The Design of Supercritical Flow Channel Junctions

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The purpose of the paper is to discuss the diagonal wave and pile-up problems which result from the intersection of two open channels, each carrying water at supercritical velocities. Methods are set forth for the determination of the height of wall pile-up which occurs under conditions of design flow and under the unbalanced conditions of flow in only one of the channels. For the determination of pile-up height for these conditions it is necessary to refer to laboratory curves which are included.

A channel junction design is suggested which has minimized the wave problem in laboratory experiments. The junction differs from the simple channel junction in two ways. The main channel is widened at the junction to accommodate the increased discharge contributed by the side channel, and a baffle, which is actually only an extension of the main channel wall, is extended downstream in front of the side channel. This baffle is recommended to be tapered at a slope equal to the greater of the Froude numbers in the main or side channels, divided by 50. Smaller rates of taper are acceptable, but more pronounced tapers result in pile-up problems on the channel sides. This type of junction results in almost no wave problem under design conditions of flow in both channels and it results in low pile-up depths for conditions of flow only in one channel or the other.

•SUPERCRITICAL, or high velocity, flow differs from subcritical flow primarily because supercritical flow does not exhibit backwater effects. Disturbances created in subcritical flow are felt both upstream and downstream from the point of origin of the disturbance. However, in the domain of supercritical flow, disturbances do not travel upstream but can only travel downstream. Hydraulic engineers familiar with supercritical flow are well aware of the diagonal waves which propagate and reflect back and forth across the channel downstream from disturbances such as changes in alignment, transition sections, channel contractions, and channel expansions.

It is only natural for the hydraulic engineer to expect diagonal wave difficulties at, and downstream from, the junction of two supercritically flowing channels. Figure 1 shows a laboratory example of the nature of the diagonal wave pattern, which results from supercritical flow channel junctions, and the resulting wall pile-up and downstream wave propagation. The purpose of the research reported in this paper is to better define the diagonal wave difficulties and pile-up problems so evident in Figure 1, and to find ways to reduce or eliminate these wave problems.

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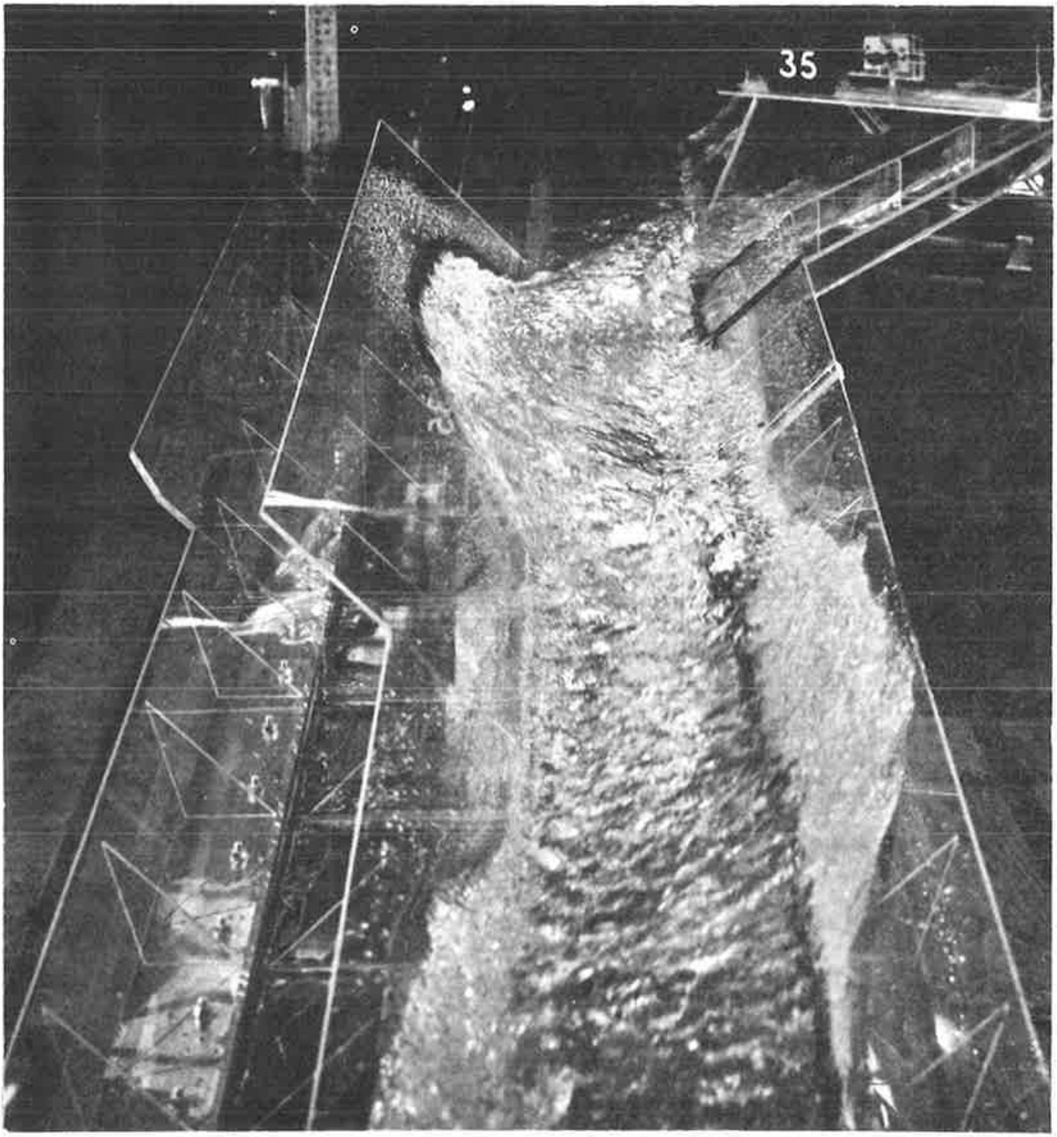

Figure 1. Wave problems created by a simple junction of supercritical flows in trapezoidal channels.

INTRODUCTION TO SUPERCRITICAL FLOW DISTURBANCE PHENOMENA

The following discussion is not intended to be comprehensive but is only intended to give the reader the necessary background to allow him to better understand the problems and the reasoning behind the solutions to the problems discussed in this paper.

The term "positive disturbance" will be applied to a disturbance which creates a wave of greater depth than that of the water feeding the wave. The type of positive disturbance which is most informative to consider is that shown in Figure 2. This type of phenomenon has been studied by Ippen (1), and by Ippen and Harleman (2). The factor which produces this disturbance is the channel's wall being deflected inward to-

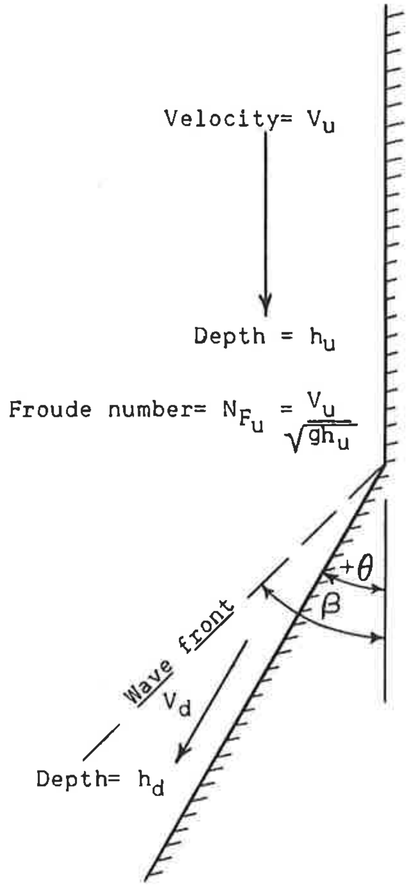

Figure 2. Effects of a simple, positive disturbance on supercritical flow.

ward the oncoming flow, resulting in a change in the direction of the velocity vector of the flowing water and an increase of depth. The velocity vector is deflected through the same angle as that of the wall deflection θ but the increase in depth occurs as the water passes through a diagonal wave which stands at an angle β to the projection of the original channel wall. The angle β is always greater than the angle θ. Though the wave is indicated as a sudden increase in depth, this wave is very abrupt for large values of β and may be quite gradual for small β values (2). The abrupt, vertical wave front (Fig. 2) simply forms a basis for discussion. Ippen has shown that the following equations relate the various parameters, defined in Figure 2, of this disturbance and the flow.

$$\sin \beta = \frac{1}{N_{F_u}} \sqrt{(1/2)\left(\frac{h_d}{h_u}\right)\left(1 + \frac{h_d}{h_u}\right)} \qquad (1)$$

and

$$\frac{h_d}{h_u} = \frac{\tan \beta}{\tan (\beta - \theta)} \qquad (2)$$

Simplification of these equations does not appear possible; however, they are presented in graphical form in Figure 3.

