Determining the Relationship of Variables in Deflection of Continuously-Reinforced Concrete Pavement

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The AASHO Road Test provided excellent information pertaining to the deflection characteristics of jointed concrete pavements, but the equations are not completely applicable to continuously-reinforced concrete pavement. A comprehensive deflection study of continuous pavement has been conducted in Texas using the Road Test studies as guidelines for the design of the experiments. This is the first report on these deflection studies of CRCP.

The variables studied include load and temperature differential which were explored fully at the Road Test. Other variables studied in this experiment are transverse crack width and transverse crack spacing which are unique to continuous pavement. In addition, a pavement support term is included that encompasses varying conditions of subbase and subgrade type. These variables are studied in terms of both deflection and radius of curvature. An empirical equation for deflection of CRCP is presented which includes all of the variables considered in this study.

•THE FIRST continuously-reinforced concrete pavement (CRCP) was built in Indiana during 1938. Texas was the fifth state to build continuous pavement and today it has more mileage than any other state (1). It was not until the early 1960's that attempts were made to study the deflection characteristics of continuously-reinforced concrete pavement.

This report is on Phase I of a continuing study on the performance of continuously-reinforced concrete pavements presently being conducted by the Texas Highway Department. This report pertains to the static deflection of continuously-reinforced concrete pavement.

The first large-scale deflection study of rigid pavements was at the AASHO Road Test, providing highway engineers with a wealth of information about rigid pavement performance, but the Road Test failed to include continuously-reinforced concrete pavements in the design factorials (2). This means that the vast amount of work done there does not apply directly to the case of continuously-reinforced concrete pavement; therefore, this knowledge must come from other sources. In studying factors affecting the deflection of continuously-reinforced pavements, the AASHO Road Test results can be used as guidelines, but direct comparisons cannot be made.

Rigid pavement research at the AASHO Road Test found the following equation for the deflection of a jointed concrete pavement (3):

$$d = \frac{A_0 L}{10^{A_1 T} D^{A_2}}$$

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where

d = deflection in inches;
L = load in kips;
T = temperature differential in degrees Fahrenheit;
D = pavement thickness in inches; and
A_0, A_1, A_2 = regression constants determined from the data.

The Road Test equation takes into account only load, temperature differential, and slab thickness. For continuously-reinforced pavements this equation is inadequate because of the additional variables associated with this pavement type.

The objective of the project is to determine the deflection characteristics of CRCP under varying conditions of subbase, natural support, pavement thickness, temperature and concrete properties. The scope of this Phase I study is to include the methods of testing for deflections and to develop an algebraic expression for determining the deflection of continuous pavement for any given set of conditions.

DESCRIPTION OF THE EXPERIMENT

Selection of Test Sections

Choice of Site.—The Phase I investigation consisted of three 24-hr deflection studies on pavement sections which had not yet been opened to traffic. These three test sections were of the same design as far as pavement thickness, concrete modulus of elasticity and percent longitudinal steel are concerned. The test sections were selected because of their different supporting characteristics. The test sections were located in three different counties—Colorado, Jefferson and Smith—and will hereafter be referred to by county name. The sections in Colorado and Jefferson Counties were on I-10 and the section in Smith County was on I-20. Figure 1 shows the general location of each of the three test sites.

Description of Sections.—Each of the three selected test sections was 2500 ft long and was chosen in level terrain so that the vertical alignment would not influence the results in any way. Each section was carefully selected so that uniform soil conditions existed throughout. The primary differences between the three pavement sections were the characteristics of the subbase and the subgrade. Table 1 gives the salient features of the three test sections. The classifications of the subgrade are according to the Texas Triaxial Method (4).

Variables Considered

Pavement design involves many parameters and is, no doubt, one of the most complex of all civil engineering problems. Research at the AASHO Road Test was the first major step toward complete control and study of the variables involved in pavement design, but as mentioned earlier CRCP was not included. For ease and clarity in presentation, the variables covered at the AASHO Road Test will be designated as Road Test variables, and the ones included as a part of this investigation will be designated as CRCP variables.

Road Test Variables.—The variables investigated at the Road Test were touched on more lightly in this experiment than the unique variables of CRCP. The Road Test research did an excellent job of investigating pavement thickness and it was felt that thickness was relative, thus this phase of the experiment did not include any studies on thickness.

The other two variables—temperature differential and load—studied at the Road Test were also investigated in relation to CRCP. The load study was merely a check on theory and other research, whereas the temperature study was quite extensive, including equipment development.

CRCP Variables.—Continuously-reinforced concrete pavement has introduced two new variables—crack width and crack spacing—into the field of rigid pavement design. Also considered in this study is the effect of pavement support.

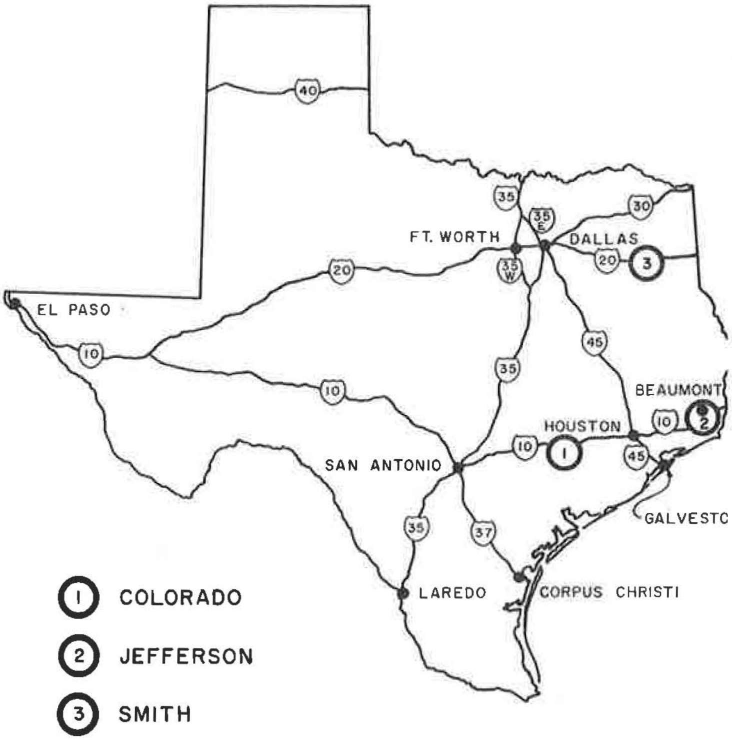

Figure 1. Location of test sites.

TABLE 1

SALIENT FEATURES OF TEST SECTIONS[a]

Highway	County	Subbase		Subgrade, Triaxial Classification
		Thickness (in.)	Type	
I-10	Colorado	6	Cement stabilized gravel[b]	Good Class 4.6
I-10	Jefferson	6	Sand shell	Poor Class 6.0
I-20	Smith	6	Fine grain	Good Class 4.0

[a] All sections are 8-in. continuously-reinforced concrete pavement with 0.5% steel consisting of A-432 #5 bars spaced at 7½ inches for longitudinal steel and A-16 #4 bars at 24-in. c-c for transverse steel.

[b] Stabilized with four percent cement by weight.

EQUIPMENT AND EXPERIMENTAL PROCEDURE

Equipment

The equipment used in the Phase I study of deflection included four Benkelman beams, a Basin beam, a specially equipped truck, special temperature equipment and a microscope. The equipment and its procedure for operation will merely be touched on in this report. For a detailed description and operational procedure of the equipment, refer to (5).

