# The Analysis of Land-Use Linkages 

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- EXAMINATION of urban travel activity patterns suggests that person trip sets may be considered to be analogous to a closed circuit movement; that is, the tripmaker generally leaves his home base, makes one or more stops, and returns to the home base. The complexity of the system in which these movements are negotiated makes it extremely difficult to deal explicitly with all relevant or potentially relevant variables. However, this very complexity suggests the use of relatively simple probabilistic models which allow the analyst to vary the components of the system without recourse to an ultra-sophisticated theoretical framework and without being handicapped by concern with a multitude of parameter changes. The Markov chain model is one such simple tool.

Let us assume that each group of land uses within the spatial structure of urban land is a member of a finite collection of states which a tripmaker may choose as a trip end. Assume further that the movement of a tripmaker is part of a process such that if he is in a given state, $i$, there is some probability, $p_{i j}$, that he will move to another given state, $j$, in any given time period, $t$. Under these assumptions, there is a simple probabilistic model which may be used to describe and analyze such a situation. This simple, time-dependent probability model is known as a finite Markov chain.

## A SIMPLE MARKOV CHAIN MODEL

As an example ${ }^{1}$ of the use of simple Markov chain models in the analysis of travel characteristics, consider the closed system of three land-use parcels shown in Figure 1 , and a tripmaker who may be on any of the three parcels. The rule of the model is that in each time period the tripmaker must leave the particular parcel on which he is located and must either travel to one of the other parcels or return to the original parcel. Using the length of the trip as a criterion (although it would be possible to use any meaningful criterion) and the parcel arrangement shown in Figure 1, we would assume that a tripmaker located on parcel 1 would be most likely to return there, may go to parcel 2, and would be least likely to go to parcel 3. If we assign probabilities to each move, their sum will be unity because the tripmaker must move in some way. The probabilities of ending a trip on each land-use parcel from each of the three possible starting positions may be expressed in a 3 by 3 matrix in which the rows designate land-use parcels of destination, and each row sums to one. A matrix of this type is known as a transition matrix; each of the land-use parcels represents a Markov "state;" and each of the elements of the matrix is known as transition probabilities. A transition matrix for the system outlined above could take the following form:

|  | 1 | 2 | 3 | $\Sigma$ |
| :--- | :--- | :--- | :--- | :--- |
| 1 | .5 | .4 | .1 | 1.0 |
| 2 | .3 | .5 | .2 | 1.0 |
| 3 | .1 | .3 | .6 | 1.0 |

[^0]

Figure 1. Closed land-use system.

Given the transition probabilities, it must also be known in what state our tripmaker will be at the beginning of the process. This is done by establishing a row or vector in which each element of the row represents the probability of the tripmaker beginning on a given land-use parcel. For example, if we know the tripmaker always begins on parcel 1 in time (0), the initial vector would take the form $(1,0,0)$. If, on the other hand, there is an equal probability that the tripmaker will begin on any of the parcels, the initial vector will appear as $(1 / 3,1 / 3,1 / 3)$. This row of values is known as a probability vector. The sum of the elements of a probability vector is always equal to one.

Let us assume that there is an equal probability that the tripmaker will start at any one of the three land-use parcels. Multiplying the initial probability vector by the transition matrix will yield a probability vector whose elements define the probability of the tripmaker being in any of the three states or land-use parcels. Thus:

$$
(1 / 3,1 / 3,1 / 3) \quad\left[\begin{array}{lll}
.5 & .4 & .1 \\
.3 & .5 & .2 \\
.1 & .3 & .6
\end{array}\right]=(.30, .40, .30)
$$

The probability vector (. $30, .40, .30$ ) becomes an intermediate probability vector establishing the probabilities that the tripmaker is on any of the land-use parcels at the end of time period (1). Furthermore, since we assume the transition probabilities remain constant through time, the probability that the tripmaker will be on land-use X at the end of the second time period may be established by multiplying the new probability vector by the matrix of transition probabilities. The relationship may be expressed by the following equation:

$$
\underset{1 \times \mathrm{n}}{\mathrm{R}^{(\mathrm{t}+1)}}=\mathrm{R}^{\mathrm{R}^{(\mathrm{t})}} \times \underset{\mathrm{n}}{1 \times \mathrm{n}} \quad \underset{\mathrm{n}}{\mathrm{P}}
$$

