Control of Traffic in Tunnels to Maximize Flow

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In 1961 a preliminary control system based on speed and flow measurements was implemented in the Holland Tunnel South Tube. A red light at the entrance portal introduced gaps into the flow in order to avoid stop-and-go waves and attempt to increase the flows.

This paper presents an inexpensive way of improving the existing Holland Tunnel System; experimental evidence is used to support the proposed change. It also presents several control logics, based on speeds and densities, which can be implemented by either use of a fixed program analog device or of a general analog computer.

A model is derived for general-purpose control of single-lane flow based on finite-state machine theory. Implementation of this system, which is more flexible than the previous one, requires a digital computer.

In the Appendix is a method of estimating section density using one or more point velocities, and the input flow is given and an approximate experimental verification is made. This method can be used as a monitor for a density measuring device based on car-count, which drifts whenever cars change lane or a count is missed.

• THIS STUDY is linked to the material previously presented by Foote (1) and a paper, also presented in this Highway Research Record, by Foote and Crowley (2).

Since 1958 The Port of New York Authority has been investigating whether peak flow through the five tubes connecting New York City and New Jersey could be increased and how the increase could be accomplished. Two of these tubes form the Holland Tunnel, three of them the Lincoln Tunnel. Each tube has two lanes; however, lane changing is normally prohibited so that traffic can be considered as single lane.

A series of experiments in the south tube of the Holland Tunnel showed that peak flow of traffic could be increased by 5 percent if gaps were manually introduced in the stream. This 5 percent increase resulted in a 33 percent decrease of congestion on the approach facilities. An experimental control system was built with fixed-logic elements so that gaps would be introduced automatically in the stream according to a predetermined program. The control box activated a red light at the entrance of the Holland Tunnel each time that certain combinations of local speeds inside the tunnel and flow rate at the entrance of the tunnel were measured. The system had a tendency to overcontrol the traffic; large fluctuations of flow characteristics were also observed. A high density period was followed by a series of red lights which caused a lower density period.

The present study examines how this experimental control system can be improved with a minimum amount of hardware. Other control logics based on density rate and speed are given. They can be implemented by fixed-program systems which are built at a reasonable cost; however, the flexibility of fixed-program control boxes is limited. Control diagrams which can be implemented with analog elements are also given. In addition, this investigation deals with a general finite-state model of a tunnel control system for digital computer control.
A discussion of various density measurement schemes is given in the Appendix. The data used throughout this study were taken in the Holland Tunnel south tube by methods described by Foote and Crowley (2). The Holland Tunnel can be divided into three zones: a 3,100-ft long downgrade, a 3,400-ft long level portion, and a 1,600-ft long upgrade. Point 0 is at the entrance of the tunnel, near the toll booths. The distances between data points 0, 1, 2, 3, 4, 5 and 6 are, respectively, 1,800, 1,300, 1,900, 1,000, 500 and 1,350 ft. Point 5 is at the foot of the upgrade. A bottleneck seems to occur between points 4 and 5. This is why data taken at point 4 are given particular attention.

NOTATION

A, B, C = categories of calculated densities, constants;

\( f_j \) = frequency of a given event in jth section of tunnel;

\( h_j, H_j \) = threshold values of density in jth section of tunnel;

i = running index for discrete instants of time \( t_i \), \( i = 0, 1, 2, \ldots \);

j = running index for tunnel section, \( j = 1, 2, \ldots, n \);

k = density, vehicles per mile;

\( k_j(m) \) = density in section j at time m;

\( k_{jj'}(m) \) = density between points j and j' at time m;

\( L_j \) = length of jth section of tunnel, feet;

m = an integer number of time units;

\( M_j \) = data point in jth section of tunnel;

n = number of tunnel sections;

\( N_j \) = number of cars passing point j per length of time;

\( O \) = origin and entrance portal of tunnel;

\( Q \) = flow, vehicles per hour;

\( \bar{Q}_j(m) \) = average flow at point \( M_j \) at time m;

\( t_i \) = a time instant or ith sampling time;

\( V \) = speed;

\( \bar{V}_j(m) \) = average speed at point \( M_j \) at time m;

\( W, w \) = threshold values of \( V \);

\( x \) = linear distance in tunnel;

HT = Holland Tunnel;

LT = Lincoln Tunnel; and

ST NL = south tube near lane.

