A Numerical Method of Orientation for the Kelsh Plotter

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•IN LARGE-SCALE photogrammetric plotting, particularly in the field of highways, the Kelsh Plotter is one of the most commonly used instruments in North America. Although recent technical development has gone far in the design of new instruments, ranging from small refinements to automatic plotting devices, and while more and more photogrammetric procedures, from instrumental to analytical aerial triangulation, are being used to obtain photogrammetric model control, the bulk of large-scale photogrammetric plans is still produced on the Kelsh Plotter.

At present a significant amount of work time is still being spent on model orientation. The ratio of time spent for orientation vs plotting may vary from 1:4 to 1:10, depending on the amount of detail in the model, while the ratio of time spent for orientation vs cross-sectioning for highway work may amount to 1:1 or 1:2.

It is therefore appropriate to suggest how the time spent for model orientation may be improved by other methods of orientation. One disadvantage in present trial-and-error procedures is the necessity of repeating the orientation procedures for relative and absolute orientation a number of times before y-parallaxes are removed and before scale and elevation differences are eliminated between model and control points. There is no doubt that a faster convergence for the relative orientation procedure and a direct setting of absolute orientation elements can be made possible by numerical orientation procedures.

Numerical orientation procedures as yet have not been applied to Kelsh Plotters since the instrument itself has no scales for the direct measurement of projector translations or rotations. Suggestions in the past have thus been along the lines of incorporating counters for projector tip, projector tilt and projector swing in the instrument, as well as making provisions for small projector translations perpendicular to the base in y and z directions. This implies the need for modification of the plotter or for a new instrument, entailing additional expense which may not be in proportion to the advantages gained.

This paper makes suggestions how numerical orientation can be applied to Kelsh Plotters without modifications of the instrument. Such numerical orientation can be applied to (a) relative orientation according to measured parallaxes, (b) absolute orientation according to control points, and (c) setting of precomputed orientation elements, determined from instrumental or analytical aerial triangulation. The latter two applications will be the most economically significant, while the first will be of interest in areas where trial-and-error relative orientation has a slow rate of convergence, such as in hilly terrain.

The introduction of any numerical orientation method will require three conditions:

1. The possibility of measuring or obtaining initial data with which calculations are to be performed. These are the y-parallaxes in the case of relative orientation, the scale and height differences for absolute orientation and the camera vector for the setting of orientation elements.

Paper sponsored by Committee on Photogrammetry and Aerial Surveys.

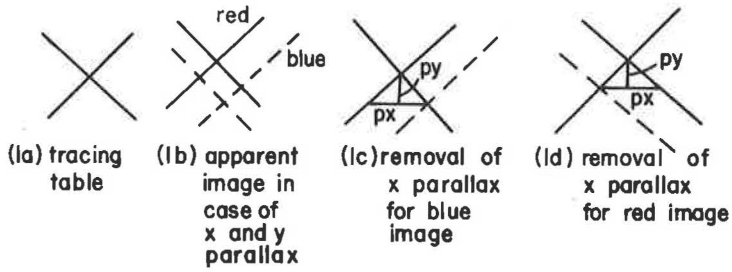


Figure 1. The use of a diagonal cross for parallax measurements.

2. The calculation of tip, tilt, swing and base length in the Kelsh Plotter system out of y-parallaxes, scale and height differences or coordinate transformations of the camera vector.
3. The possibility of introducing calculated model rotations into the instrument.

Since there is no device on the Kelsh Plotter projectors to make measurements, these have to be made on the tracing table. It is obvious that model parallaxes can be measured by dropping left and right images of identical identifiable points onto a base sheet. Linear measurements will be possible by ordinary scales with graphical accuracy (±0. 3 mm).

It is also possible to introduce model rotations as linear displacement of a point in x or y direction. Knowing the height of projection h, a model rotation can be transformed into a linear distance Δdx or Δdy by computation. An identifiable object can be dropped onto the base sheet after the floating mark has been brought to coincide with it. The linear distance can then be scaled off in the appropriate direction on the base sheet. The tracing table can be moved by this distance and the image is then brought to coincide with the floating mark using the rotational element.