where
$R=$ the probability row vector with $n_{j}$ as any element in the vector, $\mathrm{j}=(1 . . . \mathrm{n})$,
$P=$ the transition matrix with $\mathrm{a}_{\mathrm{ij}}$ as any element in the matrix,
$\mathrm{i}=(1 . . \mathrm{n})$, and
$\mathrm{t}=$ the time period designation, $\mathrm{t}=(1 \ldots \mathrm{~m})$, and t is not an exponent.
Given these assumptions, there are many questions about a particular process that the Markov chain model can answer. For example, the model can establish the probability that a tripmaker will be on each land-use parcel in any given time period, t. A Markovian approach can also establish whether or not there exists an equilibrium or balance such that the probability of being on each land-use parcel remains constant after a particular time period, $t$. Stated in another way, this means that it is possible to establish a finite number of time periods in which the tripmaker will move from any Markov "state" to all other Markov "states." This situation exists for every "regular" transition matrix. A "regular" transition matrix is such that there exists some exponent, g, such that Pg has no zero elements. The transition matrix used in our example is a "regular" transition matrix since the matrix $P$ to the first power contains no zero elements. With the knowledge that there exists an equilibrium situation, it is possible to establish the following set of equations:

$$
\begin{equation*}
r_{1}=.5 r_{1}+.3 r_{2}+.1 r_{3} \tag{1}
\end{equation*}
$$

$$
\begin{align*}
& r_{2}=.4 r_{1}+.5 r_{2}+.3 r_{3}  \tag{2}\\
& r_{3}=.1 r_{1}+.2 r_{2}+.6 r_{3} \tag{3}
\end{align*}
$$

Because Eqs. 1, 2 and 3 are not independent equations, another equation must be introduced in order to solve for $r_{1}, r_{2}$, and $r_{3}$ :

$$
\begin{equation*}
1=r_{1}+r_{2}+r_{3} \tag{4}
\end{equation*}
$$

Any two of Eqs. 1, 2, or 3 and Eq. 4 will constitute a solvable set of equations. The equilibrium vector resulting from our example is (14/46, 19/46, 13/46).

Another interesting problem which may be solved within a Markovian framework is the derivation of "mean first passage time." The mean first passage time is the number of time periods it takes the tripmaker to return to the "state" or land-use parcel from which he started. Thus, it is possible to derive the average number of stops on multipurpose trips.

All of the problems discussed above are capable of being solved through the use of computer programs currently available (2, 3). A more detailed discussion of Markov chain analysis is given by Kemeny and Snell (4).

## APPLICATION OF THE MODEL TO THE ANALYSIS OF TRAVEL BEHAVIOR

The finite Markov chain model described above may be adapted to the analysis of travel behavior through the derivation of the following tables, or matrices.

## The F Matrix

A matrix of trip origins and destinations is developed from standard O-D data. In this instance, let us assume that this matrix consists of the number of trips, $\mathrm{f}_{\mathrm{ij}}$, which start at a given land use, $i$, and end at a given land use, $j$. An example of such a matrix is shown below.


## The P Matrix

The proportion of trips, $P_{i j}$, which go from any origin, $\mathbf{i}$, to each destination, $j$, can be readily computed by dividing the number of trips from $i$ to $j$ by the total number of trips originating at land-use i. Thus:

$$
P_{i j}=\frac{f_{i j}}{\sum_{j=1}^{n} f_{i j}}
$$

In the context of the Markov chain model, $\mathrm{P}_{\mathrm{ij}}$ is defined as a maximum likelihood estimator of the probability of a tripmaker from state $i$ (in this example, land-use $i$ ) moving to state j (here, land-use j). The $P$ matrix would look just like the $F$ matrix, except that the relative frequency (i.e., the probability) of trips moving from $i$ to $j$ would be shown, rather than the absolute frequency of such movements. The sum of the $P_{i j}{ }^{\prime} s$ for any originating land use must equal 1.0 ; that is,

$$
\sum_{j=1}^{n} P_{i j}=1.0
$$

The A Matrix
If all elements of the $P$ matrix are greater than zero, i.e., if

$$
P_{i j}>0 \text { for all } \mathrm{i}, \mathrm{j}
$$

a limiting matrix, $A$, can be derived through repeated multiplication of $P$ by itself. Symbolically,

$$
\mathrm{A}=\mathrm{P}^{\mathrm{n}}
$$

As a result of repeated multiplication and as a direct consequence of the structure of the $P$ matrix, each row of $A$ is identical. One interpretation of the element, $a_{j}$, of any row is that $a_{j}$ is equal to the expected percentage of tripmakers which will be found at the j th land use at some random time during the day.

## The S Matrix

The regular Markov chain can be restructured so that one or more of the diagonal elements, $P_{i j}$, of the $P$ matrix is made equal to one. Such a Markov chain is called an absorbing chain, since once an entry is made in the $\mathrm{k}, \mathrm{j}$ cell for which $\mathrm{P}_{\mathrm{ij}}=1$, no exit is permitted. By operating upon this modified P matrix another matrix, S , can be developed; the elements, $\mathrm{u}_{i j}$, may be interpreted as the mean number of times a tripmaker will be in a nonabsorbing land use, $u_{j}$, given that it starts at a nonabsorbing land use, $u_{i}$.

