# Design Charts for Coefficients of Active Earth Pressure of Cohesionless Soils— A Rapid Method

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•THE Coulomb theory of active earth pressures of cohesionless soils against retaining walls is widely used in engineering practice. The Culmann graphical method is based on Coulomb's theory and is often employed to find the active earth pressure when values of angle of internal friction of the soil $\phi$, angle of the wall face to the vertical $\alpha$, angle of the embankment slope with the horizontal $\omega$ and the angle of wall friction $\delta$, are either known, estimated, or substituted on a trial basis (3, p. 154). The Culmann method for finding active earth pressure can be a very time-consuming process when a number of trial solutions need be made in order to arrive at an optimum or desirable set of the variables $\phi$, $\alpha$, $\omega$, and $\delta$. The Culmann method also has the disadvantage that it does not show the relations that exist between these variables and the values of active earth pressure.

A more useful design tool would be a set of information that shows the relationship between active earth pressure and the variables $\phi$, $\alpha$, $\omega$, and $\delta$, and gives ranges of values for them that would cover situations that arise in the field. Such a set of information would be a series of design charts from which active earth pressure could be obtained for any combination of the variables.

## PURPOSE

The purpose of this report was to develop a set of design curves for computing active earth pressures of cohesionless soils against retaining walls based on Coulomb's theory. The curves were to be of such a nature as to: (a) allow the designer to see the relationships between active earth pressure and the variables $\phi$, $\alpha$, $\omega$, and $\delta$, as some variables were held constant while others were allowed to vary; (b) allow the designer to arrive quickly at values of active earth pressure for a determined combination of variables; (c) encompass a wide range of possible values for each variable to allow the designer wide latitude; and (d) readily lend themselves to interpolation on the part of the designer.

## THEORETICAL CONSIDERATIONS AND ASSUMPTIONS

The Coulomb theory of active earth pressures of cohesionless soils is based on important assumptions and considerations. These must always be recognized and taken into account in any design problem.

One important assumption made by Coulomb was that the surface of failure for a backfill of cohesionless soil was a plane surface, or in a two-dimensional drawing, a straight line, which extended from the toe of the wall to intercept the backfill surface or the embankment slope. The actual failure surface has some curvature near the toe, but Coulomb ignored the curvature. According to Terzaghi and Peck, the error resulting from neglecting the curvatures is very small (3, p. 153).

Other important assumptions were made by Coulomb concerning the relative movement between the retaining wall and the soil mass required for the active earth pressure

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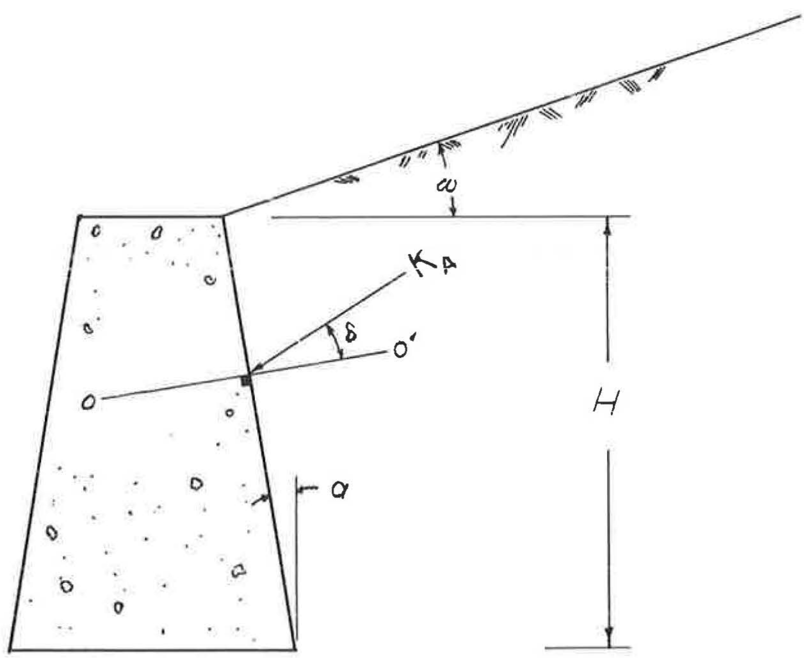
conditions to exist. Coulomb actually defines two states in which active pressures may exist (3, pp. 151, 152): (a) the state where the angle of wall friction, δ, is considered positive, and (b) the state where δ is considered negative. This report is concerned solely with the situation where δ can be considered positive.

For the active case where δ is considered positive to hold, the soil mass must move downward with respect to the wall. According to Terzaghi and Peck (3, p. 151), "The downward movement of the sand with reference to the wall develops frictional forces that cause the resultant active earth pressure to be inclined at an angle δ to the normal to the wall. This angle is known as the angle of wall friction."

## PROCEDURE

Coulomb's equation for active earth pressure and use of the IBM 7094 computer made possible collection of the data required to develop the design curves. Coulomb's




All dimensions and variables represent positive values.

$K_A$ is the coefficient of active earth pressure $P_A$.

ω is the angle the embankment slope makes with the horizontal.

α is the angle between the wall face and a vertical to the wall toe.

δ is the angle of wall friction or the angle between the reaction of active earth pressure $P_A$, and a normal to the wall, line segment 0-0'.

ϕ is the angle of internal friction of the soil.

H is the depth of the backfill, in ft, measured at the wall.

γ is the unit weight of the soil in pcf.

Figure 1.

equation for active earth pressure with positive wall friction ($\delta$ considered positive) is as follows (2, p. 496):

$$P_A = \frac{1}{2} \gamma H^2 \left( \frac{\cos^2 (\phi - \alpha)}{\cos^2 \alpha \cos (\alpha + \delta) \left[ 1.0 + \sqrt{\frac{\sin (\phi + \delta) \sin (\phi - \omega)}{\cos (\alpha + \delta) \cos (\alpha - \omega)}} \right]^2} \right)$$

The expression within the parentheses is designated $K_A$, the coefficient of active earth pressure, and the following relations may be obtained:

$$P_A = \frac{1}{2} \gamma H^2 K_A,$$

where

$$K_A = \left( \frac{\cos^2 (\phi - \alpha)}{\cos^2 \alpha \cos (\alpha + \delta) \left[ 1.0 + \sqrt{\frac{\sin (\phi + \delta) \sin (\phi - \omega)}{\cos (\alpha + \delta) \cos (\alpha - \omega)}} \right]^2} \right)$$

Figure 1 shows the significance of the variables and explains what each variable represents.

Values of $K_A$ for many combinations of the variables $\phi$, $\alpha$, $\omega$, and $\delta$ were obtained. In the several computer programs that were used, the variable $\omega$ was allowed to range from 0 to 25°, $\alpha$ was assigned values from 0° to a magnitude of $2\omega$, $\phi$ ranged from 15 to 40°, and $\delta$ was assigned values ranging from 0° to $\phi$. It was believed that these ranges give the designer wide freedom of choice.

Several methods of presenting the relationships between values of $K_A$ and the several variables were investigated. It was decided that a good method of presenting these relationships would be a series of graphs that would give the horizontal and vertical components, $K_H$ and $K_V$, of the coefficient of active earth pressure, $K_A$, for various combinations of the variables $\phi$, $\alpha$, $\omega$, and $\delta$. The expression of $K_A$ in terms of the components $K_H$ and $K_V$ would readily enable estimation of expected overturning forces and vertical forces to be encountered.

In several instances checks were made on values of $K_A$ obtained from the computer programs by means of the Culmann graphical method. The checks were necessary to ensure that the Coulomb equation was being used correctly in the computer programs.

## RESULTS

The design charts (Figs. 2-34, Appendix) represent the results of the efforts to provide the designer with a means whereby he may determine rapidly the active earth pressure for a combination of the variables $\phi$, $\alpha$, $\omega$, and $\delta$. These charts give the designer flexibility in his choice of a combination of variables and allow him to see the relationships that exist between the variables and the values of active earth pressure. They also allow him the opportunity for interpolation when necessary.

Each chart represents a situation where $\phi$ and $\omega$ are held constant. Values for $\alpha$ and $\delta$ are allowed to vary through a substantial range. The designer determines the horizontal and vertical components of $K_A$ directly from the chart for an appropriate combination of $\phi$, $\alpha$, $\omega$, and $\delta$. With $K_H$ or $K_V$ known, the designer can easily determine $K_A$ and $P_A$.

### Example

Required:

Determine the active earth pressure $P_A$ and the horizontal and vertical components of $P_A$.

Given:

$H = 30$ ft.

$\gamma = 110$ lb per cubic ft

$\omega = 15°$

$\phi = 35°$

$\alpha = 15°$

$\delta = 0°$

Solution:

1. From the design chart, $K_H = 0.450$ and $K_V = 0.121$.

2. The horizontal component of $P_A = K_H \frac{1}{2} \gamma H^2 = (0.450) \frac{1}{2} (110) (30)^2 =$ 22,300 lb/lineal ft of wall length.

3. The vertical component of $P_A = K_V \frac{1}{2} \gamma H^2 = (0.121) \frac{1}{2} (110) (30)^2 = 5{,}990$ lb/lineal ft of wall length.

4. The total active earth pressure, $P_A$, can be found from either component. From the horizontal component, $P_A = \frac{22{,}300}{\cos(\alpha + \delta)} = \frac{22{,}300}{\cos(15°)} = 23{,}100$ lb/lineal ft of wall length.

## DISCUSSION OF OBSERVED PHENOMENA

During the course of the reduction and presentation of the data, several interesting and important situations were observed.

### Cases Where $\phi$ Is Less Than or Equal to $\omega$

In cases where the Culmann graphical method was used to check values of $K_A$ for values of $\phi$ less than $\omega$, it was not possible to get a solution. The Coulomb equation was not applicable for values of $\phi$ less than $\omega$.

The physical interpretation for this failure to produce a solution on the part of both the Culmann method and the Coulomb equation is that a cohesionless soil cannot stand on a slope greater than its angle of internal friction. In reality, a cohesionless soil will not stand on a slope greater than its angle of repose, where the angle of repose represents the minimum angle of internal friction or the $\phi$ of the soil in its loosest state.

For instances in which $\phi$ is equal to $\omega$ the Culmann method does not appear to provide a solution. It is possible that the Culmann method would approach a solution for $K_A$ as trial failure wedges approached infinitely large areas. The Coulomb equation appears to provide valid values when $\phi$ equals $\omega$, although it is not probable that cohesionless soil would stand long on a slope equal to its angle of internal friction.