By choosing a point close to the principal point, expressions for tilt in flight direction (ϕ) and perpendicular to it (ω) are found as

$$\operatorname{tg} \phi = \frac{\Delta dx}{h} \text{ and } \operatorname{tg} \omega \frac{\Delta dy}{h}$$

Kupfer (4) originally suggests this procedure for the simpler case of multiplex orientation procedures. Similarly, if b signifies a distance in x direction away from the rotational center, and $\varkappa$ is the swing,

$$\operatorname{tg} \varkappa = \frac{\Delta dy}{b}$$

Assuming a graphical accuracy of ±0. 2 mm for the measurement of a point, a distance is determined with $\pm 0.2\sqrt{2}$ mm ≈ 0.03 mm accuracy. This results, for a Kelsh Plotter height of 720 mm, in a $d\phi$ and $d\omega$ of $\pm 2.6^c$ (or ±1. 4′) and, since $b/h \approx 2/3$, in 3/2 of the amount of $d\varkappa$.

NUMERICAL RELATIVE ORIENTATION

Even though a direct linear measurement of y-parallaxes to ±0. 3 mm would be possible by use of the Kelsh Plotter tracing table, it is more convenient and more accurate to use a diagonal cross (2) for the measurement of parallaxes (Fig. 1).

The cross (Fig. 1a) is superimposed or drawn on the tracing table of the Kelsh Plotter in such a way that its lines are pointed 45° off the x and y axes of the instrument. If both x and y parallaxes are present the eyes will try to merge the images with the result that two apparent images of the cross will be seen stereoscopically (Fig. 1b) in the form of two nonintersecting lines in space. The floating mark can be raised and lowered to the height of both lines (Fig. 1c, 1d) and the height difference, dh = (h/b) (px) can be measured on the tracing table counter, preferably in units in which py is to be expressed, according to

$$py = \frac{px}{2} = \frac{b}{2h} \cdot dh$$

Assuming a c-factor (= $3M_H$) of 1600 for the Kelsh Plotter and a ratio of 3:1 for the accuracy increase for points over contours ($M_H = 3m_H$) one obtains for the standard error of y-parallax measurement

$$m_{py} = \pm \frac{b}{2h} \cdot m_H = \pm \frac{m_H}{3} \approx \pm 0.02 \text{ mm}$$

which means a 15-times accuracy increase.

For computation of orientation elements one can use any existing numerical formulation, such as those by Hallert (1) or Jerie (3), with the appropriate sign changes for a chosen coordinate system.

For the system shown in Figure 2 the parallax formulas for independent pair relative orientation (the Kelsh Plotter has no by and bz) defined as corrections

$$py = dy_L - dy_L$$

become

$$py = x d\kappa_L + (x-b) d\kappa_R - h\left(1 + \frac{y^2}{h^2}\right) d\omega_R + \frac{xy}{h} d\phi_L - \frac{(x-b)y}{h} d\phi_R$$

From these, orientation formulas can be derived. As an example, Hallert's orientation formulas for measured parallaxes at 6 points (assuming a constant h) become

$$d\phi^c_L = \frac{\rho^c h}{2bd}(py_3 - py_5)$$

$$d\phi^c_R = \frac{o^c h}{2bd}(py_4 - py_6)$$

$$d\omega^c_R = \frac{\rho^c h}{4d^2}\left[2(py_1 + py_2) - (py_3 + py_4 + py_5 + py_6)\right]$$

where $\rho^c = \frac{20000^c}{\pi}$.

