A Digital Simulation of Car Following and Overtaking

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A model representing the single-lane no-passing driving situation was formulated and run on a digital computer. Although the model involves the use of a car-following equation, the simulation also includes human factors, such as reaction time lag, driver sensitivity, and the threshold of detection of relative velocity. The model allows for variation of these characteristics both between drivers and over time for each individual driver.

The study was directed to the accident prevention problem with the aim of determining the critical parameters of the driving situation, and of ascertaining the ranges of values of these parameters which define a safe or stable driving situation.

•IN the past 10 years digital simulation of automobile traffic has been increasingly employed as a research technique. The bulk of the work has been done either on simulations of urban traffic with emphasis on the intersection problem, or on freeway simulations emphasizing problems of ramp design.

Some simulations of car following have been carried out, notably in the work of Helly (5) which was aimed at the study of shock-wave development in tunnels, and in the more recent work of Howat (6), and Todosiev (10).

The simulation of car following and overtaking discussed in this report was done especially to investigate stable and unstable behavior with regard to accident-producing situations. It seems clear that a large proportion of accidents on freeways are rear-end collisions caused either by tailgating or too rapid overtaking. In this study, single-lane driving is simulated by a model that includes individual human factors and which is flexible enough to represent a variety of situations. In this way the simulation should result in an identification of the principal factors and parameters influencing safe or stable driving behavior, and a determination of the ranges of parametric values which will exclude accident occurrence.

CAR-FOLLOWING MODELS

A car-following model is essentially some form of a stimulus-response equation where the response (acceleration or deceleration) of the driver is determined by a stimulus function involving the relative velocity between his car and the car ahead, their relative spacing, the absolute velocity level, the driver's sensitivity, and many other factors, human, mechanical, and environmental.

Early car-following models were based on a simple linear relation with driver response dependent only on relative velocity. Thus (Fig. 1), the acceleration or deceleration response of the (n + 1) st car was approximated by

$$\frac{dx^2_{n+1}}{dt^2} = \lambda\left(\frac{dx_n}{dt} - \frac{dx_{n+1}}{dt}\right) = \lambda(v_n - v_{n+1}) \qquad (1)$$

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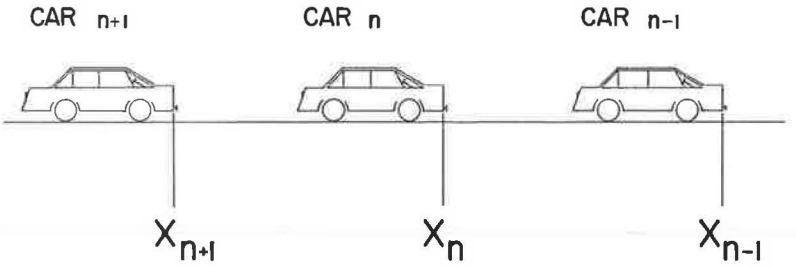

Figure 1. Car-following model.

where

λ = driver sensitivity factor (i.e., intensity of response);
x_n = position of car n; and
v_n = velocity of car n.

Later it became clear that response depended on relative spacing (inversely), on velocity level and, rather critically, on the time lag in driver response, say, T. Consequently, Herman (3), Edie (2), and others studied models of the form

$$\frac{d^2x_{n+1}(t)}{dt^2} = \alpha v_{n+1}(t) \frac{(v_n(t-T) - v_{n+1}(t-T))}{(x_n(t-T) - x_{n+1}(t-T))^p} \tag{2}$$

where

α = new sensitivity factor (λ in Eq. 1 is replaced by $\frac{\alpha v_{n+1}}{(x_n - x_{n+1})^p}$ in Eq. 2), and
p = usually 1 or 2.

Thus, in Eq. 2 the driver of the (n + 1)st car accelerates or decelerates an amount which depends on his current velocity, and on the velocity difference and separation between his car and the next at some previous instant. The lag, T, is assumed to include the times for perception, decision, and response; it has been used in the same way implicitly by other investigators.