HOLLAND TUNNEL SYSTEM IMPROVEMENT WITH MINIMUM HARDWARE

To improve the control system tested in the Holland Tunnel south tube in 1961-64 (3), one must first examine why the density and velocity exhibit a cyclic behavior. At and after time \( m \) the system senses whether or not the following quantities are above a certain threshold:

1. The maximum number \( N_4 \) of vehicles whose speed was less than 20 mph at point 4 between times \( m - 1 \) and \( m \);
2. The average flow \( \bar{Q}_4 \) at point 4 between times \( m - 1 \) and \( m \);
3. The maximum number $N_x$ of vehicles whose speed was less than 20 mph at any point $x$ between 0 and 6, excluding 4 during the time interval $(m - 1, m)$ and

4. The number $N_0$ of cars which have been entering the tunnel since time $m$, where $m = 1, 2, 3, \ldots$ minutes.

Using these data, a matrix determines at what time $t_i$, $m \leq t_i \leq m + 1$, a red light is activated at the entrance of the tunnel; at time $m + 1$, the light is deactivated and the system is reset. Unfortunately, point 4 is 6,000 ft away from the tunnel entrance (point 0), so a car traveling 40 ft/sec takes 150 sec to cover distance 04. Thus, the control actuated at time $m$ for a group of such cars would have been applied at time $m - 2.5$, say $m - 2$ to have a round figure. At time $m$, the control may or may not be beneficial. In fact, it is possible to conceive several extreme cases where the control will act exactly opposite to the way it should (Fig. 1). Due to traffic characteristics, the bad and beneficial effects of the control system have apparently compensated each other, so that no effect on the average tunnel output has been felt. Another argument in favor of this hypothesis is illustrated in Figure 2.

\[ \bar{v}_4(t) \text{ and } \frac{2000}{\bar{v}_1(t-2)} \] (where 2000 is a scale factor) vary the same way; i.e., the speed at point 1 at time and the speed at point 4, two units of time later, are in opposition of

![Figure 1](image_url)

Figure 1. Possible disadvantages of a control system based on speed.
Figure 2. Comparison of $\frac{2000}{\bar{V}_1(t-2)}$ and $\bar{V}_4(t)$.

Figure 3. Improving the existing Holland Tunnel control.
phase. This means that the control by intermittent red lights introduces not only gaps into the traffic stream, but also speed waves. Thus, the system has exhibited a tendency to overcontrol the flow.

The existing system would be improved if one of the quantities entering into the control box was sensing the effect of the control after a short time, e.g., 30 sec. As will be seen, point 1, 1,800 ft from the entrance, is an excellent data point for many purposes. Let $\overline{V}_1$ (1 min average) be introduced into the control system as a feedback in the following manner: The red light is controlled as before; if it is activated in the interval $(m, m+1)$, then $\overline{V}_1$ is examined at time $m+1$. If $\overline{V}_1 > 52.8$ ft/sec (or 0.1

Figure 4. Logic diagram for an improved Holland Tunnel control system.
ml/sec, an empirical value), then the control cannot be activated until the interval \((m+2, m+3)\). Otherwise, there is no change in the previous procedure. The vertical arrows in Figure 3 indicate where the sequence of red lights would have been interrupted with the proposed criterion. Figure 4 is a diagram of the modified control system, where \(1/A\) is a small time increment of the order of 2 sec.

Thus, with a minimum amount of additional hardware, the tendency to overcontrol and to generate speed waves would be decreased by an amount to be determined experimentally.

However, an excessive lag between the time when the need for a correction is detected and the effect of the correction is felt is still inherent in such a system. The next step is to try to predict the need for a correction so that it can be applied in due time. To this effect, the input to the control system must include a quantity which can be predicted; i.e., either output flow or section density. Data show that section density is a reliable state variable for a tunnel.

**ANALOG CONTROL SYSTEMS WITH MINIMUM HARDWARE**

To clarify the problem, a procedure which a human operator would reasonably follow is analyzed, simplified and mechanized. Density estimation methods are developed and discussed, and a control logic for a tunnel with one bottleneck is given. Finally, control diagrams for a tunnel lane with two bottlenecks are presented.

