

# Physical Properties of Granular Materials with Reference to Thermal Resistivity

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Various approaches to the engineering aspects of the problem of thermal resistivity of backfill materials for underground power cable systems are described. A study is made of the effect on the thermal resistivity of the percentage and the nature of binder added to a Rothfuchs graded granular skeleton of quartzitic sand and gravel. Addition of kaolinite in percentages up to 16 percent shows that the optimum amount is in the vicinity of 8 percent. The use of calcium carbonate, fly ash, lime and lime-fly ash as binders in an amount of 8 percent shows that the nature of the binder has little effect on the thermal resistivity of the cemented backfill material in the nearly dry state, as the value of this resistivity is controlled chiefly by the oriented water films. However, the nature of the binder becomes important as its amount increases to 16 percent and more.

The compressive strength of a nearly dry cemented backfill material containing 8 percent binder is found by experiment to increase linearly with its thermal conductivity, provided the binder does not react chemically with the quartz grains (as lime does). This experimental relation agrees with hypothetical relations derived on the basis of Debye's equation for the thermal conductivity in terms of the speed of sound. The derivation assumes a linear relation between the speed of sound in a generalized cemented granular material and its compressive strength, as has been previously established experimentally in the particular case of concrete.

A study was made of the effect on thermal resistivity of the method by which a given moisture content of a backfill material is attained, whether from the dry side or the wet side. It is shown that a material dried out from the saturated to the air-dry state has a considerably lower thermal resistivity than if it were simply mixed and densified in the air-dry state. When the material dries out, the thermal resistivity increases linearly as the logarithm of the moisture content decreases. However, when moisture is added to the dry material, the thermal conductivity increases linearly as the logarithm of the moisture content increases.

A comprehensive equation for the thermal conductivity of a generalized soil is developed to include the effect of moisture migration in both the film and the vapor phase.

•THE SCIENTIFIC and engineering study presented in this paper was occasioned by the problem faced by the electric power industry of developing economical backfill materials possessing dependable low thermal resistivities even at low moisture contents. The design of a suitable material composition or system of high dependable thermal conductivity or low resistivity involves the determination and selection of the best available constituents and their most effective combination with respect to mineral composition and granulometry. The effective and economical densification of the material with available equipment necessitates a consideration of such mechanical properties as workability and the related property of shear strength. The total engineering and scientific

problem thus involves practically all the physical properties of such backfill materials and the mutual interrelation of these properties.

The mechanical properties of granular materials were considered in a previous paper by Farouki and Winterkorn (6). The present paper contains a study of the thermal properties and of the relations between thermal and mechanical properties. It also includes the development of a comprehensive equation for the thermal conductivity of a soil, which takes into account the phenomenon of moisture migration.

### THERMAL PROPERTIES OF GRANULAR MATERIALS

If a backfill material is to have a good thermal conductivity, it should possess a continuous granular skeleton composed of quartz grains, because quartz is the common rock mineral having the best thermal conductivity. A high density of the material must be easily obtainable, which means that the material should be well-graded. To prevent segregation of the different grain sizes during handling and densification, a small amount of binder, such as clay, should be added. Since this binder usually has a lower intrinsic thermal conductivity than the quartz grains, there is an optimum percentage of binder. Additional amounts would only result in interference with the packing of the granular skeleton, and would therefore cause a decrease in the thermal conductivity. A series of experiments was performed to determine the optimum percentage of binder. Another series of experiments was performed using different binders to determine the effect of the nature of the binder. In both series the binder was added to the same type of granular skeleton which consisted of a quartz granular material made up according to the Rothfuchs (22) gradation to obtain a high density at a good workability.

The thermal conductivity of a granular material with a binder may have different values at the same density and moisture content, depending on whether this moisture content is approached from the saturated or completely dry state (32). Experiments were performed to investigate this phenomenon.

#### Method of Measuring Thermal Conductivity

The method used to measure the thermal conductivity is the transient method of Stalhane and Pyk (24) which was used by Van Rooyen (26). The essential feature of this method is the thermal probe which contains a heating wire and a thermocouple, and is located in the center of the soil. Readings of the temperature rise of the probe with time are taken, the time being measured from the instant the heating current is on.