Depths of flow in the positive wave are often somewhat greater than the depth of flow in the region downstream from the wave.

An important aspect of positive disturbances is the resulting wave is transmitted in a straight line diagonally across the channel almost without change of shape. This wave can be directed as a concentrated disturbance in desired directions by controlling the wall angle θ. For a given Froude number, in the upstream zone, $N_{F_u} = V_u/(gh_u)^{1/2}$, however, the intensity h_d/h_u of the directed disturbance is determined by Eq. 2. So, for given upstream conditions, it is possible to change the direction of the diagonal wave by changing the wall angle θ but the disturbance intensity cannot be controlled independently of the wall angle.

The possibility of directing concentrated, positive disturbances at a desired point has great application in supercritical flow channel transitions from a wider to a narrower channel and in the area of open channel bends (1). This principle will be applied later in this paper.

A negative disturbance, in contrast to a positive disturbance, is created most simply by a change in channel wall alignment (Fig. 4). The wall deflection angle θ is negative, and the depth decreases downstream from the dashed line. However, the decrease in depth is sudden only at the break in the wall alignment.

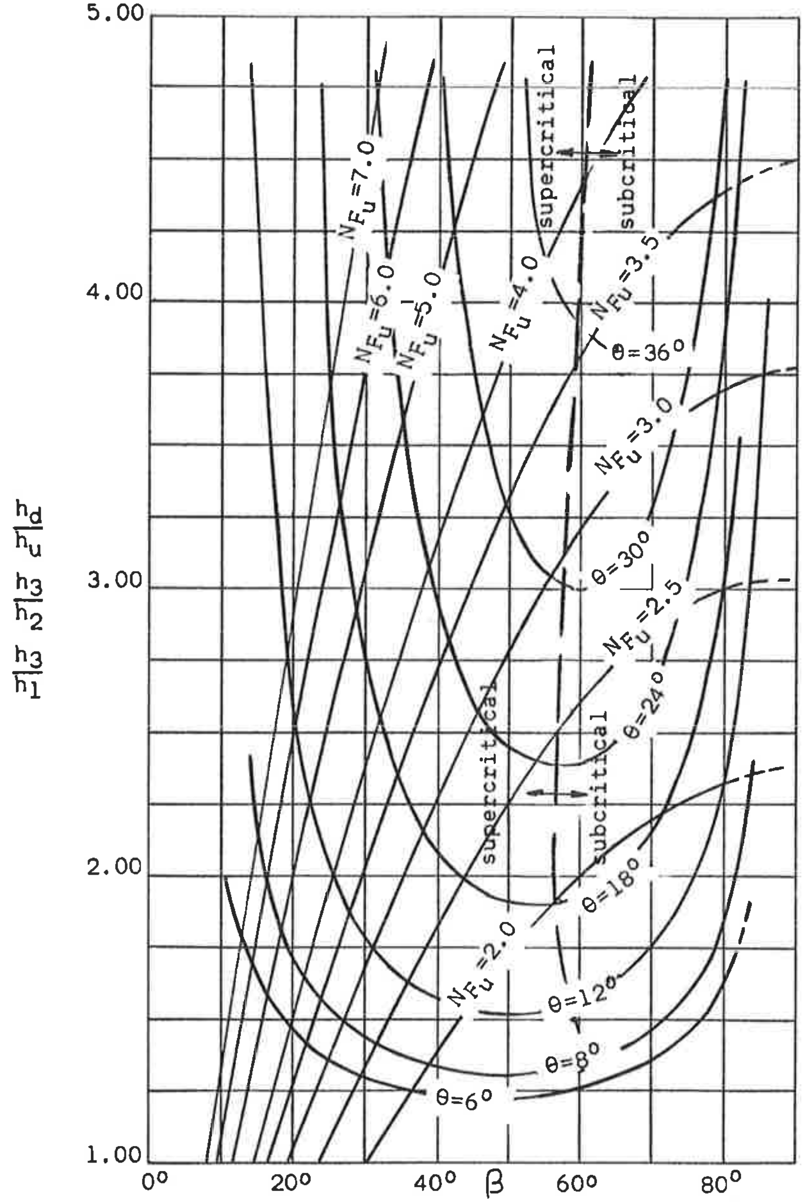

Figure 3. Plot showing relationships between θ, β, $h_{downstream}/h_{upstream}$, and upstream Froude number.

Farther out in the flow and farther downstream the change in depth is gradual. Rouse et al. (3) have investigated this at some length. The type of disturbance indicated in Figure 4 can result in a change in the direction of the velocity vector being more negative than the negative change in alignment of the wall. This can result in the creation of a positive reflection originating at some point along the wall downstream from the alignment change. This creation of a positive disturbance as a result of an upstream negative disturbance is one of the phenomena which works against the designer in many applications.

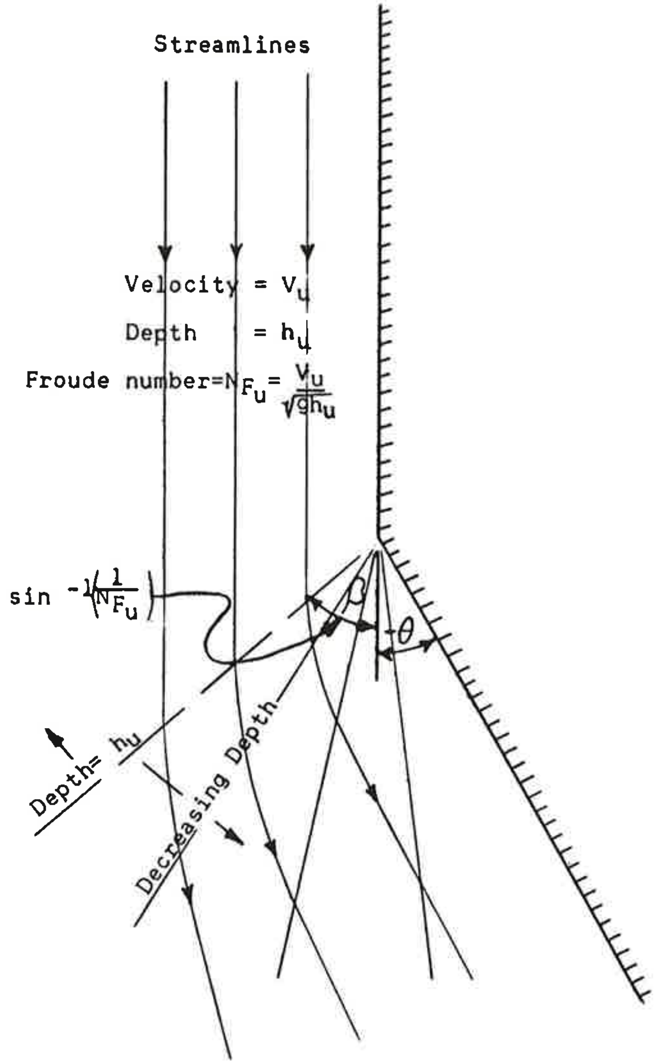

Figure 4. Effects of a simple negative disturbance on supercritical flow.

Probably the most important point to note about negative disturbances is that they are concentrated only at the point of origin, and they fan out from that point. It is not, therefore, possible to direct a negative disturbance in a concentrated form to some point away from its point of origin.

In the design of transitions, bends, and junctions, both negative and positive disturbances are created. Positive disturbances are usually directed at the point of origin of a negative disturbance in order to cancel the effects of both. However, attempts to direct a negative disturbance toward the point of origin of a positive disturbance result in only limited success.

