Application of Statistical Concepts To Accident Data

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•THE concern being expressed nationwide over the highway accident toll has generated a flood of magazine articles, promoted legislation and encouraged discussion within technical circles as to what can and should be done. As might be expected, there is no pat answer or agreement on the solution. Some believe the solutions hinge on stricter enforcement, others on more and better driver education, others on improved highways, and still others on safer vehicles. There does seem to be agreement on one point, however, and that is the lack of adequate accident records to enable agencies to establish with desirable accuracy the occurrence of highway accidents.

The purpose of this report is not to propose a method for obtaining adequate records, but to suggest methods by which better analyses of the limited accident data can be made, and above all, to guarantee that erroneous conclusions are not drawn from the data at hand.

THE CASE FOR STATISTICAL CONCEPTS

Accidents are by nature rare events (a few occurrences per million vehicle-miles of travel). The universe in which they occur is extremely large (many hundreds of millions of vehicle-miles). As a statistician would say, "We are dealing with a small sample of a large population." Given these conditions, it is very easy to draw erroneous conclusions from accident data unless well-established statistical concepts are utilized which will enable the engineer to be assured of its significance.

Most highway and traffic engineers cannot lay claim to being very well trained in statistics. Our understanding of this subject, however, should be sufficient to enable us to recognize when there is a need to utilize the training of statisticians. The following cases are cited to illustrate the need to apply statistical concepts to the analyses of accident data.

In one state, priority listings of hazardous rural and urban highway sections were prepared by listing sections based on the observed accident rates from highest on down. A rural section one mile long that had only three accidents (none involving personal injuries or fatalities) ranked number four on the priority scale. Another one-mile long rural section that experienced 186 accidents with 89 personal injuries ranked number 47 on the priority scale! In the urban listing, a 0.2-mile section that had 66 accidents and 12 injuries ranked number 13 while another 0.2-mile urban section with 123 accidents, 70 injuries and 3 fatalities, ranked number 190!

In another state, the annual state highway accident report was prepared which lists by route the accident rate for every control section. Prior to the reports being submitted, someone had gone through them and underlined in red pencil all sections that had a rate of 10 or more accidents per million vehicle-miles. Examination of these red-penciled sections showed that many of the over-10 rates were obviously not significant or worthy of closer scrutiny because only one or two accidents occurred which, when coupled with low vehicle-miles of travel, produced high rates. There were other sections with rates slightly below 10 that did appear to be significant since there were high vehicle-miles of travel present, but these were not singled out by the red pencil underlining.

Paper sponsored by Committee on Highway Safety and presented at the 46th Annual Meeting.

The highway and traffic engineers for a certain city determined that the cure for the high number of accidents at a complicated intersection was the installation of overhead sign bridges, improved signals and some limited approach widening. During the 3-month period following the improvement project, a decrease of 8 accidents was observed. A press release was prepared titled "Intersection Made Safer" showing how "_____'s most dangerous intersection at _____ apparently has been tamed." At the end of a still too short 8-month period, however, the picture changed. There were only 5 fewer accidents for a 12 percent decrease which was far short of the 38 percent needed to assure reliability. No press release or publicity was given to this.

The proper application of statistical concepts could preclude the pitfalls cited by giving the answer to such questions as, "How much variation in the accident rate should be expected as the result of normal chance variation?" or "How high must an accident rate be before it can be concluded that it definitely is above a tolerable limit that has been set?" or "How much of a reduction in the number of accidents must be experienced before it can be concluded that the improvement definitely helped?"

For the remainder of this article an attempt will be made to summarize statistical concepts that have been developed and applied to this problem area. It is interesting to note that many of these applications were developed over ten years ago, but, as yet, only limited use has been made of them.

