Deflection Factor Charts for Two- and Three-Layer Elastic Systems

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Tables of deflection factor values for three-layer elastic systems loaded with uniform circular loads have been published by A. Jones of the Thornton Research Center, in connection with the development of a fundamental method of pavement design. For convenience in the design and analysis of pavement structures, these factors should be presented in graphic form. The analytical expression of the deflection factor was modified, and the determining parameters were transformed to permit an easier and more direct process of deflection analysis. Two-layer factors were also computed as a particular case of three-layer systems. This paper presents a series of deflection factor charts for two- and three-layer systems, for a wide range of parameter values, based on Jones' tables. Several examples were computed to show the practical application of the charts.

•THE deflection beneath the center of a uniform load, p, applied on a circular, flexible bearing area of radius r (Fig. 1) resting on the surface of a uniform elastic medium of semi-infinite depth, of elastic modulus E and Poisson's ratio μ, after Boussinesq is

$$W = \frac{2(1 - u^2)\,p\,r}{E}$$

$$\text{for } \mu = 0.5 \qquad W = \frac{1.5\,p\,r}{E}$$

$$\text{for } \mu = 0.35 \qquad W = \frac{1.75\,p\,r}{E}$$

The reduction of Poisson's ratio from 0. 5 to 0. 35 increases the deflection of the uniform medium by 17 percent.

If the load is applied by a rigid plate instead of a flexible bearing area, the deflection of the uniform medium computed by the foregoing formulas should be multiplied by the factor $\pi/4$ or 0. 785. For instance

$$\text{For } \mu = 0.5 \qquad W_r = \frac{1.18\,p\,r}{E}$$

Let us now compute the deflection of the uniform medium below a certain depth, considering the layer above this depth as incompressible. The deflection at any depth is equal to the term $\frac{p\,r}{E}$ multiplied by a deflection factor F. Figure 1 shows the values of the deflection factor at various depths for flexible and rigid bearing areas. The two curves show that the influence of the type of bearing area depends on the depth at which the deflection is computed. At the surface, the deflection factor is 1. 5 for a flexible

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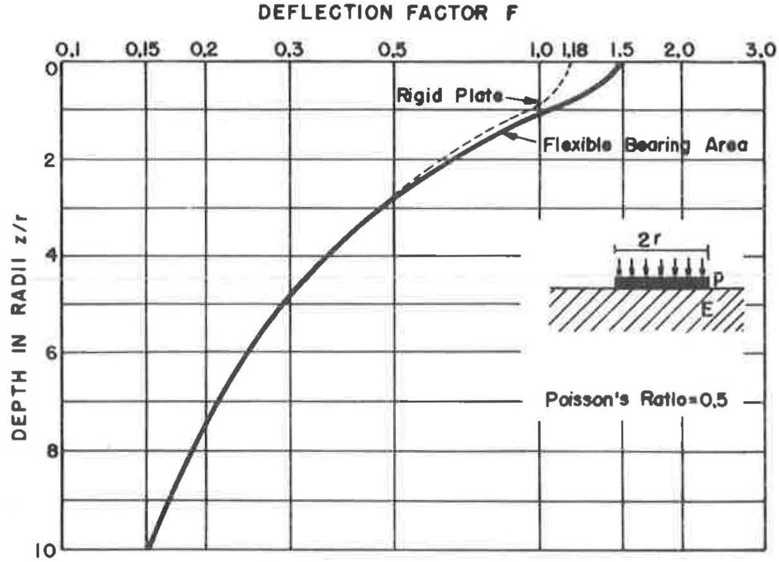


Figure 1. Deflection factor of uniform medium.

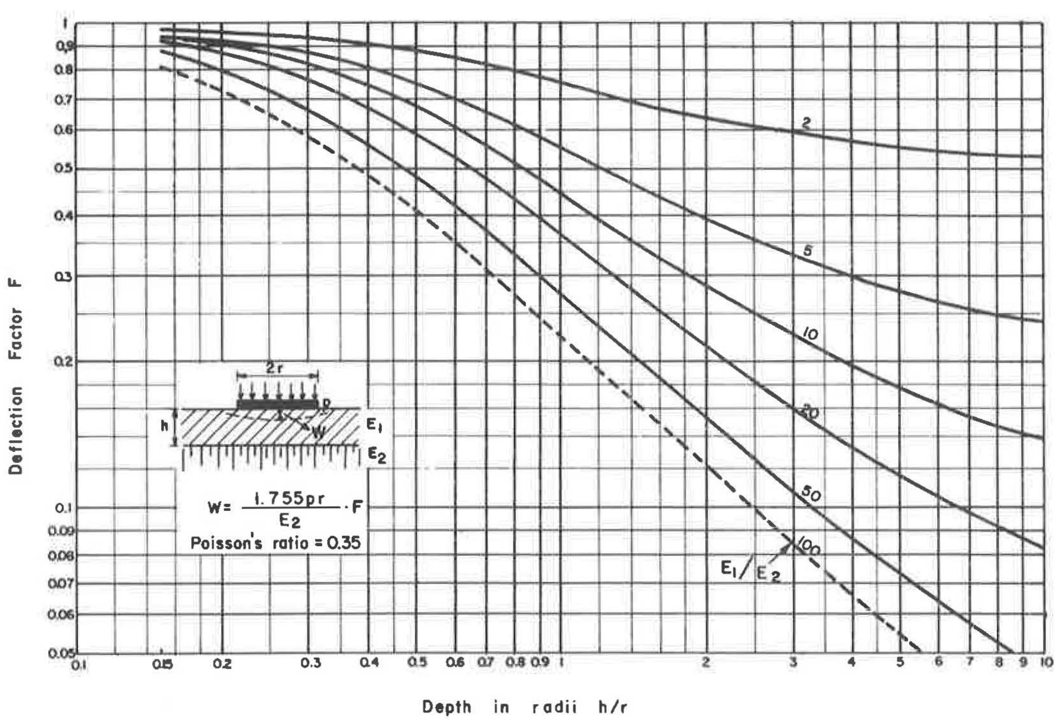


Figure 2. Deflection factor chart for two-layered elastic systems.

bearing area and 1.18 for a rigid plate (for $\mu = 0.5$). At depths greater than three times the radius the two factors are practically equal.

The deflection of a two-layer elastic system can be expressed in a form similar to the deflection of the uniform medium, but affected by an appropriate deflection factor. Assume a two-layer system such as that in Figure 2, in which the first layer is of thickness h, elastic modulus E_1, and Poisson's ratio μ, and the second layer is of semi-infinite depth, elastic modulus E_2 and the same Poisson's ratio μ, with perfect continuity between the two layers. The deflection, after Burmister (1), is

$$W = \frac{1.5\,p\,r}{E_2} \cdot F_W \qquad (\mu = 0.5)$$

The deflection factor F_W, for μ = constant, depends on the parameters E_1/E_2 and h/r. The Burmister graph (1) gives the values of the factor F_W for several values of the parameters, but allows little accuracy in the readings of F_W, especially in the region of $h/r < 1$. Moreover, the range it covers is too small for the parameter h/r (up to 6) and too large for the parameter E_1/E_2 (up to 10,000).

Jones (2) published a series of tables of deflection factor values for three-layer elastic systems loaded with a uniform circular load, in connection with the development of a fundamental method of pavement design. The Jones' deflection factors were computed for a value of Poisson's ratio of 0.35 in each layer. Two-layer systems are a particular case of three-layer systems, in which one modular ratio is equal to one. Hence, two-layer factors were computed from Jones' tables by a proper selection of parameters. For convenience and uniformity of presentation, the two-layer factors were transformed to comply with the equation

$$W = \frac{1.75\,p\,r}{E_2} \cdot F \qquad (\mu = 0.35)$$

The deflection factor F depends on the same parameters E_1/E_2 and h/r, for μ = constant. The numerical value of factor F is different from factor F_W, for the same values of the parameters, but the deflections computed by the two last equations are very close. The reduction of Poisson's ratio from 0.5 to 0.35 increases the deflection of layered systems by less than 10 percent, for the practical range of the parameters. The average increase is about 7 percent. The actual value of Poisson's ratio of pavement structures is not known, but it is likely to be between 0.35 and 0.5. This difference can be ignored in practical applications.

The second equation may be put under another form, more adequate for the determination of the moduli by load bearing tests:

$$E_2 = 1.75\,r\,\frac{p}{W}\,F$$

But p/W = k is the unit load per unit deflection or "modulus of reaction." Hence

$$\therefore\; E_2 = 1.75\,r\,k\,F \qquad (\mu = 0.35)$$

The deflection factor F varies from 0 to 1. It is inversely proportional to the load spreading ability or reinforcing effect of the pavement over the bare subgrade.

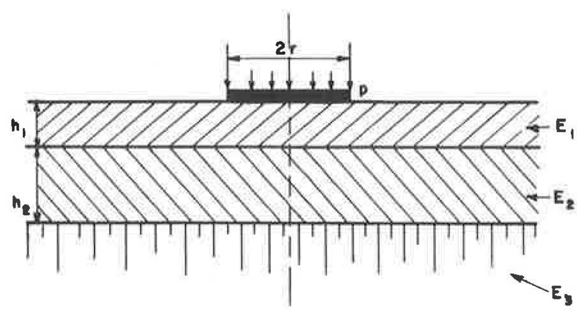
Table 1 and the graph of Figure 2 give the values of the deflection factor F for several practical values of the parameters. Parameter h/r varies logarithmically from 0.15 to 10, and parameter E_1/E_2 varies from 2 to 100. The curves for modular ratios from 2 to 50 were computed from Jones' tables. The curve for $E_1/E_2 = 100$, not computed by Jones, was drawn by logarithmical extrapolation and is accurate enough for practical applications. The graph should not be further extrapolated. All interpolations for intermediary values of E_1/E_2 should be computed logarithmically.

