Factors Influencing the Resilient Deformations of Untreated Aggregate Base in Two-Layer Pavements Subjected to Repeated Loading

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•IN RECENT years, a large body of knowledge has developed relating the cracking of asphalt-concrete pavements to the transient deflections of the pavements measured under specific axle loads (1, 2). Cracking of this type has been attributed to fatigue failures of the asphalt-concrete surfacing (1, 3) resulting from repeated stresses or strains induced by traffic loads over a period of time. Although it is recognized that the induced stresses and strains in the asphalt surfacing are associated with the curvature of the deflected pavement surface, it is to be expected that they will increase in a general way with the magnitude of pavement deflections. Thus, prediction of deflections from representative tests on paving materials in advance of construction would be a valuable first step in solving the problem of preventing load-associated pavement cracking. Recently, the California Division of Highways instituted such a procedure on a trial basis for special conditions (4).

From available data, it would appear that fatigue distress results from instantaneous and recoverable deflections in the pavement components and is not necessarily associated with any plastic or permanent deformations. Thus, it might be reasonably concluded that the deflections which produce this form of distress could be calculated, at least approximately, from an appropriate elastic theory for layered systems.

To use such a theory, however, requires appropriate values for the material properties of the paving materials (e.g., elastic modulus and Poisson's ratio). Although Poisson's ratio for most paving materials appears to lie in a comparatively narrow range (0.3 to 0.5) for the conditions of stress encountered in the pavement, the same cannot be said of the elastic moduli, or more appropriately the deformation moduli, of the constituents of the pavement. This would appear to be particularly true for untreated-aggregate-base materials.

At the 1962 International Conference on the Structural Design of Asphalt Pavements it was noted (5):

> On the basis of the results reported at this Conference there would seem to be a great need for increased emphasis on the study of base-course materials. The base-course characteristics may play a large part in determining both transient pavement deflections and curvature, yet apart from the new device, the resiliometer, there are no laboratory testing techniques available to evaluate base-course characteristics in this regard. The large scatter of values for base-course moduli reported by different authors is somewhat disturbing and the development of new procedures for evaluating the transient deformation characteristics of base-course materials, together with a systematic study of different materials and conditions, would be highly desirable.

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In recent years, a number of laboratory test procedures have been developed and used to measure the behavior of pavement materials under conditions of load similar to those created by moving traffic. These methods include repeated-load triaxial-compression tests for fine-grained soils and various dynamic tests to measure the stiffness of asphalt concrete. It appears that developments in this area are sufficiently advanced so that the repeated-load triaxial-compression test can be applied to study the behavior of aggregate bases. When used in conjunction with similar tests on subgrade soils and some form of stiffness measurement of asphalt concrete, it is possible that the results can be used to develop, within the framework of suitable theory, a measure of the response of the pavement to moving traffic.

Thus, the objectives of this research are to study and evaluate those characteristics of untreated base-course aggregates which determine the deflections of asphalt-concrete pavements under moving wheel loads, and from these evaluations to predict the transient deflections of prototype pavements for various load conditions.

Data are presented showing the results of laboratory repeated-load triaxial-compression tests on a variety of granular materials including a well-graded gravel, a uniform sand and disturbed, recompacted samples of untreated-base and subbase materials from in-service pavements. In addition, the deflections observed in repeated-load plate tests on prototype pavements, consisting of two-layer systems of the untreated gravel and a compressible, fine-grained subgrade, are presented together with the results of analytical procedures for predicting the deflections from the results of laboratory tests.

Even though it is recognized that conventional asphalt-type pavements consist of at least three layers, it would appear that definitive information on the role of the untreated aggregate in the pavement system can best be studied in as simple a structure as possible (in this instance, two layers) where the results will not be complicated by the presence of overlying layers.

The ultimate objective of such research is to provide an additional method of design to the paving engineer so that he can attempt to minimize, for the design life of the structure, the form of distress resulting from fatigue failure of the asphalt surfacing. Hopefully, the studies on granular base reported here will add to the knowledge required for the development of such a design procedure.

BACKGROUND

To provide a basis for the data, a brief summary of the existing information on laboratory-determined resilience characteristics of untreated aggregates in repeated-load triaxial-compression tests is included. In addition, a brief summary of a more detailed review (6) of the results of plate-load tests and their interpretation within the framework of elastic theory is also presented. The deformations under consideration are elastic in the sense that they are recoverable; however, they are not necessarily proportional to stress or instantaneous. Thus, in keeping with the terminology introduced by Hveem (1), recoverable deformations will be referred to as resilient deformations and the corresponding moduli as resilient moduli.

Repeated-Load Triaxial-Compression Tests on Granular Materials

Whereas repeated-load triaxial-compression tests have been performed on cohesive soils for over a decade, it is only recently that this type of test has been used to any large extent to study the resilient characteristics of granular materials.

Seed and Chan (7) investigated the effect of the duration of stress on the total deformation of soil specimens subjected to repeated loading. An increase in the duration of stress application, for intervals up to 2 min, resulted in an increase in the total deformation of the silty sand that they tested. From their data, it is also possible to show that the modulus of resilient deformation increases as the duration of load application decreases and that this increase is more pronounced for very short durations of load.

Haynes and Yoder (8) presented the results of undrained repeated-load triaxial-compression tests on gravel and crushed stone, similar to those used in the base

course of the special flexible pavement sections at the AASHO Road Test. Specimens of both the gravel and crushed stone were tested at dry densities corresponding to the average obtained in the field tests. The densities were obtained in the laboratory by impact compaction using a 5.5-lb weight falling from a 12-in. height. A 15-psi lateral pressure and a 55-psi deviator stress were used in all the tests. For the gravel, the modulus of resilient deformation was influenced both by gradation (i.e., percent passing the No. 200 sieve) and the degree of saturation, with an increase in degree of saturation causing an increase in resilience. On the other hand, for the crushed stone, the influence of gradation was small, and the degree of saturation for the range investigated (70 to 80 percent) appeared to be of minor importance.

Biarez (9) has presented results of cyclic-load triaxial-compression tests on a uniform sand (grain diameter 0.016 in.) in which the variation of the modulus of resilient deformation with mean normal stress was investigated. From the results obtained after several cycles of load, he concluded that the variation of the modulus with the mean normal stress may be stated as

$$E = K \cdot \sigma_m^n$$

where

E = the modulus of elasticity,

K = constant,

σ_m = mean normal stress = $\frac{\text{sum of principal stresses}}{3}$, and

n = exponent varying from 0.5 to 0.6.

Some repeated-load triaxial tests have been conducted by DeGraft-Johnson (10) on an air-dried, fairly rounded, well-graded gravel; in these tests the influence of void ratio and confining pressure were investigated. The most significant result of the investigation was the striking dependence of the modulus of resilient deformation on the confining pressure. For the range in conditions investigated, doubling the confining pressure resulted in a 100 percent increase in resilient modulus.

Trollope et al (11) have conducted a series of tests on sand in which an attempt was made to simulate parking conditions by subjecting soil specimens to slow, repeated cyclic loads. The effects of initial dry density, rate of deformation, lateral pressure and stress level were investigated on a poorly graded sand. The studies indicated that the modulus of the sand increased with a decrease in void ratio and an increase in rate of strain. In addition, the modulus increased with an increase in confining pressure, but was independent of the axial stress so long as a failure condition was not reached.

The Texas Transportation Institute has also investigated the behavior of granular materials in repeated loading. Based on the results of tests on partially saturated, well-graded aggregates, Dunlap (12) has suggested an equation of the form

$$M_Z = K_2 + K_3(\sigma_r + \sigma_\theta)$$

where

M_Z = the modulus of deformation measured in the direction of an applied stress, σ_Z,

K_2 = the modulus of resilient deformation for the unconfined condition,

K_3 = a constant of proportionality, and

σ_r and σ_θ = the radial and tangential stress respectively.

Coffman et al (13) have determined complex moduli for the granular materials representing both the subbase and base course at the AASHO Road Test. Over a limited range of water contents and densities, the complex modulus increased slightly with

TABLE 1

SUMMARY OF LABORATORY TRIAXIAL-COMPRESSION TESTS TO EVALUATE THE RESILIENT PROPERTIES OF GRANULAR MATERIALS

Reference	Material Investigated	Factors Investigated	Confining Pressure, psi	Deviator Stress, psi	Frequency and Duration	Number of Load Applications	Modulus of Resilient Deformation, psi
Seed and Chan (7)	Silty Sand	Duration of stress applications	14.7	23.5 and 36.0 36.0 36.0	20 per min for 1/3 sec, 2 min on, 2 min off, 20 min on, 20 min off	10,000	21,300 and 27,300 23,200 23,000
Haynes and Yoder (8)	AASHO base-course material adjusted gradation to 3/4-in. maximum size: 1. Gravel 2. Crushed stone	Percent of fines passing no. 200 sieve: 6.2, 9.1, 11.5. Degree of saturation: 70%, 85%, 100%	15.0	55	40 per min	100,000	28,000 - 63,000
University of California (10)	Niles aggregate, fairly rounded, 3/4" max. size, 5% passing, no. 200 sieve	Void ratio, confining pressure	14.2 and 28.4	20, 40 and 60	20 per min	10,000	16,700 - 54,500
Biarez (9)	Uniform sand, 0.016-in.-diam. particles	Variation of E with the applied mean normal stress	Mean normal stress: 2.2-145		Cyclic load (rate of deformation is not indicated)	~5	At σ_m = 2.2 psi-4300 At σ_m = 145 psi-71,000
Trollope, Lee and Morris (11)	Poorly graded, dry sand	Initial dry density (loose and dense); rate of deformation from 0.003 to 0.2 in. per min; lateral pressure at constant stress; effect of stress level at constant confining pressure	15 - 45	Stress level varied	Cyclic load, rate of deformation from 0.003 to 0.2 in. per min	~100	35,000 - 95,000
Texas Transp. Institute, Dunlap (12)	Graded material — 1 in. maximum size; 6% passing no. 200 sieve; molding water content, 5.5%	Variation of modulus with confining pressure	3 - 30	3.45 and 51.8	30 per min, duration 0.2 sec	130,000	30,000 - 160,000
Coffman et al (13)	AASHO base and subbase - factorial sections	Water content and dry density. Frequency.	Base - 14 Subbase - 9	Base 42 Subbase - 32	Creep-test results transformed through application of superposition principle to frequencies of 1 and 100 rad per sec	1	(Complex modulus) Subbase: 5-28,000 Base: 9-20,000

increased dry density and decreased slightly with increased water content for both base and subbase materials.

Hveem et al (14) have investigated the resilience characteristics of granular base and subbase materials using a modified stabilometer called the resiliometer. In this equipment, the deformation of a sample in repeated loading is measured as a volumetric displacement, termed the resilience value. They have presented data indicating that the resilience value at a given pressure decreases as the quality of the granular material increases. In addition, their data indicate an increase in resilience with an increase in water content for fine granular materials (e.g., silty sand).

A summary of the various investigations is given in Table 1. It will be noted that the values for resilient moduli of granular materials vary between 4,000 and 160,000 psi. In view of the wide range in values, it is desirable to discuss the factors which contribute to this variation and the relative influence of each.

The available data indicate that the resilient modulus of granular materials appears to depend on the following factors.

Duration of Stress Application and Rate of Deformation—The results of the triaxial repeated-loading tests on silty sand indicate that by decreasing the duration of the load application from 20 min to ⅓ sec while keeping the other conditions constant, the modulus of resilient deformation increased from 23,000 to 27,000 psi, or about 18 percent. The results of the cyclic-load tests on dry sand indicate that the modulus of resilient deformation increased about 20 percent when the rate of deformation increased from 0.002 in. per min to 0.040 in. per min. Both investigations show that the modulus increases with a decrease in the duration of load applications, but that, in spite of the large range of values investigated, the change in the magnitude of the modulus of resilient deformation is relatively small.

Frequency of Load Application—The results of Coffman et al indicate that the higher the frequency of load application, the higher the modulus. These increases ranged from 50 to 100 percent, depending on water content and dry density.

Type of Aggregate and Percentage of Material Passing the No. 200 Sieve—The results presented by Haynes and Yoder indicate that gravels containing 6.2 and 11.5 percent passing the No. 200 sieve exhibited almost identical rebound. The relative densities (difference between field and loose densities divided by difference between maximum and loose densities) of the compacted materials prepared from these two gradations were essentially the same. The rebound of material containing 9.1 percent passing the No. 200 sieve was up to 20 percent greater than that for material with 6.2 or 11.5 percent passing the No. 200 sieve, and the relative density was about 5 percent lower. For the crushed stone, the values of rebound were almost the same for all three gradations in spite of differences in relative densities. These results may be summarized as:

Material Tested	Percent Passing No. 200 Sieve	Rebound Modulus (psi) for Saturation of		
		70%	80%	90%
Gravel	6.2	56,000	46,500	34,000
	9.1	—	40,000	31,000
	11.5	57,500	45,000	37,000
Crushed rock	6.2	42,000	39,000	—
	9.1	39,000	29,000	—
	11.5	39,500	33,500	—

Void Ratio—A limited number of tests carried out at the University of California indicated that two specimens with slightly different initial void ratios will reach the same void ratio after several hundred load repetitions. Trollope et al indicated, however, that the difference between the moduli of loose and dense sand can be as much as 50 percent.

Degree of Saturation—The repeated-load tests reported by Haynes and Yoder indicated that by increasing the degree of saturation of a gravel from 70 to 100 percent, the modulus of resilient deformation decreased to one-half its original value. Tests on crushed stone indicated that, within the range of 70 to 80 percent saturation, the values of the resilient modulus had a small random variation not exceeding 20 percent.

Confining Pressure—All tests in which the effect of confining pressure was investigated show the large influence of this factor on the resilient modulus; e.g., the tests performed at the Texas Transportation Institute indicated that the modulus could increase by as much as 500 percent by varying the confining pressure from 3 to 30 psi. Biarez's equation also suggests the importance of mean normal stress.

Stress Level—Trollope et al concluded that the resilient modulus was independent of the stress level as long as the stress did not cause excessive plastic deformation.

In spite of these effects, it would appear that the problem of laboratory evaluation of resilient moduli (or an approximate equivalent elastic modulus) of granular materials can be somewhat simplified. In preparing specimens for test, estimates must be made for the void ratio and expected degree of saturation. The rate of load application, although having an influence, is not of major importance—a reasonable loading rate consistent with moving traffic can be utilized. Frequency, on the other hand, may influence results significantly, and some indication of the frequency of load applications should be considered. A representative number of repetitions consistent with the field conditions should also be used. The major difficulty is to define the stress condition under which the resilient behavior of the material should be measured. Because this will vary widely in the pavement base course, selection of a representative stress condition presents a major problem.

Field Tests on Paving Materials

Field tests which have been used to determine resilient moduli of materials comprising the pavement section can be divided into (a) plate-load tests—static or slowly applied loads[1]; (b) Benkelman beam tests using a loaded truck; (c) vibratory tests; and (d) plate-load tests with loads of short duration repeated many times. Results of these tests have been summarized in Table 2[2]. Of particular interest are the results for untreated granular materials. As may be seen in Table 2, reported modulus values for these materials vary from 8,000 psi (Burmister) to as high as 200,000 psi (Heukelom and Klomp). This range is comparable to that obtained for the results of repeated-load tests in the laboratory.

This variation in modulus is somewhat surprising in that the modulus of granular materials would be expected to vary less than that of the other materials comprising the pavement section. The most probable explanation for this variability is the influence of confining pressure. Thus it would appear important to know fairly precisely the stresses induced in the pavement when estimating the resilient modulus of untreated aggregates.

Stress Distribution in Pavement Sections

Generally, in determining stress distribution in pavement sections, the pavement has been represented either by a single homogeneous semi-infinite elastic solid (Boussinesq), or by a series of layers assumed to be either plates or elastic solids

[1]Procedures according both to ASTM D 1195 and D 1196 would be considered in this category, even though D 1195 is listed as a repetitive-load test.