THE LABORATORY APPARATUS

Because laboratory results will later be used in conjunction with the analysis to assist in design, and because the laboratory work paralleled and assisted in the determination of realistic assumptions to be made in the analysis, the discussion of the laboratory apparatus will precede the analysis.

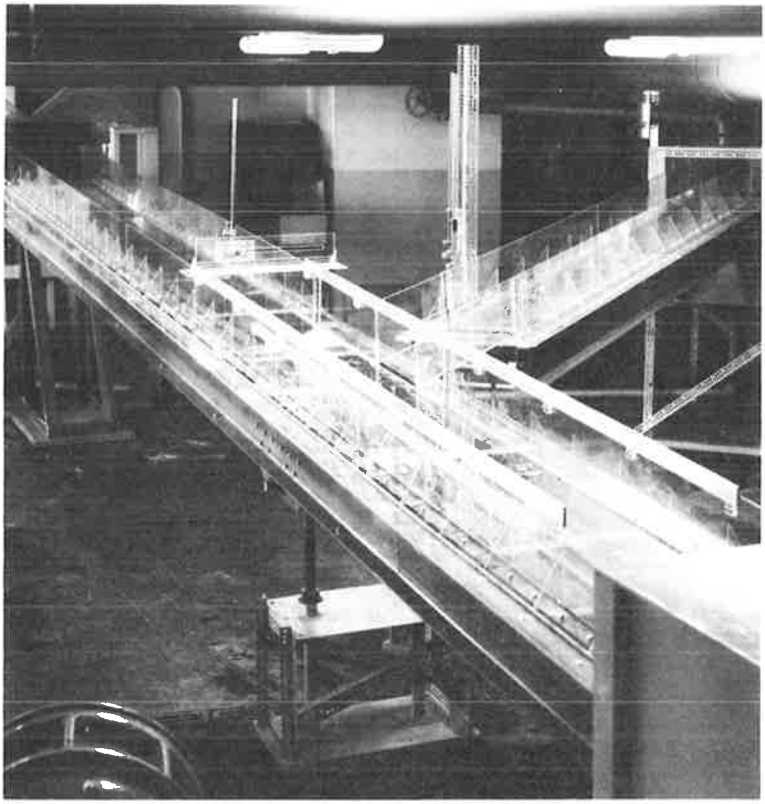

Figure 5. Laboratory channels and junction arranged for rectangular channels with a 45-deg angle of incidence.

The experiments were performed in the Oregon State University Hydraulics Laboratory. The laboratory apparatus, arranged for rectangular channel experiments, is shown in Figure 5. The main channel and the side channel were supplied with water from separate pumps. The main channel was 32 ft in length; its bottom width varied from 9 in. to a maximum of 18 in. for rectangular and trapezoidal channel experiments. The side channel varied from 18 to 22 ft in length, depending on the angle of incidence of the two channels; its width varied from a minimum of 6 in. to a maximum of 12 in. for the rectangular channel, and from a minimum bottom width of 6 in. to a maximum of 12 in. for the trapezoidal channel experiments. For the trapezoidal channel experiments the side slopes in both channels were 1:1. No attempt was made to consider other side slopes.

The slope of each of the channels could be varied independently. It was, thus, possible to vary the velocity and Froude number as well as the depth of flow in each channel independently.

The discrete angles of incidence α between the two channels were 15, 30, and 45 deg. It was felt that greater angles of incidence would be completely unrealistic.

Arbitrarily, the junction of the two channel inverts, for situations where the two channels combined with common invert elevations, was made along the line of intersection of the plane of the invert of the main channel with the plane of the side of the main channel. This required a warping of the sides and bottom of the side channel near the junction for all experiments. This was generally small, however, and did not greatly affect the depth of flow from side to side of the side channel at the point of juncture with the main channel.

Water was introduced from the laboratory piping system into box-type collectors from which the water entered the two channels. The flow distance between the upstream end of each channel and the junction appeared sufficient to create essentially uniform flow except as affected by the warping of the side channel in the vicinity of the junction.

Froude numbers for the flow in each of the channels varied from approximately 2 to approximately 7. Depths varied from 0 to approximately 0.2 ft. Velocities ranged up to approximately 10 ft/sec.

The discharge into the channels was measured by venturi and orifice meters. The depths of flow in the approaches to the junction were measured by piezometers in the bottom of each channel. Depths of flow in the disturbed sections and wall pile-up heights were measured with point gages from the top of the channel. The depth of flow in the side channel was measured just upstream from the entrance to the short warped section which terminated at the junction.

ANALYSIS AND LABORATORY RESULTS FOR SIMPLE JUNCTIONS

A diagram of a simple rectangular channel junction is shown in Figure 6. The term "simple" is applied here to mean that the two channels have common invert elevations and there are no baffles or piers of any kind to deflect the flows at the junction. Also, the main channel width is not increased downstream from a simple junction.

Figure 7 is a definition sketch of depths, Froude numbers, angles, etc., for flow conditions through a simple junction of rectangular channels.

Since the designer is usually faced with the problem of how much wall pile-up will occur and where it will occur for junctions of this type, the following discussion is aimed at a solution to this problem.

Observation indicates that the depth of flow immediately downstream from each of the diagonal jumps (Fig. 7) is the same. The depth of flow in this wedge-shaped zone is indicated as h_3, and the depths of flow in the main and side channels as h_1 and h_2. Since observation and intuition indicate that the two diagonal waves produce identical downstream depths, h_3, analysis of the problem is facilitated by the above assumption.

It should be observed that the energy level is not the same throughout the entire wedge-shaped wave area. The velocities in the main and side channels prior to the junction are probably not the same. This means that in the wedge-shaped zone the specific energy ($h + V^2/2g$) is different for water which had its origin in the main channel than for water which originated in the side channel. This difference in velocity may create an area of turbulent shear between the two velocity zones, extending diagonally across the channel in the wedge area (Fig. 8).

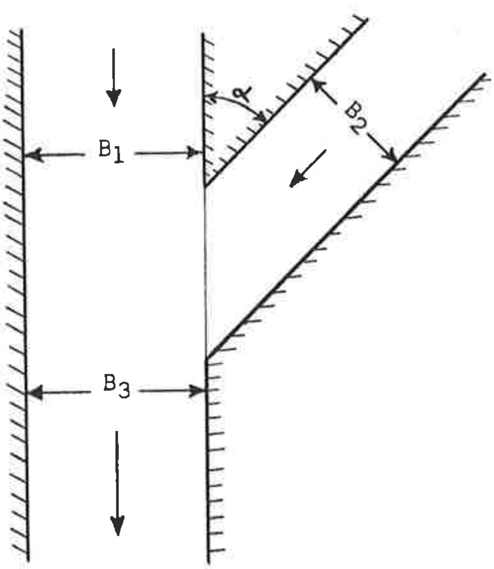

Figure 6. Plan view of simple junction of rectangular channels.

For analysis, it has been assumed that the effect of the side channel flow on the main channel flow is the same as the positive disturbance, wall effect previously described (Fig. 2). It is also assumed that the flow in the main channel has a similar effect on the flow in the side channel. The gravity effects of the channel slopes are neglected.

