line to the observed data. Although median values were estimated to the nearest 0.05 sec, precision of this visual fit cannot be described in numerical terms. The lines were drawn parallel to each other so that homogeneity of variance was obtained.

The median acceptance times for lags and for gaps in Lafayette and in Indianapolis had an equal difference of 0.10 sec. These lag-and-gap acceptances were 7.40 and 7.50 sec for Lafayette and 7.20 and 7.30 sec for Indianapolis. The acceptance times determined by the Bissell method are given in Table 9.

Median acceptance times varied only slightly for the two intersections in Lafayette. The single exception was the comparison of through movements and left turns. Drivers performing a left turn required an opening that was, on the average, 0.40 sec longer than that needed by those passing straight through the intersection. Drivers turning right had a median lag-and-gap-acceptance value of 7.30 sec. Left-turning drivers and those proceeding straight through the intersection had median acceptance times of 7.50 and 7.10 sec, respectively.

For the Indianapolis intersections the differences in lag-and-gap-acceptance times for the various movements were found to be greater than the corresponding values in Lafayette. Left-turning drivers had a median lag-and-gap-acceptance time of 7.65

TABLE 9

MEDIAN ACCEPTANCE TIMES AT STUDY LOCATIONS—BISSELL METHOD

Location	Combined Lags and Gaps (sec)			Combined Movements (sec)	
	Right Turns	Left Turns	Through Movements	Lags	Gaps
Lafayette	7.30	7.50	7.10	7.40	7.50
Indianapolis	7.35	7.65	7.05	7.20	7.30
Lafayette and Indianapolis	7.35	7.65	7.10		

TABLE 10

POOLED MEDIAN ACCEPTANCE TIMES IN LAFAYETTE AND INDIANAPOLIS AS DETERMINED BY DIFFERENT METHODS

Method	Combined Lags and Gaps (sec)		
	Right Turns	Left Turns	Through Movements
Probit	7.36	7.82	7.18
Raff	7.45	7.85	7.35
Bissell	7.35	7.65	7.10

sec, which was 0.30 sec longer than that for right-turning drivers and 0.60 sec longer than that for drivers moving straight through the intersection.

Acceptance times for the combined drivers in Lafayette and Indianapolis are also given in Table 9. Right-turning and left-turning drivers had median acceptance times of 7.35 and 7.65 sec, respectively. Drivers moving straight through the intersection had a median lag-and-gap-acceptance time of 7.10 sec.

In his field investigations, Bissell obtained median lag-and-gap-acceptance times for right turns, left turns, and through movements, of 5.25, 6.25, and 5.80 sec, respectively (1). The corresponding values from the combined intersections in the present investigation are 7.35, 7.65, and 7.10 sec. The difference of 2.10 sec between right turns was the greatest variation encountered in the comparison of the two studies.

The discrepancies in these acceptance times are probably due to different populations of drivers. The volumes on the side and main streets were larger in the Bissell investigation, and drivers might have been forced to accept smaller time intervals. However, Raff indicated that main-street traffic volumes have little influence on driver gap-and-lag acceptances. This forced gap acceptance was observed by Bissell during peak hours when side-street drivers forced themselves into the main traffic stream in which adequate gaps were not available. Bissell also noted that many drivers cruised by the stop sign without actually stopping. This fact was particularly true for right-turning drivers and may account for the differences observed in the acceptance times for this turning movement.

Comparison of Analytic Techniques

The corresponding median acceptance-time values as determined by the probit, Raff, and Bissell methods of analysis are compared in Table 10. A reasonable agreement is evident among these three analytic techniques. In general, the lag-and-gap-acceptance times determined by the probit method are smaller than those values obtained by the Raff procedure and larger than those median acceptances estimated by the Bissell method.

CONCLUSIONS

The following conclusions inferred from the findings of this field investigation are valid only for those drivers and vehicles sampled at the study intersections in Lafayette and Indianapolis. However, these locations are representative of right-angle intersections formed by two-way, two-lane urban streets. The traffic flows on the side streets are controlled by stop signs.

1. No drivers accepted any time interval of less than 2.0 sec, and only one driver was observed accepting an interval of less than 3.0 sec.

2. The overall median acceptance times for right-turn, left-turn, and through movements were 7.36, 7.82, and 7.18 sec, respectively.

3. There were no significant differences between the median lag-acceptance and the median gap-acceptance times at the four intersections.

4. In Lafayette the gap-and-lag-acceptance times for the right-turn, left-turn, and through movements were statistically equal.

5. Significant variations were found between right- and left-turning drivers and between drivers proceeding through and those making left turns for the study intersections in Indianapolis. Through-movement and right-turn acceptance times differed only slightly.

6. When the intersections in Lafayette were combined with those in Indianapolis, a difference in acceptance times was found between drivers making left turns and right turns and between those performing through movements and left turns. However, no significant difference existed between right-turning drivers and the drivers moving straight through the intersection.

7. Lag-and-gap acceptances for combined movements in Lafayette and Indianapolis were significantly different. The size of the community apparently has some effect on driver acceptance of time gaps, because this median value increased with decreasing city size.

8. Only two of the median acceptance times as determined by the Raff and Bissell methods were outside the 95 percent confidence limits for the corresponding values obtained by the probit analysis. Thus, a general agreement existed among the results from the three methods investigated.

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An Evaluation of Fundamental Driver Decisions And Reactions at an Intersection

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There has been broad interest and increasing emphasis in the development and utilization of mathematical theories and computer simulation models of traffic flow phenomena. The development and effective application of these new techniques is unalterably dependent on a fuller understanding of the fundamental parameters of vehicle and driver behavior. In this connection, there is no substitute for the measurement and analysis of real traffic behavior under actual operating conditions.

This paper reports the field measurement and analysis of fundamental driver decision and reaction parameters at a stop-signed intersection. The following research objectives were pursued:

1. A detailed examination to determine and verify the characteristics of lag and gap acceptance of drivers waiting at a stop sign.
2. Evaluation of the influence of the following traffic factors on driver decisions: (a) vehicle type, (b) pressure of traffic demand, (c) direction of movements through the intersection, (d) sequence of gap formation, and (e) conditions on the opposing side street approach.
3. Determination of the characteristics of starting delay time in accepting lags and gaps, and evaluation of the influence of certain traffic factors on these distributions.

The results strongly supported earlier findings which indicated the relationship between lag or gap size and percent acceptance is log-normal. Of the traffic factors studied, those which significantly influenced driver decisions were (a) pressure of traffic demand, (b) direction of traffic movement during periods of heavy demand, and (c) sequence of gap formation during periods of heavy demand.

Definitions of starting delay time in accepting lags and gaps were set forth. Analysis of field observations of this parameter indicated that factors which had important influence on driver decisions, namely, pressure of traffic demand and sequence of gap formation, had similar and significant effects on starting delay times.

•IN RECENT years there has been broad interest and increasing emphasis in the engineering and scientific communities in the development and utilization of mathematical theories and computer simulation models of traffic flow phenomena. As in other

Paper sponsored by Committee on Traffic Control Devices.

more mature fields of endeavor, progress on this front has been slow, difficult, and often indirect. In the beginning, this work was principally academic, but the practical value of these new tools is steadily winning a place of importance in the profession. The time lag between development by the theoretician and implementation by the practitioner, however, has been characteristically long.

Although the many theories and models are diverse in purpose and approach, all of them are inherently dependent on the availability, in one form or another, of fundamental parameters of vehicle and driver behavior. The models are only as good as the input data which they use. In this connection, there is simply no substitute for the measurement and analysis of real traffic behavior under actual operating conditions. It is ironic that, in the face of greater need, some have recognized a subordination of interest in the tedious work of comprehensive field observations of the fundamentals of behavior. Fundamental parameters must be pursued more microscopically to take into account the complexity of interactions existing in the real traffic situation. This would serve to broaden the base for theoretical accomplishments and lead to more realistic models which can be more effectively applied by the profession.

The problem of dealing with the conflict of vehicles traveling on roadways intersecting at grade has always been a primary concern of traffic engineers. Intersections at grade remain critical elements of the highway system in that they are principal sources of accidents and delays; furthermore, their capacities restrain the entire system's ability to process traffic. The most common method of controlling this conflict is the stop sign. Traffic operation and driver performance associated with stop sign control of intersections has been the subject of extensive empirical and theoretical study. Beginning with the classic work of Greenshields (1), which included both observation and sample applications of probability theory, many have carried the work forward, including Raff (2), Herman and Weiss (3), Bissell (4), and a host of others (5-11). Yet our understanding of this universal problem remains significantly incomplete.

This paper reports a limited but intensive field study and evaluation of fundamental driver decisions and reactions at a stop-signed intersection. The emphasis was not so much on the absolute values of the statistics compiled, since these were peculiar to the particular intersection studied, but rather on uncovering the degree of influence of certain traffic factors on the fundamental driver decisions and reactions.

OBJECTIVES AND TERMINOLOGY

Research Objectives

At an intersection controlled by a stop sign, where delays are for the most part encountered by vehicles on the yielding street, the overall efficiency of performance is highly dependent on the decisions and reactions of the waiting driver attempting to cross or enter the mainstream. In an effort to increase the understanding of traffic behavior, the following research objectives were pursued:

1. Perform a detailed examination of an intersection controlled by a stop sign to determine and verify the characteristics of lag and gap acceptance distributions of the waiting vehicles.
2. Evaluate the influence of the following traffic factors on the lag and gap acceptance distributions: (a) vehicle type, (b) pressure of traffic demand, (c) direction of movement through the intersection, (d) sequence of gap formation, and (e) conditions on the opposite stop-signed approach.
3. Determine the characteristics of the distributions of starting delay times in accepting lags and gaps, and evaluate the influence of certain traffic factors on these distributions.

Terminology

Definition of the following terms is necessary for an understanding of the procedures and results of this research.

A gap is considered as the elapsed time between arrival of successive main street vehicles at a specified reference point in the intersection area.

A lag is that portion of a current gap remaining when a side street vehicle arrives; in other words, the elapsed time between arrival of a side street vehicle and arrival of the next main street vehicle.

A lag or gap is either accepted or not accepted (rejected) by the side street vehicle. A lag is accepted if the side street vehicle crosses or enters the main street before the arrival of the first main street vehicle. A gap is accepted if the side street vehicle crosses or enters between two main street vehicles comprising a gap.

Starting delay time in accepting gaps is the elapsed time between arrival of the first main street car comprising the accepted gap and the complete entry into the intersection of the side street car.

Starting delay time in accepting lags is the elapsed time between arrival of a side street car and its complete entry into the intersection.

A side street car is assumed to have completed entry into the intersection when its rear bumper has crossed the line which is an extension of the near side edges of the traveled portion of the main street.

Arrival of a side street vehicle on an unoccupied stop-signed approach is considered the point in time when the vehicle either stops or reaches its lowest speed.