FLUCTUATIONS IN ACCIDENT RATES

The Office of Technical Services of the U. S. Department of Commerce in 1958 distributed a report (1) which described a procedure for determining the amount of variation in the accident rate that could be expected due to chance probability for any highway control section. The input required is the overall accident rate for the highway and the number of vehicle-miles of travel on the highway control sections. By applying the following formulas, both upper and lower control limits on the overall accident rate are established for each control section:

$$\text{Upper Control Limit} = \lambda + 2.576\sqrt{\lambda/m} + \tfrac{1}{2}m$$

$$\text{Lower Control Limit} = \lambda - 2.576\sqrt{\lambda/m} - \tfrac{1}{2}m$$

where

λ = overall accident rate for the highway, and
m = number of vehicle miles of travel on a control section.

It is possible with the use of these equations to compare the observed accident rate for each control section with the control limits to determine whether the variation from the overall rate is greater than could be attributed to chance (Fig. 1).

In 1966, S. K. Dietz discovered an error in the original equations described in the Office of Technical Services report (1) and later articles in HRB Bulletins 117 (2) and 341 (3). The Appendix contains this comment by Dietz which shows how the validity of the equations is improved if the "correction term" 0.829/m as appears in the original equations is omitted.

The coefficient of the second term (2.576) in the equation assumes a ½ percent probability that either the upper or lower control limit could be exceeded by chance variation in the observed accident rate. Other coefficients may be used, as described in the Appendix, which would increase the probability that chance fluctuation in the observed rate could cause the control limits to be exceeded.

Notice the high observed rate for Point E in Figure 1. Due to low vehicle-miles of travel on this section, the control limits differ widely from the overall rate. It could be concluded that the apparently high rate at Point E is not worthy of investigation since it is within the range of variation that could be expected by chance. By comparison, see the observed accident rate at Point D. It does not appear to be very much higher than the overall rate and certainly is much lower than Point E, but the fact that it is outside the control limit for that section indicates that its variation from the overall

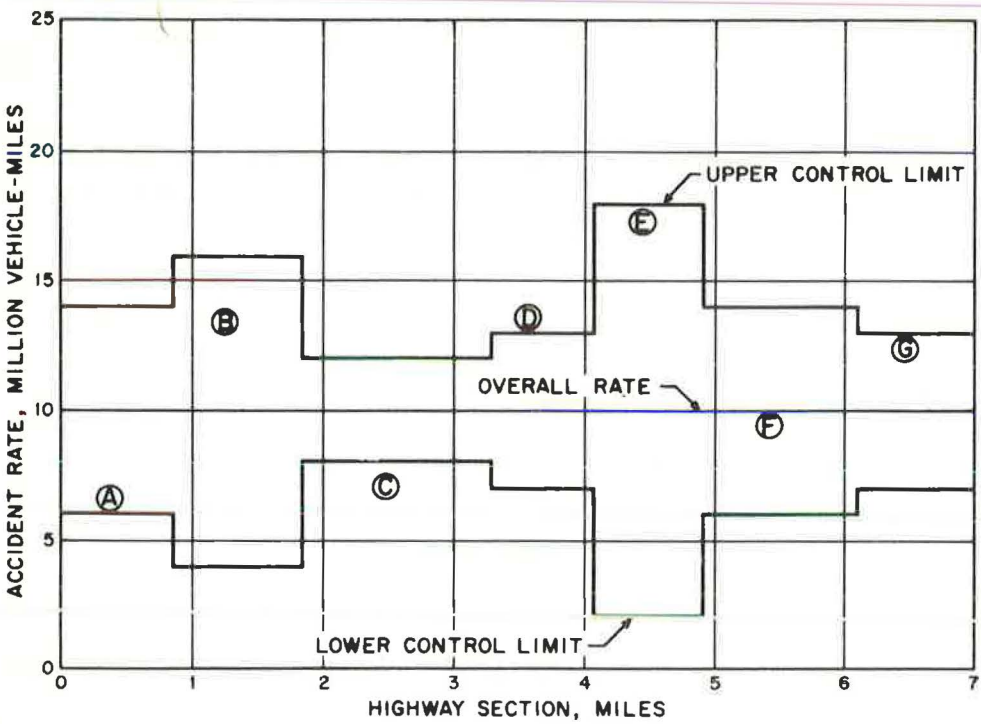

Figure 1.

rate is more than could be attributed to chance. In other words, something is definitely wrong and this section of road should be analyzed to find the reason. By the same token, the section indicated by Point C should be investigated since it can be concluded that something other than chance variation is present for this accident rate to be so much lower than average. Possibly, by finding out what is good, more insight could be gained and applied to other sections.

