HIGHWAY RESEARCH RECORD

Number 201

Photogrammetry and Aerial Surveys

6 Reports

Subject Area

21 Photogrammetry

HIGHWAY RESEARCH BOARD

DIVISION OF ENGINEERING   NATIONAL RESEARCH COUNCIL
NATIONAL ACADEMY OF SCIENCES—NATIONAL ACADEMY OF ENGINEERING

Washington, D.C., 1967   Publication 1532

Price: $2.80

Available from

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National Academy of Sciences
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Washington, D.C. 20418

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Foreword

For more than a decade the Photogrammetry and Aerial Surveys Committee has had the cooperation and assistance of numerous persons who have used aerial surveys for getting needed qualitative information and quantitative data to accomplish their engineering work. In addition, such persons have willingly shared their experiences with others by preparing and presenting papers and reports for sponsorship by the Committee. The six papers in this Highway Research RECORD are representative of the continuing contributions by committee members and others which will be helpful to every user of aerial surveys. Moreover, these papers contain the results of research, special analyses, and applications which, when utilized in future work, will contribute to progress by improving techniques and procedures, and by increasing the number of advantages to be gained from using aerial surveys.

The first paper, by Green, reports on use of the Wild Autograph, Model A7. This is a precision analog-type instrument used for (a) photogrammetric bridging to determine the horizontal position and elevation of selected points to serve as supplemental control for making measurements and compiling maps photogrammetrically; (b) measuring accurately the X, Y, and Z coordinates of points on property boundaries to comprise a cadastral survey for use in evaluating property, procuring rights-of-way, and preparing deeds; and (c) measuring and recording profile and cross sections for use in electronic computers to design highways and prepare detailed construction plans.

The second paper, by Chaves, pertains to research in use of a monocomparator to make x and y photographic coordinate measurements. The measurements are used in a computer for determining the X, Y, and Z coordinates of points selected to serve as supplemental control in the photogrammetric compilation of topographic maps and the measurement of profile and cross sections for highway design purposes.

The third paper, by Katibah, reports on research to determine the accuracies achievable and the costs incurred in the photogrammetric measurement of the position of land survey section and quarter-section corners. Several different sets of aerial photographs at two different scales were utilized. Results show that photogrammetric measurements provide adequate accuracy for cadastral surveying purposes to evaluate property, procure rights-of-way, and prepare deed descriptions.

The fourth paper, by McVay, concerns the cadastral surveying accomplished photogrammetrically by the U. S. Forest Service to measure the position of existing and found property corners and to measure the position of recovered or remonumented property corners. In addition, the points were measured from which obliterated and/or lost property corners could be measured and reset on the ground.

The fifth paper, by Rutland, is a report of procedures in the use of precision photogrammetry by the Texas Highway Department for making highway surveys. Included are the use of precision photogrammetric instruments, electronic distance-measuring instruments, precision angle-measuring theodolities, measurement on stereoscopic models of profile and cross sections, and automatic recording of the measurements for use in computers.

The sixth paper, by Pryor, pertains to the achievement of precision in surveys by the use of plane coordinates. Emphasis is given to the basic principles and procedures of adjusting the datum by zones and selected geographic areas of the State plane coordinate systems—whether a transverse Mercator or a Lambert conformal—so that distances determined from plane coordinates of points and details of maps compiled on an adjusted datum will agree within practicable limits with distances accurately measured between the same points on the ground. The techniques by which the topography of a plane coordinate zone is analyzed and an appropriate datum adjustment factor is determined on an area adjustment basis are presented.

—William T. Pryor

Contents

Application of the Wild Autograph, Model A7, in Georgia

ERNEST GREEN, Photogrammetrist, State Highway Department of Georgia

This report contains a general description of the installation and operation of a Wild A7 Autograph and a summary of results for the first year of operation in the State Highway Department of Georgia. A detailed description of aerial triangulation and a discussion of other uses of the A7 are included.

The A7 has proved to be a fast, efficient means of providing supplementary ground control. In addition, its versatility has opened new opportunities for types of projects never before possible.

•WITHIN the State Highway Department of Georgia, the cost of field control represents the largest single item in the yearly photogrammetric budget; therefore, a savings in this area could significantly reduce the overall cost of mapping. To answer this need, the Division of Surveys and Aerial Mapping purchased a Wild Autograph, Model A7 (Fig. 1), primarily for measuring supplemental horizontal and vertical control to be used in photogrammetric mapping.

The A7 was delivered in September of 1964, accompanied by two Wild technicians, who spent the ensuing eight days assembling and calibrating the instrument. Using a 30 X binocular attachment, the instrument was visually calibrated to within three microns deviation from true position on a grid plate.

In the middle of October, an engineering consultant from the Photogrammetric Division of Wild Heerbrugg arrived to train personnel in its operation. Problems which

Figure 1. Wild Autograph, Model A7.

Paper sponsored by Committee on Photogrammetry and Aerial Surveys and presented at the 46th Annual Meeting.

were encountered were only minor in nature. A new relay and cable were needed for connection of the 026 card punch to the EK3D electrical coordinate printer. The absence of major difficulties in making this system operational attest to the skill and precision of the Wild technicians who assembled it.

TRAINING

During the period of final adjustment, the training of operators was already in progress. Two experienced Kelsh plotter operators were selected for this instruction, which began with the nomenclature of the equipment and its care and maintenance. The next step was an introduction to the theoretical aspects of the A7, including the fundamentals of stereotriangulation and the theory and mathematics of photogrammetry. This indoctrination gave the trainees a good foundation for the practical instructions which followed.

The actual operation of the A7 was taught in a series of steps, beginning with the internal and external orientation of a stereoscopic model which involves the resolution of Y-parallax, scaling, and leveling. Then the procedure for tying a correctly oriented second model to the first model was taught. Once this technique was mastered, the operator had all of the basic knowledge necessary to accomplish stereoscopic bridging because further extension of a photogrammetric bridge is a matter of repeating the operation until the end of the bridge is reached.

The remainder of the training period was spent learning and practicing the methods of producing photogrammetric control: the selection and distribution of the necessary field control, the gridding of compilation sheets with the coordinatograph, the drawing of control sketch maps, and the programming of the EK3D system and the 026 card punch.

At a later date, a PUG3 point marking and transfer instrument was purchased (Fig. 2). The PUG3, although simple in operation, is extremely valuable when used in conjunction with the A7. It drills a small hole in the emulsion of the diapositive and circles it with a grease pencil. These holes are used as control points which eliminate the possibility of misidentification by the operators of the Kelsh and the A7 instruments. In addition, the PUG3 can be used to transfer accurately these control points from flight strip to flight strip or stereoscopic model to stereoscopic model.

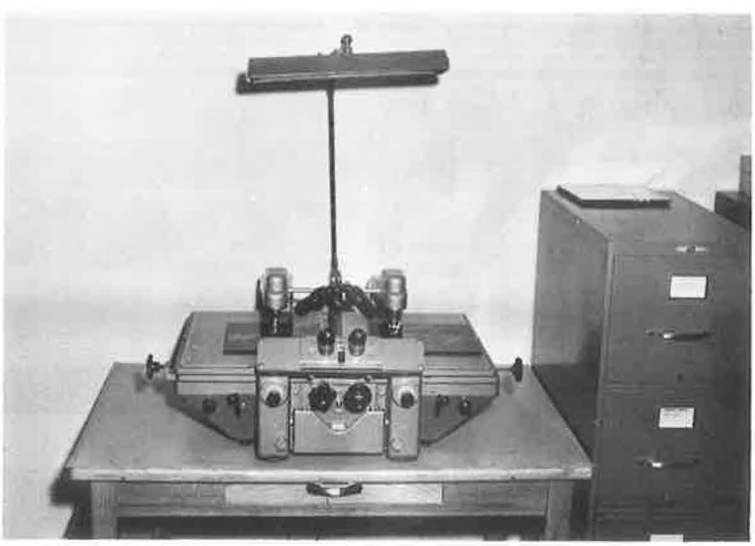

Figure 2. PUG3.

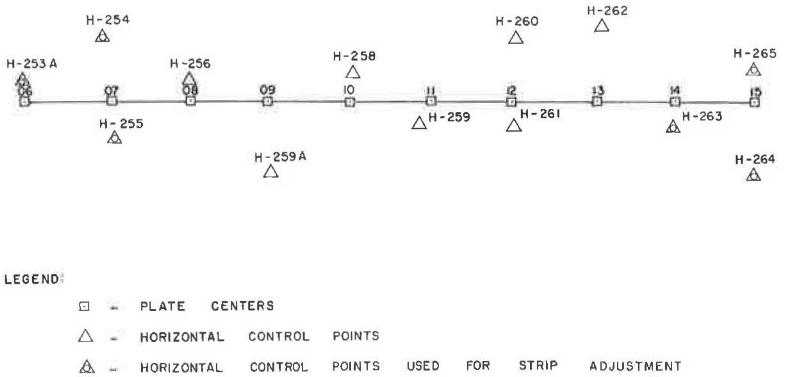

Figure 3. Distribution of horizontal control.