When more than one vehicle is waiting in queue at a stop sign, the arrival of the second or succeeding side street vehicles is defined as coinciding with the complete

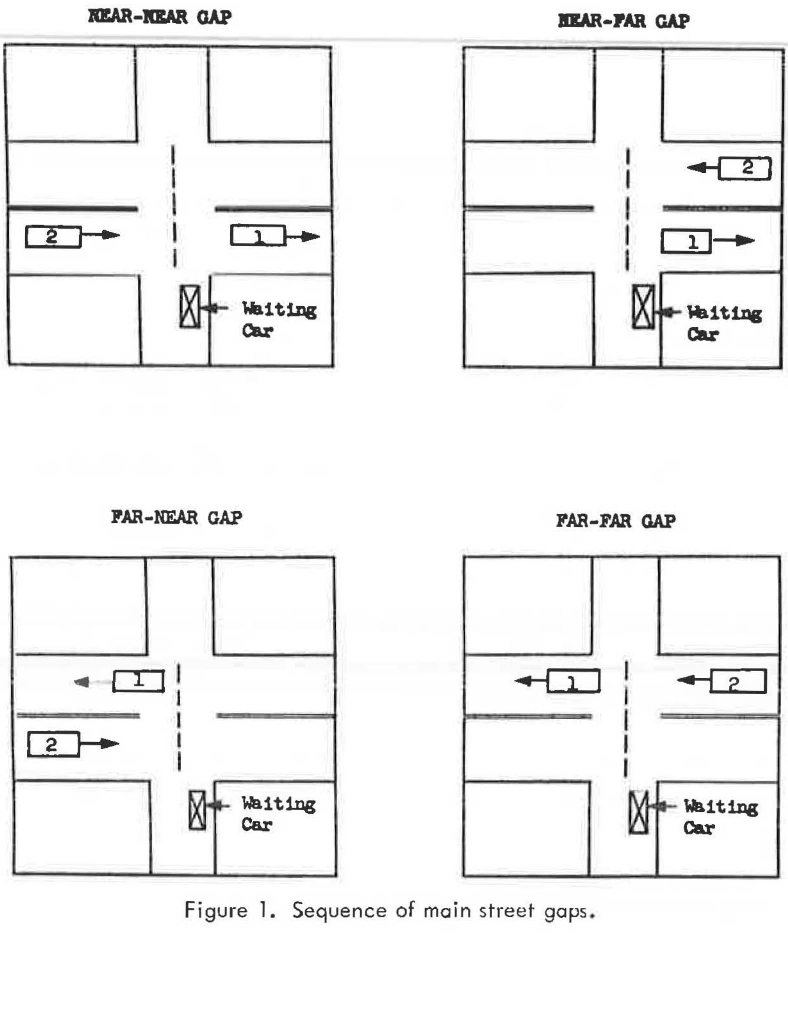


Figure 1. Sequence of main street gaps.

entry into the intersection of the first waiting car. This definition provides a beginning reference point for the measurement of the lag presented to the succeeding vehicle.

Near-side main street vehicles are those passing closer to the waiting side street car; in other words, those which approach from the waiting driver's left. Far-side main street vehicles are those approaching from the waiting driver's right.

The formation of gaps in main street traffic, therefore, is characterized by one of the following sequences: near-near, near-far, far-near, or far-far (Fig. 1).

STUDY PROCEDURE

Field data were collected at the intersection of a four-lane, undivided, intermediate-speed state highway with a two-lane, low-speed city street controlled by stop signs (Fig. 2). Traffic flow levels and fluctuations on the main highway during the day were such that observed gaps covered the full range, from those so small as to be unacceptable to all waiting drivers, to those large enough to be acceptable to all. In selecting the study site, special characteristics were avoided such as substantial horizontal or vertical curvature near the intersection, oblique crossing, severe sight distance restrictions, and one-way operation.

Two observers operated a specially devised survey device consisting of 10 push-button microswitches electrically connected to a multiple-pen event recorder. The observers manually actuated the switches to denote: (a) arrival of main street vehicles, by direction; (b) arrival of side street vehicles, by direction and vehicle type; and (c) complete entry into the intersection of side street vehicles, by direction and turning maneuver. This technique enabled gap and lag size and acceptability data, and starting delay data, to be extracted from the chart records. A total of 472 min of sample data were gathered during daylight hours on week days in fair, dry weather. The

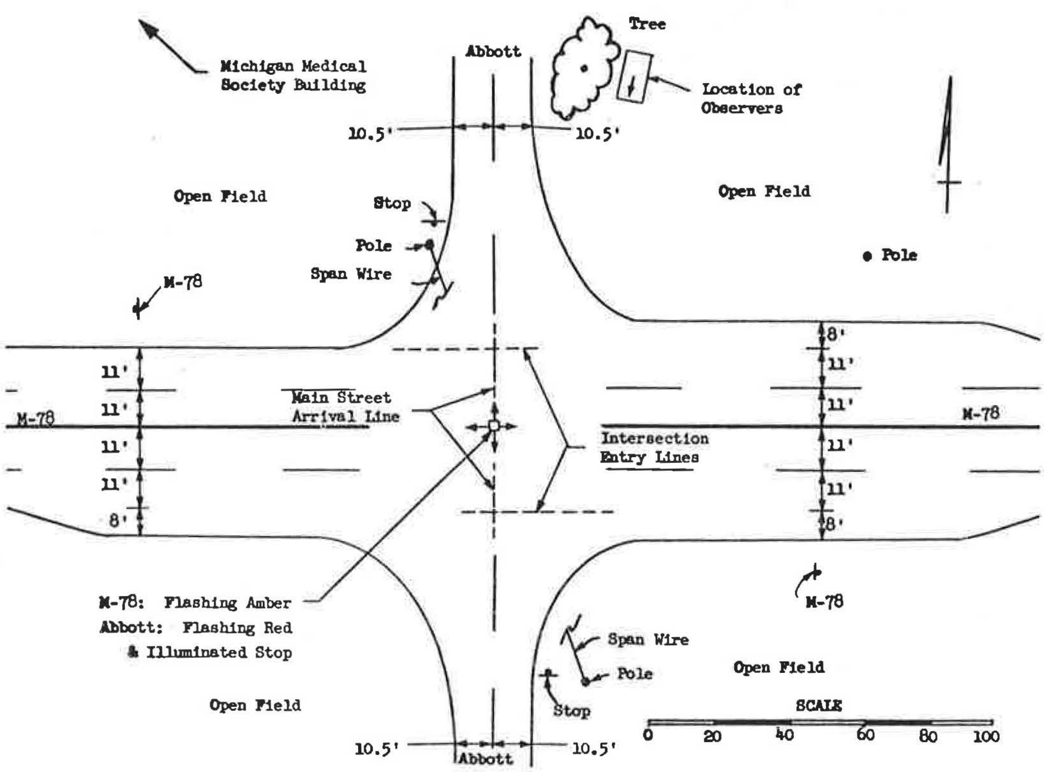


Figure 2. Condition diagram of the intersection in East Lansing, Michigan.

decisions and reactions of 1,203 separate side street vehicles, giving rise to a total of 5,179 separate lag or gap acceptance decisions, were extracted from the records.

Furthermore, the study procedure was designed to permit an effective evaluation of the influence of certain traffic factors on the lag and gap acceptance distributions. Data were stratified in the manner shown in Figure 3. Such stratification is often undertaken to safeguard against overlooking or misinterpreting the significance of a given factor caused by submerging the effects in a larger population affected by other important variables. For each category of data, the characteristics of the acceptance distributions were determined and pertinent comparisons were made.

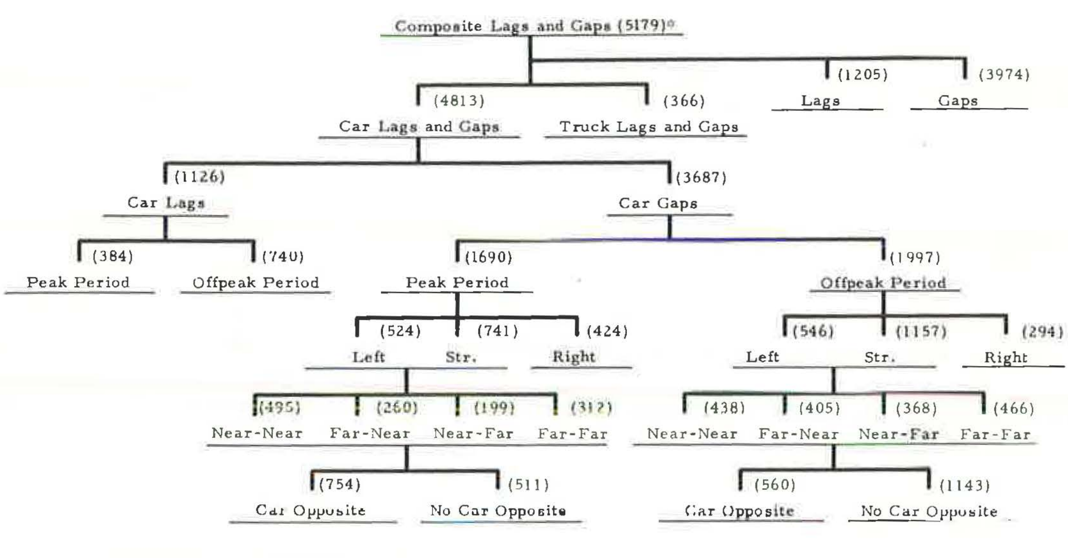
Gap and lag size data were separated into 1-sec class intervals, and for each interval the observed percent acceptance was computed. The form and parameters of the lag and gap acceptance distributions were then determined using a graphical curve fitting technique, and a specialized application of standard statistical difference tests was used to test the significance of the influence of various traffic factors on the distributions.

RESULTS

Decisions of Side Street Drivers

Form of the Acceptance Distribution. —In developing the statistical analysis methodology for this study, approaches used in earlier work of Robinson (12) and Bissell (4) were reviewed. A modified version of the earlier techniques was used to determine the form of the acceptance distribution. Rather than plotting sample percentages alone, confidence interval estimates surrounding the sample percentages were computed and plotted vs the logarithmic transform of gap or lag size. A straight line could be drawn which passed through a great majority of the confidence bands plotted on logarithmic-probability paper. This held up well for all levels of data stratification. Thus, the results gave rather strong verification of earlier findings that the relationship between lag or gap size and percent acceptance has a log-normal form.

Figure 4 shows the composite lag and gap acceptance distribution resulting from combining all driver decision data into one sample. The curve is presented in its untransformed state; that is, on a rectilinear graph. The absolute value of the median



*Note: Total sample sizes are shown in parentheses.

Figure 3. Stratification of lag and gap acceptance data.

acceptable size was 7.4 sec. Gaps or lags smaller than 4.3 sec were accepted by fewer than 10 percent of the side street drivers, and openings larger than 12.5 sec were accepted by more than 90 percent. One can use the graph to estimate the percent of vehicles accepting a given lag or gap size.

Comparison of Gaps and Lags. —The results of separating the composite data into lag acceptance and gap acceptance categories are indicated in Figure 5. To test whether the two distributions differed significantly, statistical tests were performed on the hypotheses that (a) the means were equal and (b) the standard deviations were equal. These tests were performed at the 0.05 level of significance, which means that there is only a 5 percent chance of incorrectly concluding the distributions differ, if in fact they are equal.

In this case, both hypotheses were rejected, and it was concluded that the two samples were not members of a common distribution. The gap acceptance curve had a lower central tendency, and the lag acceptance curve was more disperse. Therefore, the acceptance of gaps and the acceptance of lags should be treated separately. A rejection of either hypothesis would have caused the same conclusion. Except for very small sizes, a gap of a given size was more readily accepted than a lag of the same size. For example, a gap of 8 sec was acceptable to 60 percent of the waiting drivers, but a lag of the same size was acceptable to only 50 percent.

Influence of Traffic Factors on Driver Decisions. —Vehicle Type. The lag and gap acceptance distributions for the two classifications of side street vehicles, cars and trucks, are shown in Figure 6. From a logical viewpoint, considering the limited acceleration capability of trucks, there was reason to expect that differences would be found. In the graph the two curves are narrowly separated. However, the statistical tests led to the conclusion that this sample data gave no evidence that truck behavior and car behavior were significantly different.

If the truck-car comparison had been made separately for lags and gaps, or for offpeak and peak periods, differences might have been found. Unfortunately, the small size of the truck sample prohibited such further stratification. Until contrary evidence is found, the decision characteristics of truck and car drivers need not be handled separately.

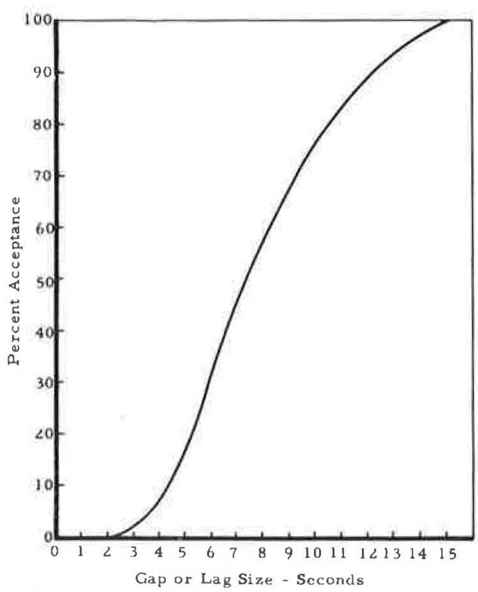


Figure 4. Composite lag and gap acceptance distribution.