## The V Matrix

The A matrix discussed previously can be operated upon to yield a matrix, V , in which each of the elements, $v_{i j}$, represents the expected variation in the expected number of stops, $\mathrm{v}_{\mathrm{ij}}$, at each land use.

## The V Vectors

Further manipulation of the $S$ matrix will yield the expected average number of stops and variances of trips with a particular land use designated as the first stop. Further operations on $S$ will also lead to the expected mean number of stops per trip set, and the variance for the system as a whole. All of the operations outlined can be carried out by using two programs developed by Marble (3).

TABLE 1
LAND USE AT ORIGIN RELATED TO LAND USE AT DESTINATION-WACO, 1964

| From To | 1. HOME | $\begin{gathered} 2 \\ \text { AGFOFI } \end{gathered}$ | $\begin{gathered} 3 . \\ \text { MFGOUR } \end{gathered}$ | 4. MFGNDU | $\begin{gathered} 5 \\ \text { TRCOOI } \end{gathered}$ | $6 .$ COMRET | $\begin{gathered} 7 \\ \text { comsve } \end{gathered}$ | $B .$ <br> WHOSAL | 9. PUBQPU | $\begin{aligned} & 10 . \\ & \text { PUBOPE } \end{aligned}$ | Sum |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. HOME | 0 | 50 | 359 | 486 | 307 | 3,729 | 1,547 | 250 | 5,511 | 283 | 12,432 |
| 2. AGFOFI | 56 | 4 | 0 | 2 | 0 | 15 | 5 | 0 | 9 | 0 | 91 |
| 3. MFGDUR | 338 | 0 | 9 | 10 | 3 | 55 | 20 | 4 | 18 | 2 | 459 |
| 4. MFGNDU | 436 | 0 | 10 | 12 | 2 | 82 | 33 | 7 | 25 | 2 | 609 |
| 5. TRCOOI | 286 | 0 | 6 | 4 | 30 | 71 | 25 | 2 | 34 | 4 | 462 |
| 6. COMRET | 4,174 | 11 | 44 | 53 | 52 | 1,148 | 285 | 51 | 333 | 52 | 6,203 |
| 7. COMSVC | 1,356 | 7 | 12 | 20 | 21 | 454 | 210 | 20 | 187 | 18 | 2,305 |
| 8. WHOSAL | 251 | 2 | 5 | 5 | 3 | 60 | 13 | 17 | 20 | 2 | 378 |
| 9. PUBQPU | 4,681 | 17 | 21 | 56 | 36 | 645 | 258 | 29 | 794 | 40 | 6,577 |
| 10. PUBOPE | 311 | 1 | 1 | 0 | 3 | 42 | 13 | 4 | 41 | 34 | 450 |

Note: Definitions of abbreviations used in Tables 1 through 6:

| 1. HOME = Home | 6. COMRET = Commercial, Retail |
| :--- | :--- |
| 2, AGFOFI = Agriculture, Forestry, Fishing | 7, COMSVC = Commercial, Service |
| 3. MFGDUR = Manufacturing, Durable | 8, WHOSAL $=$ Wholesale |
| 4, M FGNDU = Manufacturing, Nondurable | 9, PUBQPU = Public and Quasi-Public |

5. TRCOOI = Transportation, Communication, Other Industrial
6. PUBQPU $=$ Public and Quasi-Public
7. PUBOPE $=$ Public Open Space

TABLE 2
$10 \times 10$ LAND-uSE MATRIX OF TRANSItion PROBABILITIES-WACO, 1964

| From | 1. HOME | $\stackrel{2 .}{\text { AGFOFI }}$ | $3 .$ <br> MFGDUR | $4$ <br> MFGNDU | $\frac{5 .}{\text { TRCOOI }}$ | $6 .$ COMRET | $\stackrel{7}{\text { COMSVC }}$ | 8. WHOSAL | 9. PUBQPU | 10. PUBOPE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. HOME | . 00 | $.00^{\text {a }}$ | . 03 | . 04 | . 02 | . 30 | . 12 | . 02 | , 44 | , 02 |
| 2, AGFOFI | . 62 | . 04 | . 00 | , 02 | . 00 | , 16 | . 05 | . 00 | , 10 | , 00 |
| 3. MFGDUR | . 74 | . 00 | . 02 | . 02 | . 01 | , 12 | . 04 | . 01 | . 04 | . 00 |
| 4. MFGNDU | . 72 | . 00 | . 02 | . 02 | . $00{ }^{\text {a }}$ | , 13 | . 05 | . 01 | . 04 | $.00^{\text {a }}$ |
| 5. TRCOOI | . 62 | . 00 | . 01 | . 01 | . 06 | , 15 | . 05 | . $00{ }^{\text {a }}$ | . 07 | . 01 |
| 6. COMRET | . 67 | $.00{ }^{\text {a }}$ | . 01 | , 01 | . 01 | . 19 | , 05 | . 01 | , 05 | , 01 |
| 7. COMSVC | . 59 | $.00^{\text {a }}$ | . 01 | , 01 | , 01 | . 20 | . 09 | . 01 | , 08 | , 01 |
| 8. WHOSAL | . 66 | . 01 | . 01 | , 01 | -01 | . 16 | . 03 | . 04 | , 05 | . 01 |
| 9. PUBQPU | . 71 | . $00{ }^{\text {a }}$ | . $00{ }^{\text {a }}$ | . 01 | -01 | . 10 | . 04 | $.00{ }^{\text {a }}$ | , 12 | . 01 |
| 10. PUBOPE | . 69 | $.00^{\text {a }}$ | . $00{ }^{\text {a }}$ | . 00 | . 01 | . 09 | . 03 | . 01 | . 09 | . 08 |

