The Strain in Rocks in Relation to Highway Design

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•IN HIGHWAY construction rock is often the foundation on which the highway is built. In the case of rock cut or a notch in a mountain slope the walls may also be rock. A tunnel may be completely in rock. The bed of the highway is often broken rock, as is the aggregate in the surfacing material.

To the designer, stability problems are involved in the site selection and in the rocks left in place after the cut or tunnel excavation is completed. There is a further problem in the stability of the rock fill to be used and yet another problem in the permanence of the aggregate surfacing, bonded or otherwise.

Another aspect often mistakenly left to the contractor is the use of the rock properties at the point of concern when planning the blasting patterns and the sequence of operations.

This paper will deal with some of the concepts of rock mechanics and their possible importance in highway design and construction.

ROCKS

A simple but useful concept of rock is that it is a granular material consisting only of grains and glue. There is nothing else involved. The grains may be small polycrystalline aggregate from a nucleation process or from a chemical precipitate, or they may be small fragments of other rocks. The glue may be ferruginous, calcareous, argillaceous, or siliceous material which cements the grains, or, in the case of an intergrowth of crystals, the outer layers of the crystals which are in contact with each other may be considered to act as a glue. Water may be a glue in a clay or a grain in a crystal structure. Air or other fluids or gases are grains if they occupy positions within a granular mass. Such grains are functional materials in the rocks, often chemically as well as mechanically. Most rocks have the grains arranged in some packing pattern and therefore have directional properties.

Rock may be described, then, as a granular aeolotropic heterogeneous technical substance which occurs naturally and which is composed of grains of varied polycrystalline or noncrystalline materials which are cemented together either by a glue or by a mechanical bond, but ultimately by atomic, ionic or molecular bonds within the grains and the glue and at every interface of bonding (1).

Primitive Strains

All technical materials if subjected to a change in load or a force of any kind tend to adjust to the force. Such adjustment may be a movement from one place to another, called a translation, or it may be a reorientation called a rotation, or there may be a change in shape or size as in an extension or compression of a material.

In rock all the adjustment must be taken by the grains and glue. The way in which adjustment occurs will depend on the material properties of the various grains and glues and will also depend on their packing pattern and on the time involved. Other papers (2, 3) have dealt with this and the concepts will be only summarized here. In general, permanent irrecoverable movement occurs as well as elastically recoverable movement.

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The amount of each will depend on the material properties, the time involved, the packing pattern and the kind of force field. The rate at which such a material can adjust without cracking will also depend on the material properties.

Rheology

Some simple rheological models may be useful here. No real material is perfect and all materials combine in some ratio the three characteristics of elasticity, fluidity and plasticity given by the rheological equations for perfect substances as follows:

1. $\sigma = E\epsilon$, the equation for a perfectly elastic Hooke solid;
2. $\sigma = \eta \frac{d\epsilon}{dt}$, the equation for a perfectly fluid Newtonian liquid; and
3. $\sigma = \phi$, the equation for perfectly plastic flow in St. Venant solids.

where

σ = unit stress;
ϵ = unit strain;
$\frac{d\epsilon}{dt}$ = rate of strain;
E = Young's modulus;
η = coefficent of viscosity;
ϕ = yield stress;
t = a parameter representing time;
ϵ_0 = unit strain at $t = 0$;
σ_0 = unit stress at $t = 0$; and
e = Naperian base = 2. 7183.

Then for a substance below the short time yield stress

$$\sigma = E\epsilon + \eta \frac{d\epsilon}{dt}$$

represented in the spring, mass, and dashpot arrangement in Figure 1. This is a Kelvin solid. Here the full elastic strain is not instantaneous but subject to time lag as shown by the solution to the equation

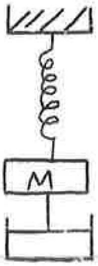

Figure 1. Kelvin solid.

$$\epsilon = e^{\frac{-E}{\eta}t}\left[\epsilon_0 + \frac{1}{\eta}\int_0^t \sigma e^{\frac{E}{\eta}t}\,dt\right]$$

if σ is constant,

$$\epsilon = \frac{\sigma}{E} + e^{\frac{-E}{\eta}t}\left[\epsilon_0 - \frac{\sigma}{E}\right]$$

and if $\sigma = 0$

$$\epsilon = \frac{\sigma}{E}\left[1 - e^{\frac{-E}{\eta}t}\right]$$

The elastic strain caused by a stress will reach a maximum only in infinite time and if the load is removed infinite time will be required for complete recovery of the elastic strain.

