Application of Three-Layer System Methods to Evaluation of Soil-Cement Bases

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A series of bearing tests on soil-cement pavements at several airports of the state of São Paulo, was made by the Instituto de Pesquisas Tecnologicas, University of São Paulo, Brazil, with the main objective of assessing the load capacity of the pavements. The findings and conclusions of this study have been reported in a previous paper. As a by-product of the study, an evaluation was made of the effective in-place elastic modulus of soilcement bases by the application of the elastic layered theory. The pavements studied are basically three-layer structures composed of soilcement base, granular soil subbase, and natural subgrade. A thin asphaltic-concrete wearing course was built after the bearing tests were performed. A first tentative analysis was made by two-layer methods, considering the two lower layers as a single layer of equivalent elastic properties, but this analysis gave erratic and unrealistic results. A new method of analysis was then developed for the interpretation of the bearing test data by the use of three-layer elastic theory, which yielded consistent and workable results. This paper reports the latter method of analysis.

A summary of the design and construction data of the airport pavements, characteristics of materials, details of tests, and reports of test data are given. The process of testing was incremental-repetitive loading, with rigid plates of three diameters. The soil-cement base modulus of reaction K (load-deflection ratio) was nearly constant and elastic for each plate for all loads after the first loading in every test. This load-deflection ratio was taken as a characteristic mechanical parameter of the pavement.

The load-deflection pattern of the pavements by the three-layer elastic theory is interpreted, and the soil-cement modulus of elasticity is evaluated. This paper presents a practical application of the three-layer deflection factor tables computed by Jones (2). The proposed method of analysis is believed to be accurate and dependable. It is, however, affected by the scatter of the field measurements. It was found that the effective elastic modulus of the subgrade underneath the pavement's structure is much greater than the value obtained by direct loading tests on the subgrade. The so-called "equivalent" single layer for substituting the two lower layers is not a valid concept for the three-layer structure. A reasonably good correlation was observed between theoretical and experimental curves of the load-deflection ratio against the inverse of plate radius. A nonlineal relationship was observed between the load and perimeter-area ratio, which follows the elastic theory.

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•THE THEORY of elastic layered systems was proposed by Burmister in 1943 (1), and was subsequently developed by others. Burmister (1) presented numerical computation of the deflection factor for two-layered systems, for the usual values of the significant parameter, in graphic form. The numerical computation of the deflection factor for three-layered systems was presented in tabular form 20 yr later, by Jones (2). Table 1 gives some typical values taken from Jones' tables. At present there is no published

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TABLE 1

$E_1/E_2 = n_1$	$E_2/E_3 = n_2$	$h_1/h_2 = H$	$h_2/r = \frac{1}{A}$	A	F	F
20	20	0. 25	1. 25 2. 5 5. 0	0.8 0.4 0.2	139.3 75.18 39.50	0. 232 0. 125 0. 066
20	20	1	1.25 2.5 5.0	0.8 0.4 0.2	67.97 35.10 18.46	0. 113 0. 059 0. 031
20	2	0. 25	1.25 2.5 5.0	0.8 0.4 0.2	36.69 23.22 13.44	0. 612 0. 387 0. 224
20	2	1	1.25 2.5 5.0	0.8 0.4 0.2	15.75 8.922 5.362	0. 263 0. 149 0. 089
2	20	0. 25	1.25 2.5 5.0	0.8 0.4 0.2	16. 98 9. 944 6. 015	0. 283 0. 166 0. 100
2	20	1	1.25 2.5 5.0	0.8 0.4 0.2	10. 92 6. 395 4. 083	0. 182 0. 107 0. 068

VALUES OF DEFLECTION FACTOR FOR THREE-LAYERED ELASTIC SYSTEMS WITH PERFECT CONTINUITY AT INTERFACES^a

^aData derived from Ref.2. All interpolations are to be computed graphically on b^{log-log paper.} Poisson's ratio = 0.35;

 \overline{F} = Jones deflection factor; and

F = normal deflection factor.

Deflection:

 $\mathsf{D} = \frac{1.5 \text{ p r}}{\mathsf{E}_3} \cdot \mathsf{F} \quad \text{or} \quad \mathsf{D} = \frac{\mathsf{p} \mathsf{r}}{\mathsf{E}_1} \cdot \overline{\mathsf{F}}$ Relationship between two deflection factors: $F = \overline{F} \cdot \frac{1}{1.5 n_1 n_2}$

computation of the deflection factor for systems of more than three layers by the elastic theory. Such computation would be exceedingly complicated because of the great number of significant parameters necessary in a general solution. To analyze pavements of more than three layers by the elastic theory, it is necessary to reduce them to a three-layer model, combining similar adjacent layers. This expedient affords a reasonable degree of analogy between the theoretical model and the real multilayer system. If a further reduction is made from a multilayer system to a two-layer model, great discrepancies may arise, depending on the values of the parameters.

The deflection under load of a uniform elastic medium, which could be considered as a one-layered system, is calculated by the Boussinesq-Love equations.

Figure 1 shows side-by-side the main equations used for deflection calculations of uniform mediums, two-layered systems, and three-layered systems. These equations together with the Burmister graph (Fig. 2) and the Jones tables given in Table 1 represent an abridgment of the theoretical information available on deflection computation



Figure 1. Equations and symbols used in this paper.

of elastic layered systems. For justification of these formulas and equations, see Ref. 3. Figure 1 also includes a list of symbols and definitions as they are used in this study.

The deflection factor computed by Jones is somewhat different from the normal deflection factor used by Burmister and most authors. The Burmister deflection factor F (originally designated by F_W) for flexible uniform loading derives from the following equations:



 $\mathbf{h}_{\mathbf{r}}$

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Two-layer system
$$D = \frac{1.5 \text{ pr}}{\text{Fr}} \cdot \text{F}$$

Three-layer system
$$D = \frac{1.5 \text{ pr}}{E_3} \cdot F$$

For rigid plate loading, substitute 1. 18 for 1. 5.

