Merging Behavior at Freeway Entrance Ramps: Some Elementary Empirical Considerations

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This paper discusses an elementary empirical analysis of merging behavior, and in particular of gap acceptance and rejection behavior at a freeway entrance ramp. The tone of the paper is purely empirical. No attempt is made to develop a theory of merging, nor to validate any existing analytical or simulation models of the merging process. Rather, emphasis is placed on the development of an improved analytical methodology and on the collection of basic empirical information of the type which might be utilized either as input to future theoretical studies, or in the design of a real-time freeway control system.

•CONSIDERABLE attention has been devoted in recent years to the analysis of gap acceptance and merging behavior at freeway entrance ramps. A selected bibliography (Appendix A) lists some 62 references to recent research in this or closely related fields.

MERGING BEHAVIOR—AN ANALYTICAL FRAMEWORK

A review of these studies suggests that most empirical research to date has been based on a "static" rather than a "dynamic" interpretation of the merging process. Gap acceptance and rejection has been described in terms of "static" measurements of mainstream time headways made at a single, arbitrarily defined point in space (conventionally, this point has been taken as the nose of the entrance ramp). No real attempt has been made either to take account of the positioning of the ramp vehicle within a given mainstream gap, nor, more importantly, to allow for the relative motions and trajectories of the merging vehicles.[1]

The implications of this comment may perhaps be most clearly illustrated by reference to Figure 1. This figure depicts the time-distance traces of a series of ramp and mainstream vehicles passing through the merge area of a freeway entrance ramp. Time is plotted continuously along the abscissa of the diagram and distance vertically up the ordinate. Mainstream vehicles are denoted by solid lines and ramp vehicles by dashed lines. Two points, X and O, are defined for each ramp vehicle trace: X denotes the point at which the ramp vehicle first encroaches onto the adjacent mainstream lane, O the point at which the vehicle finally enters that lane. The distance X-O therefore constitutes the length of the physical merging maneuver. All traces are defined with respect to the front of the vehicle in question.

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[1] There are, as may be expected, a number of exceptions to this generalization. In particular, reference is made to the empirical work of Hurst et al (32), Knox (38) and Vardon (55) and to the related theoretical analyses of Bisbee and Conan (4), Haight (24), Haight et al (25) and Jewell (34). Figure 2 summarizes graphically the results of some typical previous "static" analyses.

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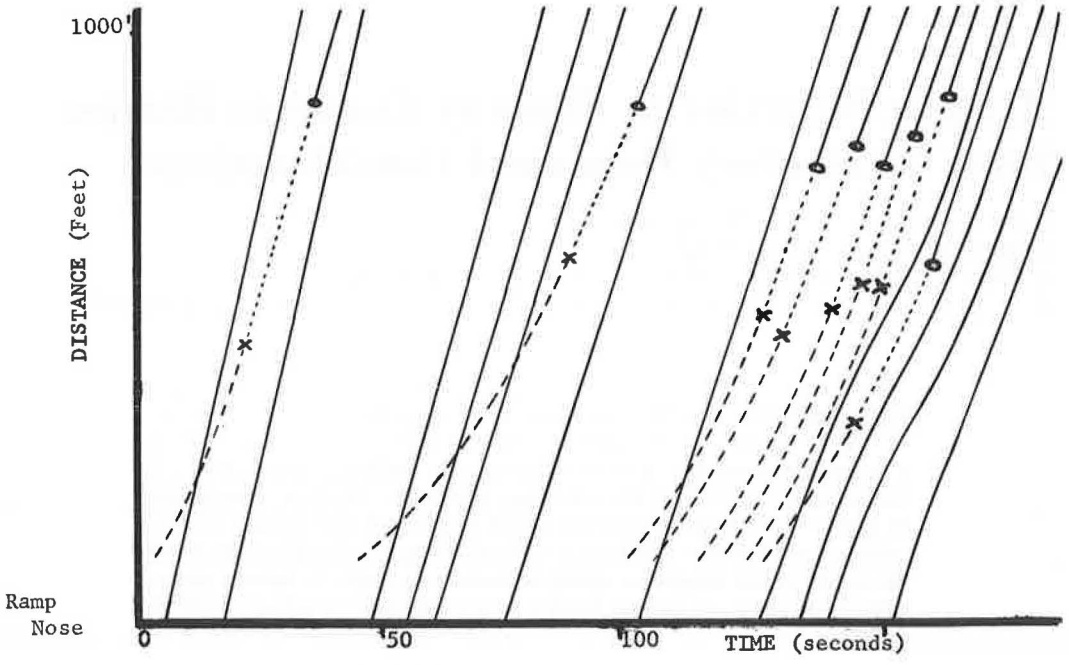


Figure 1. Time-distance diagram for ramp-freeway merge analysis.

Three different merging conditions are illustrated in Figure 1: a simple "direct merge" (Fig. 1a); a merge preceded by a rejection (Fig. 1b) and a "multiple-merge" by a platoon of ramp vehicles into a single mainstream gap (Fig. 1c). A conventional "static" analysis would describe each of these conditions in terms simply of the ramp vehicles' acceptance or rejection of the series of mainstream headways measured statically at the ramp nose. The result would be only a partial and possibly a misleading description of the merging process.

In the case of the multiple merge illustrated in Figure 1c, for example, a total of five vehicles would be recorded as having accepted a gap of 25 seconds; a sixth vehicle would be recorded as having rejected that gap in favor of the subsequent value of 13

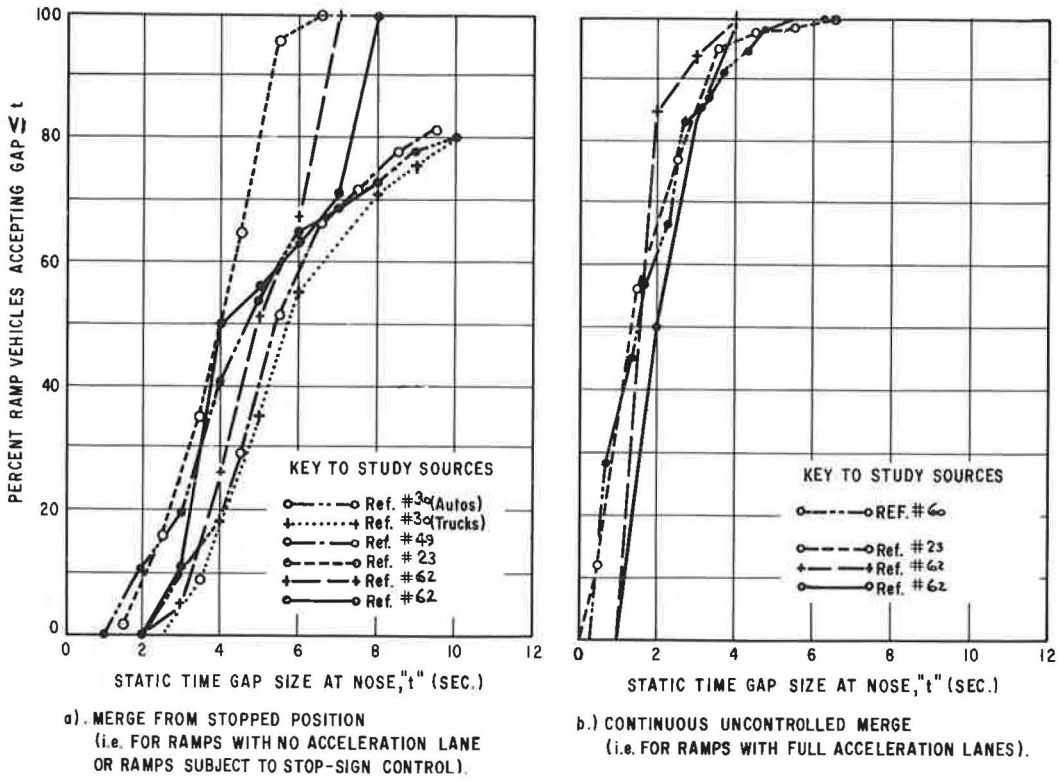


Figure 2. Summary of results of existing gap acceptance studies.

seconds. This is clearly illogical. The driver of the sixth vehicle, at the instant he reached the ramp nose, was faced, not with a gap of 25 seconds but rather with a "lead-time" to the preceding ramp vehicle of 2 seconds and a "lag-time" to the succeeding mainstream vehicle of approximately 1 second. It was this 2 second-1 second gap structure which the driver rejected rather than the original 25-second gap.[2]

The distinction between "gap size" and "gap structure" is obviously especially important in the case of a multiple merge. It is also applicable, however, to each of the single vehicle merges illustrated in Figure 1. In each case, the definition of effective lead-lag structure reduces considerably the bias introduced into a simple static analysis by the arbitrary arrival rates of the ramp and mainstream vehicles. This bias may be reduced still further by relaxing the restriction that all measurements be made at the ramp nose. It is useful, for example, to consider the values of "dynamic" lead and lag time accepted by a merging driver as he actually starts his merging maneuver (i.e., at the point X in Figure 1). Similarly, the values of accepted or rejected gaps may be evaluated "dynamically" by measuring the headway between two successive mainstream vehicles at the instant the lead vehicle passes or overtakes a ramp vehicle. Such "dynamic" measurements approximate more closely the mechanics of the merging situation in that they are

[2] "Lead-time" is defined as the time-headway between a ramp vehicle and either the immediately preceding mainstream vehicle or the preceding ramp vehicles whichever is the smaller; "lag-time" is defined as the time-headway between a ramp vehicle and the immediately succeeding mainstream vehicle. A glossary of the terms and definitions used in this paper is given in Appendix B.

defined not with respect to an arbitrary, fixed point in space but rather with respect to the observed motions of the merging vehicles.

Depending on the flow conditions in the adjacent mainstream section, dynamic and static analyses may yield similar or considerably different results. In this context, we note with Haight (25) that ramp-freeway merging is characterized in part by the merging driver's ability to adjust his environment according to his desires. By speeding up or slowing down, the driver may adjust his position relative to that of a sequence of mainstream vehicles so as to create a suitable gap for merger. In extreme circumstances he may actually force his way into a gap, causing one or more mainstream vehicles to change speed or direction in order to accommodate him. The implications of this type of behavior have been discussed theoretically by a number of writers (4, 25, 34).

Obviously, many different factors exert considerable influence on the merging process. For example, the relative speeds of the ramp and mainstream vehicles, the spatial as well as temporal dimensions of mainstream headways, the availability of alternate acceptable gaps ahead of and behind the one under consideration, the types of vehicles involved, the characteristics of the merging drivers, the ramp and merge area geometrics, etc., all may affect a given acceptance/rejection decision to a greater or lesser degree.[3] Analysis of such factors, therefore, should desirably form a primary element in any comprehensive empirical analysis of merging behavior. Many of these factors are directly measurable on the time-distance diagram shown in Figure 1.

It should be emphasized here, however, that time-distance trace analysis has its own very serious limitations. It can provide only a description (albeit a very efficient description) of the merging process; the geometrics of a time-distance diagram can tell the analyst nothing of the instant when a driver made his decision to accept or reject a mainstream gap, nor of his perception of the "size" of that gap. Apart from the work of Hurst et al (32), relatively little empiricial (or theoretical either, for that matter) attention has been paid to this aspect of merging behavior. It represents potentially an extremely fruitful field for study.[4]

merging maneuver. To be truly "acceptable," however, a merging maneuver should cause neither disruption of the mainstream flows nor undue discomfort to the merging driver. One may thus consider a possible redefinition of the phrase in terms of the delay or speed change imposed on a mainstream vehicle consequent upon completion of a merge or else in terms of the tension induced in a merging driver (measured, say, by GSR techniques) by a given merging situation. In each case an "acceptable gap" might be redefined as one whose acceptance caused neither more than a specified maximal perturbation in the mainstream flow nor more than a maximal tension level in the merging driver.

EMPIRICAL STUDIES

Collection and Analysis of Field Data

A pilot empirical study designed to test the efficacy of the analysis techniques discussed previously was conducted during the summer of 1965 at two freeway entrance ramps

[3] Haight et al (25) have shown the importance of the spatial dimension under conditions of low relative merging velocities, while Knox (38) and Vardon (55) have both analyzed the effects of relative and absolute speeds on merging behavior. Of particular interest to this discussion is a recent analysis reported by Hurst et al (32) in which comparisons are made between alternate indices of acceptable gap size. The results of this latter study suggest that complex indices which combine estimates of relative merging speed with conventional temporal measurements provide a more efficient and more stable basis for predicting gap acceptance/rejection behavior than do simple time-gap measurements alone.

[4] One interesting hypothesis worthy of research is that a merging driver evaluates the "size" of a mainstream gap in terms of the rate of change of the angle subtended at his eye by an approaching mainstream vehicle.