Also

$$\frac{d\epsilon}{dt} = \frac{\sigma}{\eta} + \frac{1}{E}\frac{d\sigma}{dt}$$

represented in Figure 2. This is a Maxwell liquid which has the solution

$$\sigma = e^{\frac{-E}{\eta}t}\left[\sigma_0 + \int_0^t E\, e^{\frac{E}{\eta}t}\frac{d\epsilon}{dt}\,dt\right]$$

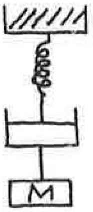

Figure 2. Maxwell liquid.

Here the rate of strain is a function of the total stress combined with the rate of stress application. Given any finite time to flow, the reaction includes flow. Given no time to flow, the reaction is elastic. The stress will vanish only in infinite time.

Given the Kelvin solid model, consider a local application of load such that a local finite mass of the material strains but influences the background mass of which it is a part to strain in the same direction at a rate proportional to the strain in the finite mass (Fig. 3). Then the finite mass will move a distance x while the background mass moves a distance y to adjust to the first movement such that

$$\frac{dy}{dt} = -\, r\, x$$

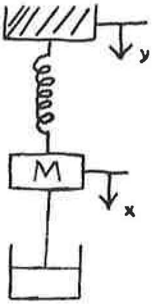

Figure 3. Kelvin solid with load.

Given

$$\sigma = E\,\epsilon + \eta\frac{d\epsilon}{dt}$$

this can be rewritten to conform to the mass-acceleration formula for such a system so that

$$F = E(x-y) + \eta\frac{dx}{dt}$$

and

$$M\frac{d^2x}{dt^2} + \eta\frac{dx}{dt} + E(x-y) = 0$$

then

$$M\frac{d^3x}{dt^3} + \eta\frac{d^2x}{dt^2} + E\frac{dx}{dt} - E\frac{dy}{dt} = 0$$

therefore

$$M\frac{d^3x}{dt^3} + \eta\frac{d^2x}{dt^2} + E\frac{dx}{dt} + E\,r\,x = 0$$

The characteristic equation is

$$\alpha^3 + \frac{\eta}{M}\alpha^2 + \frac{E}{M}\alpha + \frac{E\,r}{M} = 0$$

Let the roots be λ_1, λ_2, λ_3, or λ, $\mu - iv$, $\mu + iv$. For a cubic

$$\lambda + 2u = -\,A = -\,\frac{\eta}{M}$$

$$2\lambda\mu + u^2 + v^2 = B = \frac{E}{M}$$

$$\lambda(u^2 + v^2) = -\,C = \frac{-Er}{M}$$

The solution is $x = C_1 e^{\lambda t} + e^{\mu t} (C_2 \cos vt + C_3 \sin vt)$.

For stability both λ and μ must be zero or negative. Therefore A, B and C must all be positive. The Hurwic criterion for a stable third order system requires that $AB \geq C$, whence

$$r \leq \frac{\eta}{M} \text{ for equilibrium.}$$

Therefore when a load is applied (or removed) locally with respect to any volume of material the local deformation will propagate into the background volume and a progressive adjustment of the system will occur. Such adjustment will be acceptable or destructive depending on the value of r and on the properties of the material concerned.

It is known from tests that, in rocks, η decreases as the stress increases (4). For a given mass r must decrease when the stress is increased if the system is to reach equilibrium without destruction. This will depend to some extent on the shape of the mass as well as on the magnitude of the mass.

A fully restrained mass can accept a very large load and so develop a very low viscosity which results in a low r. The material can then adjust to a large deformation x without decoupling. For example, encapsulated water can support large loads. Triaxial compression tests are based on the use of a uniform shell of restraining pressure around the material undergoing test. The difference between the stress of restraint and the stress of the uniaxial applied causes eventual failure but the total load and the restraint are both so high that the viscosity of the material is low. Flow occurs before fracture.