The Jones deflection factor \overline{F} for uniform loading is defined by the equation:

Three-layer system $D = \frac{pr}{E_1} \cdot \overline{F}$

where

D = deflection,

p = contact pressure,

r = radius,

 E_1 = elastic modulus of top layer,

 $E_2, E_3 =$ elastic moduli of lower layers, and

 $\mathbf{F}, \overline{\mathbf{F}} = \text{deflection factors}$

The two factors are related by the equation:

$$F = \overline{F} \cdot \frac{1}{1.5 n_1 n_2}$$
$$n_1 = E_1/E_2$$
$$n_2 = E_2/E_3$$

Table 1 gives the deflection factors \overline{F} as computed by Jones, and the corresponding deflection factors F, calculated by the foregoing relationship. In this study, we will use only the normal deflection factor F, with the usual definition. When recourse is necessary to the original Jones tables, factor \overline{F} of these tables is transformed to factor F.

The Jones tables were computed for Poisson's ratio equal to 0.35^{1} . Burmister, as most authors, computed his graph for a 0.5 Poisson's ratio. In the particular case of the soil-cement bases, the value of 0.35 would probably be closer to the truth than the value 0.5. The opposite situation would occur in the case of saturated untreated granular bases and subbases. However, the influence of Poisson's ratio on the deflection is known to be small, in all instances.

Jeuffroy and Bachelez (4) also presented a numerical computation of the deflection factor for three-layered systems in graphic form, for several values of the parameters. The Jeuffroy-Bachelez theory is based on simplifying assumptions (Navier hypothesis), but it gives results close to the more exact Jones theory, for the deflections. However, the Jones tables are easier to use and cover a wider range of parameters than the Jeuffroy-Bachelez graphs.

$$D = \frac{1.755 \text{ pr}}{E_3} \cdot F$$
$$F = \overline{F} \cdot \frac{1}{1.755 \text{ n}_1 \text{ n}_2}$$
$$\overline{F} = \text{scalar free large for }$$

 \overline{F} = value from Jones tables.

Evidently the numerical value of D remains unchanged. The writer mantains the coefficients 1.5 for flexible load and 1.18 for rigid plate to preserve a formal analogy between the equations of Figure 1 for all layered systems. The final results of the analysis are not affected by the substitution of 1.5 for 1.755, or 1.18 for 1.378, in all equations.

¹The deflection coefficient for flexible uniform load to use in connection with Jones tables should actually be 1.755 and not 1.5, due to the value of Poisson's ratio. The correct deflection equations are

The application of layered system concepts and principles to the interpretation and evaluation of flexible pavements has been tried by many authors with variable success. Among others, Burmister (5) analyzed the Hybla Valley Test Track data and the WASHO Road Test data by two-layer methods. The results of this analysis are somewhat disappointing with respect to the consistency of the determination of the "in-place" moduli of elasticity of the pavement layers. Jeuffroy and Bachelez (4) analyzed the same WASHO Road Test data by three-layer methods and found more consistent results for the layers' moduli. Sowers and Vesić (6) measured the vertical stresses in subgrades beneath statically loaded flexible pavements and found great discrepancies between measured stresses and values computed by the elastic layered theory, for most types of pavements. However, for soil-cement pavements there is a good agreement between measured and theoretical computed values of the vertical stress. According to these findings, the elastic theory applies to soil-cement pavements better than to other types of flexible pavements, at least as far as the vertical stress.

The equations shown in Figure 1 are general equations, valid for any values of the parameters, except for Poisson's ratio. In the case of the three-layer systems, due to the great many parameters necessary, it is practically impossible to solve problems of deflection computation using only these general equations. To solve the specific problem of this study we have included a few graphs which are valid only for the range of parameters of our particular case, and which should not be extrapolated for other values of the parameters. However, the method of analysis outlined is believed to be general in scope, provided new graphs are drawn for the range of parameters in each particular case.

DESIGN AND CONSTRUCTION DATA

The São Paulo Institute for Technological Research performed a series of bearing tests on soil-cement pavements at eight airports in different cities of the state. Most of the airports were located in the northwestern part of the state, which is a great sedimentary basin composed of fine sandy soils of rather uniform texture. One airport was located outside this geological area, but the subgrade soil at this location is also a fine uniform sandy soil. The subgrade soils at all airports are remarkably alike. Table 2 gives the average physical characteristics of the subgrade soils. The reason for selecting soil-cement for all pavements was the ease of stabilizing these sandy soils with economical percentages of cement and the lack of granular aggregates at the airport locations.

A typical cross-section of the airport pavements (Fig. 3) is composed of asphalticconcrete wearing course of 2.5 to 3 cm (1 to $1\frac{1}{4}$ in.) of thickness; soil-cement base course of 15 to 16 cm (6 to $6\frac{1}{4}$ in.); compacted soil subbase of 60 to 61 cm (24 in.); and noncompacted subgrade. The soils used in the soil-cement base and in the compacted subbase were taken from selected borrow pits to assure uniformity, but their general characteristics are the same as the subgrade soils indicated in Table 2. The "in-place" CBR of the uncompacted subgrade, which was only lightly compacted by the normal operation of the earthmoving equipment, was near 3 percent. The soil subbase was compacted near the optimum moisture to 95 percent of the standard Proctor maximum density. The compacted subbase CBR varied from 16 to 35 percent, with an average of 25 percent. The soil-cement base was designed and built according to Brazilian standards, which follow in general the procedures recommended by the Portland Cement Association. The designed cement content was a little under 10 percent by volume, but for safety and ease of control it was specified at 10 percent in all cases. The actual construction cement content was in general slightly over 10 percent by volume. The soil-cement base was also compacted at optimum moisture to 95 percent of standard Proctor maximum density. The thin asphaltic-concrete wearing course was primarily designed as a protection against traffic abrasion and moisture infiltration, with little structural influence. The bearing tests were performed on top of the soil-cement base, before the placing of the wearing course. The wearing course is not considered in this structural analysis. The pavement cross-section can therefore be considered as a three-layered system: the first layer is the soil-cement base,

