A Conceptual Framework for Pavement Design Decisions

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•THE COMPLEX properties and behavior of highway pavement structures have resulted in the evolution of standard pavement design procedures and specifications to aid individual designers in the solution of specific problems. These standard procedures are intended to provide the basic information necessary for the solution of groups of pavement design problems having certain features in common.

The characteristic common to virtually all pavement design decisions is the uncertainty under which they are made. Uncertainty enters into pavement design decisions whenever the outcome of a particular pavement design cannot be exactly predicted. In order to simplify pavement design decisions, it has been expedient, both in actual design decisions and in analytical and empirical models of pavement behavior, to act as if the consequences of various actions could be predicted with certainty.

To overcome this uncertainty, existing pavement design procedures have relied heavily on engineering experience and judgment, much of which is of a subjective nature. This subjectivity has resulted in pavement design procedures whose underlying structure is not always clear, and which cannot easily be modified in the light of new data. Thus, an undesirable situation exists with all standard design procedures where solutions to specific design problems are based more on the forms that have been found as solutions to old problems, rather than on the particular nature of the design problem at hand. Existing pavement design methods do not allow a systematic and objective comparison of various pavement design types, and consequently pavement design decisions are biased by the personal experiences of the designer.

In addition to the uncertainty of future pavement performance, pavement design decisions are essentially economic decisions, and present pavement design procedures do not formally incorporate the economic properties of the various design alternatives. With present methods, economic considerations are implicitly expressed in the various design criteria. However, there has been a tendency to transform these criteria into rigid engineering standards that soon lose all contact with the original economic realities which entered into their development.

It is the purpose of this paper to establish the elements necessary for a rational and conceptually complete pavement design system. In particular, a formal approach to the uncertainty of future pavement performance will be developed, as well as an objective technique for incorporating the economic characteristics of pavement designs.

PAVEMENT DESIGN SYSTEM

A rational pavement design procedure, in which both the technical and economic characteristics of designs as well as the uncertainty of their future performance are objectively accounted for, should contain each of the steps defined below.

Design System Aims

The aim of the pavement design system is to select a pavement design strategy from among alternate designs whose expected present worth is a minimum with respect to the alternate designs, and whose expected life is equal to the design life.

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This definition implicitly assumes that a pavement of adequate serviceability qualities is provided throughout the design life, and that failure occurs when a limiting value of serviceability is attained. Studies by the Bureau of Public Roads (1), Saskatchewan Highway Department (2), Canadian Good Roads Association (3) and others have shown that the age at failure of highway pavements cannot be exactly predicted. Instead, a distribution of the age at failure of highway pavements has been observed for similar pavement designs. Available data suggest that the relative-frequency histograms may be approximated by a normal distribution function. This variability in the age at which failure occurs results from the heterogeneity of materials, variability in climatic conditions and loading, imperfections in the ability of analytical techniques to predict future performance, and so on.

The primary variable associated with the analysis of pavement designs is the age at which failure occurs, A. In view of the above comments, A must be considered as a random variable, $\tilde{A}$. The essential problem in pavement design is to predict the distribution function F(A), or the probability density function f(A) of the random variable, $\tilde{A}$, for each pavement design. In this investigation, random variables are shown as $\tilde{A}$, $\tilde{\theta}$, etc., using the tilde to distinguish the random variable from a particular value of the variable. Also, in taking expectations of random variables, the probability measure with respect to which the expectation is taken is indicated by naming the random variable and the conditions in parentheses following the operator, e.g., $E(\tilde{\theta}|z)$, $E(\tilde{A})$, etc.

Value Parameter

The value parameter expresses the value of each possible outcome of every design strategy to the designer. It is expressed as the present worth in dollar units of each possible outcome.

The present worth of each outcome includes the initial capital costs, annual maintenance costs, and any additional costs associated with pavement failure. The present worth of each outcome may be calculated from

$$PW = \left\{ C \cdot \frac{i(1+i)^A}{(1+i)^A - 1} + AMC \right\} \frac{(1+i)^L - 1}{i(1+i)^L} \tag{1}$$

in which

PW = present worth of outcome,
C = initial capital cost,
i = interest rate,
A = age at failure in years,
AMC = annual maintenance cost, and
L = design life in years.

Since A is a random variable, PW is a random variable. The value associated with a particular design strategy may be expressed as an expected present worth, $E[\tilde{P}W]$. In addition, since $\tilde{P}W$ is a nonlinear function of $\tilde{A}$, $E[\tilde{P}W]$ is dependent on both the mean and standard deviation of the distribution of $\tilde{P}W$.

There are no significant additional costs associated with pavement failure itself, as would be the case with highway bridge failures, where injury and loss of life must be incorporated in the measure of value.