TABLE 1

DESIGN CHARACTERISTICS OF STUDY LOCATIONS

Description	Harlem Avenue Left-Hand On-Ramp Eisenhower Expressway (Eastbound)	Nagle Avenue Right-Hand On-Ramp Kennedy Expressway (Northbound)
	(a) Geometric Characteristics	
Mainstream section	Depressed 6-lane Urban Freeway; Level, Tangent Alignment	Depressed 6-lane Urban Freeway; Level, Tangent Alignment
Acceleration lane	Parallel Type Design, 1075 ft long	Parallel Type Design, 800' long
Ramp width at nose	Single Lane Entry, 16 ft wide	Single Lane Entry, 16 ft wide
Ramp grade	-3 percent	-3 percent
	(b) Traffic Characteristics	
Ramp ADT	10,700 veh/day	5,600 veh/day
Mainstream ADT (one direction)	64,000 veh/day	55,000 veh/day
Avg. ramp flow during study	575 veh/hr (16 percent trucks)	310 veh/hr (10 percent trucks)
Avg. flow in adjacent mainstream lane during study	1,260 veh/hr	875 veh/hr

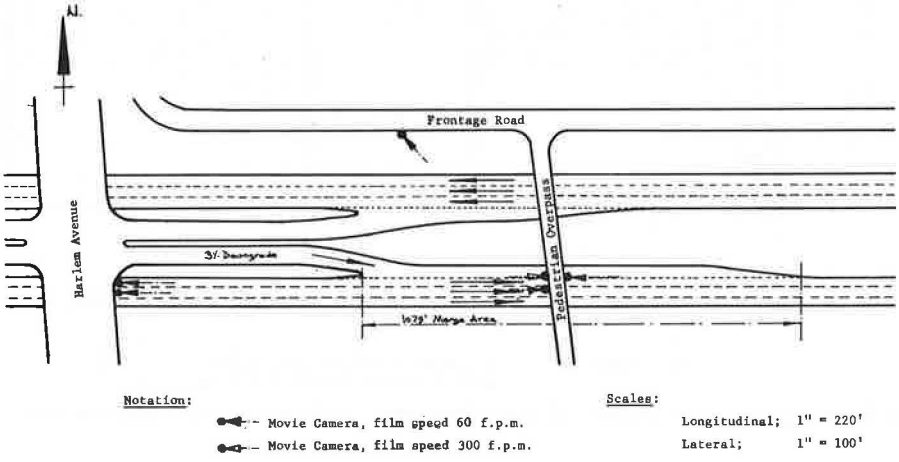


Figure 3. Harlem Ave. (eastbound) left-hand entrance ramp, Eisenhower Expressway, Chicago.

in Chicago. One of the ramps selected was located on the left-hand side of the through pavement, the other on the right. In all other respects the two ramps had similar design and traffic characteristics (see Figs. 3 and 4 and Table 1).

Field data were collected by means of synchronized, time-lapse ground photography, using a system of six time-lapse movie cameras at each ramp. The systems of cameras were synchronized by stopwatch timing, supplemented by timed runs through the study section in a marked vehicle. Figures 3 and 4 indicate the approximate positions and operating speeds of the cameras. Approximately two hours of continuous data were collected at each location, during periods of medium to high flow in the mainstream lanes.

The film data were reduced to yield information on the time-distance trajectories of all ramp and adjacent lane mainstream vehicles passing through the merge areas of the two ramps. Time measurements were made to an accuracy of $\pm 1/5$ second and distance measurements to ±5 feet. At least 10 time-distance observations were made for each individual vehicle. These time-distance data, reduced to digital form for

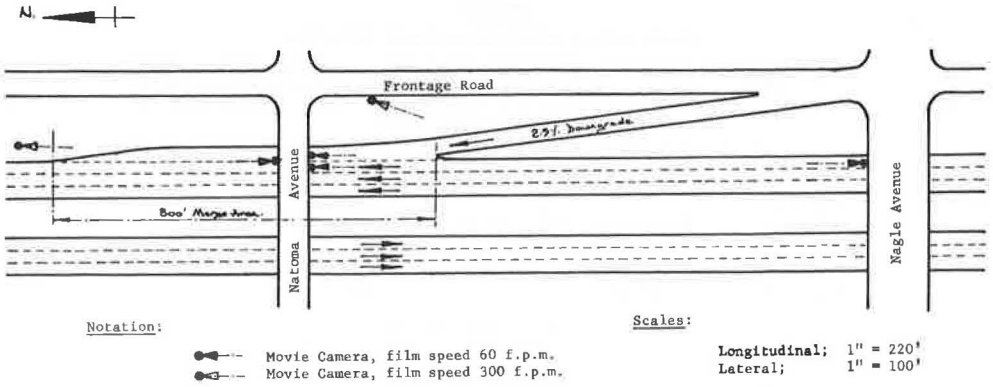


Figure 4. Nagle Ave. (northbound) right-hand entrance ramp, Kennedy Expressway, Chicago.

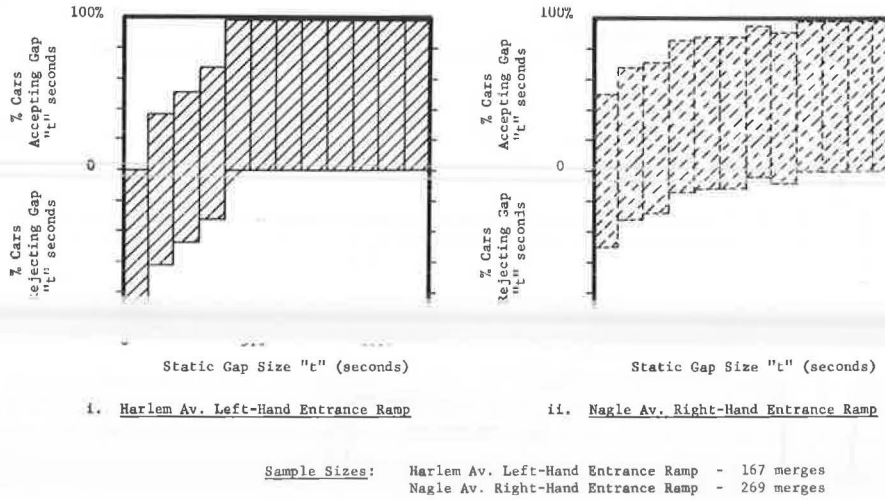


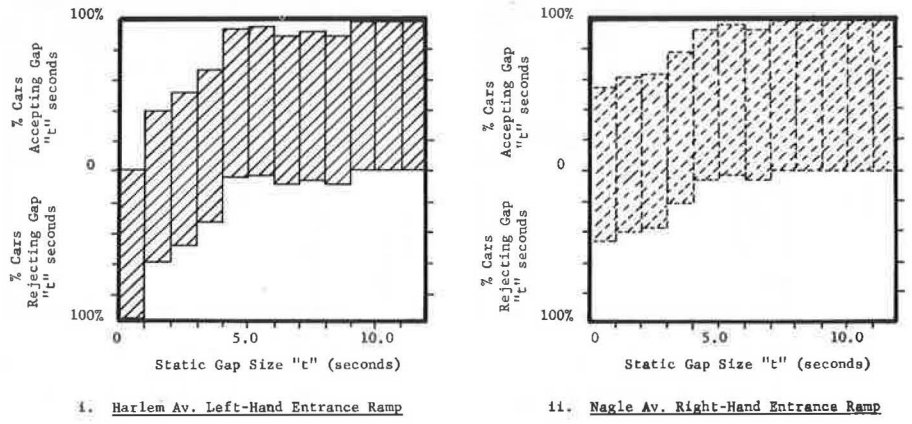
Figure 5. Acceptance and rejection of static time-gaps, single merges only, passenger cars only.

computer analysis, provided the framework for the analytical work discussed in this paper.

Structured Static Analysis

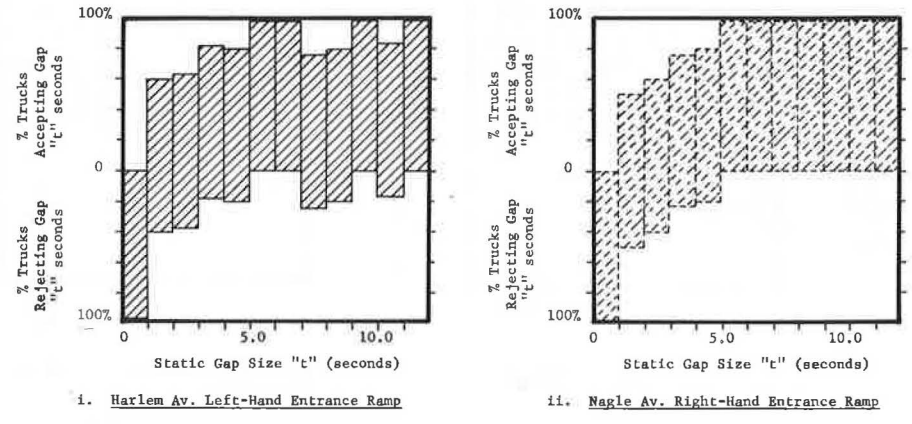
An initial static analysis indicated that there was no significant difference between the behavior of "isolated" ramp vehicles, separated by at least 5 seconds from the nearest preceding or succeeding ramp vehicle, and the behavior of "lead" vehicles in multiple-merges, separated by at least 5 seconds from the preceding ramp vehicle but by less than 5 seconds from the succeeding ramp vehicle.[5] To provide a larger sample for analysis, therefore, these two categories were combined together under the general heading "single-vehicle merges." Trailing vehicles in multiple merges (i.e., the second, third and subsequent vehicles in each platoon of ramp vehicles[5])

[5] A "platoon" of ramp vehicles was defined, arbitrarily, as a succession or group of ramp vehicles of any number greater than two, having a maximum within-group headway of 5 seconds. The efficiency of this definition is discussed in a later section.



Sample Sizes: Harlem Av. Left-Hand Entrance Ramp - 399 merges
Nagle Av. Right-Hand Entrance Ramp - 395 merges

Figure 6. Acceptance and rejection of static time-gaps, single and modified multiple merges, passenger cars only.



Sample Sizes: Harlem Av. Left-Hand Entrance Ramp - 63 merges
Nagle Av. Right-Hand Entrance Ramp - 39 merges

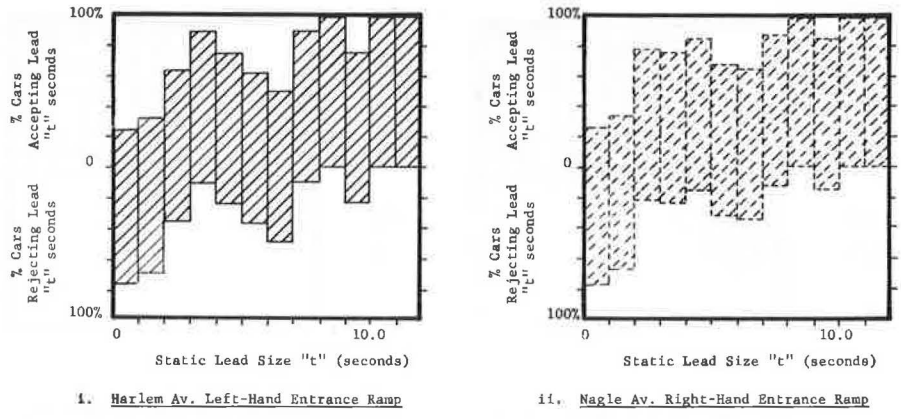
Figure 7. Acceptance and rejection of static time-gaps, single and modified multiple merges, trucks only.

were treated as a separate data sample, for which a "modified-multiple-merge-gap" was defined equal to the sum of the effective lead and lag values defined previously.

Figure 5 illustrates the simple "static" gap-acceptance/rejection distributions determined for the two ramps for single-vehicle merges, passenger cars only. Figures 6 and 7 illustrate the equivalent distributions for the combined samples of single-vehicle and modified-multiple merges involving passenger cars and trucks respectively.

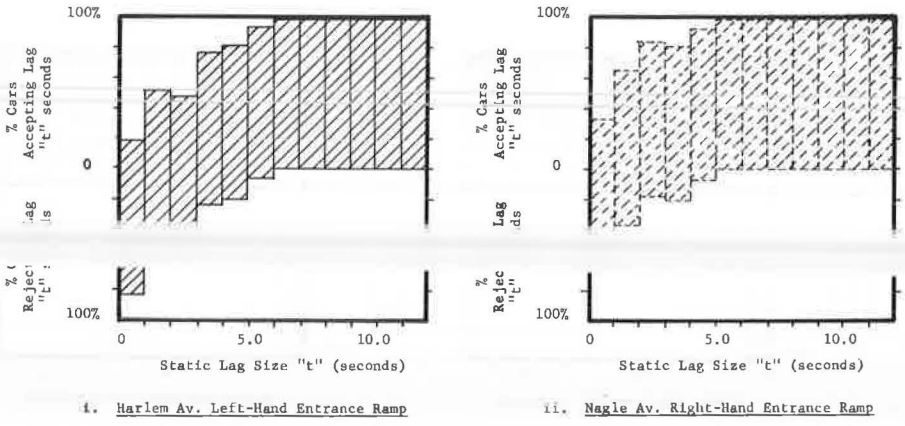
Comparison of Figures 5 and 6 suggests that there is little difference between the results derived for the two different data sets when they are analyzed in purely static terms.[6] In each case it is apparent that fewer small gaps (< 4 seconds) are accepted at the left-hand than at the right-hand ramp. Also, on the basis of Figure 7 it appears that trucks tend to require a slightly larger gap for acceptance than do cars. (This

[6] This point is discussed further in a later section.