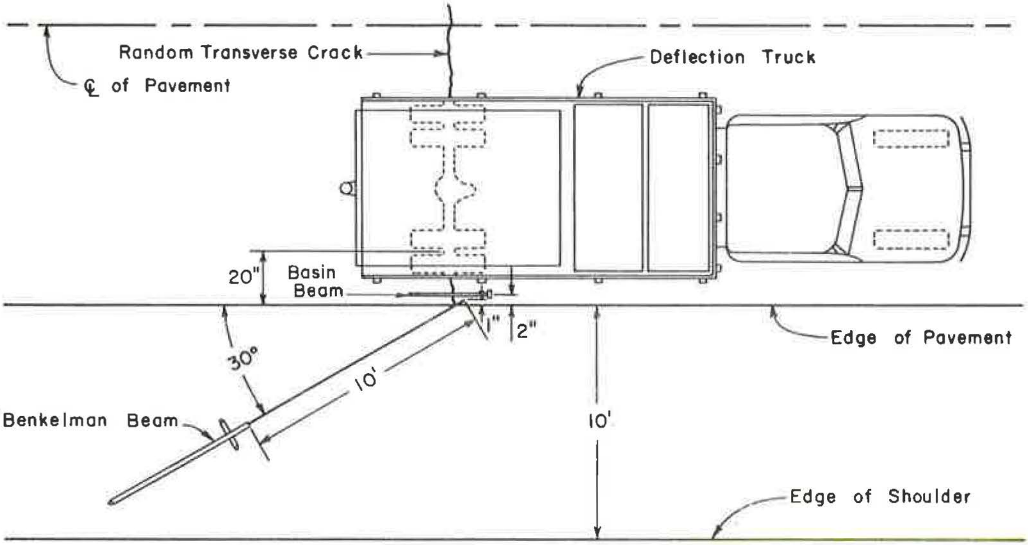

Figure 2. Plan view of equipment arrangement.

Figure 3. Basin beam and Benkelman beams as used on overnight studies.

Benkelman Beams.—Two of the four beams had 10-ft probes, and the other two had 8-ft probes. The Benkelman beams were positioned on the pavement in this study in a manner similar to that used at the AASHO Road Test (3). The beams were positioned at an angle of 30 degrees to the longitudinal edge of the pavement slab with the probe pointing toward the truck. Figure 2 shows a plan view of the position of the Benkelman beam when taking measurements.

Basin Beam.—The Basin beam, which is the instrument used to measure basin deflections in terms of radius of curvature, was designed by the Highway Design Division's Research Section and built by the shops of the Texas Highway Department. The

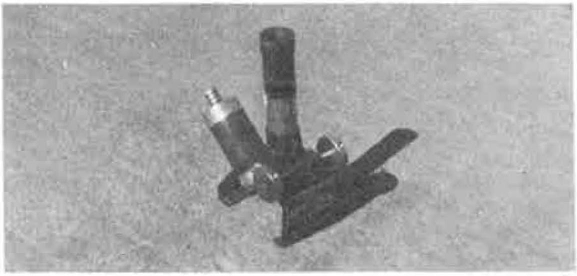

Figure 4. Microscope used to measure crack width.

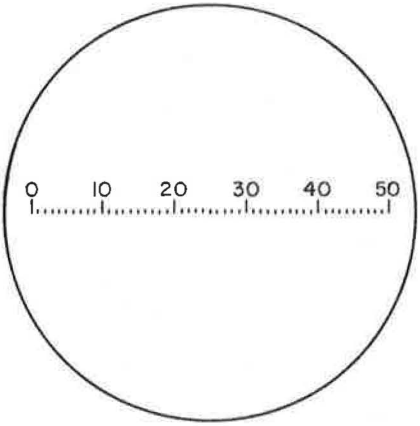

Figure 5. Graduated eyepiece of microscope.

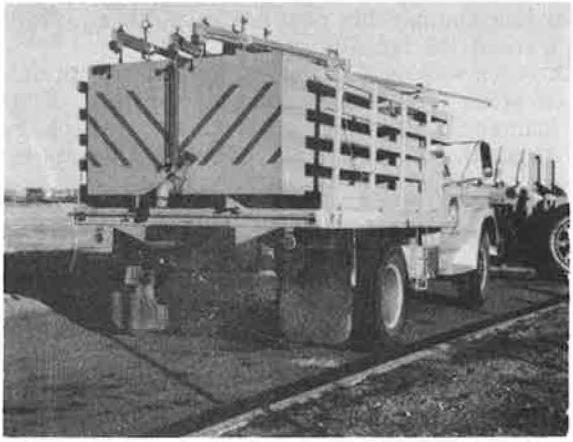

Figure 6. Completed truck carrying Benkelman beams on location.

placement of the Basin beam when taking data is shown in Figure 2 and a photograph is shown in Figure 3. The probe of the dial gage which is in the center of the beam is placed just to either side of the crack in the pavement. The radius of curvature is computed using the geometrical relationship for three points on a circle (5).

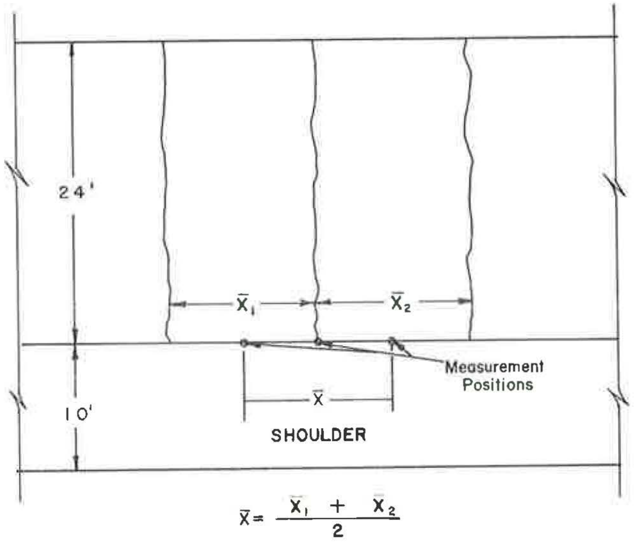

Figure 7. Selection of crack spacing.

Temperature Equipment.—The special temperature equipment used on this project was also designed by the Research Section. It consisted of a small portable 8-in. concrete slab in which two high-speed resistance thermometer bulbs were placed near the top and bottom. The leads from these bulbs were connected to a Minneapolis-Honeywell Electronik Temperature Recorder which recorded the top and bottom temperatures of the portable slab on a continuous strip chart. The development and technique for using this equipment has been covered in a previous report (5).

Microscope.—A specially fabricated microscope with a built-in scale in the eyepiece was used to measure the width of the cracks. By setting the microscope over the crack and focusing on it, the crack width could be read on the inscribed scale to the nearest 0.002 inch. Each time the microscope was used in a given location it was positioned in the same spot in order that comparable data might be taken. Figure 4 shows the microscope and Figure 5 shows the built-in scale.

Truck.—The truck which was used to deflect the pavements in this and continuing studies is a single-axle stake-type truck rated at three tons. It is equipped with a box of lead shot for dead load and also a large water tank so that the magnitude of the load can be varied. Figure 6 shows the truck with the Benkelman beams loaded in the mobile position.

Experimental Procedure

The procedures used at the AASHO Road Test were used as guidelines in developing the procedure for this experiment (3). New procedures were required to study the CRCP variables which are new to rigid pavement design.

Crack Spacing.—On each of the three sections, two small, two medium, and two large crack spacings were selected as points where deflection measurements were to be made. The crack spacings were chosen as is shown in Figure 7 where $\bar{X}_1$ was approximately the same as $\bar{X}_2$. This procedure was followed for the small, medium and large crack spacings. The small crack spacings were from one to four feet, medium from six to eight feet and large from 12 to 31 feet.

Axle Load.—The deflection truck was loaded such that the rear axle load was 18,000 pounds and the tire inflation pressure was 75 psi. The 18,000-lb axle load was adopted because it represents the maximum legal load limit on a single axle in Texas, and it is used as the basis for deriving equivalencies in the AASHO design methods (6). The center of the dual tires on the right side of the truck was kept 20 inches, ± two inches,

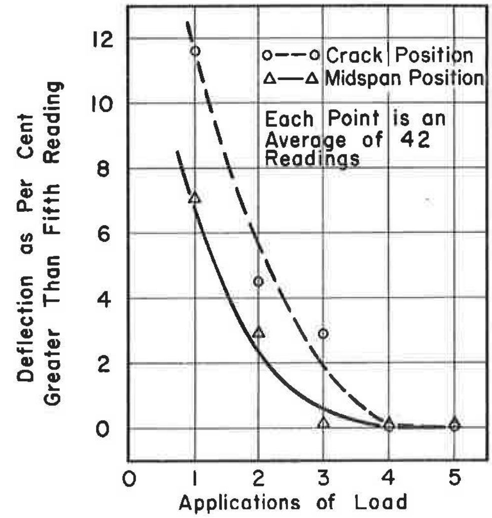

Figure 8. Applications of load vs deflection as percent greater than reading at fifth load application.