The probe used was 6 in. long and 0.035 in. in diameter and contained a copper-constantan thermocouple as well as a constantan heating wire. The material under test was contained in a 6-in. brass cylinder of 4-in. diameter.

#### Effect of Percentage of Binder on Thermal Conductivity

According to the existing empirical and theoretical formulas which show the effect of the clay content on the thermal conductivity,  $K$ , of a granular material,  $K$  decreases continuously as the percentage of clay added to the sandy material increases from zero. (See the nomogram of Makowski and Mochlinski (16) which is based on the work of Gemant (7) and Kersten (15).) The empirical formulas derived by Van Rooyen (26) also show a decrease in  $K$  as the clay content increases.

To determine the effect of increasing percentages of binder on thermal conductivity, a series of experiments was performed. Kaolinite was used as the binder

TABLE 1

#### GRAIN-SIZE DISTRIBUTION<sup>a</sup>

Sieve	Percent of Fraction
3/8 in.-No. 4	32.2
No. 4-No. 8	22.7
No. 8-No. 16	16.0
No. 16-No. 30	11.4
No. 30-No. 50	8.0
No. 50-No. 100	5.6
No. 100-No. 200	4.1

<sup>a</sup>Rothfuchs graded granular skeleton of sand and gravel.

$$p_m = p_{\max} \left[ 1 - \left( 1 - \frac{\xi}{\xi_0} \right)^{m_1} \right] \quad (2)$$

for frost depth  $\xi$  less than optimum frost depth  $\xi_0$ , and for frost depth  $\xi$  greater than the optimum frost depth  $\xi_0$ ,

$$p_m = p_{\max} \left[ 1 - \left( \frac{\xi - \xi_0}{h - \xi_0} \right)^{m_2} \right] \quad (3)$$

These relationships are illustrated in Figure 3. The dotted line in the figure indicates the theoretical case of frost depth approaching the groundwater table of constant nonfreezing temperature.

#### Subpressure, Freezing Thermal Gradient, and Frost Depth

The study also included the correlation of the maximum induced subpressure to the freezing ground surface temperature and freezing thermal gradient. The experimental evidence indicated the following relationship between the surface subfreezing temperature  $T_s$  and the maximum frost depth  $\xi$  that it can induce in the given soil

$$\xi = a_2 T_s^2 + b_2 T_s + c_2 \quad (4)$$

The maximum induced subpressure  $p_m$  can now be expressed as a function of porosity of the soil ( $n$ ) and the surface subfreezing temperature  $T_s$  by combining Eqs. 1, 2 and 4 or 1, 3 and 4 as the case may be, depending on whether the frost depth  $\xi$  is less than or greater than the optimum frost depth  $\xi_0$ . The expression will be