[2]Reference (6) contains a detailed summary of the data used in establishing Table 2.

TABLE 2

SUMMARY OF TEST METHODS TO DETERMINE THE MODULI OF IN-PLACE MATERIALS

General Test Category	Investigator	Type of Test	Test Location	Theory Used to Evaluate Moduli of Components	Criteria Used to Evaluate Moduli	Typical Moduli Values	
						Material	Modulus, psi
Plate Load Tests — Static Load	Burmister (15)	Rigid plate	Both at surface of subgrade and pavement section	Two-layer elastic solid (Burmister)	Variable; e.g., for WASHO subgrade — deformation resulting from 2nd application of 12.7-psi stress	WASHO subgrade: uncompacted	2,000 (avg.)
						compacted	5,800 (avg.)
						HYBLA Valley subgrade-compacted	2,500 (avg.)
						Subbase - (granular)	8,000 - 22,000
						Base and asphalt concrete	80,000 - 160,000
	Brown (16)	Rigid plate	At surface of pavement sections	Two-layer elastic solid (Burmister)	Total deformation of 0.2 in. for one application	Subgrade soils	3,000 - 16,000
						Base and asphalt concrete	7,500 - 21,000
							10,000 - 15,000[a]
	Corps of Engineers (17,18)	Flexible plate	At surface of homogeneous test sections	Boussinesq	Total deformation for sustained stress	Clay silt	5,000 - 25,000[b]
						Sand	20,000 - 40,000[c]
Benkelman Beam	Walker et al (19)	Benkelman Beam	Deflection of component layers of in-service pavement	Boussinesq	Deflection under 18,000 lb axle load	Subgrade (A-1 to A-6 soils)	19,000 - 40,000
						Subbase (granular open graded)	10,000 - 20,000
						Base (waterbound macadam)	18,000 - 100,000
Vibratory (Dynamic) Tests	Heukelom and Klomp (20)	Heavy vibrator and light electro-dynamic vibrator	On surface of pavement or at surface of any components	Elastic plate on elastic solid or the assumption that transmitted waves are shear waves	Dynamic deflection with heavy vibrator — phase velocity with light electro-dynamic vibrator	Soft clay	7,000
						Stiff clay, sand	28,000
						Clay gravel	56,000
						Granular base	40,000 - 200,000
						Asphalt concrete (@ 10° to 20° C)	300,000 - 1,000,000
	Nijboer and Metcalf (21)	Light electro-dynamic vibrators	On surface of section under investigation	Wave propagation in layered elastic solids	Phase velocity	AASHO subgrade: Before frost	21,000
						After frost	8,500
						AASHO subbase	~105,000
						AASHO base	420,000
	Jones (22)	Light electro-dynamic vibrators	On surface of section under investigation	Wave propagation in layered elastic solids	Phase velocity	Sand subbase	~8,500
						Wet mix slag base	14,000 - 65,000
						Subgrade (range in soil types)	3,000 - 60,000
						Asphalt concrete (B. S. 594)	200,000 - 500,000
Plate Load Tests — Repeated Load	Odemark (23)	Rigid plate cyclic load	At surface of test pavements	Odemark	Elastic deformation after 4 repetiions of load	Sand (subgrade)	~10,500
						Dry sand (subgrade)	~ 4,000
						Clay	250 - 400
						Sand (base)	3,500[d]
						Gravel (base)	3,500[d] - 13,500
						Crushed stone (base)	3,700[d] - 24,000
						Asphalt concrete	140,000
	Dehlen (24)	Rigid plate cyclic load	At various levels in pavement	Odemark and two-layer elastic solid	Slope of load vs. deformation curve — 4th load cycle	Subgrade (variable)	5,000 (avg.)
						Aggregate subbase	15,000
						Crushed rock base	35,000
	Seed et al (25)	Rigid plate cyclic or repeated load	At surface of homogeneous section of modeling clay	Boussinesq	Resilient deformation after varying numbers of load repetitions	Plasteline modeling clay (similar to saturated clay)	1,300 - 2,800

(a) Majority of data.
(b) At depth, results indicated a modulus equal to 25,000 psi.
(c) At depth, results indicated a modulus equal to 40,000 psi.
(d) These values were obtained on a clay subgrade (E = 250-400 psi) and indicate that the strength of the subgrade influences the density and hence the modulus which can be obtained in the overlying material. This point has also been more recently documented by Heukelom and Klomp.

(26, 27). A summary of investigations concerned with determining the applicability of these theories to predicting actual stress distributions indicates the following:

1. The stresses throughout a uniform clay resulting from surface loads can reasonably be determined by assuming a stress distribution according to Boussinesq. This

stress distribution also gives a fairly good estimate of stresses at greater depths in homogeneous sand layers (17, 18).

2. In a layered structure, when the ratio of the modulus of the upper layer to that of the lower layer approaches unity, the Burmister and Boussinesq solutions produce the same results, e.g., the tests with sand asphalt on sand reported by Trollope et al (11).

3. The distribution of stresses within layered structures can be estimated by Burmister's solution if the upper layers consist of concrete or soil cement. When they consist of untreated granular materials and/or asphalt concrete, on the other hand, the evidence is somewhat contradictory. McMahon and Yoder (28) indicate that the distribution of stresses is not as dependent on modular ratio (a ratio of 10 was used in the analysis) as predicted by Burmister's analysis, although a definite reduction of stress at the base-subgrade interface below that predicted by Boussinesq was observed. However, Vesic (29) found that the pattern of stresses predicted by Boussinesq was more adequate than layered-system-theory results for predicting the stress distribution in pavements, even though modular ratios of 4 were indicated by static tests on the pavement components (30, 31).

It would appear from these results that either the Burmister theory is not applicable to layered pavements consisting of asphalt concrete or that the moduli used in making these comparisons were not correct. It is possible that the modular ratio of 10 used by McMahon and Yoder and the ratio of 4 used by Vesic may be too large when considering the behavior of untreated granular bases resting on compressible subgrades. In addition, implicit in the Burmister theory is the assumption that the modulus is constant in the upper layer. Recent data would suggest, in the case of untreated materials, that this may also be incorrect.

To determine the extent to which the Boussinesq or Burmister patterns of pressure distribution occur in pavements, both the effective modulus of untreated granular material and its variation within the pavement should first be established.

From the information presented, a number of points have become apparent and can be summarized as follows:

1. The behavior of granular materials comprising the pavement section should be measured under conditions of stress which are representative of the actual conditions existing in pavements, since the magnitude of the stress influences the resilient behavior of the material.

2. Laboratory repeated-load triaxial tests would appear to provide a satisfactory means of determining the resilient characteristics of untreated granular materials.

3. Results of investigations to determine the extent to which present theories of pressure distribution are applicable to asphalt-concrete pavements containing granular bases are somewhat contradictory. Great care should be taken in selecting the modular ratios when using Burmister's analysis of layered systems; a Boussinesq distribution may be a very close approximation and it is much easier to compute.

Ideally, the solution to the problem of predicting transient pavement deflections would be obtained through studies of suitably instrumented pavements subjected to actual vehicle loads, since it is necessary for materials comprising the pavement section to be subjected to a number of repetitions of load prior to the measurement of the response. This approach has the advantage that the paving materials have been "conditioned" (i.e., the deformation under loads is comprised primarily of elastic deflection rather than a combination of comparatively large plastic or irrecoverable deformation and a smaller amount of elastic or resilient deformation). The stresses generated in the various materials comprising the structural section are those resulting from representative vehicular loads, and the time of loading (under moving wheel loads) is realistic. Initially this approach, because of the broad scope, has many difficulties, such as control of materials, size and costs. Ultimately, however, for application of techniques developed from other procedures, this type of investigation must be accomplished.

Another approach, at a more modest level of effort, is the use of the repeated-load plate test on carefully controlled field test sections. The requirements of specific numbers of load repetitions, and stresses of the same order of magnitude as those produced by loads on tires, are met by this type of test. By using suitable theories and criteria of failure, the results of this type of test could also be used in design.

Unfortunately, the use of the plate-load test for design purposes has the same disadvantage as other in situ measurements in that it can be used to evaluate the properties of the paving materials only at the time the test is conducted. Because the properties of these materials are susceptible to changes during the pavement lifetime, the testing conditions are not necessarily the most critical conditions which can occur during this time. Other disadvantages of the plate-load test are the length of time spent in performing the tests and the high cost relative to small-scale laboratory tests. Therefore, it is desirable to be able to predict the resilient deformations of the different pavement layers from laboratory test results. If this could be achieved, the critical material properties could be reproduced in the laboratory, and the resilient modulus expected from plate-load test measurements for these same conditions could be produced. Thus, the primary purpose of this investigation is to establish the possibility of predicting pavement deflections in prototype structures from the results of repeated-load laboratory tests on individual materials comprising the pavement section, with particular emphasis on the role of untreated granular materials.

LABORATORY REPEATED-LOAD TESTS ON GRANULAR MATERIALS

Emphasis in this section will be on determining the resilience characteristics of a well-graded gravel in repeated-load triaxial-compression tests, since this is the material used in the prototype tests described in a subsequent section. To illustrate, however, that the characteristic resilient behavior defined by the type of test described in this section is applicable to other granular materials, a brief indication of such behavior will also be presented for a uniform sand and representative base and subbase materials obtained from two in-service pavements in California.

Material Description

The gravel used for the prototype pavement tests was a well-graded, subrounded material from Pleasanton, California, with a grain-size distribution as shown in Figure 1.

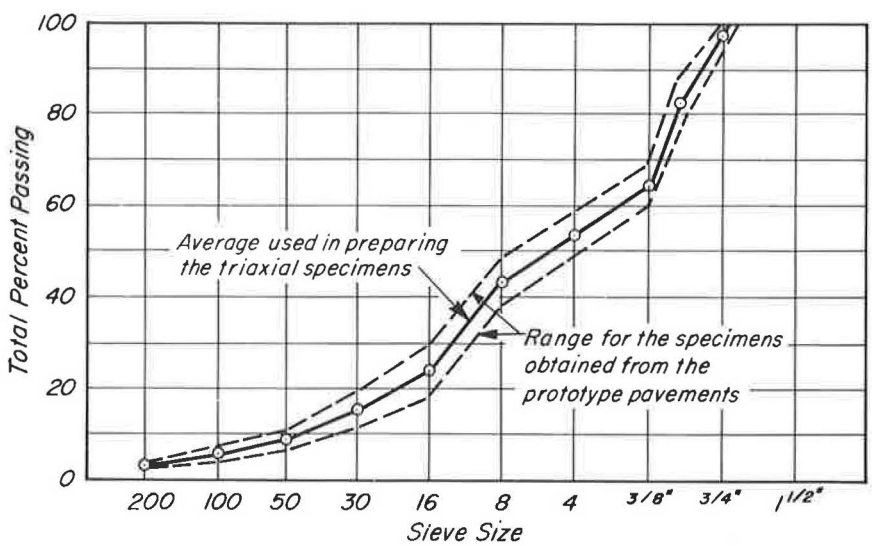

Figure 1. Grading curve, gravel base.

These results were the average obtained from analyses of several specimens taken from successive 2-in. layers of the base course of the pavements; the gradation was within the California specification limits for a Class 2 aggregate base. The specific gravity of the material retained on the No. 4 sieve was 2.75, and that of the material passing the No. 4 sieve was 2.65. Routine strength tests on the material indicated an angle of internal friction of 55 deg at a void ratio of 0.31, corresponding to the average in-place density of 139 lb/cu ft, an average CBR value of 103 and an average R value of 85. This material was tested in the air-dry condition both in the laboratory and in the field.

Equipment

A piston capable of applying comparatively large loads for short durations was necessary for the tests on granular material, particularly since triaxial specimens up to 6 in. in diameter were required for aggregate with maximum size particles up to ¾ in. or 1½ in. (sizes approaching those used in actual base courses). The loading piston developed to meet these requirements is shown in Figure 2; it was also used to apply repeated loads to the plates in the field tests on the prototype pavements.

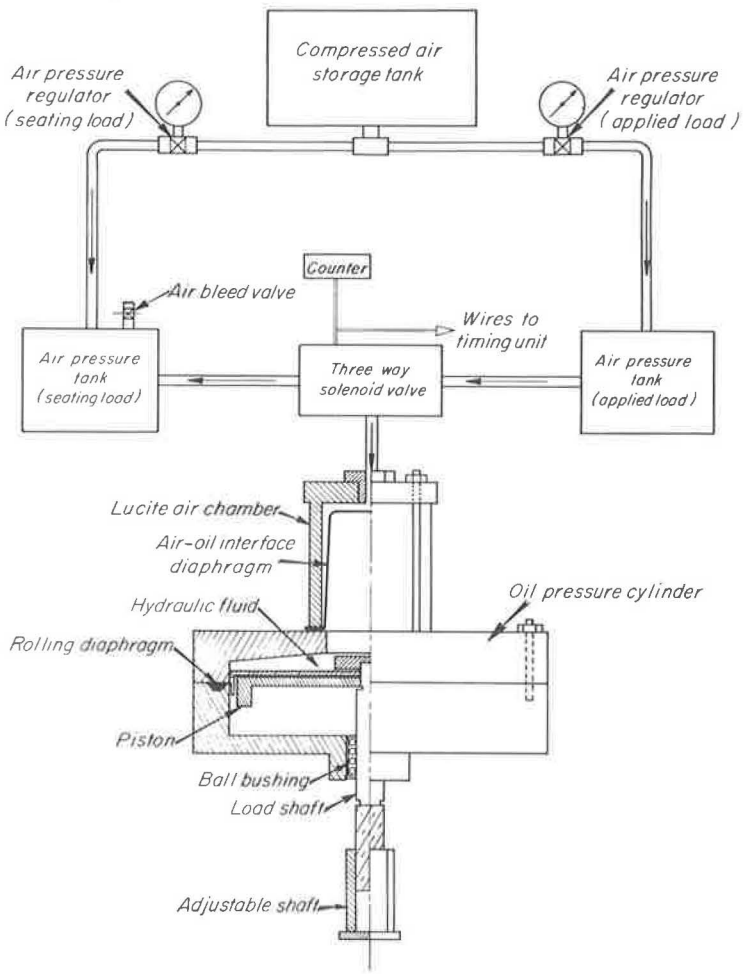

Figure 2. Large loading piston and control mechanism.

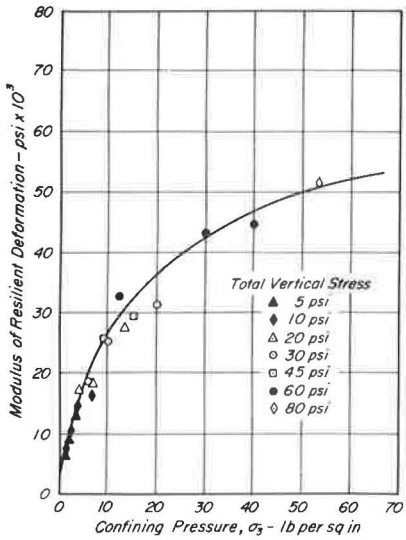

Figure 3. Relationship between modulus of resilient deformation and confining pressure for dry gravel.

The loading system was operated by compressed air stored in separate tanks at the required pressures for the seating load and the peak or applied load. By using a three-way solenoid valve, the appropriate pneumatic pressure was supplied through a bellofram seal to oil above the main piston. A ball-bushing guide was provided to reduce friction and a neoprene rolling-diaphragm seal was utilized to minimize friction and to prevent loss of oil. The volume of air between the piston and the three-way valve was reduced to a minimum to provide a rapid buildup of pressure during each load pulse. The peak load applied to the specimen was varied by regulating the air pressure, as recorded by the pressure gage. Any desired load up to 5,000 lb can be obtained. During calibration, the load was applied both statically and dynamically, and the two calibration curves were identical.

Triaxial cells capable of testing specimens up to 6 in. in diameter were used, although the majority of tests were performed with a cell in which specimens 3.9 in. in diameter and about 8 in. in height could be tested.