Characteristic	Range
Texture	
Coarse sand (2.00-0.42 mm)	0-5%
Fine sand (0. 42-0. 05 mm)	70-75%
Silt (0. 05-0. 005 mm)	10-15%
Clay (less than 0.005 mm)	10-20%
Consistency	
Liquid limit	23-30%
Plastic index	6-12%
Classification HRB classification system:	
Generally	A 2-4 (0)
Eventually Unified elegification system	A 2-6 (2)
omned classification system	BC
Compaction tests (AASHO T 134-57)	20 10 10 A
Optimum moisture	9-12%
Maximum density	1. 92–2. 00 g/cm ³
	(120-125 pcf)
Strength tests	
CBR-Typical value for uncompacted subgrade	3%
CBR-Range of values for compacted subbase	16-35%
CBR-Average value for compacted subbase	25%
Soil-cement tests	
Compressive strength (7-day curing)	$22-32 \text{ kg/cm}^2$
	(315-460 psi)

TABLE 2

AVERAGE PHYSICAL CHARACTERISTICS OF AIRPORT SOILS

the second layer is the compacted subbase, and the third layer is the uncompacted subgrade. The first and second layers have definite thicknesses, and the third layer is theoretically considered to have infinite thickness.

BEARING TESTS

Forty-four bearing tests were made with circular rigid plates of diameters of 80, 45 and 30 cm (31 1/2, 17 3/4, and 11 3/4 in., roughly). For economy's sake, and to get as much information as possible at every testing point, the three diameters of plates were successively used in every mounting of reaction load (Fig. 4). The reaction load was a box full of earth weighing up to 100 metric tons. The load was transferred to the plate by a calibrated hydraulic jack hinged to the truck frame. The loadings were measured by the calibrated pressure gage. The plate deflections were measured by two deflectometers located in diametrically opposite positions, mounted on an independent supporting beam, which rested on supports outside the deflection basin. The average of the readings of the two deflectometers was taken as the deflection at the center of the plate, for every loading. The 80-cm plate was used first at the center of the mounting. After the completion of this test, the load was removed, and new tests were made with the 45- and 30-cm plates at points to the right and left of the first test, using the same reaction load. The distance between the centers of the plates was approximately 1. 20 m (4 ft). It was later realized that this distance was not great enough to warrant independent results, i.e., the first test unfavorably influenced the second and third tests. A procedure was proposed to correct the results of the two



Figure 3. Airport pavement cross-section; bearing tests made on top of soil-cement base before placing of wearing course.



Figure 4. Mounting of bearing tests.

later tests for this influence. At one airport, a plate of 39 cm $(15\frac{1}{4} \text{ in.})$ in diameter was used instead of the 45-cm plate. The results obtained with the 39-cm plate are comparable to those of the 45-cm plate. A bearing test was made on the top of the subbase at one of the airports, using a 80-cm plate.

About half the loading tests were performed according to the Asphalt Institute Process (7). This process is as follows. After seating the plate, a small load is applied and sustained until the increase of deflection is less than 0.02 mm/min. The deflection is recorded, and the load is removed. The plate is kept unloaded until the recovery of deflection is less than 0.02 mm/min. The same load is reapplied and removed three more times, with the deflection recorded every time. A greater load is then applied and removed four times, and the deflections are recorded. The same procedure is repeated with increasing loads, until the end of the test. The test is stopped when the pavement breaks, or when the deflections are very high (over 10 mm), or when all the reaction load is used.

The remaining loading tests were performed according to the International Civil Aviation Organization Process ($\underline{8}$). This process is similar to that of the Asphalt Institute Process with one difference: every load is applied five times instead of four times.

The only loading test made directly on the subbase was performed according to the U. S. Corps of Engineers Process (9), which is a continuous loading procedure recommended for the determination of the subgrade modulus of reaction.

Load-deflection diagrams were drawn for all loading tests (Fig. 5). The soil-cement base modulus of reaction K (load-deflection ratio) was computed for every stage of loading in all loading tests. The load-deflection ratio was completely recoverable and



Figure 5. Typical load-deflection diagram of tests.

TABLE 3

TYPICAL VALUES OF LOAD-DEFLECTION RATIO FOR TEST NO. 631

Cycle 1	Initial	Elastic
1		
	30.8	
2		37.2
3		35.5
4		36.8
5		38.2
1	36, 9	
2		38.4
3		40.3
4		39.3
5		39.3
1	36.4	
2		39.8
3		39.5
4		39.5
5		40. 2
1	27.2	
2		33.7
3		34. 5
4		34.8
	$ \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \\ 1 \\ 2 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ 3 \\ $	2 3 4 5 1 3 6.9 2 3 4 5 1 3 6.4 2 3 4 5 1 2 7.2 2 3 4

Series	Test	Load-Deflection Ratio (kg/cm ² cm or kg/cm ³)			
(Airport)	N0.	\$\$\phi\$ = 80 cm	$\phi = 45 \text{ cm}$	ø = 30 cm	
1	111	37			
2	211 221 231 Mean	25 27 37 (29.7)			
3	311	32			
4	410 420 Mean	25 34 (29. 5)	54 ^a 56 ^a (55. 0) ^a	67 93 (80. 0)	
5	510 520 532a 532b 542a 542b	30 36	$ \begin{array}{c} 71 \\ 51 \\ 100 \\ 82 \\ 87 \\ 95 \end{array} $ (91)	140 120	
6	Mean 610 620 630 Mean	(33.0) 40 40 39 (39.7)	(81.0) 98 102 122 (107.3)	(130. 0) 177 123 233 (177. 7)	
7	710	31	49	97	
8	810 820 830 Mean	25 34 34 (31. 0)	80 44 31 (51.7)	193 87 136 (138. 7)	
	Overall mean Std. Dev. Coeff. of variation	32.9 5.3 16%	74.8 26.3 35%	133.3 50.2 38%	