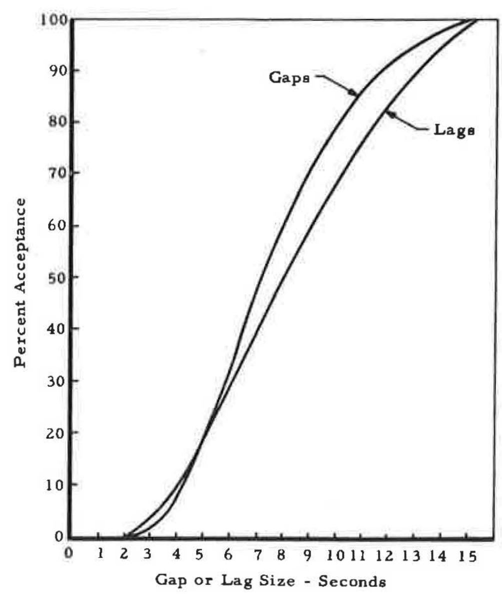


Figure 5. Comparison of lag and gap acceptance distributions.

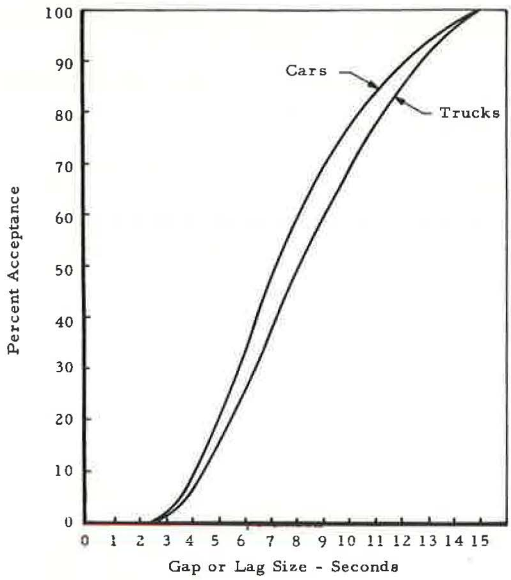


Figure 6. Effect of vehicle type on lag and gap acceptance distribution.

Pressure of Traffic Demand. Figure 7 shows the influence of pressure of traffic demand on lag and gap acceptance. Here the differences were indeed significant. The evidence indicates drivers accept smaller lags and gaps during peak periods. In other words, a greater percentage of drivers tend to accept a lag or gap of a given size during peak periods than will accept an opening of the same size during offpeak periods. For example, a lag of 6 sec was acceptable to nearly 50 percent of the peak-period drivers, but to just over 20 percent of the offpeak-period drivers.

The influence of traffic demand was more striking in the case of lag acceptance, where there was more than a 2-sec difference between the median acceptable lag during peak and offpeak periods. In the case of gap acceptance, the separation of the peak and offpeak curves was narrower but nevertheless significant.

In evaluating the influence of subsequent traffic factors, the separation of peak and offpeak data was retained.

Direction of Movement. The comparison of gap acceptance distributions for side street cars waiting to proceed straight, turn left, or turn right into the intersection is shown in Figure 8. The effect of direction of traffic movement was found to be limited during peak periods and insignificant during offpeak periods.

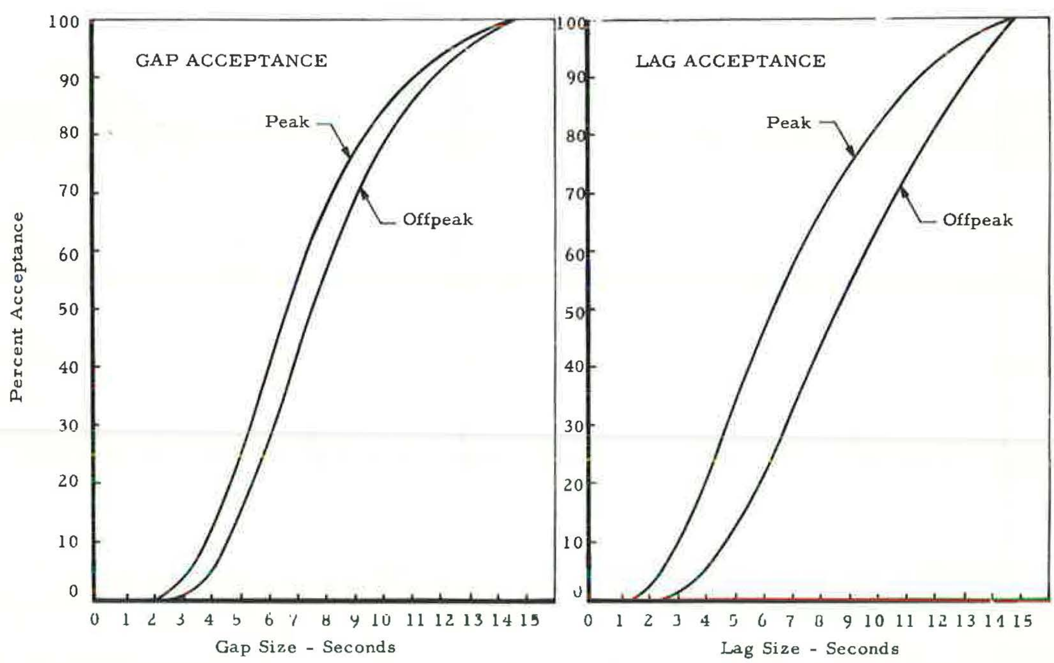


Figure 7. Effect of pressure of traffic demand on lag and gap acceptance distributions.

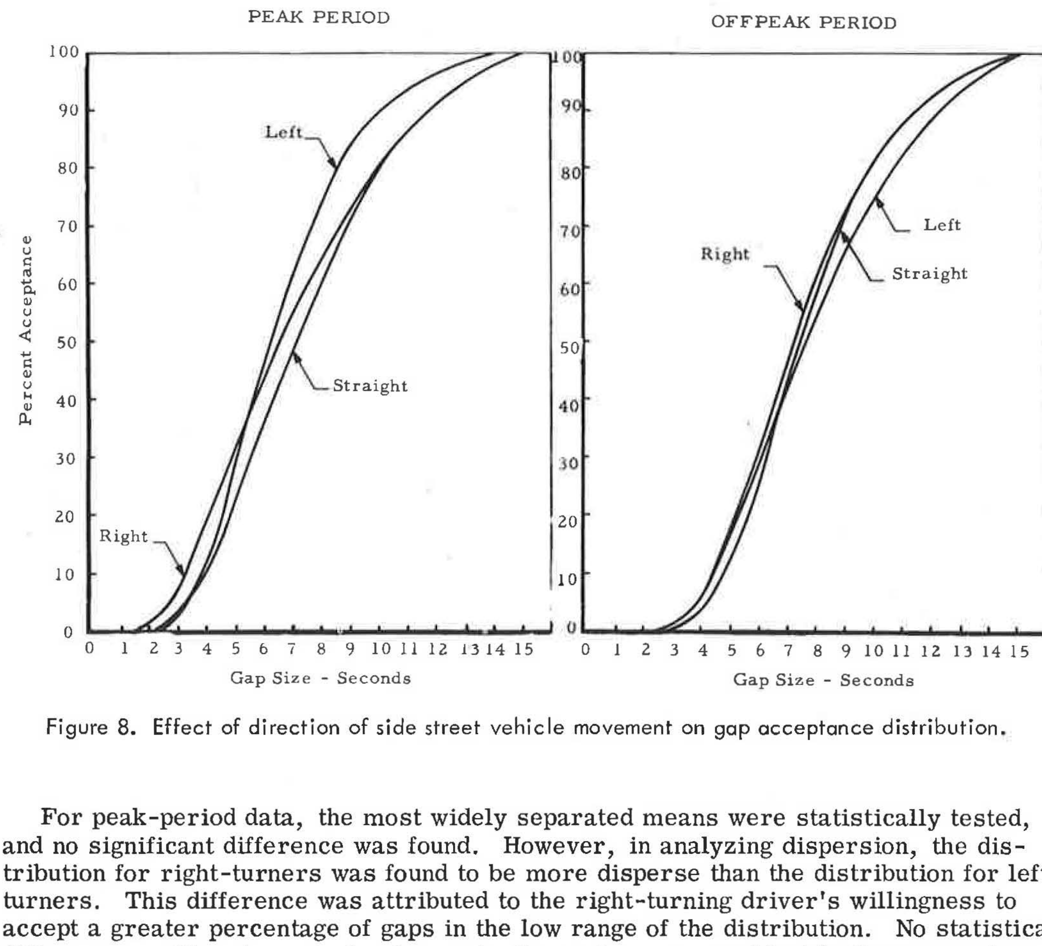


Figure 8. Effect of direction of side street vehicle movement on gap acceptance distribution.

For peak-period data, the most widely separated means were statistically tested, and no significant difference was found. However, in analyzing dispersion, the distribution for right-turners was found to be more disperse than the distribution for left-turners. This difference was attributed to the right-turning driver's willingness to accept a greater percentage of gaps in the low range of the distribution. No statistical differences, either in central value or in dispersion, were evident in the comparison of turning and straight-through drivers. It was concluded that in considering peak-period behavior it is only necessary to segregate right-turners from the others.

For offpeak-period data, the effects of direction of movement appeared even smaller. In fact, there was no evidence to indicate that the left, straight, and right gap acceptance samples did not come from a common distribution. Consequently, there is no need to make this distinction during the periods of reduced traffic demand.

Main Street Vehicle Sequence. Another factor investigated was the sequence of main street vehicles comprising the gaps presented to waiting drivers. Only the left-turning and straight-through side street cars were included in this analysis, since for right-turn decisions only those gaps in the near-side main street traffic are relevant.

Figure 9 shows that the sequence of gap formation had a strikingly significant influence on driver decisions during the peak period. The two most widely separated distributions (for near-far and far-near gaps) were more than 2 sec apart at the 50-percent acceptance level. This difference was found to be highly significant statistically. A much greater percentage of drivers accepted a given far-near gap than accepted a near-far gap of equal size. For example, a far-near gap of 6 sec was acceptable to nearly 60 percent of the waiting drivers, whereas a near-far gap of the same size was acceptable to less than 30 percent.

Still considering the peak-period data, the two inner distributions, characterizing the acceptance of near-near gaps and far-far gaps, were compared, and no significant difference was indicated. Thus, in the consideration of peak-period gap acceptance, near-far and far-near gaps should be segregated, but near-near and far-far gaps may be grouped.

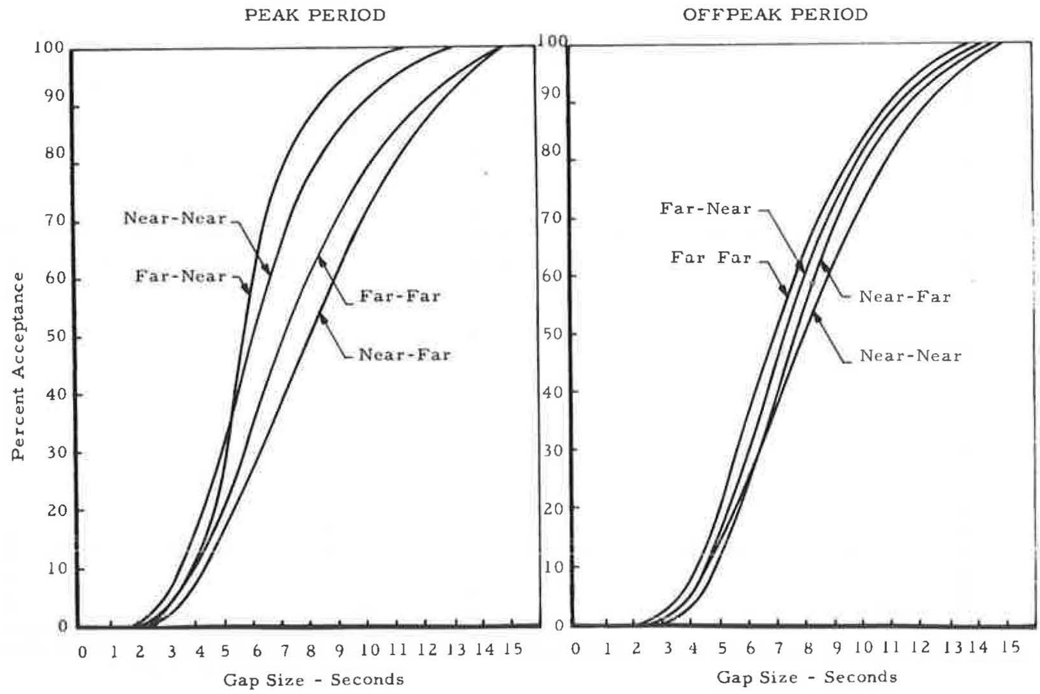


Figure 9. Effect of sequence of main street gaps on gap acceptance distribution.