There is usually no need to calculate $d\kappa^c_L$ and $d\kappa^c_R$, since these are more easily introduced by parallax elimination. Their formulas would be

$$d\kappa^c_L = -\frac{\rho c}{3b}\left[(py_2 + py_4 + py_6) + \left(3h + \frac{2d^2}{h}\right)\frac{d\omega^c_R}{c}\right]$$

$$d\kappa^c_R = -\frac{\rho^c}{3b}\left[(py_1 + py_3 + py_5) + \left(3h + \frac{2d^2}{h}\right)\frac{d\omega_R{}^c}{c}\right]$$

Then $+d\phi_L$, $+d\phi_R$, $+d\omega_R$, $+d\kappa_L$ and $+d\kappa_R$ can be computed and introduced as $+d\Delta x$ at point 1 (with ϕ_L), $+d\Delta x$ at point 2 (with ϕ_R), $+d\Delta y$ at point 2 (with ω_R), $-d\Delta y$ at point 2 (with κ_L), $+d\Delta y$ at point 1 (with κ_R).

Computation will be quite fast if the constant factors are precalculated and if a form is used in conjunction with a desk or hand calculator. The orientation elements will be obtained for $m_{py} = \pm 0.02$ mm with the following accuracy:

$$m^c_{d\phi_R} = m^c_{d\phi_L} = \pm \rho \frac{h}{bd} m_{py} = \pm 0.4^c$$

$$m^c_{d\omega_R} = \pm \rho^c \frac{2h}{d^2} m_{py} = \pm 0.8^c$$

$$m_{d\varkappa^c_L} = m_{d\varkappa^c_R} = \pm\rho^c \frac{m_{py}}{d^2 b} \sqrt{\frac{3}{2} h^4 + 2h^2 + d^4} = \pm 4^c$$

Considering that the most serious model deformation is introduced by $d\omega$ and that $d\omega$ can be set to $\pm 2.6^c$, $m_H \max = (d \cdot d\omega^c)/(\rho^c) = \pm 0.19$ mm, or 3 times the standard error of height measurement for a point. This means that one will be able to set the relative orientation elements by the outlined procedure within the c-factor limitation, but not as accurately as by careful trial-and-error procedure. The use of numerical relative orientation may therefore only be of advantage in mountainous models (where elevation differences amount to more than 3 percent of h), for which Jerie's orientation formulas (3), despite the more extensive computations, will lead to a faster and, in practice, equally accurate relative orientation.

NUMERICAL ABSOLUTE ORIENTATION

Absolute orientation, consisting of scaling and leveling of a model, is performed numerically by a comparison of model coordinates x_i, y_i, z_i, x_j, y_j, z_j, . . ., and ground coordinates X_i, Y_i, Z_i, X_j, Y_j, Z_j, . . ., of points $p_i p_j$. To find the proper scale it is necessary to introduce a base correction db to the base b (Fig. 2)

$$\frac{1}{\lambda} = \sqrt{\frac{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}{(X_j - X_i)^2 + (Y_j - Y_i)^2 + (Z_j - Z_i)^2}}$$

Suppose $1/\lambda'$ is the proper scale to be introduced. Then

$$db = b\left(\frac{\lambda}{\lambda'} - 1\right)$$

For the purpose of introducing db on the base bar carrying the projectors it is useful to attach or establish a reference mark, for example, by a needle scratch on the side portion of the base bar, and to measure distances to the projector chosen to be moved by a metal tape or any other suitable scale.

For the determination of common tilt in x direction, Φ, and common tilt in y direction, Ω, usually height differences are measured at the given control points (5).

If Δh_i denotes the difference, actual height of the point minus model height, both expressed in the same units in which measurements are made in the model, then one obtains for a geometrical configuration as shown in Figure 3:

$$\frac{\Delta h_R' - \Delta h_L}{sx} = \tan \Phi$$

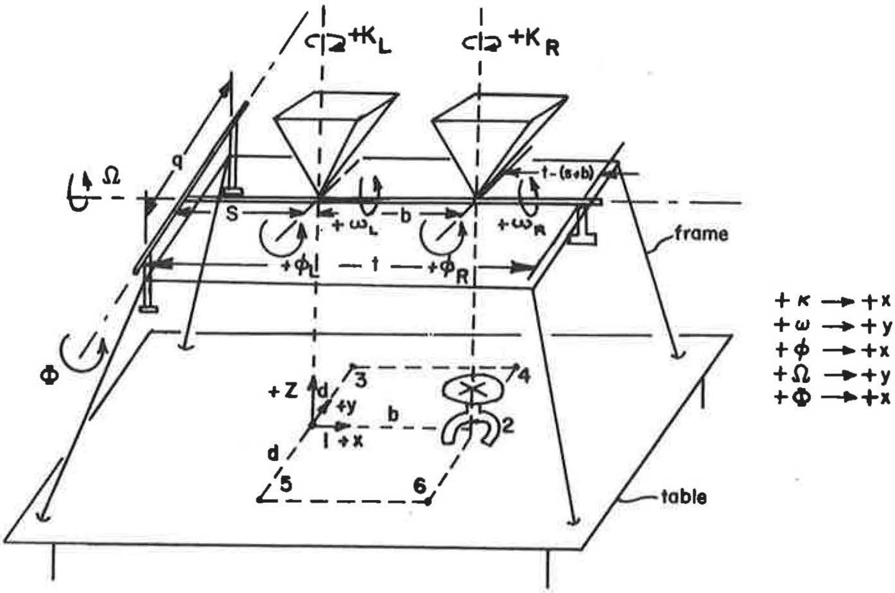
$$\frac{\Delta h_U - \Delta h_D'}{sy} = \tan \Omega$$

$\Delta h_R'$ and $\Delta h_L'$ are interpolated proportional to their distance between Δh_U, Δh_D and Δh_L, Δh_D, respectively.

If relative orientation was performed with the base bar in level position (it can be made level using a level bubble), then Ω can be introduced by moving an object located along the line between principal points by an amount $d\Delta y$ in the appropriate direction using ω_L (or ω_R) and eliminating the y-parallax by ω_R (or ω_L, respectively):

$$d\Delta y = h \cdot \tan \Omega = h \cdot \frac{\Delta h_U - \Delta h_D'}{sy}$$

Φ can be introduced, as seen in Figure 4, by



t = 890 mm
q = 430 mm
s to be set to a fixed distance of 210 mm or 220 mm
(as a mean spacing on the bar for wide angle photography)

Figure 2. Definition of coordinate axes and axes of rotation for the Kelsh Plotter.

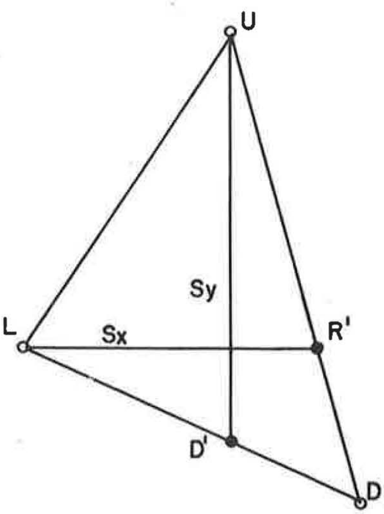


Figure 3. Absolute orientation points L,U,D, and constructed points R′ D′.

$$d\Delta x_L = \left(\frac{s}{\cos\phi}\right) + h \tan\Phi$$

if a point close to or along a line with the same x-coordinate as the left principal point is used, and by

$$d\Delta x_R = \left(\frac{s+b}{\cos\phi} - s - b\right) + h \tan\Phi$$

if a point close to or along a line with the same x-coordinate as the right principal point is used; s can be conveniently set to a fixed distance of, e.g., 210 mm (for reasons of symmetry) for all models.

On the multiplex or the balplex a common Φ rotation can of course be more conveniently replaced by individual ϕ rotations, since corrections $dbz = b \sin\Phi$ and $dbx = b(1 - \cos\Phi)$ can be made for these instruments.