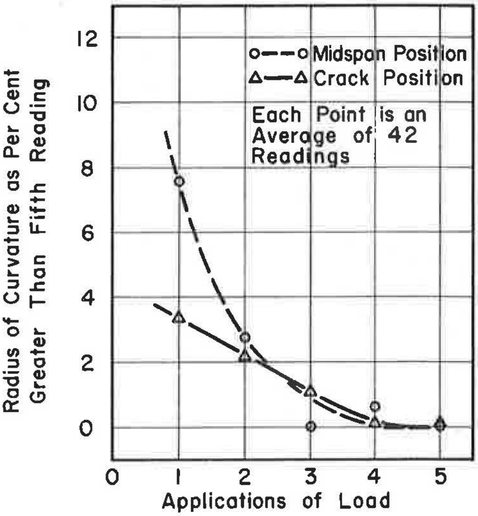

Figure 9. Applications of load vs radius of curvature as percent greater than reading at fifth load application.

from the edge of the pavement. When deflections were measured at the crack, the tire contact area of the outside tire on the right duals was centered over the crack.

Ironing.—Measurements at the AASHO Road Test and experience in Texas have shown that a concrete pavement slab is not always in complete contact with the subbase or subgrade nor is it a uniform distance from it. This phenomenon is conjectured to be due to point-to-point variations in temperature longitudinally down the slab.

A special study was initiated to determine the optimum procedure for measuring deflections with a desired degree of reproduciability. The tests were run on successive days under similar climatic conditions to avoid the variables of the environment, but the test sections were selected so as to encompass a range of support conditions. The measurement procedure consisted of placing the single-axle load in position, zeroing the dial gages, removing the load from the zone of influence and recording the Benkelman and Basin beam readings. This procedure was repeated five times at both the crack and midspan positions every 200 feet, on each of three 2500-ft test sections.

Figure 8 shows the effect of the "ironing" procedure on deflection. Each point on the graph represents the average of 42 measurements. The graph shows that the deflection at both the crack and midspan positions is relatively constant after three passes of the load. Thus at each point at which deflections were measured, the deflection truck was first passed over the area three times in order to attain the desired reproducibility of results.

Figure 9 shows that the ironing procedure had only a slight effect on radius of curvature. The ironing procedure is followed because both deflection and radius of curvature measurements are obtained at the same time.

Measuring Deflections.—The deflections were measured at three points, a, b, and c, as shown in Figure 7. The positioning of the beams for these measurements is shown in Figure 3. All measurements were made in the outside lane with the Benkelman beam probes on the pavement, one inch from the edge and at a 30-deg angle.

After beam placements, the pavement was "ironed out" by making three passes across the test area with the deflection truck. Immediately after ironing, the load was centered on the test crack and all dial gages zeroed, the load removed, and all dial gages read. Deflection readings were taken on the six selected crack spacings for a period of 24 hours on each of the three test sections. Eleven readings were taken with each beam on each crack spacing on each of the three test sections.

Eleven sets of readings on six crack spacings for three test sections produced 198 sets of data. With each set of deflection readings a crack width measurement was made using a microscope with a graduated eyepiece. A continuous recording of the temperature at the top and bottom of the pavement was made throughout each of the three overnight studies.

Deflection with Variable Load.—Deflections were measured in the outside lane by using one Benkelman beam as shown in Figure 2. The test was started with an axle load in excess of 20,000 pounds. The load was varied by reducing the level of the water being used as load. Each time the load was changed, the deflection was measured.

Basin Measurements.—Basin data were taken using the Basin beam. The pavement was ironed out by three passes of the deflection truck after which the Basin beam was placed over the crack and the center of the axle load was lined up vertically with the dial gage. The gage was zeroed, load removed from zone of influence and the dial reading was taken. This procedure was also used at the midspan position. Data were taken over a period of 24 hours so that effects of temperature would be involved. The same special temperature equipment was used here to obtain pavement temperatures as was used when measuring deflections.

PRESENTATION OF RESULTS

The variables of pavement design are studied herein in terms of deflection and radius of curvature. The variables are broken down into two groups—Road Test and CRCP. Road Test refers to variables which were considered on jointed pavements at the AASHO Road Test and CRCP refers to the variables unique to continuously-reinforced concrete pavement.

Deflection

In this section the effect of various parameters such as temperature, load, crack width, and crack spacing on deflection as measured with the Benkelman beam are discussed. First the factors investigated at the Road Test for jointed pavements are presented followed by consideration of factors applicable only to CRCP.

All relationships presented herein are for deflection at the crack position. Deflections at the crack are slightly greater than at midspan, thus for design purposes the crack deflections are analyzed. This difference in deflection is attributed to the presence of the crack.

Road Test Variables.—The studies of load and temperature on CRCP produced results which are analagous to the results of studies at the AASHO Road Test on jointed pavement.

Temperature Differential.—Slab temperature differential is truly a variable as shown by the trend in the data in Figures 10 through 12. The computed linear relationships for each set of data and the coefficients of determination are shown on the respective graphs. These graphs are for each of the test sections and are typical of the relationships fround for the other cracks. Note the inverse relationship between the two variables as was found at the Road Test. Figure 13 shows all three regression lines on one graph. The lines all have approximately the same slope, thus showing the constant relationship between temperature differential and deflection. The vertical position on the graph is indicative of the type of foundation each test pavement had.

Load.—The data shown in Figure 14 show the linear relationship between load and deflection and justify the load term in the model equation presented later herein. Westergaard and others found from their theoretical analyses that the deflection was a direct function of the load (7, 8, 9). Rigid pavement research at the AASHO Road Test also indicated that pavement deflection is a direct function of the load placed upon it (3, 10). Thus the results of this experiment substantiate both theory and other research.

CRCP Variables.—Some of the presently known variables of rigid pavement design which are unique to CRCP are crack width and crack spacing.

Crack Width.—The cracks in concrete due to lineal volume changes vary in width as temperature changes. The mid-depth temperature of the pavement computed from the average of the top and bottom temperatures of the pavement correlates well with the

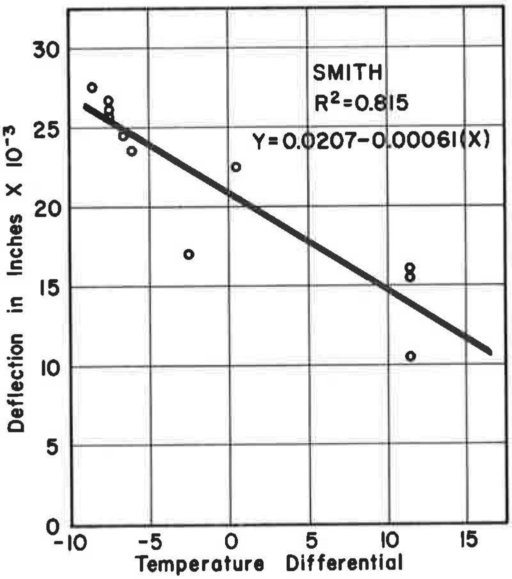

Figure 10. Deflection vs temperature differential—Smith.

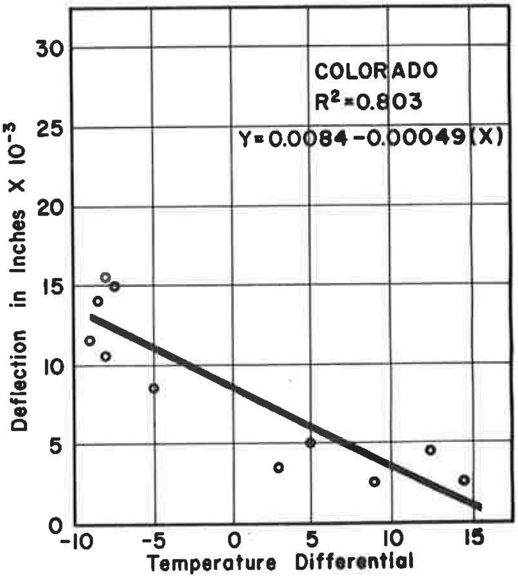

Figure 11. Deflection vs temperature differential—Colorado.