AVERAGE VALUES OF ELASTIC LOAD-DEFLECTION RATIO FOR ALL TESTS ON SOIL-CEMENT BASES

TABLE 4

^aResults measured with plate of ϕ = 39 cm; numbers between parentheses are series averages.

nearly constant for all loads after the first stage of loading, in every test. The initial load-deflection ratio for the first stage of loading was non-recoverable, and its value was of the order of 85 percent of the later deflection ratio. Table 3 gives typical values for one test. The average of the elastic load-deflection ratios for all loads after the first stage of loading, for every test, was taken as a characteristic mechanical parameter of the pavement. This average elastic load-deflection ratio is referred to as the load-deflection ratio, and indicated by the letter K, in the present analysis. It is contemplated that the elastic load-deflection ratio is the most significant parameter for the evaluation of the structural behavior of pavement under the action of repetitive loadings, such as traffic loads.

	Ø = 8	30 cm	ø	= 45 cm	¢	b = 30 cm
Series	Avg. of Tests	Avg. of Series	Avg. of Tests	Avg. of Series	Avg. of Tests	Avg. of Series
	25		54 ^a	FF 08	67	
4	34	29, 0	56 ^a	əə. 0~	93	80.0
	30		71		140	
5	36	33. 0	51	61. 0	120	130. 0
7	31	31.0	49	49.0	97	.97. 0
8	25 34 34	31. 0	80 44 31	51, 7	193 87 136	138.7
Partial mean		31, 1		54. 3		111. 5
Standard deviation		1, 4		5. 2		27.6
Coeff. of variation		5%		10%		25%
Corrected load/deflec- tion ratio (K _{eq})		31. 1		54. 3 × 1. 5 = 81. 5		111. 5 × 1. 5 = 167. 3
Ratio K _r /K _o		1		81. 5/31. 1 = 2. 62		167. 3/31. 1 = 5. 38

 TABLE 5

 SELECTED VALUES OF LOAD-DEFLECTION RATIO FOR

 COMPARABLE TESTS ON SOIL-CEMENT BASES (kq/cm³)

^aResults measured with plate of $\phi = 39$ cm.

LOAD-DEFLECTION RATIOS

Table 4 gives the load-deflection ratios measured in all tests. Each series of results corresponds to one airport (10, 11). The measured load-deflection ratios show great scatter of values. The statistical coefficient of variation is 16 percent for the 80-cm plate, 35 percent for the 45-cm plate, and 38 percent for the 30-cm plate. This large dispersion of values indicates that the results in Table 4 cannot be considered homogeneous, and therefore the overall mean is not significant. Also, there is some dispersion within the data pertaining to each airport. However, the gathering of data summarized in Table 4 required a considerable expense of energy, time, and money. It would be regretable if all this effort should be wasted. The load-deflection ratios, as experimental measurements obtained under definite conditions, are not readily useful for the design of other pavements in different conditions, unless they are analyzed, interpreted and generalized under the light of a suitable theoretical framework. It was then decided to extract a set of homogeneous and comparable data from Table 4 and to analyze these data by the elastic layered theory. The resulting numerical figures are to be regarded as tentative, as they are affected by the dispersion of field data, but the proposed theoretical method of analysis is believed to be entirely valid.

Table 5 gives the selected values of the load-deflection ratio for comparable tests on soil-cement bases. The justification for transferring data from Table 4 to Table 5 was as follows:

1. Tests of series 1, 2, and 3 were not used because these series contained data for only one diameter of plate, all for the same thickness of pavement. These data are not enough to solve the mathematical problem of layered systems.

2. Series 5 contains tests no. 510 and 520, performed according to the normal threeplate procedure, and test no. 532-a to 542-b, performed with the 45-cm plate alone. The latter group of results, obtained by a non-normal testing procedure, was not included in Table 5.

3. Results of series 6 were abandoned because they were much higher than results from all other series, for all three plates. Not only the load-deflection ratios, but also the total loads were much higher in series 6, whereas the final deflections were smaller. The causes of these differences were not readily apparent, and were not further investigated, but it is evident from the test results that series 6 represents a pavement of better quality than the other series.

4. The remaining tests of series 4, 5, 7 and 8 were considered comparable in quality of pavement and procedure of testing, and were included in Table 5. The data in Table 5 were not specifically chosen, but, rather, remained after tests which were non-typical in one respect or another were eliminated. These data are homogeneous in the sense that all tests were performed by the same procedure and the pavements tested are of comparable strength.

The data in Table 5 show marked improvement in statistical consistency over the previous table. The coefficient of variation is only 5 percent for the 80-cm plate, and 10 and 25 percent for the 45-cm and 30-cm plates, respectively. The variation of 5 percent for the largest plate was considered purely accidental, and compatible with the accuracy of experimental measurements. The average of the 80-cm plate results is statistically significant. The larger variation of the two smaller plates is attributed to the detrimental influence of the first test on the following tests, at each location. It can be concluded that the large plate data warrant a high degree of confidence, the intermediate plate data allow lesser confidence, and the small plate data deserve very little confidence. Unfortunately, three diameters of plates are needed for the mathematical solution of a three-layered system of uniform thickness. Whenever possible, the 80-cm plate data are used as the primary basis for theoretical analysis. The two smaller plates are mostly used for cross-checking the hypothesis of calculus. The averages of the results in Table 5, for each diameter of plate, are called "partial means" and represent a homogeneous type of pavement.