The article by Rudy (3) in 1962 describes the application of this technique to a route in Connecticut. The Montana Highway Department recently programmed this procedure for their IBM 1620 computer and successfully ran their 1965 accident data. The program prints out the upper and lower control limits and the observed accident rate for each highway control section. The results are incorporated in the 1965 Annual Accident Report by printing an asterisk alongside the computed accident rate for those sections that are out of control. Examination of the annual accident report shows that the procedure is not only identifying quite a few sections that are out of control with respect to the upper control limits, but also many that are "out" with respect to the lower control limit. Because it can safely be said that for those sections that are out of control the variation from the overall rate did not occur by chance, the next logical step is to conduct an investigation to find the reasons for the abnormally high and low rates.

An approach is under consideration which would assemble all possible data concerning the time and exact location of the accidents, the accident type, weather conditions, roadway alignment, cross-section details, and sight distance for those out-of-control sections into one report. A team consisting of possibly a traffic engineer, design engineer, maintenance engineer, and law enforcement officer could then study the assembled data and view the highway sections on the ground to try to determine the reason for the abnormal rates.

CRITICAL RATE ANALYSIS

A somewhat simpler application of statistical analyses to accident rate data that utilizes the same basic concepts, but approaches it from a different aspect, is being utilized in Idaho. It has been described in a 1964 report (4).

TABLE 1

MINIMUM ACCIDENT RATE TO ASSURE CRITICAL ACCIDENT RATE IS EXCEEDED
(Probability Level = 0.95)

Critical Accident Rate (acc/MVM)	Number of Accidents per Section					
	5	10	20	30	50	100
2.0	5.0	3.7	3.0	2.8	2.6	2.4
3.0	7.5	5.6	4.5	4.2	3.9	3.6
5.0	12.5	9.3	7.5	7.0	6.4	5.9
7.0	17.5	13.0	10.5	9.8	9.0	8.3
10.0	25.0	18.5	15.0	14.0	12.8	11.9

The procedure requires that a "critical accident rate" be selected. This could be thought of as the rate which has been determined to be the highest that can be tolerated. For any given critical rate, the procedure indicates the minimum accident rate that is significantly higher than the critical rate for any given number of accidents on the section. Table 1 gives values for different critical rates.

Figure 2 shows the values expressed graphically. With reference to the observed rates indicated by Points A through D and an established critical rate of 10 accidents per million vehicle-miles of travel, it can be concluded that the observed rate of 15 based on 10 accidents (Point B), and 13 based on 30 accidents (Point C), are not significantly higher than the critical rate of 10. The observed rates shown by Points A and D, however, should be attributed to something other than chance variation. The observed rate of 20 accidents per million vehicle-miles based on 10 accidents (Point A), and 14 accidents per million vehicle-miles based on 50 accidents (Point D), are significantly higher than the established rate of 10 accidents per million vehicle-miles.