In actual driving experience people do not, of course, drive continuously in strict adherence to such rules. As Michaels (7) has discussed, a driver cannot even detect relative velocity until the rate of change of angular motion of the image across the retina assumes some minimum threshold value. From Figure 2, the rate of change of angle, $\frac{d\theta}{dt}$, can be determined from the equation

$$L\theta \cong W \text{ or } \theta \cong \frac{W}{L}$$

so that

$$\frac{d\theta}{dt} \cong -\frac{W}{L^2}\frac{dL}{dt}$$

Then, since L is relative spacing,

$$\left|\frac{d\theta}{dt}\right| \cong W\left|\frac{(v_n - v_{n+1})}{(x_n - x_{n+1})^2}\right| \tag{3}$$

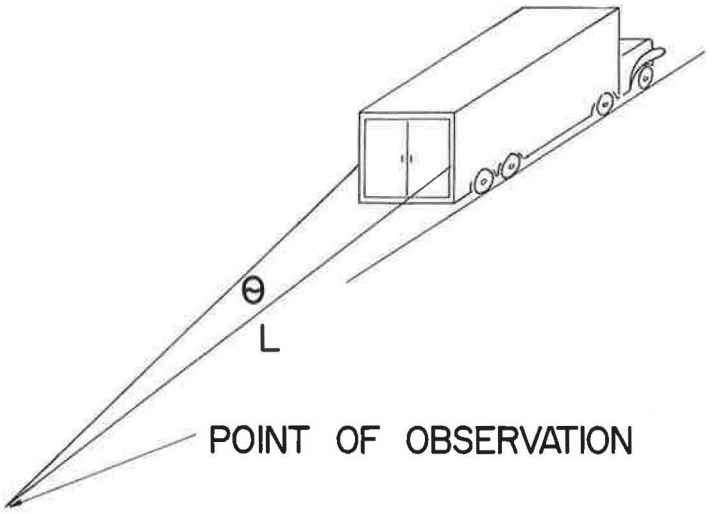

Figure 2. Angular velocity detection.

It has been shown (8) that $\left|\frac{d\theta}{dt}\right|$ is below the threshold of relative velocity detection until the quantity

$$W\left|\frac{v_n - v_{n+1}}{(x_n - x_{n+1})^2}\right| \tag{4}$$

exceeds approximately 6×10^{-4} (for x_n and W in feet and v_n in feet per sec).

Comparison of Eq. 3 with Eq. 2 reveals the extremely interesting fact that the two equations are in agreement for the case p = 2, the value recommended by Edie. It is certainly one of the most intriguing results of the current investigation to note that Eq. 3, based on human factor threshold studies, supports Eq. 2 which has been derived quite differently from empirical fits to experimental driving data. This confluence, which has not to our knowledge been noticed by previous investigators, sets the basic structure for our model.

SIMULATION MODEL

The car-following model set up for simulation is based on Eq. 2, with p = 2. This equation, however, governs the driver of each car only when the threshold test quantity for that driver, Eq. 4, exceeds a preset value for velocity threshold boundary. When the threshold boundary value is exceeded, the driver is considered in a velocity-detecting mode. Under threshold he is in a distance-detecting mode. The behavior of the simulated driver in the two modes is quite different, as outlined next.

Velocity-Detecting Mode

Over threshold, in the velocity-detecting mode, an individual driver in the simulation makes responses roughly in accordance with Eq. 2, although some overriding considerations arise. For example, the sensitivity factor, α, takes on different values for the accelerating and decelerating responses, since driver response differs in the two situations (9).

Furthermore, the values of acceleration are limited to the maximum values attainable at the current velocity (11), and deceleration is limited to given maximum values. In fact, except when the extremes of the driving situation warrant it, decelerations are limited to the more comfortable values of 8 to 11 ft per sec.

In addition, each simulated driver is assigned a certain desired velocity and attempts to keep his speed within ±15 percent or so of this value, provided there are no conflicting crises. He also has a preferred spacing (which depends on velocity levels) between his car and the car ahead.

Distance-Detecting Mode

Under threshold, a driver cannot detect relative velocity, but he can determine his actions on the basis of desired velocity, spacing, and so on. Furthermore, an actual driver does not monitor the situation continuously but samples various quantities at random intervals. Our simulated driver mirrors these responses, taking mild corrective action whenever he drifts away from the desired speed range.

Further Considerations

In our earlier models the brake light of the car ahead was not considered. When it was introduced into the model it had an excellent stabilizing influence, as it does in reality. In the model, a driver's brake light is considered to be on whenever his deceleration exceeds that caused by vehicle drag (11, p. 26), and a term representing the brake light of the car ahead is added to the threshold test quantity for the following car.