Decision Criterion

The decision criterion is a rule which specifies how value parameters should be combined to obtain a single index of value for assessing the optimality of designs. With the pavement design system, the decision criterion is simply to select that design strategy with the least expected present worth.

Design Constraints

Design constraints are the physical and economic restrictions which may limit the extent of feasible, acceptable or permissible solutions to a design problem.

The only major constraint with the pavement design system is economic, and it is a maximum limit on the expected present worth. This maximum value is dictated by the economic characteristics of the overall highway project, of which the pavement structure is only one element.

Specification of Environmental Conditions

This step involves the quantitative, but in most cases also qualitative, definition or classification of all those environmental factors which influence the performance of alternate design strategies.

This classification not only involves magnitude and number of repetitions of axle loads, but includes all those environmental factors which result in serviceability decrements. The exact nature of this load classification is dependent on the requirements of the analytical or empirical models available for the prediction of probable future behavior of alternate design strategies. It is well known that rigorous analytical and empirical models of pavement behavior are not available, and that available models are imperfect predictors of probable future behavior. It is, therefore, meaningless to attempt to predict precisely the probable distributions of axle loads, climatic conditions, materials properties, etc.

Load definition must be a classificatory procedure which attempts to specify the general environmental conditions which might be expected at each pavement location. Recent studies by the Canadian Good Roads Association reported by Wilkins (4) and others have demonstrated that only a relatively coarse classification of the pavement environment is warranted.

A classificatory approach to probable loadings is also used in structural design formulations, where regions of various environmental conditions are established. Unfortunately, many of these load classification schemes are given numerical connotations completely incompatible with the level of data on which they are based, which are essentially subjective.

Specific environmental classes are not isolated in this paper, since the appropriate number of classes will depend on a particular area. However, the type of classification scheme used by the Canadian Good Roads Association (4) would seem appropriate and sufficiently sensitive.

Design Synthesis

The synthesis of alternate designs involves the development of a set of alternate design strategies which satisfy the design system aims to a greater or lesser degree. Present efforts at innovative design are virtually nonexistent in the design of highway pavements. The unfortunate situation exists where standard designs, or slight modifications of these designs, are used as design solutions. This situation results directly from the fact that no framework has existed within which new design strategies could be evaluated relative to these standard designs. Consequently, few attempts at using new materials and pavement configurations have been forthcoming since those developed some 40 to 50 years ago.

A number of approaches to innovative problem solving have been developed within recent years, but have not been sufficiently well developed to be of immediate practical application.

Design Analysis

This step involves the prediction of the expected age at failure of the set of tentative design strategies, in the light of the physical and economic properties of each design and the environmental conditions.

It has already been established that one of the major requirements of making a rational analysis of pavement design strategies is predicting the distribution of the random variable $\tilde{A}$ associated with each design strategy. In addition, a formal procedure is required to incorporate knowledge gained about the distribution function by prior analysis or experimentation. It has become apparent in recent years (5, 6, 7) that the surface deflection of flexible pavements under a standard 9-kip wheel load is a reliable indicator of future pavement performance. In this paper, it has been assumed for purposes of illustration that a central problem in the analysis of alternate flexible pavement designs is predicting the distribution of the surface deflection f(A) for each design strategy, and then using this distribution to predict the distribution function, F(A). Any reliable indicator of future performance could be used, however, thereby allowing for the direct comparison of flexible, rigid and semirigid pavements.

The information required for this step in the design process is as follows:

1. A listing of all possible design strategies d_1, d_2, d_3, ...
2. The distributions of the age to failure of each of these strategies $f_1(A)$, $f_2(A)$, ...
3. The present worth of each possible outcome of every design strategy PW_1, PW_2, ...
4. Prior information available on the probability distributions mentioned in 2.
5. The possible experiments or analyses that may be performed before selecting a particular design strategy, in order to gain further information about the probability distributions of 2.
6. The cost of these experiments.
7. A listing of the possible outcomes of these experiments.
8. A specification of the reliability of each of these outcomes in predicting the true outcome of a particular design strategy.

The final step of the design process is to use the decision criterion to select the optimum solution to the design problem. In other words, the designer selects that pavement design strategy with the minimum expected present worth. In the next section, the well-developed principles of statistical decision theory are reviewed and used to formulate the design analysis step in a rigorous manner.

STATISTICAL DECISION THEORY

The nature of elementary statistical decision processes is well described by Luce and Raiffa (8), Bross (9) and a number of other writers. Turkstra (10) has advocated the use of these elementary formulations in structural design decisions, while Tribus (11, 12) has also proposed their use in general engineering design and reliability problems. These decision processes are usually expressed in terms of 4 basic parameters:

1. A number of alternate courses of action open to the decision maker;
2. A number of states of nature that may obtain after a particular course of action has been selected;
3. The probability measures defined over the states of nature; and
4. The desirabilities of each of the outcomes that result from combinations of specific courses of action and particular states of nature.