### Cases Where Wall Friction Is Not Desirable

Although curves giving values of $K_A$ instead of $K_H$ and $K_V$ are not presented in the design charts, these curves were plotted on a trial basis, and they revealed many instances where an increase in wall friction angle $\delta$ increased the coefficient of active earth pressure rather than decreasing it. The belief held by many is that an increase in the wall friction will always serve to reduce the value of $K_A$. Terzaghi and Peck, in discussing Rankine's theory of active earth pressure of cohesionless soils against smooth retaining walls, imply that any friction that is generated by wall roughness works in the designer's favor. These authors state that "It is shown subsequently that the roughness of the back of a wall commonly reduces the active and increases the passive earth pressure." (3, p. 144)

A case of $K_A$ increasing as $\delta$ increases is illustrated in the design chart where $\phi = 35°$, $\omega = 15°$, and $\alpha$ is assigned the value of 15°. The value of $K_A$ for $\delta = 0°$ is found to be 0.466, whereas the value of $K_A$ for $\delta - 35°$ is found to be 0.505.

### Situation Where the Active Case Ceases To Exist

In the design chart where $\omega = 25°$, $\phi = 40°$, $\delta = 40°$ when $\alpha$ was given a value of $2\omega$ or 50°, the resultant coefficient of active earth pressure, $K_A$, was found to have a vertical line of action. Therefore, $K_A$ had no horizontal component, $K_H$, and it may be concluded that a state of active earth pressure does not exist. The Coulomb equation is not applicable for this combination of the variables, and will not apply in any situation where the sum of the angles $\alpha$ and $\delta$ equals $90^\circ$. When this occurred, no points were plotted on the design chart.

## CONCLUSIONS

The design charts developed in this report are applicable to the design of retaining walls that must resist active earth pressures of cohesionless soils for cases where the angle of wall friction $\delta$ will be positive. The assumptions and limitations of the Coulomb theory must always be considered when use is made of the charts.

The major assumptions made by Coulomb were that the surface of failure for a backfill of cohesionless soil was a plane surface, and that the effect of curvature of the actual failure surface could be neglected. The relative movement between the retaining wall and the soil mass must bring about a state of active earth pressure for which the wall friction angle $\delta$ is positive. The major limitations include cases where the Coulomb equation and the Culmann graphical method fail to give solutions for $K_A$. A cohesionless soil cannot stand on slopes greater than its angle of repose. When the line of action of $K_A$ is vertical so there is no horizontal component $K_H$, the active earth pressure case does not exist.

When used with a full understanding of the Coulomb theory and the limitations to the theory, the design charts offer a means to quickly and accurately calculate earth pressures against retaining walls. They allow the designer to see the relationships between the components $K_H$ and $K_V$ and the variables $\phi$, $\alpha$, $\omega$, and $\delta$, and interpolation is possible.

## ACKNOWLEDGMENT

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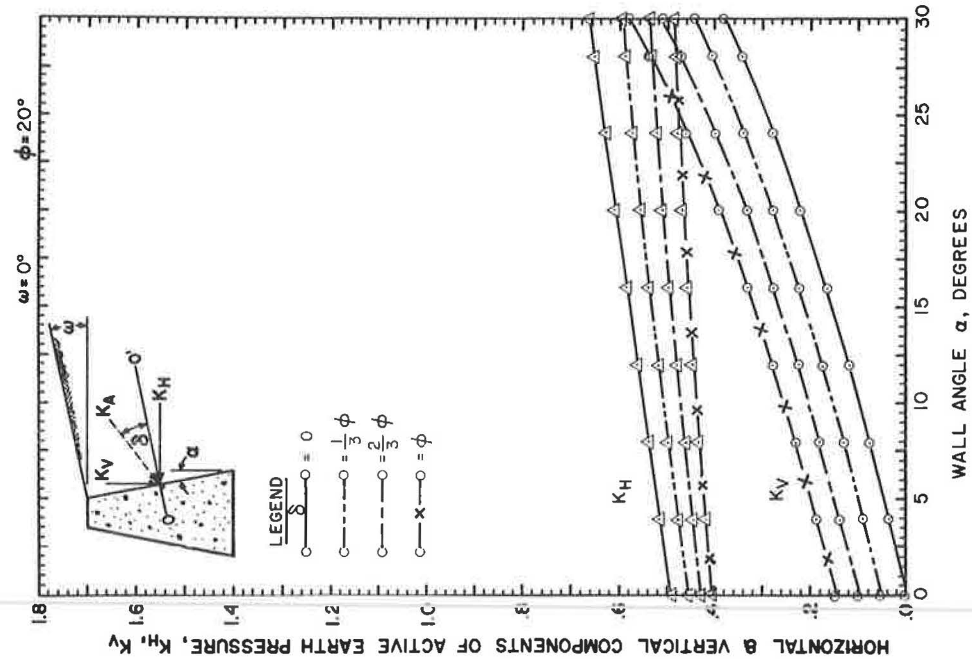
2. Taylor, Donald W. Fundamentals of Soil Mechanics. John Wiley and Sons, New York, 1948.
3. Terzaghi, Karl, and Peck, Ralph B. Soil Mechanics in Engineering Practice. John Wiley and Sons, New York, 1948.

# *Appendix*

## DESIGN CHARTS

The design charts (Figs. 2-34) are presented on the following pages.

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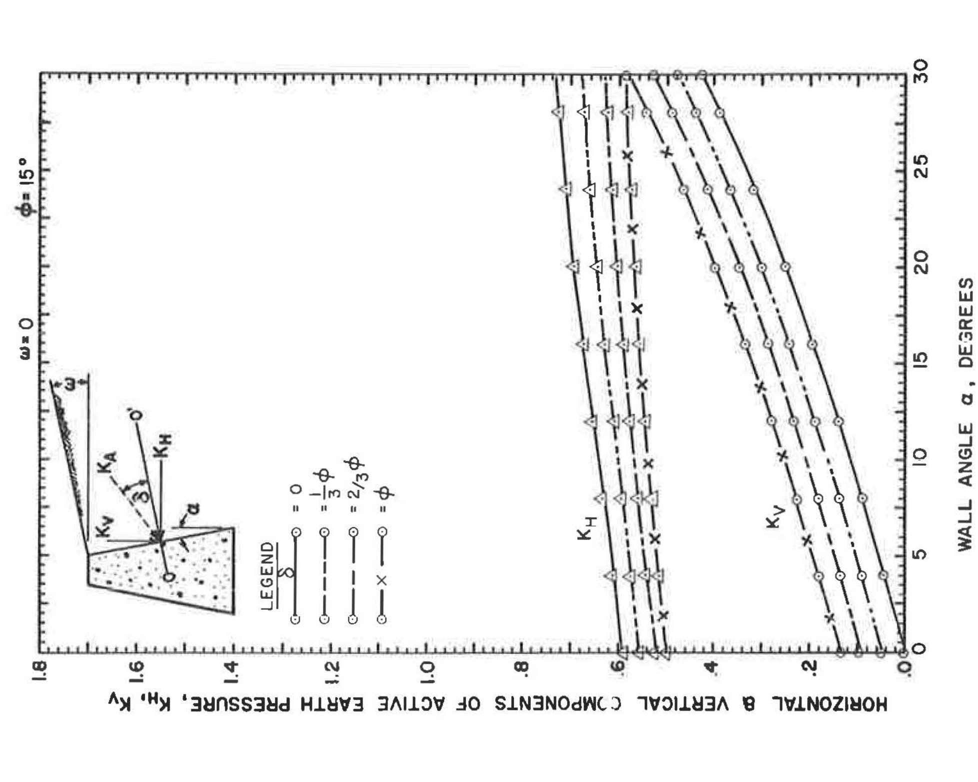


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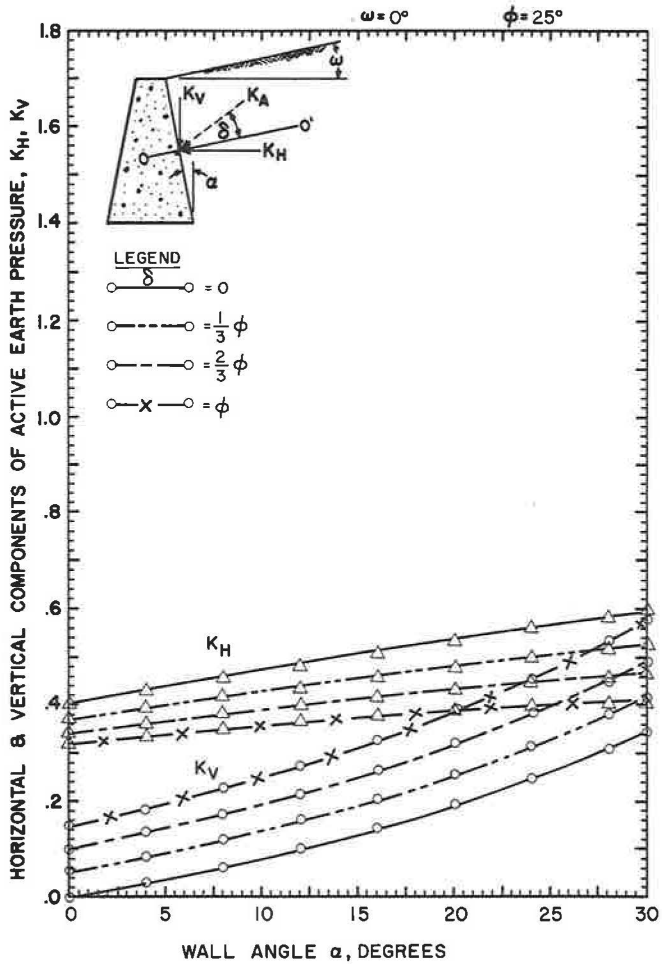


