The Analysis of Land-Use Linkages

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•EXAMINATION of urban travel activity patterns suggests that person trip sets may be considered to be analogous to a closed circuit movement; that is, the tripmaker generally leaves his home base, makes one or more stops, and returns to the home base. The complexity of the system in which these movements are negotiated makes it extremely difficult to deal explicitly with all relevant or potentially relevant variables. However, this very complexity suggests the use of relatively simple probabilistic models which allow the analyst to vary the components of the system without recourse to an ultra-sophisticated theoretical framework and without being handicapped by concern with a multitude of parameter changes. The Markov chain model is one such simple tool.

Let us assume that each group of land uses within the spatial structure of urban land is a member of a finite collection of states which a tripmaker may choose as a trip end. Assume further that the movement of a tripmaker is part of a process such that if he is in a given state, i, there is some probability, p_{ij} , that he will move to another given state, j, in any given time period, t. Under these assumptions, there is a simple probabilistic model which may be used to describe and analyze such a situation. This simple, time-dependent probability model is known as a finite Markov chain.

A SIMPLE MARKOV CHAIN MODEL

As an example¹ of the use of simple Markov chain models in the analysis of travel characteristics, consider the closed system of three land-use parcels shown in Figure 1, and a tripmaker who may be on any of the three parcels. The rule of the model is that in each time period the tripmaker must leave the particular parcel on which he is located and must either travel to one of the other parcels or return to the original parcel. Using the length of the trip as a criterion (although it would be possible to use any meaningful criterion) and the parcel arrangement shown in Figure 1, we would assume that a tripmaker located on parcel 1 would be most likely to return there, may go to parcel 2, and would be least likely to go to parcel 3. If we assign probabilities to each move, their sum will be unity because the tripmaker must move in some way. The probabilities of ending a trip on each land-use parcel from each of the three possible starting positions may be expressed in a 3 by 3 matrix in which the rows designate land-use parcels of destination, and each row sums to one. A matrix of this type is known as a transition matrix; each of the land-use parcels represents a Markov "state." and each of the elements of the matrix is known as transition probabilities. A transition matrix for the system outlined above could take the following form:

	1	2	3	Σ
1	. 5	.4	.1	1.0
2	. 3	.5	.2	1.0
3	.1	.3	.6	1.0

Paper sponsored by Committee on Origin and Destination.

¹This example was adapted from (1), pp. 33–34b.



Figure 1. Closed land-use system.

Given the transition probabilities, it must also be known in what state our tripmaker will be at the beginning of the process. This is done by establishing a row or vector in which each element of the row represents the probability of the tripmaker beginning on a given land-use parcel. For example, if we know the tripmaker always begins on parcel 1 in time (0), the initial vector would take the form

(1,0,0). If, on the other hand, there is an equal probability that the tripmaker will begin on any of the parcels, the initial vector will appear as (1/3, 1/3, 1/3). This row of values is known as a probability vector. The sum of the elements of a probability vector is always equal to one.

Let us assume that there is an equal probability that the tripmaker will start at any one of the three land-use parcels. Multiplying the initial probability vector by the transition matrix will yield a probability vector whose elements define the probability of the tripmaker being in any of the three states or land-use parcels. Thus:

$$(1/3, 1/3, 1/3) \begin{bmatrix} .5 & .4 & .1 \\ .3 & .5 & .2 \\ .1 & .3 & .6 \end{bmatrix} = (.30, .40, .30)$$

The probability vector (.30, .40, .30) becomes an intermediate probability vector establishing the probabilities that the tripmaker is on any of the land-use parcels at the end of time period (1). Furthermore, since we assume the transition probabilities remain constant through time, the probability that the tripmaker will be on land-use X at the end of the second time period may be established by multiplying the new probability vector by the matrix of transition probabilities. The relationship may be expressed by the following equation:

$$R^{(t+1)} = R^{(t)} \times P$$

$$1 \times n \qquad 1 \times n \qquad n \times n$$

where

R = the probability row vector with n_i as any element in the vector,

j = (1 . . . n),

P = the transition matrix with a_{ij} as any element in the matrix,

i = (1 . . . n), and

t = the time period designation, t = (1 . . . m), and t is not an exponent.

Given these assumptions, there are many questions about a particular process that the Markov chain model can answer. For example, the model can establish the probability that a tripmaker will be on each land-use parcel in any given time period, t. A Markovian approach can also establish whether or not there exists an equilibrium or balance such that the probability of being on each land-use parcel remains constant after a particular time period, t. Stated in another way, this means that it is possible to establish a finite number of time periods in which the tripmaker will move from any Markov "state" to all other Markov "states." This situation exists for every "regular" transition matrix. A "regular" transition matrix is such that there exists some exponent, g, such that P^g has no zero elements. The transition matrix used in our example is a "regular" transition matrix since the matrix P to the first power contains no zero elements. With the knowledge that there exists an equilibrium situation, it is possible to establish the following set of equations:

$$\mathbf{r}_1 = .5\mathbf{r}_1 + .3\mathbf{r}_2 + .1\mathbf{r}_3 \tag{1}$$

$$\mathbf{r}_2 = .4\mathbf{r}_1 + .5\mathbf{r}_2 + .3\mathbf{r}_3 \tag{2}$$

$$\mathbf{r}_3 = .1\mathbf{r}_1 + .2\mathbf{r}_2 + .6\mathbf{r}_3$$
 (3)

Because Eqs. 1, 2 and 3 are not independent equations, another equation must be introduced in order to solve for r_1 , r_2 , and r_3 :

$$1 = r_1 + r_2 + r_3$$
 (4)

Any two of Eqs. 1, 2, or 3 and Eq. 4 will constitute a solvable set of equations. The equilibrium vector resulting from our example is (14/46, 19/46, 13/46).

Another interesting problem which may be solved within a Markovian framework is the derivation of "mean first passage time." The mean first passage time is the number of time periods it takes the tripmaker to return to the "state" or land-use parcel from which he started. Thus, it is possible to derive the average number of stops on multipurpose trips.