The data of series 5 permit the establishment of a criterion for correcting the results of the two smaller plates. The average of tests no. 510 and 520 (normal threeplate procedure) is 61 kg/cm^3 . The average of tests no. 532-a to 542-b (intermediate plate alone) is 91 kg/cm^3 . These results indicate that the first test (80-cm plate), caused a weakening of the pavement, possibly due to cracking, so that the second test, (45-cm plate) produced a smaller load-deflection ratio than it should if the second test were performed over virgin pavement. The ratio between the two values gives the correction factor

Correction factor = $91/61 \approx 1.5$

In a first approximation, all results of the two smaller plates are multiplied by this empirical factor, 1.5, to obtain the corrected load-deflection ratio, K_{eq} . The values of the 80-cm plate do not need correction, of course. The corrected values agree with the theoretical curves developed in the analysis, whereas the uncorrected values fall completely out of line. This agreement between corrected and theoretical values confirms to a certain degree the validity of the correction. Nevertheless, this empirical correction is only an expedient to arrive at some tentative conclusions from a mass of experimental data that would otherwise be lost.

The following theoretical analysis shows that it is useful to study the relationship between the corrected load-deflection ratios for the three diameters of plates. The load-deflection ratio for the 80-cm diameter plate, which is always the lowest, is taken as the basic parameter K_{80} . The load-deflection ratios for the 45-cm and 30-cm



b)Profile



Figure 6. Slab effect of soil-cement bases as shown by survey of deflection basin: (a) mounting of test (plan); (b) pattern of deflection basin for three loadings (profile).

diameter plates are designated by K_{45} and K_{30} , respectively. These three values yield two relationships K_{45}/K_{80} and K_{30}/K_{80} , indicated in general terms by $K_{\rm r}/K_{\rm O}$. Evidently this ratio is equal to unity for the basic 80-cm diameter plate. Table 5 gives the $K_{\rm r}/K_{\rm O}$ ratios for the partial means.

SLAB EFFECT OF SOIL-CEMENT BASES

One of the most discussed characteristics of soil-cement bases is slab effect, i.e., the ability to distribute loads by acting as an effective rigid slab. All soil-cement bases present an irregular pattern of hair-cracking due to shrinkage and thermal variations. These cracks conceivably alleviate flexural stresses induced by applied loads, but retain the ability to transmit vertical stresses. It was not known how the cracking pattern would influence the effectiveness of the slab effect. A special series of measurements was devised to check the slab effect through the study of the deflection basin.

At the location of test no. 630, with the 45-cm plate, four additional deflectometers were installed, in addition to the two normal deflectometers located over the plate. The extra deflectometers were mounted at regular intervals on the supporting beam, with the probe point resting directly on the pavement. The farthest deflectometer was located at a distance from center of plate almost four times the radius of plate. Figure 6a shows the mounting of the special test, and the pattern of cracking before testing. Fortunately, deflectometers 3 and 4 were located on opposite sides of a visible crack. Figure 6b shows the pattern of the deflection basin for several loadings, as measured by the deflectometers. At a distance of the perimeter of the plate equal to the plate diameter, the deflection was 44 percent of the plate deflection, for the highest load. At a distance four times the plate diameter, on the opposite side of a visible crack, the deflection was 20 percent of the plate deflections. New cracks showed up under loading that were not apparent before loading. The conclusion was that soil-cement bases maintain an appreciable degree of slab effect, in spite of the cracking pattern.

INTERPRETATION OF TEST DATA

A tentative analysis was made of the test data by the theory of the two-layered elastic systems, considering the subgrade and subbase as a single layer of equivalent elastic properties. This analysis gave erratic and unrealistic results, producing values too high for the soil-cement modulus of elasticity. Two causes were thought to be responsible for the failure of the two-layer theory to explain the load-deflection pattern of the pavement structures:

1. The pavement structures are basically three-layered systems. Combining the two lower layers as a single layer is not merely a question of greater or lesser detail in the analysis; this unwarranted simplification markedly affects the computed values of the elastic modulus, in different ways for the different plate diameters.

2. The Burmister graph (Fig. 2) is not accurate enough, particularly in the region of h/r less than unity. The writers were unable to locate published tables of deflection factor values for two-layered systems. Also, the method of analysis proposed by Burmister (1), based on the shape and concavity of trial deflection factor curves, was somewhat erratic.

A new method of analysis was then developed for the interpretation of the bearing test data by the use of three-layer elastic theory. This analysis puts forward a practical application of the three-layer deflection factor tables published by Jones (2). The proposed method of analysis is believed to be accurate and dependable. Its results depend, however, on the accuracy of the measured data.

It is believed that the proposed method, particularly the analysis of the K_r/K_0 ratio, can be successfully extended to the analysis of truly two-layered systems, to avoid the difficulties discussed in the foregoing item 2.

ANALYSIS OF TEST ON SUBBASE

The only bearing test made directly on the soil subbase with the 80-cm plate was at Airport 5; it yielded a load-deflection ratio of 7 kg/cm^3 . The pavement structure tested is a two-layer system, namely the soil subbase and the uncompacted subgrade. The system parameters are

 $\begin{array}{ll} K_{eq} = & \frac{Known \ Data}{7 \ kg/cm^3} & \qquad \\ r = & 40 \ cm} \\ h = & 60 \ cm} \left\{ \begin{array}{l} h/r = & 1.5 \end{array} \right. \\ \end{array} \right. \begin{array}{l} \underbrace{Unknown \ Data}{E_1 = ?} \\ E_2 = & ? \end{array} \left\{ \begin{array}{l} E_1/E_2 = ? \end{array} \right\} \end{array}$

The mathematical problem involved is indeterminate for the known data only. The solution of the problem would require knowledge of the values of E_{eq} for other diame-



Figure 7. Three-layer deflection factor \overline{F} as function of modular ratios n_1 and n_2 and plate diameter ϕ (graph valid only for thicknesses indicated in Figure 3).



Figure 8. Ratio K_{r}/K_{o} as function of modular ratio N and plate diameter for considered three-layer system.

ters of plates, or else the value of the modular ratio E_1/E_2 . Many authors, including Peattie (12), Dormon (13), and Heukelom (14), have found that the effective modular ratio for granular non-cemented materials is always between 2 and 5. For instance, if the ratio $E_1/E_2 = 4$, from Eq. 7b (Fig. 1) and the graph in Figure 2 we have:

$$h/r = 1.5$$

 $E_1/E_2 = 4$ \therefore $F = 0.54$

1

From Eq. 3b

$$E_2 = 1.18 \times 40 \times 7 \times 0.54 \approx 180 \text{ kg/cm}^2$$

 $E_1 = 4 \times 180 = 720 \text{ kg/cm}^2$

The same result would be found computing F by the approximated Eqs. 8b and 9b, instead of taking F from Figure 2.