An important extension of these elementary decision processes involves a methodology for taking into account knowledge gained about the states of nature by experimentation or theoretical analysis before selecting a particular course of action. This formulation is much more appropriate with respect to pavement design problems than the elementary formulations mentioned above. A great deal of prior information is available in the form of theoretical and empirical models of behavior which can be used to predict future behavior. These predictions may be imperfect, but the formulation described below allows this imperfection to be formally evaluated.

The basic data required for any decision problem in which experimentation or analysis is feasible prior to the selection of a particular course of action are:

1. A listing of the possible terminal decisions d_1, d_2, ...
2. A listing of the possible states of nature that may occur after a particular course of action has been selected θ_1, θ_2, ...
3. A listing of the possible experiments or analyses that may be performed prior to the selection of a terminal decision e_0, e_1, e_2, ...
4. A listing of the outcomes of these experiments z_0, z_1, z_2, ... which the decision maker believes possible; the likelihoods of these experimental observations depend on what the true state of nature actually is.
5. A listing of the values or utilities which represent the decision maker's preferences for all e, z, a, θ combinations or strategies.
6. A listing of the probabilities which the decision maker assigns to the joint probability distribution of z, θ for each of the potential experiments or methods of analyses.

The problem facing the decision maker is to select a course of action after observing the outcome of a prior experiment. Raiffa and Schlaifer (13) describe 2 basic modes of analysis of a decision problem, and both of these methods of analysis are important in evaluating pavement design decisions. One mode of analysis is concerned with the choice of a terminal action, after an experiment has already been performed and its outcome observed. This method of analysis is known as the terminal or posterior

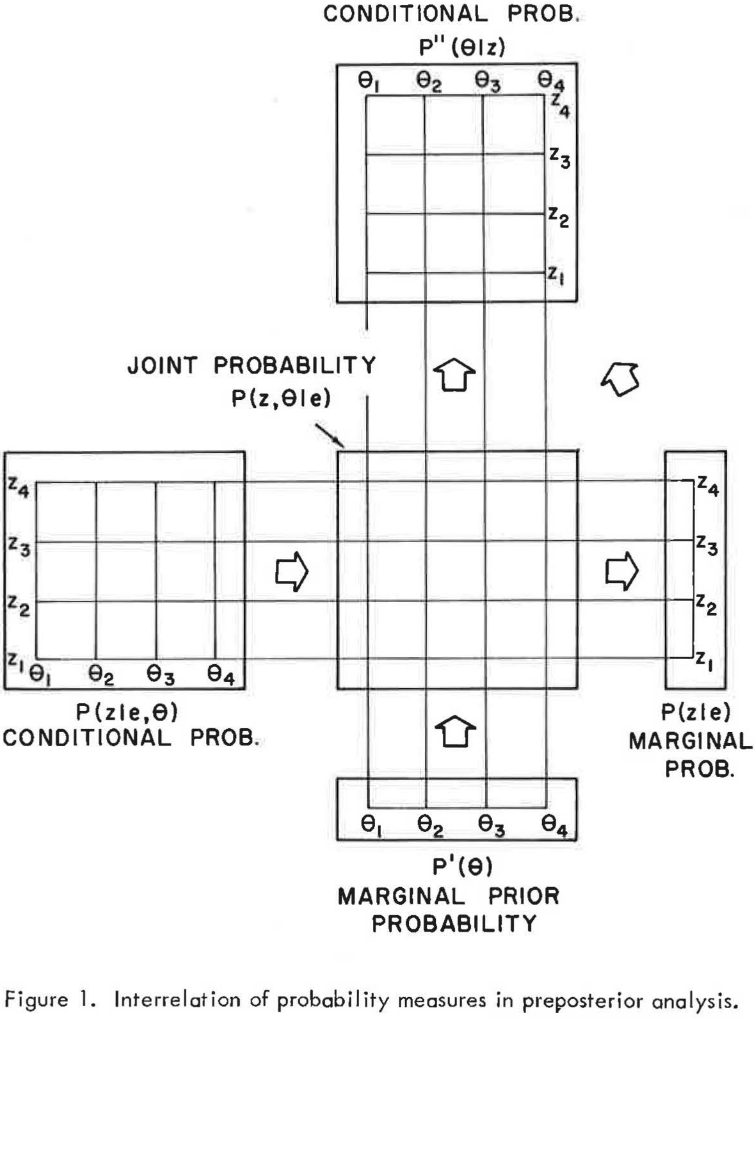

Figure 1. Interrelation of probability measures in preposterior analysis.

analysis. The second method is known as preposterior analysis, and is concerned with the selection of the most valuable experiment to be performed.

