

# Evaluating Highway Elevation Power Spectra From Vehicle Performance

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A relatively new technique for describing the condition of a highway has been developed which consists of making a power spectral density analysis of elevation measurements obtained from the longitudinal pavement profile.

In calculating the elevation power spectrum it is necessary to make certain assumptions that influence the resulting spectrum. The question thus arises as to the validity of these assumptions.

A procedure is described in this paper that was used to obtain a pavement elevation power spectrum from dynamic tire force measurements. This involved the experimental determination of tire forces which were used as a criterion of vehicle performance.

The power spectrum obtained by this procedure is compared with power spectra calculated from elevation measurements (using different assumptions) to check the validity of the assumptions.

VARIOUS INSTRUMENTS have been developed for evaluating pavement condition. Many of these devices, such as the BPR roughometer, obtain a number which is related to the roughness or smoothness of the highway profile. An investigator interested in predicting vehicle behavior (1, 2, 3) needs a more detailed description of the highway profile than is afforded by a measurement of this type. He needs a description of the highway that can be used with the vehicle characteristics to predict the behavior which he is studying.

Such a description can be obtained by making a power spectral density analysis of the longitudinal profile of a pavement (4). Unfortunately there is more than one way to make this analysis, and therefore different results can be obtained from the same data. It is the purpose of this paper to discuss this problem, and to suggest criteria by which the results of a power spectrum analysis of a highway profile can be evaluated.

## THE HIGHWAY ELEVATION POWER SPECTRUM

Power spectrum analysis has been used extensively in electrical engineering problems for which the term "power" has an obvious meaning. When spectral analysis is employed in other areas the term "variance" would be more appropriate although it is rarely used. Either term refers to the mean square value of the variable being considered.

The result of applying power spectral analysis to highway elevation measurements is a curve showing the extent to which various wavelengths are present in the highway. In addition, the area under this curve is a measure of the roughness of the highway.

If a highway was perfectly level, perfectly smooth and at zero elevation, a power spectral density analysis of the elevation measurements would result in a curve that

would be a horizontal line coinciding with the horizontal axis shown in Figure 1. This curve would indicate that no undulations of any wavelength are present in the highway, and the area under this curve would be equal to zero (all the elevation measurements would be zero).

If, however, the highway at zero elevation was level and not perfectly smooth, but with randomly distributed undulations having wavelengths varying from  $L_1$  to  $L_4$ , a power spectral density analysis of this highway would yield the curve shown in Figure 1. The total area under the curve would give the total mean square value (power) of the elevation measurements. Of considerable interest is the fact that the contribution to this mean square value from wavelengths ranging from  $L_2$  to  $L_3$  would be represented by the shaded area (Fig. 1). This area is usually referred to as the "power" associated with the wavelengths ranging from  $L_2$  to  $L_3$ . It is this property that makes the curve attractive since the wavelengths that are significant can be identified. The curve also indicates that wavelengths longer than  $L_4$  contribute nothing to the variation in the highway profile, and the same can be said for wavelengths shorter than those indicated by  $L_1$ . As a consequence only wavelengths between  $L_1$  and  $L_4$  are of any significance in the hypothetical highway under discussion.

In addition, another important property also exists. By using the curve shown in Figure 1, together with the appropriate vehicle characteristic, it is possible to predict the behavior of the vehicle on the highway. It is this latter property of the power spectrum curve that makes it very attractive to investigators interested in predicting vehicle behavior.

Although the wavelengths of undulations in a highway profile are easy to visualize and to discuss, they are not convenient to use in making power spectrum calculations.