SETTING OF PRECOMPTUED ORIENTATION ELEMENTS

Aerial triangulation techniques today and in the future will provide more and more control for absolute orientation of the models. Since, in the aerial triangulation process and its adjustment, photographs are computationally or physically oriented with respect to their relative space position,

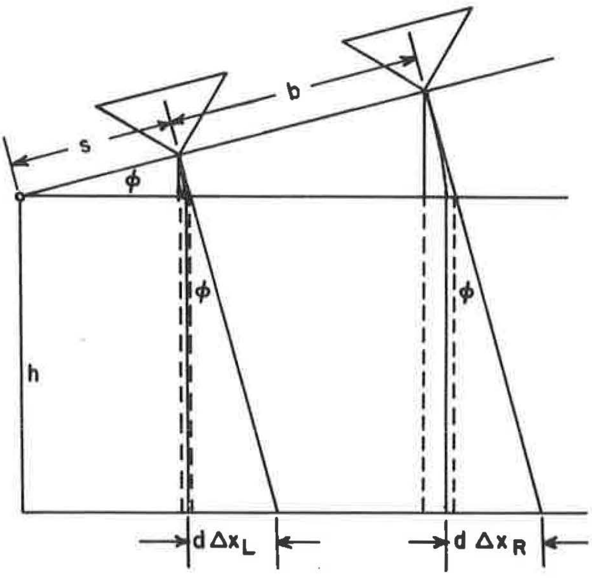


Figure 4. Introduction of Φ by $d\Delta x_L$ or $d\Delta x_R$.

it is of interest whether such orientation data can be computed and directly introduced into the Kelsh Plotter as elements of relative and absolute orientation, as a preliminary approximation by using unadjusted instrument or computation values or as computed final values derived from the adjustment.

To introduce data from instrumental and also analytical aerial triangulation, 3 problems must be solved:

1. The gimbal system of axes (primary, secondary, tertiary) of the triangulation instrument, or of the system used for formulating the analytical triangulation, has to be converted into the Kelsh Plotter system (ϕ-axis primary, ω-axis secondary, $\varkappa$-axis tertiary).
2. The vectors representing the space position of the camera axis as a function of tilt, tip and swing have to be converted together with the xyz coordinates of points from the unadjusted to the adjusted values.
3. The adjusted values for tilt, tip and swing have to be rotated parallel to the airbase, since no possibility exists of introducing actual bz and by components on the Kelsh Plotter.

The problems can be solved in the following way:

1. Photo coordinates $x'y'f$ can be converted into ground coordinates xyz by aid of an orthogonal rotational matrix:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x' \\ y' \\ f \end{pmatrix}, \text{ or } X = A\,X'$$

The coefficients a_{11} to a_{33} are functions of the sequential rotations ϕ, ω, $\varkappa$. The sequence of rotations in A can, for example, be defined as $A = A_\omega \cdot A_\phi \cdot A_\varkappa$ or as $\bar{A} = A_\phi\, A_\omega\, A_\varkappa$ in which A_i is of the form

$$A_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{pmatrix}$$

with i ranging from ϕ to $\varkappa$, taking into account the appropriate positive or negative sense of the rotation i.

If the system in which the triangulation was performed is given by A and the Kelsh Plotter system by $\bar{A}$, it is only necessary to set the individual elements $a_{11}/\bar{a}_{11}$, $a_{12}/\bar{a}_{12}$ etc., into proportion in order to determine expressions of $\bar{\phi}$, $\bar{\omega}$ and $\bar{\varkappa}$ in terms of ϕ, ω,' $\varkappa$ (1).

2. To the values $\bar{\phi}$, $\bar{\omega}$, $\bar{\varkappa}$, corrections $\Delta\phi$, $\Delta\omega$, and $\Delta\varkappa$ have to be added because of the triangulation adjustment; $\Delta\phi$, $\Delta\omega$, $\Delta\varkappa$ should be in the same sequential system as used for determination of $\bar{\phi}$, $\bar{\omega}$, $\bar{\varkappa}$. However, differences between a Eulerian system as ordinarily used in the strip adjustment of aerial triangulation will be negligible, taking into account the magnitude of the corrections.

