The Factor of Safety in Foundation Engineering

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This paper discusses the concept of the factor of safety as applied to soil and foundation engineering. Until recently the real character of the factor of safety has remained obscure, and its magnitude is generally estimated on the basis of subjective judgment rather than objective facts.

Factors of safety have been fixed rather arbitrarily. At present it is considered that the factor of safety adopted depends also on the size, shape and importance of the structure and on the degree of accuracy with which the applied loads and strength of the soil can be estimated.

The philosophical nature and content of the concept of the factor of safety are presently receiving a new look and study by the engineering profession (1-8). The application and use of the factor of safety can only be decided upon by experience.

•SOIL AND foundation engineering deals with, among other things, stability problems involving soil and/or soil-foundation-load systems. In designing a soil-foundation-load system, or any other earthwork, engineers organize the loads involved and design foundations so that the supporting soil can withstand several times as much load as will ever be imposed upon it. This is done to allow for any unexpected overloading of the soil.

Overloading of the soil may bring about intolerable settlement of the soil-foundation-load system, or rupture of soil upon exceeding its ultimate bearing capacity (viz., shear strength). In the case of rupture of slopes of earthworks, overloading brings about the exhaustion of the shear strength of the soil. To indicate the degree of safety of the soil-foundation-load system, or that of earthworks, or earth masses, engineers use the concept of "stability" characterized by the "factor of safety."

In mechanics, a general definition of stability is: the property of a body that causes it, when disturbed from a condition of equilibrium or steady motion, to develop forces or static moments that restore the original condition.

In foundation engineering, stability is a term used by civil engineers to indicate whether or not a foundation soil or an earthwork will fail under the worst service conditions for which it was designed. The concept is a kind of yardstick by which to measure the soil engineering qualities of a soil, viz., soil-foundation-load system.

It is interesting to note that the term stability encompasses some of the most basic concepts in engineering, namely, force, static moment and equilibrium. These concepts form the basis not only of soil engineering work, but also of all civil engineering work.

Unfortunately, stability, viz., safety, is a somewhat ambiguous concept. Therefore it must be specifically defined. Taking recourse to statics: if a system consisting of an acting force, F, and two opposing forces, F_1 and F_2, is in static equilibirum, then the stability, or safety, is $\eta = 1$. The equilibrium conditions are

$$\Sigma V = 0, \Sigma' H = 0 \text{ and } \Sigma' M = 0$$

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where V = vertical force component, H = horizontal force component, and M = static moment, whereby the moment point can be selected arbitrarily.

Designating the degree of safety, η, as the ratio of the resisting forces, F_R (moments M_R), to the driving forces, F_D (moments M_D), one obtains in the case of static equilibrium

$$\eta = 1.0 = \frac{\Sigma V_R}{\Sigma V_D}, \text{ or } \eta = \frac{\Sigma' H_R}{\Sigma' H_D} = 1.0, \text{ or } \eta = \frac{\Sigma' M_R}{\Sigma' H_D} = 1.0$$

If there is no equilibrium, the degree of safety is

$$\eta \gtrless 1.0$$

When $\eta > 1.0$, the system is stable. When $\eta < 1.0$, the system is unstable. In design practice the ratio η is termed the factor of safety.

Relative to the foregoing discussion, it may become apparent to the reader that stability cannot be built into an earthwork, and that stability is difficult to measure, whether in situ or in the laboratory. At best, foundation engineers try to assess the degree of stability of an engineering system, also soil-foundation-load systems, by way of a factor of safety, η. The latter is traditionally calculated as a ratio. It is a purely arbitrary, man-made number, chosen because it has worked for a long time and its worth is proved by the fact that structures built with it in mind have endured through the years. Thus, stability calculations involve the calculation of two sets of forces (or moments): (a) those that tend to produce failure, and (b) those that tend to prevent it.

Most problems of application of stability calculations in soil and foundation engineering are encountered in determining soil bearing capacity and in dealing with the stability of slopes of various kinds of earthworks. Stability analyses of soil-foundation-load systems, it may be said, have as their ultimate goal the determination of the degree of stability of the system, viz., the calculation of the so-called factor of safety.