Procedure

For the tests on the dry gravel, the samples were prepared to the desired dry density (139 lb/cu ft) by vibratory compaction, because this method minimized puncturing of membranes and crushing or degradation of the aggregate. Each sample was prepared inside a forming jacket mounted on the base of the triaxial cell attached to a table supported by rubber springs. Vibrations were then induced by compacting the specimens in two equal layers for 15 sec under a 15-lb weight. When compacting the upper layer, the cap of the specimen was inserted to obtain a flat horizontal layer.

A range in confining pressures from 1 to 53.3 psi was used for the repeated-load tests. Confining pressures up to about 11 psi were obtained by vacuum inside the membrane; for larger confining pressures, air pressure was used in the triaxial cell outside the membrane. Deviator stresses ranging from 1.7 to 40.0 psi were applied with the hydraulic-pneumatic piston at a frequency of 20 applications per minute and with a load duration of 0.1 sec. Generally, the repeated loading was continued for at least 10,000 stress applications.

Test Results for Gravel

The influence of applied stresses on the modulus of resilient deformation for dry gravel is shown in Figure 3. It will be noted the majority of tests were conducted at low confining pressures, since the change in modulus with confining pressure is greatest in this range.

Principal stress ratios used in these tests (between 1.5 and 5.0) were lower than the principal stress ratio at failure under a steadily increasing load application (approximately 11). However, one specimen was tested at a principal stress ratio of 10. At the beginning of this test and for about 800 load repetitions, essentially the same modulus was obtained as would be predicted from Figure 3. After this number of load applications, the plastic deformation increased rapidly with additional load applications, and failure occurred at approximately 1,000 load repetitions. Thus, this test would

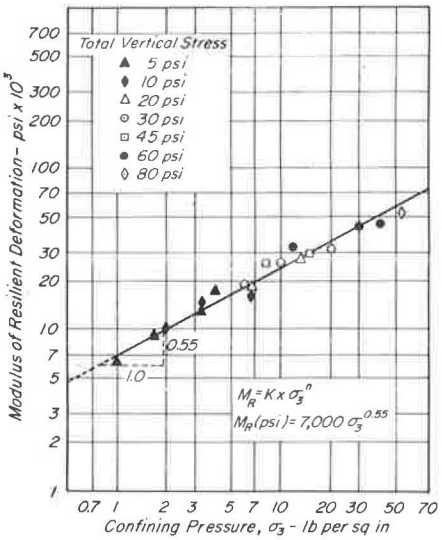

Figure 4. Relationship between modulus of resilient deformation and confining pressure for dry gravel.

appear to indicate that the relationship obtained is valid for all magnitudes of deviator stress (for the range of confining pressures investigated), as long as failure does not occur.

These results emphasize the importance of properly accounting for the actual magnitude of the confining pressure and its variation with depth in untreated bases, so that a realistic measure of the resilient characteristics of these materials throughout the layers in which they are used can be obtained. For example, for this gravel, a variation of confining pressure from 1.0 to 53.3 psi resulted in an increase in modulus from 7,000 to 50,000 psi. It will be noted that the largest increase occurs in the low pressure range, i.e., 0 to 10 psi.

The data shown in Figure 3 can be more conveniently utilized by plotting the results as shown in Figure 4. In this figure, it will be noted that a linear relationship between the logarithm of resilient modulus and the logarithm of the confining pressure is obtained; thus, the modulus M_R can be expressed by an equation of the form

$$M_R = K \cdot \sigma_3^n \tag{1}$$

where

K = material constant determined experimentally (7,000 for gravel),

σ_3 = confining pressure, and

n = material constant determined experimentally (0.55 for gravel).

The form of this equation is similar to that presented by others; e.g., Jakobson (32) has shown theoretically that for spherical particles the exponent n in Eq. 1 has a value of ⅓.

The data can also be analyzed in terms of the sum of the principal stresses. From this analysis an alternative form of the equation for resilient modulus has been developed as follows:

$$M_R = K' \cdot \theta^{n'} \tag{2}$$

where

θ = sum of the principal stresses ($\sigma_1 + \sigma_2 + \sigma_3$), and

K' and n' = experimentally determined coefficients.

The resulting plot of the data in this form is shown in Figure 5. Although Eq. 2 has not been used in conjunction with the theories presented in this paper to predict pavement deflections, it has the potential for use in analyses such as that presented by Cumming and Gerrard (33). In addition, it has the advantage from a theoretical viewpoint that it is a valid tensorial relationship, whereas Eq. 1 in terms of σ_3 is not.

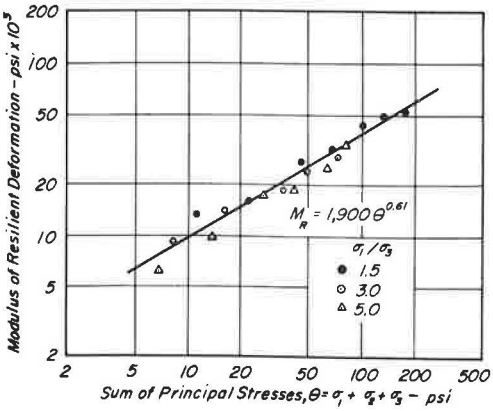

Figure 5. Relationship between modulus of resilient deformation and the sum of principal stresses for dry gravel.

Test Results for Other Materials

Although not used in the analysis of prototype pavements presented here, modulus vs confining-pressure data, such as those shown in Figure 4, have been developed for other granular materials. To emphasize that Eq. 1 would appear to be a reasonable way to represent the dependence of resilient modulus on confining pressure for a range in granular materials, data are presented in Figures 6, 7, and 8 for 4 other untreated granular materials; the data are plotted in the same form as that used in Figure 4. All of the results were obtained at the same frequency and duration of loading as the data for the gravel.

Figure 6 shows the test results for a dry, rounded, uniform sand (essentially all of the material passed the No. 16 sieve and was retained on the No. 100 sieve) from Monterey, California, compacted to a density of 101 lb/cu ft.

In Figure 7, data are presented for tests on recompacted, disturbed samples of granular base obtained from an in-service pavement near Gonzales, California. This material was compacted in the laboratory to a dry density and water content representing in situ conditions, and in this instance the degree of saturation was of the order of 60 percent.

Similar data are presented in Figure 8 for laboratory-compacted samples of both base and subbase materials from another in-service pavement near Morro Bay, California. Both materials were compacted to densities approaching those in situ and to a degree of saturation of approximately 60 percent, which was the lower limit of values measured at the time of sampling. The scatter in the data for the base course may be due in part to the fact that samples from four different locations in the pavement were used to develop the data, and no attempt was made to separate the points according to location. For the subbase material (a fine sand), a line with a slope equal to 0.33 was drawn through the available data.

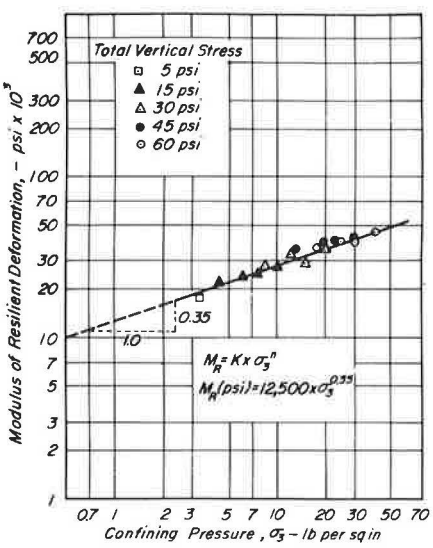

Figure 6. Relationship between modulus of resilient deformation and confining pressure for uniform sand.

Table 3 gives a summary of the coefficients K and exponents n obtained for the various materials tested in this investigation and emphasizes that the resilience characteristics of granular materials vary considerably and thus should be determined for each pavement section investigated.

In general, the data presented in this section substantiate the form of the equation relating resilient modulus and confining pressure developed for the untreated gravel. The data also emphasize the large variation which can occur in the resilience

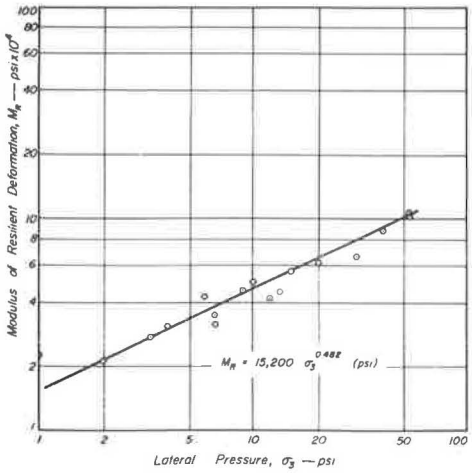

Figure 7. Relationship between modulus of resilient deformation and confining pressure, laboratory-prepared specimens of untreated base course; Gonzales By-Pass.

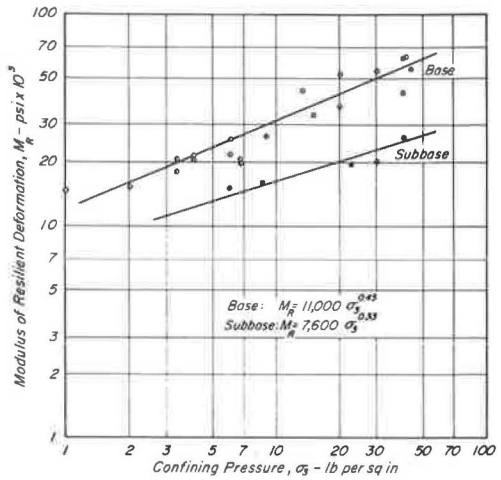

Figure 8. Relationship between modulus of resilient deformation and confining pressure, laboratory-prepared specimens of untreated base and subbase; Morro Bay pavement.

characteristics of granular materials depending on the confining pressure, a factor which plays an important role in defining the behavior of these materials in the pavement section.

TRANSIENT DEFLECTIONS IN TWO-LAYER PROTOTYPE PAVEMENTS

To investigate the resilience characteristics of the untreated gravel base course under field loading conditions, a series of tests was performed on prototype pavement sections.

The test area was paved with a layer of asphalt concrete approximately 4 in. thick. For each field test, a section 8 by 8 ft in plan was cut from the asphalt-concrete paved area. These dimensions were chosen so that the boundary conditions would have little influence on the test results. Within this 8-ft square section, the test pits were excavated to the desired depth. Final trimming of the pit was done by hand to obtain a reasonably smooth horizontal surface. The entire excavated area was then covered with a polyethylene sheet to prevent change in water content of the natural soil due to

TABLE 3

SUMMARY OF RESILIENCE DATA FOR UNTREATED AGGREGATES

Type	Degree of Saturation (%)	Constants in Eq. 1	
		K	n
Gravel	0	7,000	0.55
Uniform sand	0	12,500	0.35
Base from Gonzales	~60	15,200	0.48
Base from Morro Bay	~60	11,000	0.45
Subbase from Morro Bay	~60	7,600	0.33

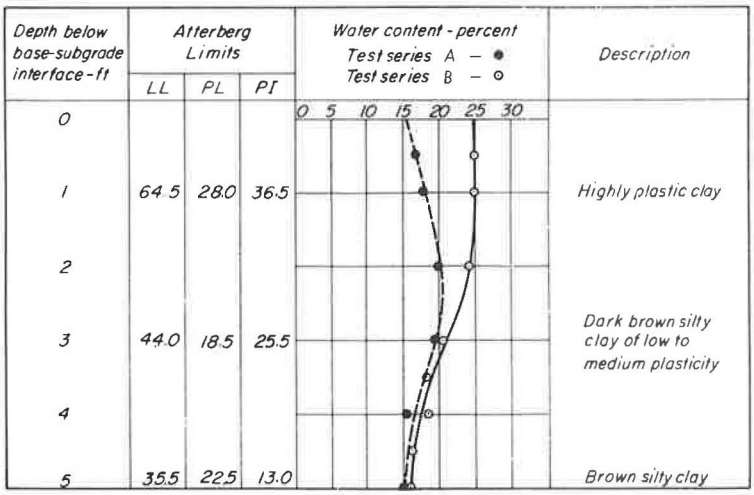

Figure 9. Subgrade characteristics.

either evaporation or absorption and to control the condition of the base course. Within this pit, a particular test pavement was constructed and tested as described in the following sections.

Six tests were performed on the following pavement sections:

1. Test Series A—subgrade comparatively dry: (a) 8-in. base, 8-in. diameter plate; (b) 8-in. base, 12-in. diameter plate; (c) 12-in. base, 8-in. diameter plate; and (d) 12-in. base, 12-in. diameter plate.
2. Test Series B—subgrade comparatively wet: (a) 8-in. base, 8-in. diameter plate; and (b) 8-in. base, 12-in. diameter plate.

A summary of the subgrade characteristics for these tests is shown in Figure 9.

Equipment

Steel plates ranging from 8 in. to 30 in. in diameter were used to apply the load to the components of the pavement section. Load was applied to the plates by means of

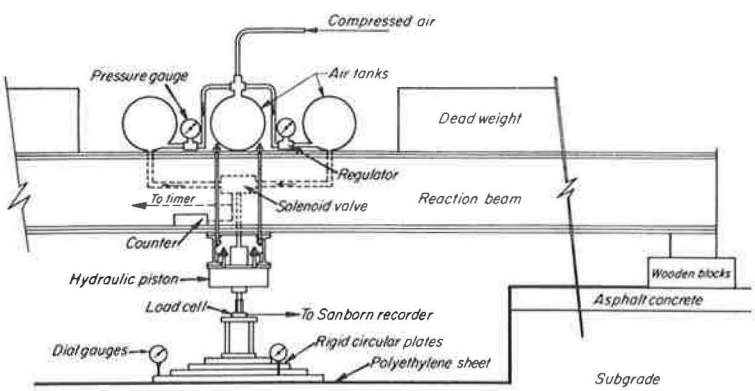

Figure 10. Repeated-plate-load test installation for test at surface of subgrade.

the same loading piston described in the previous section. As noted earlier, this piston is capable of applying loads up to 5,000 lb. A loading frequency of 20 applications per minute and loading duration of about 0.1 sec were used for the field tests. The magnitude of the loads and the shape of the load traces were constantly checked by a load cell, and were recorded on a Sanborn strip-chart recorder.

The loading piston was connected to a frame fastened to a reaction beam. The relative positions of the piston and the frame could be adjusted to center the shaft of the piston with respect to the center of the loaded plate, so as to avoid eccentricity which might result in subsequent tilting of the plate. Reaction for the load piston was provided by a steel beam loaded with either concrete cylinders or water tanks. The steel beam could be raised or lowered according to the thickness of the test section. Deflections of the pavement section were measured from a reference beam 20 ft in length, stiffened laterally to prevent sway. A schematic view of the test section set up for a test at the surface of the subgrade is shown in Figure 10.

Procedures

For the tests reported here, repeated loads were applied at the surface of the subgrade and on top of two thicknesses (8 and 12 in.) of the untreated base. Load was applied to the subgrade through a series of rigid circular plates, 18, 24, and 30 in. in diameter. Even though the subgrade was hand trimmed, it was not possible to obtain a perfectly smooth horizontal surface, and the plates were placed on a thin layer of hydrostone (maximum thickness of 0.1 in.). At least 1,000 repetitions of a particular magnitude of stress were applied. Deformations were measured by three dial gages attached to the reference beam and located at 120-deg intervals around the edge of the plate. The resilient deformation was taken to be the average of the three dial readings.

After testing the subgrade, the base courses were placed and compacted in 2-in. lifts by vibratory compaction. When the desired thickness had been constructed, repeated-load plate tests were performed using 12- and 8-in. diameter plates.

