A Statistical Analysis of Embankment Compaction

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This study statistically examined the distribution of percent relative compaction obtained with current compaction control procedures. The survey included three embankment projects, the soils of which varied from homogeneous to very heterogeneous material. Testing operations for each sampling location included two in-place density determinations by the sand volume method, and two maximim density determinations by the California impact method for each sand volume test.

An analysis of percent relative compaction results for the three projects revealed average values of 92.9, 90.5, and 93.6 percent with standard deviations of 2.4, 3.1 and 5.5 percent, respectively. The greatest dispersion in results was found to exist for the heterogeneous soils.

Factors contributing to the dispersion of percent compaction were found to be the variation inherent in the testing procedure, the soil, and in the compaction process. As the soil becomes more heterogeneous, the effects of variation within the soil and compaction process become more pronounced. This is reflected in the distribution curves for the three projects. Curves are presented which provide a comparison of field control test results and randomly sampled test results. A partial review of problems expected to be encountered in the development and use of purely statistical specifications is presented.

•THE existence of variations in embankment compaction and in the associated control tests has been recognized for a number of years, although many engineers have not been greatly concerned with the extent of variability. The lack of concern regarding variations in test results may be attributed to the type of specification most often employed—which contains a lower limit only. For this type of specification, the dispersion of results is relatively insignificant in relation to construction control procedures.

Highway engineers recently have become more aware of, and interested in, the variability in compaction, due mainly to the efforts of the U.S. Bureau of Public Roads to improve present specifications. The embankment compaction specification used at the AASHO Road Test included the statistical concept of quality control, which also helped to stimulate this interest. However, the use of statistical methods at the Road Test was primarily to insure uniformity of quality in order to better correlate road performance with quality of construction. Therefore, the main objective was to control compaction variation as much as practical rather than to determine variations obtained with usual construction procedures (1).

Data regarding the reproducibility of test methods for measuring in-place and maximum densities have been reported since about 1950. However, except for the works of Davis in 1953 (2) and Carey in 1957 (3), very little information has been published regarding variations in density of compacted embankments. One of the primary purposes of this study is to add to the knowledge concerning the statistical parameters of relative compaction.

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SAMPLING AND TESTING PLAN

The BPR Outline

The U.S. Bureau of Public Roads, through their regional workshops, presented to state highway department representatives a general outline for statistical surveys. The Bureau then left to the individual states the formulation and execution of sampling and testing plans for those particular items selected by the states for study. The general outline included the following requirements:

1. For each item being considered (in this case embankment compaction control), at least three separate construction projects should be surveyed. These three projects should represent, as nearly as possible, the range of problems and materials encountered throughout the state.
2. At least 50 sampling locations should be randomly chosen for each project.
3. Two samples should be taken at each sample location.
4. Duplicate tests should be made on each of the two independent samples taken from each location.
5. The samples should be taken, as nearly as possible, under normal field conditions by district construction inspectors.
6. The study should be independent of, and in addition to, the normal job testing and control procedures.
7. Only those materials accepted by the resident engineer should be considered in the survey.
8. Whenever possible, ASTM or AASHO test methods should be employed. When necessary to use a test method primarily of local acceptance, a similar ASTM or AASHO test should also be performed.
9. Analyses of test data should include an analysis of variance. This would include a measure of the variance between tests on duplicate samples, the variance inherent in the sampling method, and variance inherent in the material or process.

The duplicate samples from each location provided a measure of the variance inherent in the sampling process. Duplicate tests on each sample provided a measure of the variance inherent in the testing process, and the 50 test locations on each project provided a measure of the basic variance in the process or material.

Modifications of the BPR Outline

The BPR outline was general in nature and could be applied to many construction materials or processes. Because of its generality, certain modifications were necessary for physical reasons. For example, in the case of embankment compaction, the sampling and testing operations are not independent because it is not possible to split a sand volume sample to obtain two independent test results. As a compromise, two in-place density tests were made by the sand volume method at each location (Fig. 1). These tests were taken reasonably close together and never more than 3 ft apart. A sample of the soil was taken for each sand volume test and then was carefully split for maximum density determinations. Thus, at each location results of two sand volume tests and four laboratory maximum density tests were obtained.

On both Projects 1 and 2, 50 locations were sampled; but on Project 3, rain cut short the construction period and only 44 locations were sampled. It was also necessary to depart somewhat from the original request that these samples be chosen in a completely random manner. A true random sampling plan for the entire project would have required the locations to be randomly selected from the entire volume of fill material to be placed on the project, thus allowing each incremental volume an equal chance of being sampled. In accordance with construction needs, and to keep a reasonably uniform flow of work to the testing engineer, it was necessary to randomly choose one location each day from a fill area accepted as satisfactory by the resident engineer. Thus, at the end of the working day the sampler and the grade inspector established those areas or sections which had been tested and accepted by the resident engineer's personnel. The research sampler then determined the total length of these sections

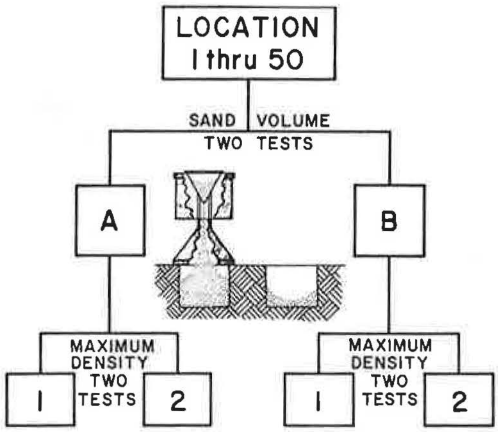


Figure 1. Testing procedure for each sampling location.

and multiplied this length by a random number taken from a table. This established a length which was readily converted to a station location. The sampler then stepped across the fill at this station to determine the width and multiplied this width by the next number from his table, thus establishing a random location for testing. The following day he repeated the process on a new area using the next set of random numbers from his table. This deviates from a true random sample because the areas from which the daily samples were chosen were not of equal size. This seemed, however, an acceptable compromise with the engineering needs. The system worked very well since one man was able to determine the location and do all necessary field and laboratory testing in one day. An example of this random sampling from an area is included in the Appendix.

Test Methods

The percent relative compaction was determined by California Test Method No. 216-F. Since this method is primarily of local acceptance, additional tests by AASHO Test Method No. T180-57C were made to provide both a comparison of results and a check of survey data. Previously reported work showed that the results of the two methods correlate with most types of soils (4).

The primary difference between these two methods is in the laboratory apparatus and procedure. The compaction mold for the California Test Method is 3 in. in diameter and the specimen height varies from 10 to 12 in. Consequently, the volume is variable. The mold of the AASHO method has a diameter of 4 in. and a constant height of $4\frac{5}{8}$ in. The tampers both weigh 10 lb and free drop 18 in. Both methods utilize 5 layers. Each layer is subjected to 20 blows in the California method and 25 blows in the AASHO method. The resulting compactive energies are approximately 33,000 and 56,250 ft-lb/cu ft for the California and the AASHO methods, respectively. Maximum densities by both methods are determined from that portion of material passing the $\frac{3}{4}$-in. sieve. Corrections for larger size material are applied to results from the California method if the percentage of larger sizes exceeds 10 percent. No corrections are applied to results from the AASHO method.

The California Test Method does not require the determination of moisture content unless the correction in unit weight is made for oversized material. As a result, information regarding moisture content was not always available. Results presented here are therefore based on wet unit weights.

PROJECT DESCRIPTIONS

The three contracts in this survey were major projects on divided four-lane highways. Projects 1 and 3 closely approximated the smallest and largest variation in soil characteristics normally expected in California. Project 2 was somewhere between these two extremes. Typical grain size distribution curves are shown in Figure 2. Table 1 includes sieve analyses, liquid limits, plastic limits, plasticity indices, and sand equivalent values.

Project One

The embankment material on Project 1 was primarily a highly decomposed granite, weathered to a clayey silty sand of medium plasticity. Embankments consisted of

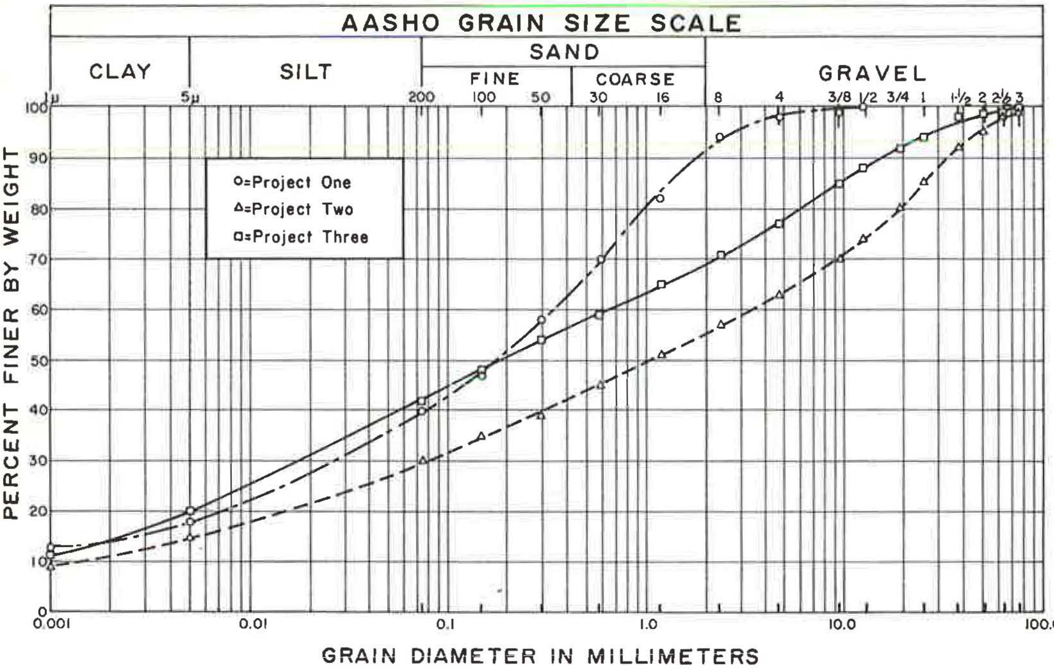


Figure 2. Grain size distribution curves.