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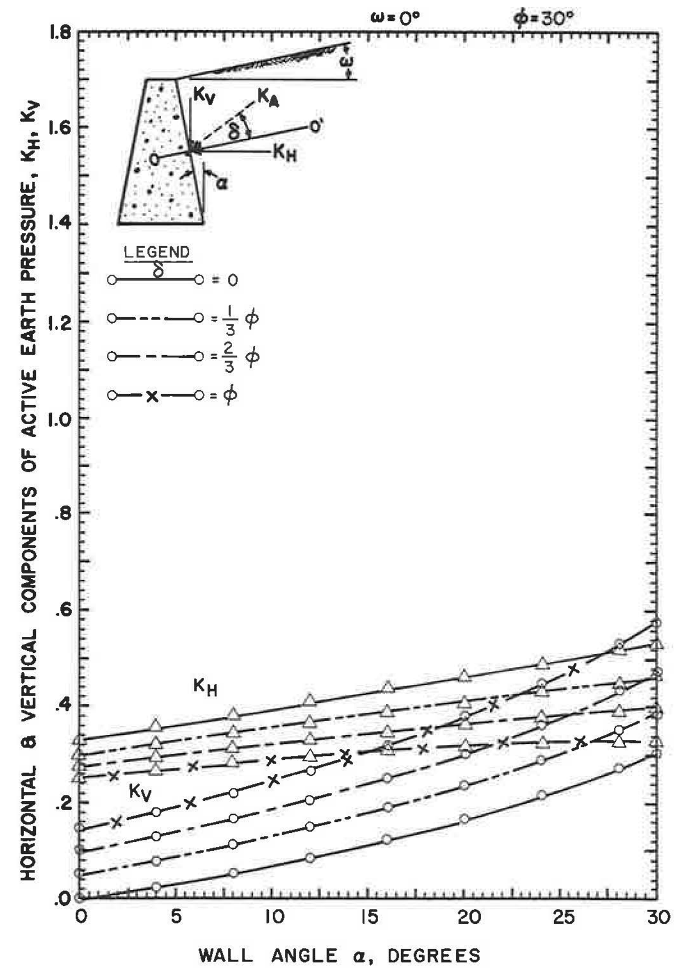


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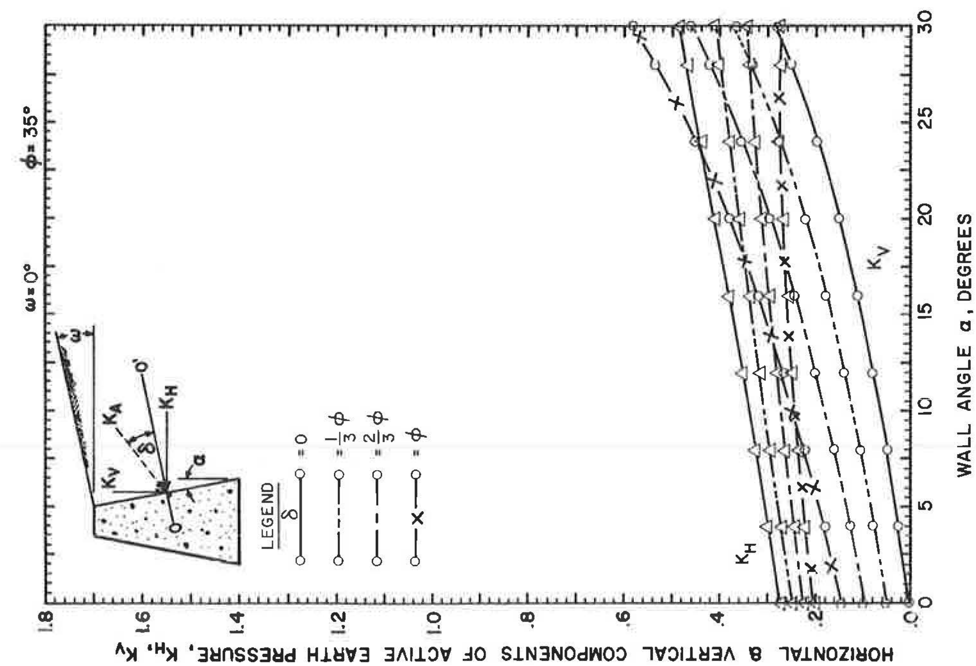


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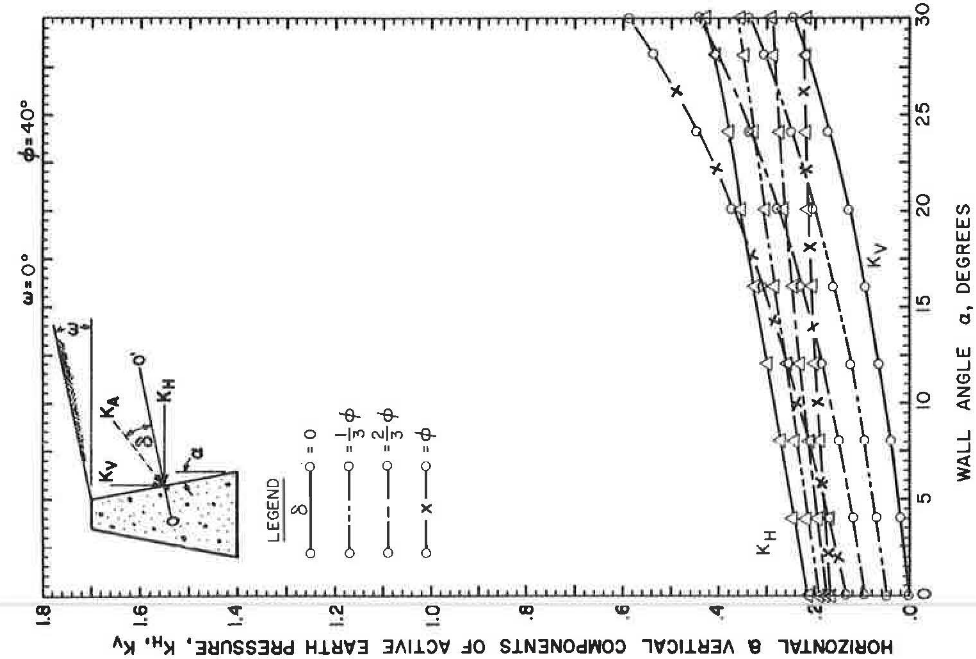


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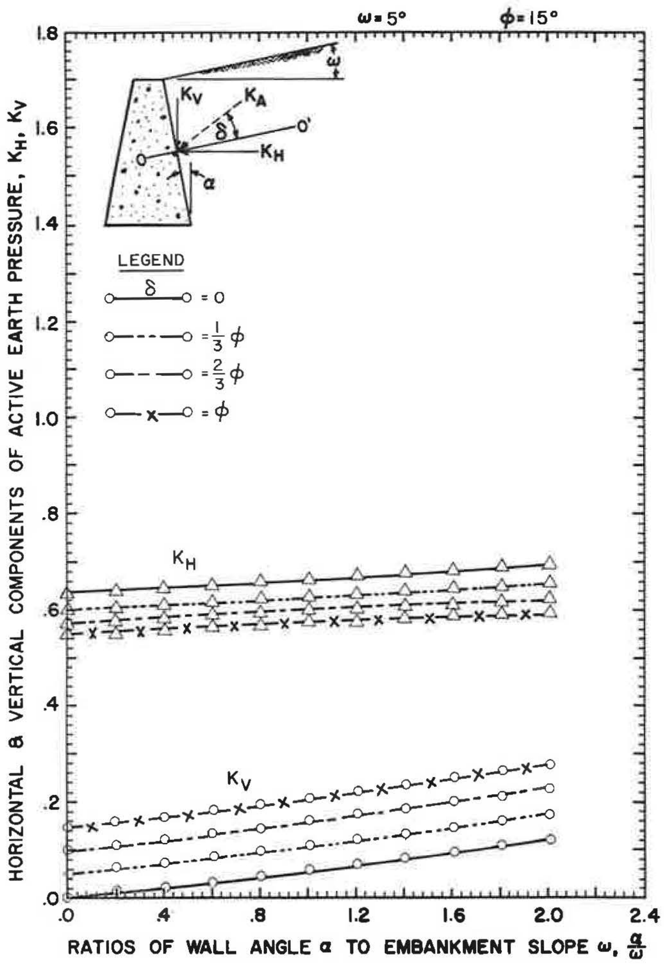


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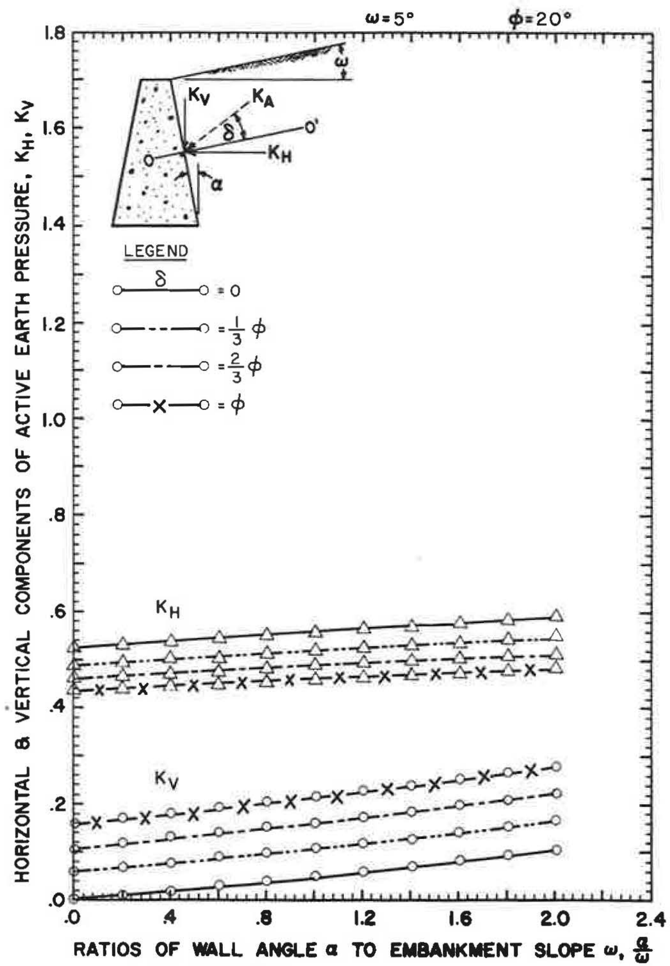