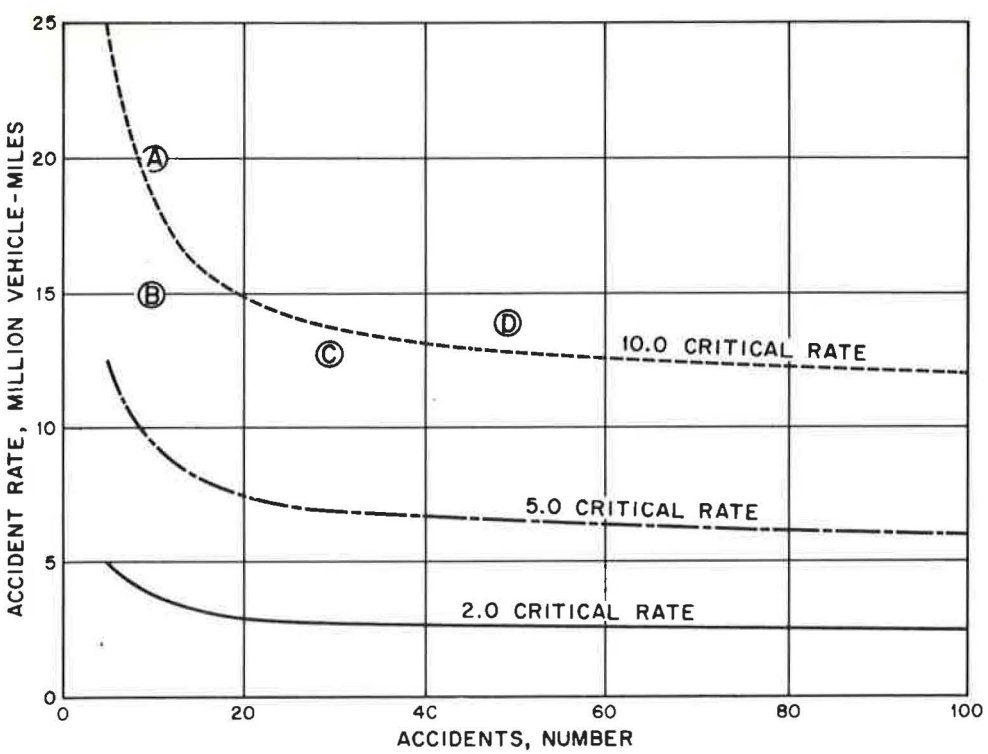

Figure 2.

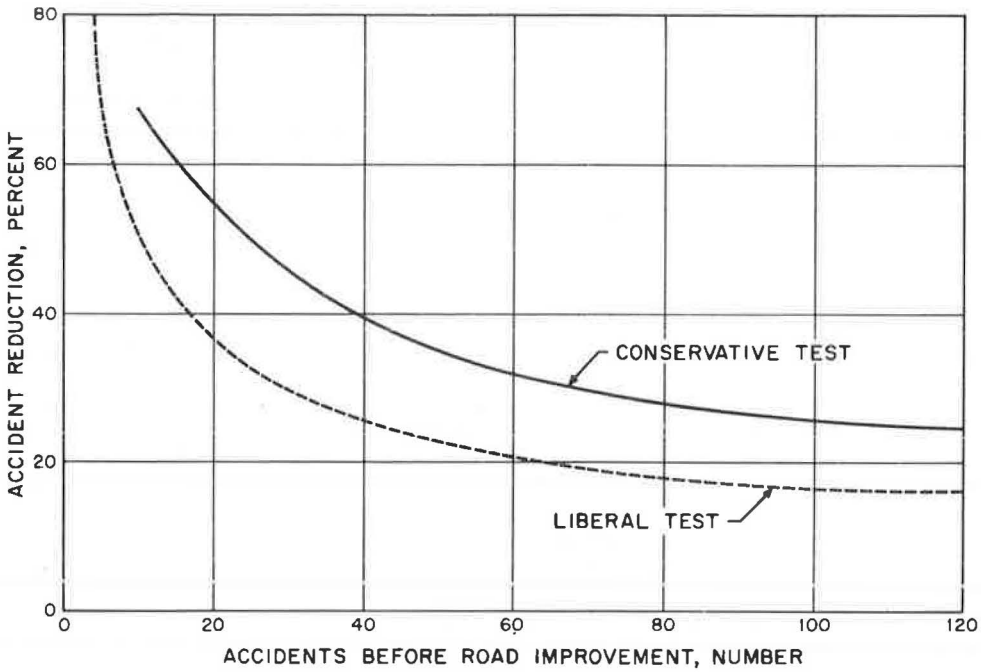

Figure 3.

While this procedure provides a quick method for identifying those sections that do have rates significantly above the predetermined "critical" or "tolerable" rate, it does not pinpoint sections with significantly low accident rates, thereby precluding the possibility of profiting from what is really good.

EVALUATING THE EFFECTIVENESS OF IMPROVEMENT PROJECTS

In 1959, R. M. Michaels published a procedure for establishing whether the percent reduction in the number of accidents "after" an improvement was made was statistically significant compared to the "before" situation (5).

