Direct Estimation of Traffic Volume at a Point

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•THE IMMEDIATE impetus for the work reported here is the growing practical need for an easy and reliable way to estimate average traffic volume, a need that is making itself felt in many quarters and which might possibly be described as urgent in an area as large, complex, and irregular as the Tri-State Region.

There are also reasons other than the immediately practical for this undertaking. Conventional traffic assignment (calculating zonal interchanges and stringing them through a network), in addition to being arduous and complicated, is an essentially unfinished process. A better instrument should be found to offer more convenient traffic estimates and also to provide more confidence in technique and, hopefully, a theoretical basis for larger problems. Although this paper does not present any fundamentally different view of travel, it does at least state a new tactic.

The following discussion is concerned with the problem of estimating the number of vehicles passing one point on a street during a fairly long period of time, such as one day. It seems likely that the mathematics could be rephrased to yield turning movements as well as simple volumes; with some ingenuity perhaps the calculations could be extended to a system-wide set of estimates, and possibly the concepts could be enlarged to cover other modes of movement. However, none of these problems has been very well thought out yet, and they are not considered here. Also, discussion of the applied aspects of the ideas developed here are mostly reserved for future reports.

DERIVATION OF TRAFFIC VOLUME EXPRESSION

The argument leans heavily on certain directional symmetries imputed to the traffic system: all streets are two-way (for every section of street going in one direction there is another section a negligible distance away allowing movement in the opposite direction at the same speed), the total long-term (i. e., daily) flow past a point in one direction is the same as that in the other direction, and one or two other considerations. This should not prevent handling one-way streets in practice through considerations of reasonableness. Also, it is assumed throughout that every moving vehicle is coming from an origin and going to a destination, and that the terms "origin" and "destination" are concepts suitable for all uses to which they are put.

Imagine a street running north and south and a point of interest on the street at which traffic volume is to be calculated. Put a single hypothetical destination at a point of interest which is placed in such a way that it is completely accessible to both northbound and southbound vehicles and does not in the least interfere with traffic. The introduction of this destination changes nothing in the traffic situation except that sometime in the course of a day an additional vehicle has to occupy it; therefore it is necessary to subtract one from the traffic volume when it is finally computed.

There is a certain probability, P_n, that any northbound vehicle at the point of interest will stop at this hypothetical destination and another probability, P_s, that any southbound vehicle will stop. These probabilities multiplied by their respective traffic volumes, Q_n and Q_s, give the expected number of vehicles that will accept the destination, and that expected number must, of course, be one:

$$P_n Q_n + P_s Q_s = 1 \tag{1}$$

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or if $Q_n = Q_s$

$$P_n + P_s = \frac{1}{Q} \tag{2}$$

This uncomplicated proposition expresses a definite relationship between the probability of a vehicle leaving the traffic stream and the number of vehicles in the stream, and might serve as a point of departure for lines of reasoning quite different from the one followed next.

If P_n and P_s can be evaluated, then Q is determined and the problem is solved. Every northbound vehicle approaching the point of interest has declared, by being where it is, its intention of finding a destination in some fairly well-defined geographical region lying generally north of the point of interest. Assume that these vehicles are distributed among destinations within this north domain according to some function of position relative to the point of interest, so that

$$dP = CF\,dV \tag{3}$$

where dP is the probability that a vehicle approaching the point of interest will go to one of dV destinations clustered around a point at which the function F has a definite value. C is a constant determined by the condition that the vehicle must find its destination in the north domain:

$$\int_n dP = C\int_n FdV = 1, \text{ so } C = 1/\int_n FdV \tag{4}$$

The symbol $\int_n$ denotes integration over the entire surface of the north domain; in general, $\int_D FdV$ may as well be called a domain integral and be replaced by the symbol I_D. Thus the probability of having a destination in some particular region R within the north domain is, integrating Eq. 3, I_R/I_n.

At this point only two things need be stipulated about the function F: it should have a finite value at the point of interest and it should be of such a form that any domain integral will be finite, no matter how large the domain (assuming that the density of destinations is never infinite). These conditions are not restrictive; any sensible function would fulfill them.