TABLE 1

INDEX PROPERTIES AND GRADATION OF EMBANKMENT SOILS

Identifying Properties	Project 1 Avg.	Project 1 Range	Project 2 Avg.	Project 2 Range	Project 3 Avg.	Project 3 Range
Sieve analysis (% passing)						
3 in.			99	97-100	100	
2½ in.			98	96-100	99	99-100
2 in.			95	92-100	99	98-100
1½ in.			92	85-96	98	96-99
1 in.			85	78-91	94	91-97
¾ in.	100	99-100	80	73-88	92	86-95
½ in.	100	98-100	74	66-84	88	79-93
⅜ in.	99	97-100	70	62-81	85	72-91
No. 4	98	96-99	63	55-77	77	59-88
No. 8	94	89-97	57	48-69	71	52-80
No. 16	82	76-88	51	43-63	65	45-76
No. 30	70	62-77	45	37-59	59	38-70
No. 50	58	50-66	39	32-53	54	33-65
No. 100	47	40-54	35	28-47	48	27-59
No. 200	40	34-49	30	24-43	42	23-54
5 micron	18	15-20	15	10-23	20	11-27
1 micron	13	12-14	9	6-15	11	3-17
Sand equivalent	15	12-17	12	9-15	10	7-17
Liquid limit	25	21-28	33	32-34	29	22-34
Plastic limit	15	14-16	21	19-22	17	15-20
Plasticity index	10	7-13	12	11-13	12	7-17

Number of tests made to determine above items: Project 1, 10 tests per item; Project 2, 10 tests per item; Project 3, 7 tests per item.

shallow fills across valley terrain. Most of the main line fills were only 2 or 3 ft in height with one short section reaching 14 ft. The soil was in a fairly dry natural state and water had to be added. The project was 4.1 mi in length and the total embankment involved only 350,000 cu yd.

Project Two

Project 2 was in a region of rolling terrain where the material was predominantly a medium plastic red clayey silty sand containing lenses of stream-rounded aggregate with cobbles up to 6 in. in diameter not uncommon. Some aggregate was well dispersed throughout the fines, and it was possible to excavate this material without blasting.

The total length of the project was 5 mi and the height of the embankments varied from 3 or 4 ft to a maximum of 26 ft. A total of 1,200,000 cu yd of embankment was placed on this project.

Project Three

Project 3 was in the Franciscan Formation, which is characterized by landslides as well as erosion. Many of the landslides in this area are still active and the slip surfaces are characterized by wet, low-strength material. Even some of the harder materials had a relatively high moisture content, a common characteristic of the sandstone and sheared shale of the Franciscan Formation. Blasting was often required during excavation. Some fills were so rocky that they had to be excluded from this study, while others were predominantly clay and silt. On some fills the contractor found it necessary to blend dryer materials with the wet, heavy clays in order to achieve a satisfactory water content. The project was 4.5 mi in length and had 1,760,000 cu yd of embankment. Height of fills varied from 3 to 38 ft.

DISCUSSION OF RESULTS

California Impact Test vs AASHO Test T 180-57C

In addition to the statistical survey tests, which were performed by district personnel according to the California test procedure, further tests for maximum density were made by Headquarters Laboratory according to both the California and AASHO procedures. These additional tests were made to provide a means for comparing maximum density results as obtained by the two methods, since the California method was primarily of local acceptance.

On Project 1, two tests by the AASHO Test Method T180-57C were made at each of 26 sampling locations. These were performed at the job site by Headquarters Laboratory personnel. On Projects 2 and 3, ten and seven sampling locations were selected, respectively, and material was shipped to the Headquarters Laboratory for testing.

TABLE 2

COMPARISONS OF MAXIMUM DENSITY DETERMINATIONS
(California 216-F vs AASHO T180-57C)

Computed Quantity	Project 1	Project 2	Project 3
Number of Locations	26	10	7
Avg. Dist. (Calif. method)	141.5	140.2	147.7
Avg. HQ (Calif. method)	—	140.7	146.6
Avg. HQ (AASHO method)	140.8	141.8	147.0
Average Difference:			
Dist. results (Calif. method) minus HQ results (Calif. method)	—	+ 0.5	- 1.0
Dist. results (Calif. method) minus HQ results (AASHO method)	+ 0.72	+ 1.6	- 2.1

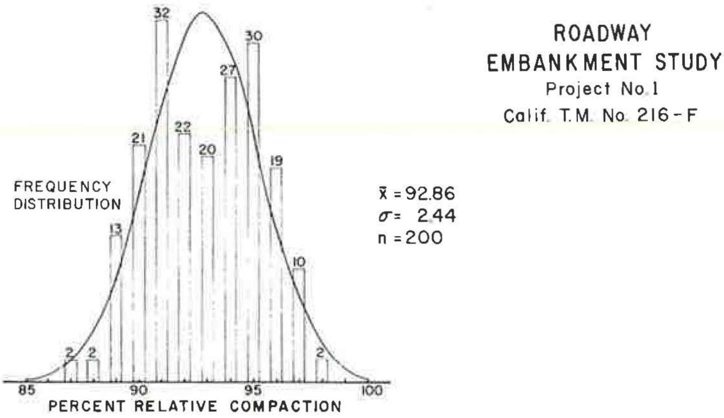


Figure 3. Relative compaction histogram, Project 1.

One maximum density determination by each method was performed on the material shipped from each sampling location. Results of these tests are given in Table 2.

For all practical purposes, the California and AASHO methods produce approximately equal average results. However, this conclusion is based on a small number of tests. The findings are in agreement with earlier published information (4).

Percent Relative Compaction

The percent relative compaction distributions for the three projects are shown in Figures 3, 4, and 5. Each figure consists of a frequency histogram of the actual survey data and a normal curve. The normal curve for a particular project represents the most probable distribution for all possible test results from that project. No explanation other than random variation was found for the bimodal distribution shown in Figure 3 or the non-normal distribution shown in Figure 5.

The plots show significant differences in the dispersion of relative compaction results for the three projects. This dispersion of compaction values about their average could be due to several factors, all of which affect both maximum and in-place density values. These factors include the variation in soil properties and the nonuniformity of field compaction conditions within the area tested. For example, local variations in the soils of Projects 1 and 2 were appreciably less than the variation for Project 3.

A portion of the dispersion may be inherent in the basic testing process. In-place and maximum densities of a particular soil are related; therefore, the practice of expressing one as a percentage of the other would seemingly compensate for variations

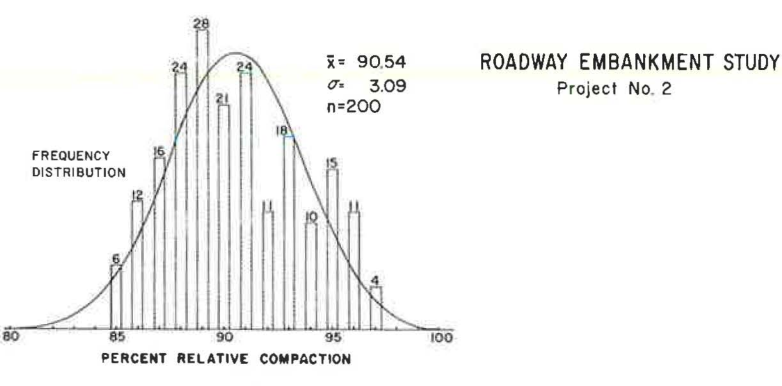


Figure 4. Relative compaction histogram, Project 2.

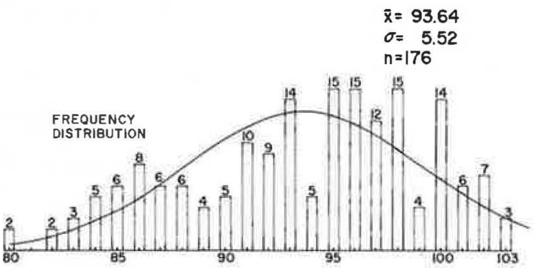


Figure 5. Relative compaction histogram, Project 3.

in magnitudes of the two and result in a fairly constant value for relative compaction throughout a project. In many instances, however, laboratory compaction is not entirely representative of field compaction. Consequently, values for the two densities often do not change in the same ratio, even within small areas for highly variable soils, thereby causing variations in relative compaction values.

The effects of soil variations on compaction were observed in 1953 by Davis (2). His statistical findings were from 29 construction locations on 23 earth dam projects. Davis reported standard deviations ranging from approximately 1.8 to 5.0 percent relative compaction with all 29 locations averaging 3.3 percent. The standard deviation for embankment soil under flexible pavement sections of the AASHO Test Road was approximately 1.85 percent relative compaction (1). This low standard deviation, however, was obtained with much more rigid control and a greater number of tests than would be economical for normal construction projects. Another factor contributing to the low standard deviation was the extremely uniform soil used on the Test Road. The standard deviation reported by both Davis and AASHO are in general agreement with the standard deviations determined for the three projects reported here.

The percentages of tests in this study failing to meet the minimum compaction requirement are comparable to previously reported data. Results from the AASHO Road Test, for example, indicate that approximately 8.5 percent of all embankment material tested failed to meet the lower specification requirement (1). Statistical estimates indicate that the percentages of failures in Davis' data vary from about 10 to 25 percent, with a few as high as 45 percent.

Numerical values from Figures 3 through 5 are summarized in Table 3. All values were computed from the special survey data only. The data in Table 3 illustrate the dependence of percent failing on the relationship between average and standard deviation.

TABLE 3

SUMMARY OF RELATIVE COMPACTION RESULTS

Quantity or Characteristics	Project		
	1	2	3
Number of sampling locations	50	50	44
Number of relative compaction determinations	200	200	176
Range of relative compaction results (percent relative comp.)	87-98	85-97	80-103
Average compaction (percent relative comp.)	92.9	90.5	93.6
Standard deviation (percent relative comp.)	2.4	3.1	5.5
Percentage of compaction tests less than spec. limit of 90 percent relative compaction	8.5	43.0	23.9