The procedure considers the Poisson distribution as an appropriate approximation of the accident probability for what is called the "Liberal Test," and the chi-square test to determine whether the before and after samples differ significantly for what is called the "Conservative Test." One of the main advantages of this procedure is that the engineer can test for significance knowing only the number of accidents before the improvement was made and the percent reduction after. The test involves spotting this point on a graph (Fig. 3) to see whether the reduction is enough to be considered significant.

It is regrettable that a simple check, such as the one mentioned, has not been used more widely to permit definite conclusions to be drawn regarding the effectiveness of spot improvement projects. A recent memorandum from the U. S. Bureau of Public Roads (6) has drawn attention to the importance of this in evaluating the effectiveness of spot improvement projects. The September 1966 issue of "Traffic Engineering" contains a reprint of the procedure (7). With this recent publicizing of the procedure, it is hoped more use will be made of it by engineers.

CONCLUSION

The value of and need for application of the foregoing statistical concepts to analyses of improvements which were reported in recent issues of "Traffic Engineering" is

worthy of comment. In one article (8) an author states: "While the small sample to date is not too significant statistically. . . ." A check of the percent reduction in accidents (adjusted for differences in the vehicle-miles of travel) with the procedure recommended by Michaels shows that the reduction in the number of accidents is statistically significant by the "conservative" test.

Another article (9) states: ". . .the improvement in the accident experience clearly indicates that the. . .has eliminated, as much as possible, the hazards involved. . . ." Checking the percent reduction shows that the reduction is not statistically significant.

By contrast, the authors of another article (10) state: "This result indicates that the program, as currently followed, does reduce the accidents significantly." This is one of the few articles on the subject where the writers have indicated that statistical tests to validate their conclusions have been made. It is evident that the authors of this article went to the necessary effort to assure that the conclusions they stated were valid.

The awareness of this need has recently been brought out by T. N. Tamburri in an excellent article (11) dealing with the problems of accident research. He states: "Another reason can be that the researchers do not establish controls that discriminate between chance variation. . . that is, the researchers may not understand the statistical and practical limitations of the data. The sixth pitfall, then, is the failure to establish statistical controls."

As traffic and highway engineers we have our work cut out for us. We cannot study each and every highway section in detail but, instead, will have to rely on statistical or quality control concepts, as have other disciplines, to weed out the relevant data from the mass of information. This will permit us to concentrate our limited professional manpower and financial resources in those areas that offer the greatest promise of "payoff." Certainly we cannot allow ourselves to draw unsound conclusions from accident data!

We are in a fortunate position. The techniques have been developed; all we have to do is apply them. A procedure must be found, however, to accelerate the use of these concepts over the pace of the past. The application of control limits to accident-rate data which Montana has just made operational was published originally in 1958; the critical-rate analysis procedure which Idaho started using in 1964 was developed in 1957, and the method for evaluating the effectiveness of improvement projects which has been emphasized in 1966 was published in 1959.

ACKNOWLEDGMENTS

The author especially wishes to express his gratitude to Stephen K. Dietz of Westat Research Analysts, Inc. for his efforts in discovering the error in the original equations and in reviewing the text of the Appendix for accuracy. The review of Dietz' work by Jesse Orlansky of the Institute for Defense Analyses, leading to agreement on the correction, was helpful in assuring that the equations indicated in this report are accurate.

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Appendix

CORRECTION OF EQUATIONS FOR UPPER AND LOWER CONTROL LIMITS

The use of statistical quality-control techniques for detecting accident rates which are significantly above average is presented in the report by Norden et al (2). The expression for approximating the upper control limit is given by the equation:

$$\text{Upper Control Limit} = \lambda + 2.576\sqrt{\lambda/m} + 0.829/m + \tfrac{1}{2}m \tag{1}$$

Or in general:

$$\lambda p = \lambda + k\sqrt{\lambda/m} + 0.829/m + \tfrac{1}{2}m \tag{2}$$

where $k = 2.576$ for $P = 0.995$ and

$$P = \text{Prob. [rate} \leq \lambda p] \tag{3}$$

Eqs. 1 and 2 would actually be more accurate if the 0.829/m term were omitted, and become exact as $\lambda m \to \infty$ (12):

$$\lambda p = \lambda + k\sqrt{\lambda/m} + \tfrac{1}{2}m \tag{4}$$

Table 2 compares the approximations of Eq. 2 vs Eq. 4 with the true value from Eq. 5 for cases where the average number of accidents, a, varies from about 0.3 to 13 accidents for the 90 percent and 95 percent probability levels.