The probability of a northbound vehicle taking the destination at the point of interest becomes

$$P_n = \frac{I_o}{I_n} \tag{5}$$

where I_o contains only that destination at the point of interest. As everything can be framed in exactly the same way from the southbound point of view,

$$P_s = \frac{I_o}{I_s} \tag{6}$$

But F can always be scaled to make I_o equal one, so that substituting Eqs. 5 and 6 into Eq. 2 gives

$$\frac{1}{Q} = \frac{1}{I_n} + \frac{1}{I_s} \tag{7}$$

or

$$Q = \frac{I_s I_n}{I_s + I_n} \tag{8}$$

However, Eq. 8 does not really amount to much as it stands. If F is assumed to be a descending function of simple distance-like parameters such as travel time, cost, etc., then Eq. 8 shows no sensitivity, or rather a perverse sensitivity, to competing facilities. For example, if the street of interest is a run-of-the-mill arterial and a parallel expressway a quarter of a mile away is opened up, the domain integrals in Eq. 8 will probably grow larger, leading to the result that an expressway competing with an arterial causes the volume on the arterial to increase. Plainly the network configuration must somehow enter into the distribution. But although it is hard indeed to think of network configuration as an explicit parameter, there is a tolerably easy revision of the distribution concept that amounts to the same thing.

Consider again the northbound stream of traffic at the point of interest, this time in the presence of a nearby expressway. Presumably, the stream is full of vehicles that have recently left the expressway. But these vehicles are not free to find a destination anywhere in the north domain. The fact that they have left the expressway implies that they are going to some subregion which excludes all places more easily accessible by remaining on the expressway. In general, any stream of traffic may be regarded as being composed of free vehicles able to go anywhere and fixed vehicles restricted by some event in their past history to a lesser destination domain. These lesser domains are referred to as n' and s', that is, north prime and south prime domains.

There is no reason why the fixed and free vehicles should not be subject to totally different distributions. However, it is highly plausible to suppose that the fixed distribution is the same function as the free but that it falls to zero everywhere outside the prime domain. This meets the essential condition that the two distributions tend to be the same, because the prime domain tends to be coextensive with the main domain, and it is helpful in other respects as well. Making this supposition, the probability of a vehicle being free and going to some group of destinations dV is $A_n (F/I_n)\,dV$, and the probability of a vehicle being fixed and going to the same group is $(1-A_n)(F'/I_{n'})\,dV$, where A_n is the fraction of free vehicles in the northbound stream and F' = F everywhere within the north prime domain but F' = 0 everywhere outside the prime. So the total probability of any vehicle going to this destination group, an amplified version of Eq. 3, is

$$dP = \left[A_n \frac{F}{I_n} + (1 - A_n) \frac{F'}{I_{n'}} \right] dV \tag{9}$$

and Eq. 5 becomes

$$P_n = I_o \left[A_n/I_n + (1 - A_n)/I_{n'} \right] \tag{10}$$

(remember that the point of interest itself is always inside the prime domain); also Eq. 6 expands into

$$P_s = I_o \left[A_s/I_s + (1 - A_s)/I_{s'} \right] \tag{11}$$

Once more, consider the stream of traffic northbound at the point of interest. All of these vehicles have originated somewhere in the south domain and have made their decision to terminate in the north domain. But generally there are regions in the south domain from which it is easier to get into the north domain by a route other than one leading past the point of interest, so that vehicles in the stream coming from these regions must be headed for some special part of the north domain, not merely to the north domain at large, and are by definition fixed. It can be argued that the special part they are going to is simply that region in the north which communicates with the south most easily via the point of interest, because if they were going somewhere else it would usually be possible to construct a better route than the one through the point of interest. On the other hand, vehicles in the stream which have originated in that part of the south where easiest passage to the north is through the point of interest exhibit no overt special intentions, other than ending up in the north domain, and are free.

So the fixed and free vehicles have been defined in terms of where they originate or, an equivalent way of looking at it, in terms of the route possibilities they have declined.

Furthermore, it appears that the destination area for northbound fixed vehicles—the north prime domain—is the same as the origin area for southbound free vehicles and the destination area for southbound fixed vehicles—the south prime domain—is the same as the origin area for northbound free vehicles. This is an important simplification without which troublesome complications set in. It should be regarded as an approximation to real behavior.