TEST AREA

A nine-model strip of aerial photographs at a scale of 500 ft/in. was selected for training the instrument operators and for testing the accuracy of the stereotriangulation program. This area was fully controlled, originally for mapping purposes; therefore, the placement of control was less than optimum for bridging (Figs. 3, 4).

A comparison was made between the horizontal positions and elevations determined by stereotriangulation and by conventional survey methods. The root mean square error in the horizontal was 1.34 ft, in the vertical 0.45 ft. The bridge spanned a distance of 11,560 ft.

A portion of the derived root mean square errors was probably caused by the distribution of control points and by the operator's lack of experience. It was concluded from this test, however, that all equipment was functioning correctly and that the two operators had performed the stereotriangulation in a precise manner.

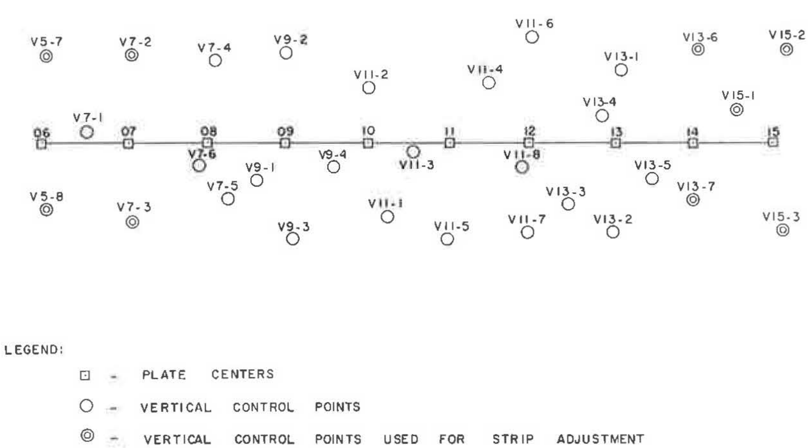

Figure 4. Distribution of vertical control points.

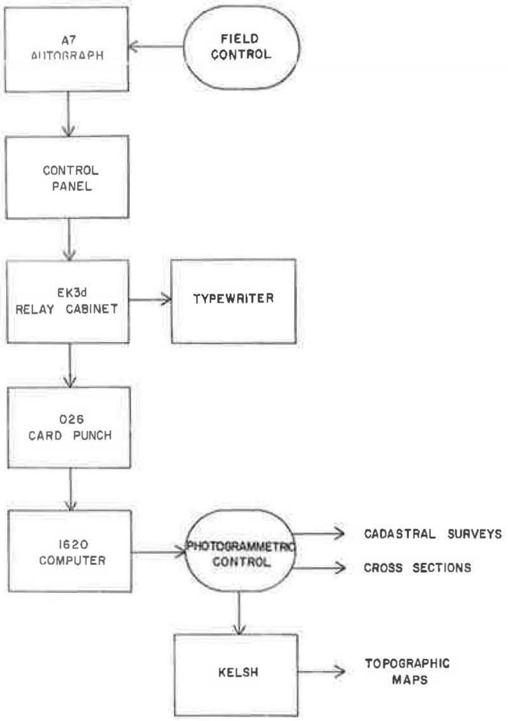

Figure 5. Production flow chart.

PRODUCTION OPERATION

Having completed the initial phases, the A7 was put on a full-time production schedule. During the next ten months, 497 stereoscopic models were bridged. Several problems cropped up during this period.

The first aerotriangulation adjustment program used evidently still needed to be debugged, as it was rejected by the computer and never worked satisfactorily. At this point, a change was made in the program.

Errors have occurred because of failure to enter an earth curvature factor when working with small-scale photography (1000 ft/in. or smaller). It is tempting to disregared this correction because the computer program is entirely separate and unusually time-consuming. When bridging with photography of 500 ft/in. scale, the insignificance of the correction makes it uneconomical to apply, but if the scale of photography is much smaller than this, the greater distance covered by a bridge introduces an objectionable amount of error.

For a continuing check of accuracy, two separate methods are used. When access is not difficult, additional control points are surveyed by the field party and withheld from the input data. In addition to this, profiles are measured in the field to check the map manuscript compilation. Through the use of these safeguards, a satisfactory standard of quality is maintained (Fig. 5).

CADASTRAL SURVEYS

As use of the A7 continued, a new facet of its value became apparent. Not only does it provide a fast, accurate, and economical method of obtaining ground control, but it is so efficient that in photogrammetric bridging work it had no trouble in outdistancing the map compilation using Kelsh plotters. As this gap widened, new areas of use were sought. It was at this point that the instrument's versatility became of value.

One of the earliest uses, other than stereotriangulation to which the A7 was put, was measuring the plane coordinate position of property corners for the Right-of-Way Department. Even this program is expanding, however, and should produce even more sophisticated results in the future.

With the computer program currently in use, the Right-of-Way Department is provided with the parcel number, the coordinates for each property corner, the bearings and distances between continuous corners, and the total area in acres and in square feet.

With X and Y coordinates for all property corners and X and Y coordinates of the P. I.'s, the Division Right-of-Way Department can, through use of a simple computer program, find the intersection of any and all lines and all acreages necessary to complete a right-of-way map.

CROSS SECTIONS

The A7 has completed measurement of cross sections for several survey projects and has proved to be an excellent instrument for this purpose. While stereoscopically viewing the image of the ground, the instrument operator energizes the EK3D electrical coordinate printer, which activates the 026 card punch. An IBM punch card is then automatically prepared for the IBM 1620 computer. By passing the data electronically from the A7 to the computer, the possibility of human error in transcribing long figures by hand is virtually eliminated.

Using a profiloscope in the procedure of punching cross section on cards makes a very efficient operation. The plotted centerline can be observed on the scope by the operator seated in the viewing position of the A7, eliminating the need for a table man. The cross section lines and distances are seen on the scope also.

The cross section conversion program converts the machine coordinates to ground coordinates which become the input data for the earthwork program the Highway Department has been using very satisfactorily in the past.

A list of all projects stereotriangulated from January 1, 1965, to September 30, 1965, is given in the Appendix. The final column shows the root mean square error (RMSE) for each set of coordinates.

FUTURE OPERATIONS

At the present time, planning is under way for construction of a building to house the photogrammetry department under one roof. The department presently occupies two buildings. The new building will double the present floor space, which will be needed for additional equipment. The new avenues opened up by the introduction of the A7 to measure cross sections and make property surveys has brought on the need for an addition of four Wild Autographs, Model A8, to enable the department to meet the demands of all six Division offices in the state. The A7 is currently working two shifts per day and is unable to keep up with the demands of one Division, in addition to the department's normal mapping program, which depends on the A7 for bridged control.

Procurement of a data plotter is also included in the department's 1967-68 budget. This equipment will have an integral part in the preparation of property right-of-way maps and in the plotting of cross sections.

Space in the new building will be allotted for this additional equipment. A communication system, 1050 series, will also be installed connecting the Aerial Surveys Department online to the computer located in the State Highway Department General Office in Atlanta. The computer is the IBM 360 system, which is to be in operation in early 1967.

Cadastral Surveys

The output of the present computer program in use is in the process of being increased to include X and Y coordinates on the intersection of highway right-of-way and property lines, station numbers, and offset from centerline. The remainder of acreage in each parcel of land to the right and left of the right-of-way will be given, besides acreage taken.

Digitized Terrain Model

With the stereoplotter linked directly to the computer, many new possibilities are now open to the photogrammetrist. One of these is the digitized terrain model.

In the DTM system, an arbitrary X-axis is selected and cross section data are punched in along measurement lines at right angles to this axis. This, in effect, places the stereoscopic model inside the computer. From this point, any number of centerlines can be considered and earthwork quantities computed. The computer, in fact, can select the best line from the topographic standpoint. Through the other programs of the DTM system, this line can be computed on every axis, X-Y, X-Z, Y-Z. With the addition of an automatic plotting device, these views can be drawn as profiles. With proper programming, much of the work in location and design can be accomplished by the computer.

The future for this type of system appears unlimited. Information such as land value, soil and geologic conditions, and design criteria could be entered into the computer, which could then make the best possible highway location, free of prejudice and human error in judgment. The A7 is ideally suited for instituting a system of this type.

CONCLUSION

The Wild Autograph, Model A7, has proved to be a fast, efficient means of providing supplementary ground control. In addition, its versatility has opened new opportunities for types of survey projects never before possible.

In the economic sense, it is paying for itself. In 1964, field survey expense represented 49.20 percent of the total photogrammetric operating expense. In 1965, after one year of use, the A7 had reduced this cost to only 34.25 percent. Actually, these figures do not present an accurate comparison of field survey cost. Since purchase of the A7, the field crews have been surveying for purposes other than topographic mapping, such as control points for photogrammetrically measuring cross sections and the X and Y coordinates of property corners by use of the A7. Consequently, this means that more work is being accomplished with a smaller budget.