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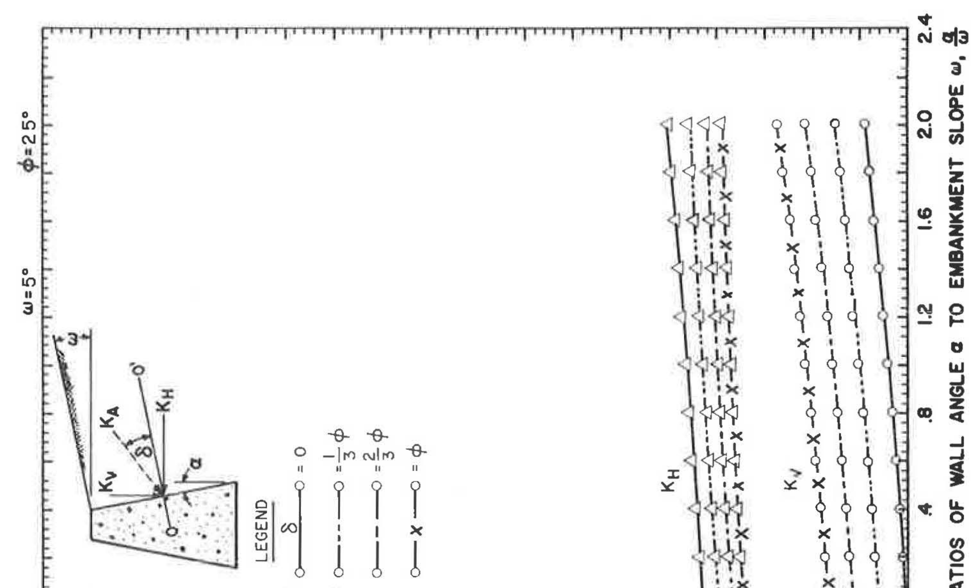


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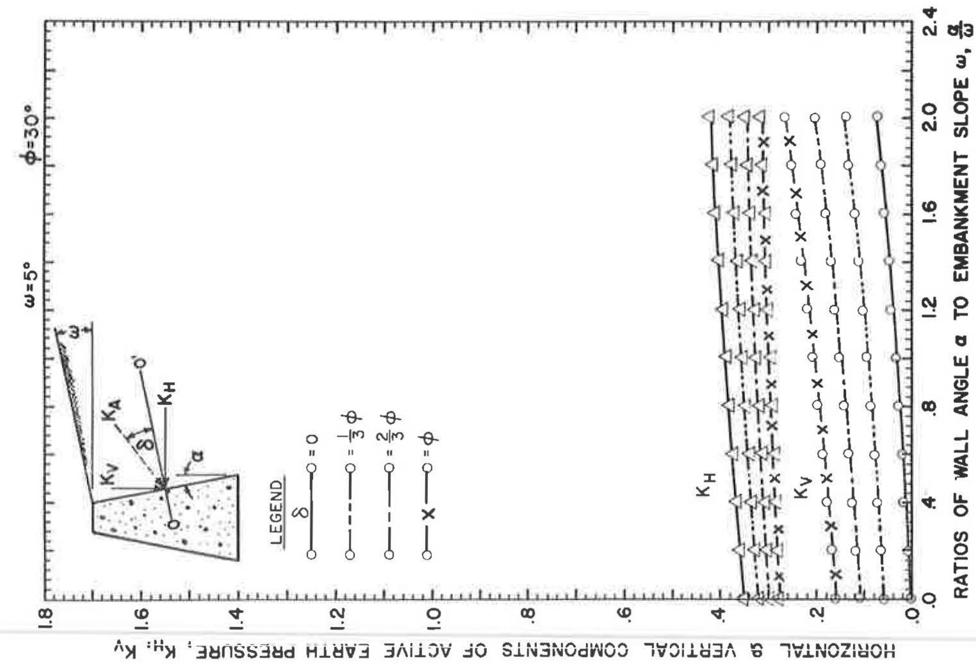


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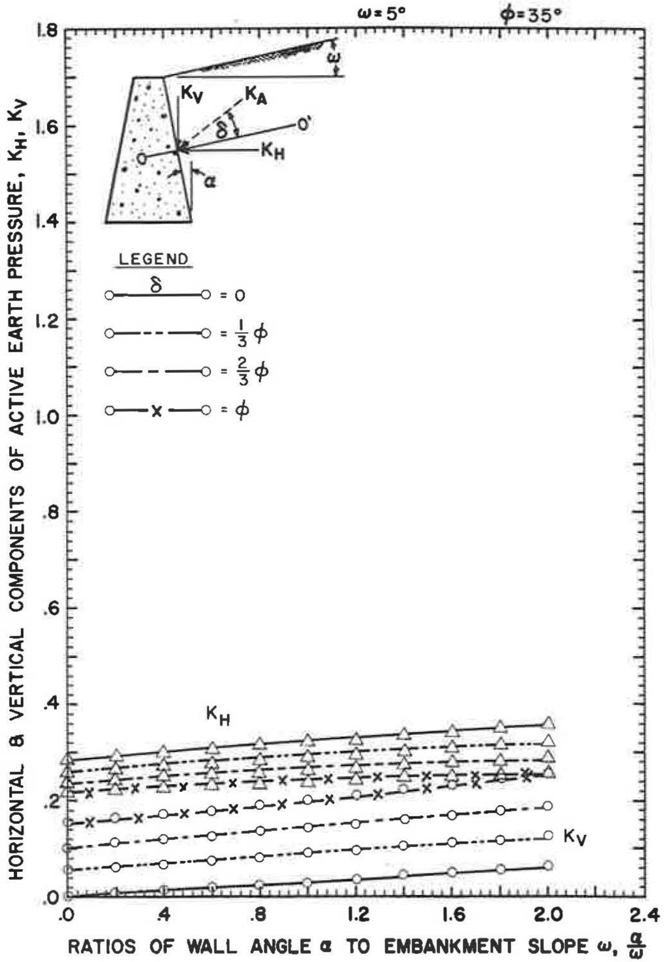


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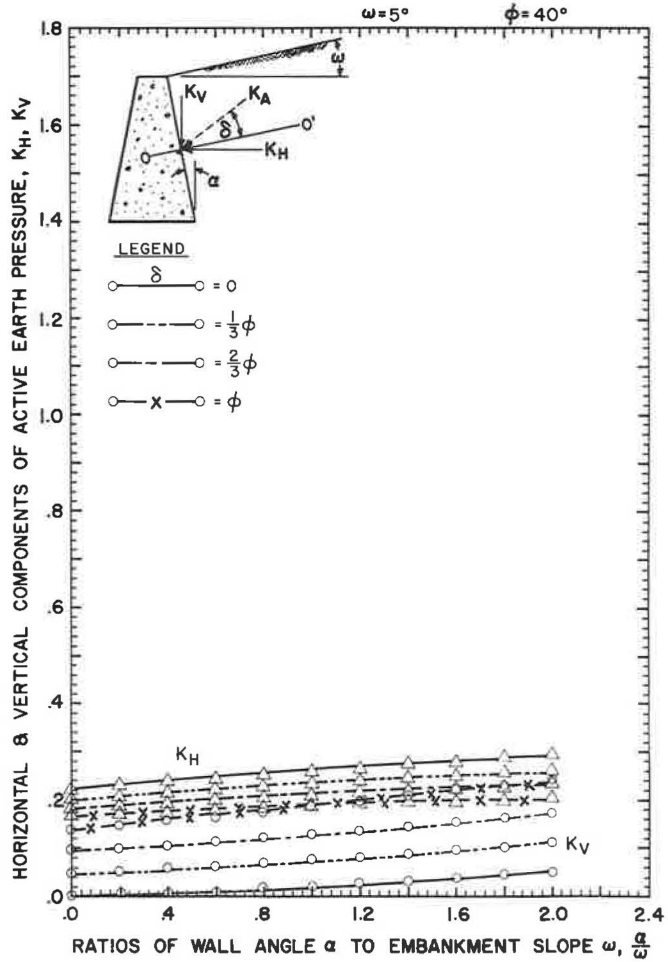


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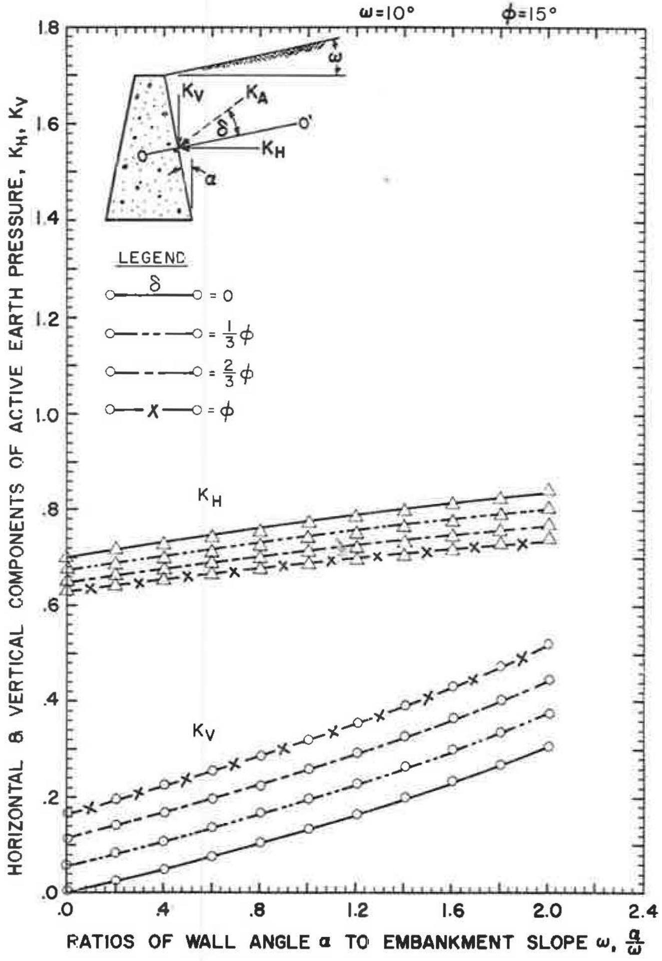