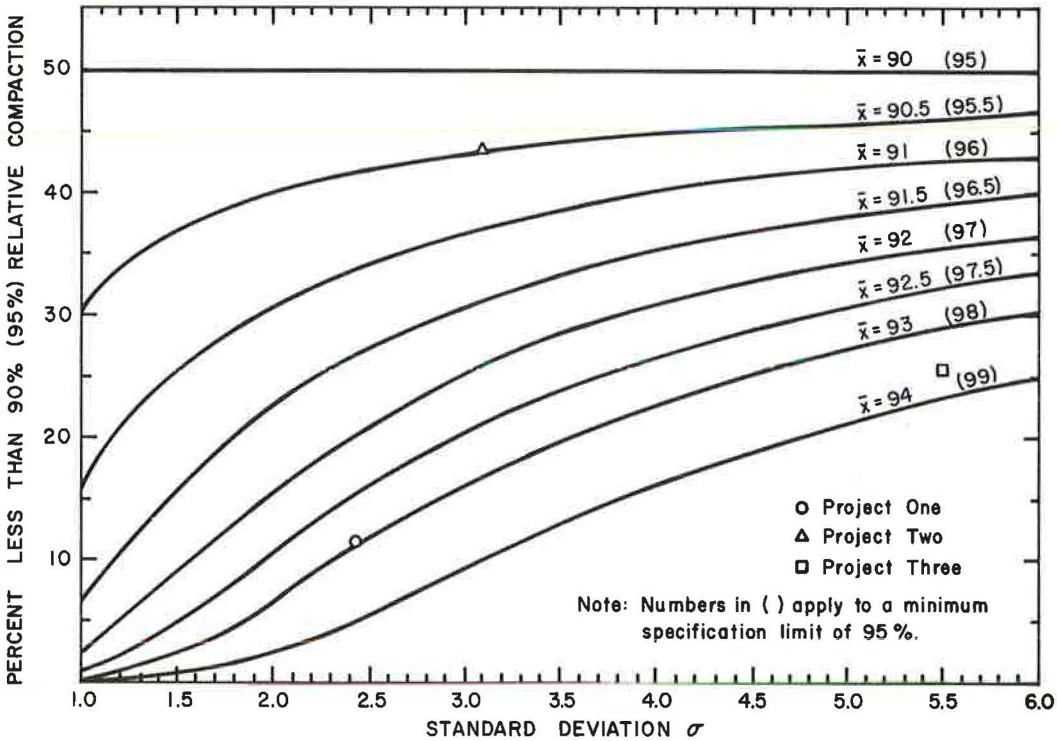
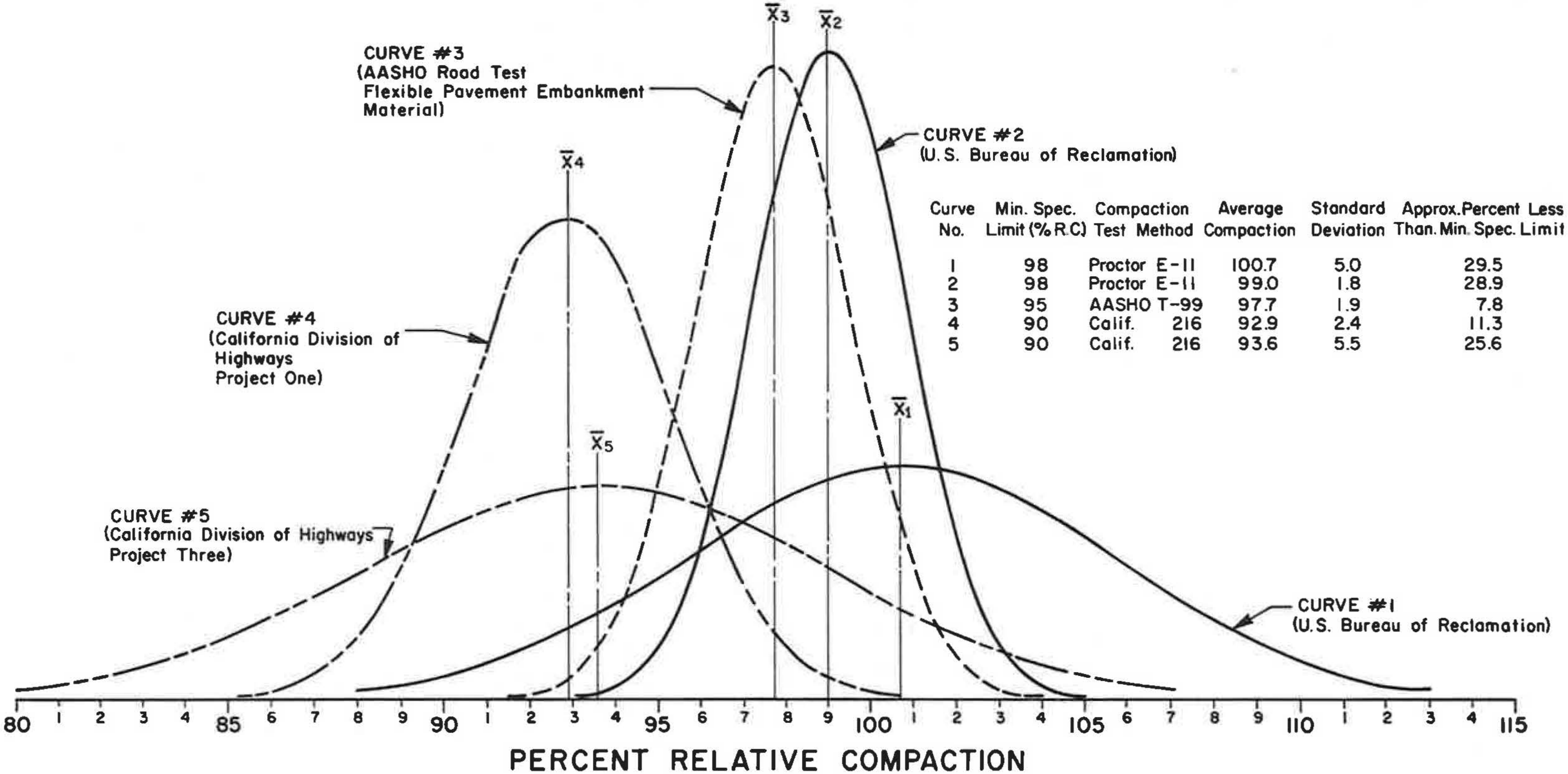


Figure 6. Relationship between standard deviation, average and percent less than lower compaction limit.

For example, comparing the values of the average, standard deviation, and percent failing for the projects shows that the percentage of failures tends to decrease with an increase of average, and increase with an increase of dispersion, as measured by the standard deviation (Fig. 6). The curves in Figure 6 show the percent failing plotted against standard deviation, with average as a parameter. Although the curves are theoretical in nature and are based on normal distributions, they produce values very close to those in Table 3 for the three projects surveyed.

Figure 6 shows that with the three soils tested with the measured standard deviations, the following overall averages would have to be obtained, if no more than 10 percent of the finished embankment is to be below the 90 percent specification limit: Project 1, 93 percent; Project 2, 94 percent; and Project 3, 97 percent. Within the limits of experimental error, Project 1 meets this criteria; however, the average values for both Projects 2 and 3 would have to be increased by 3 percent, if no more than 10 percent is to be below specifications.

Figure 7 shows some selected normal distribution curves superimposed on the same scale. This illustrates to some extent the relative dispersion in percent compaction for projects of different organizations. Curves 1 and 2 were prepared from the data reported by Davis (2) and represent two of the 29 construction locations. The minimum specification limit for this earthwork was 98 percent relative compaction, as determined by the Bureau of Reclamation's Standard Proctor Compaction Test Designation E-11. Curve 3 represents the AASHO Road Test embankment material for flexible pavement sections. Upper and lower specification limits of 100 and 95 percents were employed for this project. Maximum densities were determined by AASHO Test Method T-99. Curves 4 and 5 are those of Projects 1 and 3 of this study (Fig. 3 and 5). The minimum specification limit for these latter two curves was, of course, 90 percent using the



Curve No.	Min. Spec. Limit (% R.C)	Compaction Test Method	Average Compaction	Standard Deviation	Approx. Percent Less Than Min. Spec. Limit
1	98	Proctor E-11	100.7	5.0	29.5
2	98	Proctor E-11	99.0	1.8	28.9
3	95	AASHO T-99	97.7	1.9	7.8
4	90	Calif. 216	92.9	2.4	11.3
5	90	Calif. 216	93.6	5.5	25.6

Figure 7. Comparison of normal distribution curves from three organizations.

TABLE 4

EFFECTS OF AVERAGING INDIVIDUAL READINGS

Project	Results of Individual Determinations			Results After Averaging		
	n	X̄	σ	n	X̄	σ
1	200	92.9	2.4	50	92.9	2.2
2	200	90.5	3.1	50	91.0	3.0
3	176	93.6	5.5	44	93.7	4.5

California method. It is interesting to note in comparing curves 1 and 2 with curves 4 and 5 that the flatter curves, representing greater dispersion, have higher averages than the corresponding steeper curves.

When comparing the curves in Figure 7, it should be noted that they were calculated from the data of three organizations, all of which have different specifications and test procedures. This factor, plus the fact that the soil characteristics for some of the projects represented by the curves are not known, greatly limits the ability to make meaningful direct comparisons between the curves. However, Figure 7 does indicate that various agencies are obtaining similar variations in embankment compaction results for widely different material types.

The effects of averaging the in-place and the maximum density values on the resulting relative compaction distribution are given in Table 4. These relative compaction results were computed from the average of two in-place densities and the average of four maximum densities per location.

The reduction in dispersion given in Table 4 would be an important consideration in the enforcement of specifications. For example, for Project 1 a range of three standard deviations, ±6.6, would be acceptable providing tests were averaged as detailed to obtain the acceptance value. On the other hand, if acceptance is to be based on a single test, a range of ±7.2 must be established to allow for the wider dispersion.

Precision of Test Method

Distributions of maximum and in-place density test data are shown in Figures 8 through 20. Figures 8 through 13 and 17 through 20 show frequency histograms of all the survey density test results. Figures 14 through 16 show maximum density and percent relative compaction against roadway stationing.

The histogram of maximum densities for Project 1 (Fig. 8) exhibits a concentration of test results within the 140-148 lb/cu ft range, which results in a skewed distribution that appears to have been constructed from two distinct sets of data. The explanation for the skewness was found in Figure 14, which shows two distinct soil types located at different stations along the roadway. Test results for both maximum and in-place densities were separated into two groups each, based on Figure 14, and plotted as histograms in Figures 17 through 20.

Figures 17 and 18 are easily recognized as the two parts of Figure 8. No similar breakdowns of soil type by location for Projects 2 and 3 were observed from Figures 15 and 16. These figures show the test results to be dispersed appreciably, but the range and approximate mean appear to be fairly constant throughout both projects.

The maximum density plots for Projects 2 and 3 (Figs. 9 and 10) reveal a wide range in test values for both projects. The histogram for Project 2 approaches a normal distribution, although the same cannot be said for Project 3.

The local scatter in maximum density values may be taken as a good indication of the variation in soil homogeneity when comparing different projects or areas. For example, the 8 lb/cu ft range between stations 390 and 431 for Project 1 (Fig. 17) represents a relatively homogeneous soil when compared to the 28 lb/cu ft range for Project 3. Figures 11 through 13 show the distributions of in-place densities to have even greater dispersion than the maximum densities.

It should be pointed out again that, since the California Test Method usually does not require that the moisture content be determined, all densities are recorded as wet weight. It is realized that dry weight determination would provide additional information, but it is observed that wet weights do provide a good comparison of the uniformity of the three projects.

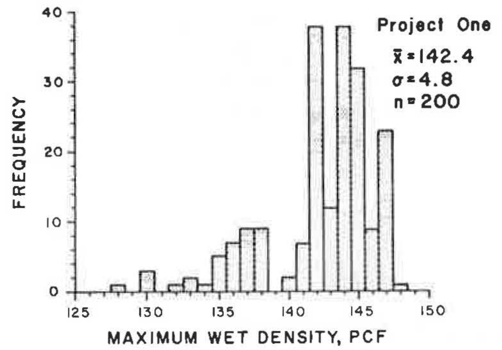


Figure 8. Maximum density histogram, Project 1.