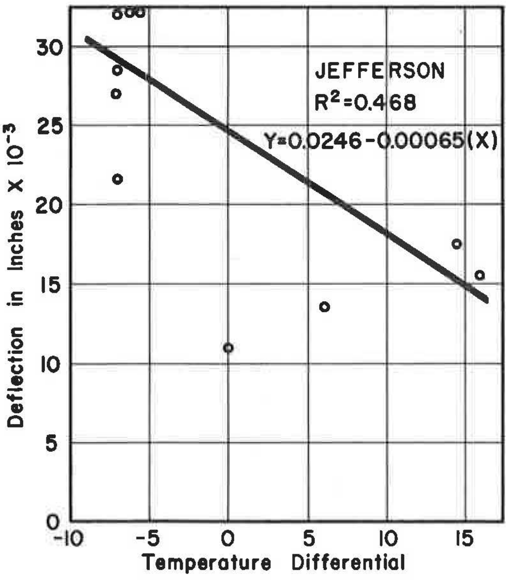

Figure 12. Deflection vs temperature differential—Jefferson.

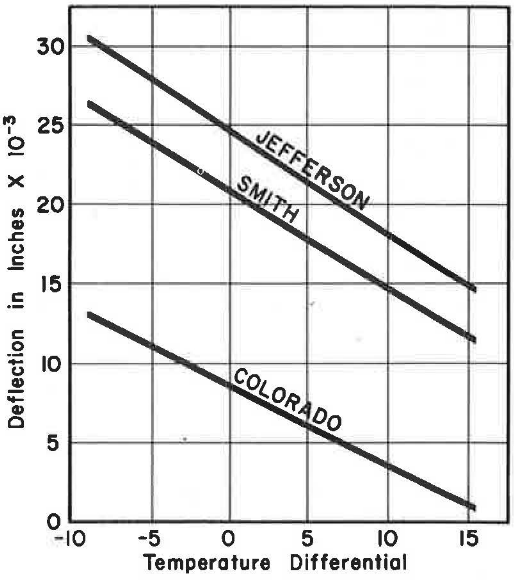

Figure 13. Comparison of regression lines (deflection vs temperature differential) for three projects.

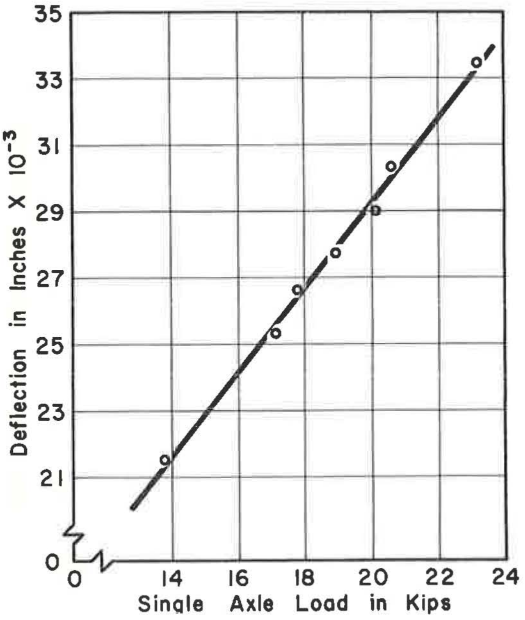

Figure 14. Deflection vs single-axle load.

crack width, thus indicating that the crack width is a function of temperature (11, 12). Figures 15 through 17, which are for like crack spacings, show how the mid-depth temperature affected the crack width on each of the three test sections. These relationships for each test section are typical of relationships found at other cracks. These data corroborate other work in that the crack width increases as the temperature decreases (11, 12, 13). The Smith and Colorado tests involved crack widths of 10 to 35 thousandths of an inch, whereas the Jefferson test section had cracks ranging up to only 10 thousandths of an inch in width. Figure 18 shows the regression lines for mid-depth temperature and crack width for the three test sections in order that a relative comparison can be made. Crack width and mid-depth temperature experienced the same relationship on the Smith and Colorado tests, but the Jefferson test showed the crack width was only slightly affected by temperature.

The crack width might be thought of as a measure of load transfer since the load is transferred by aggregate interlock, and the degree of aggregate interlock is dependent upon the crack width. As a crack closes, the load transfer increases, and the pavement deflects less because of the increased rigidity or degree of slab continuity. Figures 19 through 21 are typical portrayals of how the deflection increases with increases in crack width on each of the projects. Figure 22 gives a relative comparison of the regression lines for equal crack spacings for each of the projects. The same trend is present in all three test sections, but a variation in crack width has a much more pronounced effect in Jefferson County. Data taken from CRCP test sections located throughout the state shows that this relationship between crack width and deflection does exist. Crack width is a function of percent longitudinal steel. Deflection has been found to be a function of percent steel which in turn is a measure of the crack width (14).

As discussed earlier, the mid-depth temperature of the pavement is a relative indicator of crack width; therefore, it may be used as an indirect measure of crack width. Figure 18 shows that as the mid-depth temperature increases the crack width decreases. This same phenomenon is true for deflection. Figures 23 through 25 show the relationship between mid-depth temperature and deflection. Figure 26 shows all three regression lines and shows that as mid-depth temperature increases the deflection decreases in all cases.

Crack Spacing.—The crack spacing on a continuously-reinforced concrete pavement varies at random unless a preformed crack spacing has been provided for in the design and construction. Its relationship to deflection is shown in Figure 27.

Radius of Curvature

Radius of curvature is inversely proportional to the stress in concrete pavement, and studies have shown that it may be used as a relative measure of stress (5). In all subsequent analysis the reader's attention is drawn to the fact that the greater the radius of curvature the smaller the stress. Radius of curvature measurements were not made at the Road Test as such, but pavement stresses due to wheel load were studied in the form of strains which were obtained by use of electrical strain gages.

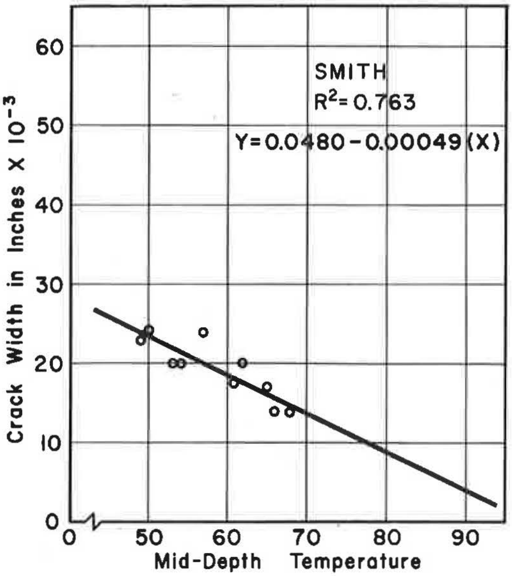

Figure 15. Crack width vs mid-depth slab temperature—Smith.

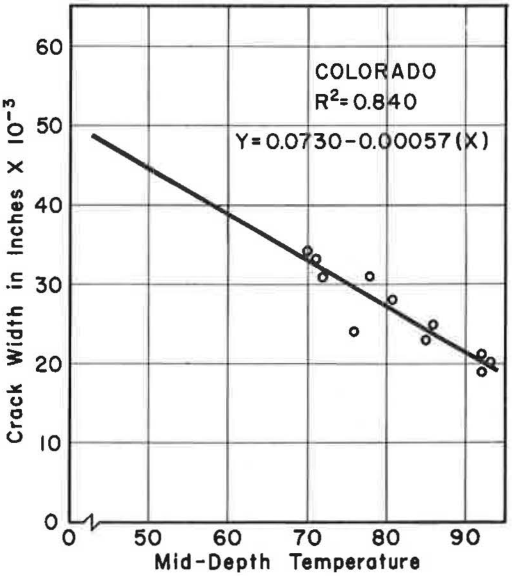

Figure 16. Crack width vs mid-depth slab temperature—Colorado.