Sample Sizes: Harlem Av. Left-Hand Entrance Ramp - 167 merges
Nagle Av. Right-Hand Entrance Ramp - 269 merges

Figure 8. Acceptance and rejection of static lead values, single merges only, passenger cars only.



Sample Sizes: Harlem Av. Left-Hand Entrance Ramp - 167 merges
Nagle Av. Right-Hand Entrance Ramp - 269 merges

Figure 9. Acceptance and rejection of static lag values, single merges only, passenger cars only.

last conclusion should be interpreted with extreme caution owing to the small sample size.)

Figures 8 and 9 illustrate the static acceptance/rejection distributions for "lead" and "lag" values, again based on single-merge data only. The lead distributions at both ramps are almost rectangular, suggesting a roughly equal probability of acceptance for all values above a minimum of 1 second. The lag distributions, however, exhibit a more exponential form, indicating that the likelihood of acceptance increases as the value of static lag increases. As in the case of the gap size analyses discussed above, there is an evident tendency for small lags to be accepted more readily at the right-hand than at the left-hand ramp.

In any binary response situation, such as that represented by a ramp vehicle's acceptance or rejection of mainstream headways, a number of alternate indexes of "critical response level" may be defined. In the following discussion it is assumed that the "median-acceptance-value" (i.e., that value of lead, lag or gap which has a

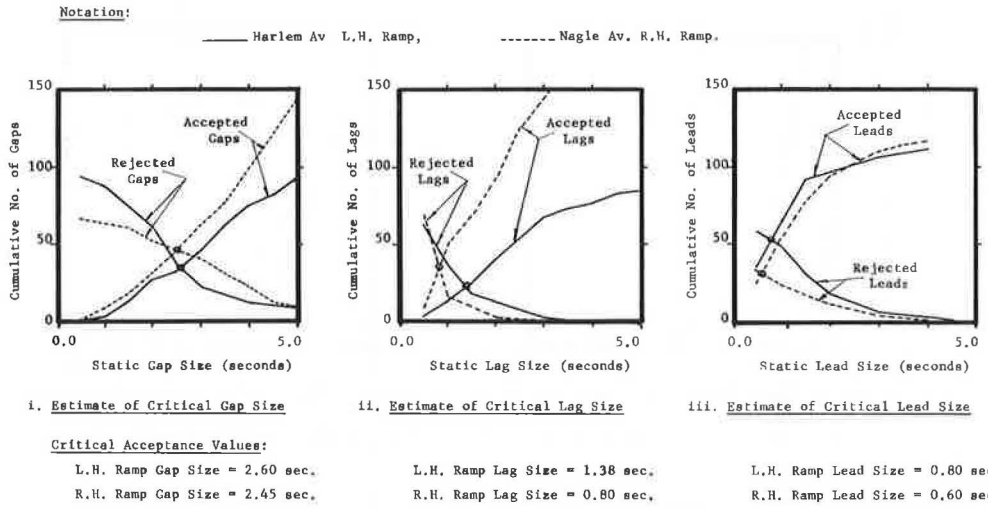


Figure 10. Raff analysis: Estimation of static gap, lag and lead acceptance values, single merges only, passenger cars only.

TABLE 2

CRITICAL GAP, LEAD AND LAG ACCEPTANCE VALUES, SINGLE VEHICLE MERGES ONLY

Method	Subject	Harlem Avenue Left-Hand On-Ramp	Nagle Avenue Right-Hand On-Ramp
Raff Analysis	Critical Gap	2. 60 sec.	2. 45 sec.
Probit Analysis	Critical Gap 95% Conf. Lim.	2. 63 sec. 2. 28-3. 03 sec.	1. 88 sec. 1. 22-2. 62 sec.
[illegible]t Analysis	Critical Lag	1. 38 sec.	0. 88 sec.
[illegible]t Analysis	Critical Lag 95% Conf. Lim.	1. 11 sec. 0. 86-1. 33 sec.	0. 71 sec. 0. 47-1. 07 sec.
Raff Analysis	Critical Lead	0. 80 sec.	0. 60 sec.

50 percent probability of acceptance) may be used as a simple, parametric index of critical acceptance behavior.[7]

Two separate empirical techniques were employed to estimate this assumed critical value: a graphical approach due to Raff, and a standard probit analysis.[8] Raff's method

[7]Obviously, the 50 percentile acceptance value implies an equal likelihood of acceptance or rejection. The arbitrary selection of this value may well be criticized on the grounds that a higher probability of acceptance than 50 percent is desirable. More importantly, however, the critical values determined here are general values for the ramps in question, aggregated over all drivers. A more realistic analysis might either develop a critical acceptance function for individual drivers or for statistically significant groupings (in the sense of a maximized ratio of between-group variance to within-group variance) of drivers. Alternatively, the simple analysis discussed here might be stratified according to driver or vehicle characteristics, speed levels, etc. The development of a more realistic empirical acceptance/rejection function is discussed in detail in the final section of this paper.

[8]Raff's method was first developed in: Raff, M. S., and Hart, J. W., A Volume Warrant for Urban Stop Signs, Eno Foundation for Highway Traffic Control, 1950. A simple discussion of probit analysis is given in: Experimental Statistics, Handbook No. 91. U. S. Department of Commerce, National Bureau of Standards, U. S. Government Printing Office, August 1963.

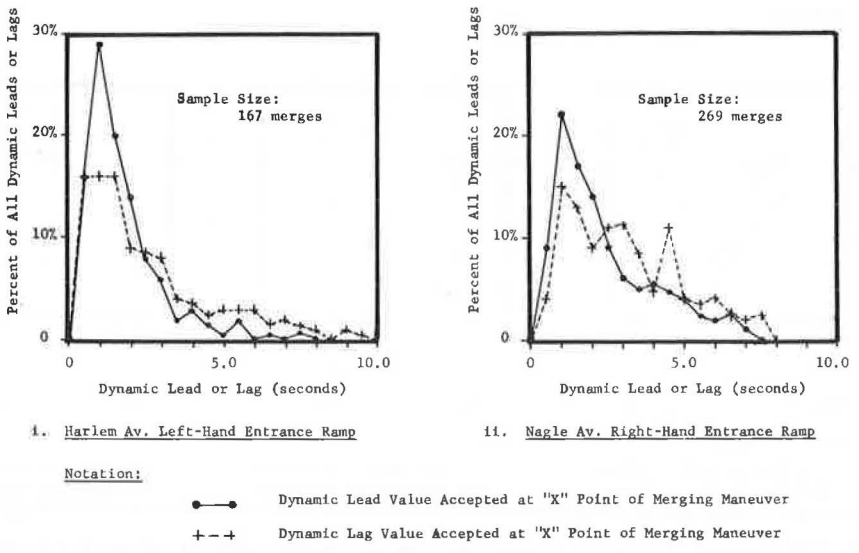


Figure 11. Distributions of dynamic lead and dynamic lag times at the commencement of the merging maneuver (X point), passenger cars only, single merges only.

involves the plotting of two cumulative curves for each set of data, one indicating the number of accepted gaps (leads, lags) smaller than a set of given values, the other the number of rejected gaps (leads, lags) greater than these values. The intersection of the two cumulative curves then provides an estimate of the median acceptance value as defined above. Probit analysis involves a slightly more sophisticated analytical

objects, $a_1, a_2, \ldots, a_k$, were subjected to a set of K stimuli $X_1, X_2, \ldots, X_k$, yielding K separate binary responses, $r_1, r_2, \ldots, r_k$. In the case of gap acceptance data, the set of stimuli are the available mainstream gaps, leads or lags; the objects are the merging vehicles and the set of binary responses the merging drivers' acceptances or rejections of the given gaps (leads, lags). The value of median critical response is then estimated simply by assuming that the distribution of response levels follows a normal distribution. In the case of gap and lag observations a close approximation to normality may be achieved by transforming the time scale logarithmically; lead values, however, cannot be so transformed. Probit analyses therefore were performed only on the samples of gap and lag observations.

Figure 10 and Table 2 summarize the results of probit and Raff analyses applied to the samples of single-vehicle merges, passenger cars only. Note that in each case the critical acceptance values for the left-hand ramp are higher than the equivalent right-hand ramp values. Note also that the critical lead values obtained from the Raff analysis are lower at both ramps than the equivalent critical lag values. Probit analysis yields estimates of both the critical median acceptance value and the critical response variance. The values of the latter (see Table 2) are virtually the same in all four cases, suggesting that though the level of response varies vetween a left- and right-hand ramp and between gap and lag values, the nature of the response mechanism is essentially the same.

Both analyses yield consistently higher values for critical acceptable gap size than for critical lag size. The sum of critical lead and critical lag values, however, exceeds the critical gap value at both study locations, suggesting that though a driver may accept critical values of either lead or lag time, he is unlikely to accept both simultaneously. Finally, it is relevant to note that the estimates of median acceptance value derived from the graphical analyses are in all cases contained within the 95 percent confidence limits for the equivalent probit analysis estimates.

A Pseudodynamic Analysis

The preceding discussion has been based entirely on "static" measurements made at the ramp nose. Figure 11 in contrast, illustrates the distributions of accepted lead and lag values measured "dynamically" at the start of the merging maneuver (i.e., at the point X in Figure 1). Again the analysis is based on data for single-vehicle, passenger car merges only.

The marked skewness and clearly defined modal values of the lead distributions illustrated in Figure 11 suggest that one may interpret these values as approximate estimates of the minimum dynamic lead time required by a merging driver.[9] Equivalent interpretation of the modal lag values is clearly not valid.

Figure 12 illustrates the set of cumulative frequency distributions derived from the curves in Figure 11, determined in this case for both dynamic and static observations. In the case of the accepted lag curves, the distributions of static and dynamic observations are significantly different from each other (at the 5 percent level) for each of the two ramps. The equivalent pairs of lead distributions, however, are not significantly different.

The difference between the sets of lag distributions suggests that the static lag values (which are largely dependent on the arbitrary arrival rates of the ramp vehicles) cannot be used to predict the size of acceptable dynamic lags at the actual point of merger. This point is perhaps emphasized more clearly by Figure 13. Note in this figure that the standard error of estimate of "Y" (dynamic lag) on "X" (static lag) is 1.68 seconds, indicating a considerable degree of instability between individual static and dynamic accepted-lag measurements. Note also that although the general tendency is for lag values at the "X" point to be smaller than those measured at the ramp nose, this tendency is reversed at extremely low values.

All of the data in Figure 13 were collected during periods of free flow on the mainstream section, when mainstream speeds exceeded ramp vehicle speeds. In a more congested situation where the reverse speed condition obtains, a totally different relation might apply, with lead rather than lag acceptance critical. It should also be noted that the poor correlation between static and dynamic lag values does not necessarily imply a serious instability of mainstream gap size over the length of the merge area. Rather, it implies an adjustment of gap structure (i.e., lead and lag values) which is suppressed in the case of the lead measurements by definitional bias.

In all cases accepted lead and lag values, both static and dynamic, tend to be highe at the left-hand than at the right-hand ramp. In considering this result, it should be borne in mind that an average of 52 lane changes per hour was observed at the righ'-hand ramp in which vehicles moved from the right-hand lane into the center lane. At the left-hand ramp, the equivalent figure was 91 lane changes per hour from the left lane into the center lane. This lane changing was frequently associated with the acceptance by a merging vehicle of a relatively small static lag value.

Speed and Volume Effects, Merge Length Analysis

A crude estimate of the influence of merging speed on gap acceptance behavior may be obtained simply by stratifying the data of Figure 12 according to a number of preselected relative speed groupings.[10] The results of such an analysis are illustrated

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[9] This in turn suggests the existence of a minimal or "buffer" lead-time between two merging vehicles, which may take on slightly different values depending on the characteristics of the drivers. Such a buffer is implicit in the familiar car-following models of stream flow.

[10] The analysis is crude in the sense that it incorporates only the coarsest levels of stratification with no regard to the absolute speeds of the vehicles involved. It is also based, necessarily, on observations of acceptance and rejection behavior which lie within rather than on the critical speed/acceptance envelope. A more meaningful approach, precluded in this case due to lack of data, might have been to perform critical response level analyses for each of a set of ramp/mainstream speed groupings.