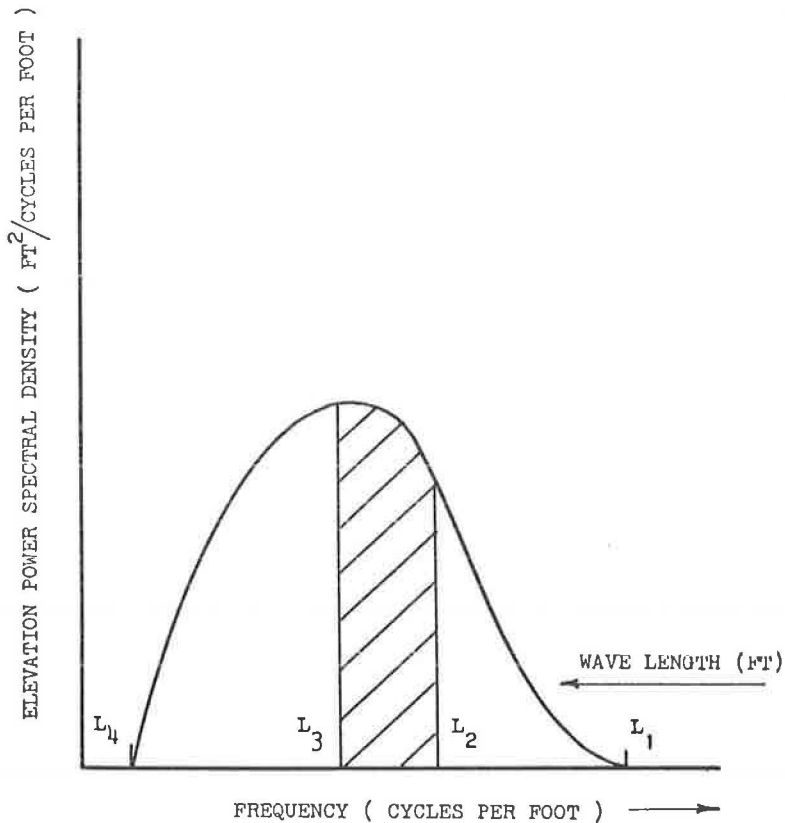


Figure 1. Elevation power spectrum for a hypothetical highway.

As a consequence a distance-based frequency is used in which frequency (in cycles per foot) is employed rather than wavelength. This quantity is the reciprocal of the wavelength and hence is easily related. The ordinate of the highway elevation power spectrum is in units of feet squared per cycle per foot and these units come naturally from the mathematical calculation. The area under the curve will therefore have the units that result when ordinate and abscissa are multiplied together. The resulting units are those of feet squared which would be associated with the mean square value of the elevation measurements.

It is not possible from a power spectrum curve to identify the amplitude of a single wavelength. Instead, it is possible to determine the mean square value of a range of wavelengths such as previously discussed.

A highway elevation power spectrum is therefore a curve. Information for calculating the ordinates of this curve can be obtained from either elevation measurements of the longitudinal profile, or from data obtained by a device that is sensitive to the highway profile.

The elevation power spectrum curve for a rough highway will have larger ordinates and a larger enclosed area than the power spectrum for a smooth highway. In addition, those wavelengths that contribute to the roughness can be easily identified. These properties, together with the fact that this characteristic can be combined with the vehicle characteristics to predict vehicle performance, make the power spectrum a valuable tool for the solution of certain highway problems.

The computational procedure for calculating a power spectrum is not included in this paper since such information can be obtained elsewhere (5).

### COMPUTATION PROBLEMS

One of the problems involved in making a power spectral density analysis of a highway profile arises from the enormous range of wavelengths present in most highways. If a highway goes over a hill and down into a valley, a wavelength of several miles may exist that will have an enormous amplitude. On the other hand, extremely small wavelengths having very small amplitudes exist in a rigid highway in the form of brush marks in the concrete. In making a power spectral analysis it is therefore important to select that range of wavelengths that is significant for the problem at hand, and to make the analysis so that these wavelengths can be carefully studied.

Since "power" is related to amplitude, it is possible to estimate roughly the relative power of different wavelengths (a hill as compared to brush marks) by considering their respective amplitudes.

Selecting the wavelengths of interest usually involves more than just a consideration of amplitude. In a previous paper (3) those wavelengths were identified that were significant in determining the dynamic tire forces created by a passenger vehicle as it traveled over a highway. This required a knowledge of the natural frequencies of the vehicle suspension system as well as the selection of a vehicle velocity.

In general, wavelengths in the highway ranging from 4 ft to approximately 100 ft in length were found to be significant. Wavelengths within this range are therefore considered significant.

The horizontal distance between adjacent elevation measurements must be selected before the rod and level survey can be conducted. Many factors influence this decision and different investigators have used different distances. In all surveys conducted for the authors a distance of 1 ft has been used.

As the horizontal distance between elevation measurements is increased, the ability to measure the power associated with the shorter wavelengths is lost. This power is not eliminated from the analysis, but will appear in the power spectrum and will be erroneously attributed to other wavelengths (aliasing).

A problem of paramount importance in calculating a highway power spectrum from elevation measurements is that of removing the effects of very long wavelengths having large amplitudes (and power) such as are introduced by a hill and a valley. Moreover, if the highway is not at sea level the undesirable effect of an infinite wavelength is introduced. Long wavelengths are very undesirable because they generally distort the

measurement of the power that is associated with the shorter wavelengths. Since these effects are all contained in elevation measurements it is virtually impossible to calculate a satisfactory highway elevation power spectrum directly from elevation data.

One method for dealing with this problem is illustrated in Figure 2. The original grade line of the highway is a section of a very long wavelength having a large amplitude. This was the original pavement profile when the pavement was perfectly smooth, and at that time the ordinates (and hence the area) of the power spectrum would ideally have been equal to zero. After years of use the pavement surface changed, and the present profile is represented by the irregular curve. If the highway power spectrum calculation is based upon the deviation of the present profile from the original grade line it will provide a measurement of the present roughness of the pavement. One problem, therefore, is determining these deviations accurately.