Appendix

STEREOTRIANGULATED PROJECTS, JANUARY 1-SEPTEMBER 30, 1965

Bridging

Job	No. Models	Photog. Scale	Total Distance	RMSE	
				X & Y	Z
SR No. 5	6	1-500	10,800	(0.86)	(0.01)
I-75	10	1-500	18,000	(0.45)	(0.39)
I-75	7	1-250	6,300	(0.48)	(0.18)
I-485	8	1-250	7,200	(0.45)	(0.24)
I-75	16	1-500	28,800	(1.03)	(0.65)
I-75	12	1-250	10,800	(0.61)	(0.20)
I-85	12	1-250	10,800	(0.34)	(0.19)
I-485	6	1-250	5,400	(0.68)	(0.03)
I-485	9	1-500	16,200	(0.53)	(0.44)
I-75	11	1-500	19,800	(2.04)	(0.29)
Columbus, 3rd Ave.	18	1-250	16,200	(1.02)	(0.13)
City of Gainesville	12	1-500	21,600	(1.32)	(0.35)
I-485	5	1-250	4,500	(0.71)	(0.18)
City of Gainesville	12	1-500	21,600	(1.36)	(0.35)
City of Gainesville	21	1-500	37,800	(1.11)	(0.18)
I-16	8	1-500	14,400	(0.88)	(0.41)
Briarcliff Rd.	6	1-500	10,800	(0.89)	(0.06)
Sugar Hill	10	1-500	18,000	(1.10)	(0.19)
I-16	19	1-500	34,200	(3.14)	(0.61)
I-16	10	1-500	18,000	(1.08)	(0.58)
I-16	11	1-500	19,800	(1.13)	(0.43)
I-16	17	1-500	30,600	(1.53)	(0.56)
I-95	14	1-500	25,200	(3.43)	(0.42)
Douglas Bypass	4	1-500	7,200	(0.55)	(0.12)
Douglas Bypass	7	1-500	12,600	(0.57)	(0.23)
I-485-F-056	13	1-500	23,400	(1.42)	(0.18)
I-485-F-056	12	1-500	21,600	(1.08)	(0.29)
I-485-F-056	12	1-500	21,600	(0.47)	(0.33)
I-16	5	1-500	9,000	(0.49)	(0.43)
I-16	20	1-500	36,000	(1.36)	(0.50)
Brunswick East Bypass	4	1-500	7,200	(0.32)	(0.09)
Brunswick East Bypass	11	1-500	19,800	(0.64)	(0.33)
I-485	5	1-250	4,500	(0.62)	(0.08)
Jasper to Fairmount	23	1-1000	82,800	(1.54)	(0.71)
Jasper to Fairmount	12	1-1000	43,200	(3.83)	(0.93)
Jasper to Fairmount	8	1-1000	28,800	(3.57)	(1.14)
Memorial Drive	8	1-250	7,200	(0.33)	(0.01)

Cross Sections

I-485 cross sections at each 100-ft interval across the centerline from Station 121+00 thru 285+00, ranging in length from 600 to 800 ft per station.

Memorial Drive cross sections at each 50-ft interval across the centerline from Station 77+00 thru 319+50, ranging in length from 300 to 500 ft per station, and Station 19+50 to 59+50, in length ranging from 300 to 500 ft per station.

Property Corners (X & Y Coordinates)

Job	No. Models	Photog. Scale	No. Corners	Total Distance
I-75 Clayton County	14	1-500	121	25,600
I-485 Fulton County	2	1-250	131	1,800

Aerial Analytic Triangulation Investigation on Interstate 80 in Wyoming

JESSE R. CHAVES, Aerial Surveys Branch, Highway Standards and Design Division, U. S. Bureau of Public Roads.

Results are reported of an investigation of aerial analytic triangulation along a 20,000-foot segment of Interstate Highway 80 in Wyoming. Eleven aerial photographs taken at a scale of 1:6000 were analytically bridged using x and y coordinates measured on glass plate transparencies of the photographs with a Nistri monocomparator. A Wild PUG point transfer and marking instrument was also utilized.

The mathematical method of analytic triangulation employed is a modified version of the one originally developed by the U. S. Coast and Geodetic Survey. The system consists of four separate parts: plate coordinate refinement, relative orientation, cantilever assembly, and cantilever strip adjustment. This analytic method was programmed in Fortran language and the computations were made with the IBM 1401 and 7010 electronic computer systems.

CONVENTIONAL analog aerial triangulation with first-order bridging instruments and ground control surveys are employed as a means of providing the ground position and elevation of points (supplemental control) needed for the absolute orientation of stereoscopic models in photogrammetric instruments. Large-scale topographic maps with a small contour interval can then be compiled for use by highway location and design engineers. With the introduction of electronic digital computers, analytic (mathematical) bridging became feasible as a means of providing the supplemental control for small-scale mapping purposes. Although well understood and initiated by a few leading engineers before 1960, use of the analytic approach to bridge control for highway engineering mapping has only recently attracted the attention of several highway organizations. There is need, therefore, to evaluate the analytic approach for extending surveyed ground control in order to determine the accuracies which can be attained through use of primarily available photogrammetric equipment and materials in highway organizations.

The investigation of analytic aerial triangulation for highways was justified for these reasons:

1. Conventional ground control surveys are expensive, time consuming, and difficult to accomplish under some circumstances. Ground control extended analytically would minimize the number of points whose position and elevation would otherwise have to be surveyed on the ground.
2. Electronic digital computers are available in most highway organizations.
3. The comparator required to measure x and y photographic plate coordinates for the analytic method can be procured at much less cost than conventional analog optical train bridging instruments.
4. Training requirements for the successful operation of comparators used to measure the x and y coordinates of image points on glass plate transparencies of the aerial

Paper sponsored by Committee on Photogrammetry and Aerial Surveys and presented at the 46th Annual Meeting.

photographs are much less than they are for operation of the optical train bridging instruments.

5. The analytic method of aerial triangulation has the potential accuracy required to accomplish mapping photogrammetrically for highway location and design and offers greater flexibility than the conventional analog types of bridging instruments.

The mathematical procedures employed for this particular evaluation of analytical photogrammetry were developed by the U.S. Coast and Geodetic Survey (1). [This method of relative orientation and cantilever assembly has now been replaced by a method called "Three-Photo Aerotriangulation" (2).] The method with modifications was programmed in the Fortran language and a preliminary investigation was made in 1964 on an 18,000-ft segment of Interstate Highway 66 in Fairfax County, Virginia (3). Results of this work were reported in which seven photographs, taken with a Wild 6-in. focal length aerial camera at a scale of 1:8400 were analytically bridged (4). A monocular comparator was used to measure x and y coordinates of natural images and targeted points on the photographic glass plate transparencies of the aerial photographs. Second-degree cantilever strip adjustment using three horizontal and six vertical control points yielded root-mean-square errors on test points of 0.41 ft for the horizontal and 0.71 ft for the vertical ground coordinates.

Encouraged by the results of this preliminary work, this investigation was begun with the following major objectives:

1. To analytically bridge 11 photographs (10 stereoscopic models) taken at a scale of 1:6000 (500 ft to 1 in.) with a 6-in. focal length aerial camera;
2. To evaluate the effect of analytically bridging photographs which have been drilled with a Wild PUG point transfer instrument;
3. To develop computer programs written in the Fortran language to reduce coordinates of image points on the photographic glass plates which have side fiducial marks, and apply a polynomial curve-fitting technique to compensate for the effects of radial lens distortion;
4. To determine the density and distribution of ground control needed for adequately adjusting a strip of 10 stereoscopic models;
5. To determine the degree of strip adjustment needed for a strip of 10 stereoscopic models;
6. To analyze photographic materials and photogrammetric instruments, equipment, and methods as sources of error in the analytic system of bridging;
7. To revise an existing cantilever adjustment program originally written for use on an IBM 1401 for the IBM 7010 system; and
8. To make recommendations for improving and implementing the analytic method and to suggest research needed in this field.

INSTRUMENTS, EQUIPMENT, MATERIALS, AND PROCEDURES

Aerial Camera and Photography

Eleven aerial photographs at a scale of 1:6000 were selected from a flight strip taken in July 1963 by Continental Engineers, Inc., for mapping a corridor for Interstate Highway 80 between Green River and Rock Springs in southwest Wyoming. The eleven photograhps utilized covered a strip of topography approximately 4,500 ft wide and 20,000 ft long, having a light-to-moderate brush cover. The photographs were taken from an average flight height of 3,000 ft with a Zeiss RMK A 15/23 aerial camera equipped with a Pleogon lens having a calibrated focal length of 152.45 mm ± 0.02 mm and a maximum aperture of f/5.6. The average value of radial lens distortion based on determinations made on two radii does not exceed ± 5 microns (see Fig. 4). The distortion values have been determined within an accuracy of two microns. The distance between the fiducial marks in both directions is 226.00 mm ± 0.02 mm.

The negative film used had an estar base from which diapositive plates (Kodak Aerographic Positive Plates, Improved, 0.25 in. thick) and photographic prints were made using a LogEtronics CP 18 automatic dodging printer.

Photograph Preparation and Image Selection

The image points used in the triangulation experiment were those for which ground control data were available. These control points had been surveyed for use in compiling topographic maps of a corridor along Interstate 80 in Wyoming. The points measured were images of targets and images of natural objects which were selected in accordance with the mapping needs of the project. Pass points for each stereoscopic model were selected in the usual rectangular pattern in the six classical locations. Two or three additional points were selected in the area of triple overlap of the photographs of each two adjacent stereoscopic models in order to insure that a sufficient number of acceptable points were available for scale adjusting one stereoscopic model to another in the cantilever assembly.