When the restraint is small as in the case of the lateral surface of a sample in an ordinary compression test, η is large, r is large and the characteristic rate dy/dt will require failure to occur for a relatively small strain x.

Therefore, each material in a given transient state will have a characteristic rate of adjustment within which the system will tend to reach a new equilibrium. If the rate is exceeded the system will become unstable and failure will occur either by excessive deformation or by brittle rupture or by both.

Relaxation

Rock is a granular material containing grains of varied yield points, and the grains are often oriented or prestressed so that some tend to yield before others. It is readily apparent from rheological considerations that, if such a medium is loaded, elastic strain energy can be stored in the system and that, if unloaded, some of it will be recovered at once; some will be recovered over a period of time, but a large amount will not be recovered as long as the medium remains a coherent, interreacting, bonded mass.

For example, consider two coil springs of different length, or of different spring constants or of different axial orientations. If the springs are subjected to a given load, then embedded in a polymerizing plastic, a condition of transient equilibrium will be reached between the springs and the bonding plastic after the plastic has set. The springs will tend to push out to their original shape, but will do so in different amounts and perhaps in different directions. One might be in compression, the other in tension and the plastic between in shear. Very high energies can be conserved in such a system.

If a cut is made so that the geometry of the mass is affected, a new state of equilibrium will be required and movement will occur accordingly. Continued cutting would create continued movement.

All rocks have been subjected to changing loads and conditions during formation and since. A rock is, therefore, a product of its own history and all rocks must contain more or less conserved elastic strain energy. Its present condition is a transient one. Therefore when a new free face is created on a rock there will be movement toward a new state of equilibrium. The movement will be in two phases, an immediate elastic rebound largely volumetric, and a time-dependent further relaxation. This will occur first in the actual new surface and will consist of expanding, moving, and rotating grains. As the outer layer relaxes the next layer has less restraint and it will relax somewhat but not as much as the first layer. As relaxation progresses, unless the grains are completely decoupled, a strain gradient is established from the outside into a region where the original strain condition is approached. Relaxation is a reversal of the condition of strain originally imposed on the rock.

This can be useful to engineers because, if a rock sample of known orientation is cut and instrumented so that strain relaxation measurements can be taken continuously on three mutually orthogonal faces, then the relaxation patterns, the relative magnitudes of the relaxation strains, the statistical directions of the principal strains and the rates of relaxation are measurable (5).

Thus it is immediately possible to define the force field in relative magnitudes and directions acting on the rock mass in its present transient state. The strain ellipsoid and the strain trajectories can be plotted. The orientation of the planes along which the rock will break and their relative order of importance can be defined.

Because rocks are granular heterogeneous aeolotropic technical substances with packing patterns and with inherent strain patterns it is necessary to assess their engineering and rheological properties with regard to the location and the time at which the assessment is made.

The so-called engineering "constants" are not constant. Young's modulus is a variable. It is different in different directions in the same rock (6, 7), and it changes with load. As a substance is compressed under restraint it will eventually approach its limiting relative volume. If no further strain can occur E must be a very large number. It must vary under loading both up and down as the load fluctuates. The strain in a rock under a given load is different in the principal directions and so is E. Because of packing patterns there are stages in the loading of a rock when the volume must increase under increased load. This follows from the packing theory for granular media and direct measurements confirm the theory. Similarly Poisson's ratio is as much a function of grain wedging as it is of anything else. If the wedge surfaces are at 45 deg then a μ of 1 could be expected without consideration of plastic flow (3).

HIGHWAY FOUNDATION

A highway is seldom straight for great distances nor does it parallel the strike of the rocks in its foundation except locally on occasion. It passes over various kinds of rock in various states of stress and with various amounts of contained recoverable elastic energy. A rock cut enters a rock and penetrates into some deeper stratum, then back to the surface again during which it crosses well-defined strain gradients. A cut will have different conditions on each wall because of the rock characteristics. For example, the rock may dip out of one wall and into the other. The force field thrust may be out of one wall and into the other.