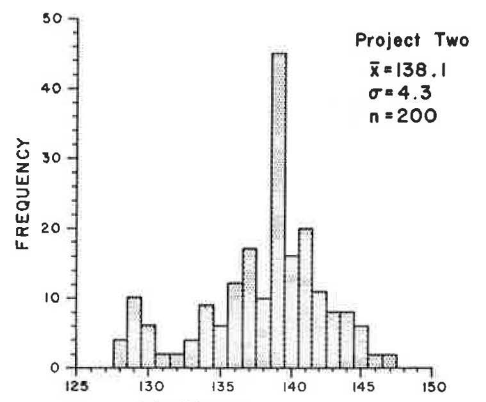


Figure 9. Maximum density histogram, Project 2.

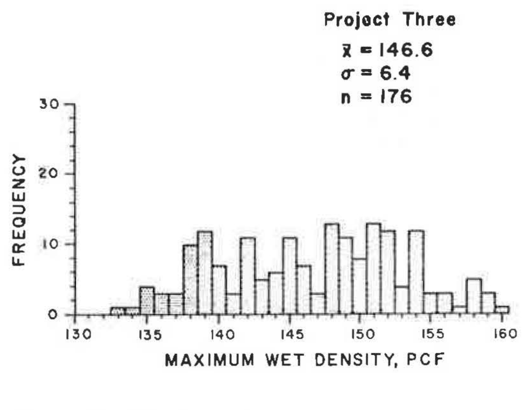


Figure 10. Maximum density histogram, Project 3.

In-Place Density Variations

The maximum recorded variations between two in-place density tests from the same location were 6, 10, and 28 lb/cu ft for Projects 1, 2 and 3, respectively. Actually the one variation of 28 for Project 3 was probably due to some assignable cause and a more realistic maximum would be the next lower observation, i.e., 18.

On Project 1, a shallow fill 4,100 ft long (station 390 to 431) was constructed with unusually homogeneous soil. Thirty-eight pairs of duplicate sand volume tests were performed on this one fill. The standard deviation of the variation between these adjacent sand volume determinations was 2.25 lb/cu ft. This means that 95 percent of the time in this type of soil we could expect the results of two sand volume tests in juxtaposition to agree within the limits of 4.5 lb/cu ft. These results are in close agreement with one study performed on carefully processed uniform soils (8).

On Project 3, where the material was extremely heterogeneous, the standard deviation of the variations between adjacent sand volume tests was 5.96 lb/cu ft. Therefore, for this type of soil, we can expect the results of any two adjacent sand volume tests to agree within the limits of 11.9 lb/cu ft 95 percent of the time.

On Project 2, where the variability of the material is somewhere between the other two projects, the standard deviation of the variation between adjacent sand volume tests was 3.13 lb/cu ft. For this type of soil we can expect results of two adjacent sand volume tests to agree within the limits of 6.3 lb/cu ft.

Remember that these seemingly large variations include not only the inaccuracies within the sand volume test itself, but also the variation in the material and compaction process within the small areas from which the pairs of adjacent tests were taken.

Mention should be made that the California Division of Highways has conducted experimental investigations into the use of nuclear testing equipment. It has been generally concluded that the nuclear equipment has about the same reproducibility as the sand volume test. One study indicated that nuclear surface gage readings could be reproduced within 9 to 10 lb/cu ft

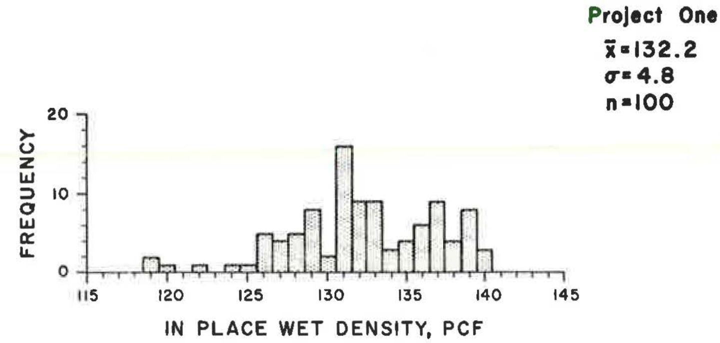


Figure 11. In-place density histogram, Project 1.

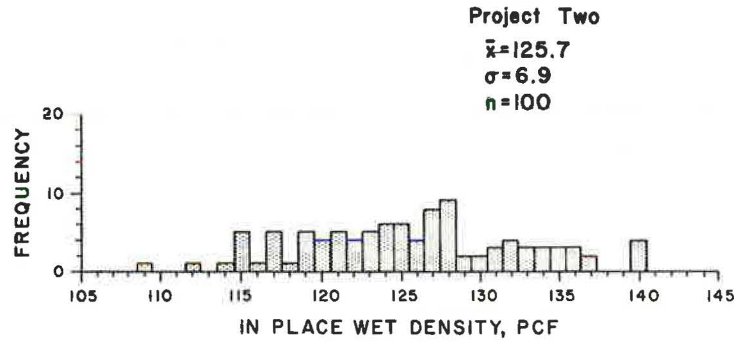


Figure 12. In-place density histogram, Project 2.

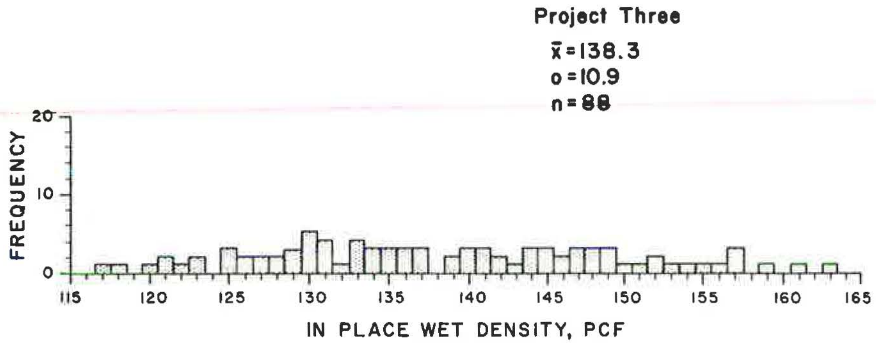


Figure 13. In-place density histogram, Project 3.

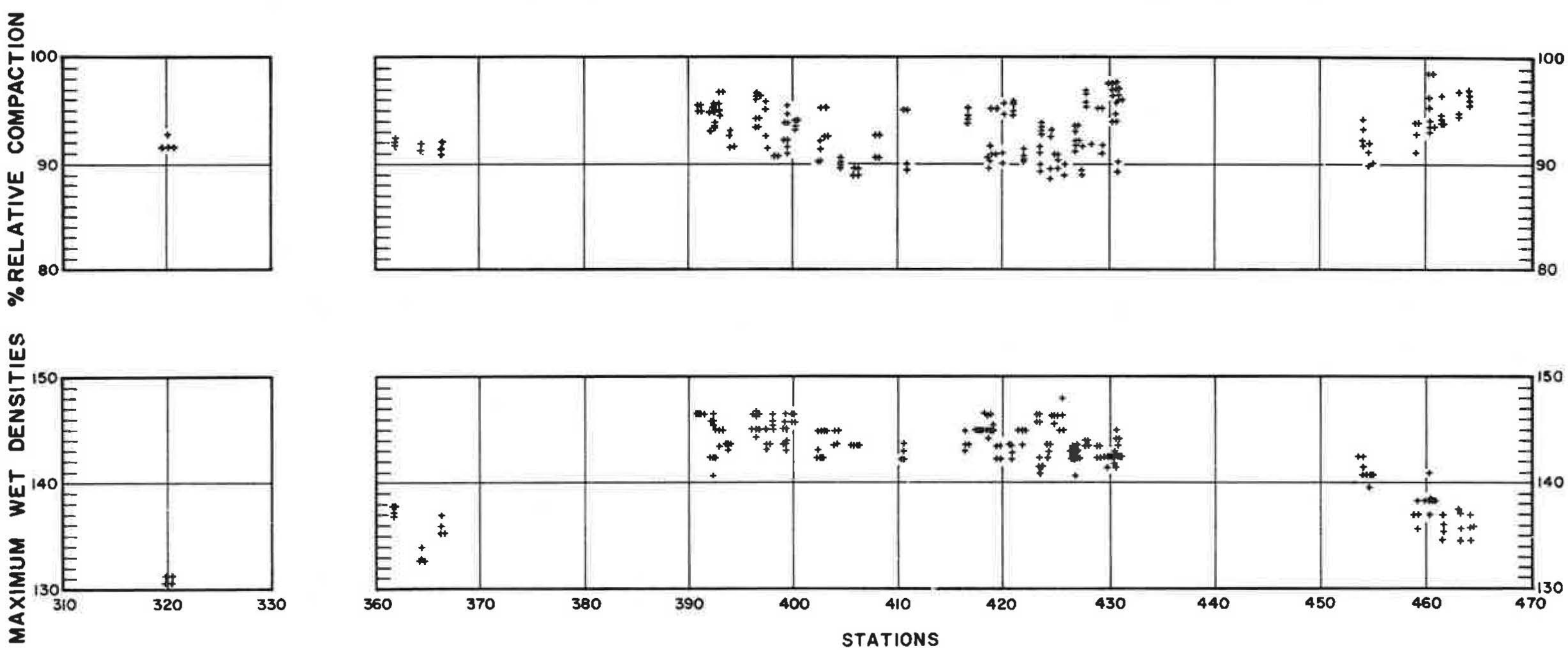


Figure 14. Dispersion of maximum density and relative compaction values related to roadway stationing (Project 1, n = 200).