Preposterior Analysis

The central problem with respect to both forms of analysis is to assign either directly or indirectly a joint probability measure $P(\theta, z|e)$ to the joint distribution of θ, z for each experiment. In other words, the decision maker must define the reliability of each possible experimental outcome in predicting the true state of nature for each experiment or method of analysis.

The joint probability measures determine 4 other probability measures which are defined below and shown in Figure 1 for a simple case involving a single experiment:

1. The marginal probability measure $P'(\theta)$ on the states of nature that the decision maker would assign to Θ prior to knowing the outcome z of the experiment e.
2. The conditional probability measure $P(z, e|\theta)$ on the space Z of experimental outcomes for a given experiment e, where θ is the true state of nature.
3. The marginal probability measure $P(z|e)$ on the space Z for all θ, and for a specified e.
4. The conditional measure $P''(\theta|z)$ on the state space for a given e and z, which is the probability measure the decision maker assigns to the state space posterior to knowing the outcome z of the experiment e.

The interrelationship of these elements is best shown in the form of a tree diagram of the type shown in Figure 2, which is an abstraction of a simple decision situation in which there is only one possible experiment e_1. Experiment e_0 represents the special case in which no experiment is performed before selecting a course of action.

The manner in which numerical values of the probability measures are arrived at in a particular design problem will be dependent on the particular problem. The most efficient techniques for arriving at probability measures for the pavement design system are discussed later. For the present, it is convenient to assume that the prob-

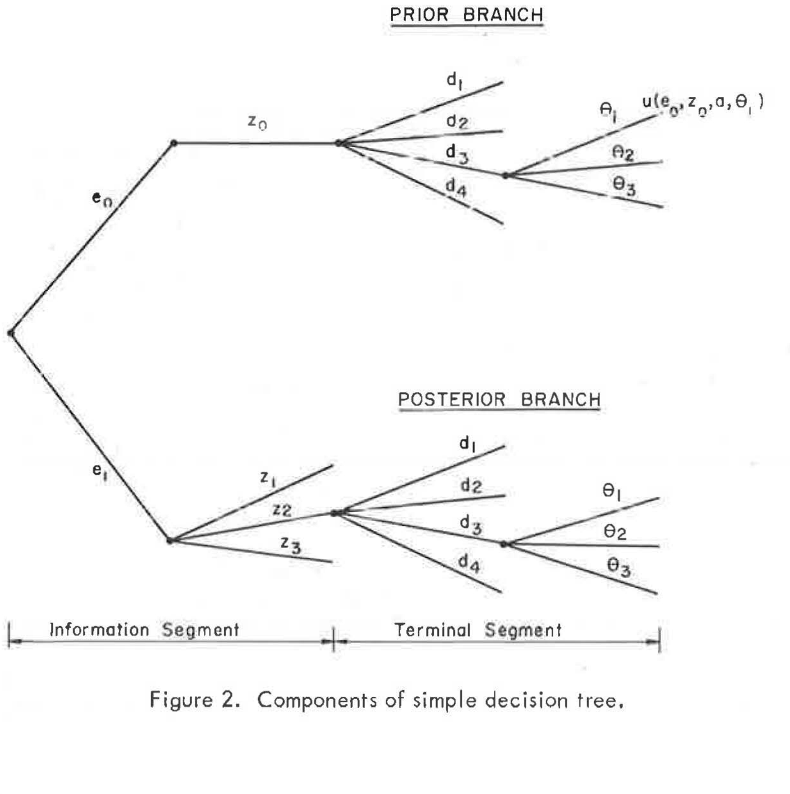

Figure 2. Components of simple decision tree.

ability measures have been defined, and the preposterior form of analysis proceeds by working backwards from the end of the decision tree to the initial point.

It begins by determining the terminal action that should be selected if an experiment has been performed and a particular outcome observed. Since the state of nature is a random variable, only the expected utility associated with each terminal action may be computed

$$u(e, z, d) = E(\theta'' | z) [u(e, z, d, \tilde{\theta})] \quad (2)$$

and the optimal terminal action is given by that action with the maximum expected utility

$$u(e, z) = \max_d u(e, z, d) \quad (3)$$

Similarly the outcome of each experiment is a random variable, and the utility of each experiment can only be computed as an expected utility

$$u(e) = E(z | e) [u(e, \tilde{z})] \quad (4)$$

The experiment with maximum expected utility can be readily selected, and the utility associated with the optimal strategy of the decision problem selected

$$\begin{aligned} u &= \max_e u(e) \\ &= \max_e E(z | e) \left\{ \max_d E(\theta'' | z) [u(e, z, d, \theta)] \right\} \end{aligned} \quad (5)$$

The posterior mode of analysis also has as its ultimate aim the description of an optimal strategy. It arrives at the same strategy as the preposterior form of analysis, but it arrives there by a different technique (13).