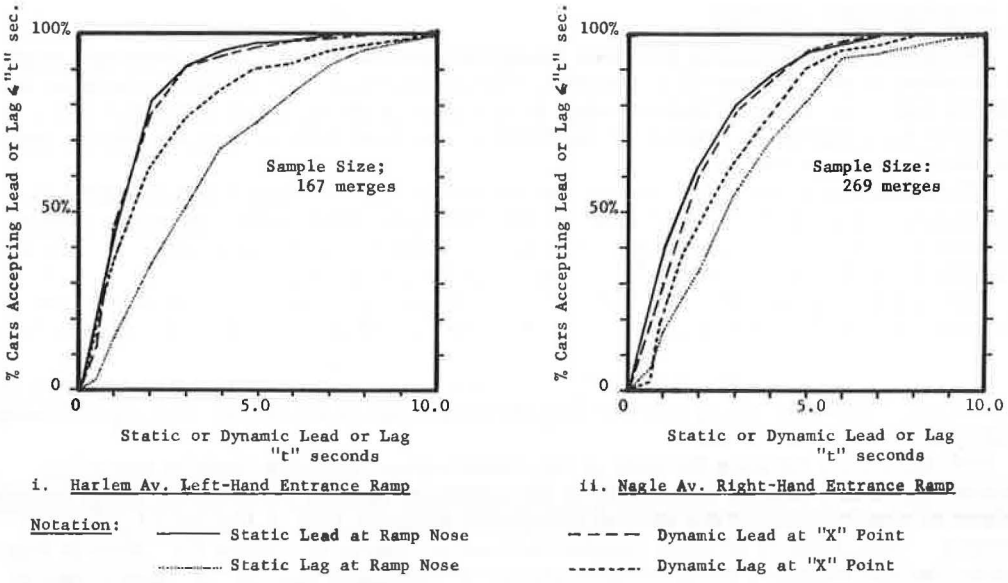
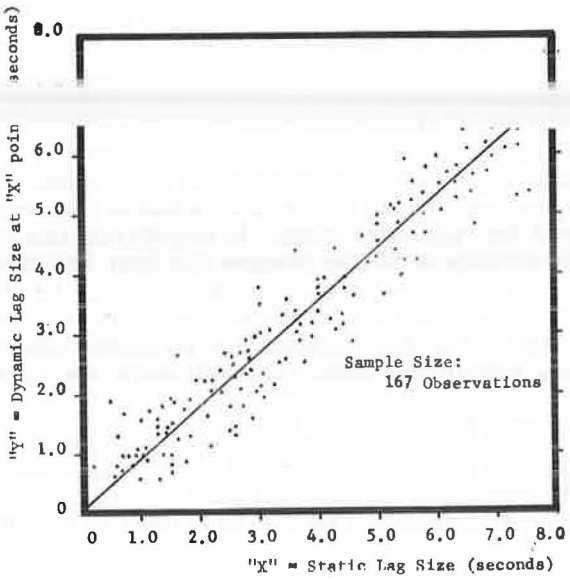


Figure 12. Cumulative distributions of static and dynamic lead and lag values, single merges only, passenger cars only.



Regression Equation: $Y = 0.056 + 0.873X$

$(R^2 = 0.88$, Std. Error of Est. = 1.68 sec.)

Where: X = Value of Static Lag Accepted by merging vehicle, measured at the ramp nose

Y = Value of Dynamic Lag Accepted by the same merging vehicle, measured at the "X" point

All data for Nagle Av. Right-Hand Entrance Ramp, Kennedy Expressway, Chicago

Figure 13. Stability of static vs dynamic lag acceptance, single merges only, passenger cars only.

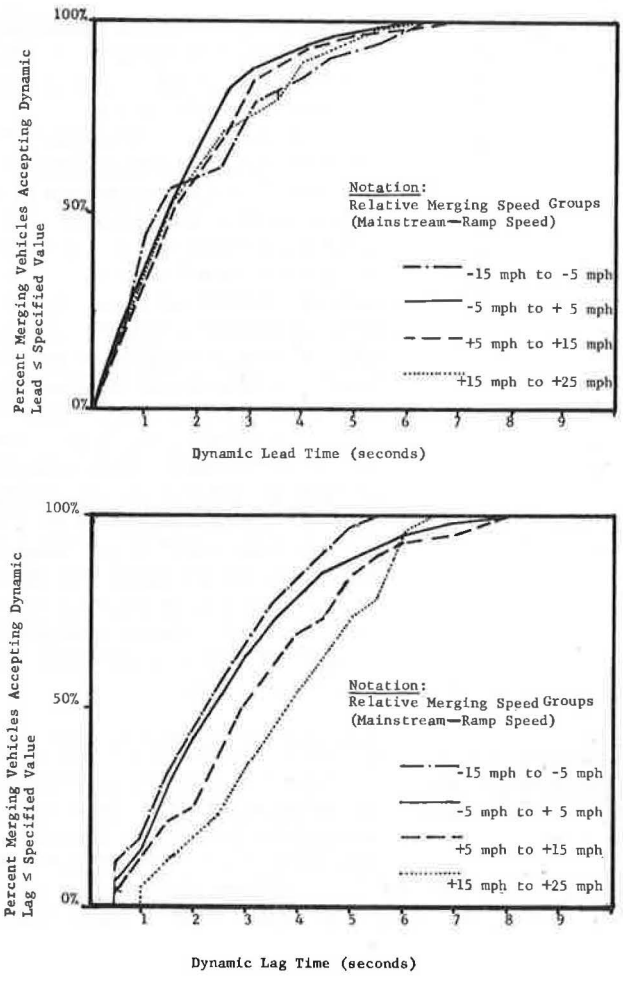
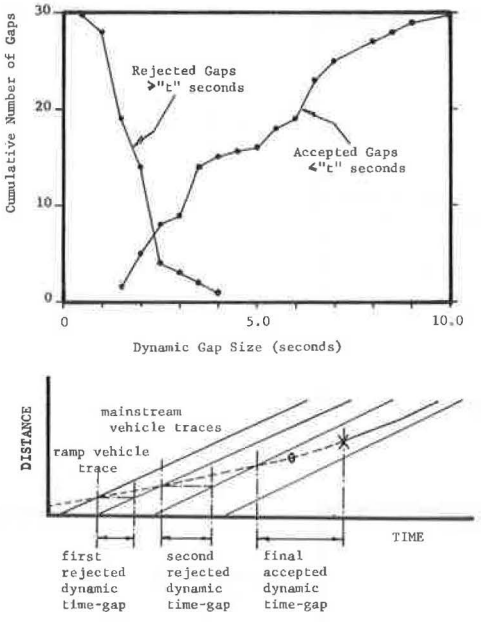


Figure 14. Effect of relative merging speed on dynamic lead and lag acceptance (all data for right-hand ramp only).

graphically in Figure 14. It is immediately apparent that relative merging speed exerts a considerable influence on the distribution of accepted dynamic lag values, the proportion of vehicles accepting small lag values decreasing rapidly as the relative speed of the mainstream to the ramp vehicle increases. The equivalent effect in the case of dynamic lead values is much less pronounced. In neither case does allowance for mainstream speed level, again by 10 mph increments, exert any significant influence on the results. This may well, however, be due to the extremely small sample size rather than to any basic speed/acceptance relationship.

Table 3 illustrates the relatively insignificant influence of mainstream volume, expressed in this case as a minute flow rate in the adjacent lane, on merging behavior.[11] Note that merge length and point of merger are virtually independent both of each other and also of relative merging speed. The slightly higher average merge length at the left-hand ramp may be a reflection of a longer acceleration lane. The higher standard deviation of merge lengths at that ramp, however, suggests a basic difference in merging behavior, due possibly to driver hesitancy at the left-hand terminal or to the

[11] An equivalent stratification of the curves in Figure 12 according to three levels of mainstream flow also indicates that volume exerts no significant influence on the merging process.



Note: Dynamic Gap Size defined as the time-gap between a ramp vehicle and the next successive mainstream vehicle, measured at the instant the ramp vehicle is overtaken by a preceding mainstream vehicle.

Figure 15. Raff analysis: Acceptance and rejection of dynamic time-gaps, single merges only, passenger cars only (all data for Harlem Ave.

tendency noted in a previous paper (61) of some drivers to travel to the end of a left-hand ramp acceleration lane before completing their merge.

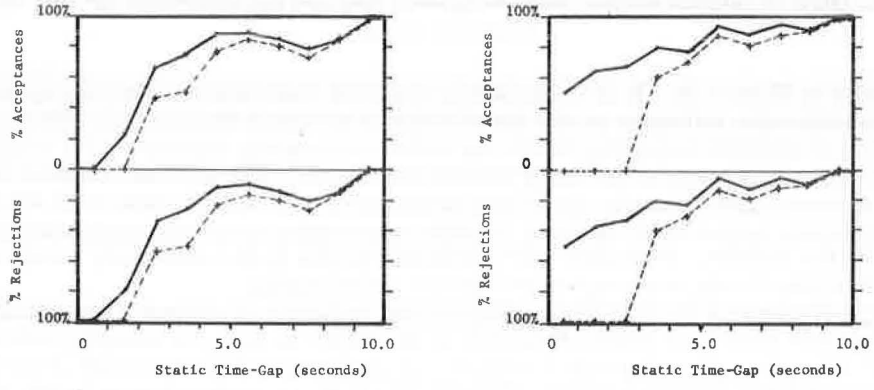
By definition, it is not possible to consider dynamic rejection behavior with reference to measurements made at the X and O points. If, however, one redefines a dynamic time-gap as suggested earlier as the time-headway between a ramp vehicle and the next, succeeding mainstream vehicle measured at the instant when the ramp vehicle is overtaken by a leading mainstream vehicle (Fig. 15), then a modified dynamic form of acceptance/rejection analysis is possible. Figure 15 illustrates a Raff analysis of a sample of 55 dynamic gap acceptance and rejection observations at the Harlem Avenue left-hand ramp. Based on this extremely small sample of data it appears that the critical dynamic acceptance value at that ramp is approximately 2. 25 seconds, compared with an equivalent static value of 2. 60 seconds.

Analysis of Multiple Vehicle Merges

Approximately 33 percent of all merges observed at the right-hand ramp and 57 per-

category of multiple-vehicle merges.

Figure 16 depicts a straightforward static analysis of these data, based on the technique developed by Pearson and Ferreri (49). Note that, apart from a marked tendency for



i. Harlem Av. Left-Hand Entrance Ramp

ii. Nagle Av. Right-Hand Entrance Ramp

Notation:

Single Vehicle Merges, i.e. one ramp vehicle only entering the mainstream

Two Vehicle Merges, i.e. two ramp vehicles accepting a single mainstream gap

Figure 16. Multiple merge analysis: Acceptance and rejection of static time-gaps by multiple vehicles.

TABLE 3

CORRELATION OF MERGE LENGTH AND POINT

Category	Values of Simple Correlation Coefficient	
	R. H. Ramp	L. H. Ramp
Merge length vs relative merging speed	0. 01	0. 08
Merge length vs through volume	0. 10	0. 09
Merge length vs point of merge	0. 03	0. 06
Point of merge vs through volume	0. 06	-0. 10
Point of merge vs relative merging speed	-0. 08	-0. 02
Average merge length	138 ft	179 ft
Standard deviation of merge length	59 ft	115 ft

single vehicles to accept smaller time-gaps than pairs of successive vehicles, there is relatively little pattern in the data.

Figures 17 and 18 illustrate a simple dynamic analysis of multiple vehicle merges for the Harlem Avenue left-hand ramp. Insufficient data were available to duplicate this analysis for the right-hand ramp. The broken lines in Figure 17 represent the distributions of modified-static leads accepted by the first, second, third and fourth vehicles respectively. In this case, all of the lead values were measured statically at the ramp nose. For all but the first vehicle in a platoon, these lead values represent also the headways between successive ramp vehicles as they pass the ramp nose. Figure 18, in contrast, compares the acceptance of dynamic lead values in single-vehicle merges with that for the trailing vehicles in multiple merges. All measurements in this case were made at the X point.

The distributions of both static and dynamic values accepted by the lead vehicle in a multiple merge (Figs. 17 and 18) are significantly lower than the equivalent acceptance distributions for successive trailing vehicles. There is no significant difference, however, among the acceptance distributions for the second, third and fourth vehicles in line in a multiple merge. These results have certain implications in terms of the comparative efficiency of isolated and multiple vehicle merges. In particular, if one defines "merge-efficiency" as the average time required per vehicle to merge into the mainstream, either singly or in a platoon, and one accepts the values for critical lead

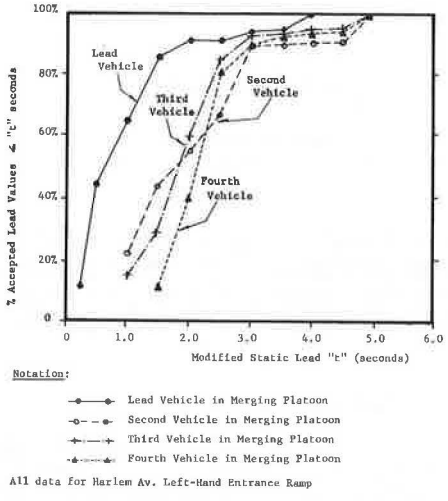


Figure 17. Multiple merge analysis: Cumulative distributions of accepted lead values by successive vehicles in a merging platoon.