Jones' original deflection factors (2) for three-layer systems conform with the equation

TABLE 1

DEFLECTION FACTOR VALUES FOR TWO-LAYERED ELASTIC SYSTEMS
(Poisson's ratio = 0. 35)

h/r \ E_1/E_2	1	2	5	10	20	50
0.156	1.000	.966	.939	.925	.908	.869
0.312	1.000	.930	.867	.818	.756	.654
0.625	1.000	.839	.693	.600	.512	.405
1.25	1.000	.711	.494	.388	.307	.227
2.5	1.000	.614	.356	.250	.183	.126
5	1.000	.558	.279	.176	.117	.0732
10	1.000	.529	.240	.138	.0836	0.466



$$W = \frac{1.755\, p r.}{E_3} \cdot F$$

Poisson's ratio = 0.35

$$\frac{E_1}{E_2} = n_1 \qquad \frac{E_2}{E_3} = n_2 \qquad n_1 \cdot n_2 = N$$

$$F = f\ (h_1/r\ ,\ h_2/r\ ,\ n_1\ ,\ n_2)$$

W = deflection beneath the center of circular uniform load
r = radius p = unit load
h_1, h_2 = thicknesses of layers
E_1, E_2, E_3 = elastic modulii of layers
F = deflection factor

Figure 3. Deflection parameters of three-layered elastic systems.

$$W = \frac{pr}{E_1} \cdot \overline{F}$$

$$\overline{F} = \int_0^{\infty} \frac{1 + \sum_{i=1}^{\infty} p_i(x) \exp q_i(x)}{1 + \sum_{i=1}^{\infty} r_i(x) \exp q_i(x)} \cdot \frac{J_1(Ax)}{x}\, dx$$

The functions $p_i(x)$, $q_i(x)$ and $r_i(x)$ are polynomials with coefficients depending on the non-dimensional parameters, J_1 is a Bessel function, $A = r/h_2$ and the other parameters are shown in Figure 3. The factor $\overline{F}$ as such has no immediate physical meaning. Jones computed and tabulated the factor $\overline{F}$ in function of the parameters

$$E_1/E_2 = k_1,\ E_2/E_3 = k_2,\ h_1/h_2 = H,\ r/h_2 = A$$

For convenience and ease of application, Jones' three-layer factors were also transformed to comply with the following equation, similar to the two-layer equation

$$W = \frac{1.75\, pr}{E_3} \cdot F \qquad (\mu = 0.35)$$

The three-layer deflection factor F, for μ = constant, depends on the parameters defined in Figure 2

$$E_1/E_2 = n_1,\ E_2/E_3 = n_2,\ h_1/r,\ h_2/r$$

The tables and graphs in Figures 4 to 18 give the values of the three-layer deflection factor F for all combinations of the following parameter values

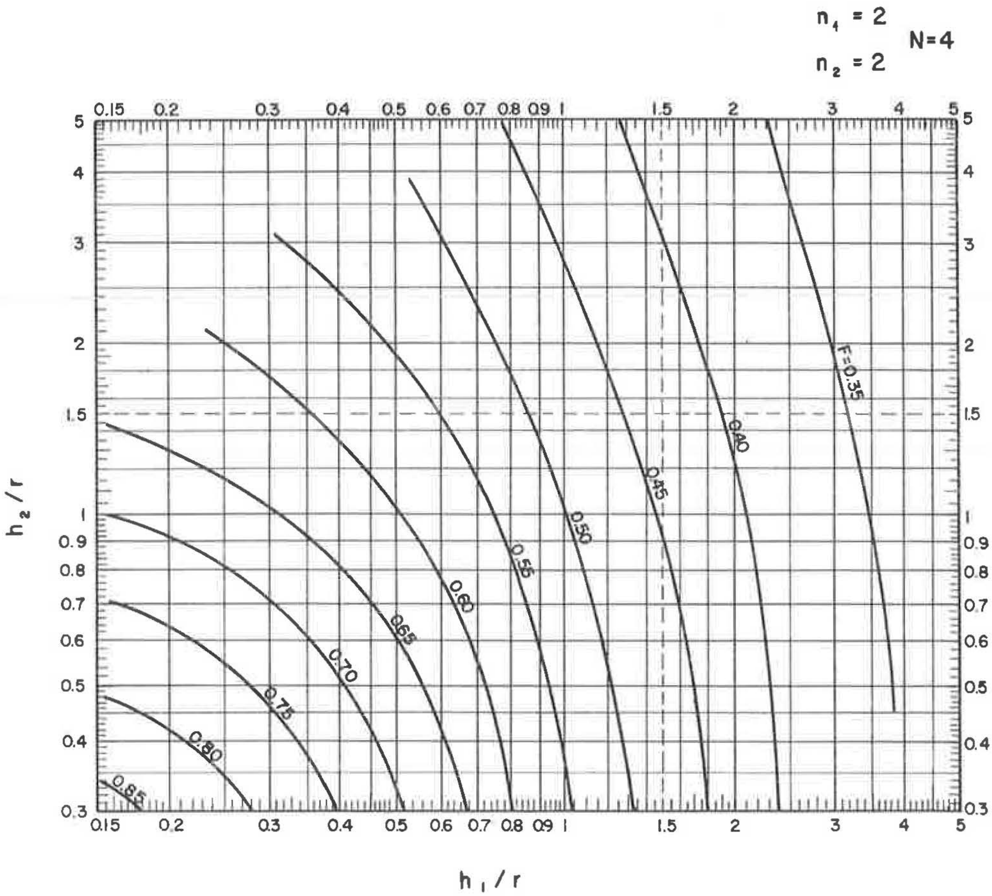
$$\begin{aligned} n_1 &= 2, 5, 10, 20, 50 \\ n_2 &= 2, 5, 10 \\ h_1/r &= 0.15 \text{ to } 5 \\ h_2/r &= 0.3 \text{ to } 5 \end{aligned}$$

Combining the graphs in Figures 4 to 18 with the graph of Figure 2 it is possible to interpolate for values of n_1 or n_2 between 1 and 2.

Jeuffroy and Bachelez (3) proposed an approximate method of three-layer system deflection calculation. The deflections computed by the Jeuffroy-Bachelez method are close to the values computed by the Jones method.

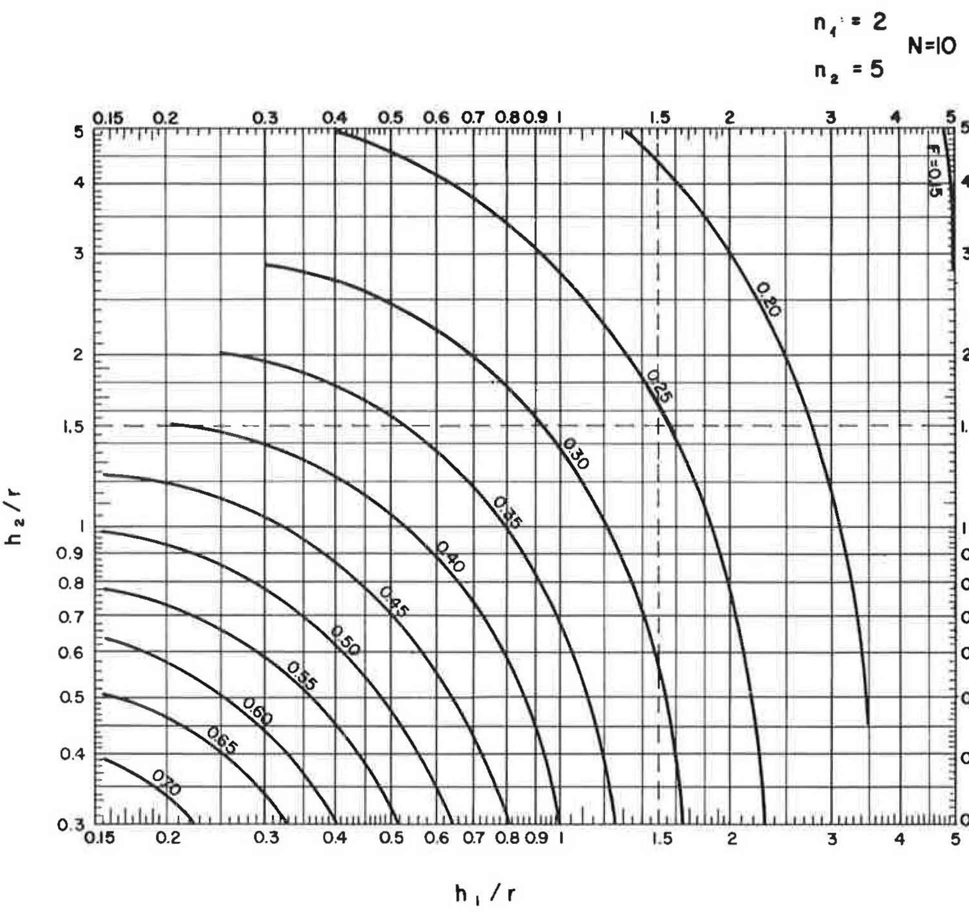
All deflection factors given for two- and three-layer systems were computed for the case of a flexible bearing area. For the case of a rigid plate, the computed deflections should be multiplied by a "bearing factor." The exact value of this factor cannot be determined at this time. It can be safely stated that for the layered systems of interest in pavement design the bearing factor must be between $\pi/4$ and 1, probably closer to 1. From analogy with the uniform medium (Fig. 1), it is evident that the surface layers have a greater influence on the difference between deflections of rigid and flexible bearing areas. If the surface layers are relatively stiff, this difference should be small. Taking into account the overall inaccuracies of modeling the pavement by an elastic layered system, it is suggested that the same deflection factors should be tentatively used in deflection analysis of pavement systems loaded with rigid plates.