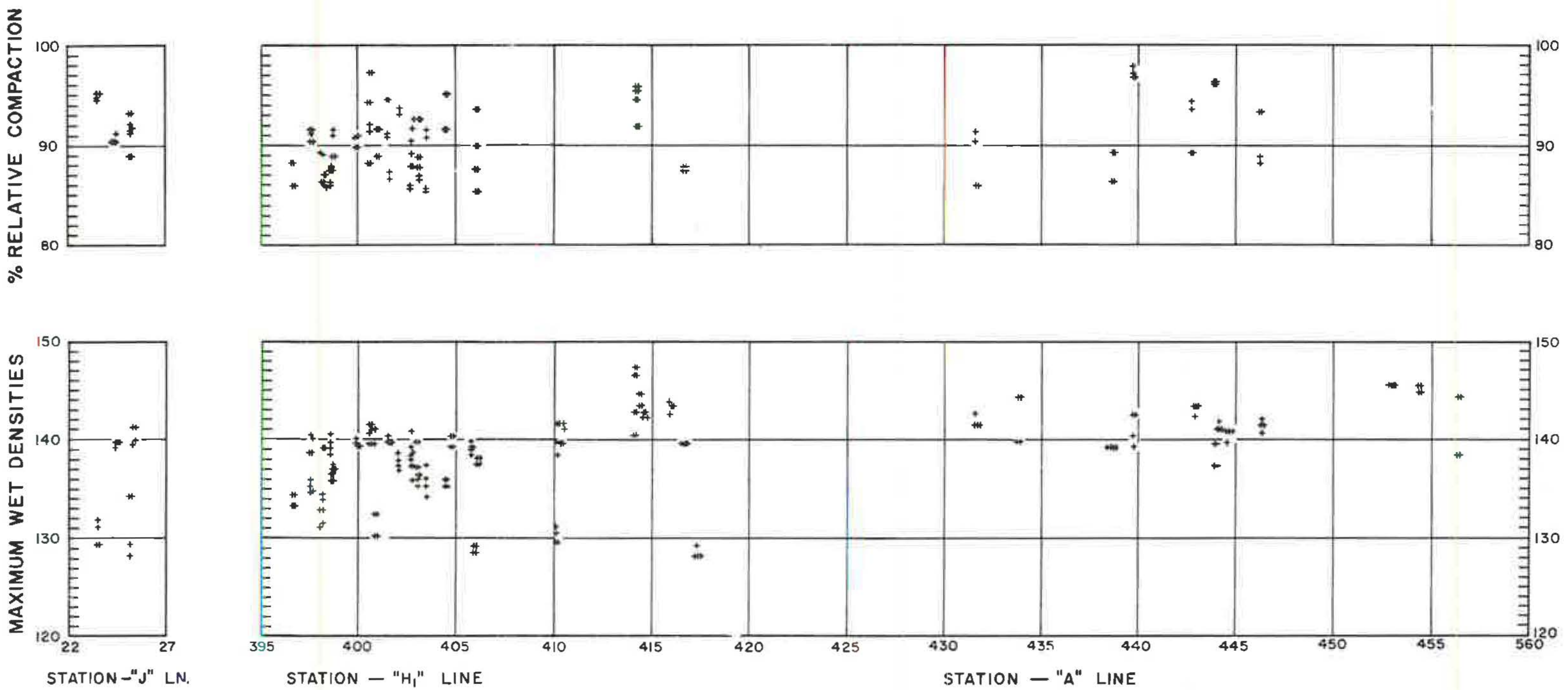


Figure 15. Dispersion of maximum density and relative compaction values related to roadway stationing (Project 2, n = 200).

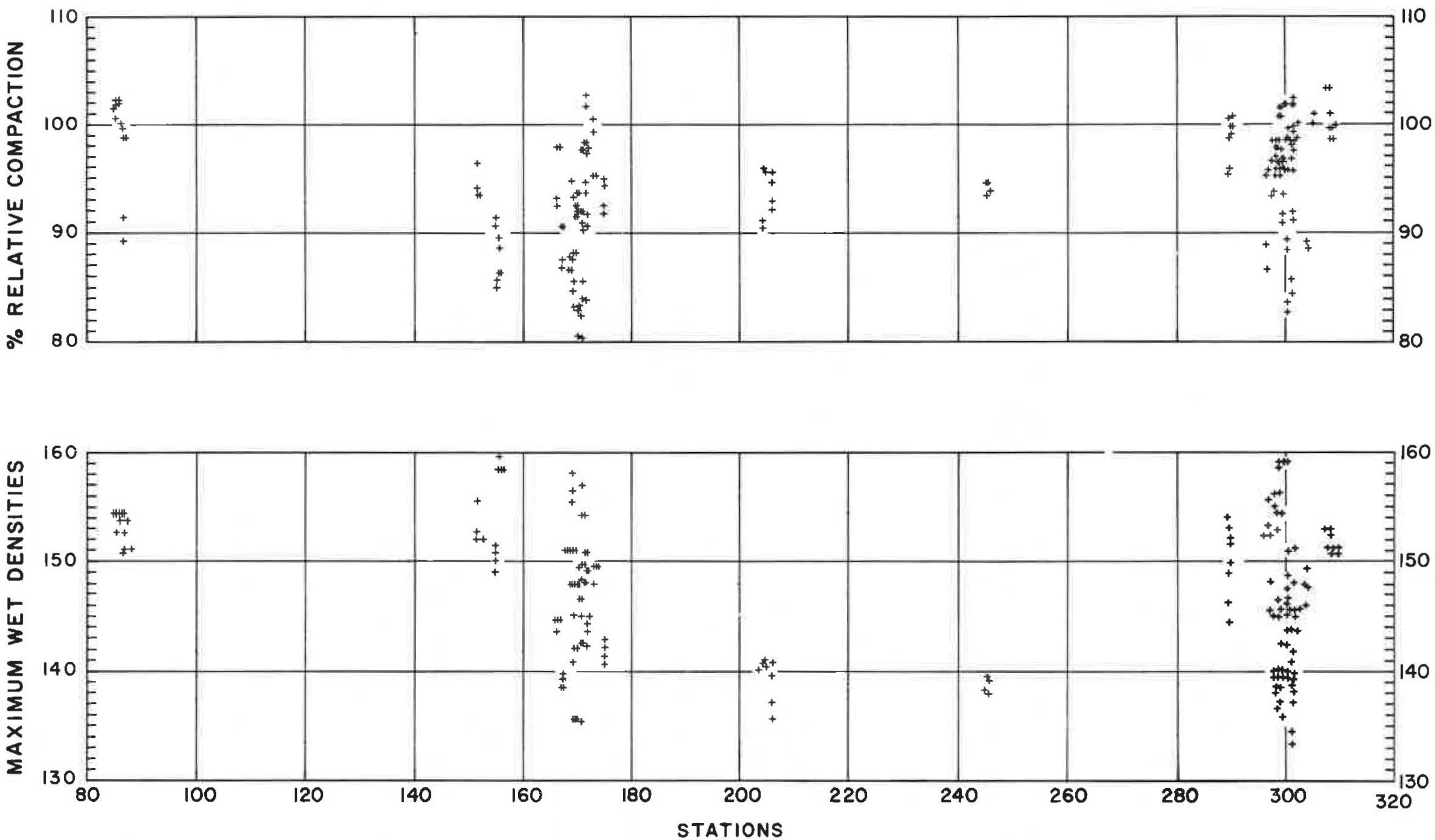


Figure 16. Dispersion of maximum density and relative compaction values related to roadway stationing (Project 3, n = 176).

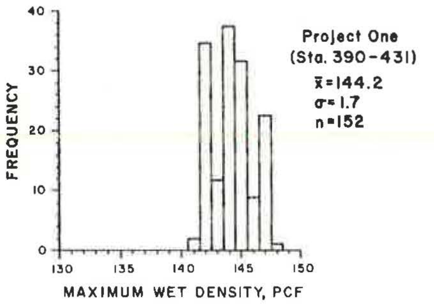


Figure 17. Histogram of maximum density by soil type, Project 1, stations 390-431.

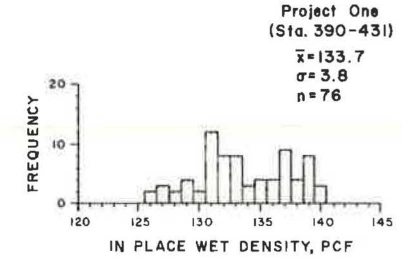


Figure 18. Histogram of in-place density by soil type, Project 1, stations 390-431.

90 percent of the time (11). A later study showed test results from the Lane-Wells Road Logger, a continuously recording mobile nuclear device, to be reproducible within 5 lb/cu ft, while the sand volume test was reproducible within 4 lb/cu ft on the same material. Both of these statements were at a 90 percent confidence level (12).

Maximum Density Variation

An analysis of the survey data revealed that the maximum density test had a standard deviation between carefully split samples of 0.6 and 1.2 lb/cu ft for the materials in Projects 1 and 3, respectively. We can then say that for the materials in Projects 1 and 3 the maximum density test was accurate within 1.2 and 2.4 lb/cu ft, respectively, 95 percent of the time. These values appear to be in very good agreement with previously published data (9).

The standard deviation for Project 2 was only 0.37 lb/cu ft. Further analyses indicated that this low value was due to an assignable cause and was not a true measure of the difference between split samples. It is concluded, however, that the individual determinations of maximum density are reasonably accurate, thus assuring the validity of the overall distribution of the relative compaction determinations.

Relative Compaction Variation

Although the variations in the in-place and maximum density test results would appear to cause very large variations in percent compaction values, such is not necessarily the case. The variations in density test results cannot be added directly. They must be combined according to the probability of occurrence (14). When the values obtained from the computer were combined in this manner, it was found that for the one fill in Project 1, two adjacent determinations could be expected to agree within 3 percent relative compaction 95 percent of the time. For Project 3, two adjacent

Figure 19. Histogram of maximum density by soil type, Project 1, except stations 390-431.

Figure 20. Histogram of in-place density by soil type, Project 1, except stations 390-431.

determinations could be expected to agree within 7 percent relative compaction 95 percent of the time. Note that the previous statements apply only to adjacent tests or tests made within a small area, such as a sampling location used in the survey reported here.

CHARACTERISTICS OF PRESENT FIELD CONTROL PROCEDURES

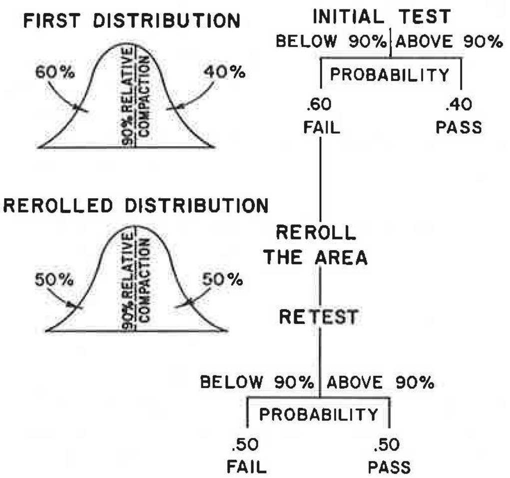
Effects of Resampling

The accepted practice of only resampling and retesting at locations which fail to meet specifications increases the probability of obtaining a test result within specification limits. This effect may be explained by a hypothetical example as shown in Figure 21. For simplification, it will be assumed that the decision to pass or require additional work will be based on one test result, and further that the distribution of percent compaction of the particular lift being considered is such that 40 percent of all possible test results would be above the minimum specification limit. Thus, the probability of one test result falling below the specification limit is 60 percent.

If the test result falls below the specification limit, some action, such as rerolling, may be taken after which another test is made. Assuming that the additional work has altered the distribution of percent compaction of the lift to the extent that 50 percent of all possible test results would be above the specification limit, a retest would now have a 50 percent chance of passing.

The total probability of accepting this fill based on this sampling and reworking procedure must be obtained from the probabilities of both the first and second tests. Therefore, in this example, the total probability of the second test result passing is 70 percent (Fig. 21).

The example is very similar to the usual procedure in actual practice. If the initial test result is only slightly below the lower limit, a check test is sometimes made. If it is considerably below the lower limit, the contractor would be asked to perform some additional work. Even with additional rolling the soil density may be altered only to a limited extent and the resampling procedure is still affecting the probability of the lift passing.