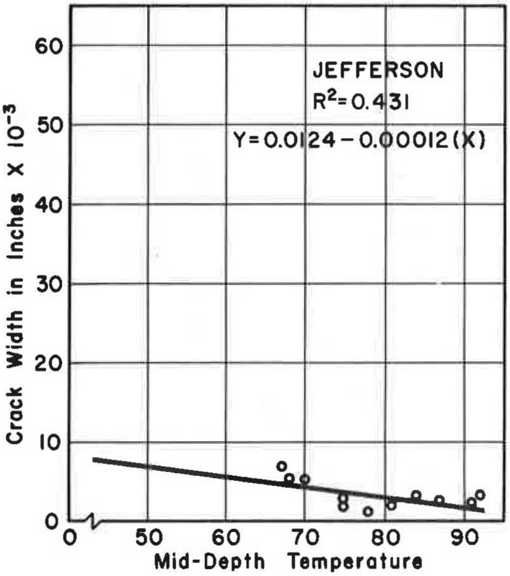

Figure 17. Crack width vs mid-depth slab temperature—Jefferson.

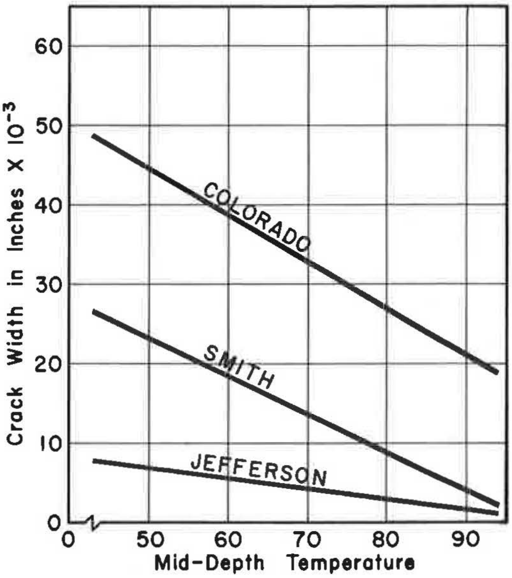

Figure 18. Comparison of regression lines (crack width vs mid-depth temperature) for three projects.

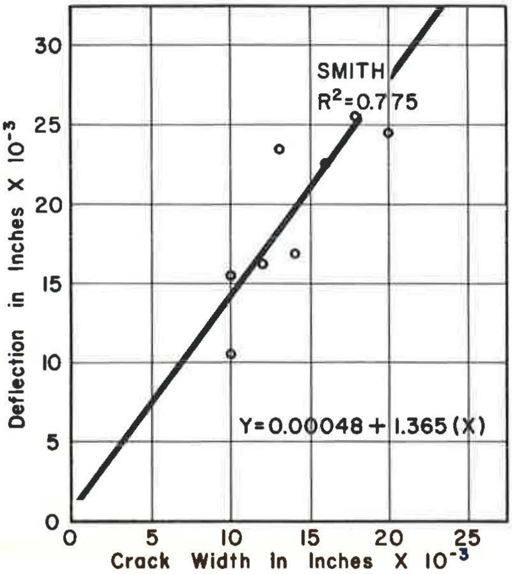

Figure 19. Deflection vs crack width—Smith.

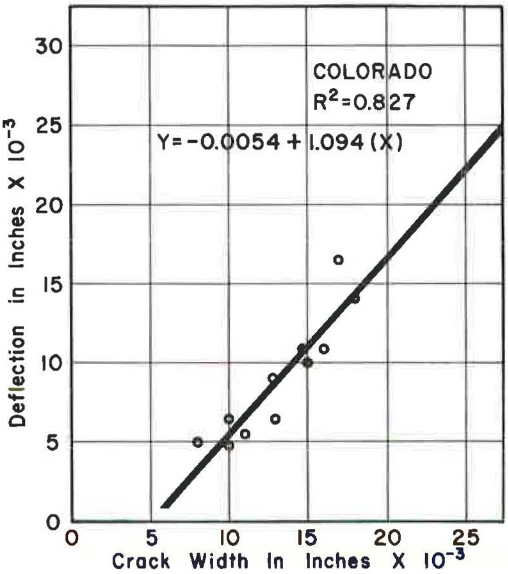

Figure 20. Deflection vs crack width—Colorado.

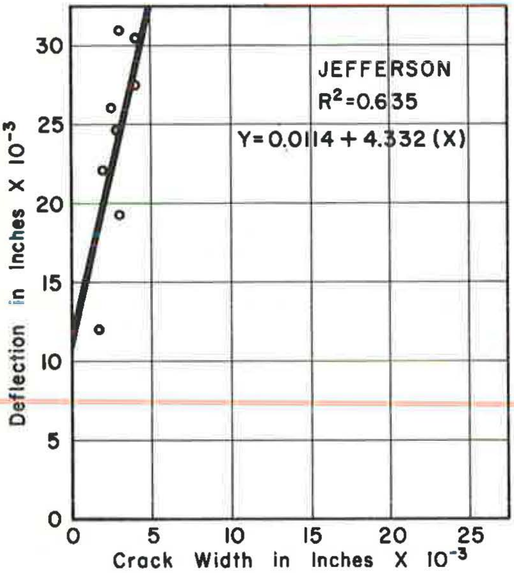

Figure 21. Deflection vs crack width—Jefferson.

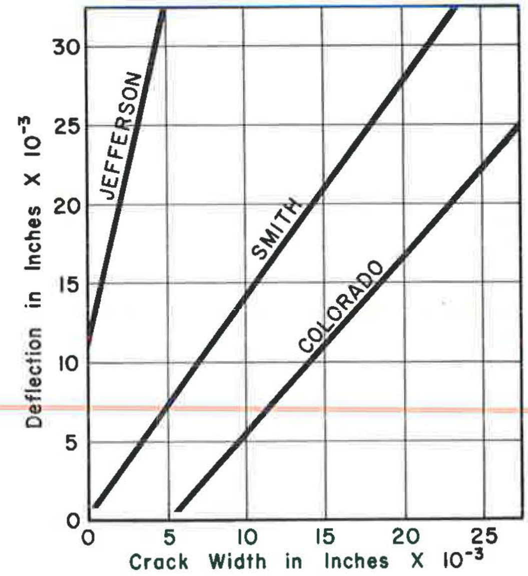

Figure 22. Comparison of regression lines (deflection vs crack width) for three projects.

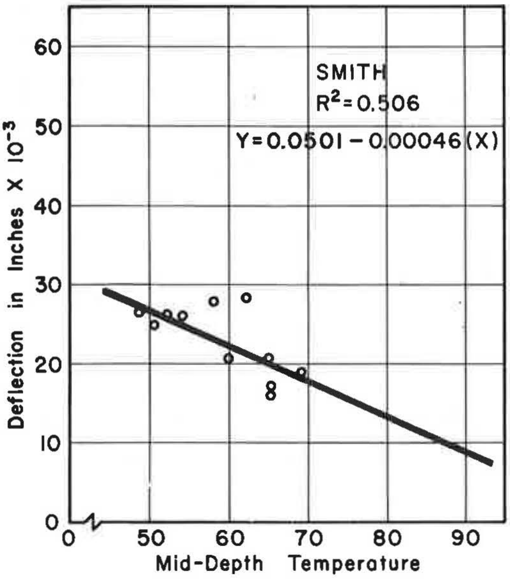

Figure 23. Deflection in terms of mid-depth temperature—Smith.

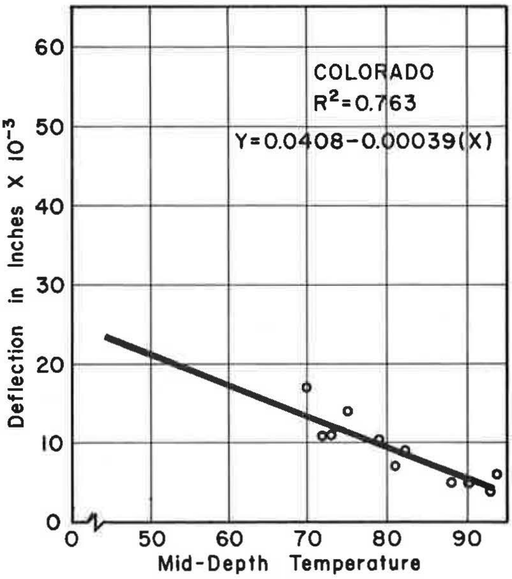

Figure 24. Deflection in terms of mid-depth temperature—Colorado.