${ }^{\mathrm{a}}$ Less than .005.
Note: See Table 1 for definition of abbreviations.

TABLE 3
ONE ROW OF THE LIMITING MATRLX A (PERCENTAGE OF THE GROUP THAT WILL BE FOUND IN A PARTICULAR STATE AT SOME RANDOM TIME DURING THE DAY) GENERATED FROM MATRIX P OF THE $10 \times 10$ LAND-USE MATRIX

| 1 |  |  |  |  |  | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| HOME | AGFOFI | MFGDUR | MFGNDU | TRCOOI |  |  |  |  |
| 40.6 | 0.3 | 1,5 | 2.1 | 1.5 |  |  |  |  |
| 6 | 7 | 8 | 9 | 10 |  |  |  |  |
| COMRET | COMSVC | WHOSAL | PUBQPU | PUBOPE |  |  |  |  |
| 20.8 | 7.8 | 1,3 | 22.8 | 1.4 |  |  |  |  |

Note: See Table 1 for definition of abbreviations.

TABLE 4
PREDICTED NUMBER OF STOPS ON WACO PERSON TRIPS

| Starting State | Mean | Variance |
| :---: | :---: | :---: |
| 2. AGFOFI | 1,6 | .8 |
| 3. MFGDUR | 1.4 | .6 |
| 4. MFGNDU | 1.4 | .7 |
| 5. TRCOOI | 1.6 | .8 |
| 6. COMRET | 1.5 | .7 |
| 7. COMSVC | 1,6 | .9 |
| 8. WHOSAL | 1,5 | .7 |
| 9. PUBQPU | 1.4 | .7 |
| 10. PUBOPE | 1.5 | .7 |
| SYSTEM | 1.5 | .7 |

## ANALYSIS AND SOME PRELIMINARY RESULTS

The models outlined were applied to land-use and trip data for the Waco, Texas, area. Four F matrices were developed: (a) a $10 \times 10$ trip purpose matrix; (b) a $10 \times 10$ major land-use matrix; (c) a $21 \times 21$ commercial landuse matrix; and (d) a $28 \times 28$ major and commercial land-use matrix. The results generated by the application of the two Markov models to these data are given in Tables 1 through 18. The tables corresponding to the matrices described in the previous section are as follows: F matrices-Tables 1, 7, and 13; P matrices-Tables 2, 8, and 14; 1 row of the A matrices-Tables 3,9 , and 15 ; the vectors, $V^{\prime}$-Tables 4,10 , and 18 ; S matrices-Tables 5, 11, and 16; and V matrices-Tables 6, 12, and 17. Results using the $28 \times 28$ land-use matrix are not illustrated in the tables.