All of the problems discussed above are capable of being solved through the use of computer programs currently available (2, 3). A more detailed discussion of Markov chain analysis is given by Kemeny and Snell (4).

APPLICATION OF THE MODEL TO THE ANALYSIS OF TRAVEL BEHAVIOR

The finite Markov chain model described above may be adapted to the analysis of travel behavior through the derivation of the following tables, or matrices.

The F Matrix

A matrix of trip origins and destinations is developed from standard O-D data. In this instance, let us assume that this matrix consists of the number of trips, f_{ij} , which start at a given land use, i, and end at a given land use, j. An example of such a matrix is shown below.

Land Use		Land	Use at	Desti	nation	
at Origin	1	2		j		n
1	f ₁₁	f ₁₂		f _{lj}		fln
5		f ₂₂		f _{2j}		f _{2n}
i	f _{il}	f.,12		f _{ij}		f _{in}
n	f _{nl}	f _{n2}		f _{nj}		f _{nn}

The P Matrix

The proportion of trips, P_{ij} , which go from any origin, i, to each destination, j, can be readily computed by dividing the number of trips from i to j by the total number of trips originating at land-use i. Thus:

$$P_{ij} = \frac{f_{ij}}{\sum_{j=1}^{n} f_{ij}}$$

In the context of the Markov chain model, P_{ij} is defined as a maximum likelihood estimator of the probability of a tripmaker from state i (in this example, land-use i) moving to state j (here, land-use j). The P matrix would look just like the F matrix; except that the relative frequency (i.e., the probability) of trips moving from i to j would be shown, rather than the absolute frequency of such movements. The sum of the P_{ij} 's for any originating land use must equal 1.0; that is,

$$\sum_{j=1}^{n} P_{ij} = 1.0$$

The A Matrix

If all elements of the P matrix are greater than zero, i.e., if

$$P_{ii} > 0$$
 for all i, j

a limiting matrix, A, can be derived through repeated multiplication of P by itself. Symbolically,

 $A = P^n$

As a result of repeated multiplication and as a direct consequence of the structure of the P matrix, each row of A is identical. One interpretation of the element, a_j , of any row is that a_j is equal to the expected percentage of tripmakers which will be found at the jth land use at some random time during the day.

The S Matrix

The regular Markov chain can be restructured so that one or more of the diagonal elements, P_{ij} , of the P matrix is made equal to one. Such a Markov chain is called an absorbing chain, since once an entry is made in the k, j cell for which $P_{ij} = 1$, no exit is permitted. By operating upon this modified P matrix another matrix, S, can be developed; the elements, u_{ij} , may be interpreted as the mean number of times a tripmaker will be in a nonabsorbing land use, u_j , given that it starts at a nonabsorbing land use, u_i .

The V Matrix

The A matrix discussed previously can be operated upon to yield a matrix, V, in which each of the elements, v_{ij} , represents the expected variation in the expected number of stops, v_{ij} , at each land use.

The V Vectors

Further manipulation of the S matrix will yield the expected average number of stops and variances of trips with a particular land use designated as the first stop. Further operations on S will also lead to the expected mean number of stops per trip set, and the variance for the system as a whole. All of the operations outlined can be carried out by using two programs developed by Marble (3).

					TA	BLE 1				
LAND	USE	AT	ORIGIN	RELATED	то	LAND	USE	АТ	DESTINATION-WACO,	1964

From	То	1, HOME	2. AGFOFI	3. MFGOUR	4. MFGNDU	5. TRCOOI	6, COMRET	7. COMSVC	8. WHOSAL	9. PUBQPU	10, PUBOPE	Sum
1.	HOME	0	50	359	486	307	3,729	1, 547	250	5, 511	283	12,432
2.	AGFOFI	56	4	0	2	0	15	5	0	9	0	91
3.	MFGDUR	338	0	9	10	3	55	20	4	18	2	459
4.	MFGNDU	436	0	10	12	2	82	33	7	25	2	609
5.	TRCOOI	286	0	6	4	30	71	25	2	34	4	462
6.	COMRET	4.174	11	44	53	52	1.148	285	51	333	52	6.203
7.	COMSVC	1.356	7	12	20	21	454	210	20	187	18	2.305
8.	WHOSAL	251	2	5	5	3	60	13	17	20	2	378
9.	PUBQPU	4.681	17	21	56	36	645	258	29	794	40	6, 577
10.	PUBOPE	311	1	1	0	3	42	13	4	41	34	450

Note: Definitions of abbreviations used in Tables 1 through 6;

J. HOME = Home
AGFOFI = Agriculture, Forestry, Fishing
M FGDUR = Manufacturing, Durable
M FGRUU = Manufacturing, Nondurable
TRCOOI = Transportation, Communication, Other Industrial

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COMRET = Commercial, Retail COMSVC = Commercial, Service WHOSAL = Wholesale PUBQPU = Public and Quasi-Public PUBOPE = Public Open Space 10

10 × 10 LAND-USE MATRIX OF TRANSITION PROBABILITIES-WACO, 1964													
From To	1, HOME	2. AGFOFI	3. MFGDUR	4. MFGNDU	5. TRCOOI	6. COMRET	7. COMSVC	8. WHOSAL	9. PUBQPU	10. PUBOPE			
1. HOME		.00a	.03	.04	.02	.30	.12	.02	, 44	.02			
2, AGFOFI	. 62	.04	.00	.02	.00	.16	.05	.00	. 10	.00			
3. MFGDUR	.74	.00	.02	.02	.01	. 12	.04	.01	.04	,00			
4. MFGNDU	.72	.00	.02	.02	.00a	. 13	.05	.01	.04	.00 ^a			
5. TRCOOI	_ 62	.00	.01	.01	.06	.15	.05	.00 ^a	.07	.01			

.01

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19 20

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.03

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TABLE 2

PUBQPU PUBOPE aLess than .005.