**A Human Control Analog**

A person in charge of maximizing the flow through the given tunnel lane is placed in front of the 18 television pictures of the present I6 ST surveillance system. Let this person have the possibility of impeding flow independently through any of the 18 sections at which he can look. A sensible control procedure which he can follow is:

1. Look at all the screens, beginning at the downstream end of the tunnel.
2. If cars are stopped or going very slowly in section \(j, 1 \leq j \leq 18\), observe whether there exists a sufficient number of sizable gaps upstream from \(j\), so that this disturbance can be absorbed.
3. If the disturbance is likely to propagate backward and impede (i.e., slow down or stop) the traffic upstream from section \(j\); otherwise, continue scanning the screens.
4. Cease control as soon as the disturbance has disappeared.

Such subjective control cannot be programmed for a machine. The sentence "a sufficient number of sizable gaps upstream from \(j\)" must be replaced by an inequality between measurable quantities, such as densities. Figure 5 represents an analog to the human control process just described. The quantity \(H_j\) is a critical density in sections 1 to \(j\), above which shockwaves have been observed to propagate backward 80 percent of the time.

On the other hand, the system can be made "learning," with a procedure analogous to the one a person would follow. After a while, the person in charge of the control will know where to look for bottlenecks. If the shockwave formation frequency in section \(j\) is \(f_j\), \(\sum\limits_{j=1}^{18} f_j = 1\), he will tend to look at screen \(j\) with a relative frequency, \(f_j\).

This system is more efficient than the one in which sections are scanned sequentially, and can be applied to any single-lane traffic situation.

When a tunnel lane possesses one or two fixed bottlenecks, such as the Holland Tunnel south tube near lane and the Lincoln Tunnel south tube near lane, one of the control systems presented later can be used. Since all these systems necessitate a knowledge of section density, \(k\), methods of obtaining \(k\) will first be investigated.
One-Bottleneck Tunnel Control

Taking the Holland Tunnel as an example, let the bottleneck start at point 4. Let $H_{04}$ be the density in section 04 at time $m$, above which the frequency of shockwave occurrence at time $m + t$, $t \leq 2$, is greater than 0.95.

Let $h_{04}$ be the density in section 04 at the time $m$, above which the frequency of shockwave occurrence at time $m + t$, $t \leq 2$, is less than 0.10.

The simplest control procedure consists in keeping $k_{04}$ below $H_{04}$. If $k_{04} > H_{04}$ then the control should impede vehicles from entering until $k_{04} < h_{04}$.

The densities $H_{04}$ and $h_{04}$ may be determined from existing data. Naturally, the threshold values 0.95 and 0.10 can be changed. Edie (4) has investigated the generation of stop-start waves and is currently examining the question in further detail. The order of magnitude of $H_{04}$ is 55 veh/mi. It is reasonable to postulate that both $H_{04}$ and $h_{04}$ depend upon the speed at point 4 (or downstream of it). At high speeds, shockwaves may
occur for lower density than at low speeds. A control diagram for the case when the limit densities $H_{04}$ and $h_{04}$ depend upon $V_4$ is shown in Figure 6. Traffic composition, weather and other factors have a definite influence on the threshold values $H$ and $h$. This is the subject of other studies presently being undertaken at the Port Authority.

**Two-Bottleneck Tunnel Control**

Given a roadway tube with bottlenecks A and B, such as the Lincoln Tunnel south tube (5); let this tube be divided into sections OA and AB. Let P and Q be two points

![Figure 6. Logic diagram for control of density with varying bounds.](image-url)
slightly upstream of A and B, respectively. The control diagram suggested for the one-bottleneck tunnel can be applied successively to sections OA and AB. The resulting chart, in which the densities are either estimated using $V_p$ and $V_Q$ or measured by counting vehicles at O, P and Q is shown in Figure 7.

Various methods to measure or estimate density are discussed in the Appendix. In particular, Figure 11 can be used to estimate $k_{OA}$ or $k_{MB}$.

![Figure 7. Control of two-bottleneck tunnel using two speeds and two densities.](image-url)
In the case when the functional relationships \( H(V) \) and \( h(V) \) are not readily obtainable, but vehicle speed must be taken into account, Figure 8 can be used. Here, an upper limit \( W \) and a lower limit \( w \) to the speed \( V \) must be given in addition to two constant bounds \( H \) and \( h \) on \( k \). The control system requires only one density measurement \( k_1 \) and two speed measurements \( V_1, V_2 \); the second density \( k_{AB}(t_i) \) is taken as \( k_{OA}(t_i - 1) \). A third speed near the tunnel exit can easily be incorporated into the system (Fig. 8).