Drilling and Measuring

All image points, targeted and natural, used in the triangulation were predrilled with a Wild PUG3 point transfer instrument equipped with drills having a diameter of 60 microns.

The x and y coordinate measurements were made with the Nistri Monocomparator, Model TA1/P, provided with both digital readout and typewritten outputs (Fig. 1). The comparator had been calibrated a few months before measurements were made. The least reading on this comparator is 1 micron. The diapositive plates were measured with the emulsion side down under a 10 X magnification. The objective lens on this particular instrument was equipped with a 20-micron measuring mark. Measuring marks of other sizes are available from the manufacturer of the instrument. The coordinate output of this particular comparator was in a left-handed system. Provision was made in the coordinate reduction program to change the coordinate system into a right-handed system, whereby all values increased along the y-axis away from the observer and to the right along the x-axis. A simple wiring modification can be made at the factory to produce output directly in the right-handed system. Measurements were made in an air-conditioned room at 72 F. Periodic checks were made for possible instability. The instrument and accessory equipment exhibited excellent stability throughout the measurement operations. It took about one hour to measure an average of 25

Figure 1. Nistri Monocular Comparator, Model TA1/P, and accessory equipment.

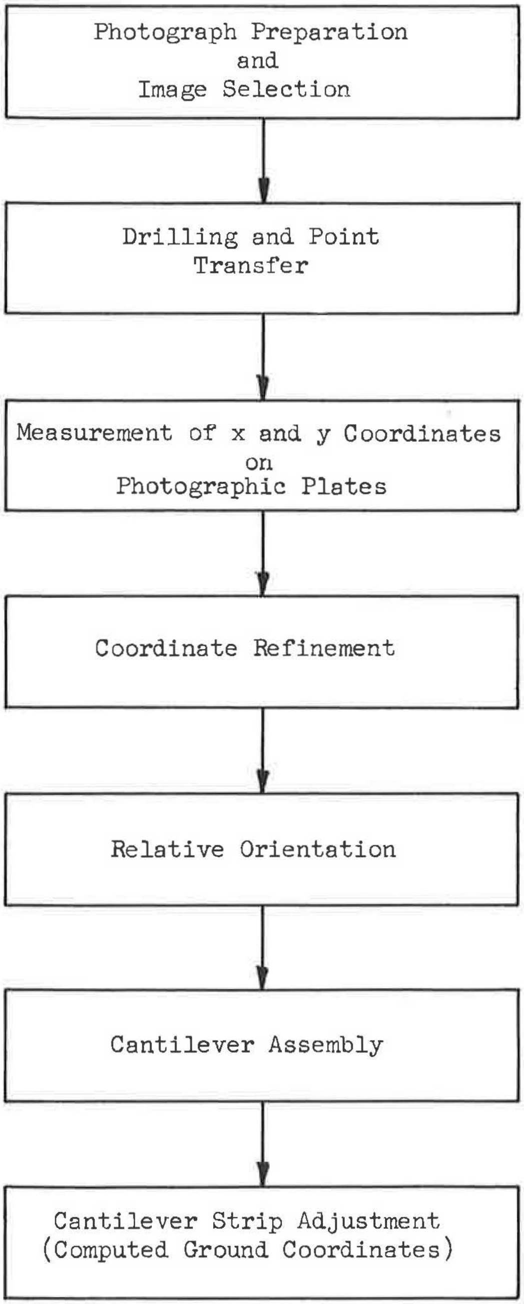

Figure 2. Flow chart of aerial analytic triangulation.

drilled holes per photographic plate and the 4 fiducial marks. Each of the drilled holes was measured 3 times, and each of the fiducial marks, 6 times. The mean of these measurements was accepted as "true" x and y coordinates for each point. The measured points were always approached with the measuring mark from the same direction to avoid the possibility of screw backlash, although screw backlash was found to be only 2 or 3 microns in magnitude.

Computers

Two electronic digital computers were used for making the mathematical computations of the analytic bridge. The cantilever strip adjustment program, which yields the X, Y, and Z ground coordinates of each measured point, was used in an IBM 7010 computer having a 60K digital storage capacity, while all other programs were used in the IBM 1401 with a 12K digital core memory.

Control Survey

Basic ground control was surveyed in a closed traverse approaching second-order accuracy. Points identified in the Appendix with the prefix SW were included in this travese. All other ground-surveyed points are assumed to be of at least third-order accuracy, although no survey closure checks were actually made. The surveying was accomplished using the Electrotape and Tellurometer electronic distance-measuring instruments, and a Wild T-2 Theodolite.

Computations

Figure 2 shows a generalized flow chart of analytic aerial triangulation used in this investigation. The following sections describe the basic computational concepts and procedures used.

Coordinate Refinement

In order to render the measured x and y coordinates suitable for performing the analytic triangulation, certain mathematical operations are required. They are: averaging of each set of coordinate measurements; conversion from a left-handed to a right-handed coordinate system; mathematical translation and rotation of the measured photographic plate coordinates; film deformation compenstaion; and radial lens distortion correction. These computational items are discussed in subsequent sections.

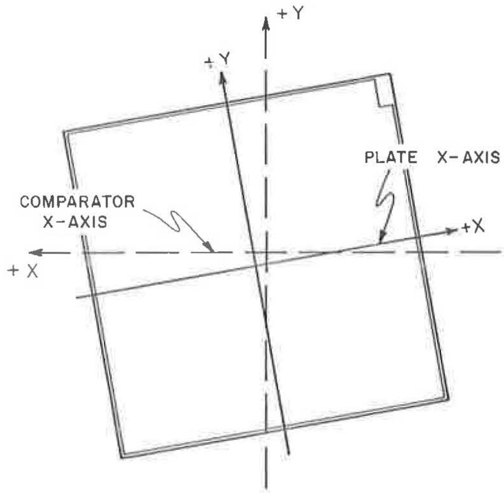

Figure 3. Relationship of diapositive plate to comparator axes at time of measuring.

Plate Coordinate Averages

The measurements made with the monocomparator are recorded in typed form. Card punching was performed directly from the typewritten record of the comparator measurements. Average values for the three measurements made on each of the image points (drilled holes) and six measurements made on each of the fiducial marks were computed with two separate computer programs. In these programs, the comparator measurements of the left-handed coordinate system are converted to a right-handed system by subtracting all x coordinates from an arbitrary constant of sufficient magnitude.

Plate Translation and Rotation

When diapositive plates are placed on the comparator stage for measurement, the x and y axes of the photographic plate must be physically oriented parallel to the respective axes of the comparator, or some means must be provided for mathematical rotation so the two coordinate systems are coincident (Fig. 3). The orientation of a plate so that its axes are precisely parallel to the comparator axes is a time-consuming procedure. Consequently it is expedient to place the plate on the comparator so that their respective axes are more or less parallel, and then mathematically translate and rotate them to coincidence based on the instrument-measured coordinates of the measured fiducials. The mathematical rotations are accomplished by using standard rotation equations from analytic geometry. The resulting translated and rotated coordinates for all points measured on each photographic plate are then referenced to the principal point of the photograph and are ready for further treatment.

Film Deformation Compensation

Plastic films are subject to dimensional change between the time of photographic exposure in the aerial camera and printing the diapositives on optically flat glass plates. Therefore, some means of compensating for the movement of images is necessary. For cameras equipped only with 4 side fiducial marks, the only feasible means of compensation is to compare the distances between the marks in 2 directions on the printed diapositive with those of the aerial camera itself. This distance between fiducial marks in the aerial camera may either be furnished by the manufacturer or measured on a diapositive plate (flash plate) previously exposed directly to the aerial camera. This method of compensation for film deformation was utilized even though it is recognized as being inadequate. Two scale factors were developed for each plate based on the distances between the fiducial marks reported by the manufacturer and those determined for each plate in the x and y directions. The x and y coordinates of all measured points were then multiplied by the respective film deformation correction factors.

Radial Lens Distortion Correction

Figure 4 shows the average radial lens distortion curve for the Zeiss Pleogon lens. Positive values of lens distortion result in the displacement of an image radially outward from the center of the photograph; for negative values the displacement is radially inward. Corrections for this displacement must be made to the x and y coordinates of each measured point on the photograph.

An equation for the lens distortion curve shown in Figure 4 was determined by means of a polynomial curve-fitting program. This program generates an approximating

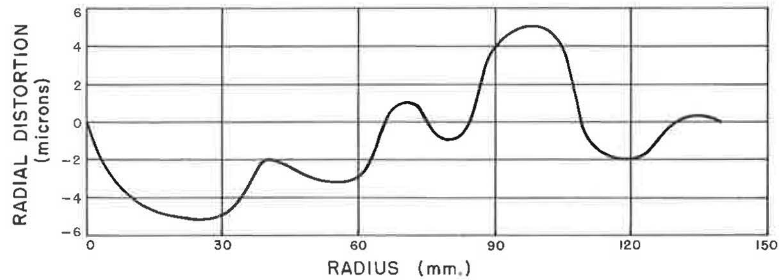

Figure 4. Average radial distortion curve for Zeiss Pleogon lens.

polynomial using the least squares technique. Coefficient terms of the polynomial curve are determined which are used to compute the amount of radial lens distortion for any given radial distance from the center of the photograph. Actual computation of the distortion is accomplished by means of another short program using the appropriate coefficients. The x and y coordinates of each point are then corrected for the effects of image displacement caused by lens distortion. This particular distortion curve had numerous points of inflection. Consequently, the curve was divided into two segments, 0 to 70 mm and 70 to 140 mm, in order to obtain sufficiently accurate polynomials. An equation for each of the two segments of the total curve was determined using radii and distortion data for 15 and 30 points, respectively, as electronic computer input data.