Different investigators have approached this problem in different ways. For this reason alone it is possible to obtain different power spectra from the same set of elevation measurements. One purpose of this paper is to indicate how the behavior of a vehicle can provide criteria whereby the "correct" technique can be identified.

Two techniques for determining the deviations of the pavement profile from the original grade line of the highway are discussed. In one method the first step is to obtain the average elevation  $\bar{Y}$  from the entire set of elevation measurements  $Y_i$ . This value is then subtracted from each elevation measurement in turn to obtain the values indicated by  $Z_i$ . A running mean, established by a selected number of these quantities, is then subtracted from the value of  $Z_i$  at the midpoint of this mean to obtain the deviation  $X_i$  (Fig. 2). The values of  $X_i$  are then used to calculate the power spectrum.

The other technique for calculating  $X_i$  uses the values of  $Z_i$  as previously determined. In this case a second-order curve is fitted to the values  $Z_i$  by the least squares technique, and this curve is considered to represent the original grade line of the highway. The ordinate of this curve at each station is subtracted from the corresponding value  $Z_i$  to obtain the value of  $X_i$ . These values of  $X_i$  are then used to calculate the power spectrum.

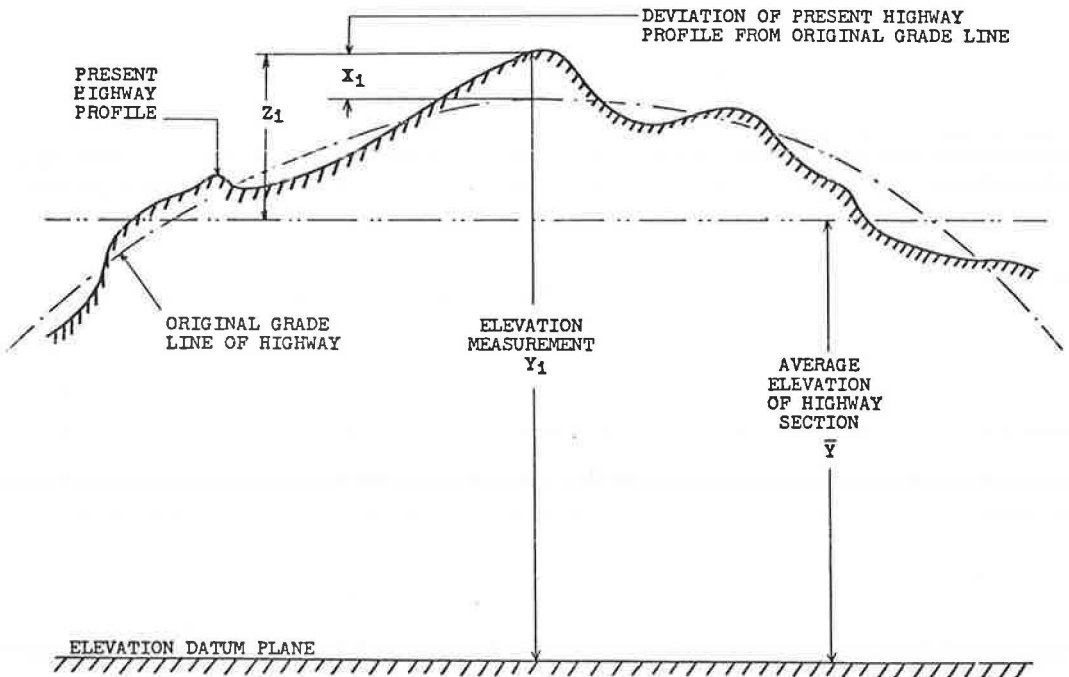


Figure 2. Relationship between elevation measurements and highway deviations.

The values of  $X_i$  obtained by these 2 methods will not be identical, and thus different power spectra will be obtained from the same set of elevation measurements  $Y_i$ . Moreover, additional techniques have been employed by other investigators to obtain values of  $X_i$  from the elevation measurements.

Investigators familiar with spectral techniques may indicate that data prewhitening procedures should be based primarily on a consideration of the entire frequency domain spectrum, and that these considerations should dictate the prewhitening procedure to be used. It should be recognized, however, that decisions concerning frequency domain filters also require the exercise of judgment, and that differences of opinion in this matter may also exist, giving rise to different spectra from the same basic data.

Another problem encountered is that of determining the appropriate length of highway to be used in this type of analysis. In general it is desirable to have as long a length of highway as possible, but physical and economic considerations often limit the length of the test section that can be considered. Generally speaking, the longer the length of highway that is analyzed the more statistically reliable will be the resulting calculations. A compromise is thus involved between statistical reliability and the number of elevation measurements that can be made. Such a compromise will also influence the power spectrum calculation.