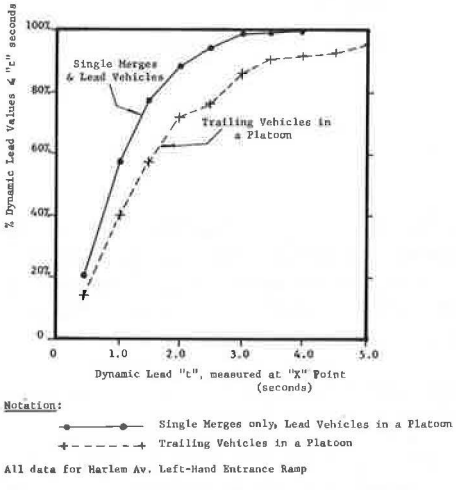


Figure 18. Multiple merge analysis: Cumulative distributions of dynamic lead values for single merges and trailing vehicles in a multiple merge.

TABLE 4

INPUT VARIABLES FOR PRINCIPAL COMPONENTS ANALYSES

Analysis Groups I & II	Analysis Groups III & IV
Group I—Nagle Ave., Direct Acceptance Group II—Harlem Ave., Direct Acceptance	Group III—Nagle Ave., Rejections Group IV—Harlem Ave., Rejections
1. Vehicle type of leading vehicle	1. Vehicle type of leading vehicle
2. Vehicle type of merging vehicle	2. Vehicle type of merging vehicle
3. Vehicle type of lagging vehicle	3. Vehicle type of lagging vehicle
4. Mainstream 1-minute volume	4. Ramp 1-minute volume
5. Ramp 1-minute volume	5. Ramp 1-minute volume
6. Accepted gap at nose	6. Accepted gap at nose
7. Accepted lead at nose	7. Absolute speed of merging vehicle at nose
8. Accepted lag at nose	8. Relative speed of merging vehicle to leading vehicle at nose
9. Absolute speed of merging vehicle at nose	9. Relative speed of merging vehicle to lagging vehicle at nose
10. Relative speed of merging vehicle to leading vehicle at nose	10. Number of rejected gaps at nose
11. Relative speed of merging vehicle to lagging vehicle at nose	11. Rejected gap at nose
12. Second previous mainstream gap	12. Rejected lead at nose
13. First previous mainstream gap	13. Rejected lag at nose
14. First following mainstream gap (excluding acccptcd gap)	14. Second previous mainstream gap
15. Second following mainstream gap	15. First previous mainstream gap
16. Third following mainstream gap	16. First following mainstream gap (excluding rejected gap)
18. Accepted lead at X-point	18. Third following mainstream gap
19. Accepted lag at X-point	19. Accepted gap at X-point
20. Absolute speed of merging vehicle at X-point	20. Accepted lead at X-point
21. Distance nose to X-point	21. Accepted lag at X-point
	22. Absolute speed of merging vehicle at X-point
	23. Distance nose to X-point

and lag time derived earlier, then it appears that a multiple merge is slightly, but only slightly, more efficient than an equivalent series of isolated merges. This conclusion, of course, takes no account of the disruptive effect of a sequence of multiple merges on traffic flow in the mainstream, a factor of considerable importance in comparing the two types of merging.

A Multivariate Approach to the Analysis of Merging Behavior

If nothing else the preceding discussion has at least emphasized the essential multivariate character of merging behavior. A broad range of variables has been defined, all of which may exert a potentially significant influence on the merging process. Some of these variables may be determined from a simple time-distance trace analysis, others may not.

In an attempt to reduce the dimensionality of the analytical problem and at the same time to identify some of the basic collinearities within the data, a varimax rotated

TABLE 5

CORRELATION MATRIX, MEANS AND STANDARD DEVIATIONS FOR SINGLE MERGES WITH DIRECT ACCEPTANCE
NAGLE AVENUE, RIGHT-HAND ON-RAMP, KENNEDY EXPRESSWAY (NORTHBOUND)

Variable		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	Mean	Std.Dev.
VEH TYPE/LDG.	1	1.0	.00	.12	.05	-.01	-.07	-.04	-.07	-.02	-.02	-.08	-.06	.09	.09	.06	-.11	-.06	-.02	-.06	-.04	-.05	0.20	0.40
VEH TYPE/MRG.	2		1.0	.04	-.16	.06	.06	-.02	.08	-.13	.09	.08	.01	.04	.03	.00	.17	.06	.06	.04	.15	.03	0.13	0.34
VEH TYPE/LAG.	3			1.0	.00	.09	-.08	-.06	-.07	.04	-.07	-.07	.01	-.05	-.01	-.11	-.03	-.07	-.07	-.05	.06	.00	0.20	0.40
THRU. VOL.	4				1.0	.12	-.40	-.22	-.34	.12	-.11	-.18	-.14	-.29	-.44	-.25	-.32	-.40	-.25	-.33	.05	.06	14.7	3.5
RAMP VOL.	5					1.0	-.02	-.10	.04	-.12	.03	-.04	-.02	.00	-.03	-.19	-.03	-.05	-.09	.00	-.16	.12	6.1	2.3
ACC. GAP NOSE	6						1.0	.59	.83	-.16	.31	.14	.00	.13	.12	-.08	.13	.98	.61	.82	-.07	-.24	6.69	4.53
ACC. LEAD NOSE	7							1.0	.04	-.06	.26	.11	.00	.18	-.04	-.07	.09	.59	.94	.07	.04	-.14	2.42	2.56
ACC. LAG. NOSE	8								1.0	-.15	.21	.09	.00	.05	.18	-.06	.10	.81	.10	.96	-.13	.21	4.32	3.63
ABS. SP. NOSE	9									1.0	-.64	-.62	-.01	-.12	.07	.07	.01	-.16	-.13	-.11	.42	.05	72.3	13.4
REL. SP. LDG NOSE	10										1.0	.50	.04	.25	-.06	-.05	-.03	.32	.30	.19	-.23	-.05	11.8	16.1
REL. SP. LAG NOSE	11											1.0	.01	.13	.01	.00	.16	.12	.14	.05	-.20	-.08	8.3	16.9
PREC. GAP #2	12												1.0	.12	-.08	-.07	-.09	.02	-.01	.02	-.02	-.04	4.11	3.28
PREC. GAP #1	13													1.0	.07	-.09	.01	.15	.17	.06	.01	.00	4.40	3.71
SUCC. GAP #1	14														1.0	.11	-.06	.12	.01	.15	-.04	-.10	4.58	3.98
SUCC. GAP #2	15															1.0	.08	-.10	-.09	-.06	.03	-.07	4.09	3.63
SUCC. GAP #3	16																1.0	.14	0.9	.12	-.01	-.06	3.95	3.30
ACC. GAP X-PT.	17																	1.0	.63	.82	-.07	-.24	6.87	4.53
ACC. LEAD X-PT.	18																		1.0	.07	-.08	-.10	2.91	2.59
ACC. LAG. X-PT.	19																			1.0	-.03	-.23	3.97	3.53
ABS. SP. X-PT.	20																				1.0	.37	74.1	14.9
DIST. N - X	21																					1.0	261.	106.

TA 6

CORRELATION MATRIX, MEANS AND STANDARD DEVI ONS FOR SINGLE MERGES WITH DIRECT ACCEPTANCE
HARLEM AVENUE, LEFT-HAND ON-RAM ISENHOWER EXPRESSWAY (EASTBOUND)

Variable		1	2	3	4	5	6	7	8	9	10	12	13	14	15	16	17	18	19	20	21	Mean	Std.Dev.
VEH TYPE/LDG.	1	1.0	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	0.00	0.00
VEH TYPE/MRG.	2		1.0	-.05	-.08	.01	.18	.13	.15	.21	.04	-.03	.08	.05	.00	-.12	.35	.27	.28	-.24	.05	0.19	0.39
VEH TYPE/LAG.	3			1.0	-.12	.04	.07	-.08	.11	-.03	.06	.01	-.03	.13	.16	.07	.09	-.05	.14	-.12	-.06	0.01	.09
THRU. VOL.	4				1.0	-.26	-.21	-.14	-.18	-.06	.03	-.19	-.22	-.29	-.33	-.15	-.17	-.11	-.14	-.10	-.10	18.3	4.1
RAMP VOL.	5					1.0	.09	-.10	.15	-.13	.04	.04	.32	.08	-.05	.07	-.11	-.12	-.07	-.22	.01	12.1	4.2
ACC. GAP NOSE	6						1.0	.48	.92	-.13	-.04	.02	.02	-.03	-.10	-.07	.69	.51	.56	-.17	.05	6.60	4.90
ACC. LEAD NOSE	7							1.0	.09	.07	-.08	-.06	-.02	.08	.02	.04	.57	.91	.19	.06	.09	1.51	1.93
ACC. LAG. NOSE	8								1.0	-.18	-.01	.04	.03	-.07	-.12	-.10	.52	.17	.55	-.22	.01	5.09	4.33
ABS. SP. NOSE	9									1.0	-.48	.07	-.10	-.09	.12	-.01	-.16	-.12	-.13	.58	.12	51.9	7.30
REL. SP. LDG NOSE	10										1.0	-.13	.07	.16	-.03	.00	.00	.00	.00	-.19	.01	23.1	12.0
REL. SP. LAG NOSE	11											.13	.05	.21	.01	.02	.05	.02	.05	-.27	-.02	23.7	12.5
PREC. GAP #2	12											1.0	.10	-.09	-.11	.05	.05	-.08	.11	.12	.02	3.10	3.31
PREC. GAP #1	13												1.0	-.01	.01	.05	.00	.02	0.02	-.07	.01	3.71	3.68
SUCC. GAP #1	14													1.0	.06	-.18	.18	.06	.19	-.04	.11	3.93	4.43
SUCC. GAP #2	15														1.0	-.09	-.04	.04	-.08	.16	-.10	3.96	4.69
SUCC. GAP #3	16															1.0	-.01	.05	-.05	-.09	-.10	3.39	2.93
ACC. GAP X-PT.	17																1.0	.60	.89	-.20	-.13	5.99	4.73
ACC. LEAD X-PT.	18																	1.0	.18	-.14	.10	2.42	2.17
ACC. LAG. X-PT.	19																		1.0	-.18	-.22	3.57	3.83
ABS. SP. X-PT.	20																			1.0	.19	61.8	9.70
DIST. N - X	21																				1.0	309.	167.

TABLE 7

CORRELATION MATRIX, MEANS AND STANDARD DEVIATIONS FOR SINGLE MERGES WITH REJECTIONS
NAGLE AVENUE, RIGHT-HAND ON-RAMP, KENNEDY EXPRESSWAY (NORTHBOUND)

Variable		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Means	Std Dev.
VEH TYPE/LDG.	1	1.0	-.27	.21	.11	.02	-.04	.08	-.21	-.01	-.19	.26	.28	-.14	.23	.32	.02	-.10	-.06	-.04	-.29	.00	.38	-.17	0.24	0.44
VEH TYPE/MRG.	2		1.0	-.17	-.05	.07	-.01	.21	-.04	.04	-.16	.10	.11	-.10	.23	-.07	.08	-.18	-.16	.01	.33	-.04	-.22	-06	0.19	0.40
VEH TYPE/LAG.	3			1.0	-.01	.03	-.16	-.01	-.16	-.03	.10	-.03	.02	-.22	-.01	.27	-.11	-.18	-.19	-.16	.04	-.17	.23	.10	0.11	0.31
THRU VOL.	4				1.0	.23	-.41	-.09	-.04	.01	.09	-.25	-.23	-.08	-.05	-.16	-.25	-.51	-.38	-.47	-.41	-.43	-.12	-.14	15.6	3.9
RAMP VOL.	5					1.0	.07	.03	-.05	-.09	.00	.03	-.03	.29	-.17	.03	-.12	-.07	.07	.00	-.28	.04	-.02	-.12	6.8	2.6
ACC. GAP (NOSE)	6						1.0	-.05	.03	.02	.05	.00	-.08	.37	.17	.16	.74	.36	.18	.97	.34	.97	.31	.01	4.61	3.78
ABS. SPEED	7							1.0	-.68	-.65	-.30	.09	.15	-.32	.34	-.19	.08	.06	.13	.00	.02	-.01	.22	.25	64.4	9.0
REL. SP. LDG.	8								1.0	.53	.13	-.06	-.09	.15	-.18	.01	-.02	.04	-.12	.01	-.02	.01	-.24	-.29	18.6	13.8
REL. SP. LAG.	9									1.0	.05	-.08	-.13	.26	.03	.10	.01	-.23	-.14	.00	.11	-.02	-.25	-.33	22.0	13.0
NO. REJ. GAPS	10										1.0	-.07	-.12	.23	-.17	-.03	-.27	-.04	.02	.02	.05	.02	-.05	.20	1.14	0.42
REJ. GAP (NOSE)	11											1.0	.98	-.03	.05	.06	-.09	.09	.10	.12	-.08	.14	.08	-.22	3.40	2.36
REJ. LEAD (NOSE)	12												1.0	-.24	.09	-.02	-.07	.02	.02	.04	-.07	.05	.08	-.21	2.66	2.43
REJ. LAG (NOSE)	13													1.0	-.17	.37	-.06	.33	.39	.37	-.05	.39	.01	.01	0.74	0.51
PREC. GAP #2	14														1.0	-.10	.29	-.09	.07	.20	.20	.18	.38	.13	3.19	2.23
PREC. GAP #1	15															1.0	.00	.21	.05	.17	.10	.16	.24	.16	3.37	2.37
SUCC. GAP #1	16																1.0	.05	-.12	.71	.22	.71	.26	.06	4.22	3.18
SUCC. GAP #2	17																	1.0	.48	.40	.24	.38	.28	.09	5.12	4.19
SUCC. GAP #3	18																		1.0	.23	.17	.21	.09	.07	5.19	4.48
ACC. GAP X-PT.	19																			1.0	.36	.99	.37	.03	4.86	3.71
ACC. LEAD X-PT.	20																				1.0	.23	.02	.41	0.97	0.52
ACC. LAG. X-PT.	21																					1.0	.38	-.03	3.89	3.55
ABS. SP. X-PT.	22																						1.0	.33	64.9	12.5
DIST. N - X	23																							1.0	334.	103.