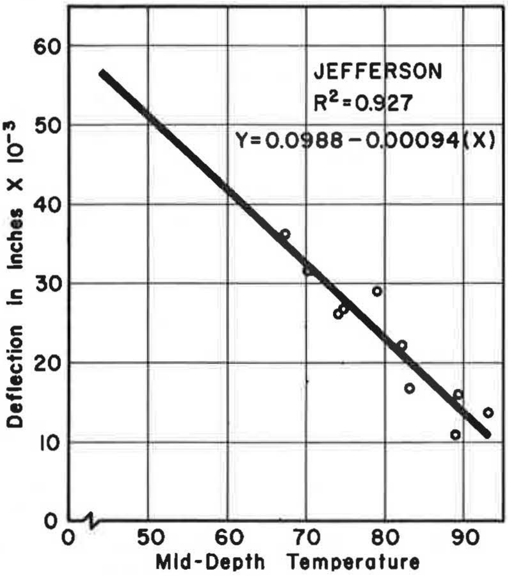

Figure 25. Deflection in terms of mid-depth temperature—Jefferson.

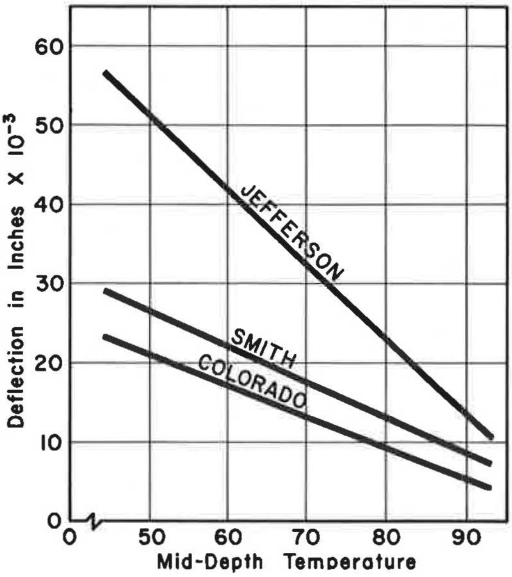

Figure 26. Comparison of regression lines (deflection vs mid-depth temperature) for three projects.

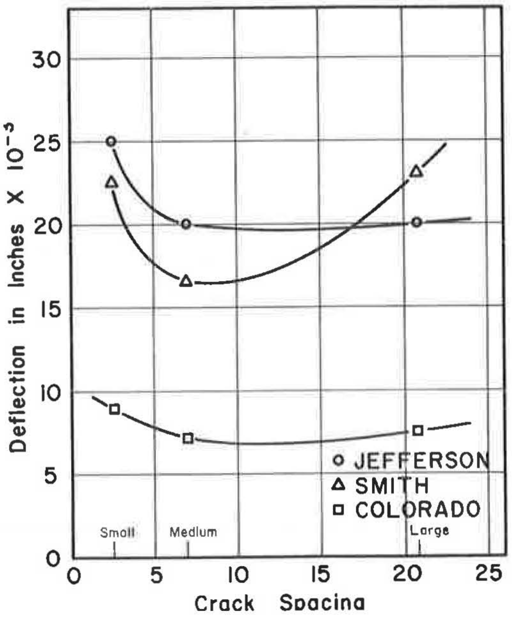

Figure 27. Deflection vs crack spacing.

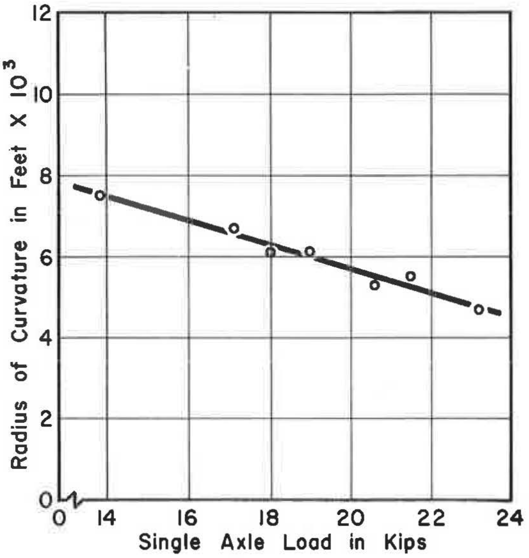

Figure 28. Single-axle load vs radius of curvature.

Road Test Variables.—Studies of single-axle load vs radius of curvature in this experiment indicate an inverse linear relationship between the two. Figure 28 shows the linear relationship between load and radius of curvature. This same linear relationship was found at the Road Test in terms of load and strain. Radius of curvature is an inverse function of stress, which is a direct function of strain, thus the analogy in results does exist.

This investigation showed that radius of curvature or stress is not related to slab temperature differential. This is definitely true for the midspan measurement position. The two groups of data points in Figure 29, representing two typical projects, show that there is no relationship between radius of curvature and temperature differential. This contradicts findings made at the Road Test where it was found that the slab temperature differential had a slight effect on the pavement stress. Their studies indicate that the temperature differential influenced slab stress ¼ to ½ as much as it did deflection. At the present time no rational explanation can be given for this apparent discrepancy in findings other than that the sensitivity of the Basin beam is less than electrical strain gages.

Figure 30, which shows the data for the crack position, indicates that the temperature differential might be related to radius of curvature. This illusory relationship of temperature differential and radius of curvature is covered in more detail in the next section.

CRCP Variables.—As discussed previously, crack width is a function of temperature because concrete volume changes are functions of temperature. The mid-depth temperature of the pavement might then be thought of as an indicator of crack width. Figure 31 shows the relationship of the radius of curvature to the mid-depth temperature (crack width) at midspan on two different test sections. Again there is no relationship between temperature and radius of curvature as was the case when comparing temperature differential to radius of curvature. Figure 32 shows a relationship of temperature to radius of curvature at the crack position for two test sections and indicates that temperature has an effect on the radius of curvature, but it must be kept in mind that the continuously-reinforced pavement is cracked and that increases or decreases in temperature have a direct effect on the width of the cracks.

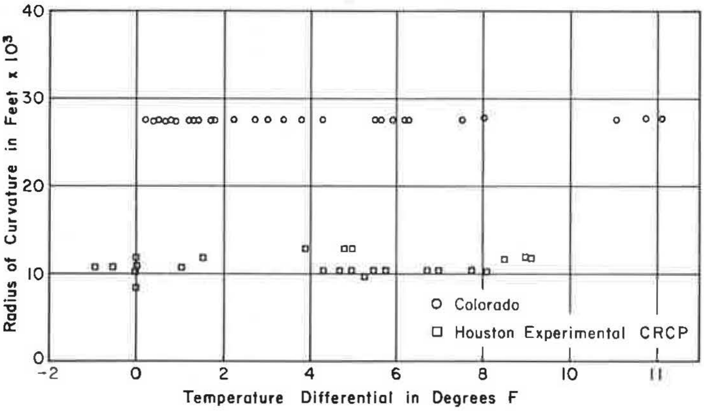

Figure 29. Temperature differential vs radius of curvature at midspan position.

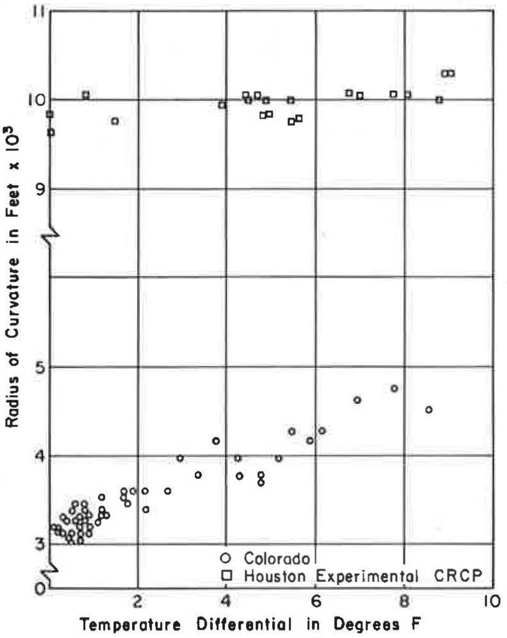

Figure 30. Temperature differential vs radius of curvature at crack position.