Relative Orientation

Relative orientation may be defined as reconstruction of the perspective conditions existing between a pair of photographs when they were taken (5). It actually consists in determining three rotational (omega, phi, kappa) and two of three translational (bx, by, bz) elements which define the attitude and positions of one photograph with respect to another, providing there is a sufficient common area of overlap in line of flight.

The method of relative orientation (1) used here depends upon enforcing a geometric condition where the photographic image, perspective center, and the object in the stereoscopic model are on a straight line (colinear). For a given stereoscopic model, there are a number of such lines (a pair for each image), but, because of various errors, it is impossible to enforce all colinear conditions completely. Since the left-hand photograph of a pair is assumed to have no tilt in this method of analytic relative orientation, small corrections are allowed to be made to the measured x and y coordinates of the right-hand photograph in a least squares manner. Each stereoscopic model was oriented independently using 12 image points, and required three computer iterations for completion of the relative orientation. The third and usually last iteration yielded x, y, and z coordinates for each point on the stereoscopic model and orientation data, which were utilized in the assembly of independently oriented stereoscopic models to form a strip. The lack of intersection of pairs of lines (corresponding rays) for all the image points in each stereoscopic model were printed out as residual y-parallaxes. These values were reviewed at the completion of the orientation. Substitutions were made for points having unusually high residuals, and a new orientation was performed.

Cantilever Assembly

After completing successively the relative orientation for each of the stereoscopic models, the individual models were "tied together" into a continuous strip. This assembly of models is accomplished by successive mathematical transformation of the model coordinates of each model in the strip. The three transformations, which were performed in order, are rotation, scaling, and translation. These mathematical computations yielded the strip coordinates. (The strip coordinates are analogous to those

TABLE 1

SCALE FACTORS USED TO COMPENSATE FOR FILM DEFORMATION

Photographic Plate No.	X-Axis	Y-Axis
1-40	0.999646	0.999646
1-41	0.999602	0.999611
1-42	0.999690	0.999690
1-43	0.999717	0.999788
1-44	0.999673	0.999805
1-45	0.999712	0.999797
1-46	0.999699	0.999814
1-47	0.999646	0.999735
1-48	0.999717	0.999761
1-49	0.999673	0.999766
1-50	0.999602	0.999797

TABLE 2

AVERAGE ABSOLUTE RESIDUAL Y PARALLAX FROM RELATIVE ORIENTATION

Model Number	Y Parallax (microns)
1	5.9
2	8.8
3	9.6
4	5.0
5	6.0
6	8.2
7	9.4
8	7.7
9	7.7
10	9.7

obtained from an analog triangulation instrument.) The first stereoscopic model in the strip was arbitrarily considered to be at the desired scale and its coordinates in the proper system. Therefore, the mathematical transformations were performed only on the second and succeeding models. Scaling in this cantilever system was accomplished by comparing slope distances between two image points, which occur in adjacent stereoscopic models (the common overlap area of three photographs), and then adjusting the model being attached to another by means of a scale factor.

Adjustment of Cantilever Strip Coordinates

The adjustment of cantilever strip coordinates is the last computational step which yields the X, Y, and Z ground coordinates desired. The mathematical method of strip adjustment used in this investigation is described elsewhere (6). The adjustment is fully applicable to strip coordinates derived from either analog optical train photogrammetric bridging instruments or to coordinates derived from measurements made with comparators. This method of adjustment attempts to correct for curvature of the strip (azimuth), twist or cross tilt, BZ fall off, scale change along the x and y axes of the strip, and local tilt of the strip in the x and y directions. Cumulative errors in a strip tend to be systematic and can be corrected by polynomials. Both second- and third-degree polynomial adjustments were applied to determine their effectiveness.

TABLE 3

SCALE FACTORS USED IN CANTILEVER ASSEMBLY

Models	Scale Factors
1 and 2	0.91743098
2 and 3	0.96043167 0.95981338
3 and 4	0.95736429
4 and 5	0.91459298 0.91491979
5 and 6	0.95755554 0.95778659
6 and 7	0.91712976
7 and 8	0.91668318 0.91630765
8 and 9	0.91052712 0.91084330
9 and 10	0.90574928

The following data are required as input in order to accomplish the vertical and horizontal adjustment of the strip coordinates:

1. A card containing the number of vertical and the number of horizontal ground control points used in the adjustment;
2. One card containing the x and y cantilever strip coordinates of a point near the center of the first stereoscopic model and another point near the center of the last model in the strip;
3. The strip x, y, and z coordinates of all points to be used as a basis for the

TABLE 4
COMPUTED STRIP COORDINATES OF POINTS IN TRIPLE OVERLAP AREAS

Point No.	X	Y	Z
1-40-B	0.98092 0.98092	0.22099 0.22100	-1.60790 -1.60809
1-42-B	0.94122 0.94121	0.71363 0.71364	-1.59301 -1.59322
1-42-L	1.93575 1.93575	0.60043 0.60049	-1.49853 -1.49885
1-42-K	1.94781 1.94780	0.14704 0.14712	-1.57810 -1.57766
1-42-D	1.93855 1.93855	-0.49565 -0.49581	-1.54782 -1.54813
1-42-G	2.88542 2.88542	-0.17437 -0.17442	-1.54318 -1.54327
1-42-J	2.92091 2.92092	0.19175 0.19172	-1.52242 -1.52299
1-44-F	3.81770 3.81770	0.81302 0.81318	-1.51911 -1.51931
42-2	3.82037 3.82037	-0.14599 -0.14601	-1.53610 -1.53600
1-44-D	3.80635 3.80636	-0.64514 -0.64507	-1.53752 -1.53737
1-44-K	4.78011 4.78013	0.59229 0.59277	-1.49192 -1.49278
1-44-G	4.79534 4.79537	-0.01733 -0.01719	-1.51885 -1.51975
41-2	5.67728 5.67728	0.11931 0.11932	-1.49878 -1.49888
1-48-F	7.53923 7.53923	-0.20051 -0.20049	-1.53959 -1.53961
38-1	7.53990 7.53990	0.59161 0.59178	-1.51810 -1.51848
1-50-E	8.45788 8.45788	0.48609 0.48586	-1.48191 -1.48140
1-50-A	8.46795 8.46795	-0.11971 -0.11996	-1.53105 -1.53033

adjustment for which the ground X, Y, and Z coordinates are known;

4. The horizontal and vertical ground control data for the points used in 3; and

5. The cantilever strip coordinates of all points in each model of the strip for which ground coordinates are needed to establish supplemental control.

Three horizontal and five vertical control points are the minimum number required to make a second-degree adjustment, and four horizontal and seven vertical control points are needed to make a third-degree adjustment.

DISCUSSION OF RESULTS

Film Deformation

Results obtained by comparing distances between fiducial marks in both directions on the measured photographic plates with distances between the same marks recorded on the camera calibration certificate showed remarkable uniformity in the deformation of estar base photographic film. The dimensional change in both the x and y photographic plate axes was about the same (Table 1). The computed linear factors for film deformation were in all cases less than unity, indicating the estar base photographic film had expanded rather than shrunk in both directions. The average computed factors from all of the x and y measurements of fiducial marks on the separate photographic plates were 0.999676 and 0.999746, respectively.

This method of correction, based on the distance between side fiducial marks, is known to be less than adequate, since film deformation is random and nonlinear in nature. No better alternative was believed possible for compensating for film deformation when only the four side fiducial marks could be measured. Film deformation represents one of the sources of error in the analytic system of aerial triangulation. Recently developed scale-stable base films, such as estar base, have certainly contributed toward minimizing this source of error.

One possible solution to this problem is the use of glass plates exposed directly in the aerial camera. This would eliminate need for film distortion compensation. No greater accuracy can be expected, however, since glass plate cameras use smaller formats and position accuracy is proportional to the scale of the photograph. The use of reseau equipped cameras offers another possibility (7). There are, however, some practical considerations at present which limit the use of these two techniques. One approach currently being employed is the use of aerial cameras with eight rather than four fiducial marks. This procedure permits more adequate mathematical restitution of points displaced by film movement (8).

It should be noted that the distance between fiducial marks is reported by the manufacturer to an accuracy of only ± 20 microns, and the diameter of the fiducial mark holes is 250 microns. Measuring the precise center of such fiducial marks using a measuring mark only 20 microns in diameter, is, in itself, somewhat uncertain.