Figure 8. Control of a two-bottleneck tunnel using two speeds and one density.
DIGITAL-COMPUTER CONTROL

Finite-State System Representation of a Tunnel Lane

All the control systems presented previously are only moderately sophisticated and can be implemented with solid-state or analog logic. However, an on-line digital computer control can handle a large amount of information with more flexibility in the input-output procedures than a fixed logic or analog system, and is bound to provide for a better control procedure. In addition, experiments, both in the laboratory (simulation) and on-line, are easy to perform with digital devices. A digital control requires no prior knowledge of the dynamics of the system in opposition to an analog control.

The conceptual framework of finite-state machine theory will be used to present the control system. Martin (6) had made several steps in this direction. A good text on the theory of finite-state machines is by Gill (7). The proposed finite-state model is essentially a black box which, when in a certain state, responds at every unit of time to a given excitation (or input) with a predictable output. Here, the output will be a set of control times.

The input to this model consists of the number of cars which are entering the tunnel between time m and m + 1, and of the average speed at several points 1, 2, ..., n in the tunnel. The state of the system is the set of 1-min average densities in sections (0, 1), ..., (1, n).

For every unit of time, a control time of the tunnel entrance is determined by the largest density in the length OX, where X = 1, 2, ..., n. Another control time at point j is determined by comparing speed, density and flow in section (j - 1, j) with corresponding quantities in section (j, j + 1). Here, a very conservative control policy is applied. The control device in section j > 1 is activated if and only if the three conditions \( V_j > V_{j+1}, k_j > k_{j+1}, \) and \( k_j + 1 \frac{V_j}{V_{j+1}} > Q_0 \) are simultaneously satisfied so that no false alarm is likely to occur. Once flow through section j is impeded, it is assumed that flow through all sections upstream of j must be impeded in a similar fashion, so that vehicles do not pack up inside the tunnel.

In addition to an output function, a finite-state machine is characterized by a "next state" function; i.e., the state at time m + 1 is uniquely determined by the state and input at time m. It is clear that the density in section j at time m + 1 can be determined from the speed and density in sections j - 1 and j at time m; the density in section 1 is determined by the entrance rate.

This finite-state machine merely determines a set of control times at fixed time intervals. The implementation of the control is done separately and independently. A considerable amount of flexibility is thus available.

Finite-State Model

Let the tunnel be divided into n sections of length Lj ending, respectively, at points 1, 2, ..., m. Let the density averaged in the time interval (m - 1, m) in section (j - 1, j) be k_j (m) and the speed averaged in the same time interval at a control point M_j located slightly upstream of j be V_j (m). As before, N_0 (m) cars have entered the tunnel in the time (m - 1, m) (Fig. 9a).

The finite state control system is represented in Figure 9b and is defined as follows:

1. \( I_0 \) is the set of non-negative integers \( \leq 120 \).
2. The input alphabet, state set and output alphabet are Cartesian products of the type \( I_0 \times I_0 \times \ldots \times I_0 \), i.e., sets of a certain size whose elements are integers between 0 and 120.
3. The input at time m is the ordered \((n + 1)\)-tuple (ordered set of size \(n + 1\)), \( N_0, V_1, \ldots, V_n \).
4. The state at time m is the ordered n-tuple (ordered set of size n), \( k_1, k_2, \ldots, k_n \).
5. The output at time m is the ordered n-tuple (ordered set of size n), \( T_0, T_1, \ldots, T_n - 1 \), where \( T_j \) is the time in seconds during which traffic through point j is impeded, beginning at time m.
6. The output function, which uniquely relates the output at time m to the input and state at time m, is given by
INPUT: \( N_0(m) \), \( \{V_j(m)\} \)

\( j = 1, ..., n \)

STATE

\( \{k_j(m)\} \)

\( j = 1, ..., n \)

OUTPUT

\( \{T_j(m)\} \)

\( j = 1, ..., n \)

(a) ACTUAL CONDITIONS AT TIME \( m \)

(b) CONDITION OF FINITE-STATE MACHINE MODEL AT TIME \( m \).