An approximate formula is now proposed to calculate an "equivalent modulus" $E_{1,2}$ that can be substituted for moduli E_1 and E_2, for the same deflection. The three-layer system is thus reduced to an equivalent two-layer system, composed of one layer of



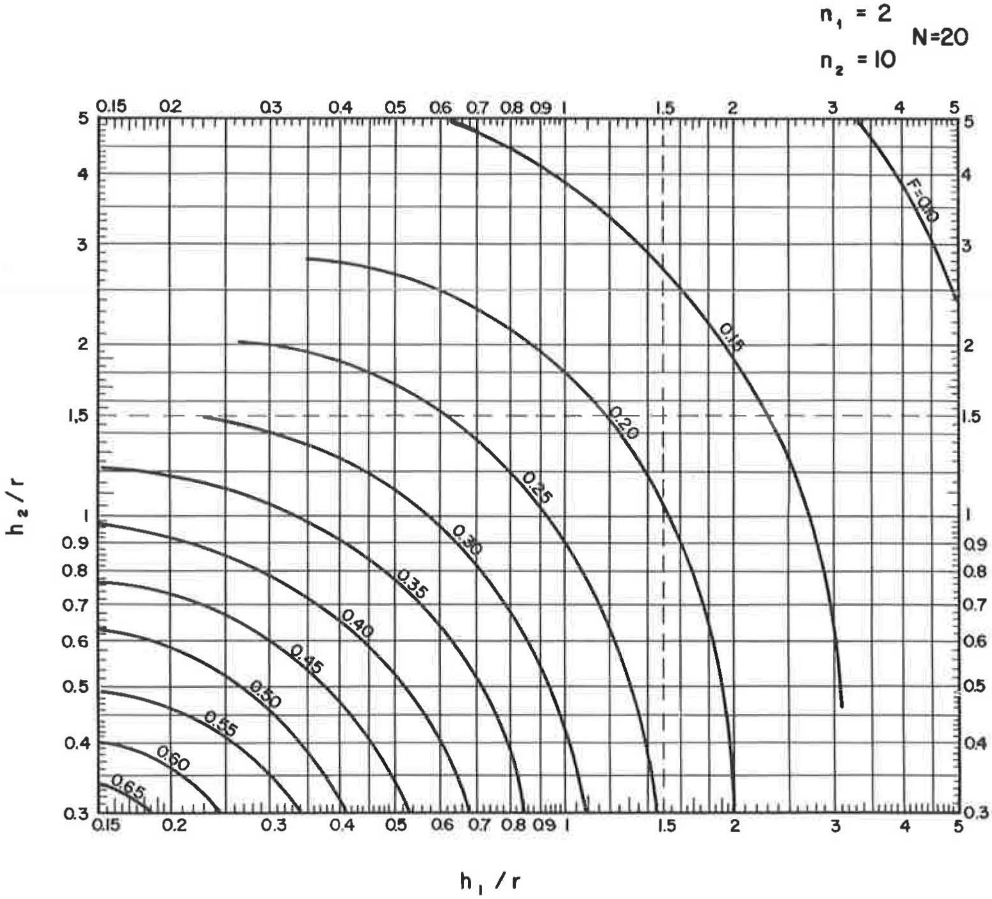
h_1/r / h_2/r	0,156	0,312	0,625	1,25	2,5	5
0,312	.858	.789	.662	.510	.394	—
0,625	.772	.717	.616	.489	.387	.323
1,25	.669	.633	.560	.460	.375	.319
2,5	—	.564	.508	.428	.360	.314
5	—	—	.470	.400	.343	.306

Figure 4.



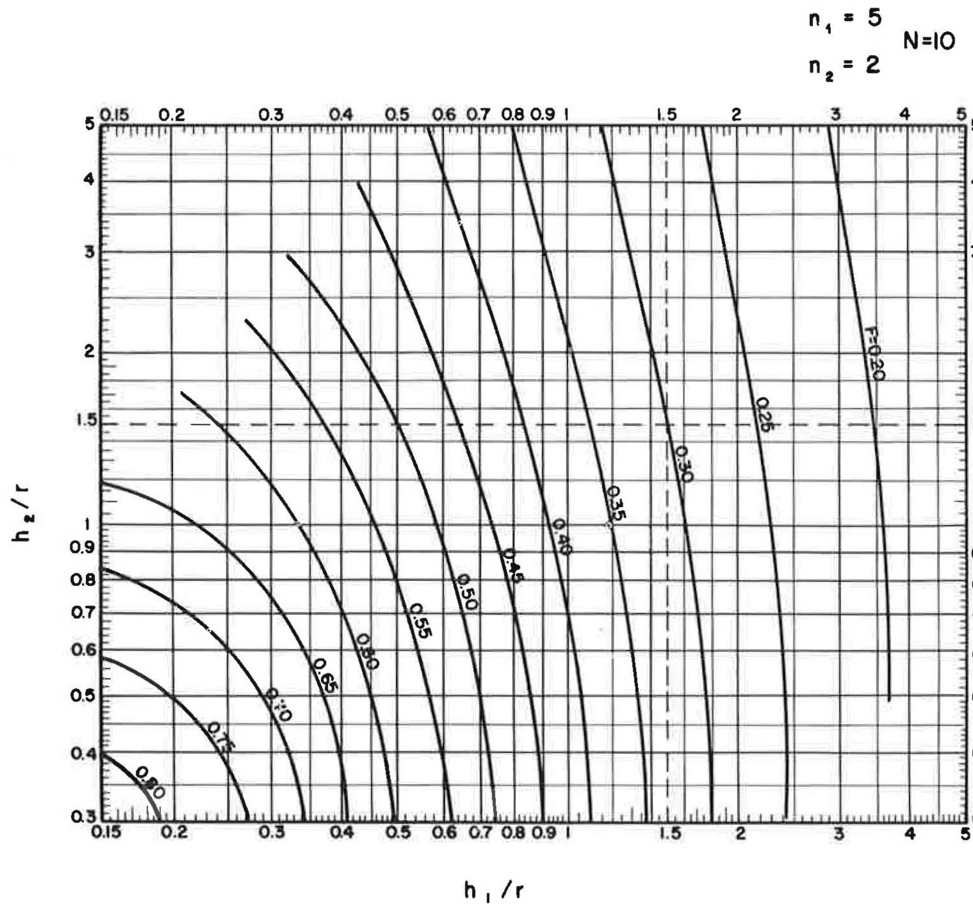
h_1/r \ h_2/r	0,156	0,312	0,625	1,25	2,5	5
0,312	.747	.651	.505	.352	.240	—
0,625	.601	.537	.438	.324	.231	.171
1,25	.449	.416	.359	.284	.215	.166
2,5	—	.320	.287	.240	.194	.158
5	—	—	.235	.202	.172	.148

Figure 5.



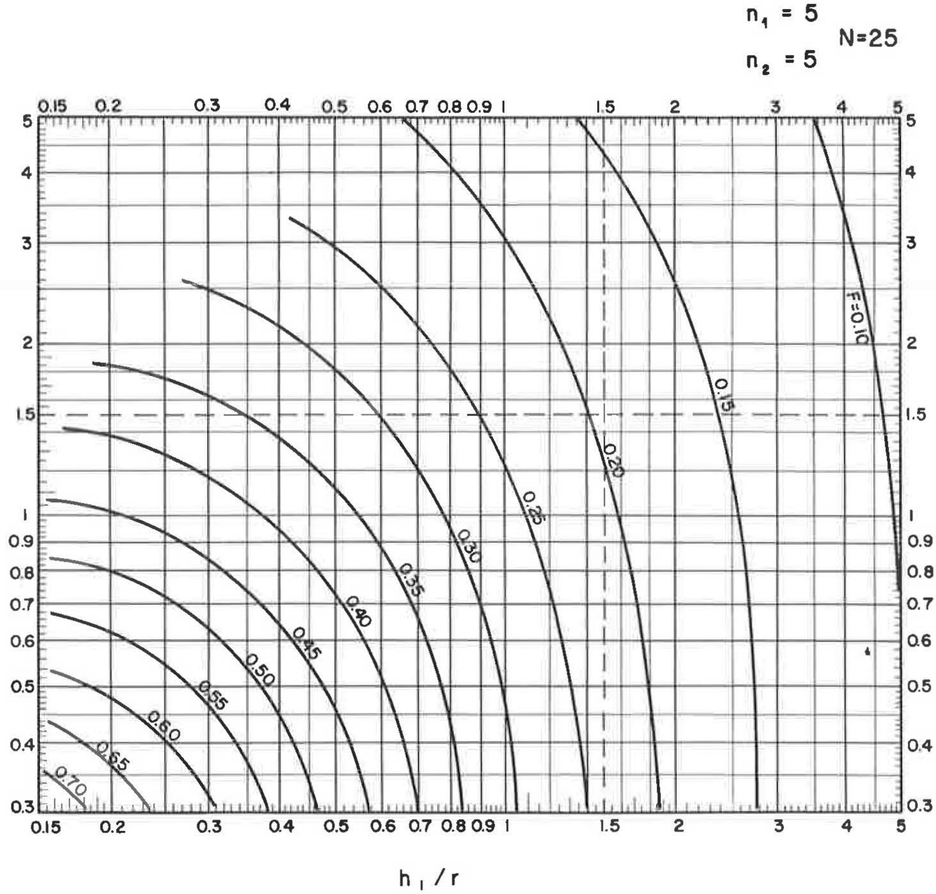
h_1/r \ h_2/r	0,156	0,312	0,625	1,25	2,5	5
0,312	.664	.559	.415	.274	.174	—
0,625	.500	.436	.346	.246	.165	.112
1,25	.344	.315	.267	.207	.150	.108
2,5	—	.221	.198	.165	.130	.100
5	—	—	.149	.128	.108	.0903

Figure 6.