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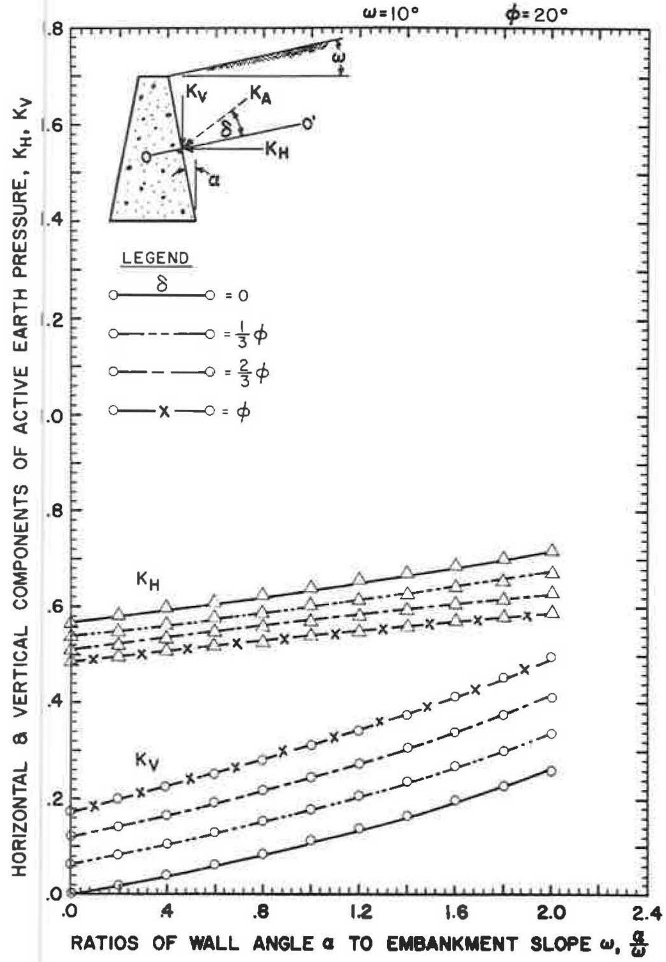


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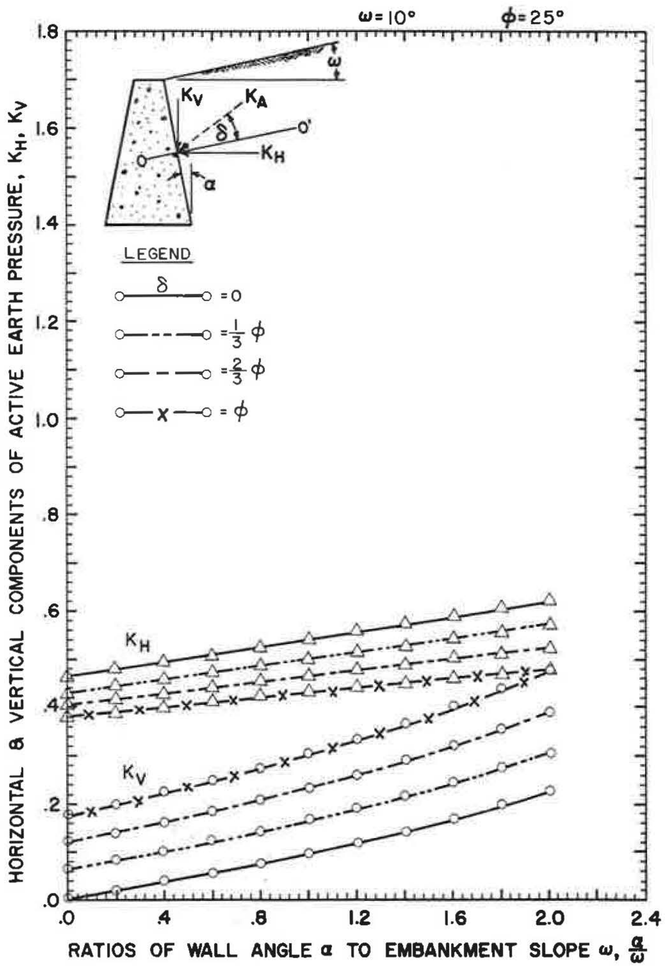


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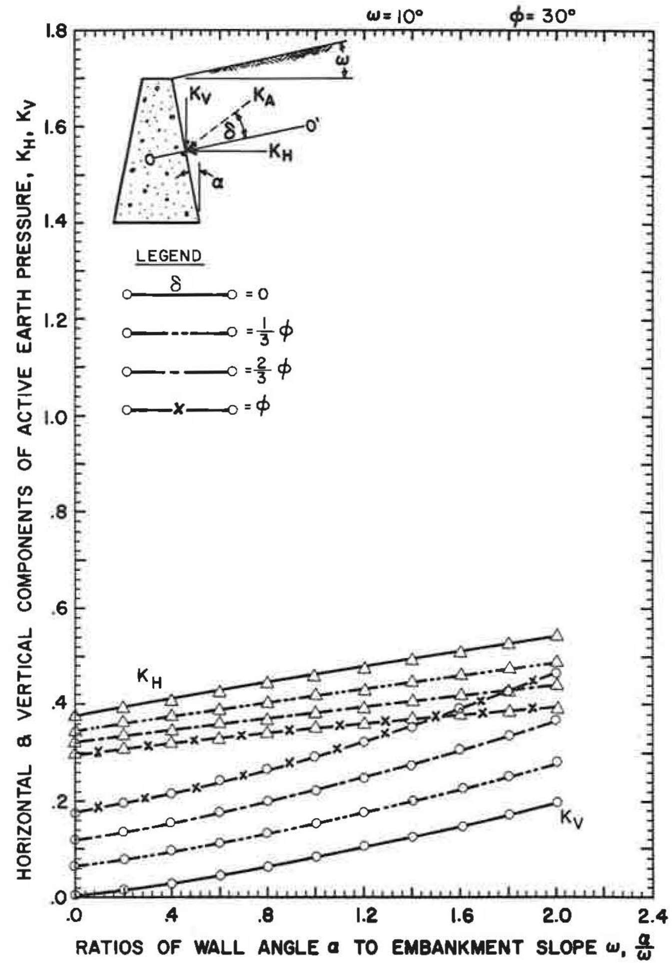


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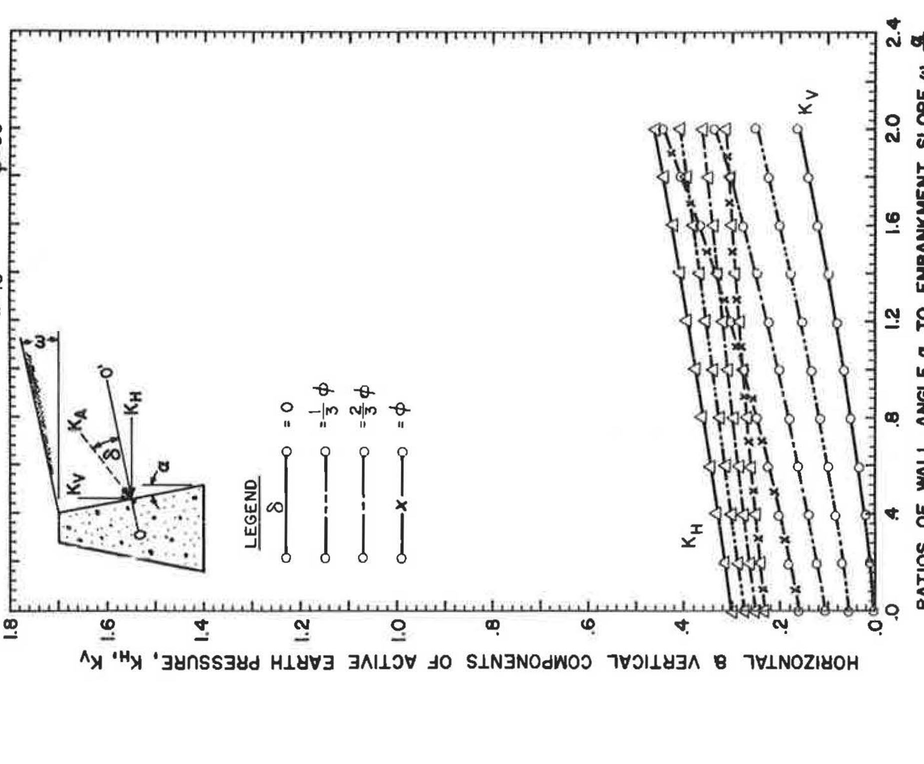


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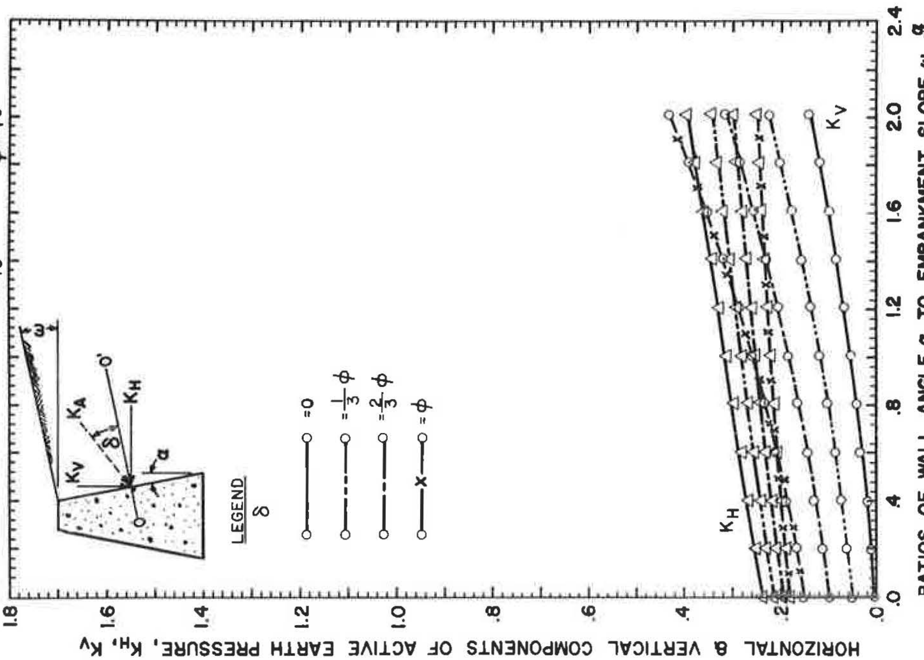