The wave angles β_1 and β_2 of Figure 7, can be determined, subject to the assumptions above, by combining Eqs. 1 and 2, written for each channel, with the facts that the value of h_3 is the same for the main channel water as it is for the side

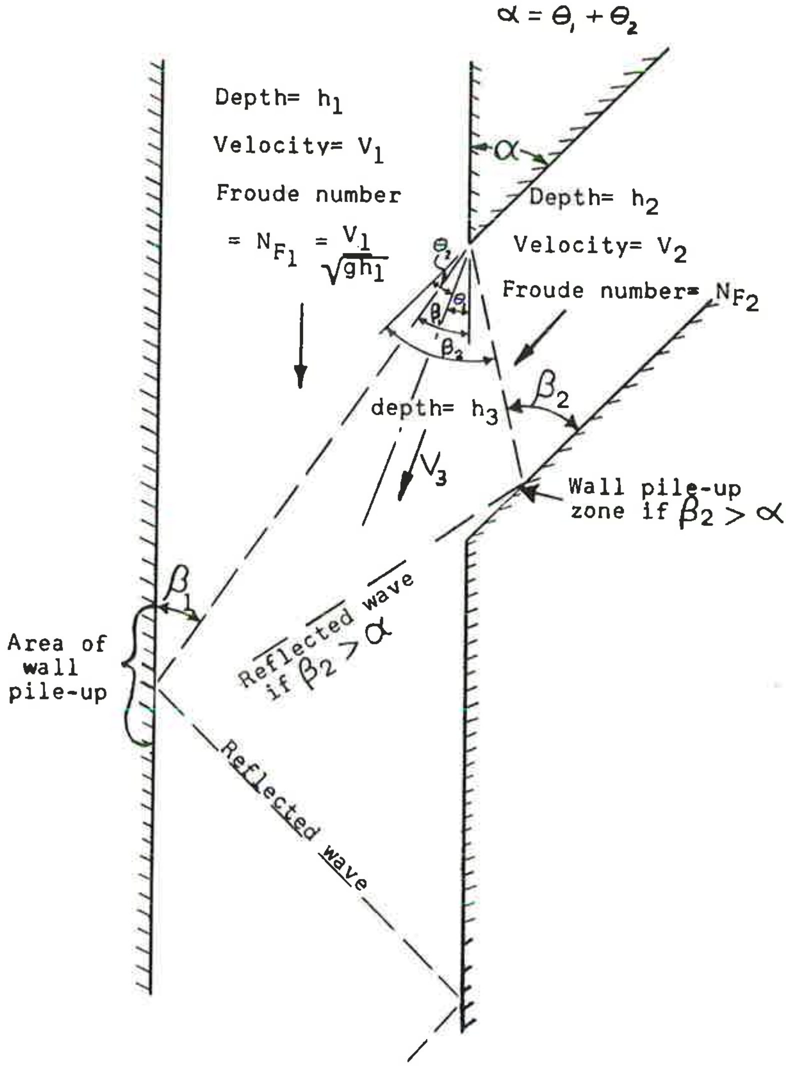

Figure 7. Definition sketch of a simple junction of rectangular channels.

channel and the sum of the two θ angles must equal the angle of incidence of the two channels. The equations are as follows:

$$\sin \beta_1 = \frac{1}{N_{F_1}} \sqrt{(1/2)\left(\frac{h_3}{h_1}\right)\left(1 + \frac{h_3}{h_1}\right)} \qquad (3)$$

$$\sin \beta_2 = \frac{1}{N_{F_2}} \sqrt{(1/2)\left(\frac{h_3}{h_2}\right)\left(1 + \frac{h_3}{h_2}\right)} \qquad (4)$$

$$\frac{h_3}{h_1} = \frac{\tan \beta_1}{\tan (\beta_1 - \theta_1)} \qquad (5)$$

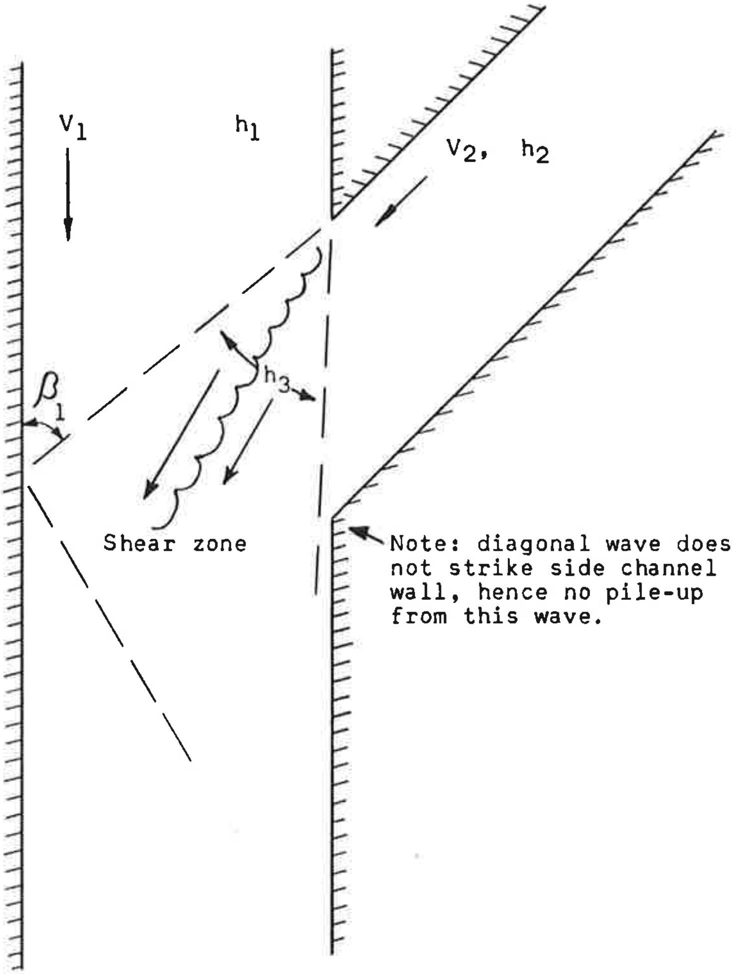

Figure 8. Shear zone between different velocity levels in the h_3 depth zone.

$$\frac{h_3}{h_2} = \frac{\tan \beta_2}{\tan(\beta_2 - \theta_2)} \tag{6}$$

$$\frac{h_3}{h_1} \div \frac{h_3}{h_2} = \frac{h_2}{h_1} \tag{7}$$

$$\theta_1 + \theta_2 = \alpha \tag{8}$$

It must be remembered that the designer would know the depths and velocities of approach for each of the channels and would know a proposed angle of incidence (which he may desire to change). The satisfaction of Eqs. 3 through 8 can be rapidly accomplished by trial and error use of Figure 3.

The wall pile-up which results from the reflection of the diagonal waves off the wall of the main channel or off the wall of the side channel is the thing of interest to the