The numerical technique whereby the power spectrum is calculated from the experimental data is also subject to personal judgment. It is not the purpose of this paper to go into great detail as to how these calculations are made but rather to discuss criteria whereby the validity of such calculations can be judged. In making the power spectrum calculation, however, it is necessary to calculate a relationship known as the autocovariance function. Such a function, when calculated, will consist of several ordinates plotted versus a distance parameter. One problem associated with this part of the calculation lies in the selection of the number of ordinates that will be used to represent the autocovariance function. More technically speaking, the maximum lag value that will be employed in generating this function is also a matter for individual judgment.

Without going further into mathematical details, there are other places in the calculation of a highway power spectrum in which discretion must be employed. Clearly a criterion is needed to evaluate the results of a calculation in which individual judgment must be exercised.

## DETERMINATION FROM VEHICLE PERFORMANCE

The condition of a pavement will be reflected in the elevation measurements, as discussed. The pavement condition will also influence the behavior of a vehicle moving over the pavement. We shall now show how the performance of a vehicle can be used to obtain criteria by which the elevation power spectrum, calculated from elevation measurements, can be judged.

The performance of a vehicle moving over a pavement can be measured in many different ways. Some investigators measure vertical vehicle acceleration, some measure the relative displacement between the body and the unsprung mass of a vehicle and some measure strains in different parts of the vehicle. The measurement of vehicle performance used in this paper is the dynamic tire force, obtained by measuring changes in the air pressure in a tire as the vehicle travels over the pavement section under consideration (6). Other indications of vehicle performance can also be used, and it is possible that better results can be obtained if such are employed.

If a linear system experiences an input that can be characterized by a power spectrum, it is possible to calculate the corresponding power spectrum of the output (7). This technique requires the determination of a quantity known as a transfer function.

The transfer function of a system can be determined experimentally by subjecting the system to a sinusoidal input of known frequency and amplitude and determining the amplitude (and phase) of the output. Under these conditions the ratio of the amplitudes of the output and input is calculated, and this ratio is the value of the transfer function at the selected frequency. This process is repeated at different frequencies to obtain additional values for the transfer function.