6.7. COMPET COMSVC 8. WHOSAL

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10.

Note: See Table 1 for definition of abbreviations.

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TABLE 3

ONE ROW OF THE LIMITING MATRIX A (PERCENTAGE OF THE GROUP THAT WILL BE FOUND IN A PARTICULAR STATE AT SOME RANDOM TIME DURING THE DAY) GENERATED FROM MATRIX P OF THE 10 \times 10 LAND-USE MATRIX

1	2	3	4	5
HOME	AGFOFI	MFGDUR	MFGNDU	TRCOOI
40.6	0.3	1.5	2.1	1.5
6	7	8	9	10
COMRET	COMSVC	WHOSAL	PUBQPU	PUBOPE
20.8	7.8	1,3	22.8	1.4

Note: See Table 1 for definition of abbreviations.

TABLE 4 PREDICTED NUMBER OF STOPS ON WACO PERSON TRIPS

Starting State	Mean	Variance
2. AGFOFI	1.6	.8
3. MFGDUR	1.4	.6
4. MFGNDU	1.4	.7
5. TRCOOI	1.6	.8
6. COMRET	1.5	7
7. COMSVC	1,6	9
8. WHOSAL	1,5	.7
9. PUBQPU	1.4	.7
10. PUBOPE	1,5	.7
SYSTEM	1.5	.7

Note: See Table 1 for definition of abbreviations.

ANALYSIS AND SOME PRELIMINARY RESULTS

The models outlined were applied to land-use and trip data for the Waco, Texas, area. Four F matrices were developed: (a) a 10×10 trip purpose matrix; (b) a 10×10 major land-use matrix; (c) a 21×21 commercial landuse matrix; and (d) a 28×28 major and commercial land-use matrix. The results generated by the application of the two Markov models to these data are given in Tables 1 through 18. The

tables corresponding to the matrices described in the previous section are as follows: F matrices-Tables 1, 7, and 13; P matrices-Tables 2, 8, and 14; 1 row of the A matrices-Tables 3, 9, and 15; the vectors, V'-Tables 4, 10, and 18; S matrices-Tables 5, 11, and 16; and V matrices-Tables 6, 12, and 17. Results using the 28×28 land-use matrix are not illustrated in the tables.

TABLE 5 PREDICTED NUMBER OF STOPS BY LAND USE AT FIRST STOP TRIP-WACO, 1964

Firs	Stops At at d Use	2. AGFOFI	Ĵ. MFGDUR	4. MFGNDU	5. TRCOOI	6. Comret	7. COMSVC	8. WHOSAL	9. PUBQPU	10, PUBOPE
2.	AGFOFI	1.05	.00 ^a	.03	.00 ^a	.26	.08	.00 ^a	.14	.00 ^a
3.	MFGDUR	.00 ^a	1.02	.03	.01	.18	.06	.01	.07	.01
4.	MFGNDU	.00 ^a	.02	1.02	.01	.20	.08	.02	.07	.01
5.	TRCOOI	.00 ^a	.02	.01	1.07	.25	.08	.01	.12	.01
6.	COMRET	.00 ^a	.01	.01	.01	1.27	.07	.01	.09	.01
7.	COMSVC	.00 ^a	.01	.01	.01	.30	1.12	.01	.13	.01
θ.	WHOSAL	.01	.02	.02	.01	.24	.06	1.05	.09	.01
9,	PUBQPU	.00 ^a	.01	.01	.01	.16	.06	.01	1,16	.01
10.	PUBOPE	.00ª	.00 ^a	.00 ^a	.01	.16	,05	.01	.13	1.08

aLess than .005.

Note: See Table 1 for definition of abbreviations.

TABLE 6 VARIANCE IN NUMBER OF STOPS BY LAND USE AT FIRST STOP ON TRIP-WACO, 1964

Fire	Stops At st d Use	2. AGFOFI	3. MFGDUR	4. MFGNDU	5. TRCOOI	6. COMRET	7. COMSVC	8. WHOSAL	9. PUBQPU	10. PUBOPE
2.	AGFOFI	,05	.01	.04	.01	.26	.11	,10	.17	.01
3.	MFGDUR	,00 ^a	.02	.03	.01	.18	.08	,02	.09	.01
4.	MFGNDU	.00 ^a	.02	.07	.01	. 20	.09	.02	.09	.01
5.	TRCOOI	,00ª	.02	.02	,07	.25	.11	.01	.14	.02
6.	COMRET	.00ª	.01	.02	.02	.29	.09	.02	.11	.02
7.	COMSVC	.01	.01	.02	.02	,29	.11	.02	.16	.02
8.	WHOSAL	.01	.02	.02	.02	.23	.08	.05	.11	.01
9.	PUBQPU	.00 ^a	.01	.02	.01	.17	.08	.01	. 15	.01
10.	PUBOPE	.00ª	.01	.01	.01	.17	.07	.02	. 14	.09

aLess than ,005.

Note: See Table 1 for definition of abbreviations.

				,										
From To		1 HOME	2 WORK	3 PERBUS	4 MEDDEN	5 SCHOOL	5 SOCREC	7 CHMODE	8 EATMEA	9 SHOP	10 SERPAS	Sum		
1	HOME	00	3230	1231	181	2031	2444	23	294	1800	1953	13, 187		
2	WORK	2920	701	199	24	11	106	1	474	205	205	4,891		
3	PERBUS	1113	145	430	12	16	153	2	70	299	89	2,330		
4	MEDDEN	157	13	18	8	2	25	1	8	40	10	282		
5	SCHOOL	1574	32	25	10	22	122	3	54	57	41	1,940		
6	SOCREC	2617	54	108	18	49	652	1	86	216	156	3,957		
7	CHMODE	18	2	0	2	5	2	0	1	1	2	33		
8	EATMEA	340	440	50	2	44	85	1	6	72	53	1,093		
9	SHOP	2216	60	173	12	4	211	0	71	686	124	3, 551		
10	SERPAS	1680	368	116	19	59	168	1	63	200	641	3, 315		