Figure 9. Finite-state machine model of tunnel control.

\[ T_0(m) = C_0 \max_{1 \leq x \leq m} \left[ \frac{1}{x} \sum_{j=1}^{x} k_j - H(x) \right] \]

if \( \sum_{j=1}^{x} k_j > x H(x) ; T_0(m) = 0 \) otherwise,

\[ T_j(m) + C_1(k_j V_j - k_j + 1 V_j + 1) \]

if the three following inequalities are simultaneously satisfied:

\[ k_j > k_j + 1, \quad V_j > \bar{V}_j + 1 \quad \text{and} \quad k_j + 1 \bar{V}_j + 1 > Q_o \quad \text{for} \ j > 1; \]

all quantities are evaluated at time \( m \); \( Q_o \) is a constant of the order of 1,000 veh/hr.

\( T_j(m) = 0 \) otherwise, where \( C_0, C_1 \) are constants; in addition, the inequalities \( T_0 \geq T_1 \geq T_2 \geq ... \geq T_j \) must be satisfied.

7. The next state function, which uniquely relates the state at time \( (m+1) \) with the input and state at time \( m \), is obtained by subtracting the flow out of section \( j \) from the flow into it:

\[ k_i(m+1) = k_i(m) + (N_o - V_i k_i) \frac{1}{L_i} \]

\[ k_j(m+1) = k_j(m) + (V_j - 1 k_j - 1 - V_j k_j) \frac{1}{L_j} \quad \text{for} \ j > 1 \]

where all the variables on the right-hand side are taken at time \( m \), and the numbers \( k_j, j = 1, ..., n \), rounded off to the nearest integer.
Thus, once the initial state of the system is given, the response of the machine to any input sequence

\((N_0, \bar{V}_1, \ldots \bar{V}_n)_1, (N_0, \bar{V}_1, \ldots \bar{V}_n)_2, \ldots\)

is uniquely determined.

**Example**

To illustrate the finite state machine model, let \(n = 2\). The following quantities and equations are needed to obtain the output \((T_0, T_1)\) at time \(m\):

\[
H(1) = 75, \quad H(2) = 65 \text{ veh/mi}
\]

\[
T_o(m) = \max (k - 75, 0), \quad \text{with } C_o = 1 \text{ for } T_o \text{ in sec}
\]

\[
\frac{(k_1 + k_2)}{2} - 65
\]

\[
T_1(m) = (k_1 \bar{V}_1 - k_2 \bar{V}_2) 10^{-2}
\]

if \(k_1 > k_2\) and \(\bar{V}_1 > \bar{V}_2\)

and \(k_4 \bar{V}_2 \geq Q_o, \quad Q_o = 1,000 \text{ veh/hr.}\)

\[= 0 \text{ otherwise, with } T_1 \geq T_o.\]

The following quantities and relations are used to obtain the next state, i.e., the state at time \(m + 1\):

\[L_1 = 3300 \text{ ft, } L_2 = 4800 \text{ ft; time unit = 60 sec}\]

\[k_1 (m + 1) = k_1 (m) + \left(\frac{N_o - \bar{V}_1 k_1}{60} \right) \left(\frac{5280}{3300}\right)\]

\[k_2 (m + 1) = k_2 (m) + \left(\frac{\bar{V}_1 k_1 - \bar{V}_2 k_2}{60} \right) \left(\frac{4800}{3300}\right)\]

Six columns of the transition table of the machine are shown in Table 1. In the output subtable, three outputs are circled; also the three corresponding states in the next state subtable are circled. The detail of the computation of the circled elements is

a. **Output subtable**

**First column**

\[k_1, k_2 < 65 \quad T_o = 0\]

\[k_1 < k_2 \quad T_1 = 0\]

\[T_o = T_1 = 0\]

**Second column**

\[T_o = \max (k_1 - 75 = 15) \quad T_o = 15\]

\[\left(\frac{1}{2} (k_1 + k_2) - 65 = 0\right)\]
<table>
<thead>
<tr>
<th><strong>TABLE 1</strong></th>
<th><strong>FINITE STATE MACHINE MODEL—EXAMPLE OF A TRANSITION TABLE</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>OUTPUT SUBTABLE: ((T_0, T_1)) sec. AT TIME (m)</strong></td>
</tr>
<tr>
<td><strong>State at (m) ((k_1, k_2))</strong></td>
<td>Input at (m) ((V_0, V_1, V_2))</td>
</tr>
<tr>
<td>(k_1 \leq k_2)</td>
<td>(50, 40)</td>
</tr>
<tr>
<td></td>
<td>(50, 60)</td>
</tr>
<tr>
<td></td>
<td>(60, 70)</td>
</tr>
<tr>
<td></td>
<td>(70, 80)</td>
</tr>
<tr>
<td>(k_1 &gt; k_2)</td>
<td>(80, 70)</td>
</tr>
<tr>
<td></td>
<td>(70, 60)</td>
</tr>
<tr>
<td></td>
<td>(80, 60)</td>
</tr>
<tr>
<td></td>
<td>(80, 50)</td>
</tr>
</tbody>
</table>
Since
\[ V_1 > V_2, \ T_1 = 0 \]
\[ T_O = 15, \ T_1 = 0 \]