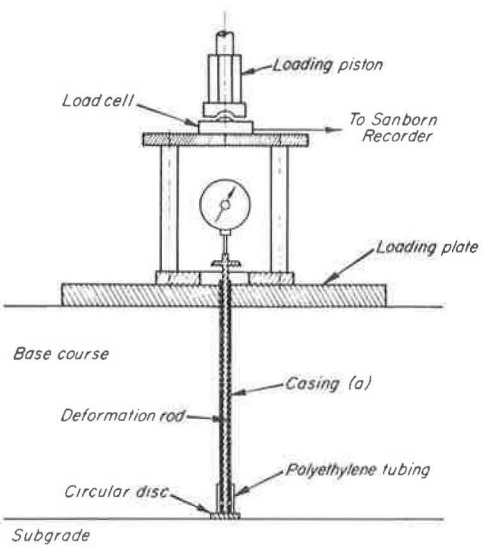

Figure 11. Device that measures deflections of individual layers of the pavement.

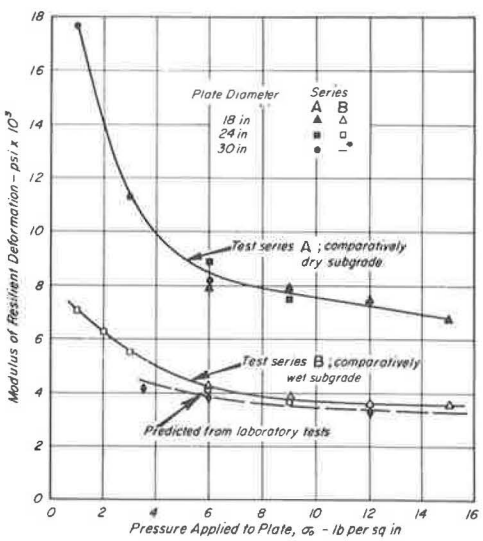

Figure 12. Relationship between resilient modulus of the subgrade determined by repeated-plate-load tests and applied pressure.

TABLE 4

RESILIENT MODULI DETERMINED FROM LABORATORY REPEATED-LOAD COMPRESSION TESTS

Laboratory Test, Stress σ_d (psi)	Stress in Plate σ_o Corresponding to σ_d (psi)	Resilient Modulus (psi)
1.0	3.5	4150
1.7	6.0	3900
3.5	12.0	3300
5.2	18.0	3200

Each stress was repeated several thousand times in the plate-load tests; after a few hundred repetitions, however, the plastic deformation generally did not increase appreciably with number of repetitions and the resilient deformation approached a constant value. Thus, the resilient deformation at 1,000 load repetitions was considered suitable for evaluating the resilient characteristics of the various layers.

Deformations of the components were measured independently (Fig. 11). The resilient deformation of the aggregate layer was evaluated by subtracting the resilient deformation of the subgrade from the total resilient deformation beneath the plate. Deformations were determined by measuring the movement of a small rigid disc 1 in. in diameter and ¼ in. thick resting on the subgrade (Fig. 11). The disc was welded to an adjustable vertical rod passing through the center of the loaded plate; measurements were taken on a smooth, flat, circular plate ½ in. in diameter, attached to the top of the rod.

To insure that this vertical rod moved freely with respect to the base course, it was placed in a thin steel casing with gaps to allow for deformation (Fig. 11). The friction between the inner rod and the outer casing was eliminated by placing a thin layer of grease in the annulus. When the inner rod was displaced, a gap between the plate in contact with the subgrade and the outer casing was formed, into which sand grains or fines tended to penetrate; this situation was avoided by covering the gap with thin polyethylene tubing.

Test Results

Subgrade—A summary of the plate-load tests performed directly on the surface of the subgrade for the two test series is shown in Figure 12. Comparison of the resilient modulus data for series A with that for series B illustrates the influence of water content near the surface of the subgrade on its resilient behavior.

The moduli shown in Figure 12 were determined using the equation for a rigid plate:

$$E = 1.18 \frac{\sigma_o \cdot r}{\Delta} \tag{3}$$

where

E = modulus of elasticity (in this case resilient modulus) of the material,

σ_o = pressure applied to surface of plate,

r = radius of the plate, and

Δ = resilient deflection of plate.

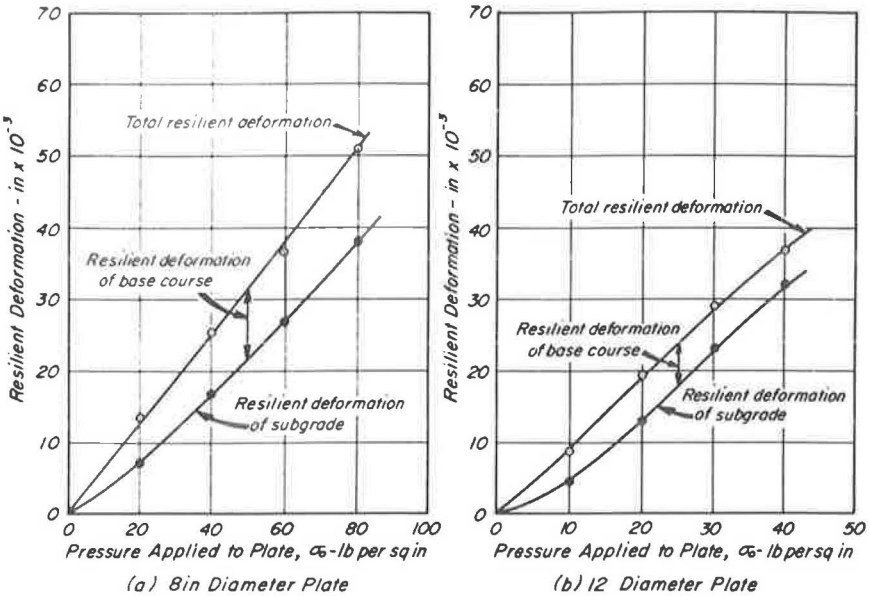

Figure 13. Relationship between applied pressure and resilient deformation of components of a two-layer system consisting of the natural subgrade and 8 in. of base; test series A.

Also shown in Figure 12 are moduli determined from repeated-load triaxial-compression tests (25) on specimens trimmed from undisturbed samples obtained at a 6-in. depth below the subgrade surface at the conclusion of test series B. The laboratory specimens were subjected to as many as 100,000 repetitions of deviator stresses ranging from 1.0 to 5.2 psi in undrained compression tests conducted using a frequency of 20 stress repetitions per minute and a duration of loading of 0.1 sec. Moduli of deformation were determined from these tests as the ratio of the repeated stress to the induced resilient strain.

To plot the moduli from the laboratory tests in Figure 12, the relation between pressures applied in the plate-load test and the stresses used in the undrained triaxial-compression tests (25) was utilized (i.e., $\sigma_d = 0.29\sigma_o$, where σ_d is the deviator stress in the repeated-load test and σ_o is the corresponding plate pressure). For convenience, the comparable pressures are summarized in Table 4. The moduli predicted from the laboratory tests follow the same trends as those observed in test series B. Since the test specimens were obtained 6 in. below the surface where the water contents are highest, the laboratory-determined moduli would be expected to give slightly lower values than the field tests. The comparison, however, is extremely encouraging and lends support to the use of the repeated-load triaxial-compression test as a means for testing fine-grained subgrade materials.

Because the field plate-load tests at the surface of the subgrade covered a wider range in applied stress (particularly in the low stress range) than would be accomplished with available laboratory repeated-load equipment, the relationships between resilient modulus and applied stress developed from the field tests have been used in the analyses of the prototype pavement behavior. It should be noted, however, that, with suitable equipment, laboratory tests would provide results equally suitable for use, as evidenced by the comparisons between field and laboratory values shown in Figure 12.

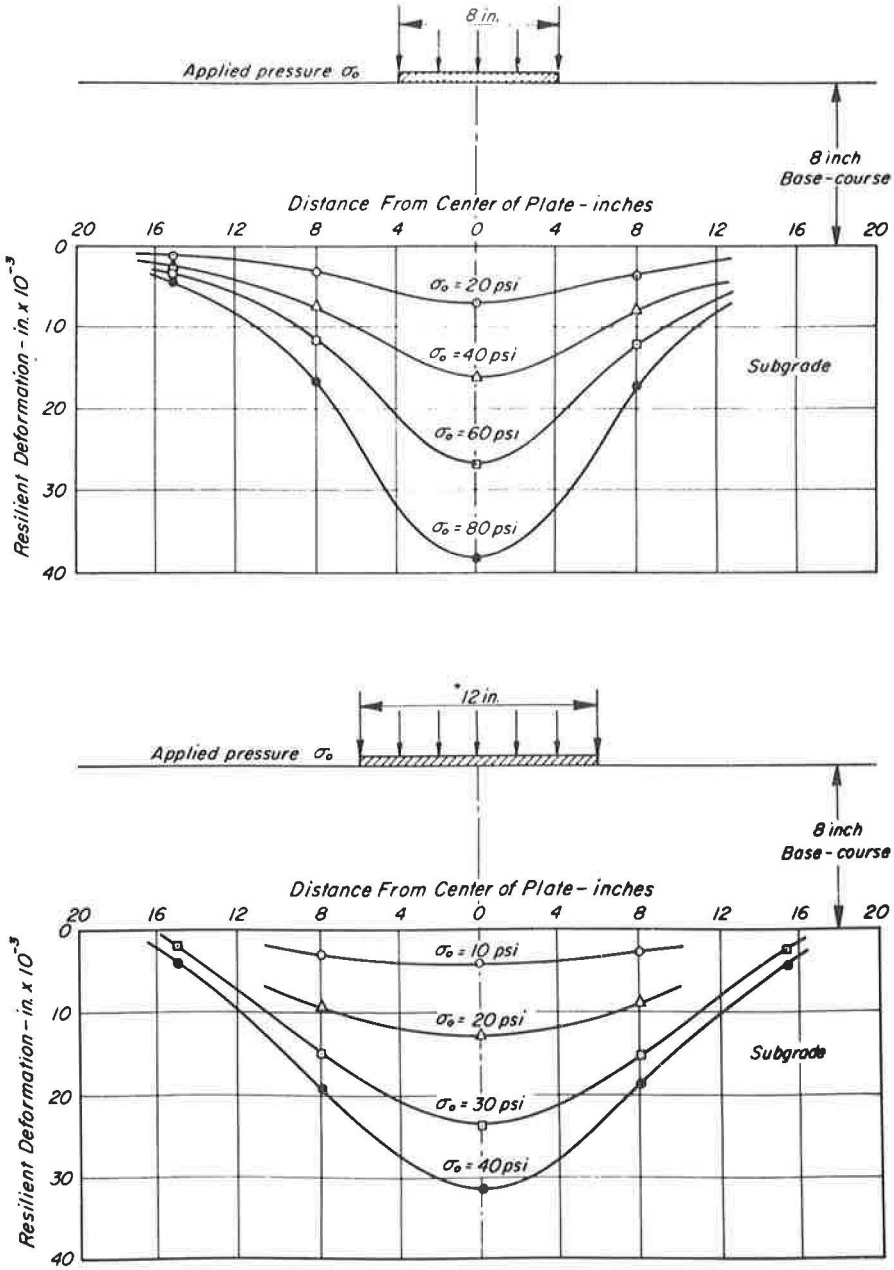

Figure 14. Radial variation of resilient deformation at base-subgrade interface for repeated-plate-load tests at surface of 8-in. base; test series A.

Two-Layer System—Results of the tests at the surface of the two-layer systems are shown in Figures 13 through 17. Figures 13 and 14 illustrate the patterns of resilient deformations at the surface of an 8-in. layer and at the surface of the subgrade in test series A, due to surface loads applied by 8- and 12-in. diameter plates. As noted in Figure 14, deformations at the surface of the subgrade were measured at

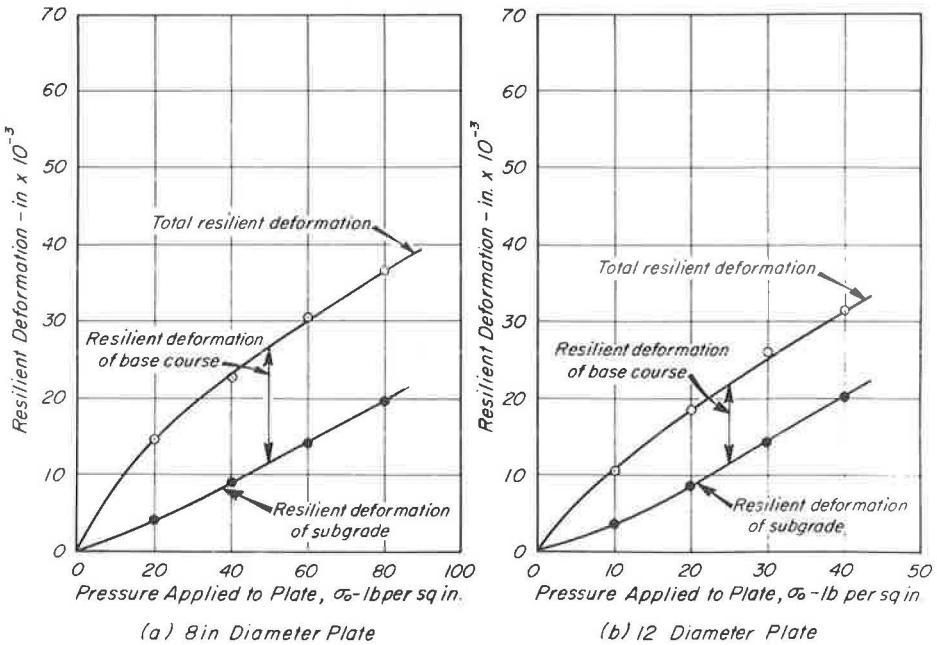

Figure 15. Relationship between applied pressure and resilient deformation of components of two-layer system consisting of the natural subgrade and 12 in. of base; test series A.

radial distances of 8 and 16 in. as well as directly under the center of the plate. Similar data are shown in Figures 15 and 16 for tests at the surface of a 12-in. aggregate layer, also for test series A.

Figure 17 shows the relationship between resilient deformations of the subgrade and base course and applied pressure for 8- and 12-in. diameter plate tests conducted on a test section involving an 8-in. gravel base in test series B.

Evaluation

While the emphasis in this paper is on the behavior of granular materials, the tests on the subgrade shown in Figure 12 illustrate certain points worthy of note:

1. These data indicate that the resilient modulus is dependent on applied stress and varies in the same manner as shown by Seed et al (25) for laboratory repeated-load tests on subgrade soils. At stresses less than 10 psi, such as can be expected in the subgrades of well-designed asphalt-concrete pavements, the variation is considerable. Thus it is evident that, to estimate the modulus of the subgrade, the stresses within the subgrade must be known.
2. The data also demonstrate the influence of water content on the resilient modulus of the subgrade and emphasize the importance, when predicting pavement deflections, of considering the changes in subgrade water content that are likely to occur during the life of the pavement. These results also indicate the inadequacy of the plate-load test since it is only capable of measuring the soil conditions at time of test—which is generally not the most critical state that the material will attain.

For the tests at the surface of the two-layer systems, the data indicate the following factors.

Influence of Applied Pressure on Resilient Deformation of Base Courses—The test results indicate a comparatively large increase in the deformation of the base course when the pressure at the surface is increased from 0 to about 10 psi (see Fig. 15). A smaller increase, on the other hand, is obtained with an increase from 10 to 20 psi.