THE CONCEPT OF THE FACTOR OF SAFETY

In proportioning structural elements and in performing stability calculations of structural elements and systems, it is customary to introduce the so-called factor of safety. The term factor of safety, η, is a deep-rooted concept in the field of civil engineering design, particularly in those branches of structural design which are based on the ultimate strength of the material.

The factor of safety supposedly should safeguard against (a) possible overloading, (b) errors introduced by simplifications and approximations in design methods and procedures, and (c) variations in quality of materials. Materials having unpredictable nonuniformities necessitate a careful evaluation of their strength. This pertains forcefully to soil as a construction material. Here a factor of safety is introduced.

In general, the factor of safety should also take care of (a) the imperfection of human observations and actions (objective uncertainty), and (b) the imperfection of intellectual concepts devised to reproduce physical phenomena (subjective ignorance). Therefore some call the factor of safety the "factor of ignorance." This latter term is, however, used more in the context of uncertainty, for example, when the engineer does not know enough facts, or when exact knowledge about some factor in design is lacking. Thus, the factor of safety should provide for contingencies which affect the design and construction of a structure. Besides, the choice between safety and economy is usually a problem with which engineers must cope. The engineer is also confronted with the problem of how much importance should be attached to safety and how much to economy, thereby introducing another factor of uncertainty in the design and stability calculations.

Some Definitions of the Factor of Safety

Whereas the term factor of safety, η, of a system can be expressed in general as

$$\eta = \frac{\text{favorable quantities}}{\text{unfavorable quantities}}$$

or

$$\eta = \frac{\text{favorable forces}}{\text{unfavorable forces}}$$

or

$$\eta = \frac{\text{favorable moments}}{\text{unfavorable moments}}$$

the definitions of the "favorables" and "unfavorables" of the ratio forming the safety factor, as well as those of the factor of safety itself, vary considerably. For example, "the factor of safety is the margin allowed for unexpected occurrences which might otherwise cause instability or failure of soil and/or earthwork;" or "the factor of safety is a ratio of forces resisting failure to the forces tending to bring about failure;" or "the ratio of the ultimate strength of the soil to its maximum expected stress is (conventionally called) the factor of safety of that soil." Theoretically, the factor of safety is the strength of the structure divided by the loading. In any event, the factor of safety should provide for the contingency of external causes weakening the soil's supporting capacity.

Freudenthal (1, 2, 3), an authority on engineering design criteria, considers the factor of safety to be directly related to the conventional concept of allowable stress, involving a comparison between a computed maximum stress and the strength of the material. Thus the existence of a margin between the two is implied. This margin of safety, as defined by Freudenthal, is "the subjective striving on the part of the designer for an adequate measure of safety as well as consciousness of the limitations of his knowledge and the arbitrariness of his own assumptions" (1).

Svensson (4) writes: "The factor of safety has been used by engineers to cover the unknown gap between the apparent strength of the material used and the apparent load applied."

Another definition describes the stability of soil as the load which the soil can support without excessive deformation. Exactly what is to be understood by excessive deformation may turn out to be a very moot question.

These and many other definitions of the factor of safety or stability show that the factor of safety can be an ambiguous term. One gets a notion that the factor of safety is, thus, a kind of device to over-design a soil system. This brings us to the question of the various factors affecting the resultant factor of safety, and the magnitude of the factor of safety to use in design and stability calculations.

Some Variables Affecting the Factor of Safety

Some of the variables necessitating the introduction in design and stability calculations of a certain margin are as follows:

1. Assumptions made in mode and nature of loading.
2. Nonuniformity in composition of materials.
3. Insufficient knowledge and limited or meager analytical comprehension in testing of (soil) materials, measurement, knowledge of the (shear) strength properties and deformation of materials and structures, as well as in sampling of undisturbed soil samples. A single component such as the shear strength of soil in turn depends on a

large number of other factors such as reconnaissance of the soil, soil sampling, quality of tests made, temperature, pore water present in the soil, and quality of interpretation, to mention but a few.