TA 8

CORRELATION MATRIX, MEANS AND STANDARD I ATIONS FOR SINGLE MERGES WITH REJECTIONS
HARLEM AVENUE, LEFT-HAND ON-RAM ISENHOWER EXPRESSWAY (EASTBOUND)

Variable		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	Means	Std.Dev.
VEH TYPE/LDG.	1	1.0	-.06	.00	.12	-.13	-.01	.02	-.09	-.03	.13	.	08	.07	-.07	.01	-.10	-.03	.06	.06	.10	.04	-.15	.27	0.02	0.13
VEH TYPE/MRG.	2		1.0	.00	-.02	.04	.09	-.15	.15	.15	-.11	.	39	.30	-.09	-.02	.12	-.18	-.11	-.11	.01	-.11	-.28	-.02	0.16	0.37
VEH TYPE/LAG.	3			1.0	.00	.00	.00	.00	.00	.00	.00	.	00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	0.00	0.00
THRU VOL.	4				1.0	-.25	-.03	.04	-.17	-.25	.19	-.	31	.11	-.40	-.08	-.21	-.15	-.04	.02	-.31	.10	-.09	-.04	17.8	3.8
RAMP VOL.	5					1.0	-.07	-.24	.29	.26	-.34	.	06	.20	.17	.05	.23	.16	-.16	-.17	-.03	-.17	-.09	-.07	13.8	4.1
ACC. GAP (NOSE)	6						1.0	-.21	.14	.17	.35	.	01	.06	.11	.01	.68	-.06	-.08	.63	.28	.58	.09	.06	5.02	3.62
ABS. SPEED	7							1.0	-.46	-.46	-.30	-.	03	-.28	-.26	.08	-.15	-.02	.30	-.11	-.32	-.03	.27	-.08	49.2	5.4
REL. SP. LDG.	8								1.0	.83	-.01	.	13	.09	.19	-.08	.28	.00	-.21	.07	.12	.04	-.24	-.32	24.8	12.3
REL. SP. LAG.	9									1.0	-.13	.	30	.01	.14	-.03	.34	.22	-.22	.15	.07	.13	-.20	-.34	25.8	12.5
NO. REJ. GAPS	10										1.0	.	06	.20	-.13	-.10	-.25	-.17	-.05	.29	.40	.19	-.13	.43	1.49	1.54
REJ. GAP (NOSE)	11											1	92	.42	-.03	-.10	-.07	.04	-.11	.11	.12	.08	-.21	.20	2.43	1.75
REJ. LEAD (NOSE)	12												1.0	.03	-.06	-.07	-.08	.07	-.08	.18	.13	.16	-.11	.09	1.28	1.59
REJ. LAG (NOSE)	13													1.0	.03	-.07	.01	-.04	-.13	-.13	.03	-.15	-.28	.33	1.14	0.70
PREC. GAP #2	14														1.0	-.08	.30	.08	.00	.00	.32	-.08	.00	-.01	4.36	4.71
PREC. GAP #1	15															1.0	-.02	-.06	-.15	-.05	-.12	-.02	.16	-.06	3.54	3.94
SUCC. GAP #1	16																1.0	.11	-.09	.32	.13	.29	.05	-.28	3.78	3.31
SUCC. GAP #2	17																	1.0	-.20	-.11	.02	-.12	.04	-.12	3.32	2.86
SUCC. GAP #3	18																		1.0	-.12	-.10	-.10	.03	-.09	3.51	2.82
ACC. GAP X-PT.	19																			1.0	.26	.97	.09	.01	3.92	2.44
ACC. LEAD X-PT.	20																				1.0	.01	-.25	.32	1.05	0.62
ACC. LAG. X-PT.	21																					1.0	.15	-.07	2.87	2.36
ABS. SP. X-PT.	22																						1.0	.07	57.8	12.1
DIST. N - X	23																							1.0	503.	212.

TABLE 9

VARIANCE ACCOUNTED FOR BY UNROTATED FACTORS

Factor No.	Direct Acceptance Merges			Merges With Rejections		
	Eigen Value	Percent Variance Accounted For		Eigen Value	Percent Variance Accounted For	
		Individual	Cum.		Individual	Cum.
1	3.985	19.9	19.9	3.653	16.6	16.6
2	2.566	12.8	32.8	2.854	13.0	29.6
3	1.741	8.7	41.5	2.603	11.8	41.4
4	1.699	8.5	50.0	1.974	9.0	50.4
5	1.444	7.2	57.2	1.676	7.6	58.0
6	1.294	6.5	63.6	1.314	6.0	64.0
7	1.108	5.5	69.2	1.276	5.8	69.8
8	1.071	5.4	74.5	1.045	4.7	74.5
9	0.918	4.6	79.1	0.980	4.5	79.0
10	0.863	4.3	83.4	0.855	3.9	82.9
11	0.753	3.8	87.2	0.716	3.3	86.1
12	0.704	3.5	90.7	0.673	3.1	89.2
13	0.546	2.7	93.5	0.613	2.8	92.0
14	0.406	2.0	95.5	0.453	2.1	94.0

factor analysis was applied to the sets of time-distance information collected at each of the two entrance ramps.[12] To avoid problems due to unequal sample sizes, separate analyses were performed for the samples of direct acceptance merges and for merges involving one or more rejections. Twenty-one pertinent variables were identified for analysis in the case of the direct acceptance samples, and 23 for the equivalent sets of rejection observations. The variables selected are listed in Table 4. Total samples of 224 merges (67 rejections, 157 direct acceptances), and 169 merges (57 rejections, 112 direct acceptances) were analyzed at the right- and left-hand ramps respectively.

Tables 5 through 8 show the matrices of simple-correlation-coefficients derived for each of the four basic analysis groups—right-hand ramp, direct acceptances; right-hand ramp, rejections; left-hand ramp, direct acceptances and left-hand ramp, rejections. Based on these correlation matrices, four rotated factor analyses were performed, yielding four reduced sets of orthogonal variates or "factors." For the sake of brevity only the results of the left-hand ramp analyses will be discussed here.

Table 9 indicates the amount of variance accounted for in each of the two left-hand ramp analyses by the first eight and the first fourteen orthogonal factors respectively. Note that in each case the first eight factors explain approximately three-quarters of the total variance. Hence, one may realistically consider replacing the original sets of 21 and 23 variables by revised, orthogonal sets of 8 variates.

[12] A detailed discussion of factor analysis would be out of place in this paper. For such a discussion the reader is referred to: M. G. Kendall, A Course in Multivariate Statistics, Griffin's Statistical Monograph Series, London, 1957; and W. E. Cooley and P. R. Lohnes, Multivariate Procedures for the Behavioral Sciences, Wiley, New York, 1962. Essentially, factor analysis permits one to identify collinearities within a multidimensional data set, and then to reduce the basic dimensions of the set by defining a new set of orthogonal or independent factors, based upon the original set of variables but fewer in number. These factors are chosen so that each in turn explains a maximal amount of the total residual variance. The character of the new set of orthogonal factors—that is, the parameters represented by each factor—may be evaluated by examining the loading of each new factor on the original set of variables. The process of rotation merely improves the efficiency of this interpretation; it does not affect the basic factor structure.

TABLE 10

ROTATED FACTOR LOADINGS—DIRECT ACCEPTANCES

SINGLE MERGES WITH DIRECT ACCEPTED GAPS AT HARLEM AVENUE

ROTATED FACTOR MATRIX

		FACTOR NUMBER	1	2		4	5	6	7	8
SUM SQUARES OVER VARIABLES			2.893	2.380	51	1.578	1.595	1.475	1.316	1.219
VARIABLE NO.	NAME	COMMUNALITY 8 FACTORS								
1	ICY	0.000	0.000	0.000	00	0.000	0.000	0.000	0.000	0.000
2	IC	0.627	0.095	0.047	65	-0.075	-0.205	0.251	0.659	0.017
3	ICZ	0.487	0.253	0.094	70	0.530	0.065	0.168	-0.254	-0.091
4	VS	0.740	-0.136	0.036	64	-0.601	0.455	0.149	0.042	-0.317
5	VR	0.653	0.140	0.090	77	0.055	-0.755	-0.078	-0.080	-0.091
6	GN	0.950	0.896	-0.028	61	-0.025	-0.084	-0.078	0.005	-0.053
7	HDN	0.916	0.149	-0.082	31	0.037	0.071	-0.118	-0.015	-0.022
8	HGN	0.919	0.947	0.004	06	-0.044	-0.127	-0.038	0.013	-0.051
9	SN	0.692	-0.141	-0.682	31	0.183	0.210	-0.311	-0.130	0.124
10	SRDN	0.784	-0.048	0.879	20	0.022	0.025	-0.072	-0.036	-0.027
11	SRGN	0.799	-0.008	0.889	22	0.077	-0.018	-0.018	-0.026	-0.029
12	GN1	0.682	0.054	-0.130	99	-0.114	-0.129	-0.012	-0.087	0.784
13	GN2	0.552	-0.084	0.026	57	0.003	-0.708	0.039	0.089	0.173
14	GNL1	0.593	-0.010	0.342	45	0.476	0.061	-0.250	0.368	0.212
15	GNL2	0.682	-0.187	-0.138	66	0.742	-0.035	0.102	0.075	-0.237
16	GNL3	0.722	-0.127	0.050	16	-0.029	-0.173	0.299	-0.703	0.204
17	GX	0.941	0.641	0.086	53	0.104	0.139	0.259	0.237	0.252
18	HD_X	0.938	0.179	0.046	44	-0.012	-0.004	-0.019	0.044	-0.102
19	HGX	0.871	0.690	0.080	48	0.135	0.174	0.331	0.268	0.369
20	SX	0.690	-0.239	-0.390	18	0.223	0.288	-0.512	-0.069	0.283
21	DNX	0.670	0.006	0.026	03	-0.118	-0.112	-0.792	0.058	-0.036

TABLE 11

ROTATED FACTOR LOADINGS—MERGES WITH REJECTIONS

SINGLE MERGES WITH REJECTIONS AT HARLEM AVENUE

ROTATED FACTOR MATRIX

		FACTOR NUMBER	1	2	3	4	5	6	7	8
SUM SQUARES		OVER VARIABLES	2.570	2.909	2.383	2.311	1.945	1.737	1.312	1.228
VARIABLE NO.	NAME	COMMUNALITY 8 FACTORS								
1	ICY	0.246	-0.074	-0.004	0.114	0.375	-0.135	-0.009	0.134	0.225
2	IC	0.708	0.153	-0.024	0.499	-0.202	-0.067	0.487	-0.391	0.013
3	ICZ	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	VS	0.800	-0.044	0.060	-0.335	0.109	-0.783	0.153	-0.087	0.164
5	VR	0.553	0.248	-0.134	-0.078	-0.327	0.270	0.419	0.291	-0.166
6	GN	0.836	0.066	0.858	-0.069	0.074	0.184	0.201	-0.105	-0.004
7	SN	0.710	-0.669	-0.094	0.121	-0.310	-0.178	-0.324	-0.013	0.076
8	SRDN	0.815	0.875	0.069	0.077	-0.149	0.124	0.011	-0.011	-0.003
9	SRGN	0.865	0.827	0.150	0.247	-0.211	0.107	-0.069	0.193	-0.007
10	KRG	0.783	0.098	0.219	-0.115	0.813	-0.103	0.031	-0.198	0.022
11	RGN	0.946	0.088	0.031	0.928	0.109	0.028	0.244	0.030	0.057
12	RLDN	0.950	0.079	0.069	0.962	0.042	0.054	-0.082	0.048	0.016
13	RLGN	0.757	0.054	-0.066	0.136	0.195	-0.067	0.826	-0.014	0.073
14	GN1	0.629	0.119	0.032	-0.136	-0.042	0.759	0.065	0.044	0.105
15	GN2	0.678	-0.054	-0.044	-0.031	-0.094	-0.022	-0.059	-0.137	-0.801
16	GNL1	0.844	0.155	0.622	-0.130	-0.448	0.363	0.266	0.071	0.087
17	GNL2	0.762	0.053	-0.072	0.037	-0.089	0.086	-0.029	0.853	0.099
18	GNL3	0.656	-0.317	-0.122	-0.056	-0.160	0.138	-0.267	-0.406	0.507
19	GX	0.899	0.068	0.892	0.118	0.203	-0.040	-0.204	-0.006	-0.013
20	HDX	0.724	0.190	0.148	0.055	0.571	0.570	0.001	-0.054	0.099
21	HGX	0.883	0.021	0.886	0.108	0.060	-0.192	-0.212	0.008	-0.040
22	SX	0.573	-0.474	0.205	-0.118	-0.177	0.085	-0.250	0.189	-0.395
23	DNX	0.779	-0.373	-0.046	0.124	0.695	0.145	0.320	0.047	-0.113

TABLE 12

INTERPRETATION OF MAJOR FACTORS

Factor	Direct Acceptance Merges	Merge With Rejections
1	Accepted Lag/Gap	Speed Index
2	Speed Index	Accepted Lag/Gap
3	Accepted Lead/Gap	Rejected Lead/Gap
4	Mainstream Flow Condition—Availability of Alternate Gaps	Previously Rejected Gaps
5	Mainstream/Ramp Volume	Mainstream Flow Condition
6	Point of Merge	Rejected Lag
7	Ramp Vehicle Type/Alternate Gaps	Ramp Vehicle Type/Alternate Gaps
8	Mainstream Flow Condition	Mainstream Flow Condition

Tables 10 and 11 summarize the rotated factor loadings for the first eight orthogonal factors for each of the two data groups. The first row in each table indicates the percentage of the total variance accounted for by each individual rotated factor; the remaining rows indicate the individual factor loadings. Based upon these loadings certain tentative interpretations may be placed on the major factors.