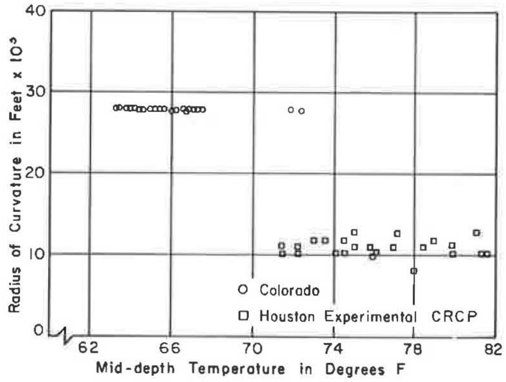

Figure 31. Mid-depth temperature vs radius of curvature at midspan position.

Both temperature differential and mid-depth temperature studies have shown that temperature increases cause an increase in radius of curvature. This phenomenon can be attributed to crack width. If the crack is closed by temperature increase, the pavement begins to react as if the crack were not present. Thus, the radius of curvature does not change with temperature at midspan and the changes in radius of curvature at the crack were caused by changes in crack width rather than temperature.

The crack spacings in this experiment were classified as small, medium, or large as stated earlier. To evaluate crack spacing as a variable, an average deflection condition is selected for a crack width. A comparison of these deflections for the various cracks reveals the influence of crack spacing. Figure 27 shows how the crack spacing affected the deflection in this experiment. The relative vertical position of the curves will vary as the crack width changes. In general, the deflection decreases as the crack spacing increases until a range of five to ten feet is reached. Beyond this range the deflection increases as the crack spacing increases. These data indicate that an optimum crack spacing is in the range of five to ten feet. These observations

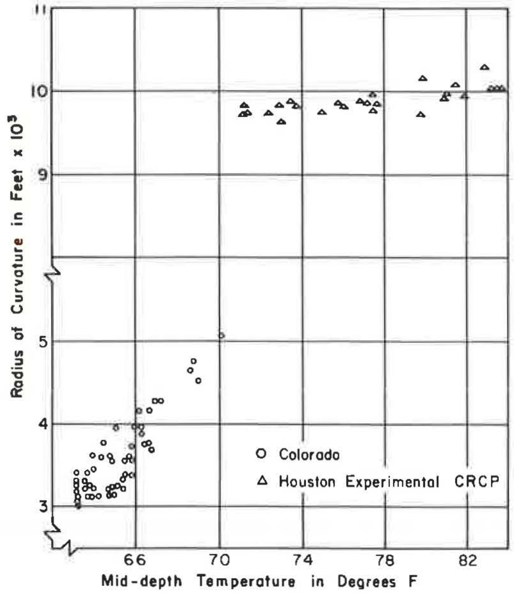

Figure 32. Mid-depth temperature vs radius of curvature at crack position.

tend to verify work reported previously in connection with preformed crack spacings where it was found that the optimum crack spacing was approximately five feet (14).

Development of Equation

Much work was done at the AASHO Road Test to develop equations to predict the deflection of jointed concrete pavements. For continuously-reinforced pavements only one equation will be developed here since the deflection at the crack and mid-span positions are approximately the same with the crack deflection being minutely larger.

Model Selection.—The model selected for the deflection of continuously-reinforced concrete pavement is a modification of the AASHO Road Test model. By adding the CRCP variables, crack width and crack spacing, to the AASHO equation a model would be obtained that was based on considerable research and would also allow direct comparisons. The model selected for continuous pavement was of the form:

$$d = \frac{A_0 L \Delta X^{A_2} \overline{X}^{A_3}}{10^{A_1 T} D^{1.178}}$$

where

d = deflection in inches;
L = single-axle load in kips;
ΔX = surface crack width in inches;
$\overline{X}$ = crack spacing in feet;
T = temperature differential in degrees Fahrenheit between $^3/_4$ inch and 1 $^1/_8$ inches from the top and bottom of the slab, respectively;
D = slab thickness in inches; and
A_0, A_1, A_2, and A_3 = constants computed from the data.

The depth term to the 1.178 power is a result of the rigid pavement research at the AASHO Road Test (3). This power was included in the equation because this experiment did not include a study of pavement thickness. Axle load was studied in terms of deflection, but was not included as a full factorial variable on the test sections. Therefore, in all subsequent regression work an axle load of 18,000 pounds and a pavement thickness of eight inches were inserted in the Road Test equation. These numbers were moved to the left side of the equation and combined with deflection to form the dependent variable. The constants derived from this regression analysis then reflect load and pavement thickness and are directly comparable to the Road Test Equation.

Regression Analysis.—A multiple regression analysis was made on each of the three sets of data from the Colorado, Jefferson and Smith test sections. Through the regression analysis the regression analysis constants in the model were computed. The constants are given in Table 2. The constant A_0 is relative to the pavement support. The Colorado test section had a stabilized subbase, whereas the other two sections did not. Therefore, A_0 for Colorado was less than that for Jefferson or Smith.

The constant A_1 compares very well with the constant on temperature differential in the AASHO equation. In the AASHO equation A_1 is equal to 0.0075 for a single-axle

TABLE 2

CORRELATION CONSTANTS FOR CRCP MODEL OBTAINED FROM MULTIPLE REGRESSION ANALYSIS

MODEL	CONSTANT \ TEST SECTION	COLORADO	JEFFERSON	SMITH
ORIGINAL	A_0	0.028502	0.035803	0.048189
	A_1	0.016679	0.007481	0.010489
	A_2	0.402499	0.119260	0.352490
	A_3	-0.146090	-0.121316	0.028986
MODIFIED	A_0	0.003993	0.016675	0.011115
	A_1	0.015571	0.003881	0.010899
	A_2	7.637913	9.908250	5.207664
	A_3	-0.161818	-0.100224	-0.029021

TABLE 3

STATISTICAL ANALYSIS OF TEST SECTIONS

TEST SECTION	STANDARD ERROR OF THE ESTIMATE	COEFFICIENT OF DETERMINATION	COEFFICIENT OF CORRELATION
Colorado	±0.00205	0.611	0.782
Smith	±0.00267	0.641	0.801
Jefferson	±0.00294	0.343	0.586

load and an edge condition, and the magnitude varies from 0.0075 to 0.015 for various conditions of reinforcement and load position, i.e., edge or joint. The constants obtained for the CRCP model are within this range, with the Jefferson County value being identical to the Road Test edge condition. Therefore, it may be deducted that a continuous pavement responds to slab temperature differentials in the same manner as a jointed pavement.

The constant A_2 reflects the crack width. For the Jefferson test A_2 is small compared to the other two, as was the case of the actual crack widths.

The constant A_3 turned out negative on the Colorado and Jefferson tests. The model indicates a direct relationship between crack spacing and deflection which is not exactly true for the test sections. Figure 27 shows that as crack spacing increases on the Jefferson and Colorado tests the deflection decreases for the bulk of the data, thus explaining the negative A_3.

Modification of Model.—The first selected model was so arranged that if the crack width was zero, the deflection was also zero which is an erroneous boundary condition. Also, the relationship between crack width and deflection was found to be linear; thus the model was slightly modified to correct for these discrepancies. In the modified model ΔX^{A_2} term was changed to $10^{A_2 \Delta X}$ since this function approaches a linear expression and it also satisfies the boundary condition.

Before the multiple regression analyses were rerun on the modified model some of the data points were deleted. With some of the obvious erroneous data (due to bad readings) removed, the multiple regression was rerun, the results of which also appear in Table 2. The only constant to change a large amount was A_2, and again the differences were relative to the magnitudes of the crack width. Also A_3 turned out to be negative for the Smith test as is the case for the other two sections. For the modified model the standard error of the estimate and the coefficients of determination and correlation are presented in Table 3.

DISCUSSION OF RESULTS

Three equations were found for deflection, one for each of the three test sections. The primary difference between the three test sections was in the foundation material or support, i.e., the subbase and subgrade. With soil-supporting characteristics being the only difference in the three pavements, it thus becomes the means whereby the results from the three overnight tests can be combined into one equation.

Soil Support

To tie the three overnight deflection studies together, the term "soil support" was formulated and defined as

$$SS = \left(\frac{U}{T_{sg}}\right)^{1/4}$$

where

SS = soil support;
U = unconfined compressive strength of subbase materials in psi at an age of seven days; and
T_{sg} = Texas triaxial classification of subgrade material.

This form was selected because studies being conducted parallel to this study in connection with subbase support show this model gives the best correlation. Furthermore, logical reasoning would lead to the hypothesis that as the unconfined compressive strength of the subbase increases, the degree of support increases. As the triaxial classification of a material increases, the material is actually weaker and of a poorer quality for use as a highway building material; thus, as the triaxial classification increases, the degree of support decreases (4).