Design Example

To illustrate the application of the statistical principles to the previously outlined pavement design process, a simple example involving the evaluation of a new pavement design is given below.

Assume that a standard pavement design for an asphaltic-concrete pavement has evolved in a particular area for certain subgrade soil conditions, traffic loadings, and environment. The expected present worth of providing this standard design is $60,000 per mi for a 20-yr design life. The expected present worth is arrived at by summing the products of the relative frequency of each age to failure and the present worth of providing a pavement assuming that it fails at each of these ages. An alternate pavement design is under consideration for the same conditions of service, and experience in other areas suggests that it could be provided for a present worth of $36,000 per mi for a 20-yr life.

Further, assume that previous experience with this type of pavement in other areas indicates that satisfactory performance has been obtained about 70 percent of the time for similar service conditions. It is possible to obtain additional information about the probable behavior of the pavement type under local conditions by constructing a test section and performing load tests on this section. Previous load testing programs have indicated that if the deflection of the pavement under a standard 9-kip wheel load is less than 0.030 in., satisfactory performance is obtained in about 90 percent of the cases. Deflections greater than 0.030 in. have indicated unsatisfactory performance about 90 percent of the time.

This design problem resolves to one of deciding whether to obtain local information by constructing a test section which will provide more definite information about the probable behavior of pavements constructed with local materials, and then, on the basis of this initial action, whether to accept or reject the tentative design. The design problem is most readily illustrated in tree form of the type shown in Figure 3. The elements of this tree are as follows.

1. Actions:
   d_1—accept new design
   d_2—reject new design
2. States of nature:
   θ_1—satisfactory performance
   θ_2—unsatisfactory performance
3. Experiments:
   e_0—no experiment
   e_1—experiment
4. Experimental outcomes:
   z_0—dummy outcome of e_0
   z_1—deflection <0.030 in. more favorable to θ_1
   z_2—deflection >0.030 in. more favorable to θ_2
5. Utilities
   The utility or value associated with each possible strategy of the decision tree (Fig. 3) may be readily determined from the cost data given above. For example,

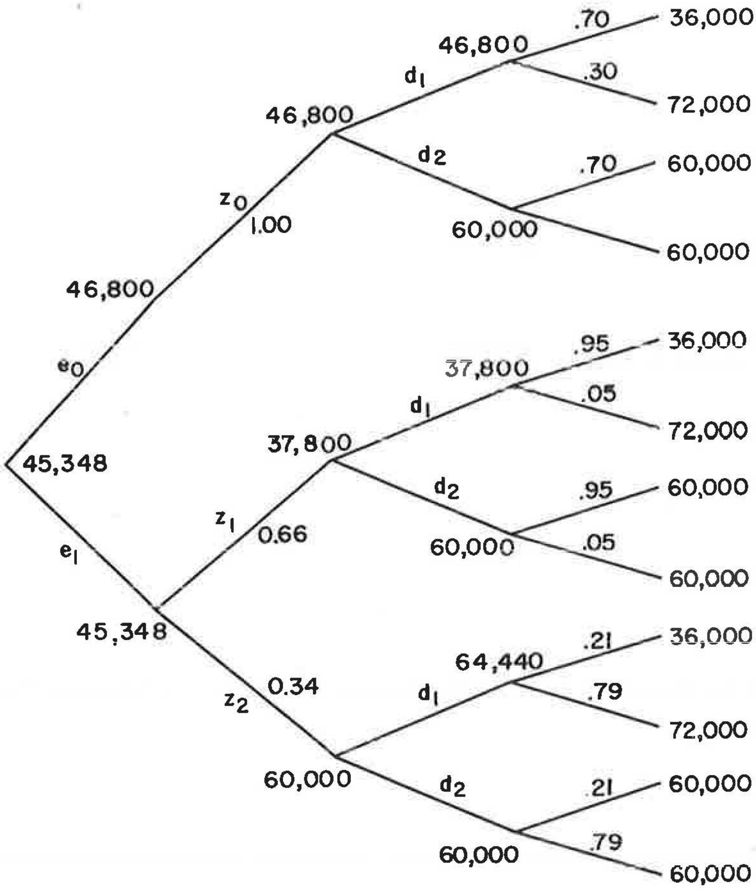

Figure 3. Preposterior analysis of pavement design decision.

if the strategy is selected which involves no experimentation and accepting the new design, the present worth would be \$35,000 per mi if it performs satisfactorily, and \$72,000 if it is unsatisfactory. The additional \$37,000 per mi in the latter case arises from the cost of reconstruction of the failed pavement. If the new design is rejected, then the present worth of providing the standard design is \$60,000 per mi. The cost of constructing the test section has not been included in the posterior branch, since this cost is trivial in this particular example. However, it could easily be shown as an additional annual cost.