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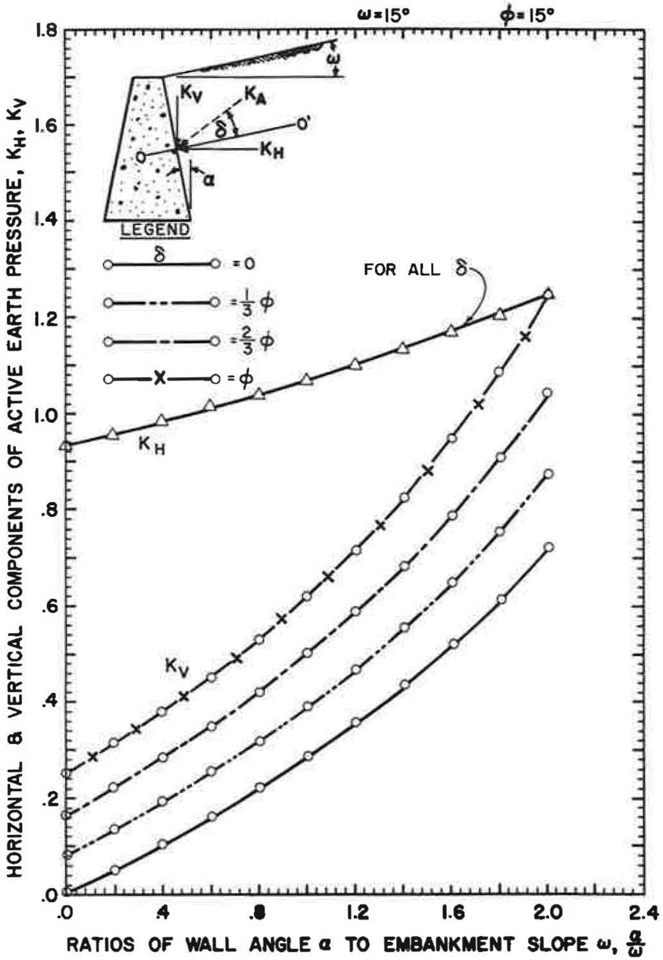


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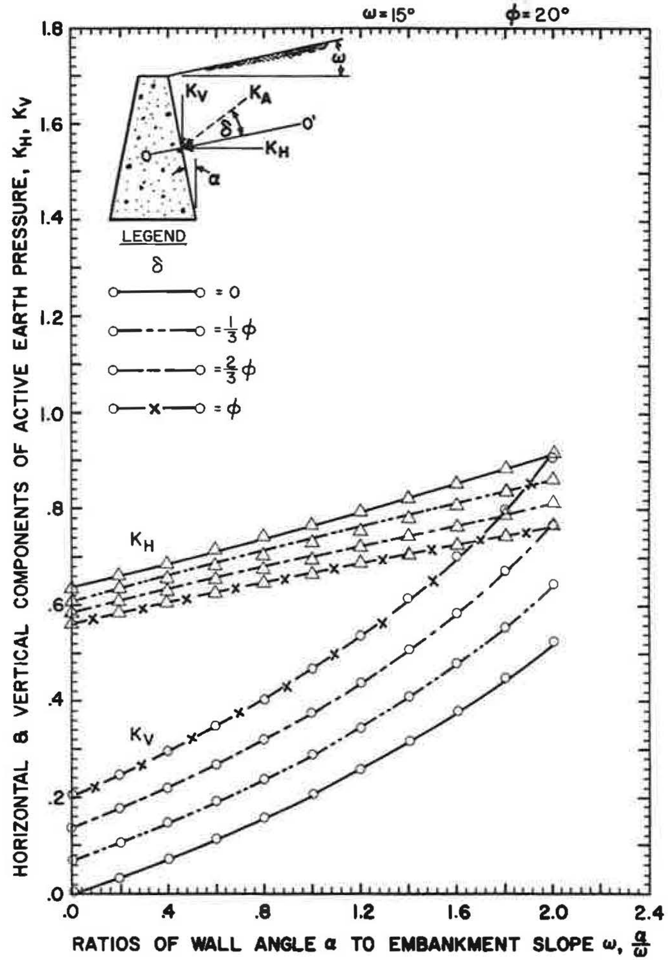


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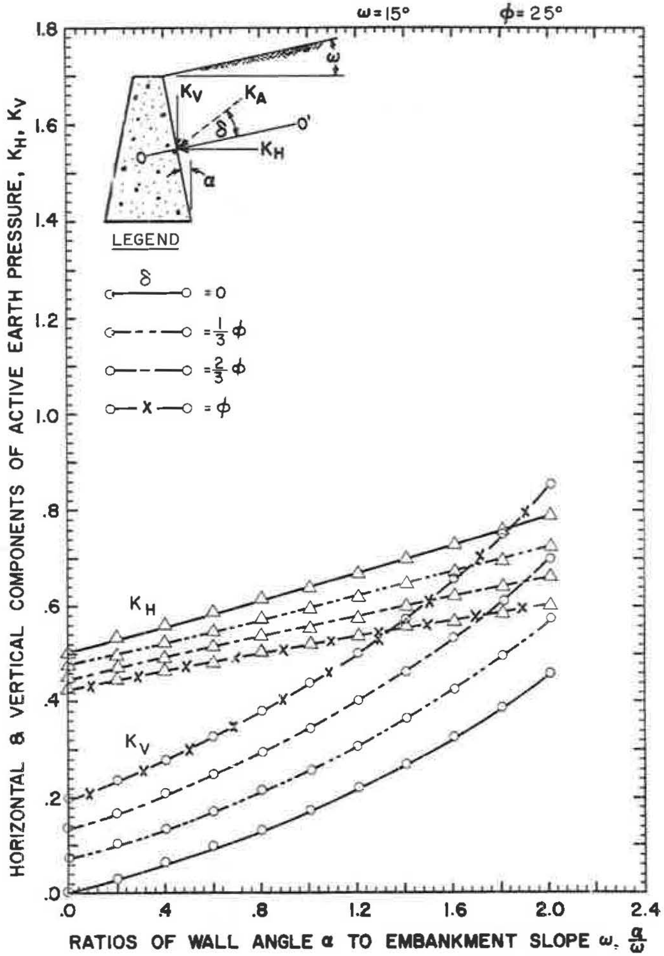


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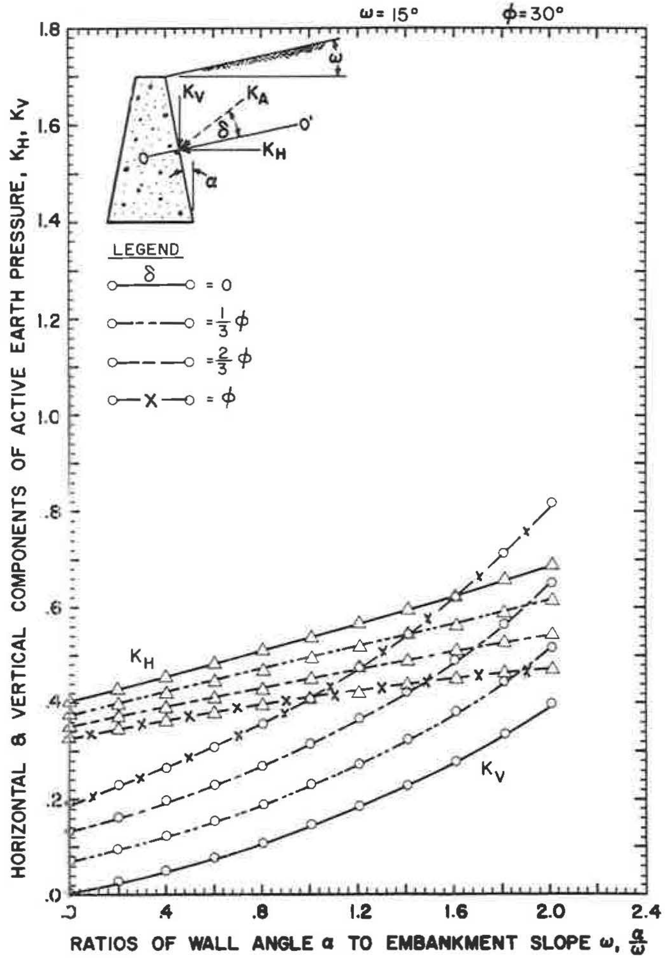


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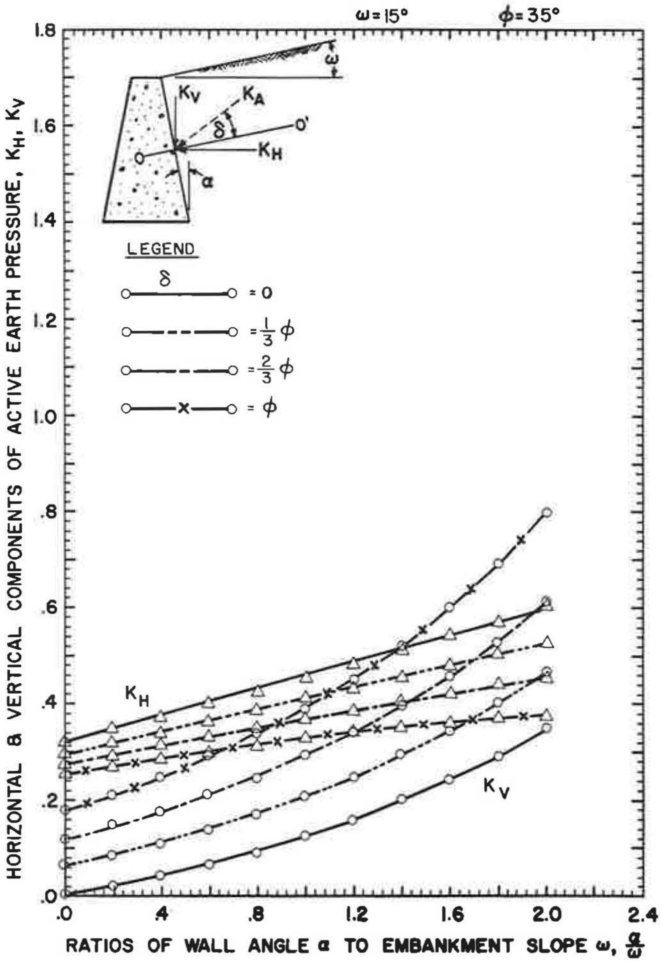