A similar analysis of the effect of main street vehicle sequence was made for off-peak traffic. In Figure 9, the four offpeak distributions are in much closer proximity. The most widely separated pair of values for mean and standard deviation was selected for testing, and no significant differences were found. Therefore, during periods of reduced traffic demand, gap acceptance data need not be segregated on the basis of sequence of gap formation.

Conditions on the Opposing Side Street Approach. The final traffic factor considered was the presence or absence of one or more vehicles waiting on the opposite side street approach. It was assumed that this factor is irrelevant to drivers turning right into the main stream; hence, only left and straight vehicles were included in the analysis. The results (Fig. 10) show the gap acceptance curves, under the conditions of (a) no car opposite and (b) one or more cars opposite, to be in very close proximity. By statistical inference, there was no evidence to indicate that the decisions of waiting drivers were significantly affected by conditions on the opposing approach.

Reactions of Side Street Drivers

Starting Delay Time Distributions. —Starting delay time in accepting gaps and lags at a stop-signed intersection can be considered analogous to starting delay time of the first vehicle in queue at a traffic signal. It is an important parameter in both theoretical study and simulation of traffic behavior at intersections, particularly at any time when more than one vehicle is waiting in line at the stop sign. Both the central tendency and dispersion of starting delay times are of interest. It was rather surprising to find no past reports of such measurements.

Starting Delay Time in Accepting Gaps. Starting delay time in accepting a gap was previously defined as the elapsed time between arrival of the first main street car comprising the accepted gap and the complete entry into the intersection of the side street car. A total sample of 703 such starting delay times were extracted from the multiple-pen chart records. Data were segregated into 0.5-sec class intervals, and

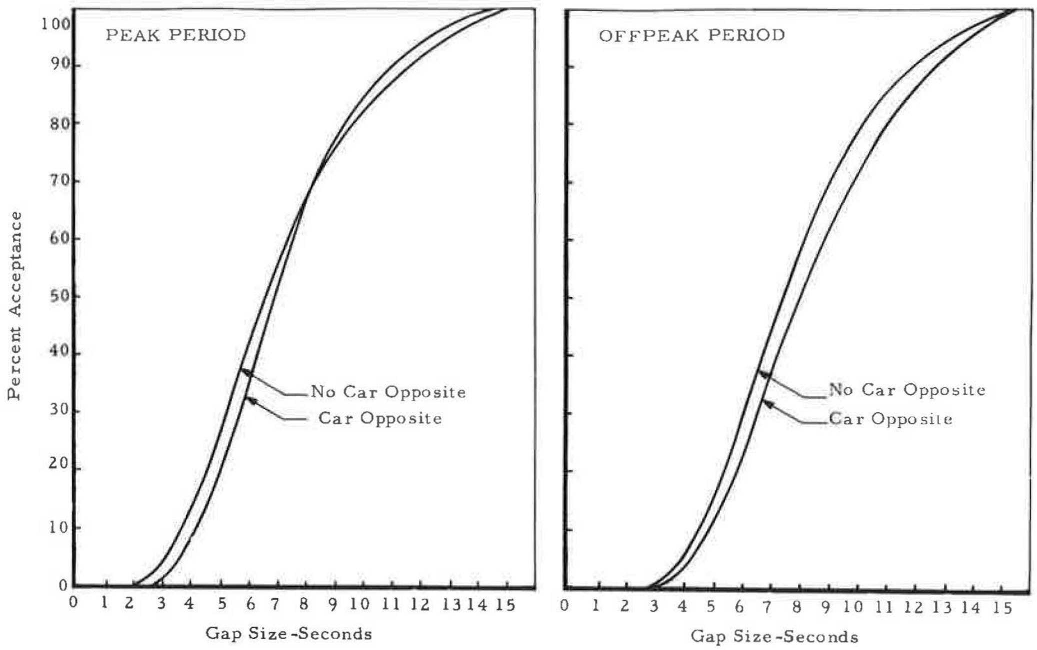


Figure 10. Effect of conditions on the opposite side street approach on gap acceptance distribution.

the resulting frequency distribution is shown in Figure 11. The composite sample presented included data from both peak and offpeak traffic periods. The distribution appears approximately normal except for a long tail to the right. Observed values ranged from virtually 0 to more than 9 sec. The median was 2.8 sec, and there were more observations in the 2.5- to 3-sec class than in any other. The 15 percentile and 85 percentile of the sample were 1.8 and 4.4 sec, respectively.

Starting Delay Time in Accepting Lags. Starting delay time in accepting lags was defined as the elapsed time between the arrival of a side street car and its complete

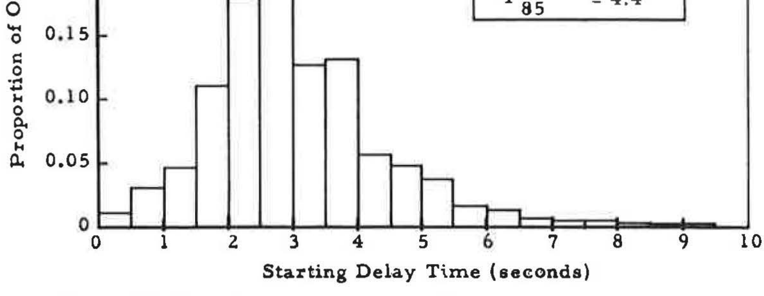


Figure 11. Distribution of starting delay times in accepting gaps.

entry into the intersection. In the special case where more than one vehicle is waiting on the side street approach, the arrival of the second, or succeeding, vehicle was defined as coinciding with the complete entry into the intersection of the first vehicle in queue.

Frequency distributions of starting delay time for first vehicles in queue and for succeeding vehicles are shown in Figure 12. One immediately notes that the two distributions are different. Because the distribution for succeeding vehicles was skewed and the other approximately normal, the standard difference tests were not performed. However, it is obvious by inspection that starting delay time for succeeding vehicles was smaller and less disperse than for the first vehicle in queue.

The statistical properties of starting delay of succeeding vehicles in accepting lags did not differ markedly from the previously presented properties of starting delay in accepting gaps. Conversely, starting delay for first vehicle in queue lag acceptances was significantly higher and more disperse than starting delay for gap acceptances.

Influence of Traffic Factors on Starting Delay Times. —Pressure of Traffic Demand. The sample of starting delay times in accepting gaps was segregated on the basis of period of the day to reflect different intensities of traffic pressure (Fig. 13).

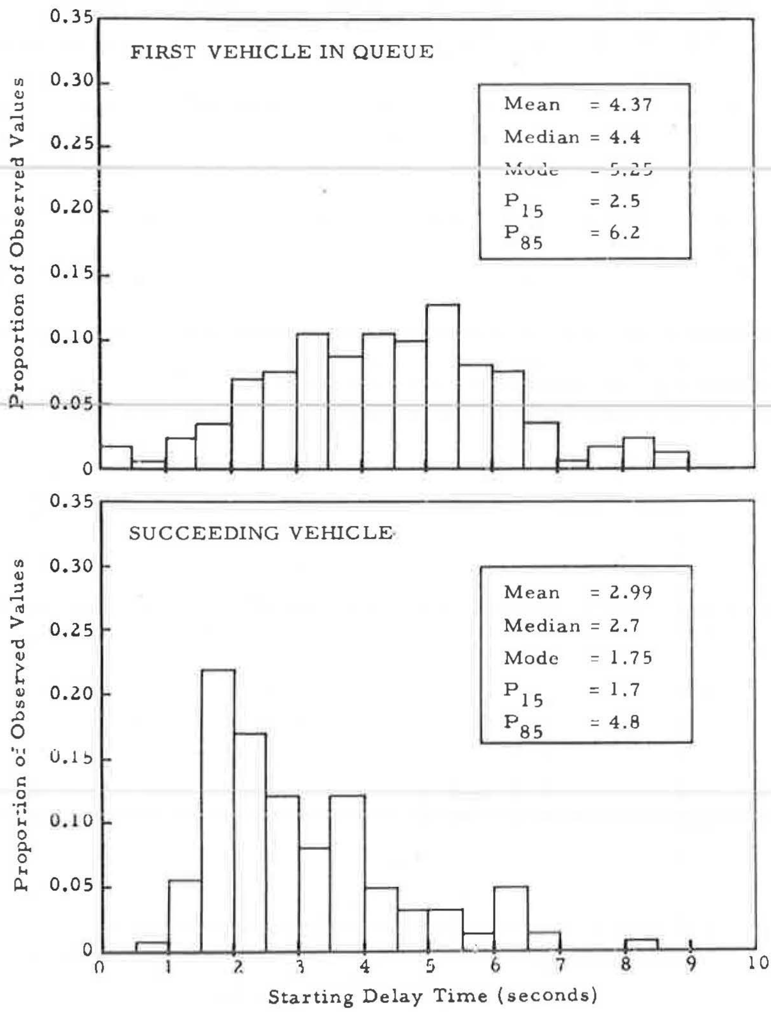


Figure 12. Distribution of starting delay times in accepting lags.

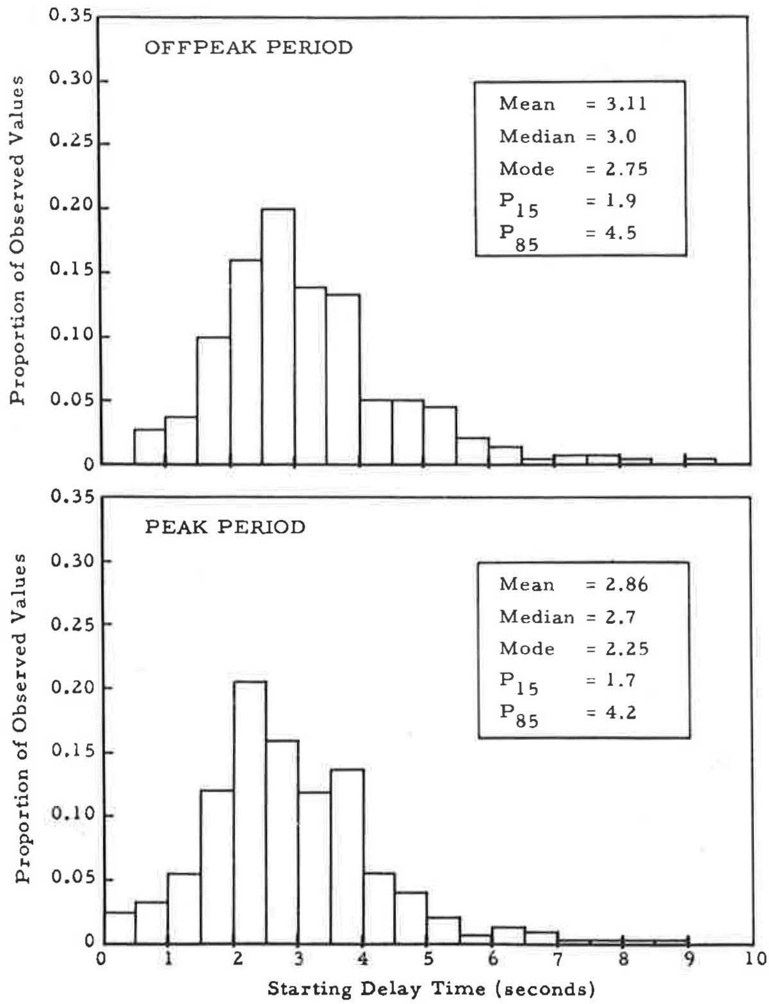


Figure 13. Effect of pressure of traffic demand on starting delay time in accepting gaps.