The immediate problem is to generate probability measures over the joint probability distribution of θ and z. The prior marginal probability measures over the states of nature are

$$P'(\theta_1) = 0.70$$
$$P'(\theta_2) = 0.30$$

The conditional probability measures of obtaining the experimental outcome Z when θ and e exist have also been defined and are given by

$$P(z_1|e_1, \theta_1) = 0.90 \qquad P(z_2|e_1, \theta_1) = 0.10$$
$$P(z_2|e_1, \theta_2) = 0.90 \qquad P(z_1|e_1, \theta_2) = 0.10$$

where the first statement means the probability of obtaining the outcome z_1, when e_1 is the experiment and θ_1 the true state of nature, is 0.90. The joint probability measure on the $\Theta \times Z$ space is given by the product of the prior and conditional measures, since they are independent events

$$P(z_1, \theta_1|e_1) = 0.70 \times 0.90 = 0.63$$
$$P(z_2, \theta_1|e_1) = 0.70 \times 0.10 = 0.07$$

$$P(z_2, \theta_2|e_1) = 0.30 \times 0.90 = 0.27$$
$$P(z_1, \theta_2|e_1) = 0.30 \times 0.10 = 0.03$$

The marginal probability measures of the outcomes z_1 and z_2 of the experiment may be determined by summing over z_1 and z_2, respectively, from the above joint measures

$$P(z_1|e_1) = 0.63 + 0.03 = 0.66$$
$$P(z_2|e_1) = 0.27 + 0.07 = 0.34$$

From Bayes' theorem $P''(\theta|z) = \dfrac{P'(\theta)\, P(z|\theta)}{\Sigma\Theta P(z|\theta)}$ the posterior probabilities over the states of nature are then determined

$$P''(\theta_1|z_1) = 0.63/0.66 = 0.95 \qquad P''(\theta_1|z_2) = 0.21$$
$$P''(\theta_2|z_1) = 0.05 \qquad P''(\theta_2|z_2) = 0.79$$

These posterior probabilities of state represent a revised set of probabilities given an experiment and a particular result of this experiment. The probability measures have been indicated on the appropriate elements of the decision tree (Fig. 3).

The optimum strategy can now be established by calculating the expected annual cost of each possible action, and selecting that action with the least expected annual cost. For example,

$$u(e_0,\ z_0,\ d_1) = 0.7 \times 36,000 + 0.3 \times 72,000 = \$46,800$$

and

$$u(e_0,\ z_0,\ d_2) = \$60,000$$

Therefore the strategy $e_0\ z_0\ d_1$ represents the optimum action of the prior branch. By proceeding in a similar manner with the posterior branch, it can easily be verified that the optimum strategy for the decision situation shown is to construct a test section, and to accept the new pavement design if a deflection less than 0.030 in. is obtained, since this strategy results in the least expected annual cost.

While the design situation examined above may be somewhat trivial, it is typical of many pavement design situations. An actual experiment may not be performed, but experimentation may consist of some form of analysis, in which known materials properties are used along with a theoretical or empirical model. In certain design problems, it is possible that the available historical evidence bears much more directly on the posterior measure $P''(\theta|z)$ and the marginal measure $P(z|e)$, than on the complementary measures $P(z|e,\ \theta)$ and $P'(\theta)$. Instead of computing the posterior measures, the prior measures are calculated, and the utility or value of carrying out experimentation or analysis may be evaluated, as opposed to using a standard design.

ANALYTICAL FORMULATION

The application of the preposterior analysis to a simple pavement design problem was illustrated above, wherein the required computations were carried out numerically. In the majority of pavement design problems numerical analysis would be extremely tedious because of the relatively large sample and state spaces. Raiffa and Schlaifer (13) have shown that both the preposterior and posterior modes of analysis may be formulated analytically, providing the following conditions are met: (a) the experimental outcome can be described by a sufficient statistic of fixed dimensionality, and (b) the prior probability measure $P'(\theta)$ on Θ is conjugate to the conditional probability measure $P(z|e,\ \theta)$ on the sample space Z.

It was established that, as a first approximation, the distribution of the age at failure, f(A), and the distribution of the surface deflection, f(Δ), may be considered to be normally distributed. Raiffa and Schlaifer have developed the appropriate distribution theory for a normal data generating process in which the above conditions are valid. The major points of the distribution theory are reviewed briefly, and it is shown how the pavement design system previously formulated may be expressed in terms of these principles.