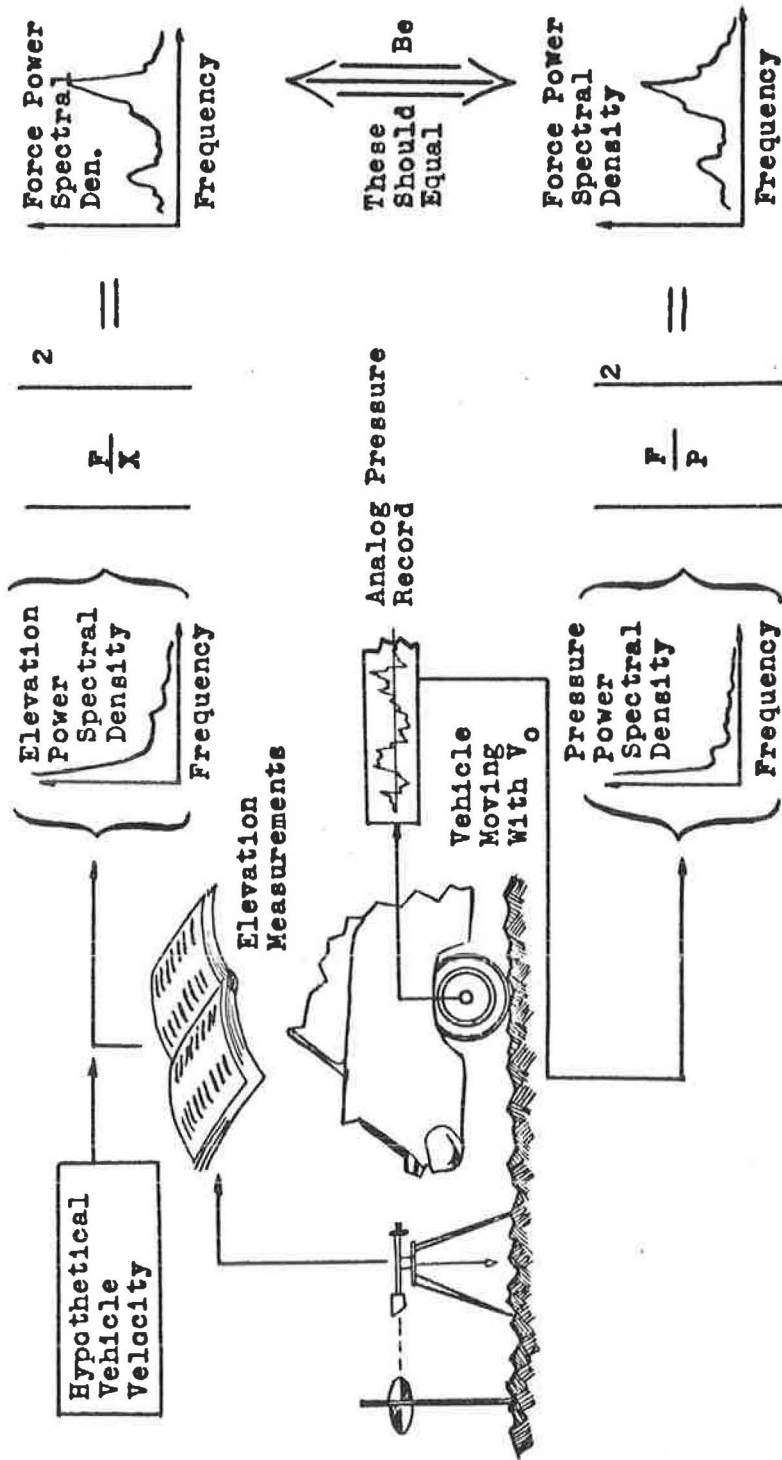


Figure 3. Two methods of calculating force power spectral density function.

The 2 transfer functions needed for this investigation are shown in Figure 3. The ratio  $F/P$  indicates that the force  $F$  on the tread of the tire is taken to be the output, while the differential air pressure  $P$  in the tire is the input. This relationship is necessary to determine the tire force from the tire pressure.

The other transfer function is shown by the ratio  $F/X$ . Here the output is the tire force  $F$  as previously, but the input is the displacement of the tread of the tire, represented by  $X$ . This relationship is necessary to determine the tire force from the highway elevation power spectrum.

The output power spectrum of a linear system is equal to the input power spectrum multiplied by the square of the transfer function. This process is shown in Figure 3 for 2 different inputs.

Considering first the problem of determining the dynamic tire force experimentally, records of tire pressure versus time can be determined experimentally from the vehicle moving over the highway. From these records it is possible to calculate a pressure power spectrum as shown by the arrow (Fig. 3). The tire pressure is related to the tire force, and this relationship ( $F/P$ ) can be determined experimentally in the laboratory. The power spectrum of the dynamic tire force can be computed by multiplying the pressure power spectrum by the relationship between the tire force and the tire pressure (Fig. 3). As may be expected, certain assumptions are also involved in computing the pressure power spectrum. There is one large difference, however, between making this calculation and computing a power spectrum from elevation measurements. This difference arises because there are no long wavelengths present in the tire pressure measurements, such as in the elevation measurements; therefore, the pressure power spectrum can be computed with greater accuracy than the elevation power spectrum.

It is also possible to compute the dynamic tire force theoretically. If elevation measurements are made of the highway in question then it is possible to compute an elevation power spectrum. (It is this relationship that introduces the greatest error because of the problems inherent in this calculation.) If the relationship ( $F/X$ ) between tire force and tire displacement is known, it is possible to calculate the force power spectrum from the elevation power spectrum (Fig. 3). This force power spectrum should check with the one obtained from the experimental pressure measurements.

A comparison of these force power spectra thus provides a criterion whereby the accuracy of the elevation power spectrum can be judged.

Since the vehicle characteristics have been determined, it is possible to start with the tire pressure spectrum and determine an elevation power spectrum. First, the force power spectrum is obtained from the experimental tire pressure measurements. The vehicle characteristics in the form of the  $F/X$  ratio are then removed, and an elevation power spectrum results. This is the elevation power spectrum experienced by the car, and it should be the same as that calculated from the elevation measurements. A direct comparison can thus be made of the 2 elevation power spectra, and the accuracy of the spectrum computed from elevation measurements can be judged by using as a criterion the spectrum obtained from the vehicle performance.

The question may be raised whether the relationships between tire force and tire displacement and between tire force and tire pressure are affected by the speed of the vehicle. Laboratory tests indicate that the amplitude of the vehicle motion will influence these characteristics as will the magnitude of the applied forces. Thus these ratios, so important in the calculation of the desired power spectra, are themselves subject to change. This is to be expected since the vehicle suspension system is nonlinear. How much do these characteristics change, and are changes in one characteristic compensated by changes in another?

As an initial check on these questions, it was decided to run the test vehicle at different speeds on the same length of highway. In all cases the vehicle would be excited by the same highway profile and the elevation power spectrum should be the same at all speeds. Increasing the vehicle velocity would, however, increase the magnitude of the applied forces and hence at different speeds different tire pressure records would be obtained. To what extent would consistent behavior be observed on the part of the vehicle?

Figure 4 shows the results of this initial test, in which the vehicle was operated at speeds of 30, 40, 50, and 60 mph over the same length of highway. The curves indicate that the elevation power spectrum experienced by the vehicle is virtually the same at speeds from 40 to 60 mph, but at 30 mph there is some deviation. However, the close agreement in the results was gratifying because there were many possibilities for error. As a result of these tests it was decided to operate the vehicle in the range of 40 to 60 mph since a slight change in vehicle velocity would have relatively little effect on the resulting elevation power spectrum.