Eq. 5b gives the following value for E_{eq} :

$$E_{eq} = 180/0.54 \approx 330 \text{ kg/cm}^2$$

The same value of E_{eq} would be found if the two-layer system were considered as a uniform medium of modulus E and L/D ratio K = 7 kg/cm³. From Eq. 3a we have:

 $E = 1.18 \times 40 \times 7 = 330 \text{ kg/cm}^2$

If the calculated values of E_1 , E_2 , or E could be used in the solution of the threelayer system, the problem would be much simplified. Unfortunately this substitution is not valid, even if the modular ratio $E_1/E_2 = 4$ is supposed to hold true. A multilayer system (Fig. 1B and 1C) can be replaced by an equivalent uniform medium (Fig. 1A) for the condition of $K = K_{eq}$, but this substitution is valid only once for the entire system. It is not valid to replace the two lower layers by one supposedly "equivalent" single layer within the three-layer system; and neither is it correct to use data measured on the two-layer system in the calculation of the three-layer system. The main reasons are (a) the stress and strain distribution would not be the same in the two cases, and the theoretical equations would not apply after the replacement; (b) the confining effect of the top layer is not present in the two-layer structure alone; (c) the compaction of the top layer is partially transmitted to the lower layers, producing an increase in the density and in the value of the effective modulus. This last effect is very important at the airport pavements tested, due to the sandy nature of the soil and the use of vibratory rollers, in the compaction of the soil-cement base. Consequently, the three-layer system moduli should be computed from measurements made on the complete structure. Tests made on the lower layers alone are of no avail for this purpose.

ANALYSIS OF SERIES 5 TESTS

Series 5 is analyzed first because it contains much useful data in addition to the test on the subbase. The pavement structure is a three-layered system (Fig. 3).

Geometric Parameters

				Test ø80	Test ¢45	Test ¢30
r	=			40	22. 5	15
h1	Ξ			15	15	15
h2	=			60	60	60
н	=	h_1/h_2	=	0.25	0.25	0.25
Α	Ξ	r/h_2	=	0.67	0.37	0.25

Bearing Test Results

	Ø80	ø 45	ø30	
K_{eq} (measured) =	33	61	130	
K_{eq} (corrected) =	33	61 × 1.5 ≃ 91	$130 \times 1.5 = 19$	95
$K_r/K_o =$	1	91/33 = 2.76	195/33 =	5.91

75

Unknown Elastic Parameters

As previously mentioned, the modular ratio n_2 is always between 2 and 5. It is shown later that the selection of any value for n_2 between 2 and 5 is not too critical for the computed value of the base modulus E_1 , which is the primary objective of this analysis. We have two independent equations relating the unknown parameters to the known data:

Eq. 3c:
$$E_3 = 1.18 \text{ r } K_{eq} \text{ F}$$

Eq. 7c: $F = f [n_1, n_2, H, A]$

The symbol f in Eq. 7c represents an extremely complex differential function, but it has been computed in tabular form (2). Some typical values are given in Table 1. As factor F is also unknown, it can be eliminated reducing the two foregoing equations to one:

 $E_3 = 1.18 r K_{eq} \cdot f [n_1, n_2, H, A]$

Applying this equation to the test results, with three plate diameters we have a system of three equations with three variables:

The mathematical problem is therefore determinate. However, due to the complexity of function f of Eq. 7c, the system must be solved by trial methods. A set of values of E_3 , n_1 , and n_2 are sought that simultaneously satisfy the three equations of the system. A practical way to do this is to adopt tentative values for n_1 and n_2 , and compute the corresponding values of E_3 . When the three values of E_3 given by the three equations are equal, the trial values n_1 and n_2 plus the computed value E_3 are a solution for the system. The base modulus E_1 can then be easily calculated. A difficulty of the trial method of solution is that the measured values of the L/D ratio K_{80} , K_{45} , and K_{80} are affected by an experimental error. The computed values of E_3 are never equal, but show a dispersion as the L/D ratios. The best solution must be found by statistical criteria.

After a few trials, the following solution was found adequate for series 5:

$$n_1 = 30$$
 $n_2 = 4$ $N = 120$

Let us check this solution, to demonstrate the trial method employed. First, the values of F corresponding to these values of n_1 and n_2 are computed for the three diameters by interpolation in Jones tables (2). Next, the corresponding values of E_3 are calculated by Eq. 3c:

The three values of E_3 are close enough to justify the given solution. The dispersion of values of E_3 is less than the dispersion of L/D ratios. Now calculate E_2 and E_1 using the average value of E_3 , or better the value for the $\emptyset 80$ plate:

 $\begin{array}{rcl} E_1 &=& 120 \ \times \ 593 \ \cong \ 71,000 \ \mathrm{kg/cm}^2 \\ E_2 &=& 4 \ \times \ 593 \ \cong \ 2,400 \ \mathrm{kg/cm}^2 \\ E_3 \ \cong & 600 \ \mathrm{kg/cm}^2 \end{array}$

Let us check the influence of the modular ratio n_2 on the computed value of E_1 , maintaining constant the overall ratio N = 120. Repeating all calculations gives

The variation of E_1 for the possible values of n_2 is of the same order of the dispersion of measured L/D ratios. This conclusion warrants a simplification of the calculus. If an intermediate value is adopted for n_2 , the problem will be reduced to the calculation of two variables, namely N and E_3 .

The value of the subgrade modulus $E_3 = 600 \text{ kg/cm}^2$ measured on the three-layered system is much greater than the value measured in the test made directly on the subbase (180 kg/cm²), even higher than the equivalent modulus corresponding to the subgrade-subbase ensemble (330 kg/cm²). This increase in the subgrade modulus value can be explained by the three causes mentioned in the analysis of the test on the subbase.