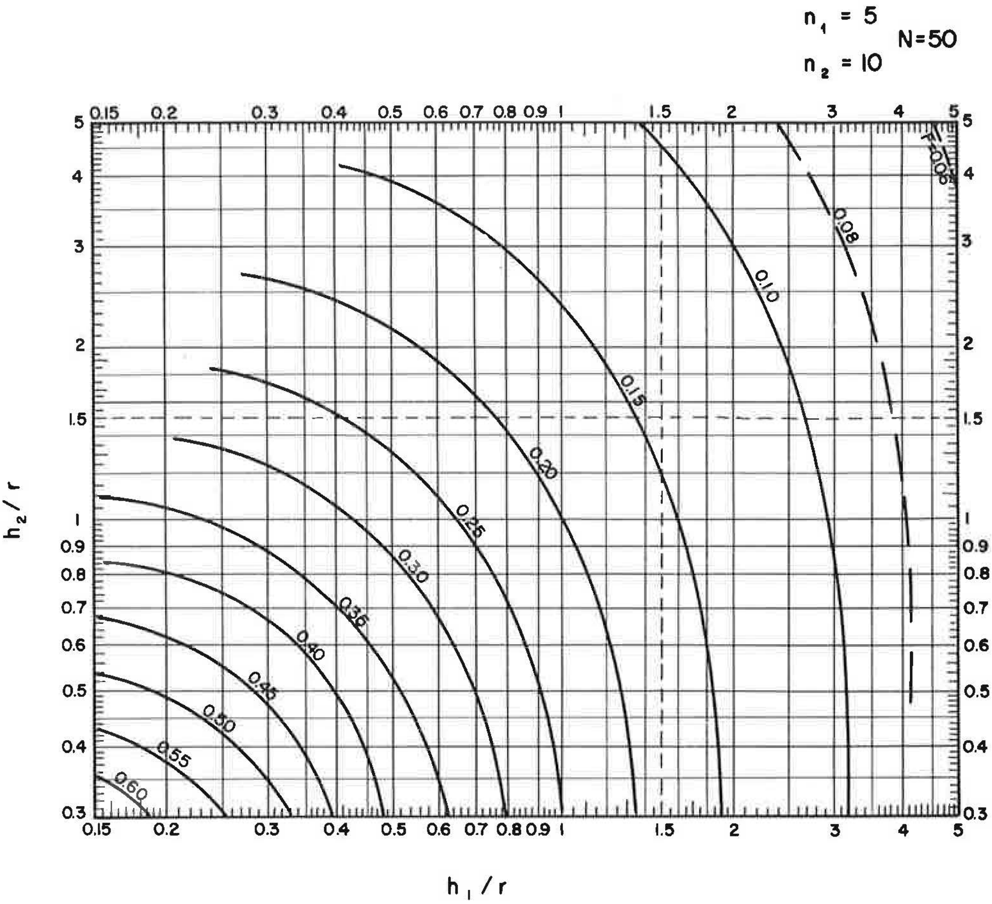
h_2/r \ h_1/r	0,156	0,312	0,625	1,25	2,5	5
0,312	.830	.733	.556	.372	.246	—
0,625	.745	.669	.522	.359	.242	.174
1,25	.648	.593	.476	.339	.235	.172
2,5	—	.528	.430	.313	.224	.168
5	—	—	.394	.289	.211	.163

Figure 7.



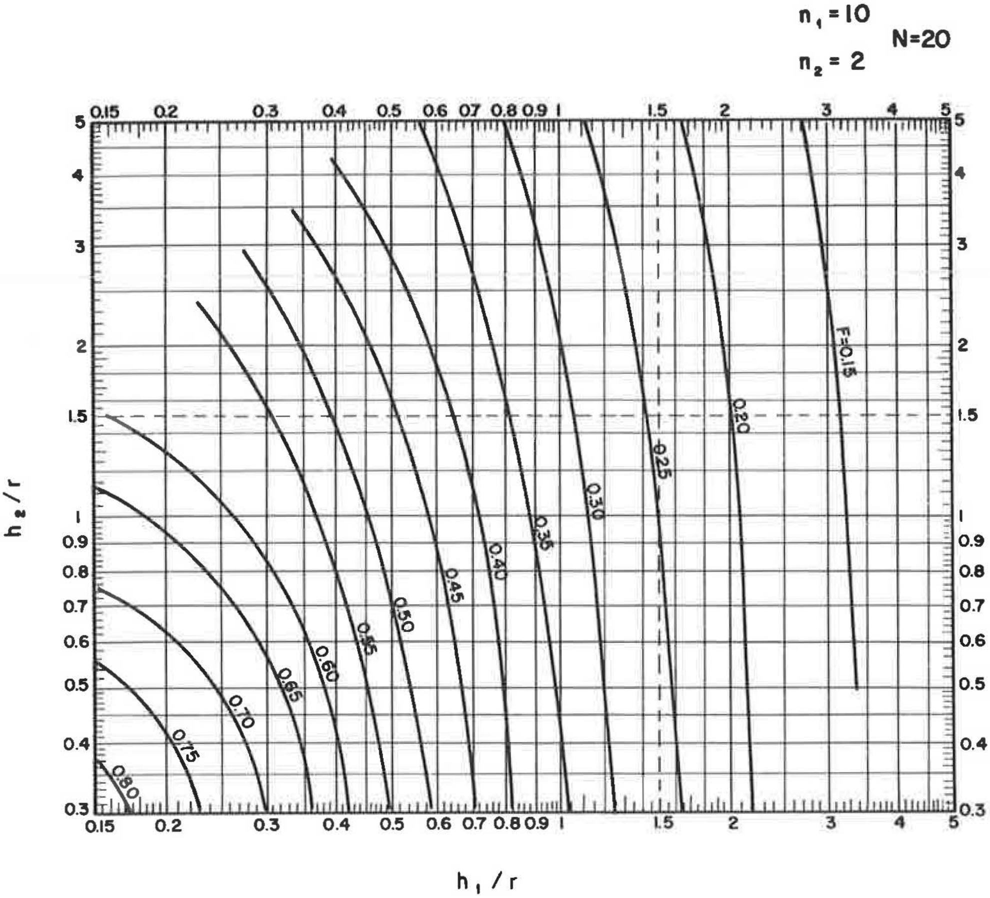
h_1/r \ h_2/r	0,156	0,312	0,625	1,25	2,5	5
0,312	.713	.598	.427	.268	.162	—
0,625	.572	.497	.378	.250	.157	.101
1,25	.429	.387	.312	.223	.148	.0989
2,5	—	.299	.249	.188	.134	.0944
5	—	—	.202	.155	.116	.0871

Figure 8.



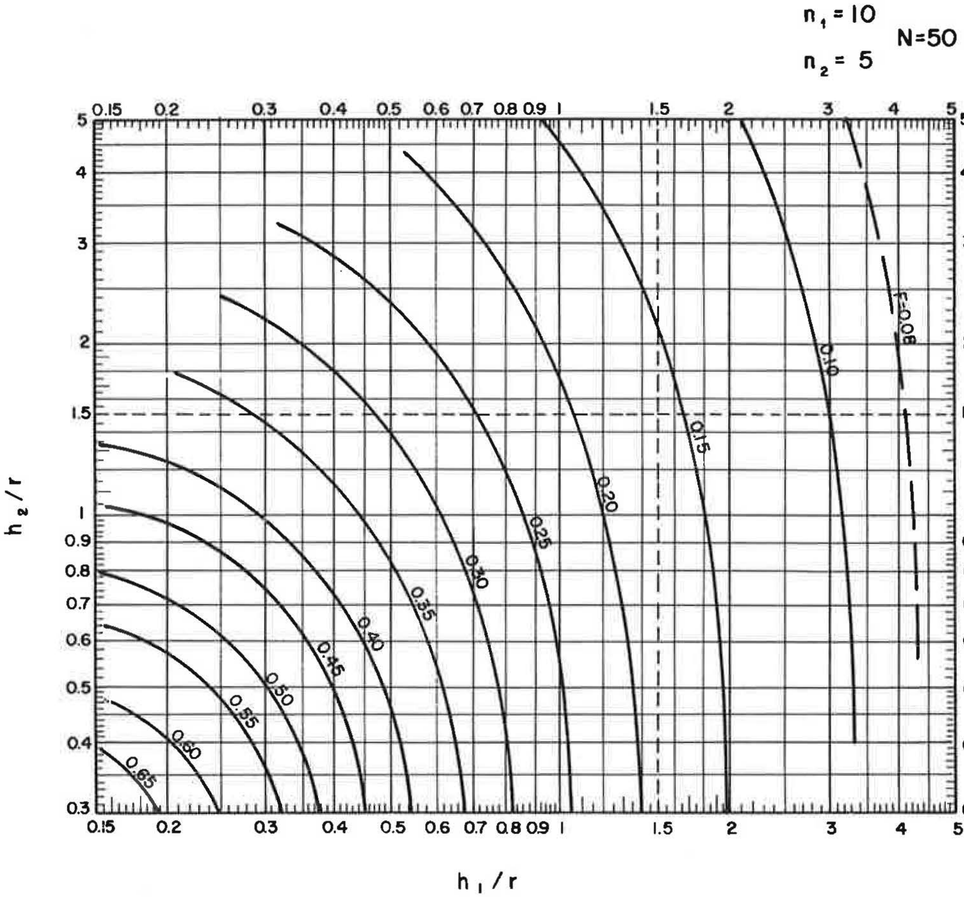
h_1/r \ h_2/r	0,156	0,312	0,625	1,25	2,5	5
0,312	.626	.508	.350	.211	.122	—
0,625	.469	.401	.301	.195	.118	.0712
1,25	.325	.291	.236	.168	.109	.0691
2,5	—	.206	.174	.134	.0954	.0649
5	—	—	.130	.102	.0779	.0579