Third column
\[
T = \max (k_1 - 75 = 5) \quad T_O = 5
\]
\[
(\frac{1}{2} (k_1 + k_2) - 65 = 5)
\]
\[
(k_1 V_1 - k_2 V_2) 10^{-2} = 20-12 \ \text{hence,} \ T_1 = 8
\]

Since \( T_1 > T_O \), one has:
\[ T_O = T_1 = 8 \]

b. Next state subtable
First column
\[
k_1 (m + 1) = 60 + \left( 15 - 1200 \times \frac{60}{5280} \right) \frac{5280}{3300}
\]
\[= 60 - 5 = 55 \]
\[
k_2 (m + 1) = 70 + (1200 - 1400) \frac{60}{4800}
\]
\[= 70 - 2 = 68 \]

Second column
\[
k_1 (m + 1) = 80 + \left( 19 - 1600 \times \frac{60}{5280} \right) \frac{5280}{3300}
\]
\[= 80 - 8 = 72 \]
\[
k_2 (m + 1) = 50 + (1600 - 1250) \times \frac{60}{4800}
\]
\[= 50 + 4 = 54 \]

Third column
\[
k_1 (m + 1) = 80 + \left( 20 - 2000 \times \frac{60}{5280} \right) \frac{5280}{3300}
\]
\[= 80 - 16 = 64 \]
\[
k_2 (m + 1) = 60 + (2000 - 1000) \frac{60}{4800}
\]
\[= 60 + 12 = 72 \]

Operational Procedure
The finite-state model produces a control policy every time unit. The state of the system is given at time zero (density in each section); from there on, entrance flow and speed at the end of each section uniquely determines a control policy to be applied in the next time interval, as well as the next state of the system. Several methods can be used to implement the control policy depending on the time scale chosen. One may
apply it every unit of time so that the policy found at time \(m + 1\) supersedes the one applied in \((m, m + 1)\). One may wish to consult the model every \(p\) units of time only. Also, one may choose to execute only selected portions of the recommended control.

Due to round-off error and noise, the state of the model may drift away from the actual state of the system; the model must be reset to zero and its new initial state given. At first, on-line experiments can be made with such a model without actually controlling the flow. The number of correct predictions of shockwave formation and of density value is a measure of the validity of the model. Beyond this, field experiments or computer simulation studies are necessary.

**Extension of the Model**

Once experiments have established satisfactory output (control) and next state functions, minimization techniques can be applied to the machine, so that a model equivalent to the original one and generally simpler may be found. Next, one can remark that it is possible to pass from a finite-state machine to a stochastic system using either one of the following two methods: (a) given the machine, define a probability distribution upon the set of inputs; and (b) given an input sequence, define a transition probability from state \(i\) to state \(j\) for all \(i, j\).

The first method can be used for simulation purposes. The second method yields a Markov chain model which has often been proposed for traffic problems. If the transition probabilities from state \((k_1, k_2, \ldots, k_n)\) to state \((k_1, k_2, \ldots, k_n)\) can be defined independently of other variables, then the Markov chain model is valid for the system presented here. An on-line experiment can answer this question.

**POINTS TO BE INVESTIGATED**

Since the purpose of a tunnel control system is to maximize peak hour flow by avoiding the formation of shockwaves, the conditions under which these shockwaves appear must be accurately studied. Preliminary investigations indicate that at least the following factors are relevant:

1. Density and traffic composition, which may be combined into a weighted density in a manner to be determined experimentally.
2. Intrinsinc speed of vehicles which is related to traffic composition (i.e., slow trucks) and drivers' desired speed.
3. Actual speed of vehicles, which is a measured quantity.
4. Environmental parameters, which include geometry and lighting of the tunnel, signalization, presence of police officers, weather, etc.