With regard to the point of interest, assume that the number of northbound vehicles originating in the south prime domain, the number of free vehicles, is equal to the number of southbound vehicles ending there. But the southbound vehicles going to the south prime domain have two components: the southbound fixed vehicles, all of which must go to the prime domain, and those of the southbound free group that happen to find their destinations in the prime. Transcribing this paragraph into notation gives

$$Q_n A_n = Q_s(1 - A_s) + Q_s \frac{I_{s'}}{I_s} A_s \tag{12}$$

and the Q's, of course, drop out if $Q_n = Q_s$. An exactly analogous equation can be written for the southbound free vehicles:

$$A_s = (1 - A_n) + \frac{I_{n'}}{I_n} A_n \tag{13}$$

Between them, these two equations determine A_n and A_s, yielding

$$\left.\begin{aligned} A_n &= \frac{r_s}{1 - (1 - r_s)(1 - r_n)} \\ \text{and} \quad A_s &= \frac{r_n}{1 - (1 - r_s)(1 - rn)} \end{aligned}\right\} \tag{14}$$

Thus

$$r_s = \frac{I_{s'}}{I_s}, \quad r_n = \frac{I_{n'}}{I_n}$$

Putting Eq. 14 into Eq. 10 and Eq. 11 and manipulating a little leads to modified forms of Eq. 5 and Eq. 6:

$$\left.\begin{aligned} P_n &= \frac{I_o}{I_n}\left[\frac{1}{1 - (1 - r_s)(1 - r_n)}\right] \\ P_s &= \frac{I_o}{I_s}\left[\frac{1}{1 - (1 - r_s)(1 - r_n)}\right] \end{aligned}\right\} \tag{15}$$

and now proceeding as in Eq. 7 produces, finally, the augmented counterpart of Eq. 8:

$$Q = \frac{I_s I_n}{I_s + I_n}\left[1 - (1 - r_s)(1 - r_n)\right] \tag{16}$$

Q is the one-way traffic past the point of interest, totaled over a long enough period of time (probably one day) for the symmetry postulates to hold.

BEHAVIOR OF TRAFFIC VOLUME EXPRESSION

So far only very weak delimitations have been imposed on the distribution function; it could be almost anything. Even so, there is a fair amount of visible character in Eq. 16.

The effective quality or competitive position of a street operates through the bracketed part of Eq. 16. The r's are the ratios of prime domain integrals to their respective main domain integrals; as the prime domains become a larger part of the main domains the r's and the entire bracketed expression grow larger. The bracketed expression achieves its maximum value, one, when the prime domains are so large as to include the entire main domains, a situation that would occur if the street of interest were, for instance, the only bridge across a long river. In this case the traffic itself, Q, would be greatest for any given I_s and I_n.

An expressway, because of its high speed, tends to have extensive prime domains, and therefore a large volume. Its extensiveness depends on its speed advantage and how far it is from other expressways. The prime domains of an ordinary arterial would usually be smaller, taking the form of strips running the length of the street and enclosing it, whereas those of a local street would be very small, pinched off after short distances. If the prime domains contained no destinations at all, the bracketed expression, and the volume, would be zero. Or, in stricter agreement with the theory, if the prime domains are so small that they include only the hypothetical destination at the point of interest, Eq. 16 reduces to a very close approximation of $Q = 1$ (a little less than one actually, expressing the slight possibility of the destination being its own origin).

The overall strength of the traffic field is measured by the left-hand part of Eq. 16, the factor in I_s and I_n. This strength increases as one or both domain integrals increase. Also, it goes to zero, taking the traffic with it, as either of the domain integrals goes to zero—a necessary property because a zero integral implies that there is no place a vehicle can go by passing the point of interest, as, for example, in the case of a street dead-ending at the ocean. For any given sum, $I_s + I_n$, the strength is maximized when $I_s = I_n$. Assuming the r's to remain constant, and without attempting a precise phrasing, this is to say that a given collection of destinations generates the most traffic at the point of interest when distributed evenly on both sides.

The domain integrals, of course, increase as destination density in the domains increases. Also, it may reasonably be suspected that they increase or decrease as destinations move nearer to or farther from the point of interest (although little is known about the distribution function). This leads to a final general inference from Eq. 16: that traffic at the point of interest tends to increase when surrounding destination masses increase or when these masses move closer, and tends to decrease when destination masses decrease or when they move farther away.

THE DISTRIBUTION FUNCTION

The function of F must be given precise definition in order to do any specific calculating from Eq. 16.

One convenient, acceptable function is

$$F = e^{-kt} \tag{17}$$

where t is travel time from the point of interest and k is a kind of natural constant. Or, more generally,

$$F = e^{-(k_1 t + k_2 u)} \tag{18}$$

where u is the cost incurred from the point of interest. Probably the simplest assumption that can be made about the distribution of vehicles among destinations is that all destinations are equally likely, subject to the constraint that average travel time must be finite even in an infinitely extensive universe of destinations.

Imagine the north domain to be divided into many cells, each containing the same number of destinations and having, therefore, the same a priori attractiveness for vehicles, and let the Q northbound vehicles at the point of interest distribute themselves among these cells so that the first cell receives q_1 vehicles, the second q_2, and so on. The Q vehicles can now be redistributed in such a way that the occupancy numbers q_1, q_2, etc., remain the same but not every vehicle is in the same cell as before.