Since both distributions appeared normal, standard tests were performed which indicated that mean starting delay time during peak periods (2.86 sec) was significantly smaller than during off-peak periods (3.11 sec).

The influence of traffic demand was much more striking in the case of starting delay time in accepting lags. Here only succeeding vehicles were considered, since during the peak period the occurrence of a vehicle arriving first in queue on an empty stop-signed approach, and accepting the lag, was practically nonexistent at the study site. Figure 14 shows that the mean starting delay time in accepting lags during the peak period was nearly 0.7 sec lower than during the offpeak period. The peak period mode was a full 1 sec lower than the offpeak mode. These differences were highly significant. On the other hand, little difference was noted in the dispersion of these two distributions.

Main Street Vehicle Sequence. From a logical viewpoint, it was expected that starting delay time in accepting gaps might be affected to some degree by the sequence of gap formation. In particular, if the first car comprising the gap was on the far side, the side street vehicle might commence motion earlier and complete its entrance more quickly than if the first car of the gap was on the near side. Figure 15 indicates that

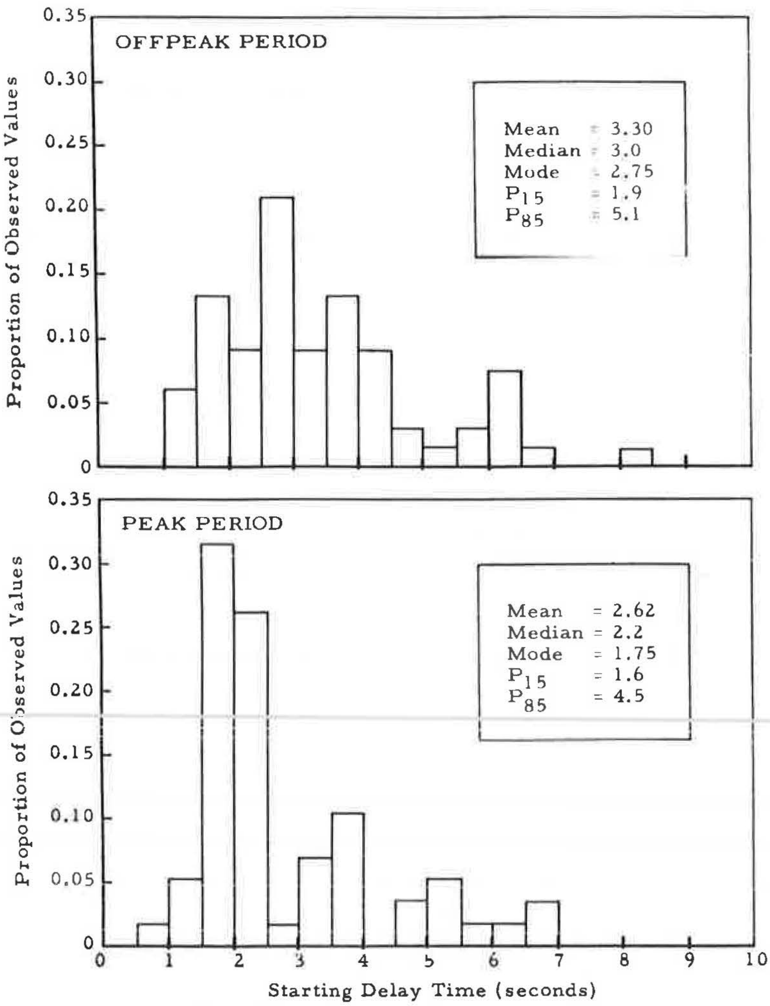


Figure 14. Effect of pressure of traffic demand on starting delay time in accepting lags (succeeding vehicles).

such reasoning was indeed valid. During both peak and offpeak periods, starting delay times were smaller when the first car of the gap was on the far side. The effects were largest during the peak period when there was nearly 1 sec difference between the means being compared. The differences were less marked but nonetheless significant during the offpeak period.

INTERPRETATION OF RESULTS

Generally speaking, the results of this research tended to verify rather than contradict that which a professional traffic engineer might deduce on the basis of logical consideration of the factors involved.

For example, the differences in gap and lag acceptance were not surprising. One might expect that a driver who has just arrived at a stop sign needs some time to orient his senses to the decision-making process. Furthermore, when such a driver is nearing the stop sign, he is often not in as advantageous a position for the critical observation of main street traffic as if he had been waiting near the intersection entry line for some period of time. These factors help to explain why a gap of a given size was more readily acceptable than a lag of the same size.

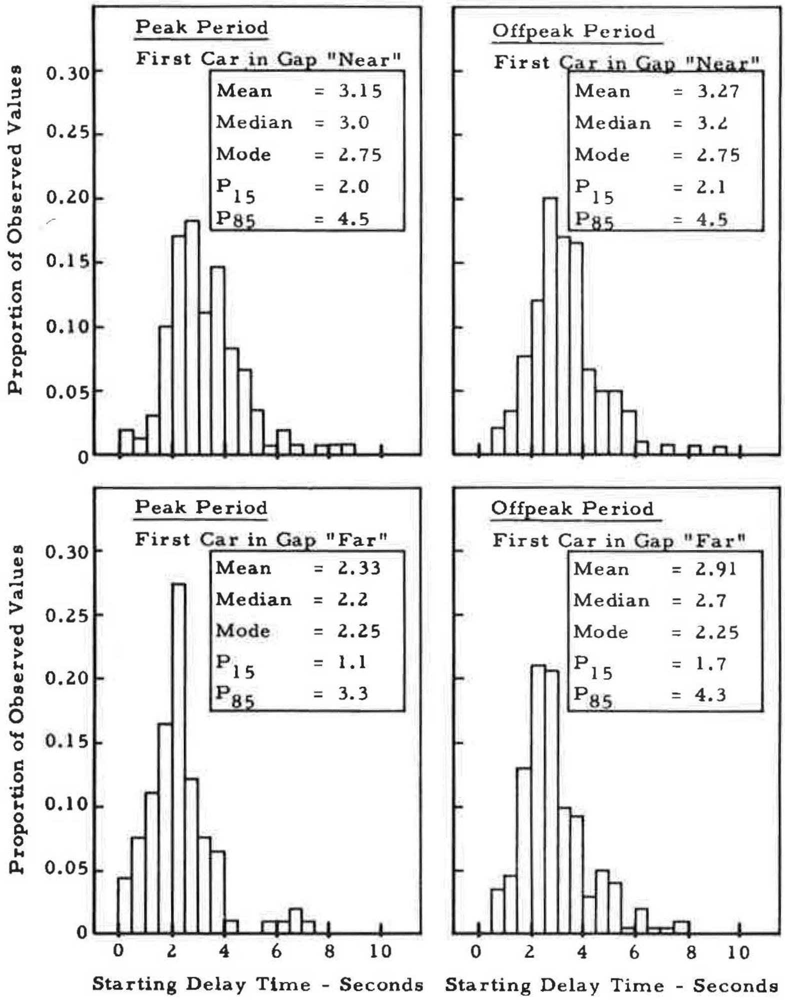


Figure 15. Effect of sequence of accepted main street gap on starting delay time in accepting gaps.

The factor which had the most striking effect on the lag and gap acceptance distributions was the pressure of traffic demand. A given lag or gap is more readily accepted during peak periods than during periods of reduced traffic demand. Several factors might be important in explaining these differences. During the peak period, many drivers are traveling between work and home. Before they reach the position of queue leader, they most likely have spent a substantial period of time in the queue. Furthermore, when they do reach the front of the line, it is likely that one or more vehicles are waiting behind. All of these factors might be expected to contribute some degree of impatience. Of possibly equal importance is the higher traffic volume, or in other words, smaller average gaps, on the main street during the peak period. The driver who rejects a marginal lag or gap may have to wait a substantial time for another opportunity that good or better.

There is also a logical basis for explaining the results relating to the sequence of main street vehicles. At least three factors are believed to be important: (a) if the first car of the gap is on the near side, it blocks the waiting driver's vision of far-side main street vehicles; (b) if the first car in the gap is on the near side, the waiting driver cannot normally begin his entry into the intersection until the near-side car has passed; (c) if the second car of the gap is on the far side, the crossing driver must travel a longer distance to clear the area of conflict. In considering a far-near gap, the favorable conditions of all three of these factors are met, whereas in the case of a near-far gap all of the unfavorable conditions are working. Near-near and far-far gaps have a mixture of favorable and unfavorable conditions.

Differences in gap acceptance due to main street vehicle sequence were evident only during the peak period. It is theorized that during offpeak periods drivers feel no special compulsion to attempt to attain the maximum performance. But during peak periods, when some degree of compulsion is working, reasonable lower boundaries on gap size corresponding to maximized performance are lower for far-near gaps, and higher for near-far gaps, than for the other types.

The study of starting delay times yielded results which can be closely correlated with the driver-decision data. The traffic factors which had important effects on driver decisions also influenced starting delay times, and in the same direction. For example, the average starting delay time in accepting gaps was lower during periods of heavy traffic demand. It is impossible to state with assurance that lower starting delays enable shorter gaps to be accepted, or, alternatively, that the decisions to accept shorter gaps cause the lower starting delays. Rather, it is probably more accurate to say that both behavior characteristics are affected similarly by common factors, such as impatience, degree of motivation, and the reduced size of main street gaps presented to waiting drivers.

Regarding another important factor, main street vehicle sequence, which similarly affects starting delay and gap acceptance, there is some reason to note a causal relationship. It is believed that a partial explanation for far-near gaps being more readily acceptable is that the position of the first car in the gap enables the side street car to start into the intersection sooner.

Comparison with Related Research

The results of this study are compared with those Bissell, Greenshields, Raff, and Herman and Weiss in Figure 16. Both Greenshields and Raff estimated central values for lag acceptance; their results are plotted on the 50-percent line. Bissell's distribution of lag and gap acceptance is shown.

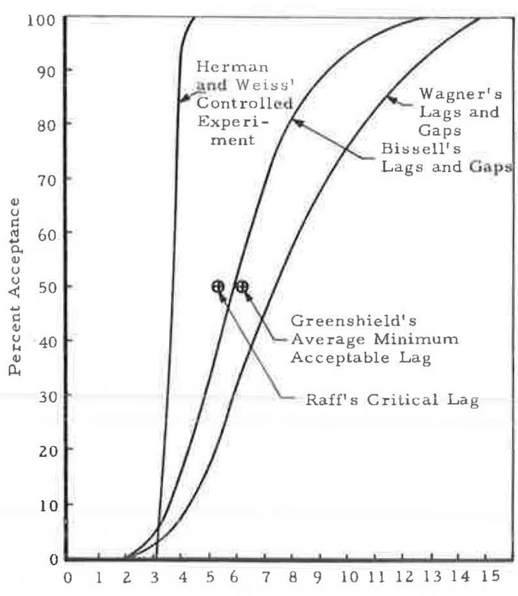


Figure 16. Comparison with related findings.

The acceptance distributions determined in this study had significantly higher central tendency than those found in any of the other studies. The variance of Bissell's lag and gap acceptance distribution, however, did not differ significantly from the present findings.

It is believed that an explanation of the differences is related to differences in the nature of the intersections studied. In particular, the main street of this study was much wider and carried higher-speed traffic than the main streets studied by the others.