Radial Lens Distortion

The polynomial curve-fitting technique was found to give adequate results based on the reliability of the input information provided. Although there were only 15 discrete radii for which lens distortion data were available at intervals of 10 mm on the plate, a smooth curve was plotted through these points (Fig. 4) and accepted as the actual distortion curve. Sufficient values of distortion for specific radii were selected from the curve and used as input data for the curve-fitting electronic computer program. All computed values of distortion, based on the computed curve, fell within less than 0.5 micron of the plotted curve. This technique of radial lens distortion compensation is well within the accuracy tolerances (± 2 microns) given by the manufacturer.

The radial lens distortion compensation program served also as a useful check on erroneous photographic plate coordinate measurements. Two points whose measured coordinates on the photographic plate were in gross error were found to have lens distortion corrections in excess of the maximum values shown in Figure 4.

Relative Orientation

Relative orientation was performed using 12 points for each stereoscopic model (2 points in each of the 6 usual areas of selection). The residual y parallaxes at each of the points were printed out by the computer and reviewed separately. The point with the largest residual was discarded and a point from its immediate vicinity was substituted in its place, then the relative orientation was again computed. This procedure was continued until no residuals larger than 25 microns remained at any given point. Table 2 shows the average absolute values of residual y parallax remaining at the 12 points in each of the stereoscopic models that were oriented. Residual y parallaxes as large as 50 microns were found for some points in the computed stereoscopic models. These larger values of parallax are due largely to errors introduced by the incorrect position of the drilled holes, but the inaccuracy attached to measuring the holes and the method of film deformation compensation are also contributing factors. The method of independent relative orientation of the stereoscopic models employed is dependent upon the intersection of only two rays (lines) from the respective photographs. Thus, no check is possible to determine the accuracy of computed points. Errors in x parallax are reflected as errors in the elevation of points on the ground. Computing the elevations is the final step of the analytic system of aerial triangulation.

Cantilever Assembly

Because of the limited storage capacity of the IBM 1401 computer used in this investigation, only 10 points per stereoscopic model could be accommodated in the cantilever assembly program. For purposes of this investigation, however, 10 points per model were found to be sufficient.

Table 3 contains a listing of scale correlation factors computed from points occurring in the triple overlap area. Wherever enough points were available, two scale factors were computed, and the average value used. Scale factors should be in reasonably close agreement. Points causing anomalies were discarded, and the strip coordinates were then recomputed using a substitute point.

Table 4 contains the strip coordinates of points occurring in triple overlap areas which were computed using data derived from the independently oriented stereoscopic models of the strip. Average values of two sets of coordinates for each point were used as the most acceptable strip coordinates for the point. It should be noted there is slightly greater disparity in computed values of the strip elevations than for the horizontal strip coordinates.

TABLE 5

SUMMARY OF ERRORS FOR FIVE THIRD-DEGREE ADJUSTMENTS

Adjust. No.	No. of Control Points*		No. of Test Points*		RMSE (ft)			Max. Error (ft)			Min. Error (ft)			Algebraic Mean Error (ft)		
	H	V	H	V	X	Y	Z	X	Y	Z	X	Y	Z	X	Y	Z
(a) Control Points																
1	4	12	—	—	0.05	0.32	0.59	-0.08	-0.49	1.28	-0.01	-0.09	0.00	-0.01	0.00	0.15
2	5	18	—	—	0.09	0.34	0.09	-0.13	-0.56	-1.93	0.02	0.08	-0.03	0.01	-0.12	-0.26
3	4	13	—	—	0.05	0.14	0.75	0.08	0.22	-1.78	0.00	0.02	0.25	0.01	0.30	-0.12
4	4	10	—	—	0.00	0.04	0.34	0.00	-0.19	-0.91	0.00	0.01	0.00	0.00	-0.09	-0.19
5	4	7	—	—	0.10	0.18	0.00	±0.13	-0.27	0.00	-0.02	-0.05	0.00	-0.02	-0.05	0.00
(b) Test Points																
1	4	12	10	40	0.60	0.59	1.52	0.95	1.21	3.60	0.14	-0.01	0.12	-0.07	0.20	0.71
2	5	18	9	34	0.61	0.42	1.42	0.86	0.80	3.18	-0.02	-0.01	0.02	0.06	0.25	0.29
3	4	13	10	39	0.54	0.52	1.67	0.91	-0.97	3.94	0.04	0.08	-0.05	-0.11	-0.15	0.19
4	4	10	10	42	0.55	0.54	1.95	-0.96	-0.93	-4.63	0.02	0.06	0.01	-0.12	-0.09	0.05
5	4	7	10	45	0.69	0.69	1.89	-1.21	-1.22	4.61	-0.14	-0.08	-0.10	-0.48	-0.17	0.39

*H and V = horizontal and vertical.

TABLE 6

SUMMARY OF ERRORS FOR FIVE SECOND-DEGREE ADJUSTMENTS

Adjust. No.	No. of Control Points*		No. of Test Points*		RMSE (ft)			Max. Error (ft)			Min. Error (ft)			Algebraic Mean Error (ft)		
	H	V	H	V	X	Y	Z	X	Y	Z	X	Y	Z	X	Y	Z
(a) Control Points																
1A	4	12	—	—	0.93	0.10	0.92	1.40	0.44	1.85	0.26	±0.05	0.10	0.00	0.00	0.02
2A	5	18	—	—	0.97	0.33	1.12	1.57	0.45	2.09	0.30	0.14	0.02	-0.01	-0.01	0.00
3A	4	13	—	—	0.41	0.12	1.25	-0.64	-0.17	2.02	-0.05	±0.01	0.07	0.00	-0.08	0.00
4A	4	10	—	—	0.51	0.10	1.21	-0.79	0.16	-1.94	-0.03	0.00	-0.39	0.29	0.00	-0.03
5A	4	7	—	—	0.81	0.23	0.84	1.20	0.38	-1.37	-0.22	0.02	0.17	-0.11	-0.11	-0.01
(b) Test Points																
1A	4	12	10	40	1.26	0.69	1.54	2.01	1.27	4.06	0.35	-0.16	-0.01	0.22	0.55	0.80
2A	5	18	9	34	1.17	0.77	1.51	2.21	1.26	3.89	0.05	0.07	0.01	0.32	0.48	0.27
3A	4	13	10	39	1.71	0.68	1.44	2.86	1.06	3.84	-0.15	-0.19	-0.03	0.58	0.10	0.27
4A	4	10	10	42	1.62	0.60	1.38	2.56	0.94	3.67	-0.36	-0.01	-0.18	0.96	0.19	-0.20
5A	4	7	10	45	1.61	0.61	1.46	-2.55	-1.47	3.84	0.43	0.09	-0.16	-1.17	-0.20	0.33

*H and V = horizontal and vertical.

Point Transfer and Marking

The accuracy with which pass points are transferred to a photographic strip materially affects results obtained. Obviously, precise x and y measurements made for a point inaccurately transferred are of no value. At the present time, there is considerable research being conducted on various aspects of the measurement procedure including evaluation of various types of point transfer and marking instruments (9). At the moment there appears to be no general agreement regarding the efficiency and reliability of the various types of instruments.

When using a monocular comparator it is a practical necessity to use some form of point transfer and marking system. Point transfer should be precise and stereoscopically correct. Measurement of plate coordinates is considerably facilitated and the possibilities of blunders due to misidentification of corresponding images are minimized. Targeted points, on the other hand, can be found and measured readily without being premarked on the diapositive plate.

Largely because of the inexperience in using the PUG point marking and transfer instrument, some errors were introduced into the analytic system on this test project. The magnitude of these errors, however, was impractical to measure. Drilled holes for the most part were nonuniform in shape and size. Some targeted control points were drilled off center. Observation with the Kelsh instrument of each stereoscopic model in the strip of photographs revealed lack of stereoscopic correspondence of the drilled holes with the ground surface for a significant number of the points. Based on this stereoscopic analysis, "digging" or "floating" points were not included as control points for the strip adjustments. A comparison was made between the sign of the errors in the computed elevations and their positions noted in the stereoscopic models as being either on, above, or below the ground surface.

There was a definite correlation between the sign of the errors in elevation and the observed elevation of the holes in the stereoscopic models. No actual elevation measurements were made because a significant number of the drilled holes could not be seen in the stereoscopic model. It should be noted, however, that the transfer of points is only one source of known error in this project.

Computed Ground Coordinates

Tables 5 and 6 summarize the errors in computed ground coordinates using second- and third-degree cantilever strip adjustments. Tables 7 through 9 of the Appendix show the horizontal and vertical errors for ground coordinates computed for 55 separate points in 10 trial adjustments. The distribution of ground control used in each of the adjustments is shown in Figures 5 through 9 of the Appendix.

The third-degree horizontal adjustments were the most accurate. Table 5 shows the root-mean-square errors (RMSE) for the horizontal coordinates of the third-degree adjustment. Values ranged from 0.42 to 0.69 ft and show no significant difference between the X and Y coordinates. The use of 5 rather than 4 horizontal control points did not significantly increase the accuracy of the computed values.

Table 6 shows the RMSE for the horizontal coordinates of second-degree adjustments ranging from 0.60 to 1.71 ft. The magnitude of the error in the X coordinates was about twice that for the Y coordinates. The errors in the computed Y coordinates in both second- and third-degree adjustments were about the same. Errors in the computed X coordinates of the third-degree adjustment were half the size of those for the second-degree adjustments.