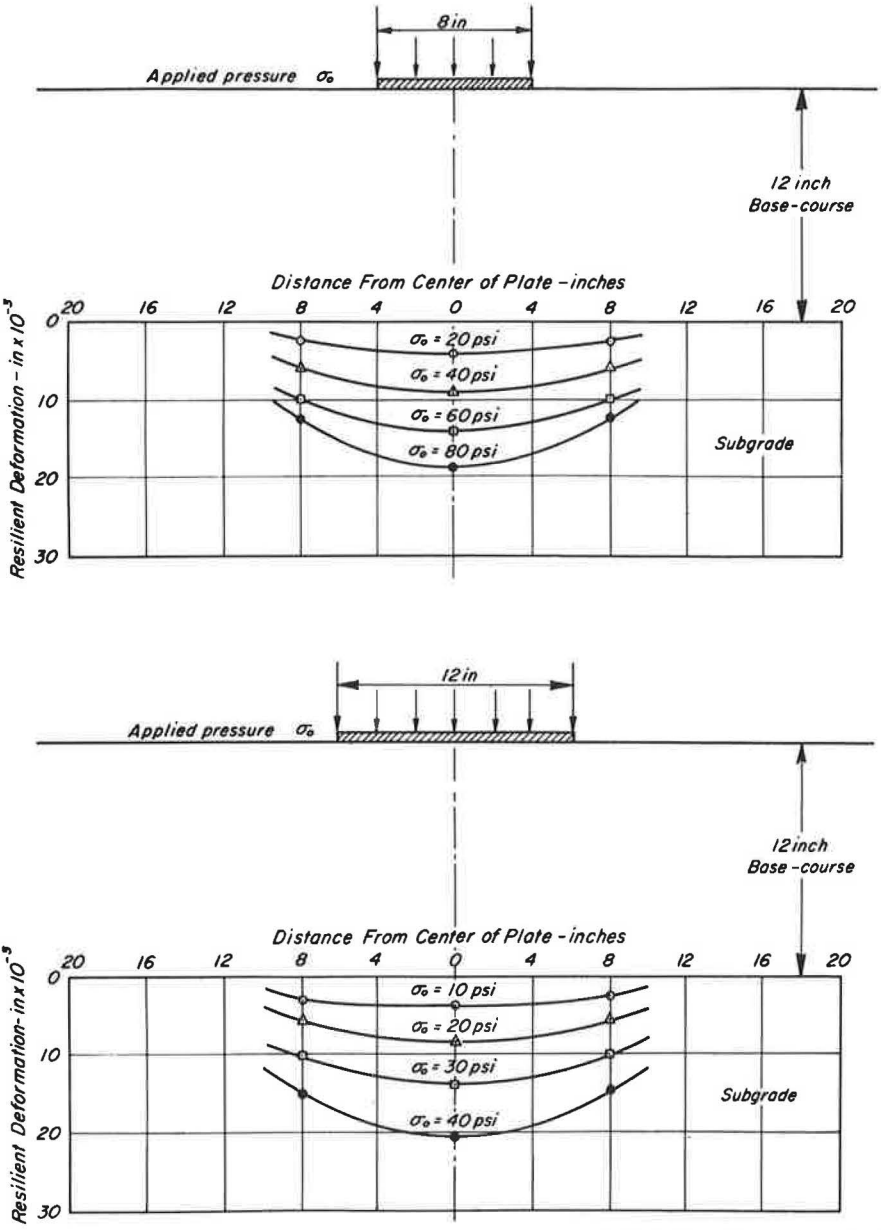

Figure 16. Radial variation of resilient deformation at base-subgrade interface for repeated-plate-load tests at surface of 12-in. base; test series A.

Because the rate of increase in resilient deformation is less than the rate of increase in applied stress, it can be concluded that the average modulus of resilient deformation of this material increases with applied pressure; this is consistent with the observed laboratory behavior reported previously.

Influence of Applied Pressure on the Resilient Deformation of the Subgrade—The resilient deformation of the subgrade measured at the base-subgrade interface increases gradually up to an applied pressure of 10 psi. When the pressure on the plate

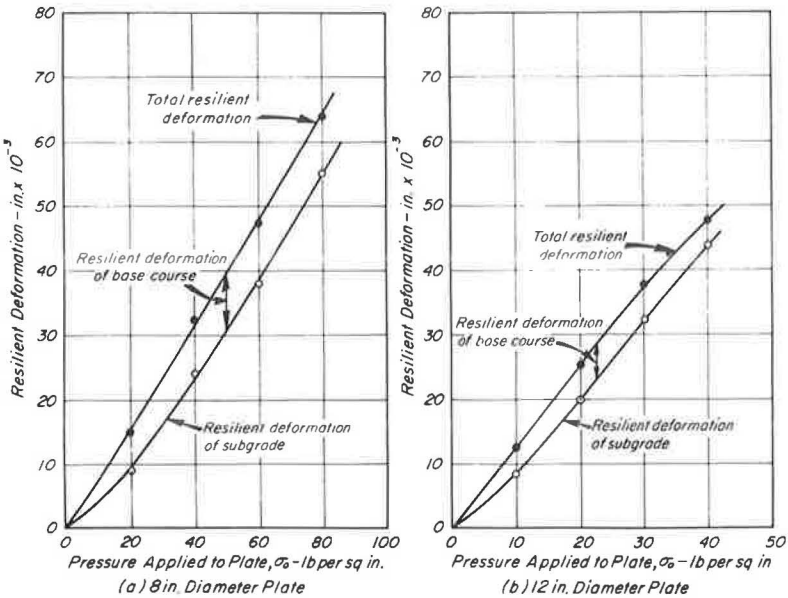

Figure 17. Relationship between applied pressure and resilient deformation of components of a two-layer system consisting of the natural subgrade and 8 in. of base; test series B.

reaches between 20 and 30 psi (e.g., Figs. 13 and 15), the resilient deformation of the subgrade increases more rapidly and almost linearly with the applied pressure. This trend indicates that the subgrade modulus is largest at low applied stresses and decreases until a level of 20 to 30 psi is reached, whereupon the modulus remains almost constant. This variation in modulus with applied pressure follows a trend similar to that obtained when testing the subgrade alone (Fig. 12).

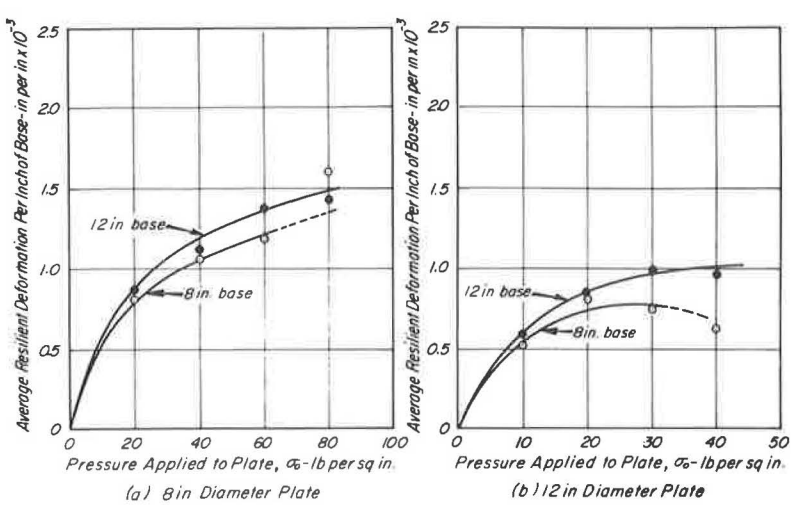

Figure 18. Relationship between applied pressure and resilient deformation per inch of base course; test series A.

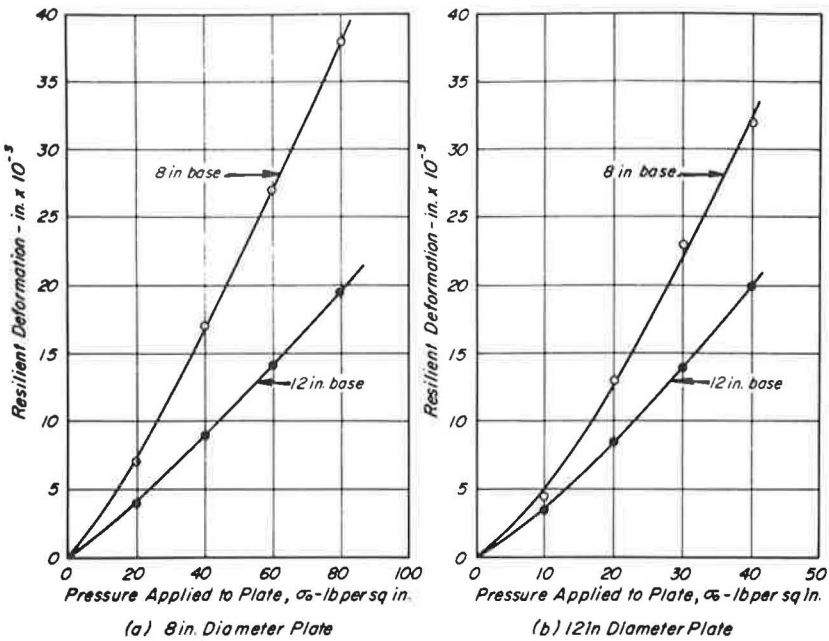

Figure 19. Relationship between applied pressure, resilient deformation of subgrade and base thickness; test series A.

Influence of Thickness of Base Course on Its Resilient Deformation—Although the influence of thickness of base can be obtained from a comparison of Figures 13 and 15, a more direct comparison is shown in Figure 18. In this figure, notice that the resilient deformation per inch of base is larger for the 12-in. base than for the 8-in. base. This pattern is in accord with data obtained in the laboratory, in that the average stress

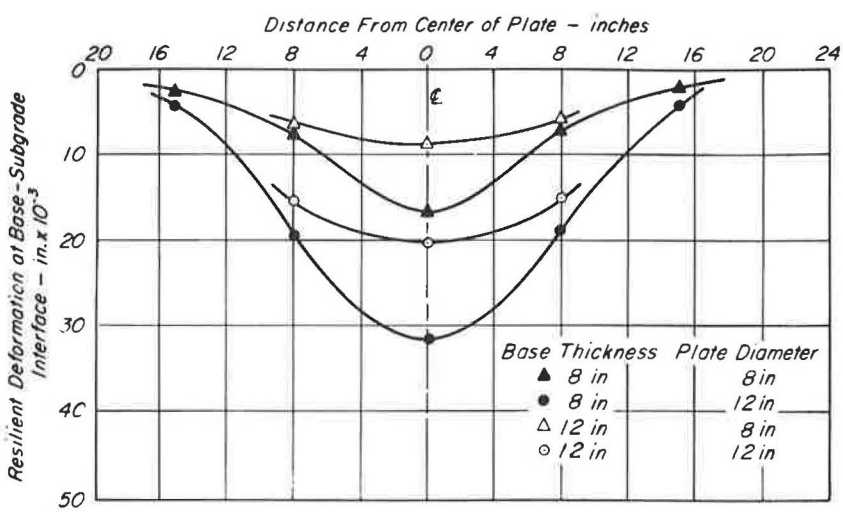

Figure 20. Influence of thickness of base on the radial variation of resilient deformation at the base-subgrade interface; test series A.

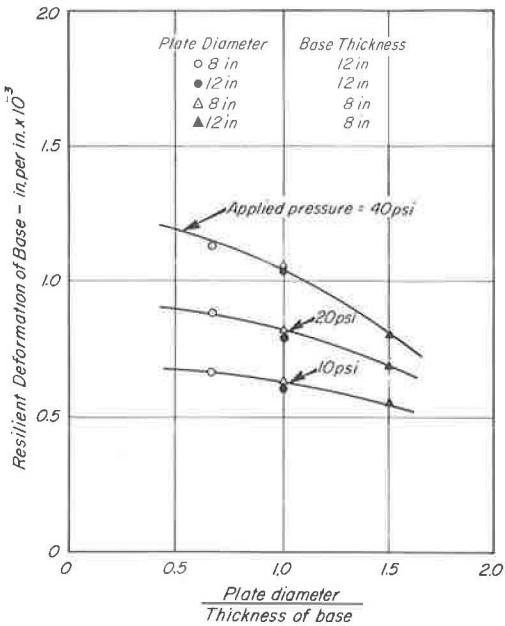

Figure 21. Relationship between the ratio of plate diameter to base thickness and the resilient deformation per inch of base; test series A.

induced in the base by a specific plate size and pressure increases as the thickness of the base decreases; thus, the modulus of resilient deformation of the aggregate increases and consequently reduces the resilient deformation.

Influence of Base Thickness on the Resilient Deformation of the Subgrade—A decrease in resilient deformation of the subgrade with increase in the base thickness is shown in Figure 19. This occurs because an increase in base thickness increases the "load spreading" capacity of the base, thereby reducing the subgrade stresses, and with reduced stress the subgrade modulus is higher; both contribute to a reduced resilient deformation. The influence of base-course thickness on the pattern of resilient deformation at the base-subgrade interface is shown in Figures 14 and 16. For comparison, the various deformation patterns corresponding to an applied stress of 40 psi have been replotted in Figure 20. The influence of the thicker base on the magnitude and distribution of the subgrade deflection is readily apparent.

Influence of Plate Diameter on the Resilent Deformation of the Base Course—The influence of plate diameter on the resilient deformation of the base course is shown in Figure 21. This figure shows the resilient deformation per inch of base as a function of the ratio of the plate diameter to base thickness. For a large ratio of plate diameter to base thickness, the confining effect is larger and the resilient deformation is correspondingly lower. It will also be noted that the resilient deformation per inch of base is essentially constant for the 8- and 12-in. bases when the ratio of plate diameter to base thickness is the same.

Influence of Plate Diameter on the Resilient Deformation of the Subgrade—For the same base thickness, the stresses induced at the subgrade-base interface are higher as the plate diameter is increased. This is shown in Figure 20 (by the increased resilient deformation) and is explained by the fact that, because of the higher stresses in the subgrade, correspondingly lower resilient moduli are developed, both of these factors leading to increased resilient deformations.

Influence of Change in Subgrade Water Content on the Resilient Deformation of the Base—A comparison of test results from series A and B for the two conditions of loading are shown in Figure 22. No significant or consistent difference is noted in the resilient deformation of the base course due to change in water content of the subgrade. These data also indicate the effectiveness of the plastic sheet in maintaining the base course in a dry condition.

Influence of Change in Water Content of the Subgrade on Its Resilient Deformation—The influence of the water content of the subgrade on tests performed at the surface of the subgrade has already been noted (Fig. 12). This influence is also important when considering the results of tests on layered systems. Figure 22 indicates that, for the change in water content which occurred from test series A to test series B, the resilient deformation of the subgrade increased on the order of 40 percent.

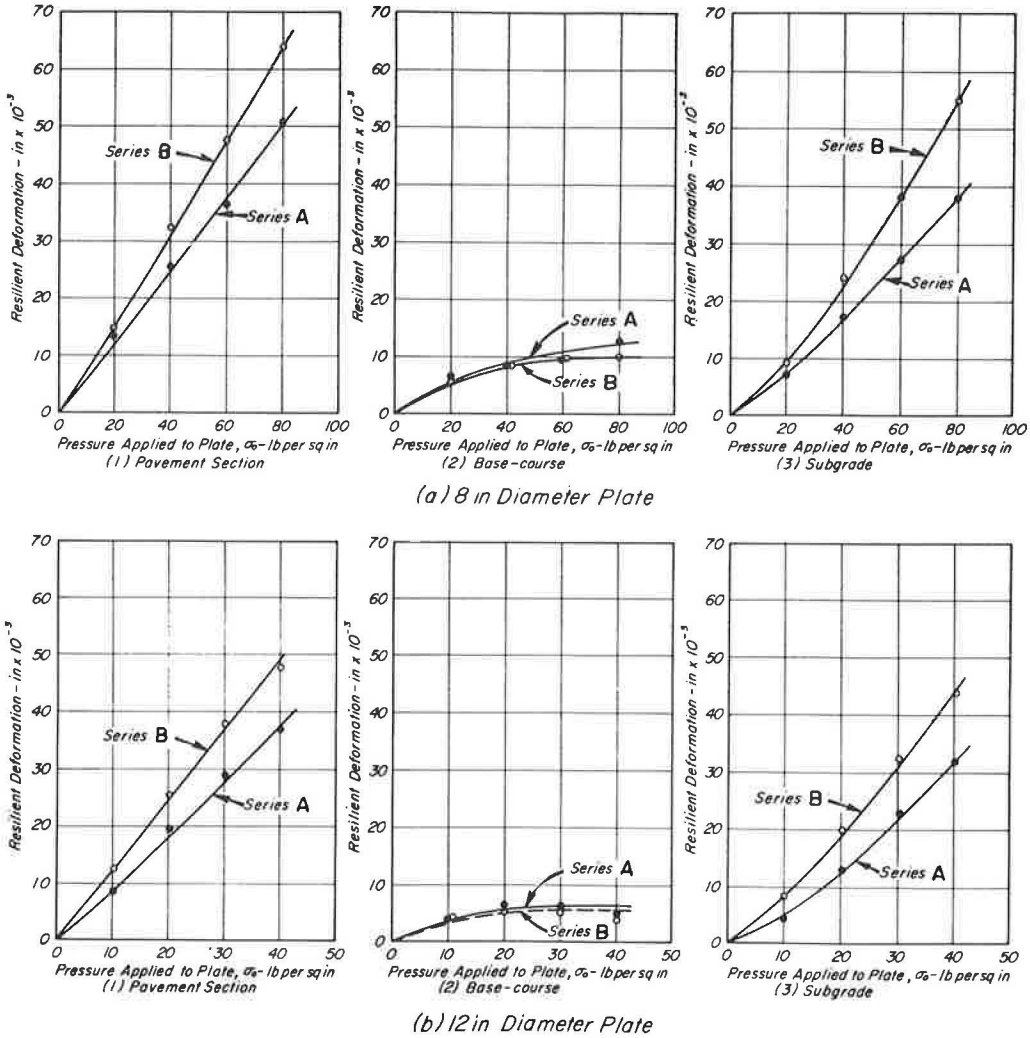

Figure 22. Comparison of resilient deformations of pavement components in two-layer system in test series A and B; 8-in. base.