Figure 9.



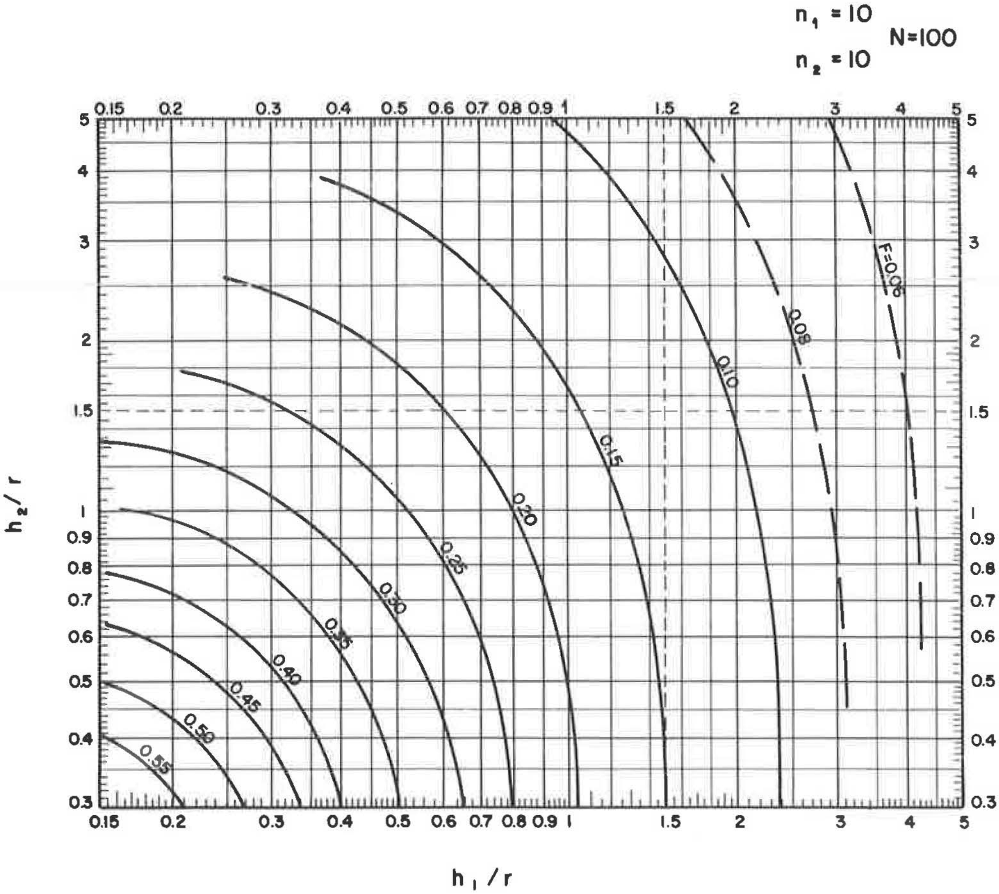
h_1/r \ h_2/r	0,156	0,312	0,625	1,25	2,5	5
0, 312	.811	.687	.483	.298	.181	—
0, 625	.729	.630	.457	.290	.178	.116
1, 25	.634	.561	.420	.275	.174	.115
2,5	—	.499	.379	.255	.166	.112
5	—	—	.346	.234	.156	.109

Figure 10.



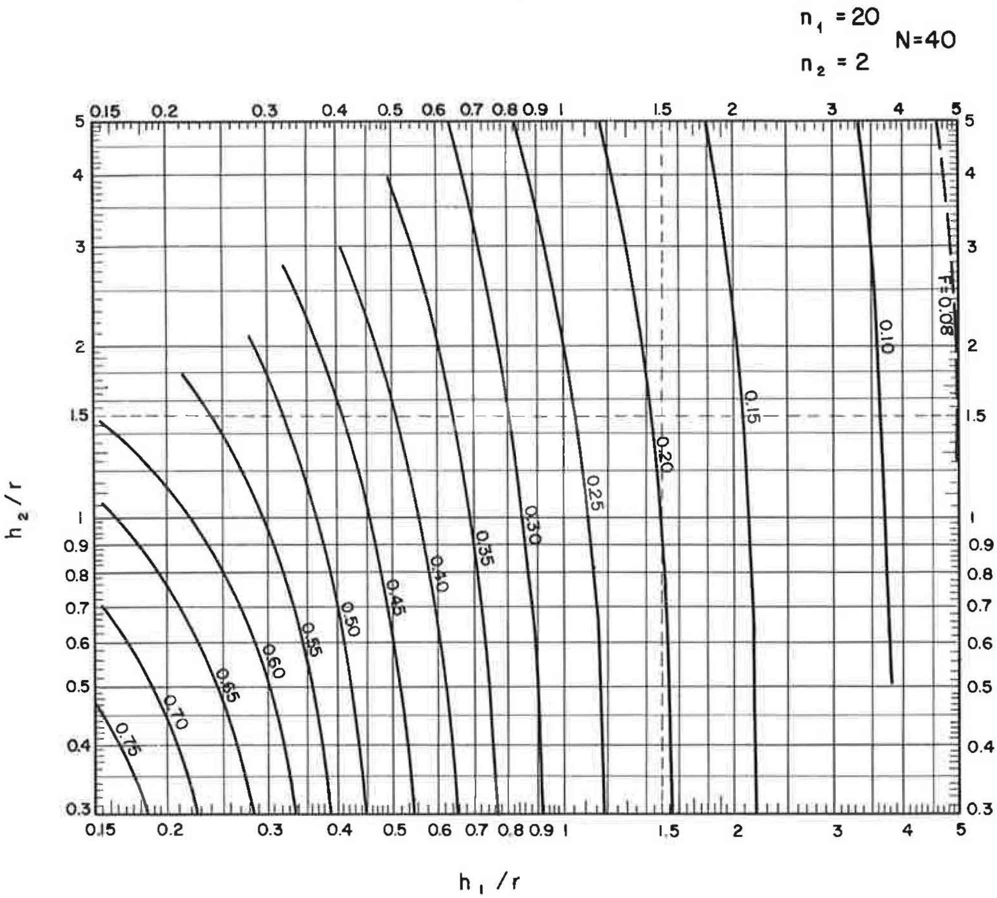
h_2/r \ h_1/r	0,156	0,312	0,625	1,25	2,5	5
0,312	.689	.553	.369	.217	.123	—
0,625	.553	.467	.334	.207	.121	.0720
1, 25	.416	.367	.281	.187	.115	.0706
2,5	—	.284	.225	.160	.105	.0679
5	—	—	.181	.130	.0912	.0629

Figure 11.