In the models discussed, the following car looks only one car ahead. In the current model the driver also considers the car two ahead and drives according to the equation

$$\frac{d^2x_{n+1}(t)}{dt^2} = \alpha v_n(t) \left[\frac{W_1 (v_n(t-T) - v_{n+1}(t-T))}{(x_n(t-T) - x_{n+1}(t-T))^2} + \frac{W^2 (v_{n-1}(t-T) - v_{n+1}(t-T))}{(x_{n-1}(t-T) - x_{n+1}(t-T))^2} \right] \tag{5}$$

where

$$W_1 + W_2 = 1$$

so that he considers the relative spacing and velocity between his car and the car two ahead. A term representing the brake light of the car two ahead has also been added to the threshold test quantity, and these two additions have led to more realistic model behavior. In exceptional cases, mainly when the car two ahead is pulling away or when the driver of the (n + 1)st car is decelerating so rapidly that he would tend to look at most only one car ahead, the effect of the car two ahead is excluded.

The reaction time, T, not only varies from one driver to another, but it also varies over time for each individual driver, depending on his driving situation. When the driver goes over threshold his reaction time drops from its larger value in the distance-detecting mode to a smaller value, giving a quickened response time. Then after a while the driver's reaction time builds up, over time, until it reaches a maximum value which depends on whether or not the driver is over threshold. Whenever an unexpected event occurs, such as a rapid deceleration of the car ahead, reaction time drops down again. Figure 3 shows a typical reaction-time behavior.

IMPLEMENTATION OF THE MODEL

Although some consideration was given to constructing a special programming language to simulate the model, the idea was rejected as too time consuming and too far removed from the main aim of the study. The Fortran programming language was used, and on the whole it has proved very satisfactory. In Fortran the individual driver or vehicle characteristics can be stored as parameters with a single index so that T (Eq. 4), for example, is the response delay time of the driver of the fourth car in the platoon. The time-dependent quantities require two indices so that a quantity such as V (Eqs. 5, 2) represents the velocity of the fifth car two time steps ago.

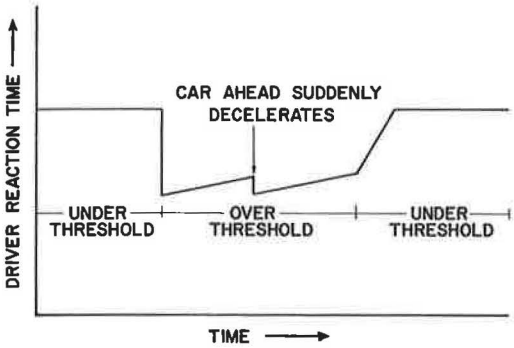

Figure 3. Driver reaction time.

The advantage of a common programming language such as Fortran is that a program can be moved with relative ease from one computer to another. The bulk of the computation has been done on an IBM 1620 computer, but the CDC 6600 computer at the Courant Institute of Mathematical Sciences of New York University is being used increasingly. Essentially the same Fortran program works on both.

The Computation

In the simulation, a platoon of cars is considered to be going along initially in a steady state at given spacings and velocities. The lead car then performs some maneuver, such as a rapid deceleration to a new velocity, and the response of the following cars is determined. At each computational time step, the response (acceleration or deceleration) of each following car in turn along the platoon is computed, and the equations are integrated numerically twice (using a simple parabolic rule) to determine the new velocities and positions. When the last car in the platoon, at that time step, has been computed, time is advanced by Δt (usually taken as 0.1 sec), and the procedure is repeated. In the course of the computation, care is taken to allow for time response lags, to test whether brake lights are on, and so on.

The considerable number of computations and decision tests that must be made require substantial computer time. On the IBM 1620-II computer, 5 sec of computer time is required for each car for each second of equivalent real time driving. Thus a half minute of real time driving for a platoon of six cars requires $5 \times 6 \times 30 = 900$ sec, or 15 min of 1620 time.

In the car-following situation, where things happen very quickly in real life, the length of run is fairly short and 1620 computation times are tolerable, but the longer overtaking studies require the faster speeds of the 6600 computer. The effective speed ratio of the two computers when running this simulation seems to be about 150 to 1.