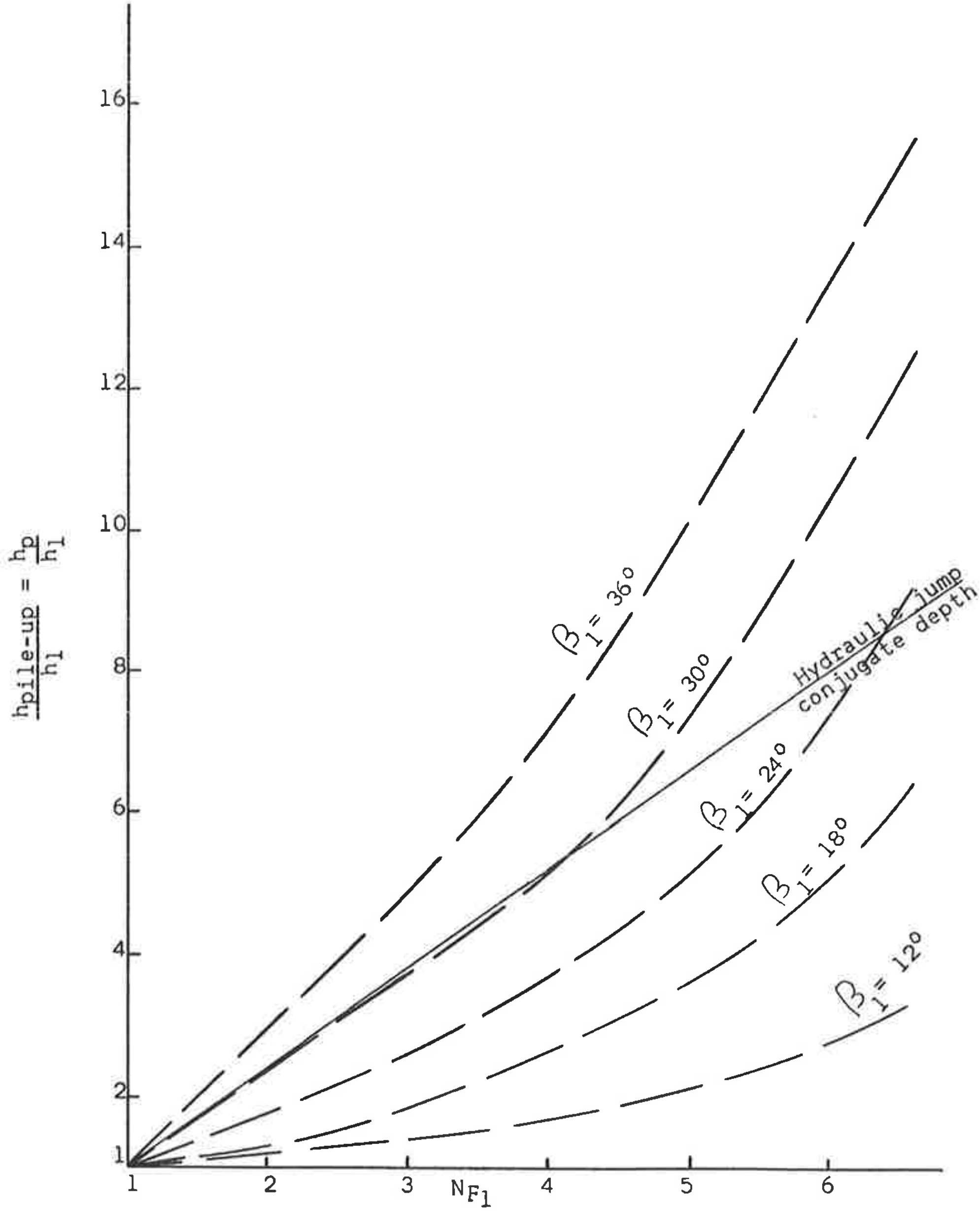

Figure 9. Experimental wall pile-up as a function of upstream depth and Froude number for $\beta_2 < \alpha$.

designer. However, the magnitude of the pile-up is a three-dimensional problem of analysis which has eluded the authors. The determination of the pile-up depth was, therefore, done in the laboratory. The results of the laboratory experiments for wall pile-up are given in Figure 9. The solid line indicates the depth which would result downstream from an ordinary hydraulic jump whose β angle is 90 deg. This is only shown to indicate the relative depths of pile-up possible.

It must be made clear that the plot of Figure 9 is only applicable under conditions where β_2 is less than α. If β_2 is greater than α, the diagonal wave of the side channel flow piles up on the side of the side channel and reflects out across the h_3 area. If it

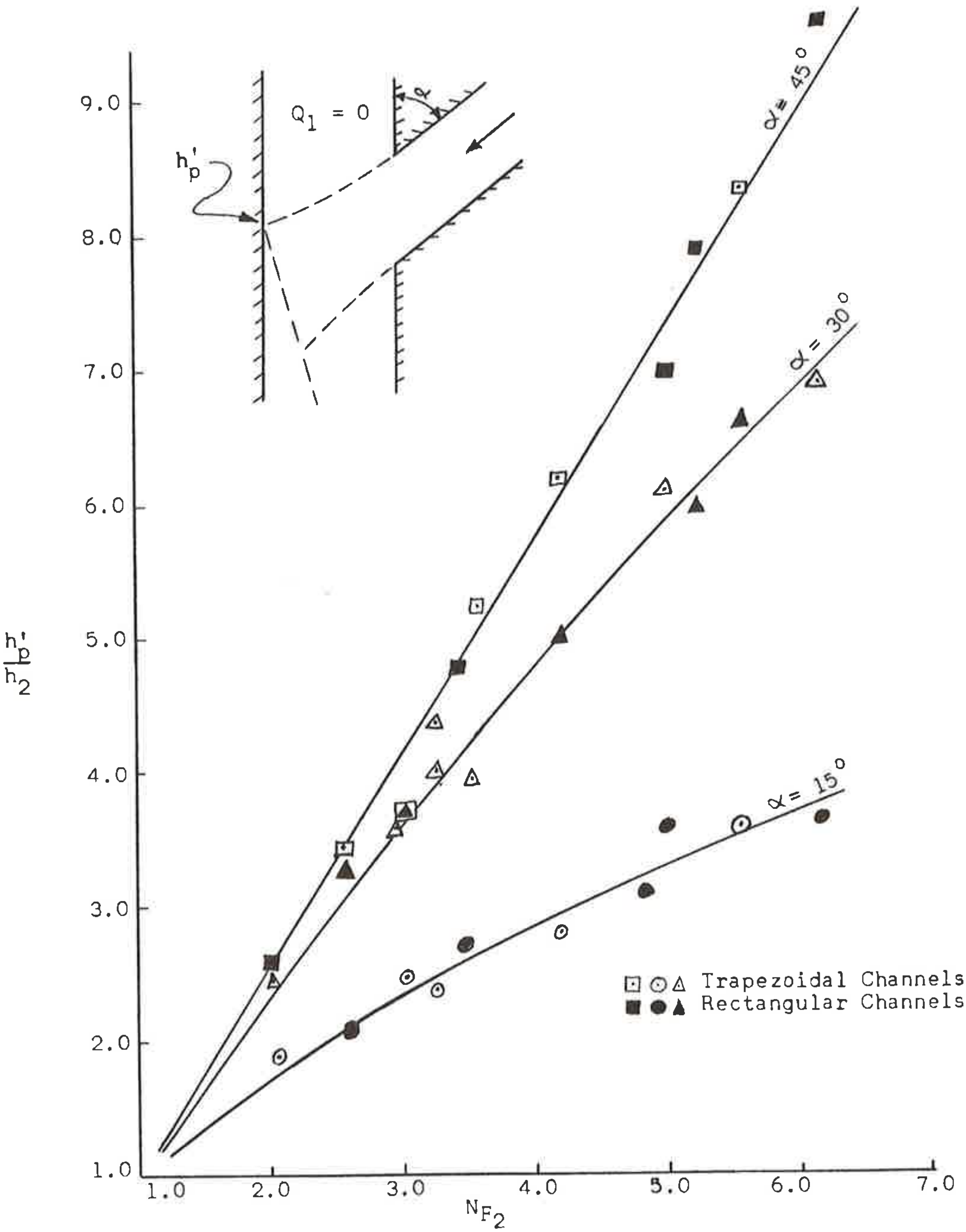

Figure 10. Relative pile-up on main channel wall resulting from flow in side channel only.

is found that β_2 is greater than α, the designer should make appropriate changes because it can be seen from Figure 9 that the pile-up on the wall of the side channel, indicated in Figure 7, can be quite appreciable for most values of $\beta_2 > \alpha$.

The best design of simple channel junctions is one in which β_2 just equals or is slightly smaller than α. This accomplishes the dual purpose of eliminating the pile-up problem in the side channel and it directs a positive wave at the beginning of a negative disturbance, thus eliminating the difficulties introduced by the negative disturbance.