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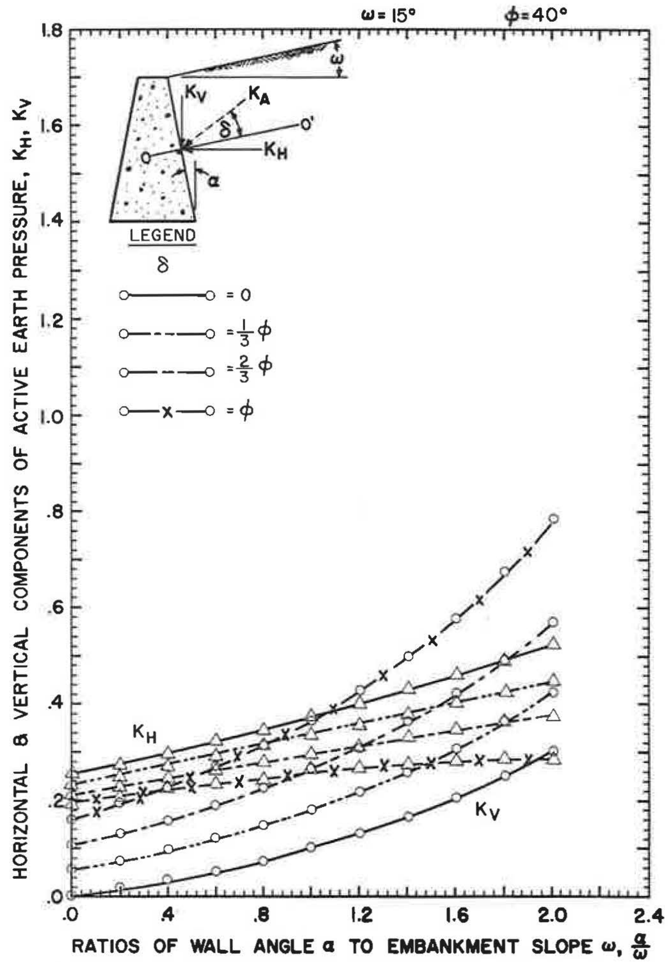


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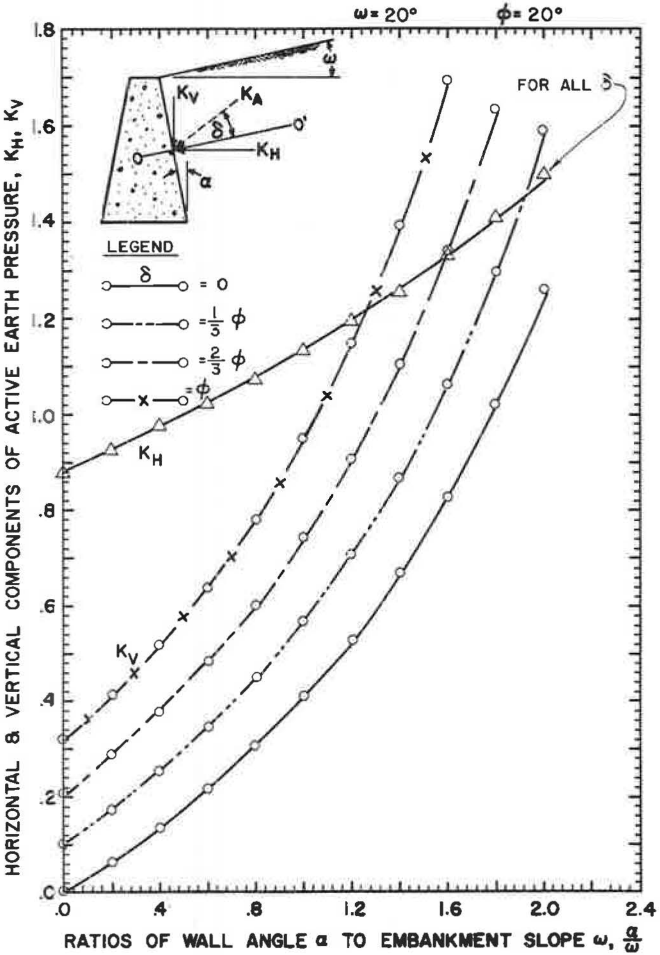


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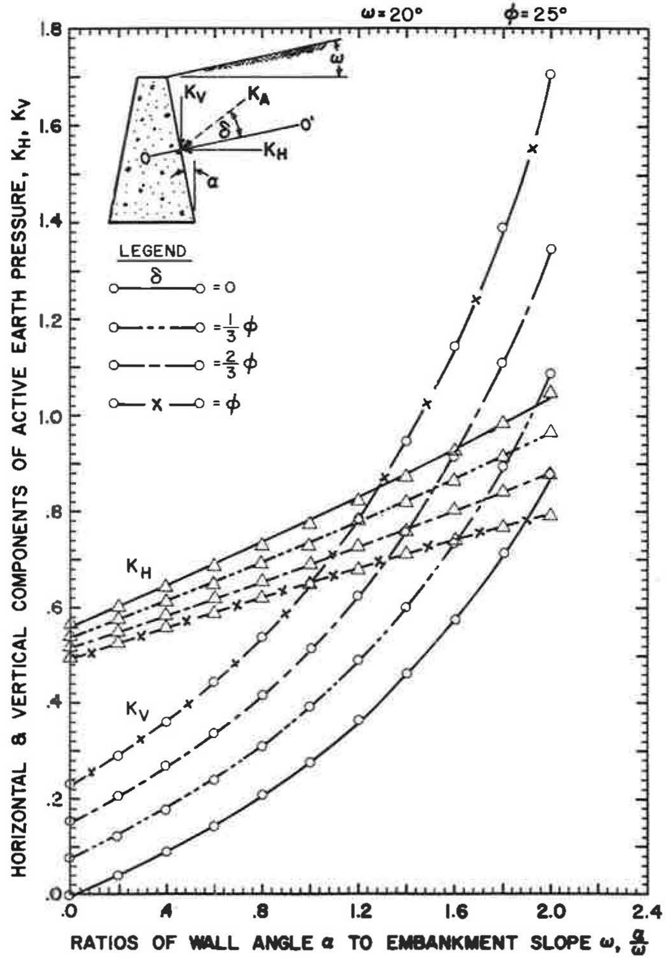


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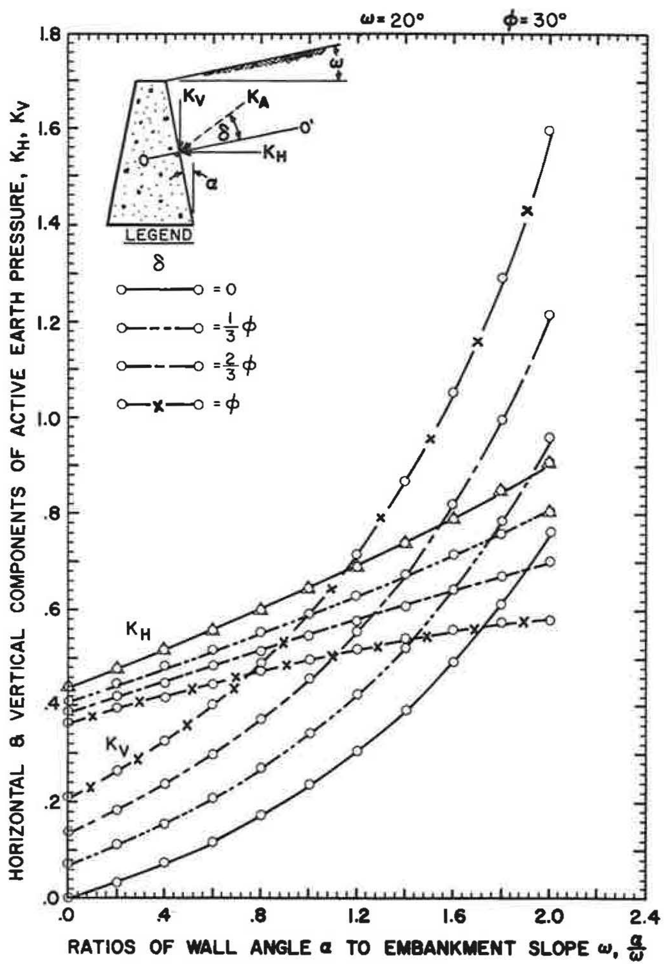


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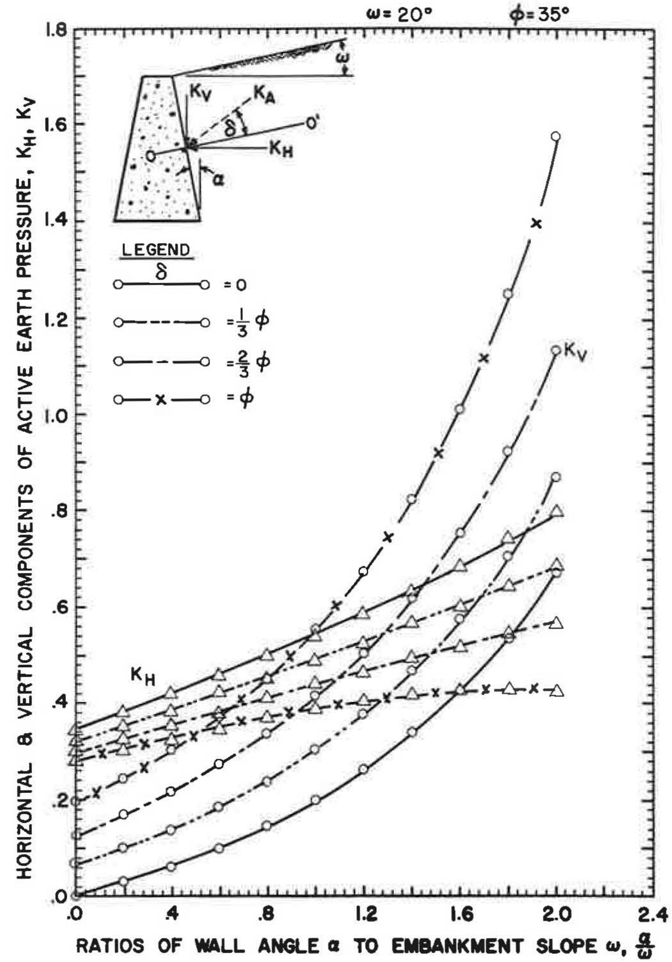