In a least squares adjustment $\Delta\phi$, $\Delta\omega$, $\Delta\varkappa$ are directly computable, and they can also be found in an interpolation adjustment by differentiation of the polynomials used. If

$$\Delta x = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4xy + a_5y$$

$$\Delta y = b_0 + b_1x + b_2x^2 + b_3x^3 + b_4xy + b_5y$$

$$\Delta h = c_0 + c_1x + c_2x^2 + c_3x^3 + c_4xy + c_5y$$

then

$$\tan \Delta\phi = \frac{\partial \Delta h}{\partial x} = c_1 + 2c_2x + 3c_3x^2 + c_4y$$

$$\tan \Delta\omega = \frac{\partial \Delta h}{\partial y} = c_4x + c_5$$

$$\tan \Delta\varkappa = \frac{\partial \Delta y}{\partial x} = b_1 + 2b_1x + 3b_3x^2 + b_4y$$

Finally the corrected coordinates $\bar{\bar{\phi}}$, $\bar{\bar{\omega}}$, $\bar{\bar{\varkappa}}$ become

$$\bar{\bar{\phi}} = \bar{\phi} + \Delta\phi$$

$$\bar{\bar{\omega}} = \bar{\omega} + \Delta\omega$$

$$\bar{\bar{\varkappa}} = \bar{\varkappa} + \Delta\varkappa$$

3. The rotations $\bar{\bar{\phi}}$, $\bar{\bar{\omega}}$, $\bar{\bar{\varkappa}}$ have to be transformed into rotations ϕ^*, ω^*, $\varkappa^*$ which, together with a common rotation Φ, can be introduced into the Kelsh Plotter. If the rotation is to be performed by an azimuthal rotation α and a common tilt Φ, then

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\Phi & 0 & -\sin\Phi \\ 0 & 1 & 0 \\ \sin\Phi & 0 & \cos\Phi \end{pmatrix} \cdot A_\phi \cdot A_\omega \cdot A_\chi \begin{pmatrix} x' \\ y' \\ f \end{pmatrix}$$

or

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A_\alpha \cdot A_\Phi \cdot A_\phi \cdot A_\omega \cdot A_\varkappa \begin{pmatrix} x' \\ y' \\ f \end{pmatrix} = A^* \begin{pmatrix} x' \\ y' \\ f \end{pmatrix}$$

It is now again possible to set $a^*_{11}/\bar{\bar{a}}_{11}$, $a^*_{12}/\bar{\bar{a}}_{12}$, etc., into proportion in order to calculate ϕ^*, ω^*, κ^* out of $\bar{\bar{\phi}}$, $\bar{\bar{\omega}}$, $\bar{\bar{\kappa}}$. It is to be remembered that α and Φ to be used in the expressions are known beforehand and can be calculated from

$$\tan \alpha = \frac{by}{bx}, \quad \tan \Phi = \frac{bz}{\sqrt{bx^2 + by^2}}$$

The calculation of Φ, ϕ^* ω^* and κ^* can be made for each successive exposure station. Thus ϕ^*_L, ω^*_L, κ^*_L, ϕ^*_R, ω^*_R, κ^*_R and Φ can be introduced in the following way: First the base bar is leveled in x direction as well as the projectors in x and y direction. Then Φ is introduced, and subsequently ϕ^*_L to κ^*_R, by calculating and introducing $d\Delta x$ and $d\Delta y$ displacements as outlined previously.

CONCLUSION

Methods have been discribed by which numerical orientation can be applied in the Kelsh Plotter. It is not attempted here to give a generalized practical evaluation of these methods, since local conditions may influence the efficiency with which numerical or trial-and-error procedures can be carried out. The numerical calculation necessary can easily be done on the slide rule for relative and absolute orientation, and the precomputation of final instrument settings from aerial triangulation can be incorporated into the adjustment which in most cases is performed on an electronic computer. In view of this, substantial time savings can be expected from numerical orientation methods.

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