Whenever the model is revised to any considerable extent, complete detailed data on the behavior of the model are printed out. The numerical values of the velocities, locations, and accelerations for each vehicle are printed at each time step, as well as the response time lags of the drivers, whether they are over or under threshold and whether the brake light is on. These data are used to check the correctness of the programming in detail, and to study the effect of various factors.

For general testing of the model, it has been found more convenient to put out data in graphical form. It is much easier to study the propagation of a disturbance down a platoon of cars by observing the plots of the velocities and the change in relative vehicle spacings than by reading numerical results. Figures 4 and 5 show a computer plot of the response of the platoon of cars to a rapid deceleration of the lead car. The sequential slowing down of each car in line is shown in Figure 4 (the curve marked by the numeral 2 represents the second car in line, etc.), and the change in relative spacing between cars is plotted in Figure 5 (where the curve marked by 2 now represents the relative spacing between the second car and the lead car, etc.). In this example, where T is 1.4 sec, the driving pattern settles down to a stable situation.

Figures 6 and 7 show the same lead car maneuver but carried out in a platoon where the response times of the following cars are so long that an accident occurs (T = 1.8 sec), a collision of cars 3 and 4. Whenever the results of a run show particular interest, as this one does, the numerical data are printed out as well.

Some phase-plane plots of response using velocity vs distance or acceleration vs velocity have been made for individual cars, but they are not included here.

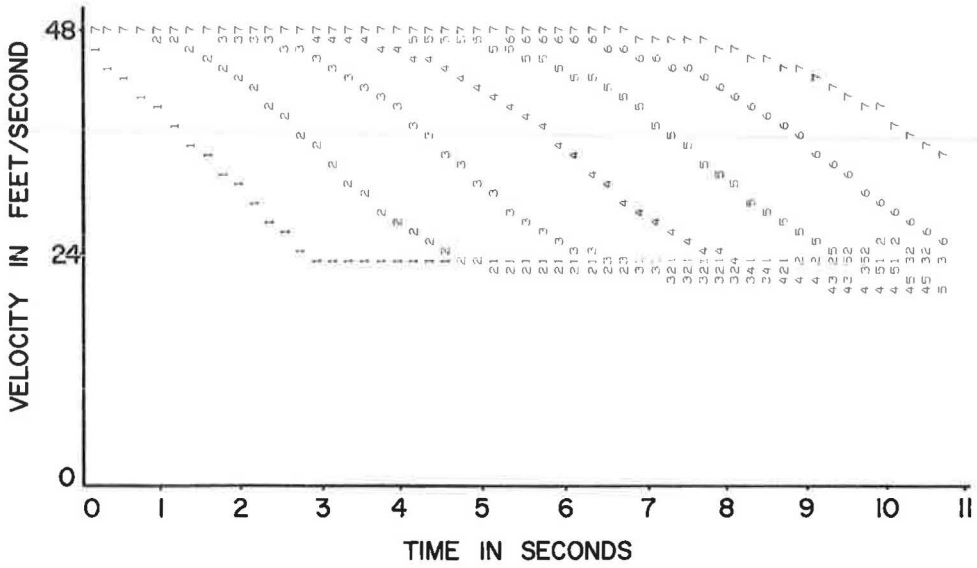

Figure 4. Individual car velocities.