A study of these four points, which unfortunately are not independent, yields the threshold values \(H(x), H(V), h(V), w, W\) and \(Q_0\) introduced previously. An adequate definition of density weighted for traffic composition is sufficient to define a first generation control system.

Once shockwave formation is understood, the first step consists in minimizing the environmental factors which cause shockwaves. The next step consists in finding actual methods of impeding traffic in a given section \(OX\). These points, together with the question of hardwave choice, will be discussed elsewhere.

**ACKNOWLEDGMENTS**

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REFERENCES


Appendix

SECTION DENSITY ESTIMATION

Generalities

Measuring the density of vehicles in a tunnel lane section is not as easy as it sounds. If one divides point flow by point speeds, wide errors appear as soon as speeds begin to fluctuate—for example, when a shockwave passes by the observation point. If an input-output count is used, the change in density from sampling time to sampling time is known accurately; however, the counter drifts each time a vehicle changes lane or a count is missed. Such an input-output count must thus be periodically resynchronized. Several methods are available to perform this latter operation:

1. The Port Authority's traffic data collection system permits an accurate measurement of vehicle length. A pattern recognition program can therefore recognize when any given train of vehicles which has entered the tunnel at a marked instant is leaving it. If N cars have entered a tunnel of length L between the passage of the last car of the sequence at, respectively, entrance and the exit portal, the exact value of the density is N/L. Observations have shown that the frequency of a pattern "long vehicle-short vehicle," such as tractor-trailer followed by a Volkswagen, is suitable for resynchronization purposes. This system is soon to be tested.

2. An alternate method for monitoring an input-output count through the use of average speed at a point and number of cars entering the section during a variable time interval is presented. This method should be at least as accurate as dividing point flow by point speed and is proposed only when wave velocities are moderately positive in the section, that is, when no stop-and-go waves are present.

3. An approximate verification of this alternate method is done using a fixed time interval—which is equivalent to using flow rate. The hand computations for the exact method would be too tedious; a verification of the range of validity of the method will be done when an on-line digital computer can be programmed to execute it.
The logic of a density meter resynchronized every ten or more units of time by a density estimator is shown in Figure 10, where the block "estimate is valid" is to be transformed into an inequality which expresses the fluidity of the flow, i.e., the presence or absence of shockwaves and/or excessively large gaps. The exact method and its approximate verification are presented using the particular example of the Holland Tunnel, where the density $k_{04}$ (or $k_{14}$) between points 0 (or 1) and 4 is to be estimated. The approximate estimate of densities is then compared with the actual value obtained by manual input-output count.

**A Method to Estimate Density in Fluid Flow Cases**

The continuity property of flow can be used to derive the following algorithm:
1. At time $t_i$, average the speed at point 1 ($1800$ ft from the tunnel entrance), $\bar{V}_i(t_i)$ during the time interval $\left( t_i - \frac{6000}{\bar{V}_i(t_{i-1})}, \ t_i \right)$, with the boundary conditions: $t_0 = 0$, $t_1 = 2\ min$. Note that $6000$ is the distance $04$ (in feet).

2. Using a time listing of vehicles having entered the tunnel up to time $t_i$, find the input $N_0(t_i)$ during the time interval $\left( t_i - \frac{6000}{\bar{V}_i(t_i)}, \ t_i \right)$. 

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Figure 11. Density estimation by averaging quantities over time intervals of varying lengths.
3. If the flow was fairly fluid, so that the average speed at point 1 is representative of the speed 04, then the number of cars in section 04 at time $t_i$ is $N_0(t_i)$. Hence the estimated value of $k_{04}$ is

$$k'_{04}(t_i) = \frac{N_0(t_i) \cdot 5280}{6000} \text{ veh/ml}$$

To illustrate the method, let $\bar{V}_1(t_i - 1) = 45 \text{ ft/sec}$. Then the average speed at point 1 during the time interval $\left(t_i - \frac{6000}{45}, t_i\right)$ or 133.3 seconds before sampling time is computed and denoted by $\bar{V}_1(t_i)$. Let $\bar{V}_1(t_i) = 52 \text{ ft/sec}$; let the number of vehicles which have entered the tunnel for the last $\frac{6000}{52} = 115.4 \text{ sec}$ be $N_0(t_i) = 48$. The estimated value of the density 04 at time $t_i$ is $48 \times \frac{5280}{6000} = 42$.0.