Of special interest is a comparison of the studies of actual traffic intersections and the controlled experimentation done by Herman and Weiss (12). These data differ markedly from the rest. No lag smaller than 3.2 sec was accepted and none larger than 4.2 sec was rejected. The point where their line crosses 50 percent corresponds to only 3- to 10-percent acceptance in the distributions of Bissell and this study. Herman and Weiss state that "these experiments were rather artificial in that the drivers were highly motivated and quickly adapt to the situation." However, their results are especially interesting and useful in that they represent maximized performance characteristics.

Future Research

Although this study was intensive, it was limited due to time and resources to only one intersection. It would seem important, therefore, to make similar studies of driver decisions and reactions, and the effects of variable traffic factors on them, at other intersections.

Certain specific items which could not be adequately handled in this study might be of interest. For example, the effects of direction of movement, gap sequence, and conditions on the opposite approach on lag acceptance could not be studied here due to inadequate sampling. Another inadequacy was the inability to make a really detailed study of the effects of conditions on the opposing approach, particularly as related to the direction of movement of the car in question and the car opposite. Although the effects of different types of vehicles on the side street were studied, no consideration was given to vehicle type on the main street.

Concluding Remarks

In the final analysis, these efforts are wasted unless the findings can be applied. Theoretical treatment and simulation both require the application of driver-decision and reaction parameters. In using either of these approaches, broad ranges of traffic variables such as turning percentages, truck percentages, directional splits, and traffic volumes must be studied. Realistic models must take into account significant changes in driver decisions and reactions associated with these variables. The key to more effective use of the new techniques is a renewed and vigorous attempt to understand more fully and document the fundamentals of traffic behavior.

ACKNOWLEDGMENTS

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Estimating Annual Average Daily Traffic from Short-Term Traffic Counts

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The purpose of this study was to evaluate a method advocated by the U. S. Bureau of Public Roads for estimating annual average daily traffic from short-term traffic counts and to determine whether existing procedures could be improved with reduced annual cost. This study pertained to rural roads carrying 500 or more vehicles per day.

Some of the first tests were conducted for the purpose of determining the most satisfactory method of grouping continuous counting stations and the computation of mean monthly adjustment factors for each group.

One of the first conclusions was that continuous count stations should be grouped on the basis of average monthly adjustment factors of several consecutive years rather than on the basis of the factors for any single year. It was further concluded that division of the states' rural roadways into five groups would be sufficient stratification of annual patterns of traffic volume variation.

Tests were made to determine the relative efficiency of seasonal control counts repeated a various number of times per year per location for establishing group assignments of roadway sections and estimating AADT. Tests were made pertaining to seasonal control counts repeated four, six and twelve times a year per location. The standard deviations of the errors of estimated AADT from seasonal control counts of four, six and twelve times per year were 3.6, 3.1 and 1.7 percent, respectively. Comparisons of the results of using various seasonal control counts to indicate group assignment of roadway sections showed no significant difference.

The Missouri State Highway Department is considering the adoption of the Bureau's method of estimating AADT using a 7-day coverage count program and seasonal control counts repeated four times a year per location. It is believed that the eventual annual savings of this method would be approximately one-half the cost of the current program.

•EARLY in 1963 the Missouri State Highway Department began an investigation of the possible advantages of the method advocated by the U. S. Bureau of Public Roads for estimating annual average daily traffic (AADT) from short-term traffic counts for rural roads carrying 500 or more vehicles per day. The purpose of the investigation was to evaluate several variations of the Bureau's method and to determine whether the existing procedure could be improved with a possible reduction in annual cost.

In general, the Bureau's method involves: (a) stratifying continuous traffic counting stations into groups of similar annual patterns of monthly traffic adjustment factors; (b) determining average adjustment factors for each group; (c) assigning all sections of the rural highway system to one of these groups; and (d) applying the appropriate average adjustment factor to any short-term traffic count to produce an estimate of

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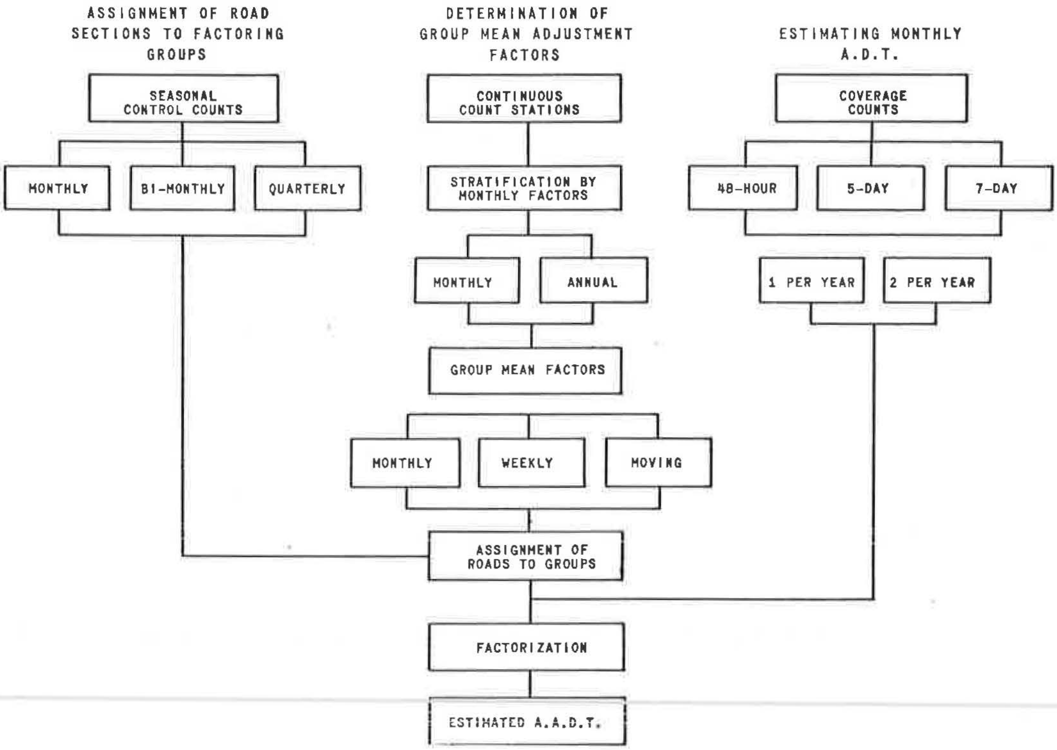

Figure 1. A plan for studying variations of the Bureau method of estimating AADT.

annual average daily traffic. The Bureau's method and procedures are described in detail in Guide for Traffic Volume Counting Manual.

The method now being used by the Missouri State Highway Department has produced useful results. This method, however, is highly subject to individual judgment. Basically, the method of estimating AADT at the location of a coverage count has been as follows:

1. Two 48-hr coverage counts, approximately 6 mo apart, are made at a particular location of interest.
2. The individual making the estimate selects a continuous count station which he believes to have a similar annual pattern of monthly traffic variations.
3. Using the data from the continuous count station, the ratios of AADT to the average daily traffic during the period of time in which each of the coverage counts were made are computed.
4. The two coverage counts are multiplied by the appropriate factors and the products averaged to produce an estimate of AADT for the particular location.

The Bureau's method as compared to the existing method has the following advantages:

1. Because of its objective nature, it can be presented in a manual of fixed procedures. With the aid of this manual, a wider range of individuals would be able to produce acceptable estimates of AADT.
2. It lends itself well to a statistical measure of accuracy.
3. It is readily adaptable to electronic data processing.

The plan for studying variations of the Bureau's method of estimating AADT is shown in Figure 1. The study was broken down into three parts:

1. Grouping of continuous count stations and the determination of group mean adjustment factors;
2. Estimating monthly average traffic using coverage counts of various lengths; and
3. Assignment of road sections to factoring groups by use of seasonal control counts.

Missouri has approximately 90 continuous traffic counting stations located throughout the state on rural roads having 500 or more AADT. Data from these stations for the fiscal year 1961-1962, and in some cases additional years, were used in performing the tests of the study.

GROUPING OF COUNT STATIONS AND DETERMINATION OF ADJUSTMENT FACTORS

The primary purpose of grouping continuous traffic counting stations, and eventually assigning most sections of roads in the state to one of these groups, is the establishment of a series of routes with consecutive road sections having similar patterns of monthly traffic volume variation.

If all roadways could be stratified into groups of identical annual patterns of traffic adjustment factors which correspond to the period of time of a coverage count, true AADT could be derived from coverage counts. It is also desirable to have the group assignment of a roadway section remain constant from year to year. These two conditions can be attained to a degree because of two fundamental characteristics of traffic patterns which have been established by many studies:

1. The pattern of monthly variations of traffic volumes persists over long stretches of highway.
2. The pattern of monthly variations of traffic volumes persists over long periods of time.

The Bureau's manual indicates that it is practical to group stations allowing a difference of 0.20 between the smallest and the largest values of factors within each month. It further indicates that by using this criterion, there should be little change in group assignment of roads from year to year.

Missouri's continuous count stations for the fiscal year 1961-1962 were grouped using the Bureau's criterion. This resulted in an excessive number of groups. When indicating the group assignment of continuous count stations on a map by the use of color codes, no reasonable pattern of continuous group assignments appeared. Other tests indicated that an appreciable number of stations would tend to change groups in the following year.

In an attempt to reduce the number of groups and to stabilize group assignment of roadway sections from year to year, continuous traffic counting stations were classified on the basis of average monthly adjustment factors of several consecutive years. The average factors of 4 yr were used. It was assumed that a gradual change of roadways from one group to another would not be too significant over a period of 4 yr. Any station which had a tendency to change from one group to another in a period of less than 4 yr would probably be noticeable because of changing conditions in that area.

Grouping continuous traffic counting stations on this basis resulted in 5 different groups in the state. Three of these groups were classified as non-recreational and two as recreational. The two recreational groups were classified as such because of the high variation of monthly adjustment factors resulting from their locations near resort areas.

Each of the five groups was assigned a color code. All continuous traffic counting stations were then plotted on a map and what appeared a very reasonable series of group assignments resulted. Groups were numbered from one through five in the order of their increasing variation of average monthly adjustment factors. Stations belonging to group No. 1 were generally located near cities and on roads where a significant amount of the travel consisted of work trips. On these roads a smaller amount of traffic volume variation occurred throughout the year than on other roads where vacation and recreational travel are more prevalent.

Group No. 2 contained over fifty percent of all the continuous traffic counting stations in the state. The average pattern of this group was very similar to the average pattern of all stations within the state. Roadways belonging to this group were not limited to any particular area in the state.

Group No. 3 contained thirteen of the ninety continuous traffic counting stations. These stations were generally located on relatively high volume roads, which during the summer months are known to carry a high percentage of vacation trips. The roads assigned to this group were not necessarily located near the resort and recreational areas of the state.

Group No. 4, which contained four stations, was located near the resort areas of the state. The two stations in group No. 5 were located in resort areas in the state. The roadways assigned to groups Nos. 4 and 5 are known to carry large volumes of weekend recreational travel during the summer months. During the winter months, the volumes on these roads are relatively small.

The four-year average monthly adjustment factors of the continuous count stations were used only to determine the group assignment of the stations. The average adjustment factors of a group, to be applied to coverage counts of a particular year to estimate AADT, were determined by averaging the factors for that year of the stations assigned to that group. If the group mean adjustment factor of four years were applied to coverage counts in any one year to estimate AADT, additional error may result due to the variation of group mean factors between years.

The use of four-year average factors to stratify continuous count stations results in more variation about the group mean factors for those stations in any one particular year than if the Bureau's method were used. However, if the Bureau's method was used, there would be a larger number of groups and also a greater tendency for stations and roadways to change groups from year to year. Thus, at the end of any one year, there would be a substantial amount of roadway sections for which the group assignment would not be known. The group assignment for these sections would have to be estimated to factor coverage counts made along the sections during the year. To estimate these group assignments, the prevailing group assignment of a number of years would possibly be used. It is believed that this would result in approximately the same variation which would have been obtained if the average of a number of years had been used to group stations.