In the case of the direct-acceptance analysis, factor 1 clearly represents a combined static and dynamic "gap/lag size" index, factor 2 a "speed" index and factor 3 a "gap/lead size" index. Factors 4, 5 and 8 each reflect the general flow conditions and availability of gaps during the merge. Factor 6 loads heavily on the point of merger and factor 7 represents an interaction between the type of merging vehicle and the availability of mainstream gaps

less clear. In this case factors 1 and 2 are reversed; the former now represents a speed index, the latter a combined accepted gap/accepted lag measure. Factors 3, 4 and 6 all reflect the merging vehicle's rejection behavior; factor 3 is loaded heavily by rejected gap/rejected lead values, factor 4 by the number of previously rejected gaps and factor 6 by rejected gap/rejected lag values. Finally, factors 5, 7 and 8 all reflect the mainstream flow conditions pertaining during the merge and also, in the case of factor 7, the influence of ramp vehicle type. These factor interpretations are summarized in Table 12.

A note of caution is appropriate here. The factors listed in Table 12 do not necessarily represent the major "traffic factors" influencing the merging process; they are merely statistical aggregations of the original set of variables which were themselves postulated to bear some relation to gap acceptance behavior. The process of factor analysis simply identifies the major correlations within the sets of variable observations and places them in groupings which are statistically independent of each other. The results of the analysis are obviously very much dependent on the selection of the original sets of variables.

The degree of collinearity within the data is not particularly high—eight independent factors are required to explain 75 percent of the total variance. Furthermore, much of the collinearity is somewhat artificial, resulting from dual observations of essentially the same parameter at slightly different points in time. If these artificial correlations are ignored it becomes apparent that there is but little interdependence within the data, the values of accepted gap size or gap structure, merging speed, number of rejected gaps, gaps, mainstream flow condition and vehicle type as evidenced by this sample data set being virtually independent. In particular, it should be noted that although accepted gap, lag and lead values load heavily on several factors, they do not load significantly on those factors reflecting merging speeds, mainstream flow conditions or previous rejection behavior. The reasons for this are perhaps not to hard to discern. As noted

in a previous section the data are derived primarily from an analysis of "noncritical" merging conditions, away from the critical acceptance/rejection envelope. It is to be expected therefore that merging speeds and accepted gap size will not be subject to high intercorrelations. The possibly surprising fact is that the inter-correlations are so low as to suggest almost total independence, implying that an original variable itself may in many cases serve as an efficient proxy for an orthogonal factor.

Finally, undue importance should not be placed on the relative ordering of the factors and on the magnitudes of the eigenvalues. These reflect more than anything else the inbuilt bias referred to previously resulting from overmeasurement of single parameters. Redefinition of the original variable set would quite probably alter the ordering of the factors considerably.

TOWARD AN EMPIRICAL MODEL OF GAP ACCEPTANCE

A number of alternative techniques suggest themselves as appropriate to the formulation of an empirical model of gap acceptance behavior. Hurst et al, for example, in a report referred to previously (32), have suggested the use of relatively complex "gap-size" indexes, incorporating estimates of relative merging speed, spatial headways and desirable safe headway in addition to the standard time-gap measurements; Haight et al (25) have developed a simple, minimum safe-headway model based on the principles of elementary dynamics; Worrall (60) and Vardon (55) have both formulated empirical descriptions of acceptance/rejection behavior as a function of relative merging speed. In this final section of the paper attention is given to a slightly different type of model, in which a linear function is developed to estimate the probability of a gap of a given "character" being accepted or rejected. Two specific formulations are discussed, one based on a simple multiple regression analysis with dummy variables, the other on multivariate discriminant analysis.

Multiple Regression Analysis with Dummy Variables

A regression function is postulated of the form:

$$P(A) = A_1X_1 + A_2X_2 \ldots A_iX_i + \ldots + A_mX_m \qquad (1)$$

where

$P(A)$ = probability of a driver accepting a gap $\begin{cases} P(A) = 1 \text{ for an acceptance} \\ P(A) = 0 \text{ for a rejection} \end{cases}$

$X_1, X_2, \ldots, X_m$ = gap characteristics (e.g., available lag size, relative merging speed, ramp vehicle type)

$A_1, A_2, \ldots, A_m$ = Regression coefficients

The function differs from a normal regression equation in that the gap characteristics X_i are expressed as "dummy" variables rather than as continuous variables, i.e., each X_i is a vector of dichotomous variables $(x_{i1}, \ldots, x_{ij}, \ldots, x_{in})$, with each element x_{ij} taking on a value of either 1 or 0 depending on the observed value of X_i, and each A_i is a corresponding vector of constants $(a_{i1}, \ldots, a_{ij}, \ldots, a_{in})$. In expanded form, therefore, Eq. 1 may be rewritten:

$$P(A) = (a_{11}x_{11} + a_{12}x_{12} + \ldots + a_{1k}) + (a_{21}x_{21} + a_{22}x_{22} + \ldots + a_{2e}x_{2e}) + \ldots + (a_{m1}x_{m1} + a_{m2}x_{m2} + \ldots + a_{mn}x_{mn}) \qquad (2)$$

Consider an empirical example where X_1 = static lag, X_2 = relative merging speed and X_3 = ramp vehicle type, with the ranges of the dichotomous variables X_1, X_2 and X_3 defined (for illustrative purposes) as:

$x_{11} \rightarrow$ 0-0.99 seconds; $x_{21} \rightarrow$ -10 to -5 mph; $x_{31} \rightarrow$ Passenger Car
$x_{12} \rightarrow$ 1.00-1.99 seconds; $x_{22} \rightarrow$ -5 to 0 mph;
$x_{13} \rightarrow$ 2.00-2.99 seconds; $x_{23} \rightarrow$ 0 to +5 mph;
$x_{24} \rightarrow$ +5 to +10 mph;

A situation in which a passenger car accepts a static lag of 2.5 seconds at a relative merging speed of +2 mph yields an empirical "dummy" observation of:

$$1 = (a_{11} \cdot 0 + a_{12} \cdot 0 + a_{13} \cdot 1) + (a_{21} \cdot 0 + a_{22} \cdot 0 + a_{23} \cdot 1 + a_{24} \cdot 0)$$
$$= a_{13} \cdot 1 + a_{23} \cdot 1 + a_{31} \cdot 1$$

Similarly, a rejection by a truck of a lag of 0.2 seconds at a relative merging speed of -9 mph yields an observation:

$$0 = a_{11} \cdot 1 + a_{21} \cdot 1 + a_{31} \cdot 0$$

Inserting a set of such dummy observations as input to a multiple regression analysis yields estimates $A_1^*, A_2^*, \ldots, A_m^*$ for the vectors $A_1, A_2, \ldots, A_m$, the individual elements $a_{11}, a_{12}, \ldots; a_{21}, a_{22}$, etc. of which may be interpreted as the partial conditional probabilities of gap acceptance, given the occurrence of a gap having the particular set of characteristics, $x_{11}, x_{12}, \ldots; x_{21}, x_{22}, \ldots$, etc.[13] For example, in the illustration above, the car driver faced with a lag of 2.5 seconds at a relative speed of +2 mph would have, based on this analysis, an average probability of accepting the gap equal to $(a_{13}^* + a_{23}^* + a_{31}^*)$, where a_{13}^*, a_{23}^* and a_{31}^* are the estimates for the elements a_{13}, a_{23} and a_{31} of the vectors A_1^*, A_2^* and A_3^* as determined from the regression equation.

The value of $P(A)^* = (a_{1j}^* + a_{2k}^* + a_{3e}^*)$ may be constrained within the range $0 \leq P(A)^* \leq 1.0$ either directly by modifying the normal regression equations or by means of an arbitrary correction due to Orcutt.[13] The simple linear form of the acceptance

achieved by using as input the set of factors derived from an orthogonal factor analysis. Of necessity the application of this method requires a relatively large and uniform data set. The small sample of empirical data available for this study precludes inclusion of a worthwhile empirical example.

Discriminant Analysis

Consider a multivariate data set consisting of m observations on n variables, with the set of observations classified into two partially overlapping groups (e.g., accepted and rejected mainstream gaps). A linear discriminant analysis then defines, first, that linear combination of the original variables which discriminates "best" (in the sense of minimizing the ambiguity of group membership) between the two observation groups and, second, provides an estimate of the probability that a given observation most properly belongs in group 1 or group 2. In the case of a gap acceptance analysis, these latter values may be interpreted directly as estimates of the probabilities that a gap having a given set of characteristics will be accepted or rejected.[14]

As in the case of the dummy variable regression approach, the analysis presumes a simple linear statistical model—in this case the linear discriminant function

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[13] A detailed exposition of the dummy variable technique is given in: Johnston, J., Econometric Methods, McGraw-Hill, New York, 1960. In essence the values a_{ij}^* correspond to estimates of cell means in an analysis of variance. The problem of heteroscedasticity arising from the dichotomous nature of the dependent variable may be at least partially corrected for by a technique due to Goldberger.

[14] The discussion and application of discriminant analysis given here is based on: Cooley and Lohnes, op. cit.

$$Z = K_1x_1 + K_2x_2 + \ldots + K_ix_i + \ldots + k_mx_m$$

where x_1, x_2, x_i, x_m are the original characteristic variables expressed in standardized form (not, in this case, as dummy variables) and K_1, K_2, ... , K_i, ... , K_m are the discriminant function coefficients. Validity of the linear form may again be approximated by using as input to the discriminant analysis standardized, orthogonal factor-scores derived from a factor analysis. The probabilities of group inclusion are then computed by evaluating the multivariate distributions of group observations as defined in the discriminant space (see Cooley and Lohnes, Chapter 7).

In contrast to the dummy variable regression technique, it is possible in this case to give a simple illustration of the analysis method based on the sample of data available for this study. It was noted earlier that relatively little collinearity existed within the original factor-analysis data set. In particular, the following 9 characteristic variables were seen to be virtually independent of one another:

Variables*	Variable Name
X_1	Ramp Vehicle Type
X_2	Ramp Vehicle Speed
X_3	Relative Speed Ramp--Leading Mainstream Vehicle
X_4	Relative Speed Ramp--Lagging Mainstream Vehicle
X_5	Static Lead Time at Nose
X_6	Static Lag Time at Nose
X_7	Size of First Succeeding Mainstream Gap at Nose
X_8	Size of Second Succeeding Mainstream Gap at Nose
X_9	Size of Third Succeeding Mainstream Gap at Nose

These variables (based on groups of 112 acceptances and 57 rejections at the left-hand ramp) were used as input into a linear discriminant analysis, yielding the linear discriminant function (LDF)

$$Z = 0.479\ X_2^* + 0.947\ X_6^*$$

where X_2^*, X_6^* are standardized values of X_2, X_6, and a level of significance of 0.05 defines the limit for variable inclusion in the LDF.

Only the values of static lag time and ramp vehicle speed contribute significantly to the power of the discriminant function in distinguishing between the acceptance and rejection groups; the remaining seven variables yield no significant increase in the discriminatory power of the function. The relative contributions of variables X_2 and X_6 to the discriminant function may be assessed by comparing their corresponding scale-vector values: $X_2' = 6.114$, $X_6' = 10.879$ (i.e., lag time is roughly twice as important, in this case, as merging speed in distinguishing between the gap acceptance and rejection groups).