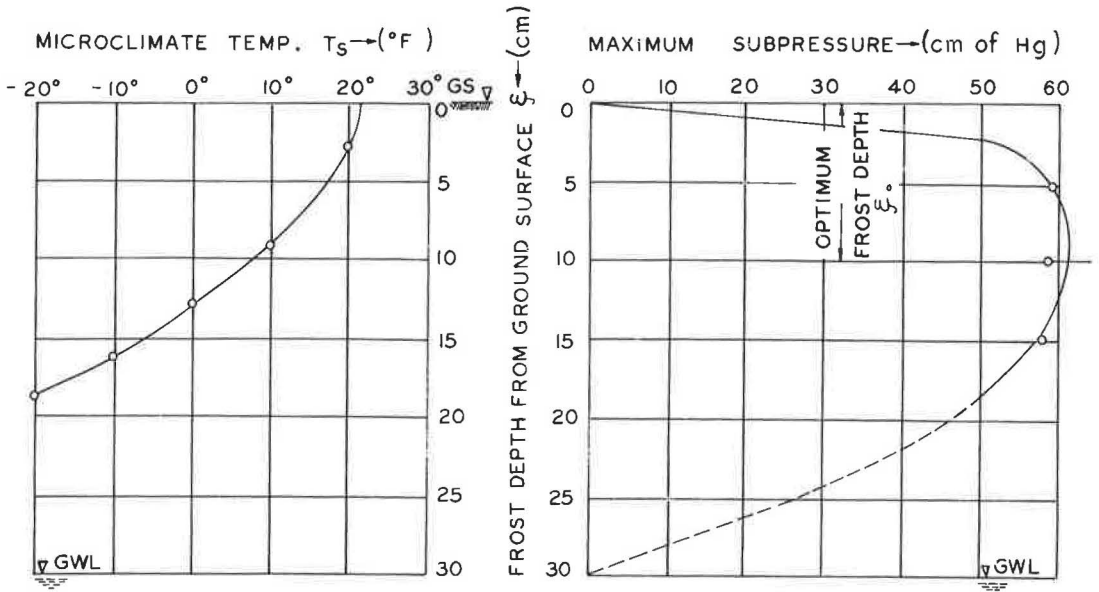


Figure 3. Relationship between maximum subpressure, frost depth, and microclimate temperature for Penn Soil (prosity 4%).

$$p_m = \left[ p_{\max_2} + \left( p_{\max_1} - p_{\max_2} \right) \frac{2}{\sqrt{\pi}} \int_0^{\frac{3(n_2 - n)}{(n_2 - n_1)}} e^{-\beta^2} d\beta \right] \left[ 1 - \left( 1 - \frac{a_2 T_S^2 + b_2 T_S + c_2}{\xi_0} \right)^{m_1} \right] \quad (5)$$

$$p_m = \left[ p_{\max_2} + \left( p_{\max_1} - p_{\max_2} \right) \frac{2}{\sqrt{\pi}} \int_0^{\frac{3(n_2 - n)}{(n_2 - n_1)}} e^{-\beta^2} d\beta \right] \left[ 1 - \left( \frac{a_2 T_S^2 + b_2 T_S + c_2 - \xi_0}{h - \xi_0} \right)^{m_2} \right] \quad (6)$$

### Subpressure Profile

The foregoing discussion relates to the maximum induced subpressure ( $p_m$ ) in a given soil of known porosity ( $n$ ) under a given freezing surface temperature  $T_S$ . However, at any instant the subpressure in the soil layer varies between ground surface and the groundwater table (both are at atmospheric pressure). It is observed that the maximum subpressure ( $p_m$ ) always occurs at the frost boundary. This is consistent with the theoretical reasoning based on the concept of the varying internal energy of

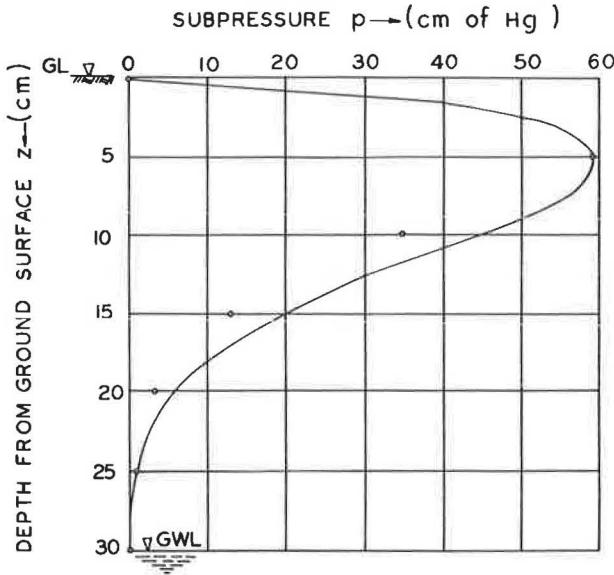


Figure 4. Subpressure profile in freezing Penn Soil (porosity 42%, frost depth 5 cm).

soil moisture at varying temperature from the ground surface to the groundwater table. The subpressure profile resembles a skew frequency distribution curve. The subpressure at any depth as a function of depth of soil layer  $z$  can be expressed as

$$p = p_m \left( \frac{z}{\xi} \right)^{k_1} \left( 1 - \frac{z - \xi}{h - \xi} \right)^{k_2} \quad (7)$$