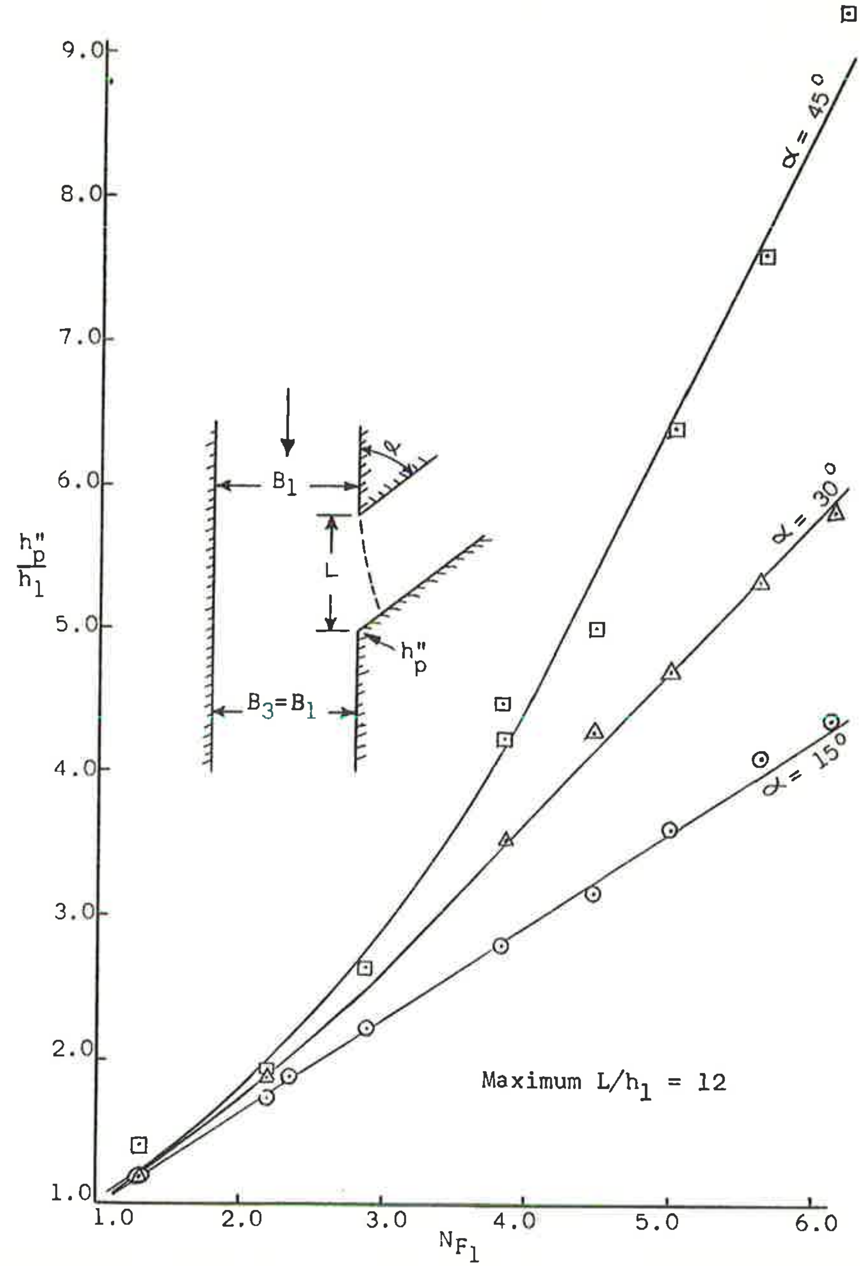

Figure 11. Pile-up at downstream corner of side channel for simple rectangular channel junctions when $Q_2 = 0$.

Experience in the laboratory with sloping channels has indicated that the angles are usually a few degrees less than those predicted by the analysis. This discrepancy, however, is small and it leads to conservative design.

With the value of β_1 known, Figure 9 can be entered to determine the wall pile-up height on the side of the main channel. The highest part of the pile-up occurs actually somewhat downstream from the point indicated by the angle β_1, and the wave is reflected

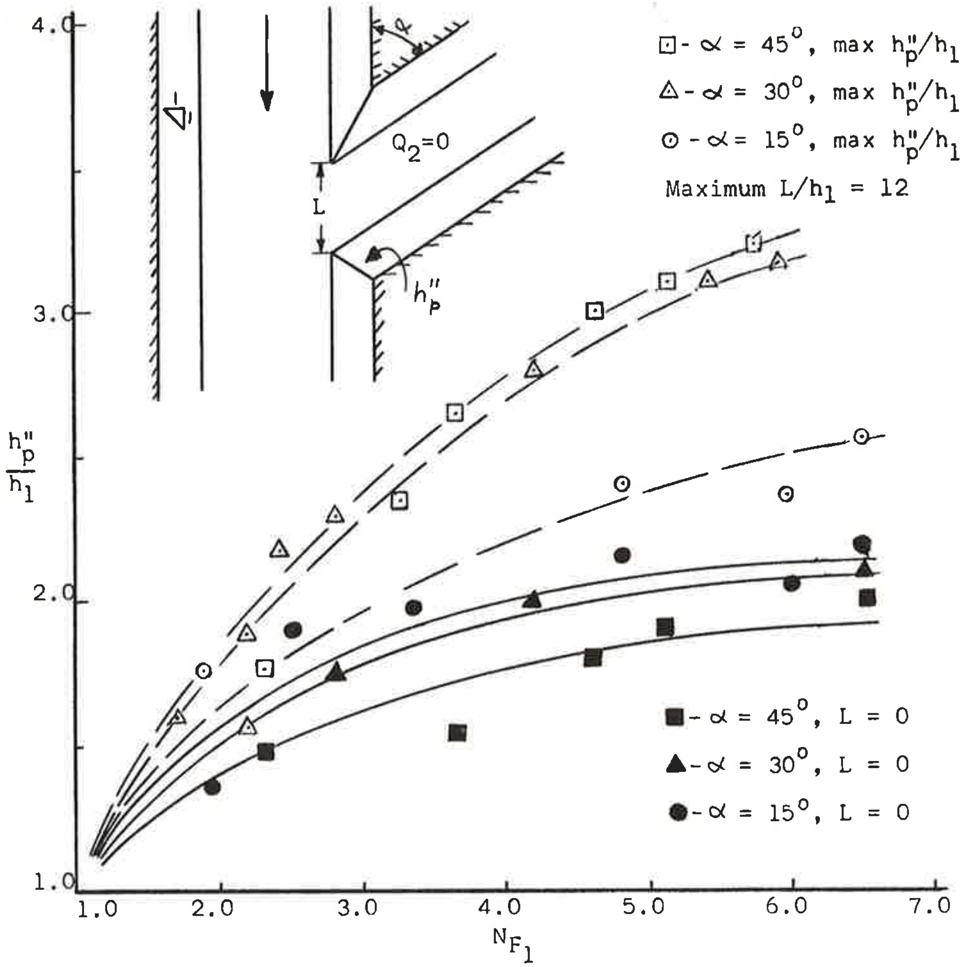

Figure 12. Plot of relative pile-up in trapezoidal channel junctions when $Q_2 = 0$; dashed lines indicate maximum observed h''_p/h_1 for all values of L studied; solid lines represent h''_p/h_1 for L = 0 (triangular side channel).

from side to side in the main channel, attenuating quite slowly. It is, therefore, necessary to design the main channel walls high enough to handle this first pile-up and subsequent reflections.

If the angle β_2 is less than the angle of incidence of the two channels, the diagonal wave, through which the side channel flow passes, does not intersect the wall of the side channel but passes harmlessly out into the main channel.

The designer must bear in mind that flows other than the design flows may occur in the channels. Laboratory experience indicates that conditions of zero flow in one channel and design flow in the other channel can create severe pile-up conditions. This can be quite serious if the junction has been designed to provide no pile-up in the side channel (β_2 less than α), because with no flow in the side channel and design flow in the main channel a good deal of pile-up may occur in the side channel.

Laboratory results of wall pile-up in the main channel resulting from flow from the side channel alone are summarized in Figure 10 for rectangular channels as well as for trapezoidal channels of 1:1 side slopes. Laboratory results indicating side channel wall pile-up with no flow in the side channel are shown by Figures 11 and 12.

Proper design considerations would include an investigation of design flow in both channels plus the possibilities of no flow in each channel while there is design flow in the other.

If the Froude numbers in the channels are small, there is the possibility of obtaining subcritical flow in the junction. In some cases this may be desirable because the diagonal waves extend across the channels at large angles and the supercritical flow problems do not exist. This point will not be discussed further here because this is a discussion of supercritical flow channel junctions.

CHANNEL JUNCTIONS WHICH MINIMIZE WALL PILE-UP PROBLEMS

Thus far, only the simple junction has been discussed. This type of junction is fraught with diagonal wave and pile-up problems which cannot be tolerated in many applications. The laboratory experiments of this study pointed the way toward what the authors feel is a rather complete solution to the pile-up problem. The principles involved in this solution are: (a) flow in the main channel should not be disturbed by

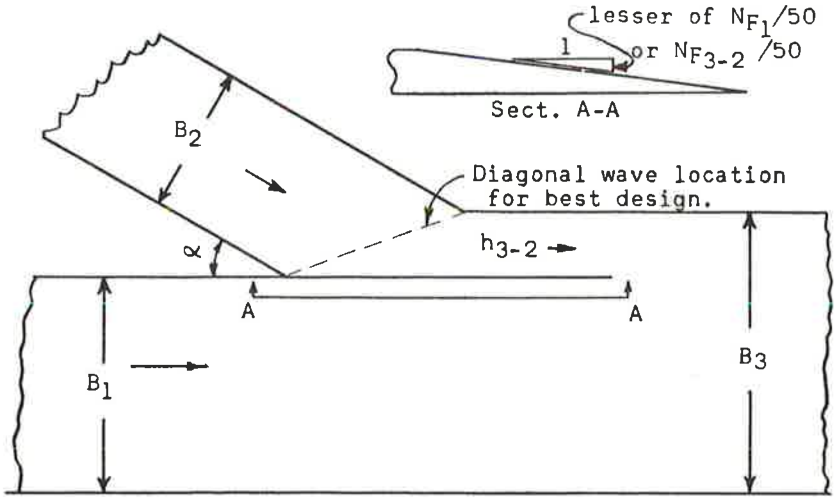

Figure 13. Sketch of improved rectangular channels junction.