Calculation of the group inclusion probabilities indicated that considerable overlap existed between the two data sets, with the rejection group lying almost entirely within the limits of the acceptance group. In particular, 30 of the 112 acceptance observations and 7 of the 57 rejection observations had associated with them probabilities which indicated that the values of their characteristic vectors (i.e., lag size, speed, etc.) were associated more closely with the complementary group. The two group centroids had associated with them complementary membership probabilities of 0.0140 (acceptances) and 0.2807 (rejections) (i.e., the centroid of the group of rejection observations, on the basis of the discriminant analysis, had a probability of 0.3696 of being associated not with a gap rejection but rather with an acceptance). This suggests either that the original 9 characteristic variables provide only an inadequate description of acceptance/rejection behavior or else that the dispersion matrices for the two observation groups are too widely divergent. The results are sufficiently intriguing, however, to warrant further extension of the simple illustrative analysis described here.

CONCLUSIONS

This paper has touched briefly on a number of elementary points relating to the analysis of gap acceptance behavior at freeway entrance ramps. In many cases the analyses have been little more than exploratory illustrations of what might be done with a larger data sample. It has been shown, however, that the development of a more detailed empirical description of the gap acceptance/rejection process is both feasible and also desirable. The ability to distinguish between those factors which exert a significant influence on merging behavior and which in some sense are susceptible to external control has considerable implications both for the design and operation of future freeway control systems and also for the formulation of future theoretical models. An initial specification of these factors represents the major contribution of this paper.

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Appendix A

BIBLIOGRAPHY

1. Athol, P. Interdependence of Certain Operational Characteristics Within a Moving
2. Athol, P. Headway Groupings. Highway Research Record 72, p. 137, 1965.
3. Baker, J. L. Determination of Discontinuities in Traffic Flow as a Factor in Freeway Operational Control. Traffic Engineering, Nov. 1961.
4. Bisbee, E. F., and Conan, M. High Density Merging. Paper presented at the 43rd Annual Meeting, HRB, Washington, D. C., 1964.
5. Brand, D. Freeway On-Ramp Simulation. SM thesis, M. I. T., June 1961.
6. Buhr, J. H. The Distribution and Acceptance of Gaps on Free Flowing Facilities. Research Report (unpublished), Northwestern Univ., 1963.
7. Bullen, A. G. R. Freeway Lane Distribution. Research Report (unpublished), Northwestern Univ., 1963.
8. Carlson, R. J. Effectiveness of Ramp Closure During Peak Hours. Proc. 13th Annual Western Section Meeting, ITE, July 1960.
9. Castillo. Simulation of Weaving Traffic on a Digital Computer. SM thesis, M.I.T., January 1960.
10. Coutts, D. W. A Study of Merging Behavior at Freeway Entrance Ramps. MS thesis (unpublished), Northwestern Univ., 1965.
11. Covault, D., and Roberts, R. Influence of On-Ramp Spacing on Traffic Flow on Atlanta Freeway and Arterial Street System. Highway Research Record 21, p. 32, 1963.
12. Dawson, R. F. Analysis of On-Ramp Capacities by Monte Carlo Simulation and Queuing Theory. PhD thesis (unpublished), Purdue Univ., 1964.
13. Dean, T. B. Acceleration Lane Lengths for Heavy Commercial Vehicles. Traffic Engineering, Vol. 27, Feb. 1957.
14. Dorfwirth, J. R. Design of Interchanges on Rural Freeways. Proc. First World Traffic Engineering Conf., ITE, Washington, 1961.
15. Drake, J. Lane Distribution of Ramp Vehicles Upstream and Downstream of Left-Hand Entrance and Exit Ramps. MS thesis (unpublished), Northwestern Univ., 1964.

16. Drew, D. R. Gap Acceptance Characteristics for Ramp-Freeway Surveillance and Control. Highway Research Record 157, p. 108, 1966.
17. Echterhoff-Hammerschmid, H. A Study of Gap Acceptance at Right-Hand and Left-Hand Ramps. MS thesis (unpublished), Northwestern Univ., 1965.
18. Foyster, M. J. Design of Acceleration Lanes at Entrance Ramps—Some Theoretical Considerations. Research Report (unpublished), Northwestern Univ., 1964.
19. Foyster, M. J. The Design of Freeway Entrance Ramps—Some Theoretical Considerations. MS thesis (unpublished), Northwestern Univ., 1964.
20. Fukotome, I., and Moskowitz, K. Traffic Behavior and On-Ramp Design. HRB Bull. 235, p. 38, 1960.
21. Gerlough, D. L. Simulation of Freeway Traffic by an Electronic Computer. HRB Proc., Vol. 35, p. 543, 1956.
22. Gervais, E. F. Optimization of Freeway Traffic by Ramp Control. Highway Research Record 59, p. 104, 1964.
23. Glickstein, A., Findley, L. D., and Levy, S. L. Application of Computer Simulation Techniques to Interchange Design Problems. HRB Bull. 291, p. 139, 1961.
24. Haight, F. A. Mathematical Theories of Traffic Flow. Academic Press, New York, 1963.
25. Haight, F. A., Bisbee, E. F., and Wojcik, C. Some Mathematical Aspects of the Problem of Merging. HRB Bull. 356, p. 1, 1962.
26. Heller, F., and Blaschke, W. Design of Interchanges on Rural Freeways. Proc. First World Traffic Engineering Conf., ITE, Washington, 1961.
27. Herman, R., and Weiss, H. Comments on Highway Crossing Problem. Operations Research, Vol. 9, No. 6, 1961.
28. Hess, J. W. Capacities and Characteristics of Ramp-Freeway Connections. Highway Research Record 27, p. 69, 1963.
29. Hess, J. W. Ramp-Freeway Terminal Operation as Related to Freeway Lane Volume Distribution and Adjacent Ramp Influence. Highway Research Record 99, p. 81, 1965.
30. Highway Research Board Committee on Highway Capacity. Highway Capacity Manual. U. S. Bureau of Public Roads, 1950.
31. Ho, Er-Chun. A Statistical Analysis of Congested Merging Traffic. Research Report 20, ITTE, Univ. of California, Los Angeles, 1961.
32. Hurst, P., Perchonok, P., and Seguin, E. Measurement of Subjective Gap Size. Report 7, Institute for Research, State College, Pennsylvania, 1965.
33. Jewell, W. S. Multiple Entries in Traffic. Jour. Soc. Ind. and Appl. Math., Vol. 2, 1963.
34. Jewell, W. S. Forced Merging in Traffic. Operations Research, Vol. 8, No. 10, 1964.
35. Jewell, W. S. Markov-Renewal Models in Traffic Flow Theory. Paper presented at Third Internat. Symp. on Theory of Traffic Flow, New York, 1965.
36. Jouzy, N. C., and Michael, H. L. Use and Design of Acceleration and Deceleration Lanes in Indiana. Highway Research Record 9, p. 25, 1963.
37. Keese, C. J., and Drew, D. R. Freeway Level of Service as Influenced by Volume and Capacity Characteristics. Highway Research Record 99, p. 1, 1965.
38. Knox, D. W. Merging and Weaving Operations in Traffic. Australian Road Research, Vol. 2, No. 2, Dec. 1964.
39. Lessieu, E. J. Operational Characteristics of High Volume On-Ramps. Traffic Engineering, Vol. 28, No. 3, Dec. 1964.
40. Levy, S. L., Carter, M., and Scheu, J. E. Investigation of the Effects of Spacing Between On and Off-Ramps. Final Report, Midwest Research Inst., March 1963.
41. Levy, S. L., and Perchonok, P. A. Application of Digital Simulation Techniques to Freeway On-Ramp Traffic Operations. HRB Proc., Vol. 39, p. 506, 1960.
42. Lewis, R. A Dynamic Programming Approach to Ramp Metering and Simulation of a Freeway System Using GPSS. Research Report (unpublished), Northwestern Univ., 1964.

43. May, A. D. Experimentation with Manual and Automatic Ramp Control. Highway Research Record 59, p. 9, 1964.
44. May A. D. Gap Availability Studies. Highway Research Record 72, p. 101, 1965.
45. Michaels, R. M. Effect of Expressway Design on Driver Tension Responses. HRB Bull. 330, p. 16, 1962.
46. Moskowitz, K., and Newman, L. Notes on Freeway Capacity. Highway Research Record 27, p. 44, 1963.
47. Norman, O. K. Operation of Weaving Areas. HRB Bull. 167, p. 38, 1957.
48. Oliver, R. M., and Bisbee, E. F. Queuing for Gaps in High Flow Traffic Streams. Operations Research, Vol. 10, No. 1, 1962.
49. Pearson, R. H., and Ferreri, M. G. Operational Study—Schuylkill Expressway. HRB Bull. 291, p. 104, 1961.
50. Perchonok, P. A., and Levy, S. L. Application of Digital Simulation Techniques to Freeway On-Ramp Traffic Operations. HRB Proc., Vol. 39, p. 506, 1960.
51. Pinnell, C. Driver Requirements in Freeway Entrance Ramp Design. Traffic Engineering, Vol. 31, No. 3, Dec. 1960.
52. Quinby, Warren S. Behavior Patterns for Merging Traffic. Student thesis, Yale Univ., Bureau of Highway Traffic, 1949.
53. Ruiter, E. R. Simulation of a Freeway On Ramp. Research Report (unpublished), Northwestern Univ., 1962.
54. Solberg, P., and Oppenlander, J. C. Lag and Gap Acceptances at Stop-Controlled Intersections. Highway Research Record 118, p. 48, 1966.
55. Vardon, J. L. Some Factors Affecting Merging Traffic at the Outer Ramp of Highway Interchanges. MS thesis (unpublished), Queen's Univ., Kingston, Ontario, 1959.
56. Wattleworth, J. A. Peak Period Control of a Freeway System--Some Theoretical Considerations. PhD thesis (unpublished), Northwestern Univ., 1963.
57. Wattleworth, J. A. Comparison of Two Metering Schemes. Research Report, Chicago Area Surveillance Project, 1964.

Vol. 31, No. 11, 1960.
59. Worrall, R. D. A Study of the Operational Characteristics of Urban Motorway. Civil Engineering and Public Works Review, London, Feb. 1964.
60. Worrall, R. D. The Capacity of Freeway Entrance Ramps. Project Report (unpublished), Northwestern Univ., 1965.
61. Worrall, R. D., Drake, J. S., Buhr, J. H., Soltman, T. J., and Berry, D. S. Study of Operational Characteristics of Left-Hand Entrance and Exit Ramps on Urban Freeways. Highway Research Record 99, p. 244, 1965.
62. Studies of Weaving and Merging Traffic. Technical Report No. 4, Yale University, Bureau of Highway Traffic, 1948.

Appendix B

DEFINITIONS

1. Time-Distance Trace—The trajectory of a vehicle defined in time and space as it passes through the merge area of an entrance ramp.
2. Time-Gap—The time headway between successive vehicles, measured with respect to the front of each vehicle.
3. Lead-Time—The time headway between a merging vehicle and the preceding mainstream vehicle.
4. Lag-Time—The time headway between a merging vehicle and the succeeding mainstream vehicle.
5. Static Analysis—An analysis based entirely on time-gap, lead and lag measurements made at the entrance ramp nose.

6. Dynamic Analysis—An analysis based on measurements of gap, lead and lag size made at varying points within the merge area, and defined in terms of the motion of the merging vehicle.
7. "X" Point—The commencement of the physical merging maneuver, the point at which a ramp vehicle first encroaches onto the mainstream lanes.
8. Dynamic Gap—The time headway between a ramp vehicle and the succeeding mainstream vehicle measured at the instant the ramp vehicle is overtaken by the immediately preceding mainstream vehicle.
9. Isolated Merge—A merge by a ramp vehicle separated by at least 10 seconds from the nearest preceding or succeeding ramp vehicle.
10. Multiple Merge—A merge by a platoon of two or more vehicles into a single mainstream gap.
11. Single Merge—A merge by either an isolated ramp vehicle or by a lead vehicle in a multiple merge; includes all instances in which one vehicle merges into a single mainstream gap plus all lead vehicles in multiple merges.
12. Modified Multiple Merge—A measure of the effective gap size for trailing vehicles in a multiple merge, defined as either the time headway between the preceding ramp vehicle in a platoon and the immediately succeeding mainstream vehicle or the time headway between the preceding and succeeding mainstream vehicles, whichever is the lesser.
13. "O" Point—The termination of the physical merging maneuver, the point at which the ramp vehicle finally enters completely into the mainstream lanes.
14. Critical Acceptance Value—The median or 50 percent response value derived from a probit or Raff analysis; that size of gap, lead or lag which is accepted 50 percent of the time.
15. Forced Merge—A merge in which the ramp vehicle forces its way into a small mainstream gap, causing one or more of the mainstream vehicles to change its speed or direction of travel.
16. Gap Stability—The variation of mainstream gap size over the length of the merge area.