Roadbed on Horizontal Rock Strata

The simple case where some excavation of loose material is made and a roadbed is built on top of or near the top of a stratum of underlying rock in a more or less horizontal orientation is not always as simple as it seems. The rock and overburden originally form a system in a state of transient equilibrium. The removal of soil acts as a disturbance to the system. The importance of such a disturbance depends on the original state of strain and on the extent of the change in the force field involved.

On removal of restraint the rock surface will tend to relax. In a simple case this may happen with no real deterioration of the rock. There is always movement and it usually varies in direction and amount from place to place. This can result in differential roadbed movement and can cause damage to a rigid road surface. A more usual case is that relaxation and movement occur as noted but in addition the rock surface deteriorates through minute or other cracking, and this may change the planned drainage or some other detail so that the road maintenance problem is accentuated. In some serious instances removal of rock cover will permit the rock to relieve by arching upward. This may happen quickly or over a period of time but in any case it creates maintenance problems. Experience in these conditions has led to the use of rock-bolting, cut-off slots, prestressing and other techniques to permit the necessary movement to take place under control. The use of such aids involves the measurement of the rock characteristics first. This is relatively cheap and quick to do but is seldom done.

Roadbed on Dipping Strata

Any removal of restraint on a rock surface in which the strata dip will cause relaxation. If the strata bedding intersects the rock surface there will be differential relaxation on the different beds because they have different properties. If this is not taken into account in the road design then one section can move relatively to another depending on the orientation of the road with respect to the strata outcrop. This can result in shear or tension cracking in the bed and in the road surface. Sometimes this has been mistaken for bed settlement except where the failure has included an increase in elevation of a point on the road. Under compression a section of the road may fold or shear or both.

Where the road is on the strata so that both bed and strata slope together and restraint is removed, there is set up a down-dip component of strain because of the tendency of the rock to flow in a down-dip direction under its own weight. This is often translated into a slump type of deformation which affects the road. However, a small elastic strain in a down-dip direction can be serious as far as the road surface is concerned. A strain of 100 microstrains can amount to a relative movement of 1.2 inches in a thousand feet. A strain of 1000 microstrains will multiply this figure accordingly. Rock relaxation of amounts in excess of 1000 microstrains are common and a down-dip component usually involves strains of this order or greater in hard rock.

If the road foundation involves cutting a notch in a slope, for example on a hillside, then the whole movement may be greatly increased. Most mountain slopes by their nature are critical or nearly so. Weathered hills are barely subcritical as far as the rocks are concerned. The angle of slope is not a function of the angle of repose of the broken rock but of the state of strain in the whole rock mass and of the material properties of the rock. Again no rational design is possible without some measured data.

TUNNELS

The design of a tunnel should consider both cost and maintenance. It is analogous to the design of a mine opening.

If the axis of the tunnel is on any other orientation than the thrust direction in the rocks then there will be a rotative movement on the tunnel as well as a tendency for the roof to shear past the abutment on the side opposite to the thrust and in the direction of the thrust component.

The expected movement in a tunnel will consist of a series of related movements. The floor will tend to relax and so will the walls and roof. This will take the form of bending longitudinally in a vertical plane, bending laterally in a horizontal plane and rotating laterally in cross section with attendant tension cracks, shears, and compressive spalling. The method of mining, the method of restraint, the design of the lining, if any, the drainage and the orientation of the tunnel will all depend on the rock conditions. These details are well recognized in the design of mines and details of one case have been published (8). Both laboratory and in situ measurements are required.

ROAD SURFACE

Surfacing materials are usually rock fragments bonded or otherwise. The fragments are characteristic of the rock mass from which they originated. All contain some strain energy.

If the aggregate is crushed rock the act of crushing will have relieved the particles of some of their elastic energy because of rebound. If the rock is piled before use the time-dependent strain energy will be restrained and will not completely relieve. Relief will continue in the road surface.

In some rocks such relief serves to crack up the fragments further and so cause deterioration of the aggregate. If the relief is considerable a form of growth concrete results. This is not necessarily chemical although chemical reaction may accompany the relaxation. Strains transferred to the enclosing cement from the expanding aggregate can cause surface spalling and attendant deterioration of the concrete.