Appropriate highway sections were selected and elevation measurements were obtained. Elevation power spectra were then computed from these elevation measurements by using the 2 different techniques for obtaining elevation deviations previously

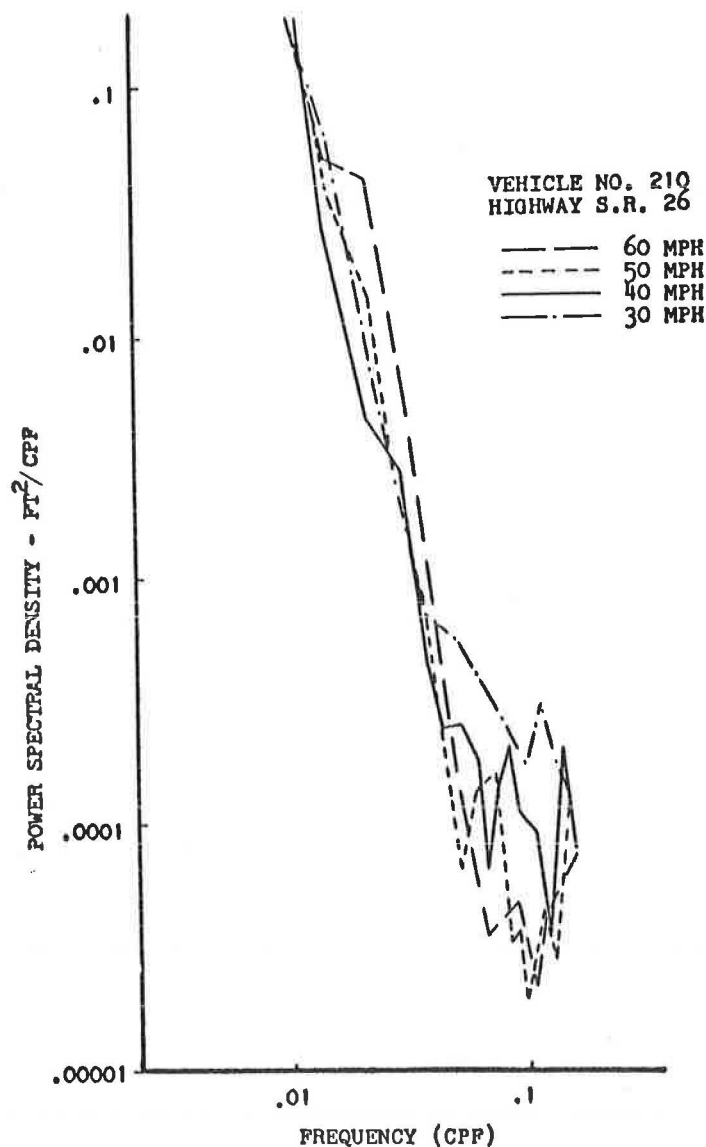


Figure 4. Elevation power spectra experienced by the vehicle at various speeds.

discussed. Typical results are shown in Figure 5 where the power spectrum curves indicate that the greatest amount of power is found in the long wavelengths. This means that long wavelengths in the highway provide the greatest excitation for producing vertical motion in the vehicle.

The curves (Fig. 5) indicate that different power spectra are obtained from the same set of elevation measurements if different procedures are used to obtain the elevation deviations. This is shown by the uppermost curve, for which the elevation deviations were obtained by merely subtracting the mean of all elevation measurements  $\bar{Y}$  from each value of elevation  $Y_i$ . (In other words the values of  $Z_i$  were used in place of the values for  $X_i$  in this calculation.) The curve shown by the solid line is the elevation power spectrum actually experienced by the vehicle, determined from the dynamic tire pressure measurements.

When the elevation power spectrum is calculated from the tire pressure measurements, large values for the power spectrum are also obtained (Fig. 5) at the longer wavelengths (low frequencies). This occurs because the ratio between tire force and tire