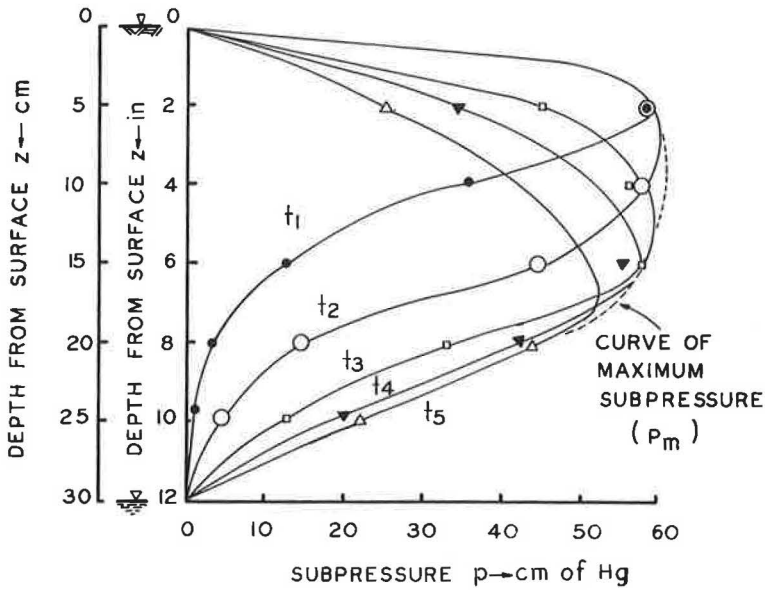


Figure 5. Subpressure profiles in the freezing soil system at different intervals of time after start of cold spell (Penn Soil, porosity 42%).

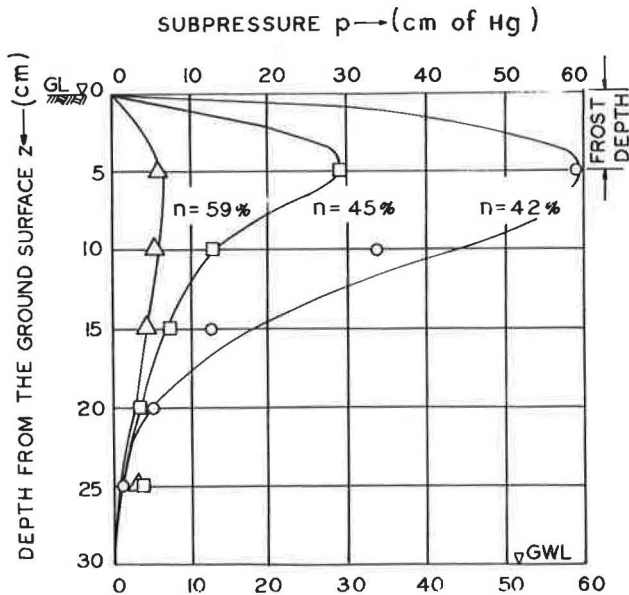


Figure 6. Subpressure profile in Penn Soil (frost depth 5 cm; different porosities).

This is illustrated in Figure 4. Figure 5 shows the relationship of subpressure profile and  $p_m$  curve. The variation of subpressure profile for the same frost depth and for the same soil but under different porosities is illustrated in Figure 6. The subpressure  $p$  can be expressed as a function of  $T_s$  and  $n$  only by substituting the values of  $p_m$  and  $\xi$  in Eq. 7 from Eqs. 4, 5, and 6.

#### Application and Limitations of the Study

The study presented in this paper has its limitations. It has only indicated the pattern of relationship. Extensive experimental data are necessary to get a practical set of quantitative charts and equations, since many of the constants in the equations presented in this paper have to be obtained from such experimentation.

There is also room for broadening the scope of the study by extending the problem into the quality of water and the pattern of the cold spell. The charts and graphs thus developed could readily be used by practicing engineers.

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