Test point 45-1 tended to have slightly more error in horizontal coordinates in 8 of the 10 adjustments because of its position outside the confines of ground control point SW-45, located near the beginning of the flight strip.

A comparison of the vertical RMSE in Tables 5 and 6 for the second- and third-degree adjustments shows no significant difference for the first three sets of trials. Twelve or more vertical control points were used to adjust these flight strips. The second-degree adjustments, however, showed a significant improvement in accuracy over the third-degree in the last two sets of adjustments where vertical control points were not as dense.

It is interesting to note that an increase from 7 to 18 vertical control points had practically no effect on the RMSE of the second-degree adjustments, but it did significantly reduce the RMSE of the third-degree adjustments. The largest vertical errors were found for test points located in areas where no vertical control points were present in their vicinity (adjustments 4, 4A, 5, 5A). The effects of density and distribution of vertical control points on ultimate accuracy are difficult to analyze. It is likely some of the drilled holes for ground control points were not in stereoscopic correspondence with the ground surface. Only those control points observed to "float" or to "dig" in the Kelsh instrument were eliminated as control points for making the strip adjustments.

The algebraic mean errors in Tables 5 and 6 for elevations of test points are positive for all trial adjustments except one (adjustment 4A). The positive sign of these mean errors tends to confirm the stereoscopic observations made of the drilled holes and the existence of systematic errors.

CONCLUSIONS AND RECOMMENDATIONS

Results of this investigation point to at least four principal sources of error in the analytic aerial triangulation system: film deformation, pass point transfer, x and y coordinate measurements of points on the glass plate transparencies, and ground control.

It is recognized that the method of compensation for film deformation is inadequate, but it is the most suitable method which can be employed for cameras equipped with only 4 side fiducial marks. It is, however, impossible to prove the amount of error in ground coordinates due to film deformation. The estar-base film is considerably more stable dimensionally than previous film bases. There are at least three possible alternatives for a more effective treatment of the film deformation problem. The first is equipping existing cameras with 4 additional fiducial marks so that a more effective mathematical restoration of displaced images can be made. A computer program is available to accomplish this. The second alternative is to employ a network of small crosses (reseau) in the focal plane of the camera. The crosses are recorded on the negative film at the instant of exposure. Displacement of images due to film distortion or lack of film flatness at the instant of exposure is compensated by comparing the measured position of the reseau crosses with their calibrated position. The third possible alternative is to use a glass plate aerial camera. This would eliminate the need for compensation due to film effects.

When measuring plate coordinates with a monocular comparator, some form of point transfer and marking technique is a practical necessity. The PUG instrument was used for this project. Largely because of lack of experience with the instrument, some error was introduced from this source. There is a divergence of opinion at present regarding the inherent accuracy of several point transfer and marking instruments. Research and evaluation of these techniques is being done by private firms, universities, and governmental agencies. Ground control and other points which are premarked with photographic targets can be reliably identified and measured without need for point transfer and marking.

The relative size of the measuring mark to that of the drilled hole appears to be a significant factor in the degree of accuracy of measurement. A 20-micron measuring mark and a fiducial hole with a diameter of 250 microns or a drilled hole of 60 microns diameter are not desirable combinations for optimum accuracy of measurement. Greater accuracy could be attained by using a larger measuring mark. There is some doubt about the adequacy of drilled holes that are 60 microns in diameter for use in map compilation with the Kelsh instrument. At a 5:1 projection ratio, a significant number of holes were not discernible in the stereoscopic models. A drilled hole of at least 100 microns in diameter is needed for use with the Kelsh instrument.

Basic ground control surveys on which aerial analytic triangulation is based should be of second-order accuracy (1/10, 000 closing error in position) or better, so results of the triangulation will not be degraded because of inferior control. Through such a practice results of the triangulation from the standpoint of accuracy can also be more realistically evaluated. Results of the aerial analytic triangulation cannot be expected to be any better than the ground control survey on which it is based.

The polynomial curve-fitting technique used to determine an equation for the radial lens distortion curve is an acceptable method. It is particularly adapted to determining the equations of curves which are smooth and have only a few points of inflection. Corrections made to the x and y measured coordinates, based on the computed curve, are within the accuracy tolerances for the manufacturer's determination of the lens distortion values.

Corrections for atmospheric refraction and adjustments for earth curvature were not considered in this investigation. Displacement of photographic images due to atmospheric refraction and earth curvature are negligible because of the relatively low flight height from which the photographs were taken and the short length of the flight strip. For bridging photographs taken from higher flight heights and for longer flight strips, appropriate adjustments should be applied for their effects (10, 11).

Use of a small-capacity computer for aerial analytic triangulation requires considerable segmentation of the computer programs and excessive card handling. Computers with larger storage capacity and greater speed must be used in order to perform the analytic operations more efficiently.

Use of mapping-scale photographs (1:6000) for analytical bridging does not appear to be the most accurate or economical approach for securing supplemental ground control data for large-scale topographic mapping. Use of the larger scale photographs requires measurement of x and y for a greater number of stereoscopic models for a given bridged distance. The greater number of intermodel ties needed tends to deform the bridged strip and thus offset the advantages gained from having a larger scale.

Most present-day aerial cameras are not designed or calibrated for analytical photogrammetry. To take full advantage of mathematical techniques to compensate for errors, some slight design changes in aerial cameras and related equipment will undoubtedly be necessary.

Future investigations of aerial analytic triangulation for highway mapping purposes may well consider one or more of the following:

1. Color diapositive plates,
2. Aerial cameras equipped with eight fiducial marks or a reseau grid,
3. Stereocomparator (12),
4. Photography scales of 1:12,000 or smaller,
5. Other mathematical methods of aerial analytic triangulation (2, 13, 14, 15, 16), and
6. Various point transfer and marking devices.

In the writer's opinion, this method of analytic aerial triangulation is capable of providing accurate supplemental control for use in compilation of large-scale maps for location and design of highways. The primary remaining problem lies in refining operational techniques and hardware which now are contributing considerable error to this method of computing the X, Y, and Z coordinates of selected points on the ground.

ACKNOWLEDGMENTS

The author extends his appreciation to the following individuals for their cooperation in this investigation: Joseph C. Knittel, Continental Engineers, Inc., Denver, for providing the photographic materials and ground control, and for drilling the photographic plates; Jack Friedman, OMI Corporation of America, Alexandria, Virginia, for use of the Nistri Monocomparator; Robert Larson and Douglas Reid, Automatic Data Processing Section, Region 15, U.S. Bureau of Public Roads, for their interest and use of electronic computing facility; William T. Pryor, Chief, Aerial Surveys Branch, for his helpful suggestions and cooperation, and Robert J. Warren, Region 9, U.S. Bureau of Public Roads, for his cooperation; and Donald N. Shaw, Aerial Surveys Branch, for his assistance.

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Appendix

TABLE 7

ERRORS IN HORIZONTAL COORDINATES FROM THIRD-DEGREE ADJUSTMENTS

Point No.	Adjustment Number									
	1		2		3		4		5	
	X (ft)	Y (ft)	X (ft)	Y (ft)	X (ft)	Y (ft)	X (ft)	Y (ft)	X (ft)	Y (ft)
45-1	0.95	-0.68	0.86	-0.74	0.91	-0.58	-0.96	-0.64	*-0.02	-0.05
SW-45	*-0.02	-0.09	*-0.05	-0.18	*-0.04	-0.09	* 0.00	-0.14	0.71	0.53
SW-44	0.14	0.31	-0.02	0.18	-0.05	0.18	* 0.00	-0.19	-0.29	0.74
SW-43	0.31	0.45	* 0.10	-0.29	*-0.03	0.22	0.02	-0.33	-0.14	0.67
42-1	-0.29	-0.61	-0.44	0.47	-0.42	0.45	-0.42	0.52	-0.46	-0.97
42-2	-0.30	1.21	-0.38	-0.01	0.04	0.79	-0.28	0.88	-0.38	-1.22
SW-41	*-0.08	0.28	*-0.13	0.08	* 0.08	0.15	* 0.00	-0.07	*-0.13	0.22
41-2	-0.80	0.10	0.75	-0.09	-0.36	-0.36	-0.49	-0.32	-0.88	-0.22
40-1	-0.73	0.12	-0.69	-0.12	-0.38	-0.47	-0.50	-0.39	-0.89	-0.08
38-1	-0.88	0.91	-0.80	0.80	-0.30	-0.41	-0.42	0.46	-1.21	-0.37
39-1	0.23	0.21	0.29	0.07	0.77	-0.22	0.64	-0.18	*-0.08	-0.27
SW-38	*-0.01	-0.49	* 0.09	-0.56	-0.49	-0.97	0.40	-0.93	-0.49	-0.91
SW-37	* 0.05	0.30	* 0.02	0.37	* 0.00	0.02	* 0.00	0.01	-0.72	0.11
BMC-131	0.68	-0.01	0.71	0.33	-0.85	0.08	0.78	0.06	* 0.13	-0.09

*Horizontal control points used to adjust strips.