TABLE 7 TRIP PURPOSE AT ORIGIN RELATED TO TRIP PURPOSE AT DESTINATION-WACO, 1964

Note: Definitions of abbreviations used in Tables 7 through 12;

 Definitions of above various deviations deviatedviations deviatintextendes deviations deviations deviati

						TABLE 8		
10	х	10	PURPOSE	MATRIX	OF	TRANSITION	PROBABILITIES-WACO,	1964

Fre	To	1 HOME	2 WORK	3 PERBUS	4 MEDDEN	5 SCHOOL	6 SOCREC	7 CHMODE	8 EATMEA	9 SHOP	10 SERPA
1	HOME	.00		.09	.01	. 15	. 19	.00ª	.02	.14	.15
2	WORK	. 60	. 14	.04	.00 ^a	.00a	.02	.00a	.10	.04	.05
3	PERBUS	.48	.06	.18	.01	.01	.07	.00 ^a	.03	.13	.04
4	MEDDEN	.56	.05	.06	.03	.01	.09	.00ª	.03	. 14	.04
5	SCHOOL	.81	_02	.01	.01	.01	.06	.00a	.03	.03	.02
6	SOCREC	.66	.01	.03	.00a	.01	.16	_00 ^a	.02	.05	.04
7	CHMODE	. 56	.06	.00	.06	,16	.06	.00	.03	.00	.06
8	EATMEA	.31	. 40	.05	.00 ^a	.04	.08	_00a	.01	.07	.05
9	SHOP	. 62	.02	.05	.00 ^a	.00a	.06	.00	,02	.19	.03
10	SERPAS	.51	.11	.03	.01	.02	_05	.00 ^a	.02	.06	.19

aLess than .005.

Note: See Table 7 for definition of abbreviations.

TABLE 9

ONE ROW OF THE LIMITING MATRIX A (PERCENTAGE OF THE GROUP THAT WILL BE FOUND IN A PARTICULAR STATE AT SOME RANDOM TIME DURING THE DAY) GENERATED FROM MATRIX P OF THE 10 × 10 PURPOSE MATRIX-WACO, 1964

1 HOME 11,3	2 WORK 14,4	3 PERBUS 6.7	4 MEDDEN	SCHOOL 6,3
6 SOCREC 11.3	7 CHMODE	8 EATMEA 3.3	9 SHOP 10.2	10 SERPAS 9,5

PR	EDICTED	NUMBER OF STOPS	S ON WACO
Sta	rting State	Mean	Variance
2	WORK	1.8	1.4
3	DEBBIIS	1 0	1 5

4	WORK	1.0	1.1
3	PERBUS	1,9	1.5
4	MEDDEN	1.8	1.3
5	SCHOOL	1.3	.7
6	SOCREC	1,6	1.1
7	CHMODE	1.7	1.2
8	EATMEA	2.2	1,6
9	SHOP	1.7	1.2
10	SERPAS	1.9	1.5
	SYSTEM	1,7	1.3

Note: See Table 7 for definition of abbreviations.

Fir	Stops At st pose	2 WORK	3 PERBUS	4 Medden	5 SCHOOL	6 SOCREC	7 CHMODE	8 EATMEA	9 SHOP	10 SERPAS
2	WORK	1.25	.08	.01	.01	.07	.00 ^a	. 13	. 10	. 10
3	PERBUS	, 14	1,26	.01	.02	.13	.00 ^a	,06	.23	.09
4	MEDDEN	.11	, 11	1.03	.01	.15	.00 ^a	.05	.23	.08
Б	SCHOOL	.05	.03	.01	1,02	.09	.00 ^a	.04	.06	.04
6	SOCREC	.05	.06	.01	.02	1.22	.00 ^a	.04	.10	.07
7	CHMODE	.12	.03	.07	.16	. 12	1,00	.06	.05	.11
8	EATMEA	.53	.11	.011	.05	. 14	.00 ^a	1.07	.15	. 12
9	SHOP	.06	.09	.01	.01	.11	.00 ^a	.04	1,27	.07
10	SERPAS	, 20	.08	.01	.03	.11	.00 ^a	.05	.13	1.27

 TABLE 11

 EXPECTED NUMBER OF STOPS BY FIRST PURPOSE AND TYPE OF STOP-WACO, 1964

aLess than .005.

Note: See Table 7 for definition of abbreviations.

		TABLE 12		
VARIANCE IN NUMBER	OF STOPS BY	FIRST PURPOSE	AND TYPE OF	STOP-WACO, 1964

Fir Pui	Stops At st pose	work	3 PERBUS	4 MEDDEN	SCHOOL	SOCREC	7 CHMODE	8 EATMEA	9 SHOP	10 SERPAS
2	WORK	.02	. 12	.01	.03	.12	.00ª	.17	.16	.13
3	PERBUS	. 22	. 27	.02	.03	. 19	.00ª	.11	. 28	. 14
4	MEDDEN	. 17	. 14	.03	.03	.18	.00 ^a	,09	.24	. 18
5	SCHOOL	.08	.04	.01	.01	. 10	.00ª	. 52	.08	.05
6	SOCREC	. 10	.08	.01	.03	.23	.00 ^a	.06	.14	.09
7	CHMODE	. 17	.07	.07	.17	. 15	,00 ^a	.09	-11	.12
8	EATMEA	. 50	.16	.02	.07	.21	.00 ^a	.02	.24	.18
9	SHOP	.12	.11	.01	.02	.14	.00ª	.07	.29	. 10
10	SERPAS	. 26	.12	.02	.04	.16	.00 ^a	, 10	.19	.29

aLess than .005.

Note: See Table 7 for definition of abbreviations.