Limestones are often sufficiently varied in their inherent strain retention so that one layer will be strongly reactive while an overlying or underlying one will not be. This is particularly so in the Paleozoic limestones in the area near Kingston, Ontario, for example.

Another aspect of roadbed preparation is the fill used and the way in which it is placed. In any rock pile, particularly freshly fragmented rock, there are two reactions of importance. The first is the reaction of relaxation on those surfaces not under load. These surfaces tend to exfoliate and to spall in relaxation. Such reactions may disintegrate the rock in a period of time and slump will occur.

The second reaction is something over which some measure of control can be exerted. At every contact point in a rock fill pile there will be a stress concentration. In a rock already subject to relaxation on the free faces the stress concentration can increase the tendency to spall and to break down the rock pieces. In rocks where relaxation is not so important there will be the problem of deterioration of the rock pieces at the corners of contact. In both cases slump will occur.

The slump can be reduced somewhat if attention is paid to the condition of placing the fill. If fill is placed by tipping off the top, then there is formed a natural stratification of successive layers inclined at the angle of repose. There is also a tendency for large lumps to segregate in rolling. Pressure along the strata will cause more yielding than pressure normal to it. If the fill is placed in horizontal layers and compacted during placement, the slump from both relaxation and stress concentration will be minimized.

MEASUREMENTS

Most of the information required to define the problems mentioned above can be obtained by direct measurements either on laboratory samples or in the field. Usually both are required. The selection of a highway site and its local development will be a compromise based on nontechnical as well as technical information but as much information as is reasonably possible should be available and should be integrated into an optimum design. In the matter of rock mechanics it is required to determine the existing condition of the rocks and then to calculate what will happen to them under the new conditions imposed by the highway construction and operation. A further problem is then to decide on any possible remedial measures to improve the performance of the structure.

Because the techniques of rock mechanics can generally make use of the same surface exposures, drill cores, and underground openings used by the geologist, it is good practice to have the structural geologist and the rock mechanics engineer work in close cooperation.

Laboratory Tests

Much information can be derived from laboratory tests of samples with known orientation. Oriented hand samples from selected points on surface and underground exposures are cut and instrumented on three mutually orthogonal planes. From the relaxation characteristics indicated by the instruments (the writer makes use of photoelastic plastic), it is possible to record: (a) statistical directions of the principal strains on each surface, (b) orientation and distribution of the planes of preferred shear, (c) general pattern of distribution of the elastically recoverable strains on each surface, (d) relative magnitudes of the principal strains on each surface and from specimen to specimen, and (e) relaxation strain-time curves for each face.

From these tests the strain trajectories in any plane in the rock mass can be plotted directly. If the statistical directions of the principal stresses are assumed to be the same as the statistical directions of the principal strains, then the strain trajectories are the stress trajectories. The principal planes are then readily identified and the relative strain energy distributed over them can be estimated. Areas of increasing strain and of decreasing strain are readily identified. Anomalous local conditions will be evident if this occurs.

The directions and orders of magnitude of the planes of preferred shear are important. These are the surfaces along which the rock will break most easily because they represent regions in which shear strain energy is concentrated. Additional load applied to increase the shear strains will cause breakage, but applied to oppose the shear strain, will reduce breakage.

Relative magnitudes and directions of the principal strains from surface to surface and from specimen to specimen will establish the direction of the current thrust in the operative force-field and will disclose strain gradients within the mass. Tunnel design requires this information specifically.

The relaxation strain-time curves give information on the speed of elastic recovery and can be used to estimate the characteristic rate of propagation of strain in the rock. This is important in planning sequences.

Laboratory tests usually include compressive, flexural and vibrational analyses. These do not supply any absolute quantities but do indicate the degree of aeolotropy in the rock and allow comparisons to be made. Such tests must be made with careful attention to the direction of the specimen axes. Relative conditions of failure, degrees of flow, relative no-load moduli and relative coefficients of viscosity can be obtained from these tests. Such relative figures will have validity in the field if account is taken of the effects of mass, restraint, and geometrical configuration.