TABLE 5
PREDICTED NUMBER OF STOPS BY LAND USE AT FIRST STOP TRIP-WACO, 1964

|  | 2. <br> AGFOFI | $\begin{gathered} 3 \\ \text { MFGDUR } \end{gathered}$ | $\stackrel{4 .}{\text { MFGNDU }}$ | $\stackrel{5}{\text { TRCOOI }}$ | $6 .$ <br> COMRET | 7. COMSVC | 6. WHOSAL | $\begin{gathered} \theta, \\ \text { PUBQPU } \end{gathered}$ | $\begin{aligned} & 10, \\ & \text { PUBOPE } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2. AGFOFI | 1.05 | $.00^{\text {a }}$ | . 03 | $.00^{\text {a }}$ | . 26 | . 08 | $.00^{\text {a }}$ | . 14 | $.00^{\text {a }}$ |
| 3. MFGDUR | $.00^{\text {a }}$ | 1.02 | . 03 | . 01 | . 18 | . 06 | . 01 | . 07 | . 01 |
| 4. MFGNDU | $.00^{\text {a }}$ | . 02 | 1.02 | . 01 | . 20 | . 08 | . 02 | . 07 | . 01 |
| 5. TRCOOI | $.00^{\text {a }}$ | . 02 | . 01 | 1.07 | . 25 | . 08 | . 01 | . 12 | . 01 |
| 6. COMRET | $.00^{\text {a }}$ | . 01 | . 01 | . 01 | 1.27 | . 07 | . 01 | . 09 | . 01 |
| 7. COMSVC | $.00^{\text {a }}$ | . 01 | . 01 | . 01 | . 30 | 1.12 | . 01 | . 13 | . 01 |
| 8. WHOSAL | . 01 | . 02 | . 02 | . 01 | . 24 | . 06 | 1.05 | . 09 | . 01 |
| 9. PUBQPU | $.00^{\text {a }}$ | . 01 | . 01 | . 01 | . 16 | . 06 | . 01 | 1.16 | . 01 |
| 10. PUBOPE | $.00^{\text {a }}$ | $.00^{\text {a }}$ | $.00^{\text {a }}$ | . 01 | . 16 | . 05 | . 01 | . 13 | 1.08 |

aLess than , 005 .
Note: See Table 1 for definition of abbreviations.

TABLE 6
VARIANCE IN NUMBER OF STOPS BY LAND USE AT FIRST STOP ON TRIP-WACO, 1964

aLess than 005.
Note: See Table 1 for definition of abbreviations,

TABLE 7
TRIP PURPOSE AT ORIGIN RELATED TO TRIP PURPOSE AT DESTINATION-WACO, 1964

| From |  | $\begin{gathered} 1 \\ \text { HOME } \end{gathered}$ | $\stackrel{2}{2}$ | $\begin{gathered} 3 \\ \text { PERBUS } \end{gathered}$ | MEDDEN | $\begin{gathered} 5 \\ \text { SCHOOL } \end{gathered}$ | $\stackrel{5}{\text { SOCREC }}$ | $\stackrel{7}{\text { CHMODE }}$ | $\stackrel{8}{\text { EATMEA }}$ | $\begin{gathered} 9 \\ \text { SHOP } \end{gathered}$ | $10$ <br> SERPAS | Sum |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | HOME | 00 | 3230 | 1231 | 181 | 2031 | 2444 | 23 | 294 | 1800 | 1953 | 13,187 |
| 2 | WORK | 2920 | 701 | 199 | 24 | 11 | 106 | 1 | 474 | 205 | 205 | 4,891 |
| 3 | PERBUS | 1113 | 145 | 430 | 12 | 16 | 153 | 2 | 70 | 299 | 89 | 2,330 |
| 4 | MEDDEN | 157 | 13 | 18 | 8 | 2 | 25 | 1 | 8 | 40 | 10 | 282 |
| 5 | SCHOOL | 1574 | 32 | 25 | 10 | 22 | 122 | 3 | 54 | 57 | 41 | 1, 940 |
| 6 | SOCREC | 2617 | 54 | 108 | 18 | 49 | 652 | 1 | 86 | 216 | 156 | 3,957 |
| 7 | CHMODE | 18 | 2 | 0 | 2 | 5 | 2 | 0 | 1 | 1 | 2 | 33 |
| 8 | EATMEA | 340 | 440 | 50 | 2 | 44 | 85 | 1 | 6 | 72 | 53 | 1,093 |
| 9 | SHOP | 2216 | 60 | 173 | 12 | 4 | 211 | 0 | 71 | 686 | 124 | 3, 557 |
| 10 | SERPAS | 1680 | 368 | 116 | 19 | 59 | 168 | , | 63 | 200 | 641 | 3,315 |

Note: Definitions of abbreviations used in Tables 7 through 12:

| 1 | HOME = Home | 6 |
| :--- | ---: | :--- |
| SOCREC = Social Recreation |  |  |
| 2 | WORK = Work | 7 |
| 3 CHMODE = Change Mode |  |  |
| PERBUS = Personal Business | 8 | EATMEA = Eat Meal |
| 4 MEDDEN = Medical Dental | 9 | SHOP $=$ Shop |
| 5 | 10 | SERPAS = Serve Passenger |