	21 MISSER	80010010101000000000000000000000000000		21 MISSER	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
	20 OFFBLD	\$00000000000000000000000000000000000000	14	20 OFFBLD	0.000 0.0000 0.0000 0.0000 0.0000 0.000000
	19 OTHPRO	H0400000000000000000000000000000000000		19 OTHPRO	28888888888888888888888888888888888888
	18 MEDDEN	88 88 88 88 88 88 88 88 88 88 88 88 88		18 MEDDEN	0.000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.000000
	17 AMUREC	4 1 1 1 0 1 0 1 0 0 0 0 1 1 1 4 0 10 1 0 0 0 0		17 AMUREC	00 00 00 00 00 00 00 00 00 00 00 00 00
	16 MISREP	2		16 MISREP	$\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 $
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13 C DESTINAT	11 MISRET	422 252 252 252 252 252 252 252 252 252	 Fluance, Insurance and Real Estate Pressoul Services R = Perssoul Services R = Businesi Services R = Automotile Regair and Services Automotile Regair and Services a Miscellaneous Regair Services C indoor Annuesment and Recreation N = Medical and Heath Services C office Buildings Sound Services M Mirdix OF TRANSTION PROBABILITIES- 	11 MISRET	.09 .01 .02 .03 .03 .03 .04 .04 .04 .04 .04 .04 .04 .04 .09 .00 .00 .00 .00 .00 .00 .00 .00 .00
TABLE 13 ND USE AT I	10 LIQBER	90000000000000000000000000000000000000	 Filaace, Jisurance and Paraon Services Business Services Business Services Address Galances Repair and Address and Realth Sevices Address and Realth Sevices Colfice Buildings Not E Miscellaneous Services Miscellaneous Services 	10 LIQBER	00 ⁴ 00 ⁴ 00 ⁴ 00 ⁴ 00 ⁵ 00 ⁵ 00 ⁵ 00 ⁵ 00 ⁵ 00 ⁶ 00 ⁶ 00 ⁶ 00 ⁶ 00 ⁶ 00 ⁶ 00 ⁶
TABLE 13 ND USE AT ORIGIN RELATED TO LAND USE AT DESTINATION, SHOPPING TRIPS-WACO, 1964	9 LUBDHA	001 001 001-4000000000000000000000000000	12 FINUNS = Finance, Insurance and Real Estate 13 PERSER = Personal Services 13 PERSER = Pusiness Sarvices 14 BUSSER = Pusiness Sarvices 15 PUTCOAL = Automobile Regair and Services, Garages, and Marine 16 MUSREP = Musicellanous Regair Services 16 MUSREP = Musicellanous Regair Services 17 MUDEN = Medical and Health Services 18 MDEN = Medical and Health Services 19 MDEN = Medical and Reciection 11 MDENE = Medical and Reciection 12 OTHPRO = Office Buildings Networkes 13 MUSSER = Miscellaneous Services 14 MUSSER = Miscellaneous Services 15 MUSSER = Miscellaneous Services 16 MUSSER A Fiscellaneous Services 17ABLE 14 COMMERCIAL LAND-USE MATHIX OF TRANSITION PROBABILITIES-WACO, 1964	9 LUBDHA	$\begin{smallmatrix} 0.0\\ 0.0\\ 0.0\\ 0.0\\ 0.0\\ 0.0\\ 0.0\\ 0.0$
IGIN RELA	8 GASSTA	812 27 24 24 24 24 24 24 20 20 20 20 20 20 20 20 20 20 20 20 20	12 1 14 1 15 2 15 2 16 2 17 4 17 4 17 4 17 4 17 4 17 4 10 2 1 9 10 2 1 9 10 4 11 1 10 4 11 1 10 4 11 1 11 1 11 1	8 GASSTA	90000000000000000000000000000000000000
USE AT OR	7 MOTVAC	234 234 234 234 234 234 234 234 234 234	Equipment COMAM EL	7 MOTVAC	.05 004 002 002 002 002 002 002 002 002 002
LAND	6 FUHFHA	8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8	and Farm	5 FUHFHA	01 00 00 00 00 00 00 00 00 00 00 00 00 0
	5 APPACC	8×0.040+00400000000	: shold Applia and Moores, e Stores	5 APPACC	$\begin{smallmatrix} & 0 \\ & $
	4 GENMER	884 888 888 888 888 888 888 888 888 888	Definitions of abbreviations used in Tables 13 through 18: 1 HOME = Rome 2 HOME = Rome 2 HOME = Roma and Drugs 2 EACDRU = Ending and Drinking Establishments 3 EACDRU = Enting and Drinking Establishments 5 APPAGC = General Merchands 5 APPAGC = Apareta and Accessories Stores 6 APPAGC = Apareta and Accessories Stores 7 MOTTAG = Apareta and Accessories. Boats and Motors, and Farm Equipment 7 MOTTAG = Aboror Service Stores 1 MUSRET = Miscellaneous Retail Stores 11 MISRET = Miscellaneous Retail Stores 11 MISRET = Miscellaneous Retail Stores	4 GENMER	.09 01 01 01 01 01 01 01 00 00 00 00 00 00
	3 EATDRI	8 241 252 222 222 222 222 222 222 222 222 22	i in Tables ing Establi andise Seeveries S es Furnishi and Accesso factoria and Accesso factoria and Accesso factoria and Accesso factoria and Accesso and Accesso and Accesso factoria and accesso factoria accesso faccesso fac	3 EATDRI	.12 .01 .03 .03 .03 .03 .03 .03 .03 .05 .05 .05 .05 .05 .05 .05 .05 .05 .05
	2 FOODRU	1, 379 1859 1859 1859 18 18 18 18 18 18 19 11 11 11 11 12 22 22 22 23 23 20 11 12 12 12 12 12 12 12 12 12 12 12 12	mittons of abbreviations used in Tables 13 throu HOME = Home HOME = Home EXODRU = Food and Drugs EATORU = Food and Drugs EATORU = Faiting and Druhking Establishments GENMER = General Merchanders Stores FUETHA = Furniture, House Accessories Stores APPACC = Agareta and Accessories Stores APPACC = Anonr Vehicle and Accessories and MOTYA.C = Moor Vehicle and Accessories and COTMA = Lumberty Buttana and Ha LUDBIM = Lumberty Buttana and Ha LUDBIM = Lumberty and Beer Establishme MISRET = Miscellaneous Retail Stores MISRET = Miscellaneous Retail Stores	2 FOODRU	27 068 068 068 068 069 069 069 069 069 069 069 069 069 069
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TABLE 13

^aLess than .005. Note: See Table 13 for definition of abbreviations.