Sufficient Statistics

The only complete way of describing the outcome of an experiment is to list all the experimental observations in the order in which they have been observed. However, for analytical purposes, the need is to establish whether there exists a set of descriptions which is simpler and easier to manipulate, yet contains all the information relative to the decision problem.

The sufficiency of a statistic depends on the particular functional form selected for a decision problem. For example, the statistics sufficient for the description of experimental outcomes generated by a normal process will differ from those sufficient for the description of a Poisson data-generating process. It can be shown (13) that the number of random variables observed, the mean value of the random variables, and the standard deviation are the statistics sufficient to describe fully a sample generated by a normal process.

Conjugate Distributions

If the prior distribution of the random variable θ has a mass function D', then it follows from Bayes' theorem that the posterior distribution of θ is given by

$$D''(\theta|z) = D'(\theta)\ell(z|\theta)N(z) \tag{6}$$

where

$\ell(z|\theta)$ = the probability, given that θ is the true state of nature, that experiment e results in the outcome z; the value assumed by $\ell(z|\theta)$ is termed the likelihood of obtaining an experimental outcome

and

$N(z)$ = normalizing constant given by

$$N(z) = \Sigma\in D'(\theta)\ell(z|\theta) \tag{7}$$

For any mass function D of θ, and if K is another function on Θ such that

$$D(\theta) = \frac{K(\theta)}{\Sigma\Theta K(\theta)} \tag{8}$$

then

$$D(\theta) \propto K(\theta) \tag{9}$$

and K is called a kernel of the mass function of θ. Similarly, if the likelihood of z given θ is $\ell(z|\theta)$, and ρ and k are functions on Z such that for all z and θ

$$\ell(z|\theta) = k(z|\theta)\,\rho(z) \tag{10}$$

then $k(z|\theta)$ is called a kernel of the likelihood of z given θ, and $\rho(z)$ is called the residue of this likelihood. That is, the kernel is that portion of the likelihood that depends on the state of nature θ.

It may be shown (13) that, using Eqs. 8 and 9, Eq. 6 may be expanded to give

$$D''(\theta|z) \propto K'(\theta)\,k(z|\theta) \tag{11}$$

If the experimental outcome can be described by a sufficient statistic y, then Raiffa and Schlaifer show that Eq. 11 may be rewritten

$$D''(\theta|y) \propto K'(\theta)\,k(y|\theta) \tag{12}$$

That is, the kernel function k is defined as a function $k(\cdot|\theta)$ with parameter θ on the reduced sample space Y. Instead, it may be considered as a function $k(y|\cdot)$ with parameter y on the state space Θ. This function when normalized and shown to be non-negative for all $\theta \in \Theta$ is called a natural conjugate with parameter y of the kernel function k. That is, the roles of the variables and parameters in the algebraic expression of the sample likelihood function are interchanged.

The mathematical simplification introduced by Raiffa and Schlaifer is to select the functional form of the prior distribution such that its kernel has the same mathematical form as the sample kernel. The 2 kernels are then combined to give a closed expression for the desired posterior probabilities.

Normal Data Generating Process

If the data generating process is normal, Raiffa and Schlaifer show that closed mathematical expressions for the sample, prior, and posterior probability distributions can be derived. It has been pointed out above that the general procedure involves the selection of a prior distribution such that the kernel of the distribution is of the same mathematical form as the sample kernel.

The normal data generating process is defined in the following manner by Raiffa and Schlaifer:

$$f_N(x|\mu, h) = \frac{h^{1/2}}{2\pi} e^{-1/2 h(x-\mu)^2} \tag{13}$$

where

μ = mean value
h is known as the precision, and
$h = 1/\sigma^2$ or $\sigma = 1/\sqrt{h}$

If the size of the sample is ν, and successive values of x are generated, then the likelihood of finding the sample is equal to the product of the individual likelihoods

$$(2\pi)^{-1/2\nu} h^{1/2\nu} e^{-1/2 h\nu s^2} \tag{14}$$

where s = sample variance. For the normal data generating process, the following 3 cases are of interest: (a) true mean of the process is known; (b) precision of the process is known; and (c) the mean and the precision are unknown.

True Mean Known. —If the true mean of the process is known, then from Eq. 14, the sample likelihood is proportional to

$$h^{1/2\nu} e^{-1/2 h\nu s^2} \tag{15}$$

The natural conjugate to the kernel of the sample likelihood is the gamma - 2 probability distribution, which is

$$f_{\gamma_2}(h|s^2, \nu) = \frac{1/2\nu s^2}{(1/2\nu-1)!} (1/2\nu s^2 h)^{1/2\nu-1} e^{-1/2\nu s^2 h} \tag{16}$$

Eq. 16 leads to

$$h^{1/2\nu''} e^{-1/2 h\nu'' s''^2} \propto h^{1/2\nu'} e^{-1/2 h\nu' s'^2} h^{1/2\nu} e^{-1/2 h\nu s^2} \tag{17}$$

and

$$\begin{aligned} v'' &= \nu + \nu' \\ s''^2 &= \frac{1}{\nu}(\nu' s'^2 + \nu s^2) \end{aligned} \tag{18}$$

where the double prime denotes the posterior sufficient statistics, and the single prime the prior sufficient statistics.