PREDICTION OF DEFLECTIONS IN TWO-LAYER SYSTEMS

To predict the resilient deformation of an untreated-aggregate base course, both the resilient moduli and the stresses within the layer of material must be ascertained. As has already been noted, the modulus of resilient deformation of base-course materials is dependent on stress and, as seen in Figure 4, can be related to confining pressure. Because the vertical and horizontal stresses induced in a pavement by a wheel load vary both vertically and horizontally, the resilient modulus within the base will vary accordingly. However, no workable theoretical solution of pavement stresses and deflections which accounts for the variation of modulus in vertical and horizontal directions is available at the present time. Thus, the variation in modulus must be taken into consideration through simplifying assumptions.

In the following analyses, it will be assumed that the modulus of resilient deformation of the base course underneath the loading plate is constant along a horizontal plane (see Appendix). Variation of the modulus in the vertical direction can be approximated by subdividing the base course into several horizontal layers, each of which is assumed to have a constant modulus throughout its thickness. The modulus of each horizontal layer may then be determined by calculating the lateral stresses in the layer, resulting from the applied load and the weight of the pavement, and selecting the corresponding modulus from the results of repeated-load triaxial tests (relating the resilient modulus to confining pressure).

Proposed Analysis

Stresses and deflections in pavements can be analyzed by assuming that the materials behave like uniform elastic materials (Boussinesq), or like layered elastic materials (Burmister). In the latter case, approximate ratios of the moduli of the layer components must be determined.

McMahon and Yoder (28) and Sowers and Vesic (30) have shown that the stresses measured in two-layer systems consisting of an untreated base course overlying a compressible soil (similar to the two-layer systems analyzed in this study), are very similar to those predicted by Boussinesq's analysis for stress distribution; results of the analyses shown in subsequent sections also support this conclusion. Therefore, it has been considered appropriate to assume, for the two-layer system, that the lateral and vertical stresses are equal to those determined by the Boussinesq theory of stress distribution.

With this assumption, the proposed procedure to estimate the resilient deformations of the system consisting of an untreated granular base and subgrade can be briefly summarized as follows:

1. To compute the modulus of resilient deformation of the base course, the horizontal normal stresses resulting from the applied load can be estimated using the expression developed from the Boussinesq solution and presented by Ahlvin and Uhlery (34):

$$\sigma_H = \sigma_o \left[2\nu A + C + (1 - 2\nu) F\right] \tag{4}$$

where

σ_H = lateral pressure,

σ_o = uniform pressure at the surface of the base course[3],

ν = Poisson's ratio, and

A, C and F = functions depending on the depth and offset of the element under consideration relative to the center of the plate—determined from tables presented by Ahlvin and Uhlery (34).

The stresses are computed along a vertical line offset from the center of the plate at a distance equal to 0.7 times the radius of the plate[4] and for values of Poisson's ratios of 0.35 and 0.50, since it appears that the values for granular materials lie between these two limits so long as the deformations are small.

The lateral confining pressure caused by the weight of the material above a particular point can be determined by assuming that it is equal to the earth pressure at rest and,

[3]The computed values are based on a uniform surface pressure applied to a circular area, whereas the measured values are obtained from rigid-plate tests. Computations which have been made for a few of the conditions analyzed in the report show that there is, at the most, a ± 20 percent change in the deflection pattern under the plate for the flexible as compared to the rigid loaded area.

[4]This distance divides the contact area into two equal parts.

in the case of granular materials, the coefficient of earth pressure at rest, K_0, is assumed to be 0.5. This stress, when added to the lateral stress induced by the applied load, is considered as the controlling stress in defining the resilient modulus of the granular material at this point.

2. The variation in horizontal normal stress with depth can then be used to determine the variation of the modulus of resilient deformation of the base course with depth from the results of repeated-load triaxial tests at appropriate confining pressures.

3. The vertical stresses under the center of the plate induced in the subgrade by an applied load are calculated to a depth of 4 radii from the surface, on the assumption of a Boussinesq stress distribution.

4. The variation in vertical normal stress with depth in the subgrade can be used to determine the variation of the modulus of resilient deformation of the subgrade from repeated-plate-load tests on the subgrade under various vertical stresses or from repeated-load triaxial tests on the subgrade material with a range in deviator stresses (6), as seen in Figure 12.

5. The variation of resilient modulus with depth can be considered by dividing the base course and subgrade into several horizontal layers, each having a constant modulus equal to the average modulus over the thickness of each layer.

6. Deflection factors, presented by Ahlvin and Uhlery (34), can be used to compute the compression of a layer of any thickness at any depth. The deflection of a particular layer along a vertical axis through the center of the plate can be determined from the expression

$$w_{z_1} - w_{z_2} = \sigma_0 \frac{1+\nu}{E} r \left[(z_1 A_1 - z_2 A_2) + (1-\nu)(H_1 - H_2)\right] \tag{5}$$

where

w_{z_1} = deflection at top of layer,

w_{z_2} = deflection at bottom of layer,

E = average modulus of the layer,

A and H = functions whose values depend on the location of the point under consideration, and

z = depth in multiples of the radius of the loaded area.

Subscript 1 refers to the upper surface of the layer and subscript 2 to the lower surface.

7. The compression of the individual layers comprising the base course can then be added to give the total deformation of the base course; the deformations of the subgrade can be computed in the same manner.

Example

As an example, an analysis is presented for a 12-in. base course loaded with a 12-in. diameter rigid plate. Confining pressures and the corresponding moduli at points on a vertical line 0.7 rad from the center of the plate have been tabulated in Table 5 for a Poisson's ratio of 0.50. A similar computation was also made for Poisson's ratio of 0.35.

In this table notice that, although the confining pressure contributed by the weight of the base is relatively small in the upper portion, it is a major contributor in the lower portion of the base, especially when the surface pressure is low. The variation in modulus with depth corresponding to the computed confining pressures is shown in Figure 23; the results are typical of the trend obtained in all cases. The modulus of resilient deformation at the surface of the base is several times that at the bottom of the base, the rate of change with depth depending on thickness of the base, plate diameter and applied pressure. In addition, also note that the resilient modulus at the bottom of the base is approximately the same as that of the subgrade (5,000 to 10,000 psi).

TABLE 5

VARIATION OF CONFINING PRESSURE AND CORRESPONDING RESILIENT MODULUS WITH DEPTH IN UPPER LAYER OF TWO-LAYER SYSTEM[a]

Depth (in.)	Confining Pressure Due to Weight of Pavement (psi)	Applied Pressure 1		10		20		30		40	
		Confining Pressure (psi)		Confining Pressure (psi)	Resilient Modulus (psi)	Confining Pressure (psi)	Resilient Modulus (psi)	Confining Pressure (psi)	Resilient Modulus (psi)	Confining Pressure (psi)	Resilient Modulus (psi)
0	0.040	0.650		6.54	20,000	13.04	29,000	19.54	34,500	26.04	39,500
2	0.120	0.315		3.27	13,500	6.42	19,500	9.57	24,500	12.72	28,500
4	0.200	0.175		1.95	10,000	3.70	14,500	5.45	18,000	7.20	21,000
6	0.280	0.105		1.33	7,500	2.38	11,500	3.43	14,000	4.48	16,000
8	0.360	0.065		1.01	6,500	1.66	9,000	2.31	11,000	2.96	13,000
10	0.440	0.042		0.86	5,500	1.29	8,000	1.70	9,000	2.12	10,500
12											

[a]System consists of 12 in. of gravel and the natural subgrade; load applied to a 12-in. diameter plate; stresses were obtained from tables prepared by Ahlvin and Uhlery for a value of Poisson's ratio equal to 0.5. Corresponding moduli were obtained from Figure 3.

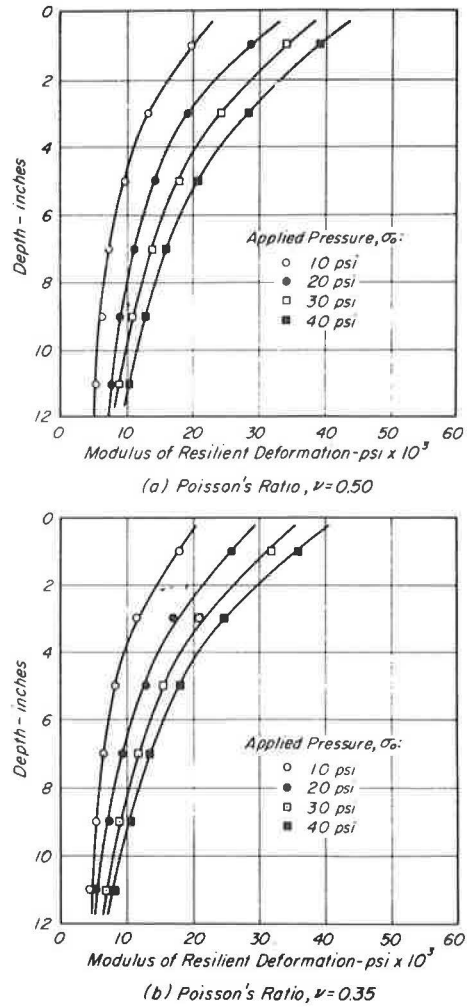

Figure 23. Variation of modulus of resilient deformation with depth in base course of two-layer system consisting of 12 in. of gravel base and the natural subgrade, 12-in. diameter plate.

By dividing the base course into 6 equal layers and using the moduli determined in Table 5, the deformation of each layer can be obtained from Eq. 5, as shown in Table 6 for a Poisson's ratio of 0.5. The total deflection of the base course is obtained by summing the deflections of the individual layers.

A comparison of the resilient deflections of the base course computed in this way for values of Poisson's ratio equal to 0.5 and 0.35 with those observed in the field tests is shown in Figures 24 and 25 for 8- and 12-in. thick gravel layers.

TABLE 6

RESILIENT DEFORMATION OF 12-IN. BASE LOADED WITH 12-IN. DIAMETER PLATE (ν = 0.5)

Layer	Resilient Deflection, in. $\times 10^{-3}$			
	At Pressure of 10 psi	At Pressure of 20 psi	At Pressure of 30 psi	At Pressure of 40 psi
From 0 to 2 inches	$\frac{10 \times 9 \times .07}{20,000} = .31$	$\frac{20 \times 9 \times .07}{29,000} = .435$	$\frac{30 \times 9 \times .07}{34,500} = .550$	$\frac{40 \times 9 \times .07}{39,500} = .64$
2 - 4	$\frac{10 \times 9 \times .105}{13,500} = .70$	$\frac{20 \times 9 \times .105}{19,500} = .972$	$\frac{30 \times 9 \times .105}{24,500} = 1.160$	$\frac{40 \times 9 \times .105}{28,500} = 1.39$
4 - 6	$\frac{10 \times 9 \times .115}{10,000} = 1.03$	$\frac{20 \times 9 \times .115}{14,500} = 1.423$	$\frac{30 \times 9 \times .115}{18,000} = 1.720$	$\frac{40 \times 9 \times .115}{21,000} = 1.97$
6 - 8	$\frac{10 \times 9 \times .110}{7,500} = 1.32$	$\frac{20 \times 9 \times .110}{11,500} = 1.720$	$\frac{30 \times 9 \times .11}{14,000} = 2.120$	$\frac{40 \times 9 \times .11}{16,000} = 2.48$
8 - 10	$\frac{10 \times 9 \times .090}{6,500} = 1.24$	$\frac{20 \times 9 \times .09}{9,000} = 1.800$	$\frac{30 \times 9 \times .09}{11,000} = 2.210$	$\frac{40 \times 9 \times .09}{13,000} = 2.49$
10 - 12	$\frac{10 \times 9 \times .06}{5,500} = .98$	$\frac{20 \times 9 \times .06}{8,000} = 1.350$	$\frac{30 \times 9 \times .06}{9,000} = 1.800$	$\frac{40 \times 9 \times .06}{10,500} = 2.06$
Total Resilient Deformation, in. $\times 10^{-3}$	5.60	7.76	9.56	11.03

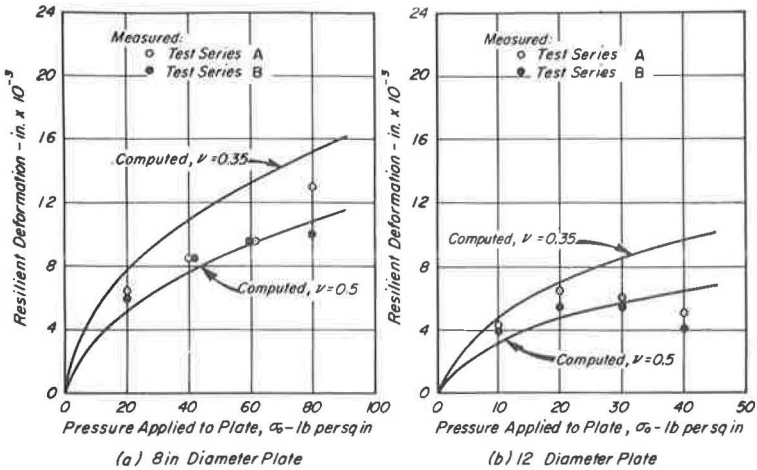

Figure 24. Comparison between computed and measured resilient deformations of base course in a two-layer system consisting of 8 in. of base and the natural subgrade; test series A and B.