ONE ROW OF THE LIMITING MATRIX A (PERCENTAGE OF THE GROUP THAT WILL BE	
IN A PARTICULAR STATE AT SOME RANDOM TIME DURING THE DAY) GENERATE FROM MATRIX P OF THE 21 × 21 LAND USE MATRIX	

1	2	3	4	5	6	7
HOME	FOODRU	• EATDRI	GENMER	APPACC	FUHFHA	MOTVAC
41.7	14 _* 9	7.6	6.1	1,3	"7	3.0
8	9	10	11	12	13	14
GASSTA	LUBDHA	LIQBER	MISTRET	FININS	PERSER	BUSSER
2,2	1,2	,2	5,3	4.0	4,8	.5
15	16	17	18	19	20	21
AUTOGA	MISREP	AMUREC	MEDDEN	OTHPRO	OFFBLD	MISSER
1,0	4	1.7	2,4	.3	5	.3

Note: See Table 13 for definition of abbreviations.

10 × 10 Land-Use Matrix

A glance at the matrix of transition probabilities (Table 2) indicates that the commercial retail land uses are highly linked with all other land uses. Quite unexpectedly the linkages between public-quasi-public land uses and the other groups are also quite high. This is most likely a measure of the magnitude of employment at the air base in Waco, and also a function of the number of school trips in the area. Within the Waco area 74 out of every 100 trips from the home may be expected to end at a commercial retail or public-quasi-public land use. If one adds the probability of going to commercial service land use, approximately 85 percent of the trips emanating from home will end at three of the ten land-use classes. As one would expect, the majority of the trips emanating from non-home bases terminate at the home base.

Table 3 gives the expected percentage of the tripmakers in any one state. The large number of trip ends concentrated in the four land uses of home, commercial retail, commercial service, and public-quasi-public are reflected in the expected percentage of trip makers found in these states. Although the figures in Table 3 indicate the expected percentage of tripmakers in any one state at some random time during the day, it would be more reasonable to interpret these figures reflecting percentages of tripmakers in any one state during the working hours of the day. We can expect that of the universe of the tripmakers in Waco, fully 92 percent will be found on the four land uses named. Table 4 gives the expected mean number of stops on multiple-leg trips starting from the nine non-home land uses, and the expected variances in number of stops before terminating at the home. This particular breakdown of the land uses yields values of total expected stops which show little variation between land-use classes. Once again, the distribution of expected trip ends (Table 5) is highly skewed in favor of commercial retail, commercial service, and public-quasi-public land uses. For example, trips which have as a first stop wholesale land use, have an expected 0.05 stops at wholesale land uses after the initial stop; 0.24 stops are expected to terminate at commercial retail land uses given the same initial stop.

10 × 10 Purpose Matrix

Table 8 gives the limiting transition probabilities for the 10×10 purpose matrix. Of trips beginning from the home base, there is an expected probability of 0.24 that people are leaving for work. The majority of trips have home as a purpose. However, the purposes of eat meal and personal business have probabilities of only 0.31 and 0.48, respectively, that the next purpose will be to go home. These low figures indicate the large number of eat meal trips which ultimately end up back at the work place. In personal business it illustrates that a large number of stops are made on personal trips (in Table 10 personal business has one of the largest number of expected stops). Other highly linked purposes are as follows:

- 1. Work and work, eat meal;
- 2. Personal business and personal business, social-recreation;
- 3. Medical dental and social-recreation;