If the base modulus E_1 is computed from the subbase L/D ratio K = 7 kg/cm², and the base L/D ratio K_{eq} = 33 kg/cm² by Eqs. 6b and 7b, assimilating the pavement structure to a two-layered system, the value $E_1 = 726,000$ kg/cm² will be found. This latter value is evidently highly unrealistic for soil-cement bases. This computation proves again that the supposedly equivalent single layer for substituting the two lower layers is not a valid concept in the three-layered structure.

The trial method described, using Jones tables directly (2), is extremely tedious because of the great number of interpolations necessary. Each value of F, corresponding to a pair of values of n_1 , and n_2 , for every diameter, requires at least eight interpolations on log-log paper. To avoid this difficulty, a simplified method of analysis was developed, based on Jones tables (Figs. 7, 8). Figure 7 shows the three-layer deflection factor F as function of modular ratios n_1 and n_2 , for the three plate diameters. Figure 8 shows the value of ratio $K_{\rm r}/K_0$ as function of modular ratio N and plate diameter. For the particular values of the geometric parameters of the pavement structures of this study, the ratio $K_{\rm r}/K_0$ is a function of N, but it is practically independent of the individual values of n_1 and n_2 . The graphs in Figures 7 and 8 apply only to the range of parameters of the parameters, new graphs should be drawn up, from the original tables.

The problem is now solved as follows:

Known Data

 $\begin{array}{rl} K_{80} &=& 33 \ \mathrm{kg/cm^2} \\ K_{45}/K_{80} &=& 2.\ 76 \\ K_{30}/K_{80} &=& 5.\ 91 \end{array}$

Unknown Parameters

 $N = ? = E_3 = ?$

From Figure 8, the two values of K_r/K_0 give two values of N:

$$N_{45}/_{80} = 130$$
 $N_{30}/_{80} = 110$

The two values of N are close enough to warrant the adoption of its average as the most probable value. Adopting an intermediate value for n_2 gives

N = 120 $n_1 = 30$ $n_2 = 4$

TABLE 6VALUES OF SOIL-CEMENT MODULUS OFELASTICITY FOUND IN SERIAL ANALYSIS (kg/cm²)

Series	n1 x n2	N	$E_1 (kg/cm^2)$	Data
1	20 × 3	60	46,600	¢80
2	20 x 3	60	37,400	Ø80
3	20 x 3	60	40,300	Ø80
4	5 x 2	10	8,700	¢80, ¢39
5	30 x 4	120	71,200	\$80, \$45, \$30
6	30 x 4	120	88,000	¢80
7	6 x 2	12	10,700	\$80, \$45, \$30
8	11×3	33	22,900	Ø80, Ø45
General			$E_1 = 40,700$	
avg.			kg/cm ²	

From Figure 7, comes the value of F for the ϕ 80 plate:

$$F_{80} = 0.380$$

Eq. 3c gives the value of E_3 :

$$E_{s} = 1.18 \times 40 \times 33 \times 0.380$$

= 592 kg/cm²

Knowing E_3 , the other moduli E_2 and E_1 are calculated as before.

The proximity between the two values of N confirms the validity of the correction factor of 1.5 previously suggested. The value N = 130 is based on measured data only, as series 5 contains test data on virgin points of pavement both for $\phi 80$ and $\phi 45$ plates. The value N = 110 is based on corrected data for the $\phi 30$ diameter. If the uncorrected test data were



Figure 9. Ratio $K_r K_o$ as function of radial ratio r_o/r for partial mean values for the considered threelayer system (curves are accurate at plotted points, only approximate between points).

used, it would not be possible to find a value of N satisfying simultaneously the two values of $K_{\rm r}/K_0$

ANALYSIS OF SERIES 6 TESTS

As previously noted, series 6 test data indicate a pavement considerably stronger than the other series. When analyzed by the same method used in series 5, series 6 data produced incompatible results. There is no value of N simultaneously satisfying the two values of K_r/K_0 , neither for the corrected nor for the uncorrected data. This incompatibility indicates that some of the hypotheses assumed in the theoretical formulation are not met in series 6. The causes of these discrepancies were not further investigated.

Using only the L/D ratio for the 80-cm plate $K_{80} = 39.7 \text{ kg/cm}^2$, and adopting the same modular ratios found in series 5, we have for series 6:

N = 120 $n_1 = 30$ $n_2 = 4$ $E_1 = 88.000 \text{ kg/cm}^2$

ANALYSIS OF ALL SERIES

The same method used in series 5 was used in analyzing test data from all eight series (Table 6).

For the three first series, comprising results for the \emptyset 80 plate only, the modular ratio of the partial mean (N = 60) was adopted. For series 4 and 8 the result of the \emptyset 30 plate was abandoned because it was too much out of line with all others. The proposed method of analysis yielded consistent results for the three plate diameters both for series 5 and series 7, where the modulus values are very high and very low, respectively. The average of all values of the base modulus is 40,700 kg/cm², but this average is not significant because the series data are not homogeneous. The most important information of Table 6 is the range of values of the soil-cement base modulus of elasticity in the pavements studied, which goes from 10,000 to 70,000 kg/cm² (150,000 to 1,000,00 psi). It is likely that the modulus is higher than this latter value in series 6.

ANALYSIS OF THE PARTIAL MEAN DATA

The partial mean refers to a homogeneous group of test data given in Table 5. The analysis of the partial mean data is aimed at finding a significant mean value for the base modulus. As the test data on Table 5 are more refined, so is the method of analysis. The basic concepts remain the same, however.

	C n	Computatio Iodular rai	n of tio N		
K_{eq} (measured) =	31. 1	54. 3	111. 5		
K _{eq} (corrected) =	31. 1	81.5	167.3		
$K_r/K_0 =$	1	2.62	5, 38		
N (Fig. 8) =	-	64	48		

The two values of N are close enough, as computed modular ratios go, to justify adopting its average as the best value. But, as the ϕ 45 data deserve more confidence than the ϕ 30 data, the most probable value of N is closer to 64 than to 48. Let us adopt: N = 60.