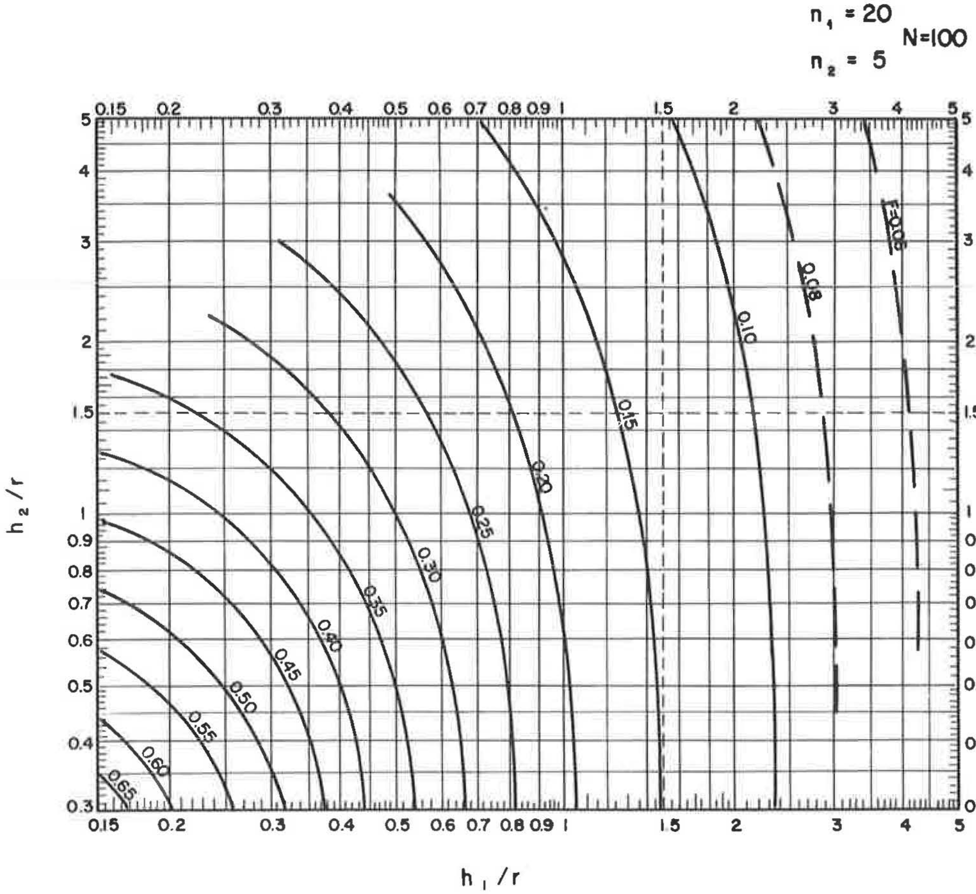
h_1/r \ h_2/r	0,156	0,312	0,625	1,25	2,5	5
0, 312	.601	.467	.302	.172	.0946	—
0,625	.450	.376	.267	.163	.0924	.0525
1, 25	.312	.275	.214	.144	.0876	.0514
2,5	—	.195	.159	.117	.0782	.0489
5	—	—	.117	.0885	.0642	.0442

Figure 12.



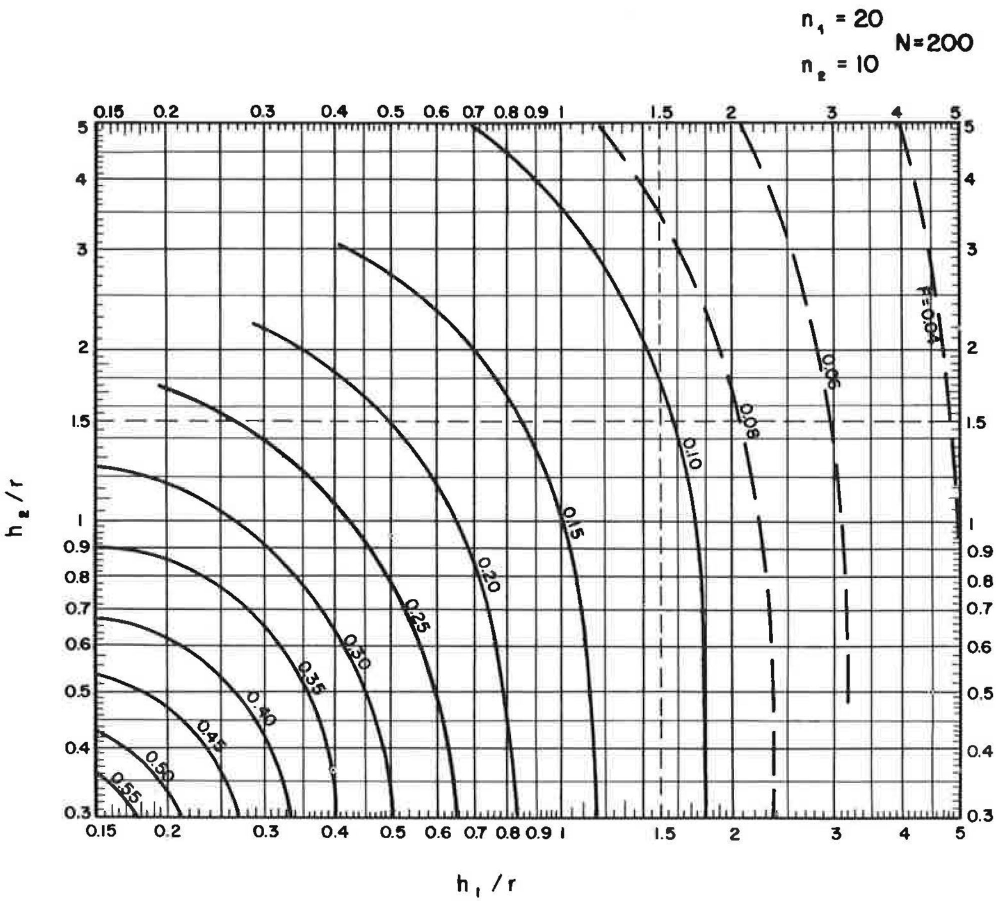
h_2/r \ h_1/r	0,156	0,312	0,625	1,25	2,5	5
0,312	.789	.629	.411	.239	.136	—
0,625	.711	.583	.394	.234	.135	.0809
1,25	.621	.523	.365	.224	.132	.0802
2,5	—	.465	.331	.209	.127	.0788
5	—	—	.300	.191	.120	.0764

Figure 13.



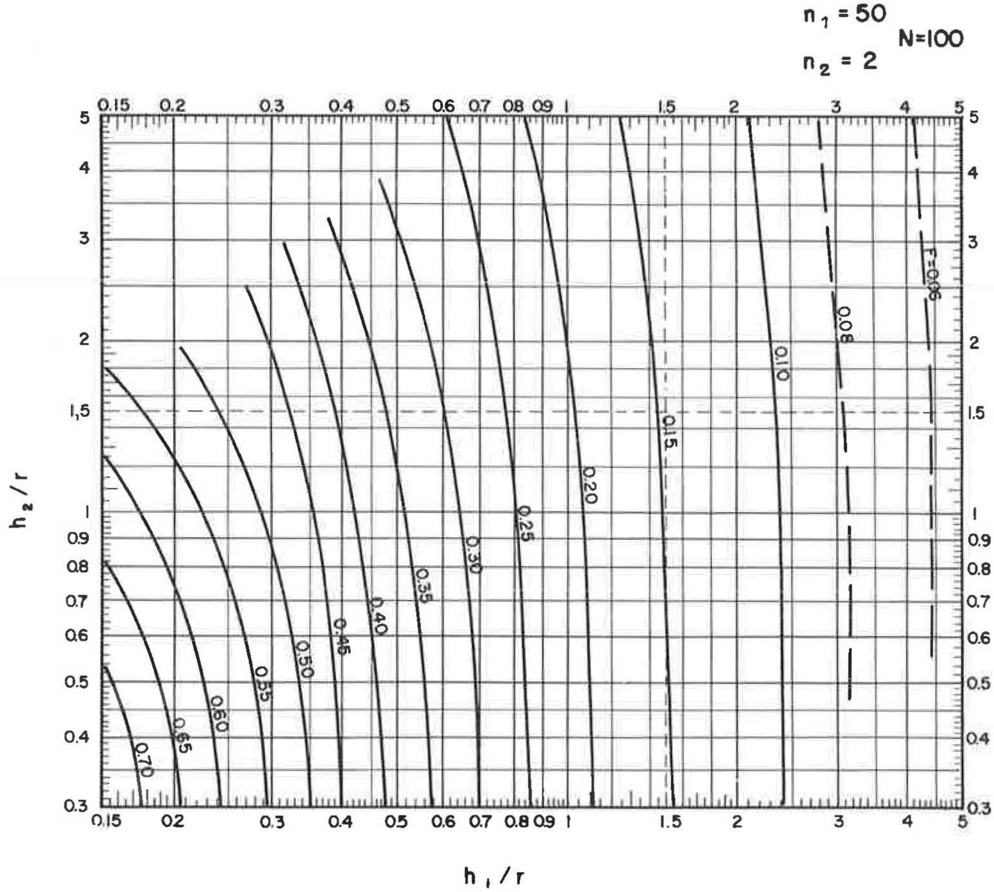
h_2/r \ h_1/r	0,156	0,312	0,625	1,25	2,5	5
0,312	.662	.501	.313	.175	.0953	——
0,625	.535	.433	.290	.169	.0939	.0528
1,25	.404	.345	.250	.157	.0909	.0521
2,5	——	.267	.202	.136	.0846	.0506
5	——	——	.161	.112	.0742	.0475

Figure 14.