4. Failure to reproduce field conditions in the laboratory when testing materials and structural prototype models.
5. Uncertainities in simplifying assumptions in static and dynamic calculations of structural and/or soil-foundation-load systems.
6. Errors in and departures from design which inevitably occur in the execution of the construction project.
7. Quality of construction work (very difficult to anticipate).
8. Mode and probability of failure on which the stability analysis and calculation of factor of safety is based. Instead of considering rupture condition, one may also consider plasticity condition, or intolerable deformation in terms of settlement of the structure, thus basing the soil-foundation system's design on the so-called proportional limit or critical edge pressure (9, 10).
9. The reduction in quality of the structure and its materials which can occur by aging, weathering and exploitation (service and use).

The mutual relationship of these and other variables to the resultant stability of a structure must be well understood if the structure is to have a reasonable degree of safety.

Loading

In soil engineering frequently the self-weight of the structure is the principal load on the soil. Its magnitude can be determined with relatively satisfactory precision. When variable lateral forces such as wind, waves, ice thrust and earthquakes apply, or where moving repetitional and vibratory loads occur, the assessment of loading, its effects, and the corresponding factors of safety are more difficult to determine than for systems with no such forces. There should be incorporated in the design of a structure as many overall safety precautions as possible against the hazards of earthquakes. Earthquakes are conspicuous examples of vibration effects on structures, and therefore a higher factor of safety is required.

Besides these externally applied loads there also should be considered the variable natural loads, such as pore water pressure, buoyancy of submerged soils, seepage pressure and the liquefaction phenomenon.

Soil Material

Whereas the properties of a man-made material such as steel are well known, the properties of materials, such as soil, provided by nature are nonuniform and as yet not too well understood. Soil conditions are less reliable than most other materials connected with a structure. The strength of the soil material refers to its resistance to shear and other physical properties associated with it. A single element in the composite factor of safety, such as the shear strength of the soil, for example, which is a fundamental factor in stability analyses, itself depends on a large number of other factors: the quality of field reconnaissance, the quality of the soil sampling, quality and type of shear tests of soil made (unavoidable errors arising from incompleteness of soil shear-testing devices and testing methodology, for example), shape of particles, particle size distribution, angle of internal friction, moisture content, pore water pressure, temperature, quality of calculations and interpretation of test results. Many of these elements themselves are functions of other independent factors.

Assumptions made regarding the angle of internal friction of sandy soils, the relative density of a sand deposit, or the compressibility of a layer of clay are more examples of variables affecting the magnitude and quality of the factor of safety. The determination of the coefficient of permeability of the soil is another vulnerable variable, and so is the unreliable information when attempts are made to predict the pore water pressure in stratified sand layers or in beds of clay containing seams of more permeable material. This is because the pore water pressures depend on unexplorable structural details of foundation soils and other earthworks.

The properties of soil vary also with season (frozen or unfrozen), position below ground surface, and location and fluctuation of the groundwater table. Clayey soils, fissured clays and silts certainly perform differently from sand under load. The causes for dispersion of stress and shear strength of a soil-foundation-load system may be almost infinite in number.

The foregoing indicates that not only do external variables (loading, for example) affect the soil material, but there are also many internal variables involved. These variables in the properties of soils are unique to them and are not encountered in a material such as concrete or steel. The complexity of the nature of soil is due partly to the fact that it is a combination of solid, liquid and gaseous phases and partly to the fact that in many cases a significant portion of the solid matter is so finely divided that its particles are of colloidal size, with the characteristic colloid-chemical properties and behavior in water in motion (electrokinetic phenomena) and adhered water films (11). In any given soil the quantities of solid, liquid and gas are subject to wide variation due to loading, or effects due to the elements such as wetting, drying, freezing and thawing. Other properties that may vary in a soil include its density, void ratio, moisture content, degree of saturation, soil consistency, and the effects due to groundwater, such as capillary action and permeability.

One sees that the factor of safety is a much more complex problem in soils than in homogeneous materials like steel, and that it will probably never be possible to describe their strength and deformation characteristics as precisely as those of steel. Hence the variable factors, as well as the resultant factor of safety for soils, are, indeed, very difficult to assign. In general, considering these variable factors, the factor of safety here should not be judged too small.