Let us try several combination of n_1 and n_2 , keeping constant the product $n_1 \times n_2 = 60$. Every pair of values of n_1 and n_2 correspond to one value of F from Figure 7 and to one value of E_3 computed by Eq. 3c:



Figure 10. Deflection factor of considered three-layer system as function of radial ratio, modular ratio, and subgrade elastic modulus (trial method solution).

n ₁ × n ₂	Computed values of $E_3 (kg/cm^2)$				
	Ø80	¢45	ø 30		
30 x 2	763	790	773		
20×3	653	662	651		
15 x 4	584	589	580		
12×5	539	532	521		

In theory, the equality of computed values of E_3 for one given combination of n_1 and n_2 would indicate that the system of equations was simultaneously satisfied, and this combination is a solution for the problem. However, in practice the absolute equality of values of E_3 is never attained because of the dispersion of measured values of K_{eq} . Practically any of these combinations would be acceptable. The values of E_3 for the \$000 plate are the most reliable. Let us take as representative values the following:

 $\begin{array}{l} n_1 \times n_2 = 20 \times 3 & N = 60 \\ E_3 \cong 650 \ \text{kg/cm}^2 \\ E_2 = 3 \times 650 \cong 2,000 \ \text{kg/cm}^2 \\ E_1 = 60 \times 650 = 39,000 \ \text{kg/cm}^2 \end{array}$

The method of analysis is shown in Figures 9 and 10. Figure 9 is a graph of ratio $K_{\rm T}/K_{\rm O}$ as function of radial ratio $r_{\rm O}/r$ for the partial mean values. The curves are accurate at plotted points, and only approximate between points. The $K_{\rm T}/K_{\rm O}$ ratio was chosen as a significant parameter because it is dependent on N but practically independent

of the values of n_1 and n_2 , within the range of parameters of our particular case. The ratio r_0/r was chosen as the geometric parameter referring to plate diameter. The basic radius is $r_0 = 40$ cm ($\phi 80$ cm).

The graph of K_r/K_0 as function of r_0/r is similar to the graph of load vs perimeterarea ratio. The r_0/r ratio, being proportional to the perimeter-area ratio, is a nondimensional number, whereas the perimeter-area ratio is numerically equal to 2/rand has the dimension of cm⁻¹. The r_0/r ratio is therefore a more adequate parameter for drawing influence curves. McLeod (15) found an empirical lineal relationship between load and perimeter-area ratio, for a given deflection, for flexible pavements with granular and asphaltic bases. Figure 9 shows a curved relationship, nearly parabolic for the three diameters, for the semiflexible soil-cement bases. According to the layered system elastic theory, this relationship should be nonlineal, as it was found to be. In theory, the concavity of the curves is inverted for diameters less than $\phi = 30$ cm, if the basic radius $r_0 = 40$ cm is kept constant. There is no experimental evidence to confirm the shape of the curves beyond $\phi = 30$ cm.

The solid lines of Figure 9 are theoretical curves for several values of N. The broken line (long dashes) is the curve of measured values, with the correction already referred to. The pointed line (short dashes) is the curve of uncorrected measured values. The shape and curvature of the corrected curve closely follow the set of theoretical curves, indicating a value of N close to 60. The uncorrected curve falls completely out of line with the theoretical curves. The shape of the corrected curve again confirms the validity of the adopted correction. The straight line for N = 1 represents the variation of $K_{\rm T}/K_0$ as function of r_0/r for the uniform medium. Theoretically, this relationship should be lineal, according to Eq. 4a. Experimental measurements reported by Stratton (16) have shown a moderate deviation from the theoretical lineal relationship. According to Stratton, the load-deflection ratio of a uniform medium (modulus of subgrade reaction) is independent of the plate diameter for diameters over 75 cm (30 in). This finding justifies the selection of $r_0 = 40$ cm as the basic radius.

Figure 10 shows the trial method for the simultaneous solution of E_3 , n_1 and n_2 values. The solid lines are experimental curves of F as function of r_0/r , for tentative values of E_3 and measured values of K_{eq} , computed by Eq. 3c. The broken lines are theoretical curves of F as function of r_0/r , for several values of $n_1 \times n_2$, independent of E_3 . Comparing the empirical and theoretical curves, the best fitting line is found, yielding the simultaneous values of the three variables. Two theoretical curves corresponding to 20 x 3 and 15 x 4 are practically parallel to the empirical curves, whereas the outside curves run in opposite directions (Fig. 10). Figure 10 yields the two best solutions:

> $n_1 \times n_2 = 20 \times 3$ $E_3 = 650 \text{ kg/cm}^2$ $n_1 \times n_2 = 15 \times 4$ $E_3 = 580 \text{ kg/cm}^2$

Any one of the foregoing solutions is within the accuracy of the measured data. The first one was taken as a typical value for the soil-cement bases.

The values of K_{eq} used in drawing the empirical curves of Figures 9 and 10 represent the final refinement of hundreds of direct measurements of pressures and deflections. The remarkable similarity between the theoretical and empirical curves on both figures points to a significant cause and effect relationship. It indicates that the theoretical interpretation of the measured data is correct, within the accuracy of the measurements.

CONCLUSIONS

The main conclusions of the study are as follows:

1. The theory of elastic layered systems is adequate for intepreting the load-deflection pattern of soil-cement bases.

2. The modulus of elasticity of soil-cement bases in the pavements studied is in the range of 10,000 to 70,000 kg/cm² (150,000 to 1,000,000 psi). There is some indication that the modulus can be higher than this value when the soil is very good.

3. The value 40,000 kg/cm² (550,000 psi) can be taken as a typical value for soilcement made with soil of 25 percent CBR and 10 percent of cement, built following sound construction practices.

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