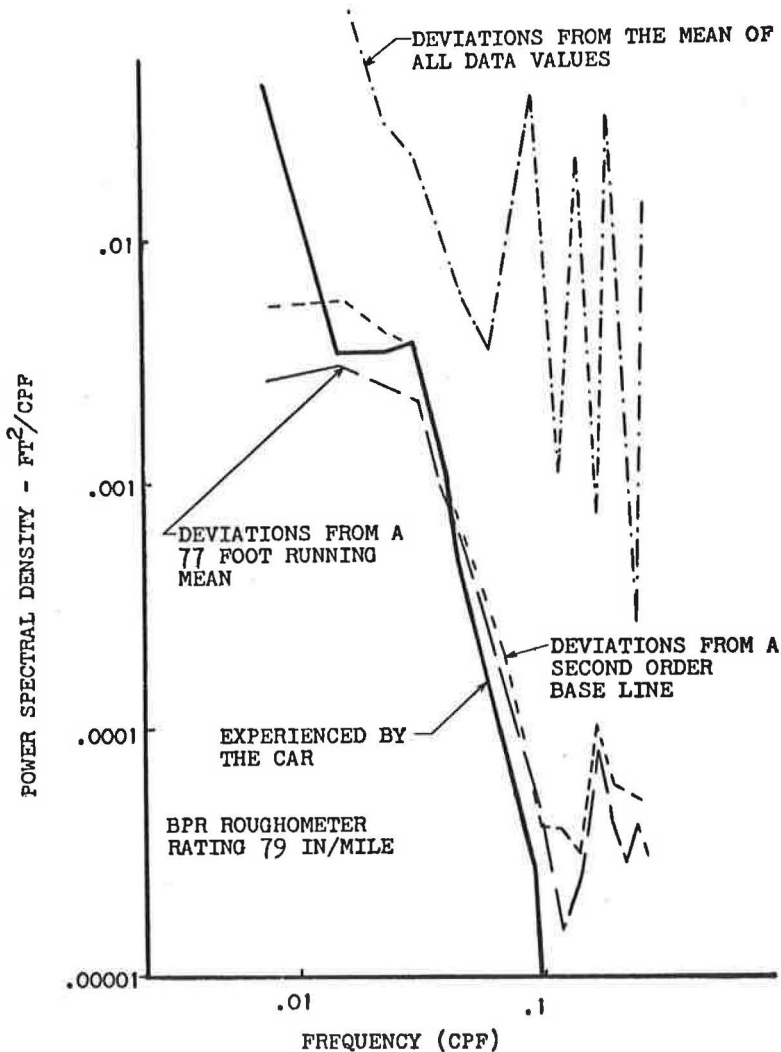


Figure 5. Elevation power spectra for U.S. 52.

displacement approaches zero as the frequency decreases. Hence, at low frequencies it is necessary to divide the ordinates of the dynamic tire force spectrum by values which are very close to zero. As a consequence the elevation power spectrum experienced by the car (obtained from dynamic tire force measurements) displays the same characteristics as the power spectra calculated from the elevation measurements. This is gratifying in terms of the trend shown, but it is difficult to compare numerical values in the low frequency region since each procedure is subject to limitations.

The power spectrum experienced by the car thus affords the criterion whereby the power spectra calculated from elevation measurements can be compared, but this comparison is confined chiefly to the higher frequency region. Since a large amount of power is found in the lower frequency region, this criterion leaves something to be desired. Nevertheless, it indicates that for the highway shown, deviations obtained from either the running mean or a second-order least squares base line yield a result close to that experienced by the vehicle. In addition, the power spectrum calculated