The supporting quality of the subbase and the subgrade bears a direct relationship with deflection as is clearly shown in Figure 33.

The support term was calculated on the basis of in-place values—compressive strength and triaxial class—and the deflection is the average of all reading on the test section. The effect of the variation of support conditions for the regression analysis of each section is reflected in the A_0 term. As shown by the dashed line in Figure 33 both of these trend lines indicate the feasibility of combining the data from the three test sections into one equation.

Subgrade classifications of good, fair, and poor alone cannot be used to explain pavement deflections. Emphasis must be placed on the supporting material immediately beneath the pavement. In cases where the subbase has been stabilized by any one of the four methods presently used by the Texas Highway Department, the deflections do not compare to deflections of a pavement with a nonstabilized subbase with the same class of subgrade. Thus it becomes important in deflection studies that stabilized

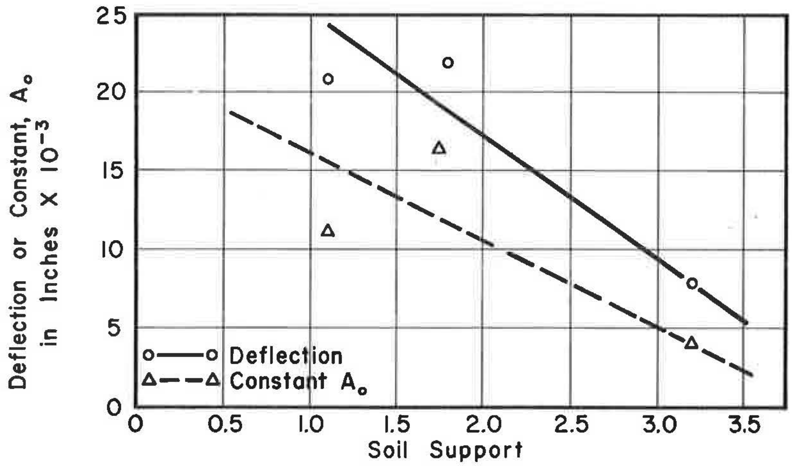

Figure 33. Comparison of deflection and the constant, A_0, as functions of soil support.

TABLE 4

RESULTS OF FINAL REGRESSION ANALYSIS

CONSTANT	COMPUTED VALUE
A_0	0.010617
A_1	0.014724
A_2	4.899716
A_3	-0.099375
A_4	0.850280
STANDARD ERROR OF THE ESTIMATE	±0.0026
COEFFICIENT OF DETERMINATION	0.901
COEFFICIENT OF CORRELATION	0.949

subbases be accounted for. In many cases subgrades are treated with lime either as a construction aid or as a desired improvement of subgrade immediately beneath subbase. When subgrades are treated with lime a second subbase is actually created.

Because deflections are inversely proportional to soil support, the new term "soil support" was placed in the denominator of the model. After the addition of the soil support variable, the model for the deflection at the crack position is

$$d_c = \frac{A_0 L \, 10^{A_2 \Delta X \frac{}{\overline{X}} A_3}}{10^{A_1 T} \, D^{1.178} \, SS^{A_4}}$$

where A_4 is a constant computed by data analysis and all the other terms are as previously defined.

Final Analysis

A multiple regression analysis was made using the above equation and the combined data from the three tests. The small crack width on the Jefferson test caused the constant, A_2, to be erroneous. The Jefferson and Smith tests were very much the same from the standpoint of deflection and support. Thus, the Jefferson data were dropped from the final analysis because of the very small crack width.

A multiple regression analysis was made on the remaining data from the Smith and Colorado tests. The results of this analysis are presented in Table 4. Thus the final equation for the deflection at the crack position is based on only two of the overnight deflection studies conducted.

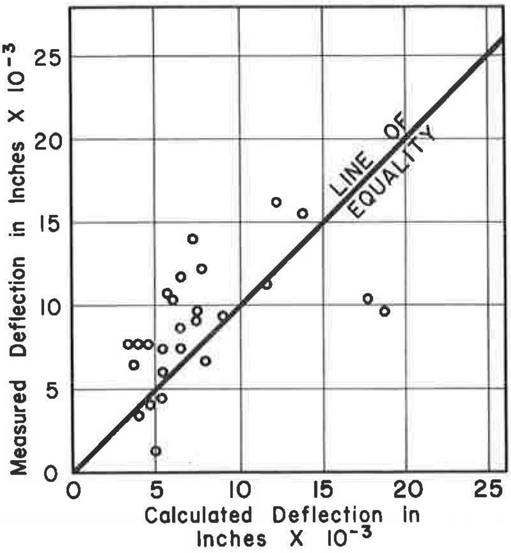

Figure 34. Measured deflection vs deflection calculated by final equation.

TABLE 5

RELATIVE IMPORTANCE OF THE DEFLECTION VARIABLES

	DEFLECTION VARIABLES							
Given Conditions * Changes in Conditions	Load (L)	Thickness (D)	Soil Support (SS)	Crack Width (ΔX)	Temperature Differential (T)	Crack Spacing ($\bar{X}$)	Calculated Deflection	Per Cent Change
	18	8	1.5	0.015	3	5	0.0116	—
SS			3.0				0.0065	−44.0
Δ X				0.030			0.0149	28.4
T					6		0.0115	−0.8
$\bar{X}$						10	0.0115	−0.8

*** The given conditions prevail other than where specified.**

The final equation for the deflection of a continuously-reinforced concrete pavement is

$$d = \frac{0.0106\, L\, 10^{4.8997 \Delta X}}{10^{0.0147 T}\, D^{1.178}\, SS^{0.8503}\, \bar{X}^{0.0994}}$$

The error in this equation is comparable to that in each of the three equations for the three individual tests. The standard error of the estimate for the final equation was 0.00263 which is very close to the values shown in Table 3 for the three tests. The coefficients of determination and correlation, presented in Table 4, indicate that the equation is valid.

To test the validity of the equation, data were taken from a statewide deflection run and deflections were computed for each test section. The calculated deflections were then plotted against the measured deflections as shown in Figure 34. The points cluster closely around the line of equality, thus showing the equation is valid.

Relative Importance of Variables

An empirical relationship depicting deflection in terms of rigid pavement variables for CRCP has been presented. The relative importance of these variables in terms of deflection is given in Table 5. This table contains a set of given conditions for which the deflection is computed using the final equation developed herein. In the table each variable, besides load and slab thickness which are the two of most importance, respectively, is doubled independently of the remaining variables to show the effect of its change on deflection. The variables are presented in order of decreasing importance. Thus, the order of the relative importance of the variables is load, thickness, soil support, crack width, temperature differential, and crack spacing.

SUMMARY

This Phase I study on the performance of continuously-reinforced concrete pavement warrants the following conclusions:

1. The deflection of continuously-reinforced concrete pavement is a function of the load applied, crack width, crack spacing, temperature, pavement thickness and the supporting characteristics of the subbase and subgrade.
2. An empirical equation has been derived that enables a designer to approximate deflections in terms of the above enumerated parameters. A designer may then use the equation to prescribe a set of conditions that will insure the pavement deflection will be less than a desirable maximum.
3. The order of the relative importance of the variables is load, slab thickness, soil support, crack width, temperature differential, and crack spacing.
4. When measuring deflections the pavement should be "ironed out" three times before taking data.
5. Concrete pavements deflect in predictable patterns that can be measured with the Benkelman beam and Basin beam if proper precautions are taken.
6. From a deflection standpoint an optimum average crack spacing appears to be in a range of five to ten feet.
7. Deflection is a direct linear function of load and radius of curvature is an inverse linear function.
8. Radius of curvature calculations need not be corrected for slab temperature differential.

Needed Research

The equation presented herein is intended to represent the best utilization of the presently available knowledge and data concerning the deflection of continuously-reinforced concrete pavements. The deflection equations are empirical and must be used as such.

An attempt has been made herein to evaluate the support provided by the subbase and subgrade, but studies should continue on this and other variables such as weather and other environmental conditions. With the advanced data processing methods available today, vast amounts of data can be handled rapidly, thus facilitating the research minded who are interested in pushing back the frontier of pavement design.

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