4. Figure 11 shows a density estimation method for section $(j - 1, j)$ of a tunnel (Fig. 9a). Discrete time intervals $(t_0, t_i)$ ... $(t_i, t_{i+1})$ ... are defined by the recurrence relationship:

$$t_{i+1} = t_i + \frac{L_j}{\bar{V}_j(t_i)}, \ i > 1, \ \text{with} \ t_0 = 0 \ \text{and} \ t_i = 1$$

For example, let $L_j = 1800 \text{ ft}$. The faster the traffic is flowing, the more often $k$ is estimated. If $\bar{V}_j = 60 \text{ ft/sec}$, $k'$ is calculated every $\frac{1800}{60} = 30 \text{ sec}$. If $\bar{V}_j = 30 \text{ ft/sec}$, $k'$ is calculated every $\frac{1800}{30} = 60 \text{ sec}$.

Estimates $k'$ of $k$ are accepted only if the flow was fairly fluid during the previous time interval. Thus, in Figure 11, the estimate $k'_j(t_i)$ of $k_j(t_i)$ is accepted only if the 0.5-min moving average of $\bar{V}_j$ over $(t_i - 1, t_i)$ was greater than 15 mph (for example). No estimate is computed if this condition is not satisfied.

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Figure 12. Smooth uncontrolled flow: density estimation, Case 1.
Figure 13. Smooth uncontrolled flow: density estimation, Case II.

Figure 14. Smooth controlled flow: density estimation.
Approximate Verification of the Proposed Method

If the speeds are around 50 ft/sec, then the sampling times are close to \( \frac{6000}{50} = 120 \text{ sec or 2 min.} \) Thus, a fairly gross approximation to the method proposed above consists in taking a fixed sampling time interval \((m-2, m)\) instead of a variable one \( \left( t_1 - \frac{6000}{V_1(t_1)}, t_1 \right) \). This is equivalent to dividing input flow by average speed at point 1, so that the estimated density is given by the formula

\[
\hat{k}_{04}(m) = \frac{N_0(m)}{120} \frac{6000}{V_1(m)} \frac{5280}{6000}
\]

Clearly, the error increases every time that \( V_1 \) departs from the value 50 ft/sec. Furthermore, the speed at point 1 is not representative of the speed 04—the former appears to be systematically higher than the latter. Accordingly, the factor \( \frac{5280}{6000} \) is dropped from the formula to yield much better estimates of \( k_{04} \).

In Figures 12 and 13 density estimated by this latter method is compared with density measured under fluid traffic conditions by manual count for two different uncontrolled flow cases. The estimate is within 10 percent of the actual value. An entrance-controlled flow case is shown in Figure 14; here, the density prediction is quite satisfactory whenever the density is increasing, i.e., after the red light has been deactivated. During the control, large gaps and high speeds cause an underestimation of the actual density; the estimated value of \( k_{04} \) is then close to the actual value of \( k_{03} \).

Extension of the Method

An extension of the method for the case of high flow with shockwaves is proposed. Empirical limits on the validity of the method are used to yield a part-time estimation of density. The method is presented in detail through two examples in report TBR5-65 of The Port of New York Authority; only a summarized version is presented here.

![Figure 15. Stop-and-go flow: part-time estimation of density, Case 1.](image)
Shockwaves create a wide variation of average point speeds (more than one order of magnitude), whereas the corresponding section density variation is relatively moderate, say ±60 percent. In order to take this experimental fact into account, section densities will be estimated during only that part of the time when speed measurements reflect neither a forward moving wave, i.e., large gaps and high speeds, nor a backward moving wave, i.e., small gaps and very low speeds.

An estimated value $k_{04}$ of $k_{04}$ is first obtained using a weighted average of three virtual densities calculated from speeds at points 1, 2, 3 of the Holland Tunnel ST NL. The comparison between the accepted estimates and the actual values obtained by manual count is shown in Figure 15. The relative error is generally less than 10 percent.

Figure 16 shows a part-time estimation $k'_{14}$ of the density $k_{14}$ using the entrance rate $N_1$ in section 1 - 2 and two speeds $V_2$, $V_3$. A weighted average of two densities, $k'_{14}$, is obtained by the method used for the case of smooth flow.

It can be noted that the estimation of $k_{04}$, which is based upon speeds at three points (Fig. 15) is closer than the estimation of $k'_{14}$ which is based upon two speeds only (Fig. 16).