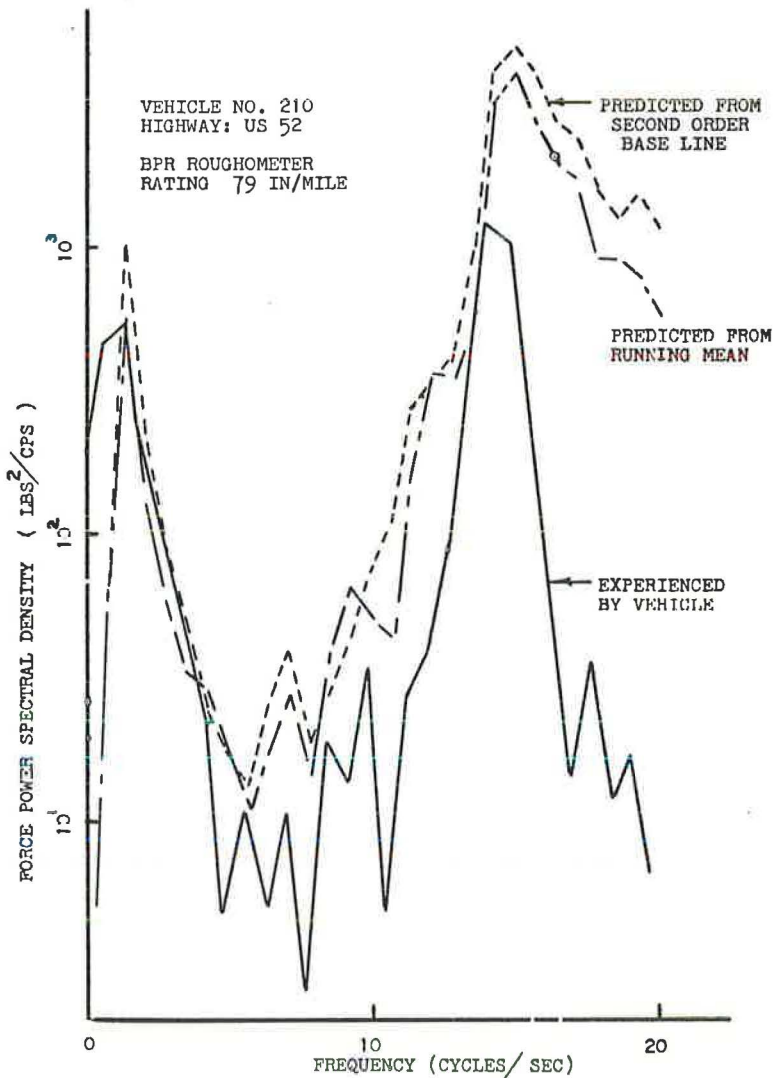


Figure 6. Comparison of force power spectra.

from the mean of all data values is based on a procedure which does not yield a highway characteristic comparable to that experienced by the vehicle.

Figure 5 also shows that at frequencies in excess of 0.1 cpf (wavelengths less than 10 ft) the power spectra calculated from elevation measurements overestimate the amplitudes of the existing undulations. This cannot be ignored, because certain vehicles are very sensitive to these wavelengths at normal operating speeds, and the use of these power spectra will result in predicted vehicle behavior that will be in excess of that encountered in actual operation.

One cause of this overestimation is the limited accuracy with which the elevation measurements were made; other factors in the calculation procedure also contribute to this error.

The existence of a large amount of power in the first frequency band is disturbing, and it is difficult to compare power spectra when the first band is included in the power calculation. It has been shown (3), however, that the vehicle is relatively insensitive to highway excitation at low frequencies, and hence a large amount of power in the highway at low frequencies does not necessarily result in a prediction of excessive forces generated by the vehicle. Keeping this in mind, it is instructive to examine the force power spectra computed from the elevation measurements and the vehicle characteristics ( $F/X$ ) compared with the force power spectrum determined experimentally from the tire pressure measurements (Fig. 6).

The force power spectrum, determined from the tire pressure measurements, indicates that 2 frequencies predominate in the vehicle behavior. The lower frequency at which appreciable forces are generated is a result of the motion of the sprung mass of the vehicle (body roll) and is indicated by the peak in the curve near 2 cps. The peak in the region of 14 cps indicates that large forces are induced that result from the motion of the unsprung mass of the vehicle (wheel hop).

The force power spectra predicted from both elevation power spectra indicate larger forces in the high frequency region than actually exist. This is a direct result of the higher values of the elevation power spectra computed from elevation measurements (Fig. 5).

Thus a highway elevation power spectrum, computed from elevation measurements, can be used with the proper vehicle characteristics to obtain the corresponding force power spectrum (Fig. 3). This can be checked with the force power spectrum determined from the tire pressure measurements, and this comparison can serve as a criterion of the accuracy of the elevation power spectrum obtained from elevation measurements.

A comparison of force power spectra, rather than of elevation power spectra, offers the advantage of curves that have a better-defined area in the low frequency region. In addition, the response of the vehicle serves to define the range of frequencies that is important, and in this way the range of wavelengths can be identified that is significant in the highway profile.

Although the results shown in Figures 5 and 6 pertain to only one highway, similar calculations have been made for other highways. In most instances the results have been very similar, and in all cases the predicted forces were larger than those measured experimentally (Fig. 6).

In general, these calculations can be made with greater accuracy for smooth highways than for very rough ones. Moreover, best results to date have been obtained for rigid and flexible highways, rather than for those of overlay construction.

## CONCLUSIONS

The elevation power spectrum of a highway can be determined from the performance of a vehicle as well as from elevation measurements. The performance of the vehicle was taken to be the dynamic tire force, but other vehicle performance characteristics such as stress or acceleration could also be used.

Results indicate that elevation deviations obtained from either second-order least squares base lines or from a running mean produce an elevation power spectrum from highway elevation measurements that is in many ways reasonably close to that experi-

enced by the vehicle. (Introducing the corresponding frequency domain filters will improve the results but will not resolve the basic problems encountered at both the high and low frequencies.) In both cases, however, the tendency is to overestimate the high frequency (short wavelength) effects. In the case of dynamic tire forces, the vehicle is very sensitive at these frequencies and hence an even greater overestimation of the forces results. This is partly due to the accuracy with which elevation measurements can be made, and partly to the procedures used in making the power spectrum calculations.

Therefore, it appears that more accurate characterizations of highway profiles can be obtained by observing the performance of a device with appropriate response characteristics as it moves over the highway profile in question than can be obtained from elevation measurements made under normal conditions. In checking the performance of such a device, however, the elevation measurements are very useful.

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