Uncertainties in Simplifying Assumptions

One of the uncertainties in simplifying assumptions underlying theories as applied to soil engineering pertains to the nature and course of stress distribution in soil. Because there is nothing better available, Boussinesq's stress distribution theory in an ideal, elastic, homogeneous and isotropic medium is also being applied to problems of stress distribution from loads in soil.

Methods of Calculating the Factor of Safety

The method used in calculating the factor of safety also has an effect on its magnitude. The equation $\eta = M_R/M_D$ is a general analytical statement of the factor of safety. However, various authors have put forward their own methods of computing η, which include considering the ratios of shear strength, s, of the soil to the shear stress, τ, or available cohesion, c, of the soil to the necessary cohesion, c_{nec}; available coefficient of internal friction of soil, tan ϕ, to the necessary coefficient of internal friction, tan ϕ_{nec}, and consideration of friction and cohesion being important but acting separately. However, all these methods reduce, in a vague way, to the equation $\eta = M_R/M_D$. What is more important is that all these methods point out that the concept of the factor of safety is a subjective one and depends to a great extent on the individual's ideas on a particular problem. Thus one notes that for the same problem the factor of safety varies from author to author, from method to method of calculation.

Analysis of soil and earthwork stability problems is largely a trial-and-adjustment procedure to determine the safety factor of an assumed design or of an actual soil mass. First, a potential failure surface is assumed, and the shearing resistance acting along the surface is calculated. The forces acting on the segment of soil bounded by the failure surfaces are determined, and then the safety factor of the segment is calculated as follows:

Safety against rotation:

$$\eta_M = \frac{\text{resisting moments}}{\text{moments causing failure}}$$

Factor of safety against any motion:

$$\eta_T = \frac{\text{forces opposing motion}}{\text{forces causing motion}}$$

Theoretically, if a large number of different segments is assumed, the smallest safety factor found for any will be the actual safety factor of the mass.

Quality of Construction; Modification in Value of Structure with Age and Use

The factor of quality of construction (workmanship, construction accuracy and reliable inspection) in earthworks and foundation engineering is very important. It is especially important in soil compaction operations of earth dams, highways and airfields, in artificial improvement of soil properties by means of compacted soil cushions, in installing sand drains, in grouting operations, lowering of the groundwater table, pile and sheet-pile driving, performance of field bearing capacity tests of soils and other construction pertaining to soil engineering. Variation of the groundwater table due to the introduction of local and regional drainage systems after erection of structures (decrease in buoyancy underneath dams and foundations), intolerable settlements and subsequent cracking of structures, increase in impounded head of water behind the dam (increase in buoyancy, decrease in stability) after it has been finished may modify the service and proper functioning of the structure.

Access of air and/or water to soil may change some soil properties for the worse, particularly decreasing the shear strength of the soil in the slopes of earthworks such as cuts for roads and canals, for example. Improper functioning of drainage structures in soil such as weep-holes, French drains, lateral drainage galleries in slopes, and galleries and drainage carpets underneath earth dams may bring about the collapse of earth retaining walls, earth slopes and earth dams.

NUMERICAL VALUES OF THE FACTOR OF SAFETY

At the start of design of an earthwork and/or soil-foundation system there always arises the question of the magnitude of the factor of safety to use in stability calculations of earth masses. Such discussions, as well as a review of the technical literature, reveal that there is no uniformly accepted value of η to use.

For most engineering construction materials and modes of loading the factor of safety is officially regulated in one way or another by building codes, city ordinances, or the like. These codes vary from one edition to another, and since there is no statutory guarantee that their compilers know what they are codifying, the ratio, strength as calculated by one arbitrary code/loading stipulated by another code (= factor of safety) is variable and no more reliable than the opinions of those members of the drafting committee who were not too busy to take an active part in drafting the code. In any case it is safe to say that the factor of safety to use in soils engineering is a much more complex problem than in those fields of engineering which deal with homogeneous materials.