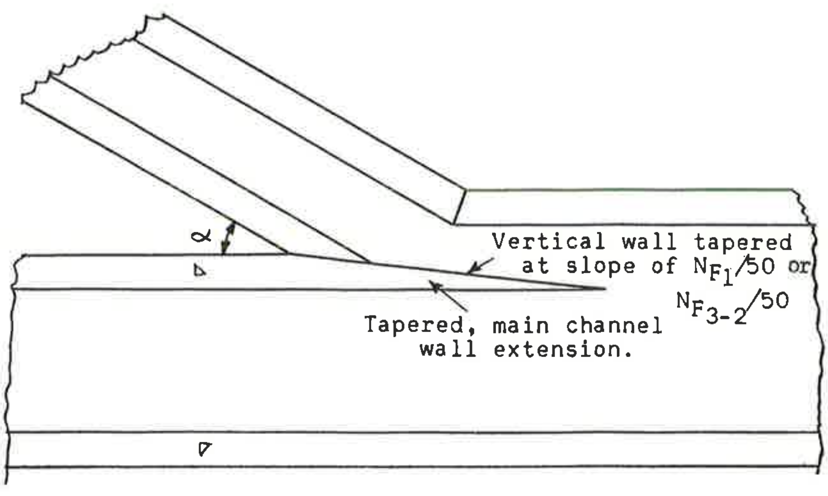

Figure 14. Sketch of improved trapezoidal channels junction; note that vertical wall has a vertical slope of the lesser of $N_{F1}/50$ or $N_{F3-2}/50$.

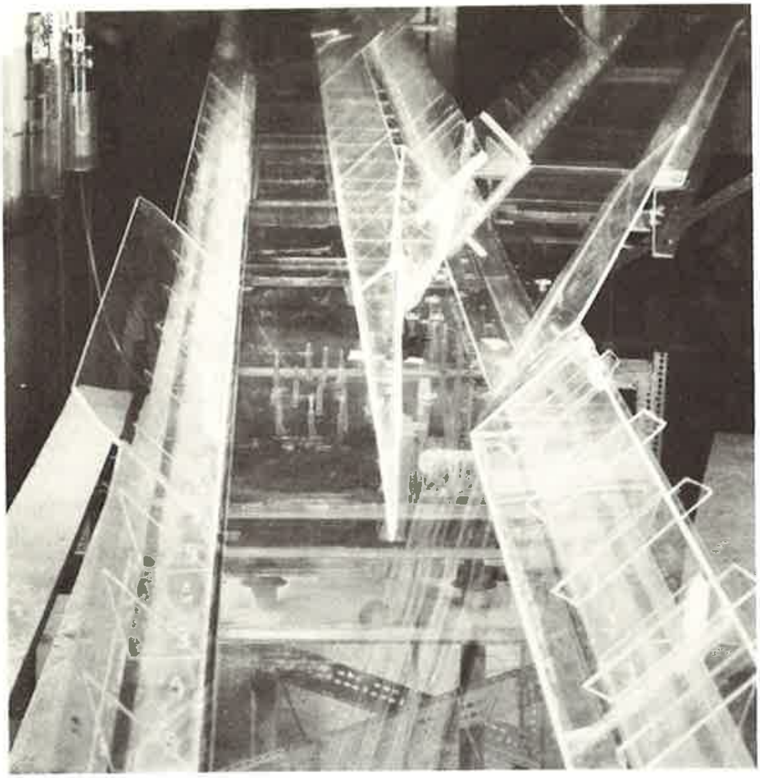

Figure 15. Improved junction of trapezoidal channels.

changes in direction of any kind, and (b) the direction of flow of the fluid entering from the side channel should be changed to that of the direction of the main channel without affecting the main channel flow.

An example of the proposed solution to the problem of wall pile-up is shown for a 15 deg, trapezoidal channel junction in Figures 13, 14, and 15. The salient features of the junction are as follows: the main channel is widened downstream from the junction to accommodate the sum of the main and side channel discharges; the side of the main channel on the side at which the side channel enters is gradually tapered in height but kept in the plane of the side of the main channel upstream; and the side channel flow is turned by means of a vertical, tapered wall placed parallel to the sides of the main channel for a rectangular channel or almost parallel for a trapezoidal main channel. One purpose of this tapered wall baffle arrangement is to remove the wall from the side of the main channel water gradually thus creating a gradual negative disturbance when the main channel flow is deeper than that entering beside it from the side channel (Fig. 16). With little or no side channel flow, gradual taper results in a gradual spilling of the main channel water against the widened side of the main channel just downstream from the side channel opening. This gradual spilling prevents a pile-up of any consequence on the main channel wall. With little or no main channel flow (Fig. 17), the side channel flow spreads in the main channel, but it enters the main channel in a downstream, rather than a cross channel, direction and does not pile-up on the main channel wall to a depth greater than the depth of design flow in the main channel.

With design flow in both channels (Fig. 18), the vertical wall still diverts the side channel flow into the direction of the main channel flow and the water surfaces of the main and side channel flows are also then matched. The side channel flow plus the tapered channel wall produce, then, the same hydrostatic pressure against the main channel flow as that of the channel wall just upstream from the junction. The main channel flow then experiences only a reduction in wall friction but no negative disturbance as it flows

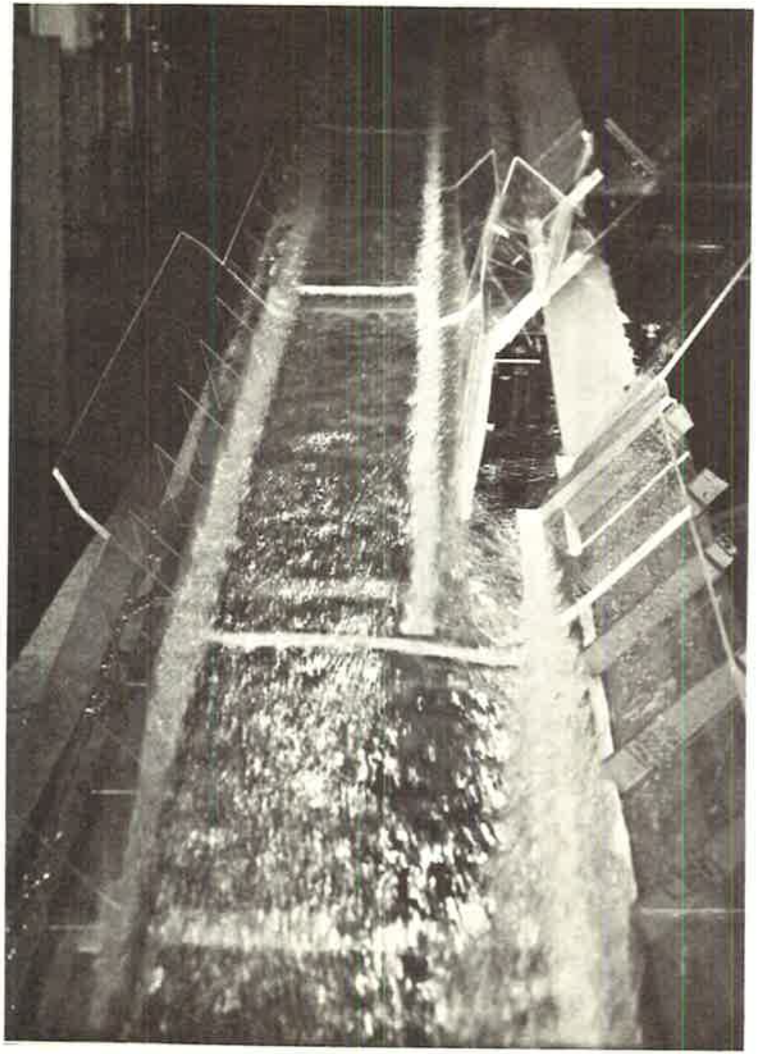

Figure 16. Flow from the main channel in an improved channel junction of trapezoidal channels; $N_{F_1} = 3.3$.