TABLE 8
$10 \times 10$ PURPOSE MATRIX OF TRANSITION PROBABILITIES-WACO, 1964

|  |  | $\begin{gathered} 1 \\ \text { HOME } \end{gathered}$ | 2 wORK | $\begin{gathered} 3 \\ \text { PERBUS } \end{gathered}$ | $\stackrel{4}{\text { MEDDEN }}$ | $\stackrel{5}{\text { SCHOOL }}$ | $\begin{gathered} 6 \\ \text { SOCREC } \end{gathered}$ | $\begin{gathered} \stackrel{?}{2} \\ \text { CHMODE } \end{gathered}$ | $\begin{gathered} 8 \\ \text { EATMEA } \end{gathered}$ | $\stackrel{9}{\text { SHOP }}$ | $\begin{gathered} 10 \\ \text { SERPAS } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | HOME | . 00 | .24 | . 09 | . 01 | . 15 | . 19 | $.00^{\text {a }}$ | . 02 | . 14 | . 15 |
| 2 | WORK | . 60 | , 14 | . 04 | . $00^{\text {a }}$ | $.00^{\text {a }}$ | . 02 | . $00{ }^{\text {a }}$ | . 10 | , 04 | . 05 |
| 3 | PERBUS | . 48 | . 06 | . 18 | . 01 | . 01 | . 07 | $.00{ }^{\text {a }}$ | . 03 | . 13 | . 04 |
| 4 | MEDDEN | . 56 | . 05 | . 06 | , 03 | . 01 | . 09 | $.00{ }^{\text {a }}$ | , 03 | . 14 | . 04 |
| 5 | SCHOOL | . 81 | -02 | , 01 | , 01 | , 01 | . 06 | .00a | . 03 | . 03 | . 02 |
| 6 | SOCREC | . 66 | . 01 | . 03 | . $00^{\text {a }}$ | . 01 | . 16 | $.00^{\text {a }}$ | . 02 | . 05 | . 04 |
| 7 | CHMODE | . 56 | . 06 | . 00 | . 06 | . 16 | . 08 | . 00 | . 03 | . 00 | . 06 |
| 8 | EATMEA | . 31 | .40 | . 05 | $.00^{\text {a }}$ | . 04 | . 08 | $.00{ }^{\text {a }}$ | ,01 | . 07 | . 05 |
| 9 | SHOP | . 62 | . 02 | . 05 | . $00{ }^{\text {a }}$ | $.00^{\text {a }}$ | . 06 | . 00 | . 02 | . 19 | . 03 |
| 10 | SERPAS | . 51 | . 11 | . 03 | , 01 | , 02 | . 05 | $.00^{\text {a }}$ | . 02 | . 06 | . 19 |

${ }^{\text {a }}$ Less than .005 .
Note: See Table 7 for definition of abbreviations.

TABLE 9
ONE ROW OF THE LIMITING MATRIX A (PERCENTAGE OF THE GROUP THAT WILL BE FOUND IN A PARTICUTAR STATE AT SOME RANDOM TIME DURING THE DAY) GENERATED FROM MATRIX P OF THE $10 \times 10$ PURPOSE

| 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| HOME | WORK | PERBUS | MEDDEN | SCHOOL |
| 11.3 | 14.4 | 6.7 | , 8 | 6.3 |
| 6 | 7 | 8 | 9 | 10 |
| SOCREC | CHMODE | EATMEA | SHOP | SERPAS |
| 11.3 | . 1 | 3.3 | 10.2 | 9.5 |

TABLE 10
PREDICTED NUMBER OF STOPS ON WACO PERSON TRIPS

| Starting State | Mean | Variance |  |
| :---: | :---: | :---: | :---: |
| 2 | WORK | 1.8 | 1.4 |
| 3 | PERBUS | 1.9 | 1.5 |
| 4 | MEDDEN | 1.8 | 1.3 |
| 5 | SCHOOL | 1.3 | 7 |
| 6 | SOCREC | 1.6 | 1.1 |
| 7 | CHMODE | 1.7 | 1.2 |
| 8 | EATMEA | 2.2 | 1.6 |
| 9 | SHOP | 1.7 | 1.2 |
| 10 | SERPAS | 1.9 | 1.5 |
|  | SYSTEM | 1.7 | 1.3 |

Note: See Table 7 for definition of abbrevlations.

TABLE 11
EXPECTED NUMBER OF STOPS BY FIRST PURPOSE AND TYPE OF STOP-WACO, 1964

${ }^{\text {a Less than , }}$, 05
Note: See Table 7 for definition of abbreviations.