In Situ Tests

Any laboratory sample by the act of preparing it is changed to some extent. The most important change is the loss of elastic rebound energy. It is generally good policy to measure both the elastic rebound and the time-dependent relaxation on the same sample on some systematic basis. This is easily done by a simple over-coring technique using a diamond drill to overcore a plastic disc previously cemented to the bottom of a hole of smaller diameter. Rebound and the following relaxation are both recorded on the same disc.

In tunnels or road cuts it is desirable to measure the amount of stress multiplication at corners. It is also useful to measure the direction of thrust and the change in rate, magnitude, and direction of the force-field affected by the working. This can be done with rock bolt dynamometers, bore hole gages or surface gages.

A useful series of measurements can be made cheaply and quickly with seismic equipment. Two kinds of information become available. From the travel times of seismic waves the thickness and condition of various rock layers can be estimated. In addition, if used in conjunction with laboratory knowledge of the directions of principal strain and the planes of preferred shear, the travel time information can be used to design blasting patterns with high productivity and low energy ratios. In several instances within the experience of the writer a very simple in situ measurement has resulted in a radical improvement in the excavation costs and character.

Seismic methods are sometimes used to determine moduli but it should be remembered that from rheological considerations a seismic test is usually a short-time test and that the rock affected by it will behave more elastically if given no time to flow, whereas in actual operation flow is usual. Therefore the moduli found will be too high except for calculating other dynamic effects. Such a modulus must be corrected for changes of load and must be used only with regard to the orientation of the direction in which it was measured.

A new procedure in use and under test for the past two years involves a combination of a bore hole gage and seismic techniques. The writer has used a high compliance photoelastic plastic plug which is glued into a bore hole at any depth desired. The plug has its own light source embedded in it and the biaxial strain field normal to the plug axis is represented by a fringe pattern resulting from the retardation of the light at the source by the strained plastic. Such a pattern gives on inspection the direction of principal strains and their relative magnitudes. If viewed from time to time any changes can be recorded.

Viewing of the plug can be direct with a telescope at distances of up to 100 ft, or as long as bends in the hole do not cut off the view. At greater depths a small bore-hole camera will record the picture and its orientation in color. The strain can be monitored at any time with a photocell and a recorder.

An additional advantage of the plastic bore hole plug is that it will also register any seismic disturbance. A photocell scanner will plot the frequency and amplitude of a series of vibrations set up by any mechanism. If a shot point is used then the time of arrival as well as the wave form characteristics and damping can be directly determined. The plug remains at the same spot and it will continue to record any time-dependent changes. If at a later date another blast is detonated at the same point as before, any change in rock characteristics since the last record will be indicated by a different reading characteristic in the gage. Some gages have now been in continuous use for three years but improved models have been used for the last two years.

It is perhaps noteworthy that the same gage and also a low-compliance high-modulus photoelastic glass plug have been used successfully to monitor the strains in concrete structures by highway engineers in England.

In situ measurements are warranted prior to construction because of the low-cost, highly reliable design data that can be obtained by these techniques. Similar measuring devices are also useful during and after construction to test the validity of the calculations and of the predictions based on earlier measurements. If everything is seen to move or strain at the predicted rates and in the predicted amounts and directions, then the structure is under control. If any unexpected condition occurs then the structure is out of control. However, the engineer will at once be aware of the problem and remedial measures may be taken well ahead of catastrophe. The effects of the remedy can be seen on the same instrument.

CONCLUSIONS

It has been the intention to point out some of the problems in highway design caused by rock conditions and to indicate the nature of simple tests that can be used to provide data to enable a more rational design to be developed where rock is concerned.

Recent applications of the instrumentation used by the writer have provided good results in England and in Canada on highway structures, both concrete and rock. The cost is low and the procedure can be operational at the site. There seems to be no valid reason for not making use of present knowledge in rock mechanics. In this respect there are now many mines in the USA and elsewhere with the design based on the rock mechanics theories mentioned here. The theories are no longer academic hypotheses but are working theories. The instruments are proven and operational. It is time to apply this knowledge on the design of any functional structure in or on rock.

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