Precision Known. —If the true mean is unknown, but the precision of the process is known, then the kernel of the likelihood is given by

$$e^{-1/2 hn(\bar{x} - \mu)^2} \tag{19}$$

where n = number of x's observed. The natural conjugate of the kernel of the sample likelihood is the normal distribution, and the distribution function of μ is given by

$$f_N(\mu|\bar{x}, hn) \propto e^{-\frac{1}{2}hn(\mu - \bar{x})^2} \tag{20}$$

Using the same symbology for prior, posterior and sample parameters, the following relationships are obtained:

$$\begin{aligned} n'' &= n' + n \\ n\bar{x}'' &= n'\bar{x}' + n\bar{x} \end{aligned} \tag{21}$$

True Mean and Precision Unknown. —If both the true mean and precision of the process are unknown, then the kernel of the likelihood is given by

$$\frac{(hn)^{1/2}}{2\pi} e^{-\frac{1}{2}hn(\bar{x} - \mu)^2} h^{\frac{1}{2}(n - 1)} e^{-\frac{1}{2}h(n - 1)s^2} \tag{22}$$

Raiffa and Schlaifer show that the natural conjugate is the normal-gamma distribution with $\nu = n - 1$

$$f_{N\gamma_2}(\mu, h|\bar{x}, s^2, n, \nu) = f_N(\mu|h, hn) f_{\gamma_2}(h|s^2, \nu) \tag{23}$$

The prior, posterior and sample parameters are related by the following expressions

$$\begin{aligned} n'' &= n + n' \\ \nu'' &= \nu + \nu' + 1 \\ n''\bar{x}'' &= n\bar{x} + n'\bar{x}' \\ \nu''s''^2 &= (\nu's'^2 + n'\bar{x}^2) + (\nu s^2 + n\bar{x}^2) - n''\bar{x}''^2 \end{aligned} \tag{24}$$

The marginal distribution of h as a random variable is gamma - 2, and the marginal distribution of μ is student.

Formulation of Pavement Design Process

The pavement design system previously formulated may be expressed in terms of the framework provided by statistical decision theory, and this involves the following steps:

1. Specify the prior probability distribution of the age at failure for each design, by the sufficient statistics.
2. Specify the conditional distribution of the surface deflection for each possible failure age, through the use of empirical or theoretical relations, for each design.
3. Estimate the distribution of the surface deflection of each design, and calculate the sample likelihood.
4. Calculate the posterior distribution of the age at failure for each design, either by the numerical or analytical procedures outlined above, for each design.
5. Calculate the present worth in dollars for each possible failure age, or establish the functional relationship between age and present worth.
6. Calculate the expected present worth of each design from the posterior probability distribution and the present worth relation.
7. Select that design with the minimum expected present worth.

The precise nature of each step will depend on the available theoretical and empirical models available. The analysis of a particular design problem may be carried out using either the preposterior or posterior modes of analysis. The mechanics of executing both these methods of analysis are well described by Raiffa and Schlaifer.

The procedure described above allows an objective and systematic comparison of possible solutions to a particular design problem. The framework which has been established is relatively free from the personal biases and immediate experiences of individual designers. Although the illustrative examples and criteria have been concerned with flexible pavements, any pavement type could be considered within this framework, as well as any parameter as a predictor of future pavement performance.

The application of this design procedure to the analysis of design decisions could be demonstrated by an example similar to that previously described. Instead of using discrete probability measures, the probability distributions could be specified, and the analytical approach described above used to evaluate the decision problem.

SUMMARY AND CONCLUSIONS

1. Current pavement design decisions are heavily biased by the personal experiences of individual designers, and the selection of an optimum solution to a pavement design problem is not always realized. No general framework has been established to date whereby systematic and objective comparisons of various pavement designs can be made.
2. The elements of a rational pavement design procedure have been established wherein both the technical and economic characteristics of designs, as well as the uncertainty of their future performance, are objectively accounted for.
3. The principles of statistical decision theory have been summarized, and the techniques for analyzing decision problems under uncertainty illustrated.
4. The pavement design system has been formulated in terms of the framework provided by statistical decision theory, and the steps basic to this formulation have been described. The framework allows an objective and systematic comparison of possible solutions to design problems which is relatively free from the personal biases and immediate experiences of individual designers.

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