The number of possible different arrangements of this kind for a particular set of occupancy numbers is

$$\frac{Q!}{q_1!\, q_2! \ldots q_n!} \tag{19}$$

The question can be asked: what set of occupancy numbers can be obtained in the most ways? This would be the set most likely to turn up at random because it can occur in more different ways than any other pattern. The set of occupancy numbers that can be obtained in the most ways is, of course, that set which maximizes Eq. 19, and it is the set in which all q's are equal.

Now the constraint that average (or total) travel time must be finite can be written

$$q_1 t_1 + q_2 t_2 + \ldots + q_n t_n = T \tag{20}$$

where t_i is the travel time to the ith cell and T is some finite constant. And the question in the preceding paragraph can be rephrased: what set of occupancy numbers consistent with Eq. 20 can be obtained in the most ways? This is a somewhat more sophisticated question, but it can be answered in essentially the same way: by determining the q's that maximize Eq. 19, although taking Eq. 20 into account. The procedure is to take the partial derivative of Eq. 19 with respect to each q_i, add to it a term proportional to the corresponding derivative of Eq. 20 (using Lagrange's multipliers), and set the sum equal to zero. This involves both manipulation and approximation, and the result is

$$q_i = e^{-\lambda t_i} \tag{21}$$

which is the same as Eq. 17 once the notation is adjusted to conform to previous usage. If Eq. 17 is used as the distribution function, I_0 in Eq. 5 and those following it naturally equals one without any further meddling.

Cost can be introduced in a completely analogous fashion by arguing that just as travel time must be limited, so must travel cost. This produces another constraint,

$$q_1 u_1 + q_2 u_2 + \ldots + q_n u_n = U \tag{22}$$

Equation 19 can now be maximized subject to both Eq. 20 and Eq. 22 and the result is equivalent to Eq. 18.

WORKING METHODS

With F defined, it becomes possible in principle to evaluate the integrals of whioh the traffic estimate is composed. The possibility in principle, however, scarcely helps when it comes time to go ahead and do it in practice, yet preserve the measure of convenience that is the most important aspect. If the conditions of destination density and network geometry were regular, the integrals could be evaluated by direct mathematical operations. In the real, unaccomodating world the integrals can still be calculated numerically, but any straightforward numerical technique would seem to be laborious.

There is a question of how much detail and precision the whole process deserves. The input information—network speeds and destination densities—is not really well defined or accurately obtainable, and the theoretical structure itself does not have the ring of final truth. Moreover, conventional assignment often yields wildly inaccurate results on the level of specific street estimates, yet is generally regarded as an acceptable methodology. In short, at present no great accuracy seems to be either possible or expected.

Further, a previous paper (1) derived traffic estimates, under drastically simplifying assumptions, from extremely rudimentary information: a single average trip-end density and single average spacings of local streets, arterials, and expressways. Although this was not a practicable procedure for a variety of reasons, it did produce estimates with a rough, order-of-magnitude realism. It is suggested that a small number of pieces of information above this bare one-point level would produce a great improvement without being too difficult.

The pieces of information might take the form of readings at points scattered throughout the region, constituting a kind of sample of the region. These readings would consist of the best route travel time from the point of interest to the point of reading and the average destination density around the point of reading. If the location of each reading point is known, the read values could be interconnected by an arbitrary interpolation and the necessary integrations performed; some description of the borders of the north, south, and prime domains would also be required. Thus the precision of the method would be directly related to the amount of work put into it, i. e., to the number of reading points. Inasmuch as the contribution of any area to traffic at the point of interest diminishes with distance, the readings can grow farther and farther apart as they move away from the point of interest. To be mathematically convenient, the reading points should lie in a regular pattern, and this pattern should be fixed so the person taking the readings is not free to make a biased choice of points.

Based on these considerations, the working method presently used is this. A template or transparent overlay is drawn showing radial lines emanating from a point; rings intersect the radials, with the spacing of successive rings becoming larger as they lie farther from the center. This template is overlaid on a map containing the street system, with speeds indicated and destination densities blocked in, with the center of the template right on the point of interest. For each intersection of ring and radial, the reader estimates best route travel time from the center, notes the ambient density, and enters these values on a form. In a separate operation, he writes down polar coordinate points (from the same template) which, when connected by straight lines, will reasonably delineate the prime domains. For simplicity, the north and south domains are considered to be demarcated by a straight east-west line (the directional terms are schematic, of course); this is a convenience of the moment, not an essential simplification, and will very likely be revised. The rule for drawing a prime domain is that the prime should include all points from which it is easier to cross the main domain line by passing the point of interest than by any other way, and should exclude all other points.