21 MISSER	.00ª	.00ª	.00 ^a	.00 ^a	.00ª	.00 ³	.01	.00	.00 ^a	.00 ²¹	.00ª	.00ª	.00ª	.00ª	.00 ^a	.00 ²	.00 ^a	.00 ²	.00ª	1.05				21 MISSER	000 000 000 000 000 000 000 000 000 00
20 OFFBLD	,00 ⁴	,00ª	.00 ^a	.00 ³	.00ª	.01	E00.	.02	.00 ^a	.00ª	.00ª	.01	.00 ^a	.00ª	.00ª	.01	.00ª	.00ª	1.03	.00 ⁴				20 OFFBLD M	00 ⁴ 001 002 002 002 002 002 002 002 002 002
19 OTHPRO	.00 ^a	.00ª	.00 ^a	.00a	.00ª	.00a	.00ª	.00ª	,00 ^a	.00ª	.00 ^a	.00ª	.00 ^a	.00 ^a	.00 ^a	,00 ^a	.00 ^a	1.06	.00ª	.00 ^a				19 OTHPRO 0	400 400 400 400 400 400 400 400
18 MEDDEN	.01	.02	.01	.02	.00 ^a	.02	.00a	.02	.00 ^a	.01	.03	.01	10.	.03	,00 ^a	.00ª	1.04	.00 ²	.02	.03				18 MEDDEN O	$\begin{smallmatrix} & 0.0 \\ & $
17 AMUREC	.00 ²	.02	.00 ^a	.00ª	.02	.00ª	.02	.00ª	.00ª	.00ª	.003	.00ª	.02	.04	.00 ²	1.03	.01	.00 ^a	.02	.03				17 AMUREC M	00 ^a 01 01 01 01 01 01 01 01 01 01 01 01 01
16 MISREP	.00 ^a	.00 ^a	.00ª	.00ª	10.	.00ª	.01	10.	.00ª	.00 ^a	.00ª	,00ª	.00 ^a	.02	1.04	.00ª	.00 ²	.00ª	.00 ^a	.00 ^a				16 MISREP A	00 ⁴ 00 ⁴ 00 ⁴ 00 ⁴ 01 01 00 ⁴ 00 ⁴ 00 ⁴ 00 ⁴ 00 ⁴ 00 ⁴ 00 ⁴ 00 ⁴ 00 ⁴ 00 ²
15 AUTOGA	.00ª	.01	,00 ^a	.00ª	.00 ^a	.00 ^a	.003	.003	.00ª	.00ª	.01	,00 ^a	.02	1.04	.00 ^a	.01	.01	.00ª	.00 ²	.00 ^a), 1964	15 AUTOGA N	$\begin{array}{c} 0.00\\$
14 BUSSER	.00ª	10.	.003	.00 ²	.00a	.00ª	.00ª	.003	.00ª	.00 ^a	.00 ^a	.00 ^a	1.00	.01	.04	.00ª	.00ª	.00 ^a	.003	.00ª			LAND USES AT FIRST STOP ON SHOPPING TRIP-WACO, 1964	14 BUSSER	$\begin{array}{c} 0.00^{a}\\ 0.01^{a}\\ 0.00^{a}\\ 0.00^{a}\\ 0.00^{a}\\ 0.00^{a}\\ 0.00^{a}\\ 0.00^{a}\\ 0.00^{a}\\ 0.00^{a}\\ 0.00^{a}\\ 0.01^{a}\\ 0.01^{a}\\$
13 PERSER	.01	.03	.02	.03	.02	.03	.01	.04	.01	.04	.04	1.05	.03	.02	.00ª	.03	,04	.00ª	.04	.00 ^a			DNIddOHS	13 PERSER	02 04 00 00 00 00 00 00 00 00 00 00 00 00
12 FININS	.01	.05	.02	.02	.03	.04	.04	.03	.04	.02	1,10	.03	.09	90.	.01	10.	.04	.00ª	.02	.01			STOP ON S	12 FININS	02 06 03 03 04 03 03 03 03 03 03 03 03 03 03 05 00
11 MISRET	.02	,04	90.	,08	.05	.02	.03	.03	.01	1.08	.06	.03	.03	.03	.05	.03	.06	.00ª	.01	.04		 17	AT FIRST	10 LIQBER	0 0 0 0 0 0 0 0 0 0 0 0 0 0
10 LIQBER	.00 ²	.00 ^a	.00ª	,00a	.00 ^a	00.	,00 ^a	.00ª	1,07	.00 ^a	.00ª	.003	.00 ^a	.02	.00ª	.01	.003	.00ª	.00ª	.00ª		TABLE 17	AND USES	9 LUBDHA	4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
9 LUBDHA	.00 ^a	.02	10.	.00 ^a	.04	.01	10.	1.07	.00ª	.01	.01	.01	.00ª	.04	.00ª	.003	.00ª	.00ª	.00 ^a	.00ª			STOPS BY LA	8 GASSTA	$\begin{smallmatrix} 00^{2}\\ 00$
8 GASSTA	.01	10.	.00ª	.01	.04	60.	1.02	.01	.00ª	.01	.02	.02	.00 ^a	,00a	.00ª	.01	10.	.00ª	,00 ^a	.03			BER OF SI	TMOTVAC	01 02 05 05 05 01 01 01 01 01 01 01 01 01 01
7 MOTVAC	.00ª	.03	.01	.01	.03	1.07	.03	.03	.04	.01	.05	.01	.01	.02	.00ª	.01	.01	.03	,00 ^a	.03			VARIANCE IN NUMBER OF	5 FUHFHA N	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
6 FUHFHA	,00 ^a	01	10	.01	1.04	10	.00 ^a	.01	,04	10	,00 ²	_00 ²	,00 ^a	.01	.00 ^a	0.03	0.03	.00 ^a	_00a	.00 ^a			VARIANO	5 APPACC I	00 ^a 011 012 014 014 014 014 014 014 014 014 014 014
5 APPACC	.01	10.	.03	1.10	.01	.01	10.	.01	.00a	.02	.01	.00 ^a	.00 ^a	.00ª	.02	,00 ^a	.01	.00 ^a	.00 ^a	.00 ^a				GENMER	01 04 01 00 00 00 00 00 00 00 00 00 00 00 00
4 GENMER	.02	-07	1.11	.20	.10	.04	.03	.04	.01	.07	.08	.05	.08	.05	.01	.02	.07	.01	.06	60.	ations.			3 EATDRI (0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
\$ EATDRI	.02	1.11	.05	.06	.13	.08	.04	70.	.09	.05	20.	.05	H.	.08	.05	.22	20.	.06	.03	.07	n of abbrevi			FOODRU	03 09 00 00 00 00 00 00 00 00 00 00 00 00
2 FOODRU	1.09	.10	.11	.17	.10	.10	.10	40.	.05	.09	80.	.11	.10	60.	.04	.06	.13	.04	.12	.08	for definitio.			HOME I	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
First Purpose	2 FOODRU	3 EATDRI	4 GENMER	5 APPACC	6 FUHFHA		8 GASSTA	9 LUBDHA	10 LIQBER	11 MISRET	12 FININS	13 PERSER	14 BUSSER	15 AUTOGA	16 MISREP	17 AMUREC	18 MEDDEN	19 OTHPRO	20 OFFBLD	21 MISSER	^a Less than .005. Note: See Table 13 for definition of abbreviations.		-	Stops First Purpose	2 FOODRU 4 GENMER 5 FOUHTHA 5 FUHTHA 6 FUHTHA 7 MOTVAC 8 LUBDHA 10 LIQBDHA 11 MISRET 11 MISRET 13 PUSSER 14 BUSSER 14 BUSSER 15 AUTOCA 16 MISREP 10 MISREP

 21
 MiSSVC
 ,11
 ,08
 ,10

 ^aLess than ,005.
 Note: See Table 13 for definition of abbreviations.