These figures indicate that the resilient deformations of the base course computed for a Poisson's ratio of 0.35 are about 40 percent higher than those computed for a Poisson's ratio of 0.50. It has been suggested (35) that, for a homogeneous material, the resilient deflection under a plate load for a material with a Poisson's ratio of 0.35 would be about 17 percent higher than when Poisson's ratio was 0.50. However, the change in confining pressure due to a change in Poisson's ratio, and the corresponding change in resilient modulus, were not considered in this latter analysis. A low value of Poisson's ratio creates lower confining pressures and correspondingly decreases the modulus of resilient deformation. When this factor is considered, the increase in

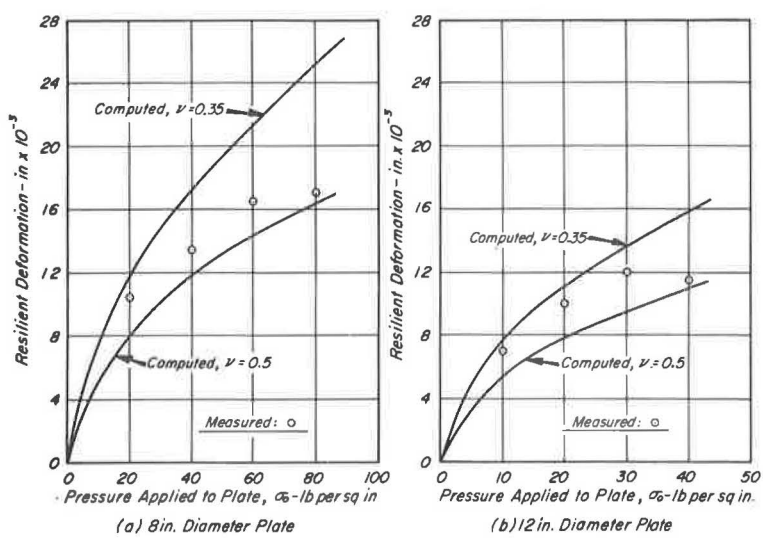

Figure 25. Comparison between computed and measured resilient deformations of base course in a two-layer system consisting of 12 in. of base and the natural subgrade; test series A.

TABLE 7

DETERMINATION OF RESILIENT DEFORMATION IN SUBGRADE—12-IN. THICK BASE, 12-IN. DIAMETER PLATE AND 20-PSI APPLIED PRESSURE

Depth from the Surface of the Pavement (in.)	Vertical Stress (psi)	Corresponding Modulus of Resilient Deformation[a] (psi)	Resilient Deformation in Layer Between Indicated Depths (in. × 10^{-3})
12	5.70	8,000	1.71
15	3.98	8,900	1.11
18	2.92	10,000	1.30
24	1.74	13,400	3.27
			Σ = 7.40

[a]From Figure 12.

resilient deflection due to a change in Poisson's ratio from 0.5 to 0.35 will change from 17 percent to 40 percent.

The measured resilient deformations of the base course fall, in general, within the range of resilient deformations predicted, although at high stresses for the 8-in. base-course tests the measured deflections are somewhat lower than those predicted; this difference may be due to the fact that at the higher stresses the plastic deformation was continuously increasing with number of load applications.

In general, the results of this approximate analysis using the results of repeated-load triaxial tests agree reasonably well with the measured resilient deflections in plate-load tests, especially if Poisson's ratios of 0.40 to 0.45 are assumed in the computations.

A similar analysis has been used for determining the resilient deformation of the subgrade. For example, the deformations in this material under a 12-in. base course

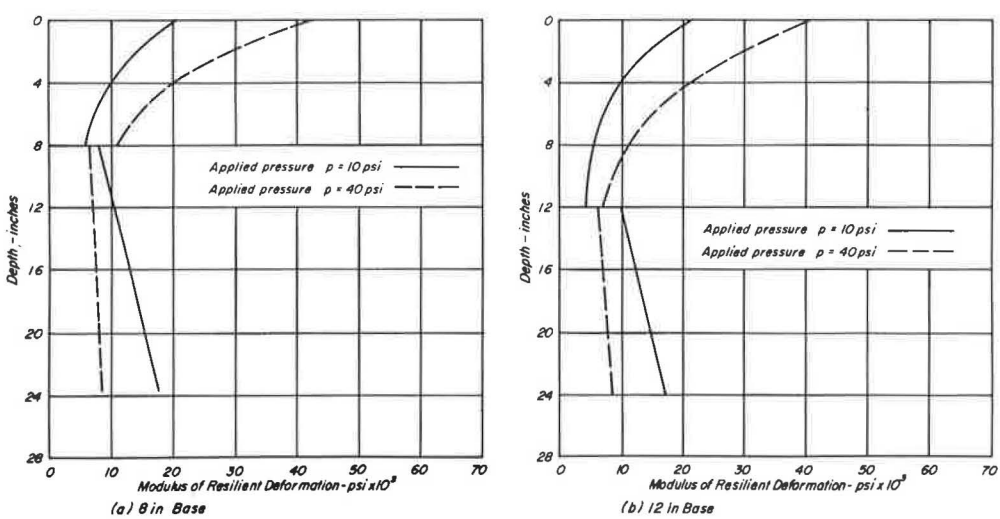

Figure 26. Variation of resilient modulus with depth beneath the center of a 12-in. diameter plate in two-layer systems (ν_{base} and $\nu_{subgrade}$ assumed equal to 0.35 and 0.5, respectively); test series A.

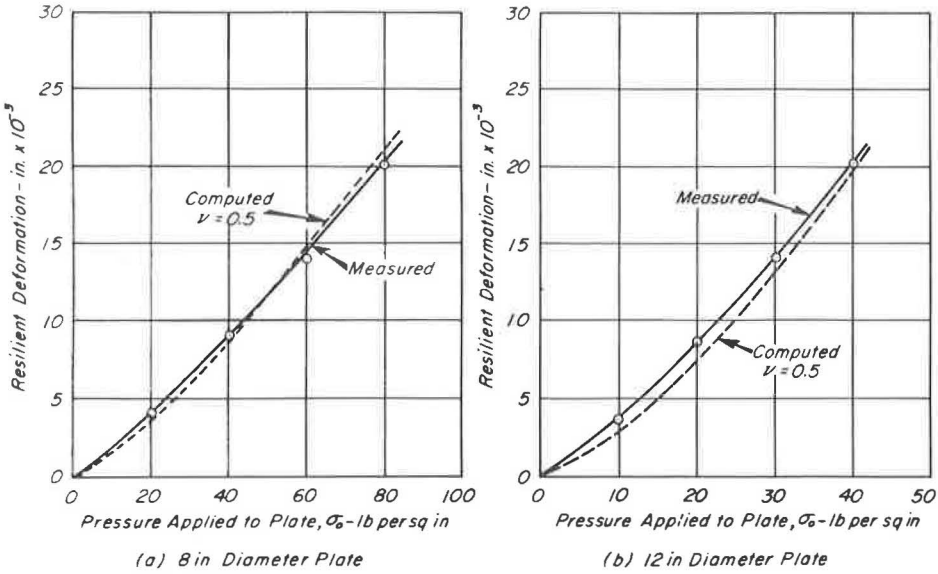

Figure 27. Comparison between measured and computed subgrade deflections for two-layer system consisting of 12 in. of base and the natural subgrade; test series A.

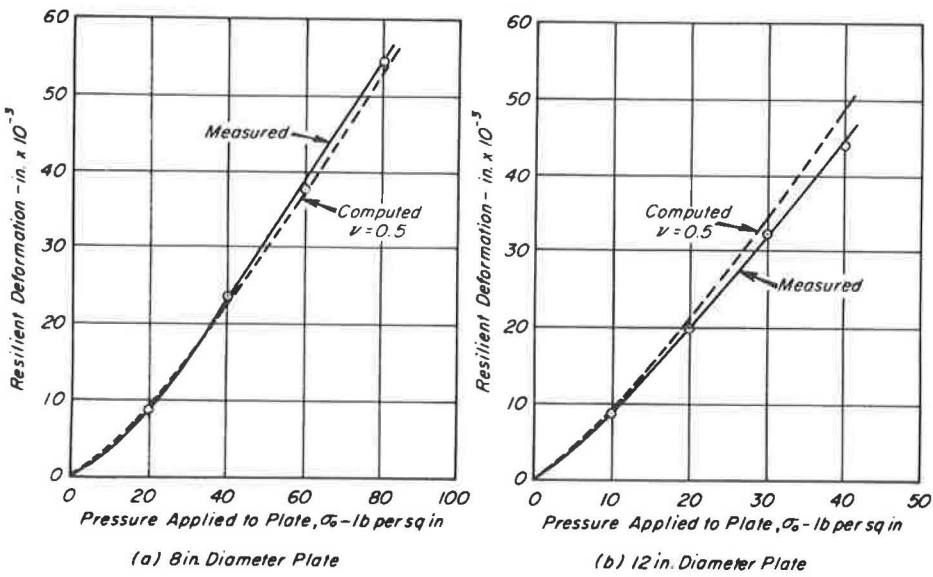

Figure 28. Comparison between measured and computed subgrade deflections for two-layer system consisting of 8 in. of base and the natural subgrade; test series B.

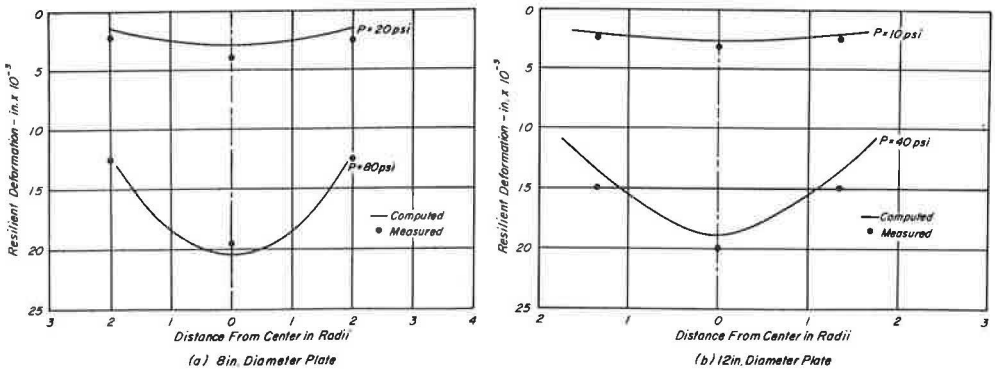

Figure 29. Comparison between computed and measured resilient deformations at the base-subgrade interface for two-layer systems consisting of 12 in. of base and the natural subgrade; test series A.

loaded by a 12-in. diameter rigid plate with an applied pressure of 20 psi are given in Table 7. For convenience, the subgrade has been divided into four layers, and the vertical stress at the top of each layer estimated from the tables prepared by Ahlvin and Uhlery. The moduli of resilient deformation corresponding to these vertical stresses were in turn estimated from Figure 12 (test series A). Deformations were then computed from Eq. 5 with a value for Poisson's ratio of 0.5.

The moduli for the subgrade determined in this way together with those obtained previously for the base course are plotted in Figure 26 to illustrate the variation in

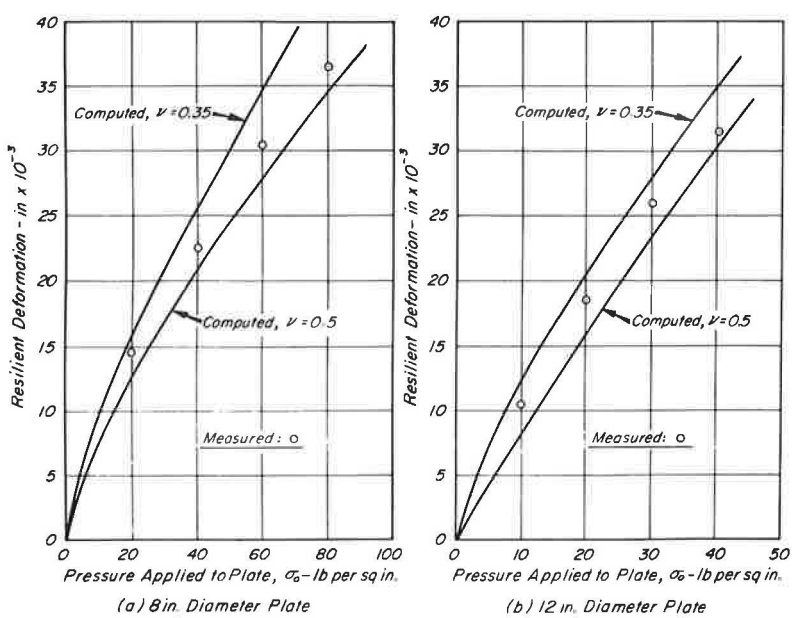

Figure 30. Comparison between computed and measured total resilient deformation for two-layer system consisting of 12 in. of base and the natural subgrade; test series A.

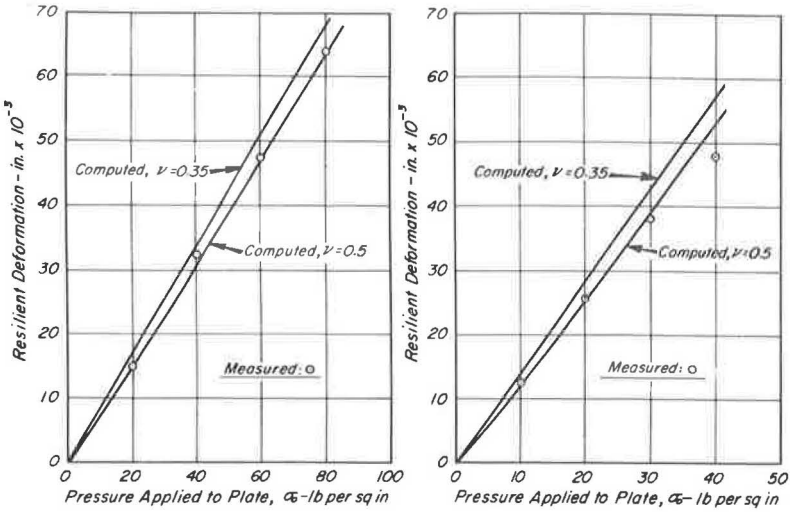

Figure 31. Comparison between computed and measured total resilient deformation for two-layer system consisting of 8 in. of base and the natural subgrade; test series B.

base-course and subgrade moduli with depth for the above conditions. It will be noted that the ratio of the modulus of the base to that of the subgrade at the interface of the two layers varies from approximately 0.4 to 1.75, depending on the surface pressure.

Comparisons between computed and measured deflections directly below the center of the plate at the surface of the subgrade are shown in Figures 27 and 28. The computed values are based on a value for Poisson's ratio equal to 0.5 and moduli determined from Figure 12. Excellent agreement is indicated for both 8- and 12-in. diameter plates.

In a similar manner, the variation of deflection at the top surface of the subgrade has been computed along a radial line, using Eq. 5. The values obtained are compared to those determined from the field tests in Figure 29; again, good agreement is indicated.

Comparisons between the computed total resilient deformations and the corresponding measured deformation are presented in Figures 30 and 31. The computed values in this case were obtained by adding the resilient deformations shown in Figures 24 and 25 for the gravel to those shown in Figures 27 and 28 for the subgrade. The predicted and measured values, as shown in these figures, are again in reasonably close agreement.

These results would indicate that the resilient deformations of two-layer systems consisting of untreated granular base and compressible subgrade soils can be reasonably predicted from the results of laboratory repeated-load triaxial-compression tests by the proposed analysis.

SUMMARY

This investigation was undertaken because of the interest in deflections under load as a measure of pavement performance, and the need for a procedure whereby pavement deflections could be predicted in advance from suitable laboratory tests.

The deflections of interest are essentially elastic in the sense that they are completely recoverable (for all practical purposes) on unloading and appear to be

approximately proportional to load. However, in order that these deformations should not be confused with elastic deformation in the classical sense, they have been termed resilient deformations.

It would appear that one of the major difficulties in predicting resilient pavement deflections is the lack of knowledge concerning the resilient behavior of untreated granular materials; hence, the major portion of this paper has been devoted to a discussion and definition of the factors influencing the resilience characteristics of these materials. It has involved the measurement of the resilient behavior of representative granular materials in the laboratory, the measurement of deflections of prototype pavements composed of one of these materials (a well-graded, partially crushed gravel) in the field, and the relating of the laboratory test results to the observed deflections of the prototype pavements.

One of the major factors influencing the resilience characterisitcs of granular materials has been the magnitude of the applied stress. Since stresses due to load vary in both the vertical and horizontal directions in a pavement section, this influence of stress on resilience should properly be accounted for in order to adequately predict the deformation characteristics of the pavement section. Accordingly, an evaluation of existing methods for computing stress distributions was also prepared. This evaluation was based on analyses for elastic media, since it is the transient recoverable deflections due to passage of the wheel load which are of interest.