The forms containing these readings are key-punched and the cards fed into an IBM 1401 computer, which performs all the complicated calculations, ending in an estimate of traffic at the point of interest. Linear interpolations are made among the point values, allowing integral terms to be computed.

These methods seem to fall within a tolerable range of labor. The readings do not seem too hard to execute, and the computing is quick and easy. A total reading time of an hour or two and computing time (1401) of 10 or 15 min seems within shooting distance. A lot depends on where the point of diminishing returns lies in the number of readings. Also, none of this should be regarded as fixed; some better working scheme might very well emerge to replace it.

CONCLUSION

The mathematical and computational forms developed here appear at this time to represent traffic behavior. A dozen or so real cases have been calculated. Although the purpose of these calculations has been to regularize the technique rather than to subject it to strict tests, and although good input information is not yet available, it seems fair to say that the results, for so early a state of evolution, are quite promising. The calculations evidently admit a full range of traffic volumes, from local street to expressway, and the practical labor is within reason. Exact results ought properly to be considered meaningless right now, and when bad they are so considered. However, the comparison with listed traffic flows is fairly good for a first trial.

A number of practical problems are turning up. In some cases, delineation of prime domains is ambiguous—different people will draw them with appreciable differences—

perhaps calling for more carefully devised rules. The working method described in the foregoing introduces various statistical problems, all of which can probably be solved, concerning the number of reading points to be used and their pattern, whether or not the prime domains should be sampled differently from the main domains, and how to avoid statistical wastefulness, i. e., taking readings where there is no real gain in information or, conversely, throwing information away because of the arrangement of the template. There are also customary minor difficulities of thinning out errors in reading and transcribing.

ADDENDUM

Since this paper was written some developments have taken place which ought to be at least briefly mentioned here.

A large-scale computer program has been written, and is now being tested, to calculate traffic volumes throughout a very large network automatically. The program is designed to produce volumes on any specified links, on all links within a specified area or group of areas, on links of a selected class (such as expressways), or on all links in the entire system. Turning movements can also be requested pretty much at will, and the program will do its best to compute them. Other options, capacity restraint among them, are imaginable and may be added.

The map-and-template method has been pursued beyond the previous discussion, but not too far beyond. Although it was convenient for making experimental calculations and remains useful for situations in which no coded network is available, it is not at all competitive with a fully computerized system once the inputs for such a system have been prepared.

Several quirks in the behavior of Eq. 16 have turned up and the exact form given here probably will not long survive. It appears, though, that most of these aberrations can be removed by a plausible modification, entailing no loss of generality and leaving the basic reasoning intact.

In summation, there seems now to be a good chance of developing a durable methodology which will improve traffic-estimating technique through the sheer force of its flexibility. The user can focus his attention, effort, and budget on that aspect of traffic estimating that concerns him, from minor detail to generalized planning. A complete set of system-wide link volumes is seldom of much interest. Most often only a relative handful of estimates, expressing local finer resolution or the results of planning changes, is really wanted, and this handful can be obtained in a few seconds of computer time, without having to run a specially scaled, full assignment. At the same time, satisfactory and systematic estimates can still be made by any one of many possible variations on the map-and-template method when tying into a big computer system is not warranted. In short, the technique looks well-tempered: the precision (not to be confused with accuracy) and extent of the calculations—or, more generally, their expense—can be made consistent with the uses to which they are put.

Also, the direct estimation point of view seems adaptable to a larger scheme. Recently, a fragment of a theory has been worked out which includes travel within a more general framework, relating activity at a site to something that might be called accessibility of the site. On the face of it, this partial theory has nothing much to do with the work described here, except that the direct estimation mathematics can be reformulated within its context. When that is done, however, the term giving rise to the most stubborn (and disturbingly fundamental) of the quirks mentioned is precisely canceled out. Moreover, it turns out that the domain integrals introduced here have a distinct kinship with quantities appearing in the newer theory, and that it might be possible to lead into a land activity model using programs and materials developed for direct estimation.

Finally, since an explicit calculation has been stated which gathers the access of a piece of road to the geography in which it is embedded into a traffic flow on the road, the natural speculation arises: might not the process be reversed? Might not the pattern of traffic flows imply the geographic spread of activities, and traffic counts be used to measure activity on a piece of land?

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