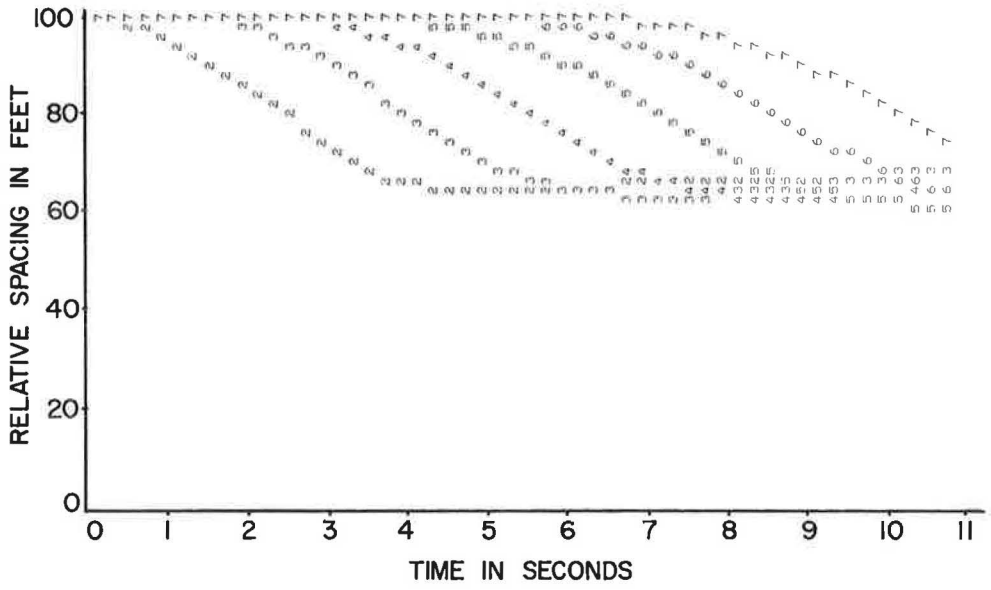

Figure 5. Relative spacing between cars.

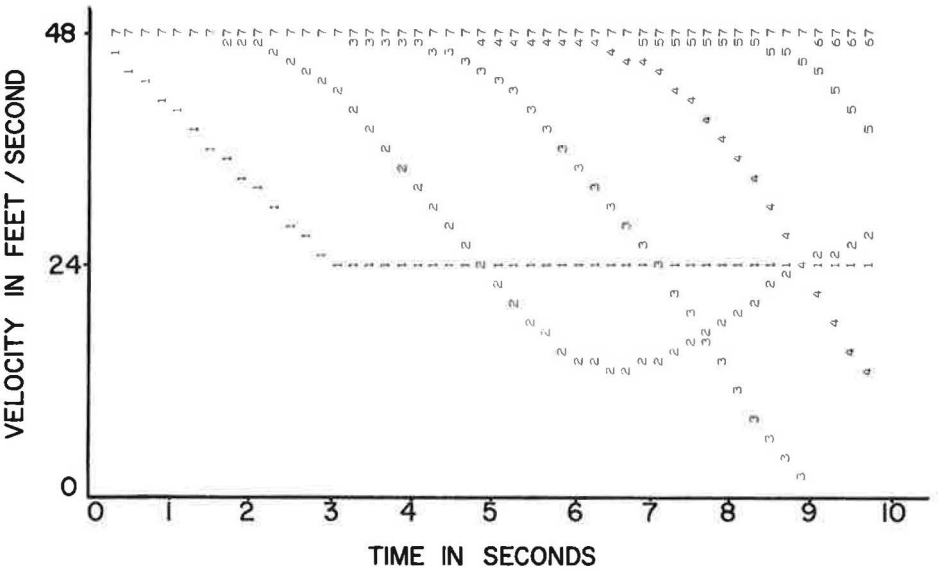

Figure 6. Individual car velocities—collision case.

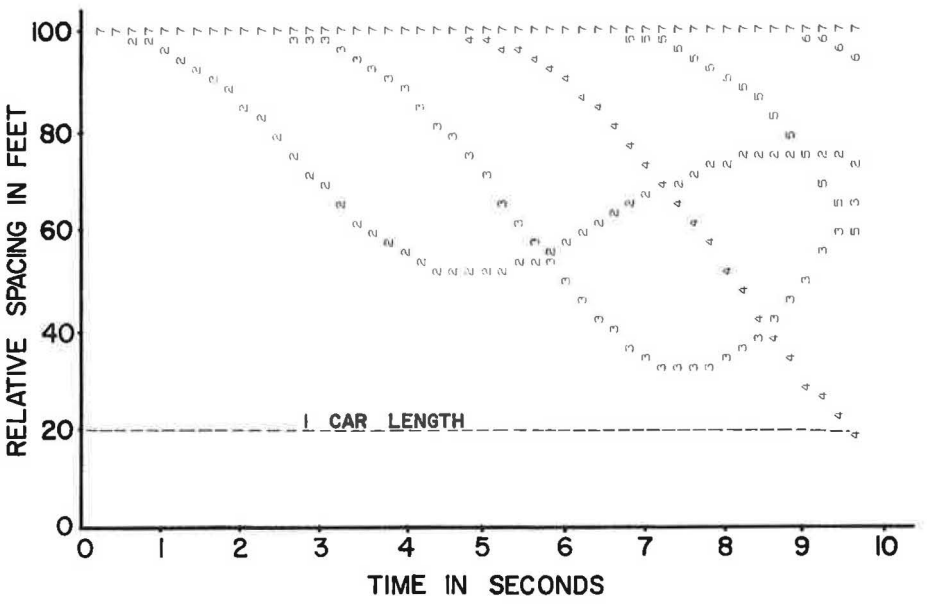

Figure 7. Relative spacing between cars—collision of cars 3 and 4.