TABLE 16 BER OF STOPS BY LAND USE AT FIRST STOP ON SHOPPING THIP-

6

TABLE 18 PREDICTED NUMBER OF STOPS ON WACO PERSON TRIPS-COMMERCIAL LAND USE

Sta	urting State	ng State Mean					
2	FOODRU	1.2	.3				
3	EATDRI	1.5	.7				
4	GENMER	1.5	.7				
5	APPACC	1.7	.9				
6	FUHFHA	1.6	.8				
7	MOTVAC	1,5	.7				
8	GASSTA	1.4	.5				
9	LUBDHA	1.5	7				
10	LIQBER	1.4	.6				
11	MISRET	1.4	.6				
12	FININS	1.6	.8				
13	PERSER	1.4	. 0				
14	BUSSER	1,5	.7				
15	AUTOGA	1.6	.8				
16	MISREP	1.3	.4				
17	AMUREC	1.5	.7				
18	MEDDEN	1.5	.7				
19	OTHPRO	1.2	.3				
20	OFFBLD	1.4	.5				
21	MISSER	1.4	. 6				
	SYSTEM	1.4	.6				

Note: See Table 13 for definition of abbreviations

TABLE 19 DISTRIBUTION OF TRIP PURPOSES BY DESTINATION FOR SEVERAL URBAN AREAS^a

Area	Home	Work	Shopping	School	Social-Rec.
San Francisco	37.7	27.3	9.2	2.8	12.2
Sacramento	31.4	33.6	10.5	2.6	10.1
Cedar Rapids	38.4	22 4	9.8	1.1	6.2
Chicago	43.3	20.3	7.6	1.9	12.7
Wacob	37.3	14.4	10.2	6.3	11.3

^aInformation from Marble (2), Table 3, p. 153. ^bAs predicted by the Markov Chaln model. 4. Social-recreation and social-recreation;

5. Change mode and school;

6. Eat meal and work, social-recreation;

7. Shop and shop; and

8. Serve passenger and serve passenger, work.

Although the commercial retail and commercial service land uses had high linkages with all other land uses, the purpose shop is not highly linked with all purposes.

Table 9 indicates that the majority of tripmakers would be traveling for four purposes: (a) to go home, (b) to work, (c) for social-recreation, and (d) to shop. Data from other studies are compared with the results of the Waco analysis in Table 19. Waco differs considerably from the other urban places in the percentage of tripmakers in the purpose states of work (much lower) and school (much higher). The presence of Baylor University in Waco could well account for the high percentage of tripmakers going to school. The rest of the percentages seem to be fairly well in line.

The purpose of shopping dominates the expected number of stops in the transition

states, given some non-home purpose first stop. Serve passenger and personal business also have large expected stop values. The variation in trip lengths (measured in number of stops) is quite large when using the purpose classification. Trip lengths vary from 2.2 stops for the eat meal purpose to 1.3 stops for the school purpose.

21 × 21 Commercial Land Use

The 21×21 commercial land-use matrix was derived from shopping trips. This particular matrix was used to decrease the level of aggregation and derive information as to the internal pattern of trip distributions and lengths within two major land-use categories.

Table 14 gives the resulting transition probabilities. It appears that food and drug, eat and drink, general merchandise, miscellaneous retail stores, finance-insurance-real estate, and personal service have the highest level of interaction with all other land uses. Further, the higher the order of the good (or service) the greater the link-ages between the same land use; i.e., a trip for a low order or convenience good, such as food or drugs, has a relatively small probability that a person will stop at another land use or activity which dispenses food or drugs. A person who is interested in obtaining a higher order good such as furniture or apparel generates a higher probability of a stop on the same trip at a similar land use (Table 15).

Table 16 indicates several interesting changes in the linkage structure given a first stop at a particular land use. For example, activities which distribute food and drugs showed a high level of interaction with most of the other land-use categories. On the other hand, when food and drug activities become the first stop on the trip, one may expect very little interaction with other types of land uses. In all other cases, food and drug activities again exhibit high levels of interaction. The categories indicated above as having high probabilities of linkages maintain that level of interaction when the model is restructured into an absorbing chain. The mean and variation in the expected length of trips which stop at a given land use are indicated in Table 18. The longest trips are associated with apparel and accessories (1.7) and the shortest with food and drug (1.2). It is clear that the number of stops is a function of distance to some extent. Higher order goods, usually found at nucleated planned or unplanned shopping centers (i.e., furniture, apparel, banking facilities, etc.) and at farther distances from the home than lower order goods, have an expectation of longer trips (again measured in number of stops).

VALUE OF THE MODELS IN CONTINUING RESEARCH

The Markov models used in the analysis thus far have demonstrated their value in the analysis of origin and destination data. Such models have considerable potential for adding meaningful insights into travel behavior on multipurpose trips. These models can also be useful in generating valuable information for use in the analysis of nonresidential trip generation. For example, linkage parameters can be used to "explain" spatial variation in the trip attraction of a given nonresidential land use.

The Markov models also add a new dimension to the analysis of household travel behavior. Given sets of socioeconomically homogeneous households, trip length (measured in number of stops), linkages, and purpose distribution information may be added to the current body of knowledge associated with household travel behavior. Timeseries data on the same sets of family units provide a base for the examination of the stability of the aforementioned relationships through time.

Finally, there is the spatial connotation that probabilistic models can be given. By using areal units, such as census tracts or traffic analysis zones, in addition to land use or trip purpose to define a given state of the Markov process, information as to the spatial distribution of trip ends may be derived. Although trip distribution is beyond the scope of the research which is currently being conducted, future research efforts might well include the development of probabilistic models which can integrate generation and distribution.

ACKNOWLEDGMENTS

The support and assistance of the U.S. Bureau of Public Roads, and the efforts of the Texas Highway Department, the Waco Area Transportation Study, and the Texas Transportation Institute of the University of Texas in obtaining and processing the data used in this analysis are gratefully acknowledged.

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