To this point, the grouping of continuous traffic counting stations has been based on the difference of annual patterns of monthly adjustment factors. This has been designated as the annual method of grouping. The Bureau manual indicates that when a computer is available, groupings can be made separately for every month during which vehicle coverage count stations are operated. Using this procedure, the continuous count stations would be grouped on the basis of the values of the monthly adjustment factors for that month. With this method, group assignments tend to vary from month to month. There is also a tendency for the number of groups to vary from month to month.

Missouri's continuous count stations were grouped using the monthly method. As in the annual method, the average monthly adjustment factors of four years were used. The number of groups per month varied from one in September to four in January.

A test was performed to find which method would yield the greater accuracy of estimates of AADT. Seven-day coverage counts were simulated from daily traffic volumes of continuous count stations selected at random. Sixty of these simulated counts were made for each month. Estimates of AADT were produced by applying the appropriate mean monthly adjustment factors derived by the annual method of grouping. The average factors of the groups from the monthly grouping of continuous count stations were applied to the same set of simulated coverage counts to compute another group of estimates of AADT. Comparisons by month and by year were made to test for a significant difference in the distributions of errors of estimated AADT. In no case was a significant difference found.

After examining the results of the comparison between the annual method and the monthly method, it was decided to adopt the annual method. Generally, the annual method is more easily understood. Unusual variations of monthly traffic volumes are more obvious when the annual pattern is examined. The annual method also lends itself better to the use of seasonal control counts.

The determination of group mean adjustment factors was related to the grouping of continuous traffic counting stations. Tests were made to determine whether group mean adjustment factors should be computed on a monthly, weekly or moving base corresponding to the length of coverage counts. In no case were the differences of the distributions of errors of estimated AADT highly significant, but all indications were that increased accuracy could be gained by using a moving base as opposed to a monthly base. A somewhat limited analysis of variance test indicated that over a year's time, the standard deviation of the percent errors of estimated AADT could possibly be reduced by approximately five-tenths of one percent by using the moving base. This, however, is assuming that coverage counts would be made during all twelve months of the year. If the winter months were not used, the difference of the accuracies would possibly not be as great. Differences in accuracies appear significant only during the winter months. It was concluded that if average adjustment factors were determined by use of an electronic computer, the additional cost of computing factors from a moving base would not be excessive. If the adjustment factors are computed manually, however, the cost of the additional possible accuracy would be too great.

ESTIMATION OF MONTHLY AVERAGE TRAFFIC

Considerable time was given to determining the probable accuracy of estimated AADT when using short-term coverage counts of various lengths. Tests were made using 7-day, 5-day, 48-hr and 24-hr coverage counts. Estimates of AADT were made for continuous count stations from simulated coverage counts. These estimates were compared to the true AADT's and a frequency distribution of errors of estimation formulated. The probable occurrence of errors within the limits of various magnitudes was stated by statistically measuring the dispersion of this frequency distribution.

The standard deviation was used to measure the probable occurrence of errors in estimates of AADT which would result from the factoring of coverage counts of various lengths. The formula for computing the standard deviation is as follows:

$$\text{Standard deviation} = \sqrt{\frac{\Sigma X^2}{N - 1}}$$

where

X = percent error of estimated AADT; and
N = number of observations in sample.

This formula varies from the conventional formula for the standard deviation, which is as follows:

$$\text{Standard deviation} = \sqrt{\frac{(X - \bar{X})^2}{N - 1}}$$

where

X = percent error of estimated AADT;
$\bar{X}$ = average percentage error of estimated AADT; and
N - 1 = number of observations, less one degree of freedom.

The formula used for this study was based on the experience in other states and in the U. S. Bureau of Public Roads that the average of percent errors differs from zero by such a small amount as to be negligible.

In actual application of the Bureau's method of estimating AADT, it is believed that the average estimated percentage errors would not be significantly different from zero in most cases. If some year-end simulation of coverage counts would possibly indicate a significant average error, all estimates of AADT could be adjusted in the appropriate direction to reduce this average error to near zero. Unless this adjustment is made, however, it is desirable to know the expected dispersion of errors from zero and the modified equation provides such an estimate.

Assuming a normal distribution of errors and a zero average error, approximately 68 percent of all errors could be expected to be within the range of plus and minus one standard deviation; 95 percent within two standard deviations; and 99.7 percent within three standard deviations.

The following outline describes the steps used to produce simulated distributions of errors of estimated AADT which could be expected from coverage counts of various lengths and types.

1. Continuous traffic counting stations were grouped using the annual method.
2. Group mean monthly adjustment factors were computed.
3. Coverage counts were simulated at continuous count stations.
4. The average 24-hr traffic volume of each coverage count was expanded to an estimate of AADT by applying the appropriate group mean monthly adjustment factor.
5. Each estimated AADT was compared to the true AADT of the particular continuous count station and the plus or minus error of estimate as a percent of true AADT was computed.
6. The standard deviation of the resulting distribution of directional percentage errors was computed using the previously mentioned formula.

In grouping continuous traffic counting stations, the four-year average monthly adjustment factors were used as previously indicated. If coverage counts are made during all twelve months of the year, it is best to group stations based on the adjustment factors of all twelve months. If coverage counts are made only during a particular part of the year, it is best to group the continuous count stations on the basis of the factors of the months involved. During this portion of the study, continuous count stations were first grouped using all twelve months and later grouped using only nine months, omitting December, January and February. Although some stations tended to change groups, the number was very small and the difference in group mean adjustment factors per month was insignificant. When 7-day coverage counts, including Saturday and Sunday, were tested, the continuous count stations were grouped on the basis of the ratio of AADT to monthly average daily traffic. When 5-day coverage counts, excluding week-end days, were simulated, the continuous count stations were grouped on the basis of the ratio of AADT to monthly average weekday traffic. There was an obvious difference between the group assignment of stations when these two methods were compared. The differences of the average monthly adjustment factors were also significant.

In the tests concerning lengths of coverage counts, the value of the standard deviation of the errors of estimated AADT is of primary importance. To measure the relative accuracy of estimated AADT's between months, separate distributions of errors by month were derived from simulated coverage counts. The standard deviations of the months were combined statistically to produce the expected overall standard deviation for the coverage count season.

A pilot sample of 25 simulated 7-day coverage counts for the month of January was used to estimate a standard deviation of the errors of estimated AADT. Based on this sample, it was estimated that 60 simulated coverage counts per month would yield a standard error of the standard deviation of one percent or less. The estimated standard deviation of a counting season would have a standard error of the standard deviation of less than one-half of 1 percent. To attain approximately the same degree of accuracy for standard deviations of 5-day and 48-hr counts, it was estimated that approximately 100 samples of simulated coverage counts would be needed in each month. One hundred samples in each month were also used for 24-hr coverage counts.

Tables 1, 2 and 3 give the standard deviations of the various distributions of simulated errors expressed as a percent of true AADT. Table 1 indicates, for various coverage count seasons, the standard deviations of errors of estimated AADT for 7-day, 5-day, 48-hr and 24-hr coverage counts. These distributions are based on the assumption that only one coverage count per year per station would be made. Table 2 gives the standard deviations, by month, for the various length coverage counts. Table 3 in-

TABLE 1

STANDARD DEVIATIONS OF PERCENT ERRORS OF ESTIMATED AADT[a]

Counting Season	Length of Coverage Counts			
	7 Day	5 Day	48 Hr	24 Hr
12 Months	10.1 (0.27)[b]	10.1 (0.21)	12.6 (0.26)	14.7 (0.30)
Mar. - Nov.	8.8 (0.27)	9.3 (0.22)	11.5 (0.27)	13.5 (0.32)
Apr. - Nov.	8.7 (0.28)	9.2 (0.23)	11.5 (0.29)	13.5 (0.34)

[a]Based on one count per station per year.
[b]Standard error of standard deviation.

TABLE 2

STANDARD DEVIATIONS OF PERCENT ERRORS OF ESTIMATED AADT BY MONTHS

Month	Length of Coverage Counts			
	7 Day	5 Day	48 Hr	24 Hr
July	7.6 (0.69)[a]	8.1 (0.57)	9.5 (0.67)	12.2 (0.86)
Aug.	7.7 (0.70)	8.6 (0.61)	9.8 (0.69)	13.1 (0.93)
Sept.	9.0 (0.82)	10.6 (0.75)	13.9 (0.98)	15.2 (1.08)
Oct.	7.1 (0.65)	8.1 (0.57)	10.7 (0.76)	10.9 (0.77)
Nov.	9.7 (0.88)	10.4 (0.74)	12.7 (0.90)	13.7 (0.97)
Dec.	12.8 (1.17)	11.9 (0.84)	15.4 (1.09)	17.9 (1.27)
Jan.	17.3 (1.58)	13.7 (0.97)	17.4 (1.23)	20.2 (1.43)
Feb.	8.4 (0.77)	9.4 (0.66)	12.2 (0.86)	15.6 (1.10)
Mar.	9.7 (0.88)	9.9 (0.70)	11.3 (0.80)	13.1 (0.93)
Apr.	9.6 (0.88)	11.1 (0.78)	13.3 (0.94)	15.6 (1.10)
May	9.0 (0.82)	9.3 (0.66)	12.6 (0.89)	14.4 (1.02)
June	9.3 (0.85)	8.1 (0.57)	9.6 (0.68)	12.4 (0.88)

[a]Standard error of standard deviation.

TABLE 3

STANDARD DEVIATIONS OF PERCENT ERRORS OF ESTIMATED AADT[a]

Months	Length of Coverage Counts			
	7 Day	5 Day	48 Hr	24 Hr
July & Jan.	9.0 (0.82)[b]	8.1 (0.57)	9.8 (0.69)	12.1 (0.86)
Aug. & Feb.	4.6 (0.42)	6.4 (0.45)	7.6 (0.54)	10.2 (0.72)
Sept. & Mar.	5.0 (0.46)	5.2 (0.37)	8.0 (0.57)	8.8 (0.62)
Oct. & Apr.	5.8 (0.53)	7.5 (0.53)	9.1 (0.64)	9.9 (0.70)
Nov. & May	5.8 (0.53)	6.5 (0.46)	8.9 (0.63)	10.0 (0.71)
Dec. & June	7.1 (0.65)	7.1 (0.50)	9.2 (0.65)	10.7 (0.76)
Year	6.4 (0.24)	6.8 (0.20)	8.8 (0.25)	10.3 (0.30)

[a]Based on average of two estimates per year per station made six months apart.
[b]Standard error of standard deviation.

dicates the expected standard deviations of the percent error if two counts per station per year, spaced approximately six months apart, were used to estimate AADT.

The tables include the values of the estimated standard error of the standard deviation. The standard error of the standard deviation is an indicator of the accuracy of the estimated standard deviations when considering their values and the size of sample from which they were computed. If the range of plus and minus one standard error from the standard deviation is established about the estimated standard deviation, the fiducial probability is approximately 68 times out of 100 that the true standard deviation falls within this range. If the range of plus and minus two standard errors from the standard deviation is established, the fiducial probability is approximately 95 chances out of 100. Using plus and minus three standard errors of the standard deviations, the fiducial probability would be approximately 997 chances out of 1,000.

The standard error of the standard deviation is also used in testing for a significant difference between two standard deviations. The formula for the standard error of the standard deviation is as follows:

$$\text{Standard error of standard deviation} = \frac{\text{Estimated standard deviation}}{\sqrt{2N}}$$

where N = sample size used in estimating the standard deviation.

A statistical comparison of the standard deviations of Table 1 for a 12-mo coverage count season indicated that no significant difference between the accuracy of 7-day and 5-day coverage counts could be expected. The values shown, however, do indicate that a significant increase in accuracy of estimates of AADT would be gained by using 7-day or 5-day coverage counts rather than 48-hr or 24-hr coverage counts. The difference of the standard deviations shown for 48-hr and 24-hr coverage counts is also significant.