In the past the factor of safety was chosen arbitrarily by the designer, utilizing all his skill and years of experience. In general, it was probably a larger factor of safety than actually needed, but it tended to comprehend the "ignorance" and uncertainties of the design. In selecting a factor of safety the designer had to keep an eye on economic considerations as well as the safety of the people who were depending on his skill.

Analyses of existing slopes of earthworks for their stability have revealed relatively small factors of safety as compared with those of other structures. A factor of safety of 1.5 or 2.5 is usually uncommon in structural engineering. However, such magnitudes of safety factors, if applied to earthworks, would make their cost so prohibitively high that they probably could not be built. On the other hand, many earth structures having a computed factor of safety as low as $\eta = 1.0$ have been demonstrated to be stable by the test of time (or else, some important "favorable" factors have been forgotten in the stability calculation).

Typical values of the factor of safety for most soil engineering work range between 1. 5 and 2. 5, as can be readily found in any textbook and in many published papers. It is interesting that almost none of these texts or papers report the origin of the factor of safety range indicated, but merely suggest the range. Lately equations were published by D. W. Taylor (13), K. Terzaghi (12), J. Ohde (14), O. K. Fröhlich (15), W. Fellenius (16), A. W. Bishop (17), N. Janbu (18) and others for calculating the factor of safety with respect to the failure of earth embankments. The values of the factors of safety calculated by these equations range between 1. 3 and 1. 7.

To avoid the various types of soil failure, the following minimum factors of safety have usually been used:

1. The factor of safety against a sliding failure must be at least 1. 5.
2. The factor of safety against a deep-seated failure must be at least 1. 5.
3. The factor of safety against a shallow shear failure must be at least 2. 0.
4. The factor of safety against rotation must be at least 1. 5.

It is interesting to note that the factor of safety to use depends also on the size (10, 11, 19), shape and importance of the structure (the more important the structure the higher the factor of safety to be used), and the degree of accuracy with which the applied loads and strength of the soil can be ascertained.

Leussink (20) described the various partial coefficients of safety in earthwork and foundation engineering, and tabulated various factors of safety from his own calculations as well as those of other authors.

Critical Edge Pressure

Besides ultimate failure, the plastic flow condition (proportional limit) may be used as the basis for calculating the safe bearing capacity of soil. For example, Fröhlich (9, 10) calculates the allowable pressure from the foundation on the soil, which he calls the "critical pressure." By critical pressure is understood that pressure at which soil particles just begin to flow out from underneath the edge of the loaded footing of the foundation. In essence, Fröhlich's critical pressure corresponds approximately to the proportional limit. Hence the soil bearing capacity may be based on the proportional limit of the soil. The calculated critical edge pressure needs no factor of safety.

PROBABILITY AND STATISTICS

The philosophical nature and content of the concept of the factor of safety are currently receiving a new look and study by the engineering profession. The most recent ideas put forward for the rational selection of the factor of safety to use are from the discipline of mathematics, particularly that of probability and statistical analysis (1, 4). Relative to loads and structural behavior, Freudenthal (2) considers a means for estimating failure through the laws of probability (or rather relative frequency). He writes (3): "The probabilities of failure or of survival for which structures are designed must be referred to the critical operating conditions of the structure; different values of probabilities are necessarily associated with different operating conditions."

"Because the design of a structure embodies certain predictions of the performance of structural materials as well as of expected load patterns and intensities," writes Freudenthal, "the concept of probability must form an integral part of any rational design" (2). Thus, instead of a factor of safety, the mathematics would be concerned with the probability of failure. In this context Svensson (4) writes that "the load and the strength of the material exhibit variations defined by certain probability functions and thus the occurrence of failure will also be defined by a probability function."

According to the above trend of thought, it becomes apparent that in the final analysis of stability of a system it is necessary to choose arbitrarily a suitable probability of failure. However, this in effect means really choosing a factor of safety. Let us recall that, figuratively speaking, the factor of safety aims at the protection of the structural system against extreme, yet reasonable, fluctuations which statistical methods indicate have the probability of occurring. Thus, according to Freudenthal, mathematics, i. e., probability, and statistics seem to be the answer to the question of what factor of safety to use. This may be realistic under complex loading conditions such as are encountered in aircraft design, but may be too unreasonably involved for the more static loading conditions such as occur in a soil-foundation-load system. Statistical analysis seeks to analyze human experience of random events. Ultimate strength design (as, for example, in concrete technology) presents some of the concepts of the statistical approach to the factor of safety (6, 7). It has also been said that the ultimate design method results in a more realistic design and one with a more accurately provided factor of safety.