OVERALL PROBABILITY

PASSING .40 + (.60 x .50) = .40 + .30 = .70

FAILING .60 x .50 = .30

Figure 21. Probability of acceptance considering resampling.

Compaction control differs sharply from the control of those items that can be sampled, evaluated, and then accepted or rejected. The construction of an embankment is often a process of working, sampling, and reworking. Complete rejection occurs only when the material is removed from one or more lifts within a fill. In such instances, the state of compaction is rejected instead of the soil, which may be recompacted under more favorable conditions.

The effect of resampling may be considered in conjunction with the question of how much reworking is required before a new universe is prepared from which a new sample may be drawn. When a lift is rerolled, a new universe is not created; the present universe is merely altered. Although the alteration may be very small, it is nearly always an improvement. In a situation requiring rerolling, the effect of resampling is still present. Here lies one of the problems in present methods of field compaction control. This also explains the discrepancy between field control records and accurate statistical estimates of the state of compaction. There

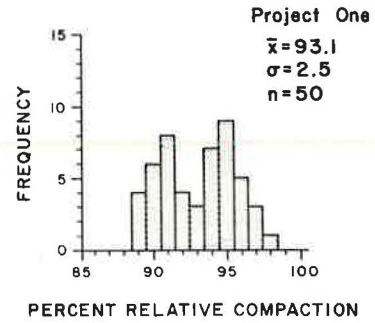


Figure 22. Relative compaction, Project 1, initial test.

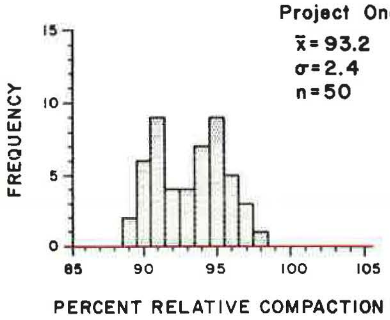


Figure 23. Relative compaction, Project 1, first retest.

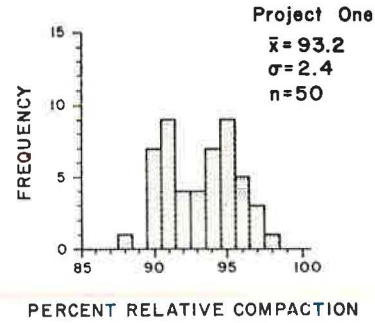


Figure 24. Relative compaction, Project 1, second retest.

appears to be no practical way of limiting the number of times an area may be rerolled and resampled.

The effect of resampling may be illustrated further by considering the data obtained from this study. Only Project 1 will be considered since this amply makes the point. Since two maximum density determinations were made for each of the two in-place density determinations per sampling location (Fig. 1), four individual percent-compaction values were obtained for each location.

To represent a field control procedure, one of the four determinations was randomly selected from each location. These values were plotted as a frequency histogram (Fig. 22). Those locations having percent compaction values less than 90 were then retested. The retesting procedure consisted of eliminating the previously selected failing values, and randomly choosing another value from the remaining three at that location. These new values were combined with the passing values of the initial selection and plotted as frequency histograms (Fig. 23). The procedure was repeated until the fourth value was used for those locations still yielding values of less than 90 percent (Fig. 24).

The initial selection or test resulted in 8 percent of the results being less than 90 percent compaction. This value was reduced to 4 percent by the first retest and to 2 percent by the second retest. The third retest, utilizing the fourth value per location, produced no further reduction in percent failing. In fact the third retest for Project 1 resulted in a histogram identical to that of the second retest. The results of these procedures for Project 3 are shown in Figures 25 through 28.

Project Control Data

Job control records for the three projects were reviewed and compared to the statistical study data to determine the effects of representing the universe by different sets of data obtained under different field conditions. The universe, in this case, is the state of embankment compaction. The different field conditions producing three sets of data include (a) the initial job control tests, which were all the first control tests whether acceptable or not; (b) the final control tests, which include all acceptable initial control tests and the last test of each series of retests made after additional rolling; and (c) the random survey data, which were obtained after the work was accepted by the resident engineer.

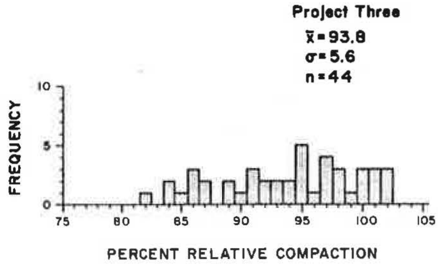


Figure 25. Relative compaction, Project 3, initial test.

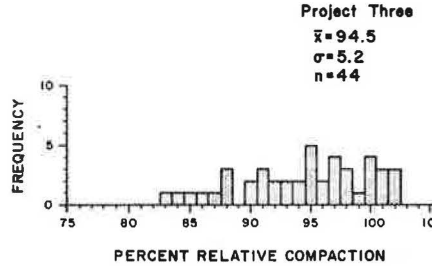


Figure 26. Relative compaction, Project 3, first retest.

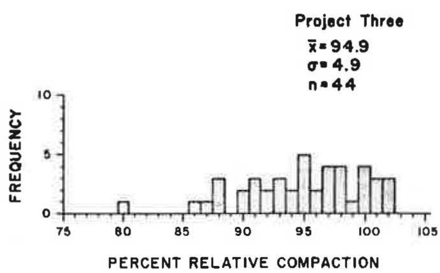


Figure 27. Relative compaction, Project 3, second retest.

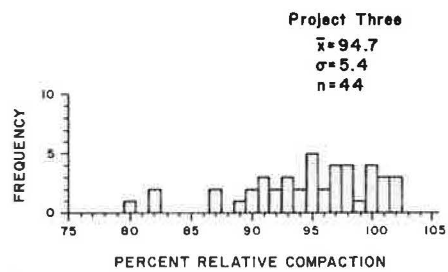


Figure 28. Relative compaction, Project 3, third retest.

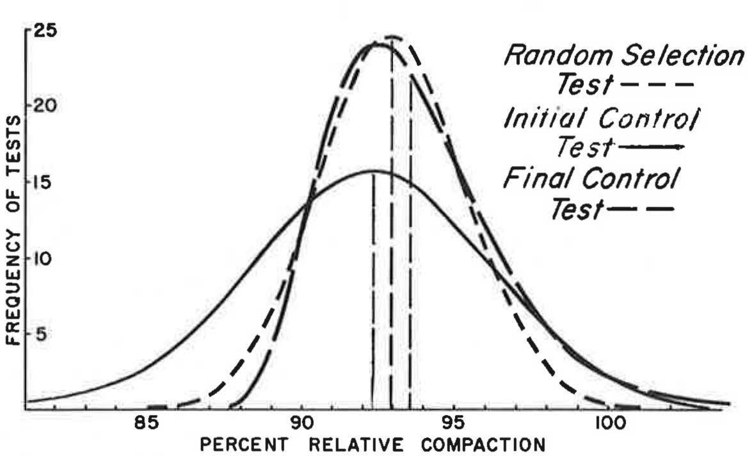


Figure 29. Random sample curves vs project control curves, Project 1.

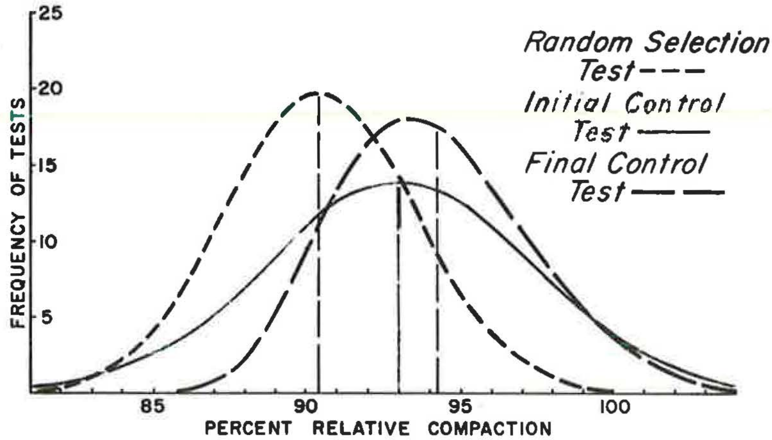


Figure 30. Random sample curves vs project control curves, Project 2.

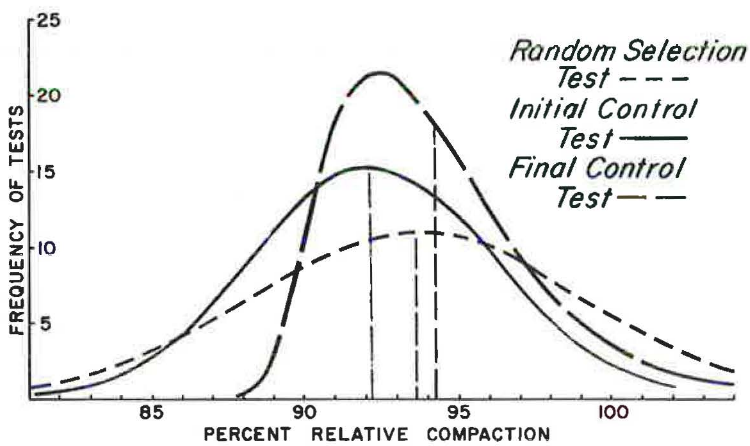


Figure 31. Random sample curves vs project control curves, Project 3.

TABLE 5

PERCENTAGE OF TEST VALUES BELOW 90 PERCENT RELATIVE COMPACTION

Tests	Project		
	1	2	3
Initial project control	23.8	17.6	25.2
Final project control[a]	1.2	2.4	1.0
Randomly located	8.5	43.0	23.9

[a]Final control tests are the last retests or the initial tests if no rerolling was required.

The differences in field conditions were primarily those with regard to rolling. For example, random survey tests were performed subsequent to all rolling operations. However, the final job control tests were not performed at randomly selected locations and the results include the effects of resampling. Initial job control tests were performed after a certain amount of rolling appeared to be sufficient, based on the judgment of the engineer and the contractor. Thus, the random survey data

TABLE 6

COMPARISON OF STATISTICAL STUDY TO INITIAL PROJECT CONTROL TESTS

Item	Project 1		Project 2		Project 3	
	Stat. Study	Initial Control	Stat. Study	Initial Control	Stat. Study	Initial Control
No. of tests	200	164	200	125	176	103
Average	92.86	92.30	90.54	93.10	93.64	92.11
Standard deviation	2.44	3.87	3.09	4.37	5.52	3.98

and the final job control data have the advantage of more rolling than the initial job control data. The final control data further have the advantage of including the effects of resampling.