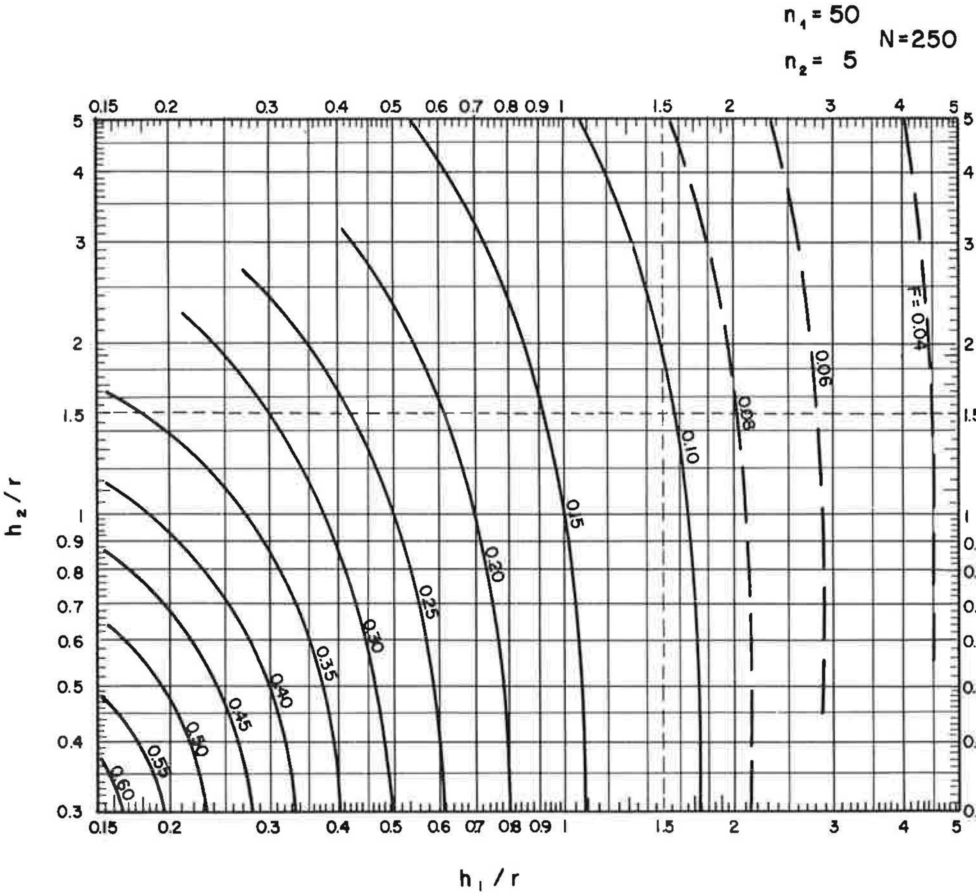
h_1/r \ h_2/r	0,156	0,312	0,625	1,25	2,5	5
0,312	.573	.419	.254	.139	.0740	—
0, 625	.434	.349	.232	.134	.0729	.0396
1,25	.301	.260	.193	.123	.0703	.0390
2,5	—	.184	.145	.102	.0645	.0377
5	—	—	.106	.0779	.0542	.0348

Figure 15.



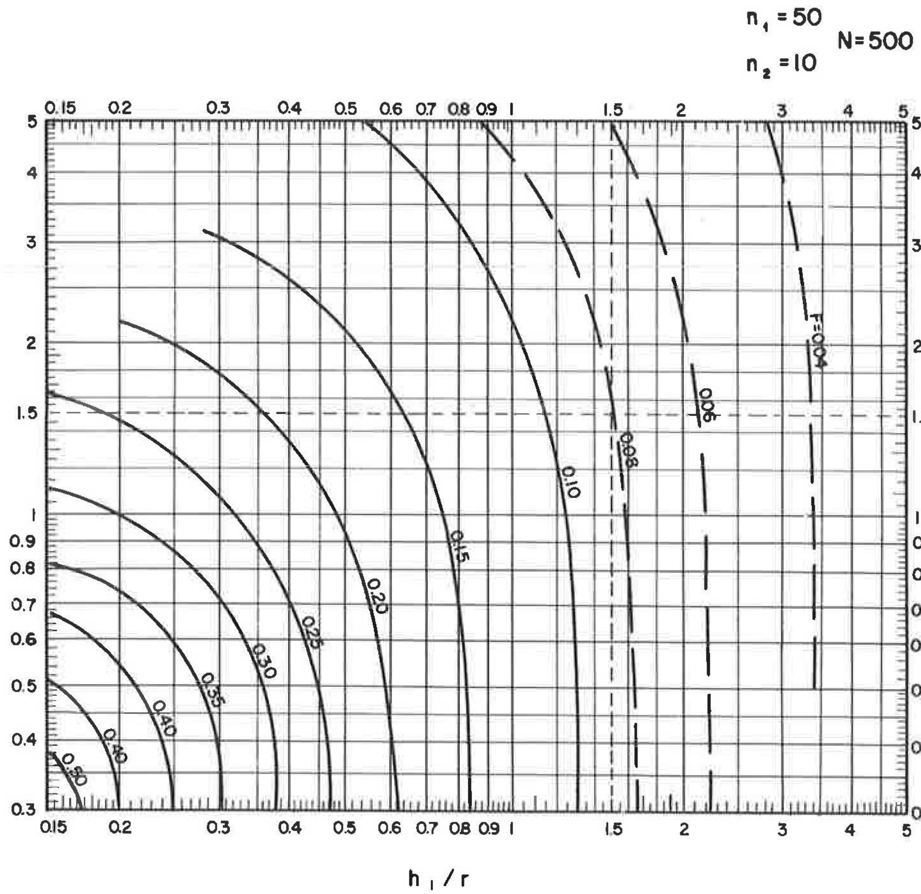
h_1/r / h_2/r	0,156	0,312	0,625	1,25	2,5	5
0, 312	.744	.538	.324	.178	.0960	—
0, 625	.677	.508	.314	.175	.0953	.0532
1, 25	.594	.461	.297	.170	.0940	.0528
2,5	—	.411	.271	.161	.0915	.0522
5	—	—	.245	.148	.0868	.0509

Figure 16.



h_1/r \ h_2/r	0,156	0,312	0,625	1,25	2,5	5
0,312	.612	.421	.244	.131	.0687	—
0, 625	.507	.378	.233	.128	.0681	.0364
1, 25	.387	.311	.209	.122	.0667	.0361
2,5	—	.241	.173	.110	.0637	.0354
5	—	—	.137	.0915	.0575	.0338

Figure 17.



h_1/r \ h_2/r	0,156	0,312	0,625	1,25	2,5	5
0,312	.524	.348	.197	.104	.0539	—
0,625	.410	.305	.187	.102	.0534	.0280
1,25	.288	.237	.164	.0966	.0523	.0277
2,5	—	.169	.128	.0847	.0496	.0272
5	—	—	.0926	.0664	.0436	,0258

Figure 18.

modulus $E_{1,2}$ and thickness $h_1 + h_2$ supported by the same subgrade of modulus E_3. The approximate formula is

$$E_{1,2} = \left[\frac{h_1 \sqrt[3]{E_1} + h_2 \sqrt[3]{E_2}}{h_1 + h_2} \right]^3$$

The term $h_i \sqrt[3]{E_i}$ is called the layer rigidity factor. The approximate formula is exact within 10 percent of the computed deflection for h_2/r not greater than 1, and within 15 percent for h_2/r not greater than 2. This formula is not indicated for h_2/r greater than 2. Making n' the modular ratio of the equivalent two-layer system and $h_1/h_2 = H$, it follows

$$n' = \frac{E_{1,2}}{E_3} = n_2 \left[\frac{H \sqrt[3]{n_1} + 1}{H + 1} \right]^3$$

The deflection of the equivalent two-layer system may be calculated by the graph in Figure 2 with the parameters n' and $(h_1 + h_2)/r$.

An analogous expression has been proposed by Palmer and Barber (4) to reduce the two-layer system to an equivalent uniform medium. Barber demonstrated that his approximate formula yielded deflections very close to Burmister's two-layer analysis (1). The validity of the rigidity factor concept both for two-layer and three-layer systems lends support to its tentative extension to multilayer systems. It is suggested that intermediate layers may be modified if the rigidity factor is kept constant. It is also suggested that multilayer systems can be reduced to equivalent three-layer systems for the purpose of deflection calculation, aggregating similar adjacent layers and computing their equivalent modulus by the foregoing formula. The subgrade should not be altered, in any case, and neither should it be included in the equivalent modulus calculation, due to its infinite depth. Further research is needed on these points.

The modular ratio E_1/E_2 for two-layer flexible pavement structures is always below 100, being closer to 100 for semiflexible soil-cement pavements (5, 6, 7, 8). The modular ratio for rigid pavements is always well above 100. Therefore, the graph in Figure 1, including values of E_1/E_2 from 2 to 100 is adequate for the structural analysis of flexible and semiflexible two-layer pavements. Rigid pavement design is based on a stress criterion and not on deflection limits. Accordingly, surface deflection computation is of little use for rigid pavement analysis, and it is not necessary to include in the graph the higher values of the modular ratio.

In the case of three-layer pavement structures, there is normally a marked difference between modular ratios n_1 and n_2. For technical and economical reasons n_1 and n_2 are always greater than one, and n_1 is usually greater than n_2. The effective modular ratio for granular non-cemented materials is always between 2 and 5 (9, 10, 11). Hence, the value of n_2 is usually between 2 and 5. The value of n_1 for flexible and semiflexible pavements is usually between 5 and 50. The three-layer deflection charts in Figures 4 to 18 provide an adequate range of modular ratios for the design and structural analysis of flexible and semiflexible pavements.

The elastic moduli to be used in the structural analysis should be measured by load bearing tests or other field tests conducted on the full pavement section. Laboratory tests on small samples or molded specimens do not correlate well with field values. Great care should be exercised in the determination of the layers moduli, for the moduli values have a critical effect on the deflections.