RESULTS AND VALIDATION

In the past year, 400 to 500 cases have been run which study various models and various aspects of these models. The responses of the cars in a platoon to different lead car maneuvers have been explored, and the behavior of the model for various ranges of parametric values has been considered.

In the initial stages of the formulation, the aim was to develop a model whose behavior was acceptable on the average, i.e., when the behavior patterns were about average for each driver. It seemed reasonable to assume that only after the basic model was acceptable could the effect of individual driver anomalies on the driving situation be determined.

The investigation of parametric values suitable for representing typical car-following phenomena has supported the essential validity of the model. The approach in validating the model in this way has been an after-the-fact verification. Thus, for a given driving parameter the model is tested over a range of values to find the value interval giving a safe, stable and realistic simulation of driving behavior; the value interval is then checked against values determined by other investigators from actual experimental driving behavior.

This type of quantitative checking has yielded, for example, the following conclusions, supporting the essential correctness of the model: (a) if velocity detection thresholds are much larger than the 0.0005 to 0.0020 radians per sec given by the psychologists (7), the model exhibits instabilities; (b) if maximum decelerations are set too small, collisions occur; (c) if the range of tolerance about desired velocity is too large, oscillations arise; and (d) if the brake lights of the cars ahead are ignored, collisions occur.

A further quantitative check of this type is given by testing reaction times. If reaction times are set longer than the 1.4 sec found empirically by various investigators (3), the platoon behavior is unstable. Figure 8 shows a plot of the minimum spacing between pairs of cars achieved during platoon response to a moderately rapid lead car deceleration of 8 ft per sec. Note that for reaction times, T, of 1.4 sec or less the disturbance does not amplify as it propagates down the line of cars—no pair of cars gets closer than approximately 38 ft. For large reaction times, each pair of cars gets closer, leading to a collision of some pair of cars further along the platoon. At present the behavior of the model seems essentially correct. Both its behavior in a qualitative sense and its agreement with quantitative values are reassuring.

Some work has been done in attempting to validate the model with actual driving data from the Port of New York Authority. However, for each car both the aerial data and tunnel data are available only at 5- or 10-sec intervals, which is very coarse in terms of our model structure and in fact also in terms of the time scales of accident occurrence. It is necessary to have data at 0.5- or 1-sec intervals in order to measure the fine structure of individual driver behavior.

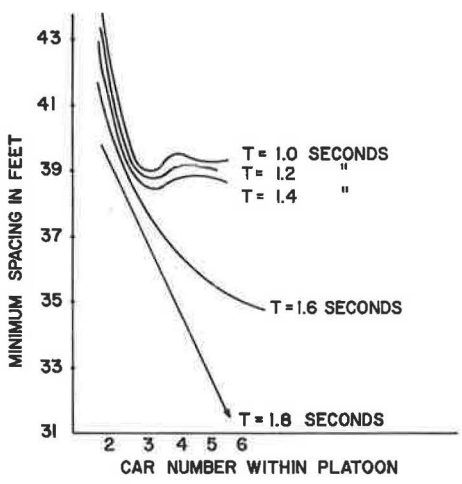

Figure 8. Effect of reaction time.

CONCLUSIONS

This research resulted in the construction of a model representing single-lane no-passing car-following driving situations. Simulation of the model on a computer shows that its behavior, on the average, corresponds very well with whatever data are known experimentally on the car-following problem. Therefore, usefulness of the simulation in projecting the situations and characteristics leading to instability and consequent accident occurrence seems assured.

Concentrated effort to validate the model will continue during the next phase of the study which will emphasize the study of the effect of individual driver differences on the driving situations leading to accident causation, especially in the overtaking case or in the inattentive car-following situation.

ACKNOWLEDGMENTS

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