The distribution curves representing the universe under the three different conditions are shown in Figures 29 through 31. These figures show that the largest average for all three projects is obtained from the final job control tests. This would be expected, based on the conditions previously stated. The curves for this set of data for all three projects are skewed to the right; i.e., the left side is relatively steep while the right side tails off. This characteristic is the result of rerolling and resampling which, in this case, eliminates extremely low values. Due to the elimination, or reduction in prominence, of the left tail of the curve and the accompanying increase in average, the total percentage of the universe less than the minimum specification limit is extremely small (Table 5).

The average of the random survey data would be expected to be greater than that for the initial control test data as may be seen in Projects 1 and 3. The reverse is shown in Figure 30 for Project 2. The relative positions of the initial-control-tests curve and random survey curve for Project 2 are believed to be the exception rather than the rule. For all three projects, these two curves are generally very similar in appearance considering the different conditions reflected by the two sets of data. Comparison of numerical values may be made by referring to Table 6.

Based on the data shown in Figures 29 through 31, the following comments regarding the effects of representing the universe by the three different sets of data may be made. The final job control data tend to produce an average greater than the true average, assuming that the true average is closely represented by the random survey. The skewed distribution of the final control data results in a misrepresentation of the dispersion of the true universe. For the projects included in this study, the random survey data distributions are believed to closely approximate the true distributions.

As explained in the preceding section, much of the bias in the control test results is due to the practice of resampling. Bias may also be introduced by the long-established practice of selecting samples by nonrandom methods. The control test samples may be selected from only those areas appearing to the sampler to be well compacted or only those areas which do not appear to be well compacted. In any case, it is well established that nonrandom sampling tends to introduce bias (5, 6).

COMPACTION SPECIFICATIONS

The Present Specification Problem

This study illustrates the need for improving present procedures for evaluating embankment compaction. Although current field control procedures may produce results that appear to be compatible with present specifications, the random survey of this study on each project revealed a percentage below the specification requirement. This discrepancy between field control data and statistical estimates has been more of a concern to non-engineering people interested in highway construction than it has to the highway engineer. Even though this difference has not always been measured, the engineer has always been aware of it and considered it within the realm of engineering judgment.

The problem then is that extensive testing of fills reveals that complete statistical compliance with present specifications cannot be achieved because no provision is made for less than 100 percent compliance. Therefore, if future specification requirements are to be enforced to 100 percent compliance, a new embankment compaction specification will be necessary. It should be one that will continue to assure that the present desirable quality level will be maintained, but with which compliance can be achieved using present acceptable construction procedures.

Review of Statistical Specifications

The California Division of Highways has reviewed two general types of specifications which may be adapted to a variety of materials or processes. They are: (a) the type presented by the Bureau of Public Roads and in further detail in Miller-Warden Associates Technical Report No. 201 (6); and (b) the type presented in the AASHO Road Test Report No. 2 (1) and in further detail in Military Standard 414, "Sampling Procedures and Tables for Inspection by Variables for Percent Defective."

From our review, it appears that the theoretical statistical specifications for on-the-job processing of manufactured materials may lead to higher testing costs with no guarantee of increased quality. Significant changes in the testing and inspection procedures could of course change this situation. From the work done by Weber, it appears that an adoption of the area concept method, similar to that which was employed for the construction control of the actual road test, may be economically feasible providing nuclear testing equipment can be employed (13).

A major portion of highway embankments are made from material taken from the cut areas. Since this is state-owned material, it is generally only possible to accept or reject the compaction work done by the contractor. This means that resampling and rerolling must be considered an accepted part of the construction process. Because neither method mentioned above has a procedure for acceptance after reworking and resampling, they would require considerable alteration before they could be successfully used in embankment control. Since quality will change with each reworking, there probably should be no limit on the number of times an area may be reworked and resampled.

Forms for New Specifications

The U.S. Bureau of Public Roads has provided the state with a five-point guide for statistical specifications (15):

1. A statement as to the desired average value of significant characteristics.
2. A definite acceptance criteria. These criteria will consist of numerical upper and/or lower limits for significant characteristics.
3. A definite number of random samples upon which the decision for acceptance or rejection will be based. The number of samples will be determined by the confidence level required, relative to material outside the tolerances.
4. A statement as to the location or point in the process where acceptance samples will be taken, and the method of sampling and testing.
5. A statement as to what action will be taken if acceptance limits are not met.

While this form provides adequate framework for a specification, it was intended to be of a general nature. When considering compaction control specifically, it is the opinion of these researchers after reviewing the data from this study and existing statistical specifications that the above outline should be modified to read as follows:

1. A statement as to the desired average value of relative compaction. (In the case of uniform material, this average should be based on some prior knowledge of the type of soil to be placed in the embankment. When dealing with extremely heterogenous

materials, it may be necessary to make day-by-day adjustments in the specified average.)

2. A definite acceptance criteria which consists of a numerical lower limit for relative compaction. (This lower limit should be established with the prior knowledge of the type of soil to be placed in the embankment.)

3. A statement defining the maximum and minimum size of the compacted area which may be considered as one lot for acceptance testing. (Present thinking is that this should be a field engineering decision as the areas can vary from a few square feet for the structure backfill to wide expanses of fill area.)

4. A definite number of random samples on which to base the decision to accept or reject the state of compaction. (Areas should be defined before tests are made.)

5. A statement as to the point in the compaction process where acceptance sampling is to be done and the exact method of sampling and testing.

6. A statement as to what action will be taken if acceptance limits are not met.

7. A statement defining a procedure for resampling of reworked areas. (This procedure should compensate for the resampling effect. It is not deemed practical to limit the number of times that an area may be resampled; therefore, some sequential sampling procedure should be considered.)

Since all present compaction measuring methods are subject to wide variation and interpretation when applied to various materials, the incorporation of theoretically correct statistical criteria, such as those listed above, probably cannot be economically justified.

California Division of Highways is presently gaining experience with a compaction control specification entitled "Method of Testing for Relative Compaction of Soils by Nuclear Method." This specification, Test Method No. Calif. T231-B, though not a true statistical specification, does incorporate one item which is found in most statistical specifications. Namely, it specifies that multiple testing shall be done in each area and that acceptance of the area shall be judged on the average of six or more test results. This specification is presently being used on 11 embankment construction projects on an experimental basis. The results of this study at this time look very promising. However, it will be approximately a year before the final evaluation can be made.

CONCLUSIONS

1. The results of this study indicate that it would be extremely difficult to prepare an embankment compaction specification based fully on statistical consideration. This is not surprising since most statistical specifications are intended to aid in making a decision to accept or reject material. In embankment construction, the engineer does not reject the embankment material after it has been judged satisfactory for the intended purpose. He accepts or rejects the state of compaction and, in this case, rejection usually means that the contractor must do additional work on the same material.

2. The variation in the statistical distribution of relative compaction values may be quite large depending on the moisture control, uniformity of compacting effort, the variation in the soils, the susceptibility of the soils to this compaction effort, and other differences. Any statistical specification must take into consideration these potential variations from project to project, particularly the variation in the soils.

3. Finished earthwork on the projects surveyed has been judged satisfactory by present engineering standards and is consistent with present specifications, based on field control requirements which include the effort of resampling. However, based on results from randomly selected samples for this survey, the earthwork quality is inconsistent with a strict 100 percent compliance interpretation of the present specifications. This leads to the conclusion that a revision of present specification requirements is necessary if statistical quality control methods are to be used to enforce construction standards.

4. Results of this study indicate that the adoption of purely statistical specifications for compaction using present testing methods (AASHO T180-C and T181-C or Calif. 216) would require an increase in the amount of testing now performed in California.

However, other research work in progress indicates that by the use of a rapid method of testing, such as nuclear testing equipment, it is practical to use statistical specifitions (13).

5. A procedure which allows retesting only of locations having unsatisfactory compaction test results, regardless of whether additional work is performed prior to the retesting, increases the risk of accepting unsatisfactory work.

6. The accuracy of the present control test procedure, California Test Method 216-F, is sufficient to measure significant variations in the percent relative compaction.

7. The distribution of relative compaction values obtained from this survey is believed to be indicative of the range of compaction currently being accepted. For very uniform, non-variable soil, the result of two adjacent relative compaction determinations can be expected to agree within 3 percent relative compaction 95 percent of the time. For highly variable heterogeneous soil mixtures, the results of two adjacent relative compaction determinations can be expected to agree within 7 percent 95 percent of the time.

8. Depending on specific conditions, a contractor must plan to average 93+ percent relative compaction in order to have substantial compliance with the present specification of "not less than 90 percent by the California Test Method 216."

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Appendix

TYPICAL EXAMPLE OF RANDOM SAMPLING FROM AN AREA

A portion of a section of roadway which is 30,000 ft long and 26 ft wide is about ready for the cement-treated base. The inspector has been asked to randomly draw 50 samples in duplicate from the section in order to survey the percent of cement in the base material.

Using the attached table of random numbers, Table A-1, the sampler chooses 50 locations in the following manner. Starting at any point on the table and proceeding up or down, but not skipping any numbers, he reads 50 pairs of numbers. In Column 4, reading down, he finds .732, .721; .153, .508; and so on to the fiftieth pair, .698, .539, which is found about midway down in Column 5.

The first, or A, decimal in each pair is multiplied by the length, 30,000 ft, and the second, or B, decimal is multiplied by the width, 26 ft. Each pair of products establishes a coordinate location in a grid system for taking duplicate samples (See Table A-2).

The sampler then plots the 50 locations (Fig. A-1), and numbers them in the order in which the samples will be taken. Should two locations fall so close together that they both could not be sampled properly, the second one is discarded. Returning to the table of numbers, the next, or fifty-first, pair of random numbers is substituted.

Figure A-2 shows how the samples are numbered for identification. Each duplicate sample will be split into two equal portions before being tested.