It is not correct to measure the subgrade modulus by load bearing tests on the subgrade alone, and then to measure the pavement layers' moduli by new tests on each superimposed layer, using in the calculations the subgrade modulus previously determined. This process gives too high and erratic values for the pavement moduli, as reported elsewhere (12). The effective in-place subgrade modulus is much higher than the modulus of the subgrade alone. The main reasons are (a) an increase in compaction of the lower layers caused by the compaction of the top layers; (b) the confining effect of the top layer; (c) the nonlinearity of the soil stress-strain curve; (d) the load spreading ability of the pavement, allowing lower stresses on the subgrade; (e) possibly, lack of agreement between the theoretical elastic model and actual pavement response. All these factors contribute in varying degrees to an increase of the effective subgrade modulus. The saturation of the subgrade by capillarity after the construction of the pavement acts in the opposite direction, but this effect is smaller than the sum of the others. Consequently, the modulus of the subgrade alone is of no avail for the calculation of the pavement moduli. All moduli should be measured simultaneously by tests conducted at the surface of the complete pavement. This determination is possible with load bearing tests with several plate diameters or several pavement thicknesses. Theoretically, the solution of a two-layer system requires at least two plate diameters (or two thicknesses) and the solution of a three-layer system requires at least three diameters (or thicknesses). The calculation of the moduli requires the solution of a system of simultaneous equations. For greater precision, the deflection factor values should be interpolated in the tables below each graph, instead of reading the values on the graphs. The problem is further complicated by the scatter of test results. A study of three-layer system moduli determination has been published (12).

A few examples of application are included to illustrate the use of the deflection factor charts.

PRACTICAL EXAMPLES

1. Suppose a pavement is composed of penetration macadam base course and light surface treatment, with total thickness of 15 cm (6 in.), resting directly on the subgrade. Assume the following values for the elastic moduli: subgrade soil 500 kg/cm² (7,140 psi); base and surfacing considered as a single layer 30,000 kg/cm² (429,000 psi). The basic wheel load is 5 long tons (11,200 lb), with contact pressure of 7 kg/cm² (100 psi) and contact radius of 15.1 cm (6 in.). Calculate the elastic deflection.

$$h/r = 15/15.1 \simeq 1$$
$$E_1/E_2 = 30,000/500 = 60$$
$$W = ?$$

The graph in Figure 2 gives the value of F by interpolation between the curves of $E_1/E_2 = 50$ and 100, for $h/r = 1$. For better accuracy, several values of F should be taken from the graph, at the intersections of the curves of E_1/E_2 with the vertical line of $h/r = 1$. An auxiliary graph should be made on log-log paper, plotting values of F vs respective values of E_1/E_2, and connecting the plotted points by a continous curve. This auxiliary graph gives the value of F for $E_1/E_2 = 60$

$$F = 0.26$$

$$\therefore\ W = \frac{1.75 \times 7 \times 15.1}{500} \times 0.26 \simeq 0.1 \text{ cm} \qquad (0.04 \text{ in.})$$

2. Design the thickness of the same pavement of Example 1 for the condition of the deflection being less than 0.05 cm (0.02 in.).

$$E_1/E_2 = 60$$
$$W = 0.05 \text{ cm}$$
$$h/r = ?$$

The required deflection factor is

$$F = \frac{0.05 \times 500}{1.75 \times 7 \times 15.1} = 0.13$$

Inasmuch as the deflection factor is proportional to the deflection, this value of F could be obtained taking half of the previous value. Now it is necessary to draw the complete curve of $E_1/E_2 = 60$ on the graph in Figure 2 interpolating between the curves of 50 and

100. The points near the probable solution, i.e., between $h/r = 2$ and 3, should be plotted by logarithmical interpolation, as in the previous example. The point at the intersection of the curve of $E_1/E_2 = 60$ with the horizontal line of $F = 0.13$ gives the solution

$$h/r = 2.3$$

$$\therefore h = 2.3 \times 15.1 = 35 \text{ cm} \qquad (13.8 \text{ in.})$$

It was necessary to increase the thickness to more than double the previous value to reduce the deflection in half.

3. Compute the elastic moduli of the same pavement, with 35 cm (13.8 in.) of thickness, from the following results of load bearing tests

Diameter		Modulus of Reaction	
cm	in.	kg/cm^3	pci
80	31.5	24.6	888
20	7.9	317.5	11,462

The equation is

$$E_2 = 1.75\ r\ k\ F$$

For ϕ 80 $h/r = 35/40 = 0.875$
$k = 24.6 \ kg/cm^3$
$F' = ?$
$E_2 = 1.75 \times 40 \times 24.6 \times F' = 1722\ F'$

For ϕ 20 $h/r = 35/10 = 3.5$
$F'' = ?$
$k = 317.5 \ kg/cm^3$
$E_2 = 1.75 \times 10 \times 317.5 \times F'' = 5556\ F''$

Comparing the two values of E_2

$$1722\ F' = 5556\ F''$$
$$\therefore F'/F'' = 3.22$$

Take from the graph in Figure 2 several values of F, at the intersections of the curves of E_1/E_2 with the vertical lines of $h/r = 0.875$ and $h/r = 3.5$, and calculate their respective ratios

E_1/E_2	F' ($h/r = 0.875$)	F'' ($h/r = 3.5$)	F'/F''
2	0.78	0.58	1.34
5	0.59	0.31	1.90
10	0.49	0.21	2.33
20	0.40	0.14	2.86
40	0.328	0.106	3.10
50	0.305	0.096	3.18
60	0.290	0.090	3.22
70	0.278	0.085	3.27
100	0.250	0.075	3.33

The points near the probable solution (E_1/E_2 more than 40) should be computed by logarithmical interpolation, as in the first example.

The computed value of F'/F'' equals the required value of 3.22 at the point $E_1/E_2 = 60$ or $F' = 0.290$. The moduli sought are

$$E_2 = 1722 \times 0.290 = 500 \text{ kg/cm}^2 \quad (7,140 \text{ psi})$$
$$E_1 = 60 \times 500 = 30,000 \text{ kg/cm}^2 \ (429,000 \text{ psi})$$

4. Calculate the deflection of the following three-layer pavement. The wheel load is 5 long tons (11,200 lb), the contact pressure is 7 kg/cm² (100 psi) and the contact radius is 15.1 cm (6 in.).

Layer	Thickness cm	Thickness in.	Modulus kg/cm²	Modulus psi
1. Asphaltic-concrete surface course	15	6	50,000	714,300
2. Stabilized base course	20	7.9	2,500	35,700
3. Subgrade	—	—	500	7,140

$$n_1 = 50,000/2,500 = 20 \qquad h_1/r = 15/15.1 \simeq 1$$
$$n_2 = 2,500/500 = 5 \qquad h_2/r = 20/15.1 = 1.32$$
$$E_3 = 500 \text{ kg/cm}^2$$

From the graph in Figure 14 : $F = 0.185$

$$\therefore \ W = \frac{1.75 \times 7 \times 15.1}{500} \times 0.185 = 0.07 \text{ cm} \qquad (0.027 \text{ in.})$$

5. Modify the thickness of the pavement of Example 4 so that the deflection is less than 0.05 cm (0.02 in.). The required deflection factor is

$$F = \frac{0.05 \times 500}{1.75 \times 7 \times 15.1} = 0.13$$

A new curve for $F = 0.13$ should be interpolated between the curves of 0.10 and 0.15 in the graph of Figure 14. All points on this curve correspond to thicknesses h_1 and h_2 satisfying the required condition of $W = 0.05$ cm. Possible combinations are as follows.

h_1/r	h_2/r
1	4.7
1.25	3
1.5	1.7
1.6	1.32

The best combination of h_1 and h_2 should be selected by considering economic and engineering aspects, taking into account the unit costs of surface and base courses. In most cases, it is cheaper to increase the base thickness. Suppose the first combination is selected

$$h_1/r = 1 \qquad h_1 = 1 \times 15.1 \simeq 15 \text{ cm}$$
$$h_2/r = 4.7 \qquad h_2 = 4.7 \times 15.1 \simeq 71 \text{ cm}$$

Total thickness: $15 + 71 = 86$ cm (33.9 in.)
If the fourth combination were preferred

$$h_1/r = 1.6 \qquad h_1 = 1.6 \times 15.1 = 24 \text{ cm}$$
$$h_2/r = 1.32 \qquad h_2 = 1.32 \times 15.1 = 20 \text{ cm}$$

Total thickness: 24 + 20 = 44 cm (17.3 in.)

Increasing the surface course from 15 to 24 cm permits reducing the total thickness from 86 to 44 cm. In this particular case, 1 cm of surface course is equivalent to 5.7 cm of base course.

The deflection of the latter combination may be checked by the approximate formula, since h_2/r is less than 2.

$$H = 1.6/1.32 = 1.22$$

$$n' = 5\left[\frac{1.22\sqrt[3]{20}+1}{1.22+1}\right]^3 = 36.2$$

$$n'/r = \frac{24+20}{15.1} = 2.91$$

Entering the graph of Figure 2 with parameters n' and h'/r F = 0.128

$$\therefore\ W = \frac{1.75 \times 7 \times 15.1}{500} \times 0.128 = 0.047 \text{ cm}$$

The result is close enough to the specified deflection of 0.05 cm.

6. If the given modular ratios are intermediate between the values of the graphs in Figures 4 to 18, it is necessary to construct an auxiliary graph of similar aspect, by a series of logarithmical interpolations between the given graphs. Consider for instance the following case.

$$n_1 = 15 \qquad h_1/r = 1$$
$$n_2 = 4 \qquad h_2/r = 1.5$$

The deflection factor is bracketed by the following values

$n_1 \times n_2$	F
20 × 5	0.175
20 × 2	0.265
10 × 5	0.210
10 × 2	0.315

The deflection factor, obtained by three interpolations on log-log paper, is

$$F = 0.208$$

If an analysis of several thicknesses is required, it is necessary to trace the auxiliary graph corresponding to $n_1 = 15$, $n_2 = 4$ for the whole range of thicknesses.

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