Essentially two methods are available: Boussinesq considered stresses and displacements in a uniform system, whereas Burmister considered a layered system. The evaluation indicated that the actual distribution of stress in pavements constructed of untreated granular materials is most closely approximated by the Boussinesq analysis. Accordingly, approximate values for deflections in these structures may be obtained by using the Boussinesq theory and the modifications suggested by Vesic to account for the variability in deformation characteristics (as measured by resilient moduli) of the granular material. This method has been used to analyze the results of tests performed on the two-layer systems studied in this investigation.

CONCLUSIONS

1. The results of repeated plate-load tests at the surface of the subgrade indicate that the modulus of resilient deformation of clay soils varies extensively with the applied pressure and water content. The resilient modulus of the subgrade soil used in the test program decreased rapidly in the range of stress between 1 and 10 psi (which is the range to be expected in the subgrade of well-designed pavements); the modulus had an effectively constant value at a stress of the order of 10 psi and over. The test results also indicated that the rate of change in modulus with stress is dependent on water content. As the water content increased, the rate of change in modulus with stress decreased. When comparing the results of two series of tests conducted with the subgrade at different water contents, the resilient modulus was reduced to about one-half the initial value as the subgrade became wetter. This change emphasizes the importance of allowing for possible variations in the modulus of the subgrade due to environmental changes during the pavement lifetime.

2. The resilient deformation of an untreated base course depends on (a) confining pressure—the average modulus of resilient deformation increases with the confining pressure; (b) thickness of base—the resilient deformation of the base increased as the thickness of base increased from 8 to 12 in.; and (c) plate diameter—when the surface pressure remained constant, the resilient deformation of the base course decreased as the diameter of the plate increased from 8 to 12 in. The resilient deformation per inch of thickness of base decreased as the ratio of plate diameter to thickness of base increased.

3. A general statement concerning the relative contribution of the base and subgrade to the total resilient deformation of a two-layer pavement section cannot readily be made on the basis of data obtained from the field tests. It appears that, because the modulus of the base increases and that of the subgrade decreases as the applied stress

increases, the resilient deformation of the subgrade becomes relatively more significant at higher magnitudes of surface-pressure applications.

4. Results of repeated-load triaxial-compression tests on dry granular materials indicate a unique relationship between the modulus of resilient deformation and the confining pressure (σ_3) so long as a shear failure does not occur. A similar relationship appears also to be valid in terms of the sum of the principal stresses.

The modulus of resilient deformation of the dry materials varies with the effective confining pressure or the sum of the principal stresses according to the equations

$$M_R = K \cdot \sigma_3^n \text{ and } M_r = K' \cdot \theta^{n'}$$

which indicates that the modulus can vary considerably over the range of stresses usually encountered in pavements.

5. The results of repeated-load tests on subgrade and base-course materials clearly show that the resilient moduli vary with stresses. When the stresses at the top and bottom of the base course are very different (i.e., when the base is loaded directly), the resilient modulus at the top of the course may be as much as four times the resilient modulus at the bottom. For the conditions used in this study, the ratio of the moduli of the base course to that of the subgrade at the interface between the two layers varied from 0.4 to about 1.75.

6. The resilient deformations computed by the proposed method from the results of repeated-load triaxial-compression tests for two-layer structures were in reasonably good agreement with the resilient deformations measured in the prototype pavements. Thus, it would appear that the results of repeated-load tests on paving materials can be used within the framework of available theories to predict transient pavement deflections.

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REFERENCES

1. Hveem, F. N. Pavement Deflections and Fatigue Failures. HRB Bull. 114, pp. 43-87, 1955.
2. Monismith, C. L. Symposium on Pavement Behavior as Related to Deflection, Part II—Significance of Pavement Deflections. Proc. AAPT, Vol. 31, pp. 231-253, 1962.
3. Monismith, C. L. Asphalt Mixture Behavior in Repeated Flexure. Rept. No. TE 65-9, Univ. of California, Berkeley, Nov. 1965.
4. State of California, Division of Highways. The Development of a Resilience Design Procedure for Flexible Pavements. Materials and Research Dept. Final Research Rept. No. M&R 431618-2, April 1966.
5. Seed, H. B., and Monismith, C. L. Moderators' Summation of Session V, Strength Evaluation of Pavement Structure Elements. Proc. Internat. Conf. on the Structural Design of Asphalt Pavements, 1962, Univ. of Michigan, pp. 1009-1011, 1963.

6. Seed, H. B., Mitry, F. G., Monismith, C. L., and Chan, C. K. Prediction of Pavement Deflections from Laboratory Repeated Load Tests. Rept. No. TE 65-6, Univ. of California, Berkeley, Oct. 1965.
7. Seed, H. B., and Chan, C. K. Effect of Duration of Stress Application on Soil Deformation Under Repeated Loading. Proc. Fifth Internat. Conf. on Soil Mech. and Found. Eng., Vol. 1, pp. 340-345, 1964.
8. Haynes, J. H., and Yoder, E. J. Effects of Repeated Loading on Gravel and Crushed Stone Base Course Materials Used in the AASHO Road Test. Highway Research Record 39, pp. 82-96, 1963.
9. Biarez, J. Contribution a l'Etude des Proprietes Mecaniques des Sols et des Materiau Pulverulents. D. Sc. thesis. Univ. of Grenoble, France, 1962.
10. DeGraft-Johnson, J. Unpublished Research Report. Soil Mechanics and Bituminous Materials Laboratory, Univ. of California, Sept. 1962.
11. Trollope, D. H., Lee, I. K., and Morris, J. Stresses and Deformation in Two Layer Pavement Structures Under Slow Repeated Loading. Proc. Australian Road Research Board, Vol. 1, Part 2, pp. 693-721, 1962.
12. Dunlap, W. A. A Report on a Mathematical Model Describing the Deformation Characteristics of Granular Materials. Tech. Rept. No. 1, Texas Transportation Institute, Texas A&M Univ., 1963.
13. Coffman, B. S., Kraft, D. C., and Tamayo, J. A Comparison of Calculated and Measured Deflections for the AASHO Road Test. Proc. AAPT, Vol. 33, pp. 54-91, 1964.
14. Hveem, F. N., Zube, E., Bridges, R., and Forsyth, R. The Effect of Resilience-Deflection Relationship on the Structural Design of Asphaltic Pavements. Proc. Internat. Conf. on the Structural Design of Asphalt Pavements, 1962, Univ. of Michigan, pp. 649-666, 1963.
15. Burmister, D. M. Applications of Layered System Concepts and Principals to Interpretations and Evaluations of Asphalt Pavement Performances and to Design and Construction. Proc. Internat. Conf. on the Structural Design of Asphalt Pavements, 1962, Univ. of Michigan, pp. 441-453, 1963.
16. Brown, P. P. Analysis of Flexible Airfield Pavements by Surface Plate-Loading. Proc. Internat. Conf. on the Structural Design of Asphalt Pavements, 1962, Univ. of Michigan, pp. 680-685, 1963.
17. U. S. Army Engineer Waterways Experiment Station. Investigations of Pressures and Deflections for Flexible Pavements, Report No. 1. Homogeneous Clayey Silt Test-Section. Tech. Memo. 3-323, Vicksburg, Miss., 1951.
18. U. S. Army Engineer Waterways Experiment Station. Investigations of Pressures and Deflections for Flexible Pavements, Report No. 4. Homogeneous Sand Testing Section. Tech. Memo. 3-323, Vicksburg, Miss. 1954.
19. Walker, R. D., Yoder, E. J., Spencer, W. T., and Lowry, R. Significance of Layer Deflection Measurements. HRB Bull. 321, pp. 63-81, 1962.
20. Heukelom, W., and Klomp, A. J. G. Dynamic Testing as a Means of Controlling Pavements During and After Construction. Proc. Internat. Conf. on the Structural Design of Asphalt Pavements, 1962, Univ. of Michigan, pp. 667-679, 1963.
21. Nijboer, L. W., and Metcalf, C. T. Dynamic Testing at the AASHO Road Test. Proc. Internat. Conf. on the Structural Design of Asphalt Pavements, 1962, Univ. of Michigan, pp. 713-721, 1963.
22. Jones, R. Following Changes in the Properties of Road Bases and Subbases by the Surface Wave Propagation Method. Civil Engineering and Public Works Review, Part I, May 1963, and Part II, June 1963.
23. Odemark, N. Investigations as to the Elastic Properties of Soils and Design of Pavements According to the Theory of Elasticity. Meddelande 77, Statens Väginstitut, Stockholm, 1949.
24. Dehlen, G. L. An Investigation of Flexure Cracking on a Major Highway. Proc. Internat. Conf. on the Structural Design of Asphalt Pavements, 1962, Univ. of Michigan, pp. 812-820, 1963.
25. Seed, H. B., Chan, C. K., and Lee, C. K. Resilience Characteristics of Subgrade Soils and Their Relation to Fatigue Failures in Asphalt Pavements.

Proc. Internat. Conf. on the Structural Design of Asphalt Pavements, 1962, Univ. of Michigan, pp. 611-636, 1963.
26. Burmister, D. M. The General Theory of Stresses and Displacements in Layered Soil Systems. Jour. Appl. Phys., Vol. 16, No. 2, pp. 89-96; Vol. 16, No. 3, pp. 126-127; Vol. 16, No. 5, pp. 296-320, 1945.
27. Whiffin, A. C., and Lister, N. W. The Application of Elastic Theory to Flexible Pavements. Proc. Internat. Conf. on the Structural Design of Asphalt Pavements, 1962, Univ. of Michigan, pp. 499-521, 1963.
28. McMahon, T. F., and Yoder, E. J. Design of a Pressure Sensitive Cell and Model Studies of Pressures on a Flexible Pavement Subgrade. HRB Proc., Vol. 39, pp. 650-682, 1960.
29. Vesic, A. B. Discussion presented at Session III. Proc. Internat. Conf. on the Structural Design of Asphalt Pavements, 1962, Univ. of Michigan, pp. 283-290, 1963.
30. Sowers, G. F., and Vesic, A. B. Stress Distribution Beneath Pavements of Different Rigidities. Proc. Fifth Internat. Conf. on Soil Mech. and Found. Eng., Vol. 2, pp. 327-332, 1961.
31. Sowers, G. F., and Vesic, A. B. Vertical Stresses in Subgrades Beneath Statically Loaded Flexible Pavements. HRB Bull. 324, pp. 90-123, 1962.
32. Jakobson, B. Some Fundamental Properties of Sand. Proc. Fourth Internat. Conf. on Soil Mech. and Found. Eng., London, Vol. 1, pp. 167-171, 1957.
33. Cumming, D. A., and Gerrard, C. M. Computation of Stresses on Pavements. Proc. Australian Road Research Board, Vol. 2, Part 2, pp. 729-743, 1964.
34. Ahlvin, R. G., and Uhlery, H. H. Tabulated Values for Determining the Complete Pattern of Stresses, Strains, and Deflections Beneath a Uniform Circular Load on a Homogeneous Half Space. HRB Bull. 342, pp. 1-13, 1962.
35. Peattie, K. R., and Jones, A. Surface Deflections of Road Structures. Proc. Symposium on Road Tests for Pavement Design, 1962, Lisbon, Portugal, pp. VIII, 1-30.

Appendix

ANALYSIS OF ASSUMPTION OF CONSTANT MODULUS IN THE HORIZONTAL PLANE FOR TWO-LAYER SYSTEMS

In the paper, two-layer systems consisting of untreated granular material and the natural subgrade were analyzed by dividing the base course into a series of horizontal layers. To simplify this analysis, the variation of the resilient modulus in the horizontal direction was assumed negligible with respect to that in the vertical direction. This assumption was based on the results of analyses, examples of which are shown in Figures 32 and 33.

In these figures, contours of resilient moduli have been plotted to show their variation in both the horizontal and vertical directions. Moduli were determined from the results of the repeated-load triaxial-compression tests using the sum of horizontal stresses resulting from the applied load and the weight of the pavement. Thus it would appear that the assumption of a constant modulus in a horizontal plane under the loaded area is justified.

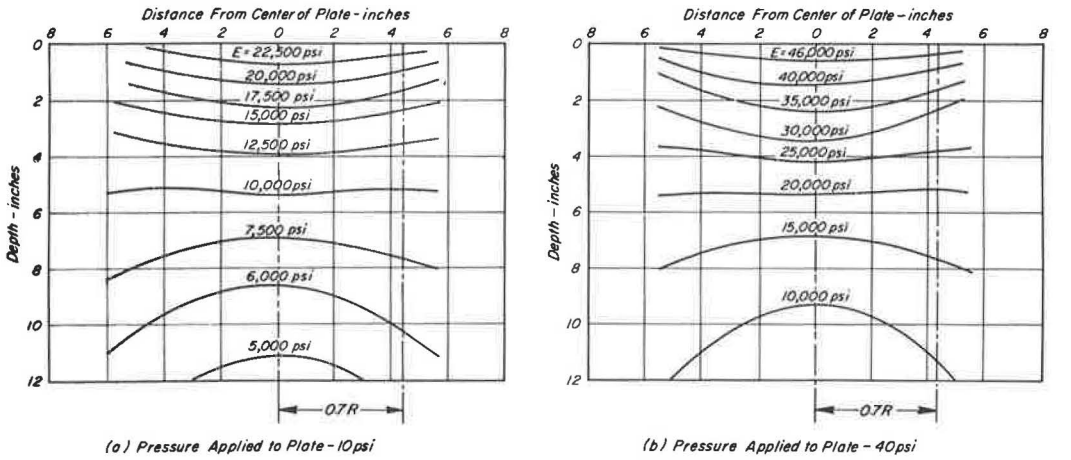

Figure 32. Variation of modulus of resilient deformation under 12-in. diameter plate in 12-in. thick base (σ_o = 20 psi, ν = 0.5, K_o = 0.5).

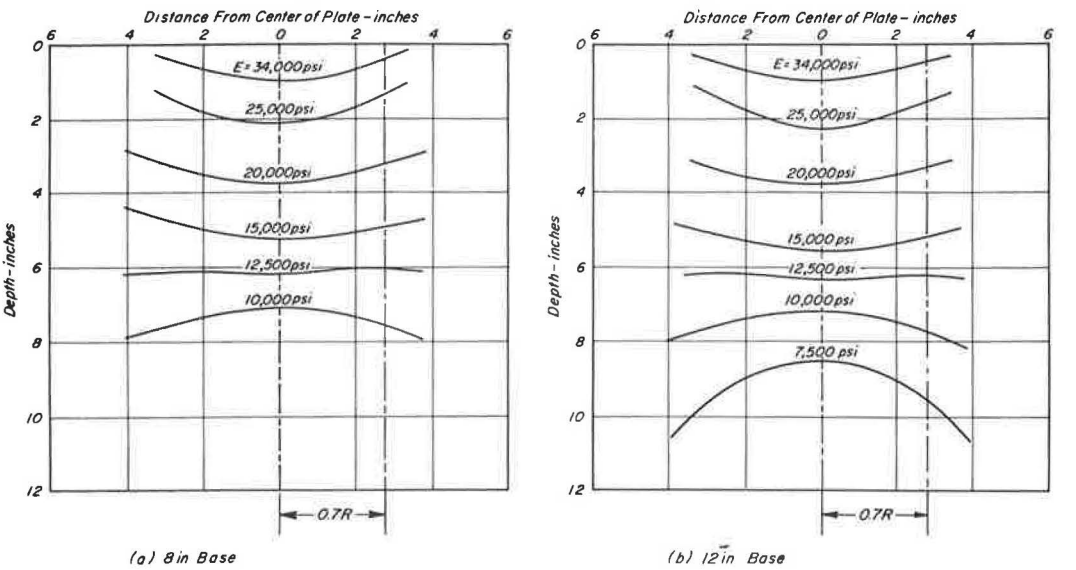

Figure 33. Variation of modulus of resilient deformation under 8-in. diameter plate in both 8- and 12-in. thick base courses (σ_o = 20 psi, ν = 0.5, K_o = 0.5).