Realizing that the choice of a suitable probability of failure "can only be examined on economical grounds where one must balance the cost of failure against the cost of increased strength to reduce the incidence of failure" (5), one would be inclined to think that probability and statistical methods definitely have their place in the determination of the factor of safety to use, especially with the advent of the electronic computer.

CONCLUSIONS

How much of the design procedures suggested by the proponents of probability calculus and statistical analysis will be adopted by foundation engineers is open to question. In the words of A. Chibaro (21), who discussed Freudenthal's paper (2), ". . . it will probably be a long time before the use of the 'factor of safety' concept will be supplanted by the use of the 'probability of failure' concept. . . . In order to use the concept of probability of failure a great mass of statistical data pertaining to load effects and strengths must be obtained and correlated."

Things become even worse when working with soil as a construction material. It appears that probability and statistics are applicable only to controlled, man-made engineering materials, such as steel and probably concrete. One field where the probability calculus and statistical analysis approach to the safety factor is clearly not applicable as yet is soil and foundation engineering. This is because of the nature of soil. There are no two soils in the world exactly the same, nor will they react in the same fashion under various conditions. Due to the nonuniformity of soils, the mathematical method cannot be applied to soils either directly or indirectly. In foundation engineering, according to J. M. Corso (22), "Seldom, if ever, are enough test data available to determine the frequency distribution of the soil properties. Furthermore, in many cases the loading effect and resistance cannot be considered as independent variables, and interrelation between the load and the resistance, including variations with time, is often so complicated as to defy rigorous analysis."

It appears as though the question of applying probability calculus and statistical analysis to soil and foundation engineering problems is still premature, mainly because of our still relatively meager knowledge of the nature of deformation of soil subject to shear stresses.

Besides, Freudenthal himself (2) says that "the foregoing does not imply that the use of the probability theory is, in itself, sufficient to make design procedures more adequate and reliable. Probability concepts and statistical methods based thereon can be used effectively only in conjunction with a thorough knowledge of the operating conditions of the structure and of its structural action. . . ."

Thus it seems that the expression "factor of safety" has become and will probably remain in the engineering lexicon as an integral term in soil and foundation engineering design, and that in this field experience and common sense cannot be dispensed with as yet in favor of probability and statistical analysis.

Until science and engineering contribute to the correct evaluation of the complex properties of soil as an engineering material and to the correct determination of its shear strength, thus enabling one to apply more sophisticated methods for determining a proper factor of safety than is now possible, it seems that we are still compelled to continue to resort to the present conventional methods in selecting a proper factor of safety to use.

SUMMARY

1. Until recent times the real character of the safety factor has remained obscure, and its magnitude is generally estimated on the basis of subjective judgment of the engineer, rather than on objective facts. The selection of a factor of safety has been arbitrarily chosen for years. It is an ambiguous term.

2. The factor of safety should provide for the contingencies of external causes weakening the strength of the soil, viz., the soil bearing capacity, and should compensate or allow for errors in assessing the various factors in the stability analysis of a soil-foundation-load system. The term factor of safety is used more in the context of uncertainty.

3. Proponents of the rational (mathematical) approach to the selection of the magnitude of the factor of safety to use feel that the mathematical discipline of probability and statistical analysis offers the key to selecting the factor of safety.

4. The above may be true for controlled, man-made materials such as steel and concrete, as only time will tell.

5. But due to the nonuniformity, inhomogeneity and anisotropy of soils and their complex properties, probability calculus and statistical analysis cannot yet be directly applied to soil and foundation engineering.

6. The factor of safety must be chosen realistically and with deep awareness of what is occurring or what may occur. Then it could be used with confidence in any deliberations and decisions. The selection of a factor of safety for foundation and earthwork design purposes still requires sound experience, common sense and careful engineering judgment.

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