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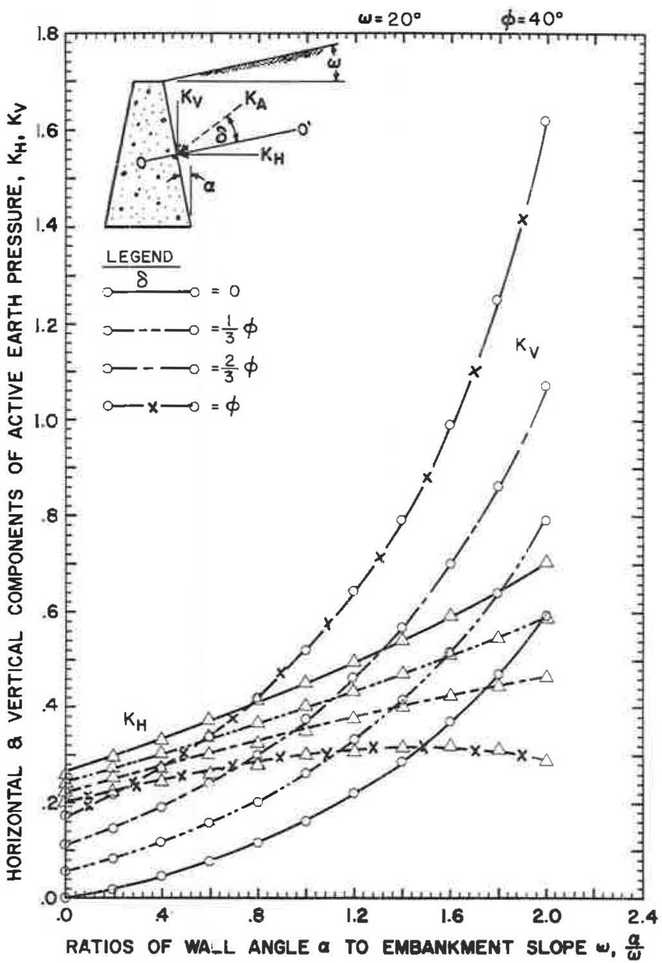


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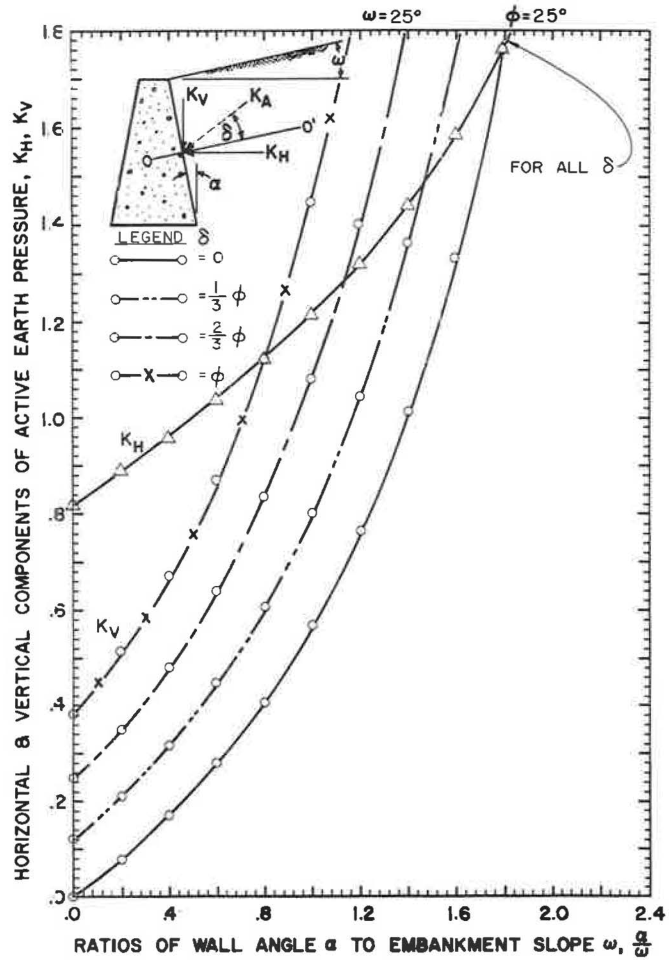


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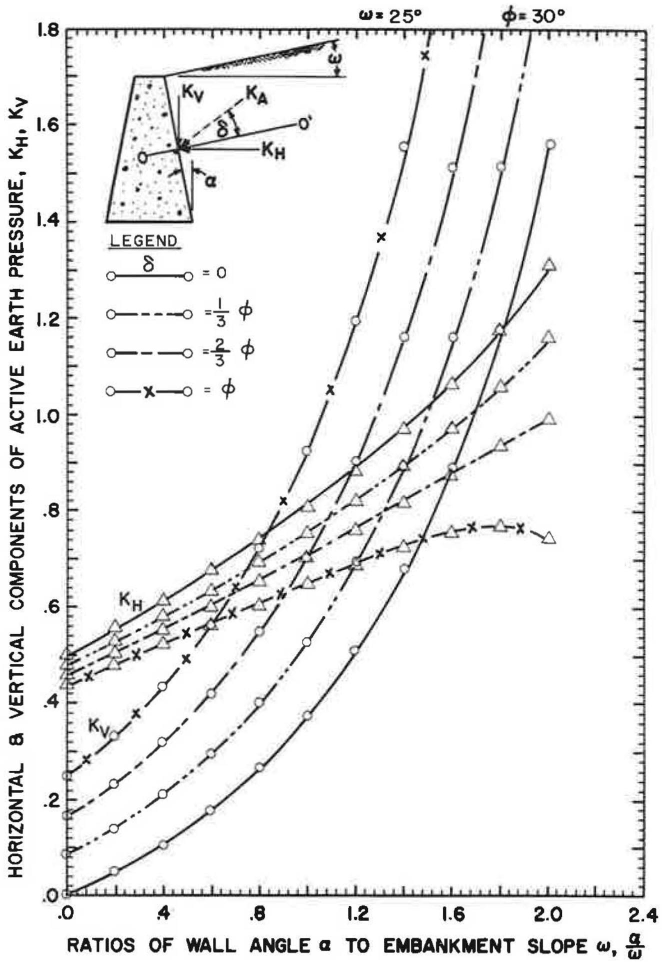


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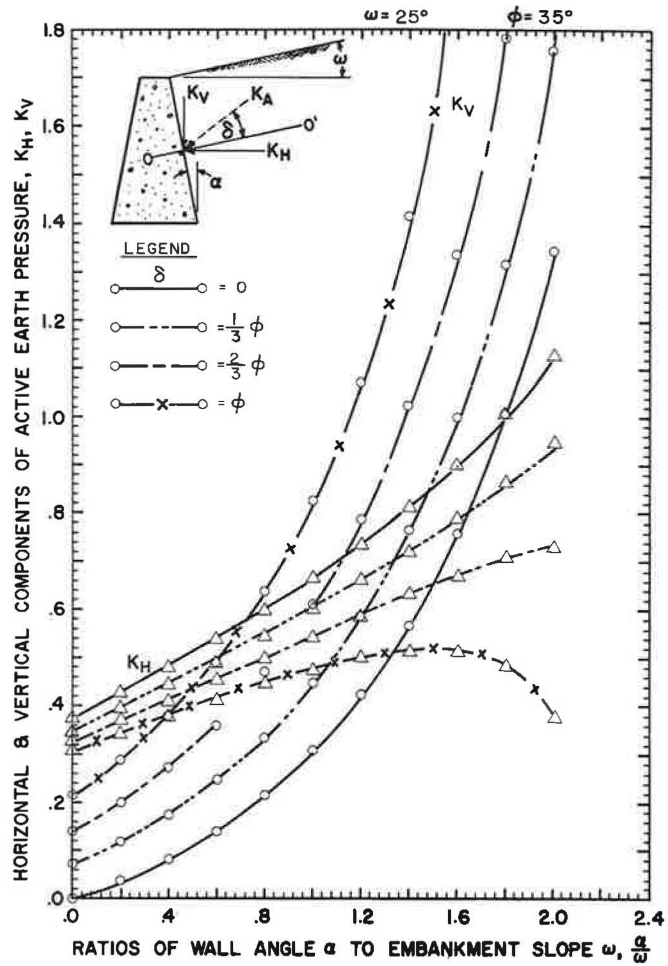


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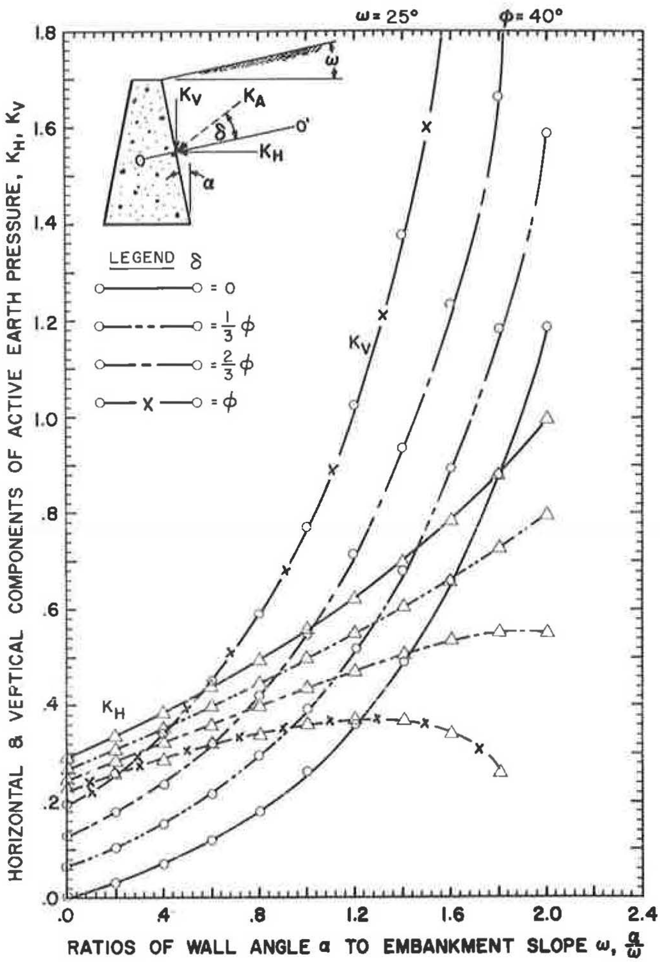

ω = 25°
φ = 40°
K_V
K_A
δ
O'
O
K_H
α
ω
LEGEND δ
= 0
= 1/3 φ
= 2/3 φ
= φ
K_V
K_H
HORIZONTAL & VERTICAL COMPONENTS OF ACTIVE EARTH PRESSURE, K_H, K_V
RATIOS OF WALL ANGLE α TO EMBANKMENT SLOPE ω, α/ω
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
.0
.0
.4
.8
1.2
1.6
2.0
2.4


Figure 34.