TABLE A-1

RANDOM NUMBERS

1		2		3		4		5	
A	B	A	B	A	B	A	B	A	B
.576	.730	.430	.754	.271	.870	.732	.721	.998	.239
.892	.948	.858	.025	.935	.114	.153	.508	.749	.291
.669	.726	.501	.402	.231	.505	.009	.420	.517	.858
.609	.482	.809	.140	.396	.025	.937	.310	.253	.761
.971	.824	.902	.470	.997	.392	.892	.957	.640	.463
.053	.899	.554	.627	.427	.760	.470	.040	.904	.993
.810	.159	.225	.163	.549	.405	.285	.542	.231	.919
.081	.277	.035	.039	.860	.507	.081	.538	.986	.501
.982	.468	.334	.921	.690	.806	.879	.414	.106	.031
.095	.801	.576	.417	.251	.884	.522	.235	.398	.222
.509	.025	.794	.850	.917	.887	.751	.608	.698	.683
.371	.059	.164	.838	.289	.169	.569	.977	.796	.996
.165	.996	.356	.375	.654	.979	.815	.592	.348	.743
.477	.535	.137	.155	.767	.187	.579	.787	.358	.595
.788	.101	.434	.638	.021	.894	.324	.871	.698	.539
.566	.815	.622	.548	.947	.169	.817	.472	.864	.466
.901	.342	.873	.964	.942	.985	.123	.086	.335	.212
.470	.682	.412	.064	.150	.962	.925	.355	.909	.019
.068	.242	.667	.356	.195	.313	.396	.460	.740	.247
.874	.420	.127	.284	.448	.215	.833	.652	.601	.326
.897	.877	.209	.862	.428	.117	.100	.259	.425	.284
.875	.969	.109	.843	.759	.239	.890	.317	.428	.802
.190	.696	.757	.283	.666	.491	.523	.665	.919	.146
.341	.688	.587	.908	.865	.333	.928	.404	.892	.696
.846	.355	.831	.218	.945	.364	.673	.305	.195	.887
.882	.227	.552	.077	.454	.731	.716	.265	.058	.075
.464	.658	.629	.269	.069	.998	.917	.217	.220	.659
.123	.791	.503	.447	.659	.463	.994	.307	.631	.422
.116	.120	.721	.137	.263	.176	.798	.879	.432	.391
.836	.206	.914	.574	.870	.390	.104	.755	.082	.939
.636	.195	.614	.486	.629	.663	.619	.007	.296	.456
.630	.673	.665	.666	.399	.592	.441	.649	.270	.612
.804	.112	.331	.606	.551	.928	.830	.841	.602	.183
.360	.193	.181	.399	.564	.772	.890	.062	.919	.875
.183	.651	.157	.150	.800	.875	.205	.446	.648	.685

TABLE A-2

COMPUTATION OF RANDOM SAMPLE LOCATION COORDINATES

Coordinate Along Roadway Centerline			Coordinate Transverse to Roadway Centerline	
Column A Random Numbers (Top Col.4A Down)	Column B Station to Be Sampled (Col.A x 30,000ft.)	Column C Order of Sampling	Column D Random Numbers (Top Col.4B Down)	Distance Form Left Edge of Roadway (Col.D x 26ft.)
.732	219+60	30	.721	19
.153	45+90	7	.508	13
.009	2+70	1	.420	11
.937	281+10	47	.310	8
.892	267+60	42	.957	25
.470	141+00	18	.040	1
.285	85+50	11	.542	14
.081	24+30	2	.538	14
.879	263+70	39	.414	11
.522	156+60	20	.235	6
.751	225+30	32	.608	16
.569	170+70	22	.977	25
.815	244+50	35	.592	15
.579	173+70	23	.787	20
.324	97+20	12	.871	23
.817	245+10	36	.472	12
.123	36+90	6	.086	2
.925	277+50	45	.355	9
.396	118+80	15	.460	12
.833	249+90	38	.652	17
.100	30+00	3	.259	7
.890	267+00	40	.317	8
.523	156+90	21	.665	17
.928	278+40	46	.404	10
.673	201+90	26	.305	8
.716	214+80	29	.265	7
.917	275+10	44	.217	6
.994	298+20	49	.307	8
.798	239+40	34	.879	23
.104	31+20	4	.755	20
.619	185+70	24	.007	0
.441	132+30	17	.649	17
.830	249+00	37	.841	22
.890	267+00	41	.062	2
.205	61+50	8	.446	12
(Column 5A)			(Column 5B)	
.998	299+40	50	.239	6
.749	224+70	31	.291	8
.517	155+10	19	.858	22
.253	75+90	10	.761	20
.640	192+00	25	.463	12
.904	271+20	43	.003	26
.231	69+30	9	.919	24
.986	295+80	48	.501	13
.106	31+80	5	.031	1
.398	119+40	16	.222	6
.698	209+40	27	.683	18
.796	238+80	33	.996	26
.348	104+40	13	.743	19
.358	107+40	14	.595	16
.698	209+40	28	.539	14

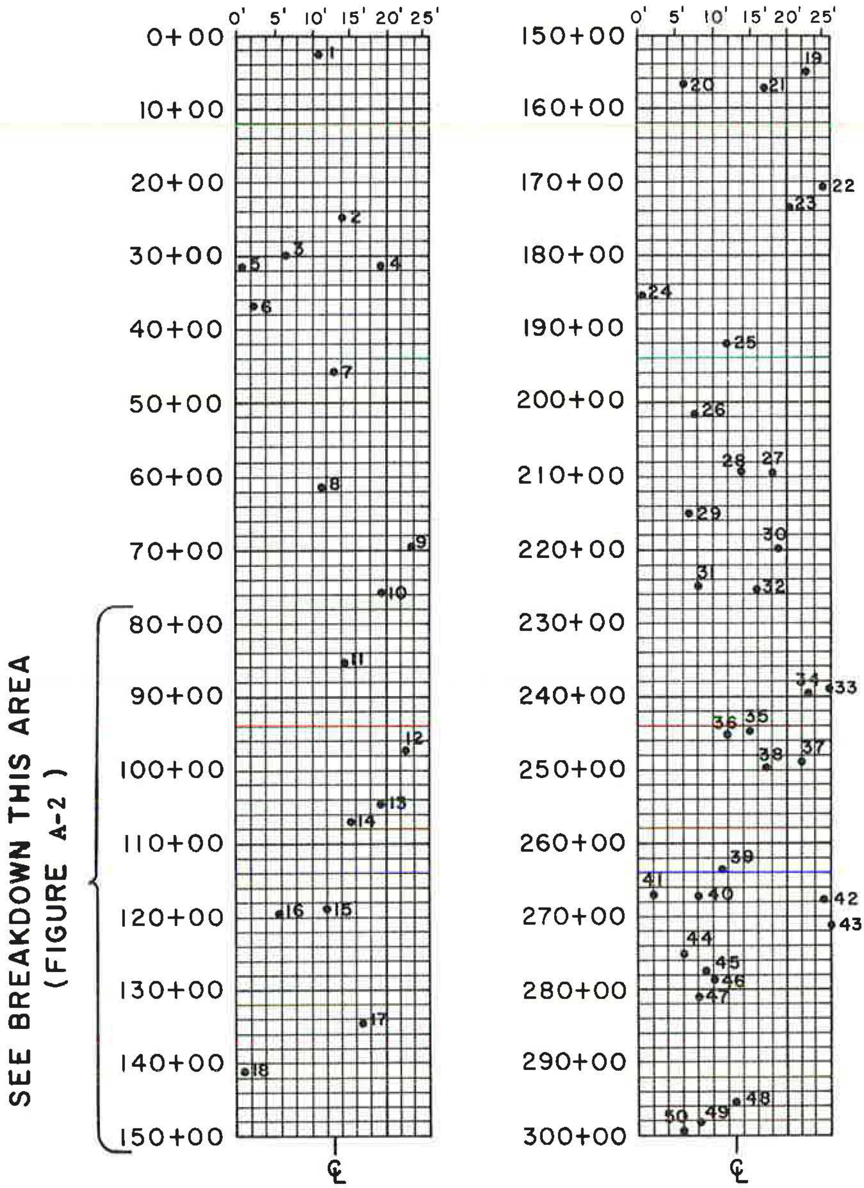

Figure A-1. Typical random sampling from area.

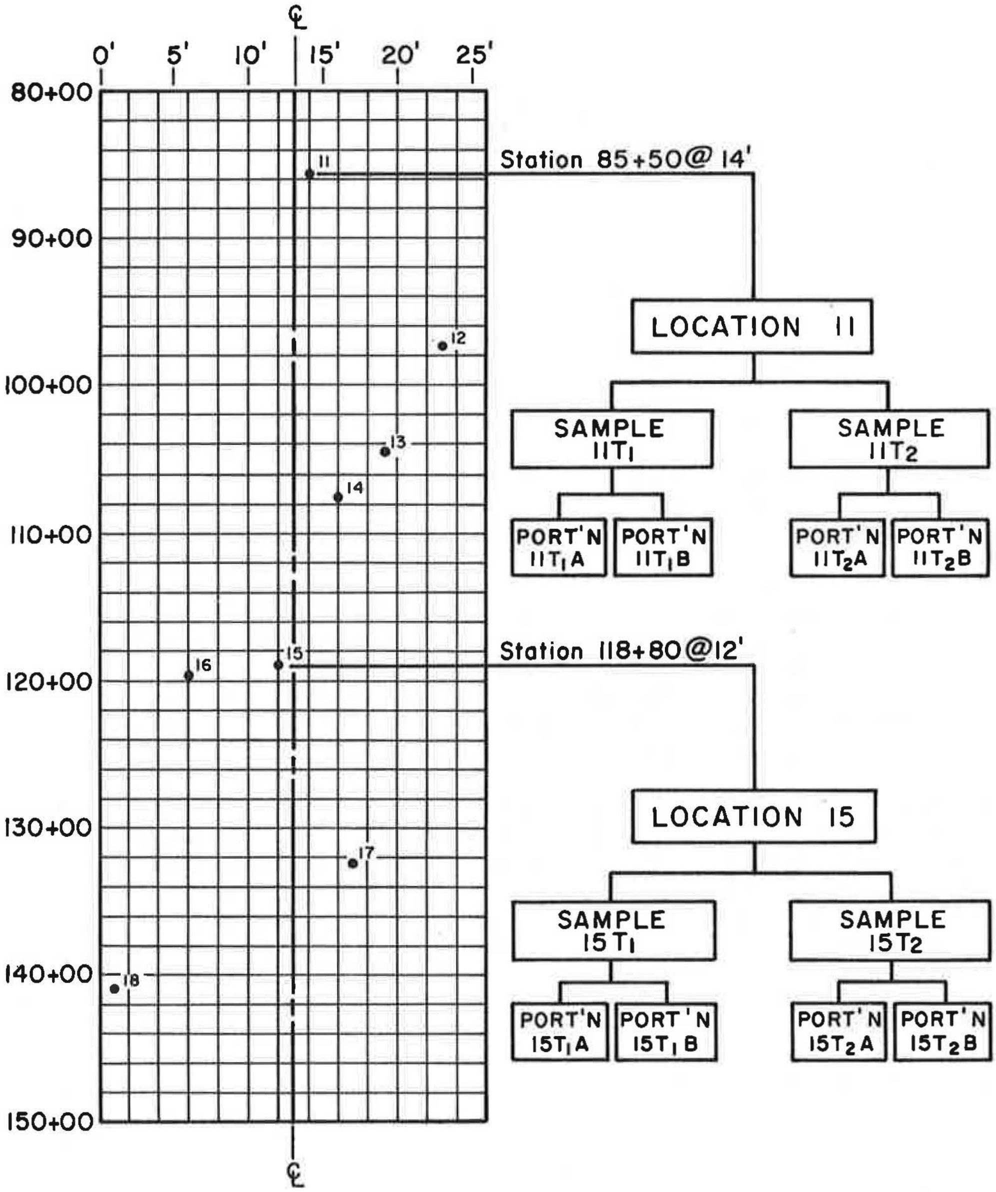

Figure A-2. Typical breakdown of location.