TABLE 12
VARIANCE IN NUMBER OF STOPS BY FIRST PURPOSE AND TYPE OF STOP-WACO, 1964
First
Purpose
a Less than , 005.
Note: See Table 7 for definition of abbreviations.
TABLE 14
COMMERCLAL LAND-USE MATRIX OF TRANSITION PROBABILITIES-WACO, 18



TABLE 15
ONE ROW OF THE LIMItiNG MATRIX A (PERCEN'TAGE OF THE GROUP THAT WILL be FOUND in a particular state at some random time durung the day) generated FROM MATRIX P OF THE $21 \times 21$ LAND USE MATRIX

| 1 | 2 | 3 | 4 | , | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| HOME | FOODRU | * EATDRI | GENMER | APPACC | FUHFHA | motvac |
| 41.7 | 14.9 | 7.6 | 6.1 | 1.3 | . 7 | 3.0 |
| 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| GAssta | LUBDHA | LIQBER | MISTRET | FININS | PERSER | busser |
| 2.2 | 1.2 | . 2 | 5,3 | 4.0 | 4.8 | . 5 |
| 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| AUTOGA | MISREP | AMUREC | MEDDEN | OTHPRO | OFFBLD | Misser |
| 1.0 | , 4 | 1.7 | 2.4 | , 3 | . 5 | . 3 |

## $10 \times 10$ Land-Use Matrix

A glance at the matrix of transition probabilities (Table 2) indicates that the commercial retail land uses are highly linked with all other land uses. Quite unexpectedly the linkages between public-quasi-public land uses and the other groups are also quite high. This is most likely a measure of the magnitude of employment at the air base in Waco, and also a function of the number of school trips in the area. Within the Waco area 74 out of every 100 trips from the home may be expected to end at a commercial retail or public-quasi-public land use. If one adds the probability of going to commercial service land use, approximately 85 percent of the trips emanating from home will end at three of the ten land-use classes. As one would expect, the majority of the trips emanating from non-home bases terminate at the home base.

Table 3 gives the expected percentage of the tripmakers in any one state. The large number of trip ends concentrated in the four land uses of home, commercial retail, commercial service, and public-quasi-public are reflected in the expected percentage of trip makers found in these states. Although the figures in Table 3 indicate the expected percentage of tripmakers in any one state at some random time during the day, it would be more reasonable to interpret these figures reflecting percentages of tripmakers in any one state during the working hours of the day. We can expect that of the universe of the tripmakers in Waco, fully 92 percent will be found on the four land uses named. Table 4 gives the expected mean number of stops on multiple-leg trips starting from the nine non-home land uses, and the expected variances in number of stops before terminating at the home. This particular breakdown of the land uses yields values of total expected stops which show little variation between land-use classes. Once again, the distribution of expected trip ends (Table 5) is highly skewed in favor of commercial retail, commercial service, and public-quasi-public land uses. For example, trips which have as a first stop wholesale land use, have an expected 0.05 stops at wholesale land uses after the initial stop; 0.24 stops are expected to terminate at commercial retail land uses given the same initial stop.

## $10 \times 10$ Purpose Matrix

Table 8 gives the limiting transition probabilities for the $10 \times 10$ purpose matrix. Of trips beginning from the home base, there is an expected probability of 0.24 that people are leaving for work. The majority of trips have home as a purpose. However, the purposes of eat meal and personal business have probabilities of only 0.31 and 0.48 , respectively, that the next purpose will be to go home. These low figures indicate the large number of eat meal trips which ultimately end up back at the work place. In personal business it illustrates that a large number of stops are made on personal trips (in Table 10 personal business has one of the largest number of expected stops). Other highly linked purposes are as follows:

1. Work and work, eat meal;
2. Personal business and personal business, social-recreation;
3. Medical dental and social-recreation;


[^1]TABLE 17
VARIANCE IN NUMBER OF STOPS BY LAND USES AT FIRST STOP ON SHOPPING TRIP-WACO, 1964


[^2]TABLE 18

| Starting State | Mean | Variance |
| :---: | :---: | :---: |
| 2 FOODRU | 1.2 | 3 |
| 3 EATDRI | 1,5 | 7 |
| 4 GENMER | 1.5 | , 7 |
| 5 APPACC | 1.7 | , 0 |
| 6 FUHFHA | 1.6 | . 6 |
| 7 MOTVAC | 1.5 | . 7 |
| 8 GASSTA | 1.4 | . 5 |
| 9 LUBDHA | 1.5 | . 7 |
| 10 LIQBER | 1.4 | . 6 |
| 11 Misket | 1.4 | .6 |
| 12 FININS | 1.6 | . 8 |
| 13 PERSER | 1.4 | 0 |
| 14 BUSSER | 1.5 | , 7 |
| 15 AUTOGA | 1.6 | . 8 |
| 16 MISREP | 1.3 | 4 |
| 17 AMUREC | 1.5 | .7 |
| 18 MEDDEN | 1.5 | - 7 |
| 19 OTHPRO | 1.2 | . 3 |
| 20 OFFBLD | 1.4 | . 5 |
| 21 MISSER | 1.4 | . 6 |
| SYSTEM | 1.4 | . 6 |