If December, January and February are eliminated from the coverage count season, there tends to be a difference between the accuracy which can be expected from 7-day and 5-day coverage counts. Although the difference between the two standard deviations is not highly significant, there is an indication that an improved accuracy would be gained from the use of 7-day coverage counts. A comparison of the values in Table 2 indicates that in most months a 7-day coverage count produces a lower standard deviation of the errors of estimated AADT. In December, January and June, the standard deviation of the errors of 7-day counts is greater than for the 5-day counts. Based on the sample size used to determine the monthly standard deviations, the differences of the standard deviations in December and June cannot be regarded as significant. The difference between the standard deviations of January, however, is significant.

Some small tests were made to determine why a 7-day coverage count produced less accuracy in January than a 5-day coverage count and seemingly more accuracy in other months. The results of the tests indicated that it is better to assume an average relationship of daily traffic in January between weekdays and weekend days rather than using a sample of only one weekend which is included when a 7-day coverage count is taken. It was concluded, that in the winter months there tends to be a significant uniform variation of weekend daily traffic from the average weekend daily traffic of the month among the various roadways of the state. If for January average group adjustment factors had been determined on the basis of the period of time in which the coverage counts were made, it is believed that the accuracy of 7-day coverage counts would have been better than 5-day coverage counts. Uniform variation between weeks of the month of all stations in the group would have been accounted for in the adjustment factor. As the summer season approaches, the variation between weekend traffic volumes within a month is not as significant as in the winter months and a sample of one weekend tends to be better than using an overall average relationship, which is assumed when a 5-day coverage count is expanded to an estimate of AADT.

Assuming that the present method used in Missouri of estimating AADT has an accuracy somewhat comparable to the value shown for 48-hr counts in Table 3, it was concluded that the same approximate accuracy could be obtained if 7-day coverage counts were made once a year per location for a 9-mo count season. The eventual cost of this procedure should be approximately one-half the cost of the present method.

ASSIGNMENT OF ROAD SECTIONS TO FACTORING GROUPS

Seasonal control counts are a necessity when using the procedure of estimating AADT recommended by the U. S. Bureau of Public Roads. The primary purpose of seasonal control counts is to assign roadways to groups of similar seasonal traffic patterns when continuous traffic counts are not available. They can also be used to estimate AADT for a particular location when a greater degree of accuracy is desired than may be expected from regular coverage counts.

Seasonal control counts at a location provide an estimate of the annual pattern of monthly adjustment factors for that particular location. The U. S. Bureau of Public Roads recommends that seasonal control counts be made either four, six or twelve times a year per location. These seasonal control counts, of seven consecutive days duration, should be spaced at approximately equal intervals throughout the year.

When seasonal control counts are made twelve times a year, an estimate of the adjustment factor for each month can be computed. If seasonal control counts are made only six or four times a year, the estimated annual pattern of monthly adjustment factors is not complete, but a sketch of the estimated monthly variation is provided.

Knowing that the cost of seasonal control counts per location increases with the number of times the location is counted per year, it was decided to investigate the difference between the results obtained when using the various types of seasonal control counts. The tests were performed in the following manner:

1. Twenty-six of Missouri's continuous traffic counting stations were used in this test. Some of the 90 previously used continuous count stations had a substantial number of days missing in some months due to various reasons making it inadvisable to use these stations in this particular test. The 26 stations provided a good proportional representation of the five groupings of stations in the state.
2. A 7-day simulated seasonal control count was made for each month for each of the 26 stations.
3. The twelve simulated 7-day seasonal control counts of each station were grouped to form six samples of various type seasonal control counts. There were three ways of simulating four control counts per year, two ways of simulating six counts per year, and one way of simulating twelve counts per year. The total sample sizes of the four, six and twelve repetitions per year were 78, 52 and 26, respectively.
4. The first test consisted of comparing the accuracies of estimated AADT's of the various types of seasonal control counts. The standard deviations of the errors of estimated AADT resulting from seasonal control counts of four, six and twelve times a year were 3.6, 3.1 and 1.7 percent, respectively. Based on the sample sizes, there proved to be no significant difference between the standard deviation of four counts a year and six counts a year. There is, however, a significant difference between the standard deviations of four counts a year and twelve counts a year, and between six counts a year and twelve counts a year.
5. For each sample of simulated seasonal control counts, the estimated annual pattern of monthly adjustment factors was computed. Monthly adjustment factors were computed by dividing the estimated AADT by the average five weekdays of each 7-day simulated seasonal control count of each month. This produced estimated ratios of annual average daily traffic to monthly average weekday traffic.
6. Each sample of simulated seasonal control counts per station was assumed to be on a road section of which the group assignment was not known. The station was then assigned to one of the five predetermined groups. These assignments were based on the similarity of the estimated annual pattern of monthly adjustment factors compared to the group mean patterns of the five groups. A least squares criterion was used to assign stations to one of these groups. This least squares method is different from the one recommended in the Bureau's manual. An example of assigning a seasonal control station to a group using the least squares method is as follows: (a) The individ-

ual monthly deviations of the estimated annual pattern from each group mean annual pattern is determined; (b) the individual deviations of each group mean pattern comparison are squared and the sum of squares determined; and (c) the road section is assigned to the group whose summation of squared deviations is the least. Because the summation of the squared deviations from a particular measurement is directly related to the magnitude of the expected distribution of errors, this method should hold errors of group assignment to a minimum. Although this method may be more accurate than a straight deviation comparison as recommended by the Bureau, it is not believed that the increased accuracy would be too significant. The least squares method, however, is relatively simple to program on a computer.

7. The percentage of wrong group assignments was computed based on the assumption that the group assignments from the original 90 continuous count stations were correct. Of the 78 samples counted four times per year, 44 percent of the assignments were made to the wrong group. For the 52 samples of six counts per year, 54 percent of the group assignments were wrong. For the 26 samples of twelve counts per year, 46 percent of the group assignments were incorrect. Based on these percentages, it can be expected that approximately 50 percent of the time a wrong group assignment will be indicated by any type seasonal control count. However, in every case where a wrong group assignment was made, it was made to a group adjacent to the true group. For example, the five group mean patterns were numbered consecutively in order of increasing variation of monthly adjustment factors. If a station had been assigned to group No. 1, and this assignment was in error, the true group assignment would be No. 2. If a station had been incorrectly assigned to group No. 3, the true group assignment would be either group No. 2 or group No. 4. The probability of the true group assignment being either one or five, in this case, is very small. Approximately 95 percent of the time incorrect group assignments were made, the true group assignment was the group which had the next least total squared deviations. Thus, when the least squares method is used, it is highly probable that the roadway involved belongs to one of two particular groups.

8. Five-day coverage counts were simulated at the test stations for the months of March through November. These coverage counts were expanded to estimates of AADT using the average factors of the true group assignment. A standard deviation of the resulting errors of estimated AADT was determined. This produced an expected range of errors which would result if group assignments from the use of seasonal control counts had been entirely correct. AADT was then estimated based on the group assign ments resulting from the seasonal control counts of the various types. The standard deviations of the errors of estimated AADT were then computed and compared to the standard deviation resulting from the use of the true group assignments. The differences of the various standard deviations were not significant and in no case did a difference exceed one-half of one percent.

It has been concluded that the increased accuracy of estimated AADT from a twelve times per year seasonal control count program does not warrant the extra cost over programs of four or six counts per year. Because the difference between the accuracies of the six and four counts per year programs are not deemed significant, the four times per year seasonal control count program is being considered for adoption by Missouri.

Although group assignments resulting from seasonal control counts may be incorrect at times, the true group assignment might be determined in a number of cases when a preponderance of one group assignment is found along a length of roadway. This is assuming that a very high percentage of the other estimated group assignments are made to groups adjacent to the group which has the preponderance.

CONCLUSIONS

The Missouri State Highway Department is considering the adoption of a 7-day coverage count program. Each coverage count location would be counted once a year and the coverage count season would be for nine months, omitting December, January and February.

Initially, a rather extensive seasonal control count program may be used. After what is believed to be sufficient coverage, approximately two to three years, the seasonal count program would be greatly reduced. An insignificant amount of control counts would then be handled as special counts as they are deemed necessary, such as an indication that a significant change in the annual pattern of a particular roadway section has taken place.

In comparing estimated annual costs of the proposed program to the existing program, the eventual savings should be approximately one-half the current annual cost.

Computer Model of Driving Behavior: The Highway Intersection Situation

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ABRIDGMENT

•A DIGITAL computer model of the perceptual, decision-making, and response processes of the driver has been formulated for the highway intersection situation. Including simulated vehicle characteristics, the model presents a simulation of human behavior in a dynamic control task. The model is completely deterministic with the possibility for inclusion of probabilistic functions when desirable. A completely determined process was selected to facilitate the study of the effects of various pertinent parameters.

Figure 1 is the summary flow chart of the computer model. Each process of this flow chart is further defined by subroutines (as numbered) which provide the detail required for the computer program. The set up block of Figure 1 provides for inputs of the following parameters which may be varied for a particular run: (a) initial location

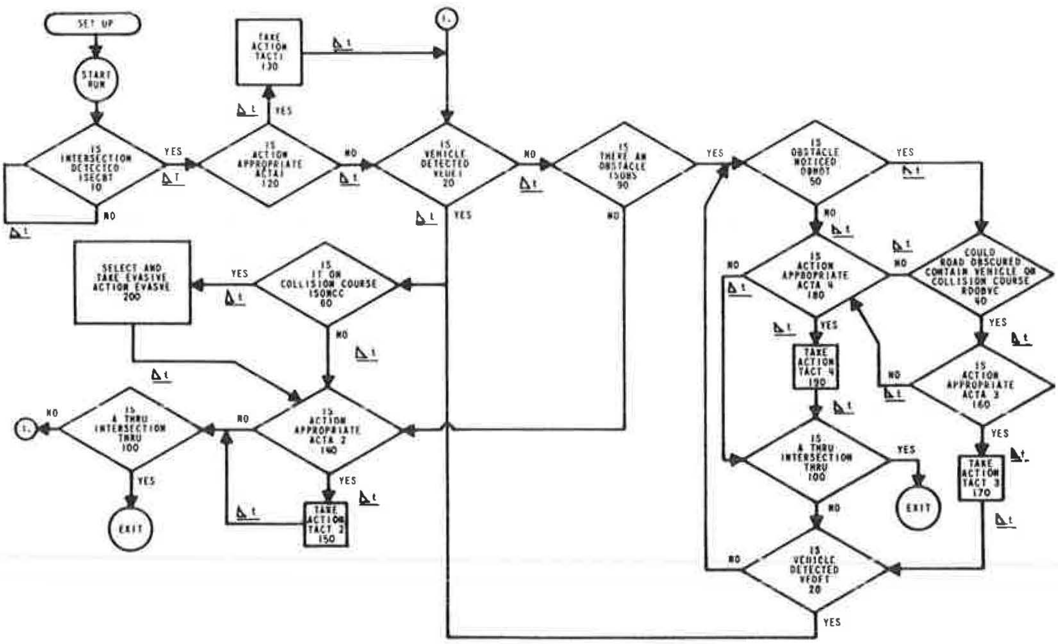

Figure 1. Summary flow chart.

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and velocity of the opposing vehicles, (b) obstacle locations, (c) maximum deceleration and acceleration capability of the modeled driver's vehicle, (d) maximum viewing distance from the modeled vehicle, and (e) pertinent perceptual and decision-making parameters and threshold values.

A parameter study of the foregoing variables was conducted to determine the effects on driver performance and accident or near-accident occurrence. Some of the parameters were more important in determining driving behavior than others. These were (a) how far ahead the driver is considering the consequences of his decisions, (b) the time required for each information process and decision, (c) the threshold for perceiving angular velocity, (d) the driver's vehicle velocity, and (e) location of the obstacle-to-view. In addition, there were significant interactions among these variables.

This technique of computer modeling of the driver-vehicle combination shows great promise as a method for examining the pertinent factors affecting performance of the driving task. The magnitude of this problem in including relevant and irrelevant perceptual inputs, logic and details of the simulated information processing and decision making, and the vehicle characteristics precludes an exclusively experimental approach. A model formulation is essential to further studies in this area. This digital computer model seems particularly suited to the problem, but the real test of the model must come from experimental validation, both of the overall output and of particular aspects.