Figure 17. Flow from the side channel only, in the improved junction of Figure 16; $N_{F_2} = 2.6$.

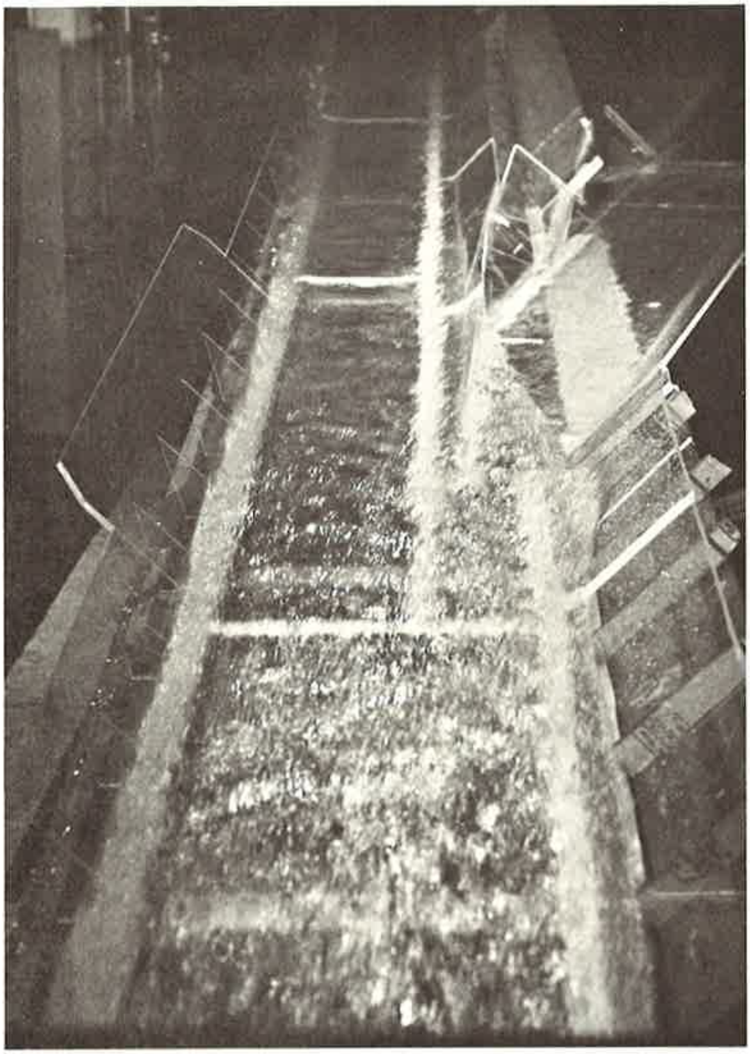

Figure 18. Design flows from both channels, in the improved junction of Figures 16 and 17; $N_{F_1} = 3.3$ and $N_{F_2} = 2.6$.

downstream. With no positive or negative disturbances impinging on the main channel flow, diagonal waves are not produced in this channel and the problem is eliminated.

In the laboratory experiments, the difference in velocity between the former side channel flow and the original main channel flow as the two begin to flow side by side in the new main channel section did not appear to create any kind of disturbance.

Though it was the original intent of the study to determine a solution to the diagonal wave problem only for rectangular channels, the solution found works equally well for trapezoidal and rectangular channels. The Froude number for trapezoidal channels, however, is defined as $V/(gh_m)^{1/2}$, where h_m is the mean depth in the channel defined by the ratio of channel cross-sectional area to the width of the channel at the water surface. With this change, Figures 3 and 9 still yield reasonable results.

Laboratory results indicate the following criteria should be met if the channel junction is to eliminate diagonal wave problems:

1. The channel downstream from the junction must be enlarged so that the depth of flow in the downstream channel is the same as in the main channel preceding the junction.

2. At design flow, the water surface of the flow from the side channel where it merges with the main channel flow must be at the same elevation as the main channel flow. If the water from the side channel is deeper than the main channel flow, a positive disturbance will be sent out into the main channel. If the water entering from the side channel is not as deep as in the main channel, there will be a negative disturbance created in the main channel flow. This ultimately results in a pile-up on the channel wall downstream from the junction.

3. If the flow in the side channel approaches the junction at an abrupt angle of incidence, as indicated in Figure 5, the diagonal wave required to change the direction of the side channel flow, must not strike the side channel wall, but should just strike the point of intersection of the side channel wall and the enlarged main channel wall. This eliminates wave problems on the wall of the side channel, and downstream in the main channel, from this source. However, since Froude numbers can seldom be predicted with complete accuracy in advance, it is wise to design according to the methods indicated here and then make provision for some pile-up along the downstream end of the side channel near the point of its intersection with the enlarged main channel.

4. Laboratory experiments indicate that the main channel on the side channel side of the junction, should be tapered as it extends from the upstream side of the junction, with a slope of no less than $N_{F_1}/50$ or $N_{F_{3-2}}/50$, which ever yields the smaller rate of taper to be most effective. $N_{F_{3-2}}$ is the Froude number of the flow from the side channel just downstream from the diagonal wave which forces that flow to change its direction to that of the main channel flow. $N_{F_{3-2}}$ instead of N_{F_2} enters the baffle design because the side channel flow which actually contacts the baffle is that which has changed direction and Froude number. $N_{F_{3-2}}$ is determined with the aid of Figure 3 and

$$N_{F_{3-2}} = \frac{h_2 B_2 \sin \beta_2}{h_1 (g h_{3-2})^{1/2} \sin (\beta_2 - \theta_2)} \tag{9}$$

which is derived simply from the continuity equation assuming the design depth of the side channel flow at the baffle to be the same as the design main channel depth, h_1.

If the preceding criteria are met, our laboratory experience indicates that the diagonal wave and pile-up problems are minimized or eliminated for the design flow in the channels. For less than design flow, diagonal waves exist, but they do not produce pile-ups which are greater than the depths of flow encountered for design flows.

The following design problem indicates that for given conditions in the main channel, the side channel must be designed to create the proper results. It is not possible to fit all combinations of side and main channel conditions to the criteria already enumerated.

Given:

Rectangular channels

$B_1 = 20$ ft, $h_1 = 5$ ft, $V_1 = 40$ ft/sec
$B_2 = 10$ ft, $h_2 = 2$ ft, $V_2 = 30$ ft/sec

Required:

Determine junction dimensions to minimize diagonal wave problems.

Solution:

$$N_{F_1} = \frac{V_1}{\sqrt{g h_1}} = \frac{40}{12.7} = 3.16$$

$$N_{F_2} = \frac{V_2}{\sqrt{gh_2}} = \frac{30}{8.02} = 3.74$$

Since the depth in the main channel must be maintained at 5 ft, h_3/h_2 for the side channel flow is 5/2. Entering Figure 3 with $N_{F_u} = 3.74$ and the depth ratio, $h_3/h_2 = 5/2$, $\beta_2 = 34$ deg and $\theta_2 = 19$ deg. Since only the side channel flow changes direction, $\alpha = \phi_2 = 19$ deg. $N_{F_{3-2}}$ is determined by means of Eq. 9.

$$N_{F_{3\ 2}} = \frac{h_2 V_2 \sin \beta_2}{h_1 (gh_{3-2})^{1/2} \sin (\beta_2 - \theta_2)} = \frac{(2)(30)(0.559)}{(5)(12.7)(0.259)} = 2.04$$

The taper of the main channel wall through the junction is sloped at the smaller of $N_{F_1}/50 = 3.16/50 = 0.0632$, or $N_{F_{3-2}}/50 = 2.04/50 = 0.0408$. The tapered side then is sloped at 0.0408 ft/ft, and, allowing one foot of freeboard, the length of the baffle is $(5 + 1)/0.0408 = 145$ ft long.

The downstream width of the main channel is

$$20 + \frac{10 \sin (34^\circ - 19^\circ)}{\sin 34^\circ} = 24.62 \text{ ft, say } 24.6 \text{ ft.}$$

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