TABLE 19
DISTRIBUTION OF TRIP PURPOSES BY DESTINATION FOR SEVERAL URBAN AREAS ${ }^{\text {a }}$

| Area | Home | Work | Shopping | School | Social-Rec |
| :--- | :---: | :---: | :---: | :---: | :---: |
| San Francisco | 37.7 | 27.3 | 9.2 | 2.8 | 12.2 |
| Sacramento | 31.4 | 33.6 | 10.5 | 2.6 | 10.1 |
| Cedar Rapids | 38.4 | 22.4 | 9.8 | 1.1 | 6.2 |
| Chicago | 43.3 | 20.3 | 7.6 | 1.9 | 12.7 |
| Wacob | 37.3 | 14.4 | 10.2 | 6.3 | 11.3 |

ainformation from Marble (2), Table 3, p, 153
bas predicted by the Markov Chaln model.
4. Social-recreation and social-recreation;
5. Change mode and school;
6. Eat meal and work, social-recreation;
7. Shop and shop; and
8. Serve passenger and serve passenger, work.

Although the commercial retail and commercial service land uses had high linkages with all other land uses, the purpose shop is not highly linked with all purposes.

Table 9 indicates that the majority of tripmakers would be traveling for four purposes: (a) to go home, (b) to work, (c) for social-recreation, and (d) to shop. Data from other studies are compared with the results of the Waco analysis in Table 19. Waco differs considerably from the other urban places in the percentage of tripmakers in the purpose states of work (much lower) and school (much higher). The presence of Baylor University in Waco could well account for the high percentage of tripmakers going to school. The rest of the percentages seem to be fairly well in line.

The purpose of shopping dominates the expected number of stops in the transition states, given some non-home purpose first stop. Serve passenger and personal business also have large expected stop values. The variation in trip lengths (measured in number of stops) is quite large when using the purpose classification. Trip lengths vary from 2.2 stops for the eat meal purpose to 1.3 stops for the school purpose.

## $21 \times 21$ Commercial Land Use

The $21 \times 21$ commercial land-use matrix was derived from shopping trips. This particular matrix was used to decrease the level of aggregation and derive information as to the internal pattern of trip distributions and lengths within two major land-use categories.

Table 14 gives the resulting transition probabilities. It appears that food and drug, eat and drink, general merchandise, miscellaneous retail stores, finance-insurancereal estate, and personal service have the highest level of interaction with all other land uses. Further, the higher the order of the good (or service) the greater the linkages between the same land use; i.e., a trip for a low order or convenience good, such as food or drugs, has a relatively small probability that a person will stop at another land use or activity which dispenses food or drugs. A person who is interested in obtaining a higher order good such as furniture or apparel generates a higher probability of a stop on the same trip at a similar land use (Table 15).

Table 16 indicates several interesting changes in the linkage structure given a first stop at a particular land use. For example, activities which distribute food and drugs showed a high level of interaction with most of the other land-use categories. On the other hand, when food and drug activities become the first stop on the trip, one may expect very little interaction with other types of land uses. In all other cases, food and drug activities again exhibit high levels of interaction. The categories indicated above as having high probabilities of linkages maintain that level of interaction when the model is restructured into an absorbing chain.

The mean and variation in the expected length of trips which stop at a given land use are indicated in Table 18. The longest trips are associated with apparel and accessories (1.7) and the shortest with food and drug (1.2). It is clear that the number of stops is a function of distance to some extent. Higher order goods, usually found at nucleated planned or unplanned shopping centers (i.e., furniture, apparel, banking facilities, etc.) and at farther distances from the home than lower order goods, have an expectation of longer trips (again measured in number of stops).

## VALUE OF THE MODELS IN CONTINUING RESEARCH

The Markov models used in the analysis thus far have demonstrated their value in the analysis of origin and destination data. Such models have considerable potential for adding meaningful insights into travel behavior on multipurpose trips. These models can also be useful in generating valuable information for use in the analysis of nonresidential trip generation. For example, linkage parameters can be used to "explain" spatial variation in the trip attraction of a given nonresidential land use.

The Markov models also add a new dimension to the analysis of household travel behavior. Given sets of socioeconomically homogeneous households, trip length (measured in number of stops), linkages, and purpose distribution information may be added to the current body of knowledge associated with household travel behavior. Timeseries data on the same sets of family units provide a base for the examination of the stability of the aforementioned relationships through time.

Finally, there is the spatial connotation that probabilistic models can be given. By using areal units, such as census tracts or traffic analysis zones, in addition to land use or trip purpose to define a given state of the Markov process, information as to the spatial distribution of trip ends may be derived. Although trip distribution is beyond the scope of the research which is currently being conducted, future research efforts might well include the development of probabilistic models which can integrate generation and distribution.

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[^0]:    Paper sponsored by Committee on Origin and Destination.
    ${ }^{1}$ This example was adapted from (1), pp. 33-34b.

[^1]:    Note: See Table 13 for definition of abbreviations.

[^2]:    