The true value of λp is

$$\lambda p \text{ (TRUE)} = c/m \tag{5}$$

where c is the solution in

$$P = \sum_{x=0}^{c-1} \frac{e^{-a}a^x}{x!} = 1 - \sum_{x=c}^{\infty} \frac{e^{-a}a^x}{x!} \tag{6}$$

for specified values of P and a.

TABLE 2

COMPARISON OF EQ. 2 AND EQ. 4 WITH THE TRUE VALUE FROM EQ. 5

Probability Level	Avg. No. of Accidents ($a = \lambda m$)	Error in Eq. 2 (%)	Error in Eq. 4 (%)
0.95	0.325	33	-8.5
0.95	1.970	12	-4.6
0.95	11.638	3	-1.4
0.90	0.530	40	-1.8
0.90	1.103	26	-1.7
0.90	12.820	4	-0.5

ALTERNATE DEFINITION OF CONTROL LIMITS

Eq. 4 provides an expression for the upper control limit, λp, which has a probability 1-P of being equaled or exceeded by chance. This is the definition used by Norden et al (2).

It is easy to forget that an accident rate is actually a discrete variable for a given value of m. For example, the accident rate for a section with m = 5 million vehicle-miles of travel during a study period can only assume the discrete values of 0, 0.2, 0.4, 0.6, . . ., etc., corresponding to 0, 1, 2, 3, . . ., etc., accidents.

If we wish to define the upper control limit, λp, as that which has a probability 1-P of being exceeded, then λp would be given by

$$\lambda p = \lambda + k\sqrt{\lambda/m} - \tfrac{1}{2}m \tag{7}$$

This is identical to Eq. 4, except that the sign of the $\frac{1}{2}m$ term is reversed.

By the same token, if we define the lower control limit, λp, as that which has a probability 1-P of being equaled or exceeded (by a more negative number) then λp would be given by

$$\lambda p = \lambda - k\sqrt{\lambda/m} - \tfrac{1}{2}m \tag{8}$$

If we wish to define the lower control limit, λp, as that which has a probability 1-P of being exceeded (by a more negative number) then λp would be given by

$$\lambda p = \lambda - k\sqrt{\lambda/m} + \tfrac{1}{2}m \tag{9}$$

ALTERNATE PROBABILITY LEVELS

The coefficient, k, of 2.576 in Norden et al (2) assumes a probability level for the upper and lower control limits of 0.990 or a 1-P of 1 percent. Eq. 2, in discussing the upper control limit only, uses a k value of 2.576 for a probability level of 0.995 or a 1-P of ½ percent. The difference is whether we are talking about the probability of either or both of the control limits being exceeded.

If we use a k value of 2.576, it would therefore mean that 1 percent of the locations which have only a "normal" or "average" accident rate could be expected to fall beyond the control limits by chance even though there is nothing out of the ordinary about them, and that ½ percent of the locations which have only a "normal" or "average" accident rate could be expected to fall above the upper, or below the lower, control limit by chance even though there is nothing out of the ordinary about them.

Other coefficients which would change the probability of labeling a rate as out of the ordinary when it is in fact "normal" could be used as follows: (a) 2.576 for 1 percent false detection of both or ½ percent of either, (b) 1.960 for 5 percent false detection of both or 2½ percent of either, (c) 1.645 for 10 percent false detection of both or 5 percent of either, (d) 1.440 for 15 percent false detection of both or 7½ percent of either, and (e) 1.282 for 20 percent false detection of both or 10 percent of either.