TABLE 8

ERRORS IN HORIZONTAL COORDINATES FROM SECOND-DEGREE ADJUSTMENTS

Point No.	Adjustment Number									
	1A		2A		3A		4A		5A	
	X (ft)	Y (ft)	X (ft)	Y (ft)	X (ft)	Y (ft)	X (ft)	Y (ft)	X (ft)	Y (ft)
45-1	1.89	-0.16	2.21	-0.54	1.72	-0.43	1.92	-0.32	*-0.22	0.02
SW-45	* 0.26	-0.05	* 0.60	-0.35	* 0.25	-0.17	*-0.59	-0.13	-1.18	0.17
SW-44	-1.04	0.25	-0.58	0.07	-0.60	-0.19	*-0.79	0.16	-2.04	0.26
SW-43	-1.54	0.30	*-0.90	0.17	*-0.64	0.01	-0.80	-0.01	-2.29	-0.19
42-1	-2.01	0.30	-1.47	0.24	-1.11	0.35	-1.22	0.22	-2.55	0.09
42-2	-1.39	0.83	-0.81	0.91	-0.15	-0.92	-0.36	0.74	-1.64	0.43
SW-41	*-0.87	0.05	*-0.30	0.14	* 0.44	-0.17	* 0.22	-0.04	*-1.05	-0.38
41-2	-0.35	0.43	0.14	0.54	1.12	0.52	0.87	0.29	-0.44	-0.18
40-1	-0.48	0.39	0.05	-0.23	0.92	-0.34	0.68	-0.56	-0.67	-0.94
38-1	0.70	1.27	1.02	1.26	2.20	1.06	1.94	0.94	0.43	0.22
39-1	1.29	0.95	1.71	1.00	2.86	0.93	2.56	0.74	* 1.20	0.13
SW-38	* 1.40	-0.44	* 1.57	-0.42	-2.69	-0.72	2.51	-0.81	0.81	-1.47
SW-37	*-0.80	0.44	*-0.98	0.45	*-0.05	-0.01	*-0.03	0.00	-2.11	-0.52
BMC-131	0.71	0.97	0.63	1.11	1.55	0.76	1.54	0.68	*-0.38	-0.22

*Horizontal control points used to adjust strips.

TABLE 9

ERRORS IN VERTICAL COORDINATES FROM SECOND- AND THIRD-DEGREE ADJUSTMENTS[1]

Point No.	Adjustment Number									
	1	1A	2	2A	3	3A	4	4A	5	5A
1-40-B	-0.75	-1.36	-0.73	-1.67	-0.36	-0.85	-0.83	-1.84	-0.80	-2.09
1-42-B	-0.82	-0.73	-1.36	-1.00	-1.79	-1.27	-2.51	-1.26	-2.57	-1.26
1-36-D	1.61	1.00	* 0.71	* 0.62	* 0.70	* 0.30	0.31	-0.29	* 0.00	* 0.17
45-1	2.99	1.35	2.94	0.96	3.94	0.65	3.88	0.75	3.68	0.41
1-40-A	-0.50	-0.13	-1.43	-0.40	-1.22	-0.75	-3.02	-0.71	-3.17	-0.75
SW-45	* 0.59	*-0.12	* 0.23	*-0.45	0.47	-0.75	* 0.05	*-0.69	-0.10	-0.91
45-2	3.60	2.67	3.18	3.40	2.83	3.17	2.13	3.17	2.10	3.05
1-42-L	-1.78	*-1.09	-2.19	-1.27	-3.05	-1.42	-3.92	-1.49	3.85	-1.46
1-42-K	*-0.05	0.29	*-0.07	* 0.02	-0.60	-0.09	-1.32	-0.18	-1.23	-0.21
1-42-D	-0.12	-0.06	-0.19	-0.44	0.57	-0.50	0.05	-0.61	0.21	-0.70
1-40-C	1.36	-0.35	2.18	-0.73	3.77	-0.80	3.68	-0.79	3.83	-1.25
SW-44	-1.29	*-1.23	*-0.54	*-1.50	*-1.78	*-1.68	2.47	-1.70	-2.44	-1.83
1-42-C	-2.52	-1.99	-3.01	-2.20	-3.78	-2.41	-4.63	-2.44	-4.61	-2.48
SW-43	*-0.03	*-0.10	*-0.03	*-0.52	* 0.34	*-0.57	*-0.18	*-0.72	* 0.00	*-0.74
1-42-G	-0.54	1.13	0.23	0.81	-0.22	0.76	-0.87	0.57	-0.75	0.75
1-42-J	3.44	4.06	0.16	3.89	2.46	3.84	1.77	3.67	1.87	3.84
3-28-J	-1.03	-0.16	*-1.38	*-0.28	-2.46	-0.38	-3.33	-0.49	-3.27	-0.41
1-42-H	0.20	1.03	-0.06	0.93	-1.12	0.87	-1.91	0.74	-1.84	0.88
1-42-F	1.00	1.18	0.80	0.72	1.02	0.67	0.47	0.47	0.64	0.57
42-1	-0.66	-0.01	0.94	-0.24	-1.67	0.31	-2.35	-0.45	-2.24	-0.40
1-44-F	1.95	2.38	2.05	2.36	1.23	2.42	0.80	-2.26	0.78	2.40
42-2	1.39	1.97	0.02	1.64	-0.50	1.62	0.05	1.36	0.13	1.73
1-44-D	0.92	2.58	*-0.29	* 2.09	* 0.91	* 2.02	*-0.44	* 1.71	0.57	2.21
1-44-A	0.54	1.19	0.29	0.97	-0.40	0.95	-0.99	0.75	-0.93	0.98
3-28-F	1.62	2.23	1.47	2.09	0.68	2.08	0.12	1.90	0.16	2.10
1-44-B	1.88	2.01	1.13	1.92	0.14	1.90	-0.53	1.76	-0.49	1.90
3-28-E	* 1.28	* 1.85	* 1.29	* 1.81	* 0.37	* 1.84	*-0.16	* 1.69	0.16	1.83
1-44-K	1.85	1.75	2.11	1.67	1.79	1.81	1.81	1.59	1.72	1.81
1-44-G	0.89	1.05	0.67	0.73	0.35	0.70	0.27	0.46	0.30	0.94
3-26-D	2.33	2.38	1.62	2.35	2.13	2.48	2.02	2.29	1.94	2.45
SW-41	* 1.04	1.47	*-0.73	* 1.16	*-0.25	* 1.16	*-0.07	* 0.87	* 0.00	* 1.29
41-2	0.69	0.34	0.68	0.01	0.73	0.12	1.05	-0.19	1.02	0.29
3-26-E	1.86	0.39	2.24	1.29	2.25	1.48	2.58	1.25	2.43	1.48
40-1	*-0.20	* 0.13	-0.98	-0.52	-1.24	-0.61	-1.32	-1.03	1.07	-0.16
1-46-E	-0.72	-1.78	-0.21	-1.90	0.42	-1.57	-1.27	-1.79	0.99	-0.68
1-48-H	*-0.47	*-1.01	*-0.79	*-1.61	*-0.50	*-1.55	* 0.00	*-1.94	0.26	-1.19
1-46-F	0.31	0.17	*-0.61	*-0.77	0.60	-0.92	-0.36	-1.44	0.12	-0.20
1-46-C	*-0.85	*-1.47	*-0.44	*-1.58	*-0.29	*-1.36	* 0.16	*-1.59	* 0.00	*-1.37
1-46-H	0.78	0.97	*-0.12	* 0.20	*-0.31	* 0.07	*-0.28	*-0.39	* 0.00	* 0.64
1-46-D	0.56	-1.25	-0.45	-1.58	-0.11	-1.40	0.47	-1.71	0.40	-1.27
1-48-F	0.74	0.11	0.35	-0.60	-0.83	-0.50	1.46	-0.90	1.76	-0.13
1-48-E	* 0.20	*-0.12	-0.84	-1.26	*-0.72	*-1.45	-0.45	-2.02	0.40	-0.58
1-50-E	1.19	0.71	* 0.99	* 0.33	1.79	0.80	2.70	-0.59	2.58	0.60
1-50-A	1.35	1.05	*-0.72	* 0.25	1.19	0.42	1.74	0.05	2.23	0.73
1-48-C	* 0.25	* 0.54	0.25	-0.91	* 1.07	*-0.51	2.02	-0.76	1.89	-0.62
1-50-D	0.13	0.05	*-1.03	*-1.30	-1.00	-1.52	-0.91	-2.00	0.41	-0.54
1-50-G	* 0.00	* 0.72	*-1.93	* 0.23	*-0.55	* 0.80	*-0.03	* 0.62	* 0.00	* 0.48
1-50-C	2.33	2.93	* 0.86	* 1.30	* 0.49	* 1.01	* 0.04	* 0.41	2.08	2.14
BMC-131	*-0.01	* 0.26	-1.38	-1.32	-0.59	-1.61	-1.82	-2.23	* 0.00	*-0.48
1-50-B	0.84	0.98	-0.14	-0.05	-0.05	-0.03	0.30	-0.39	1.21	0.52
1-50-F	0.36	0.47	-0.22	0.03	0.39	0.55	1.12	0.36	1.08	0.29

[1]A indicates second-degree adjustment.

*Vertical control points used to adjust strip elevations.

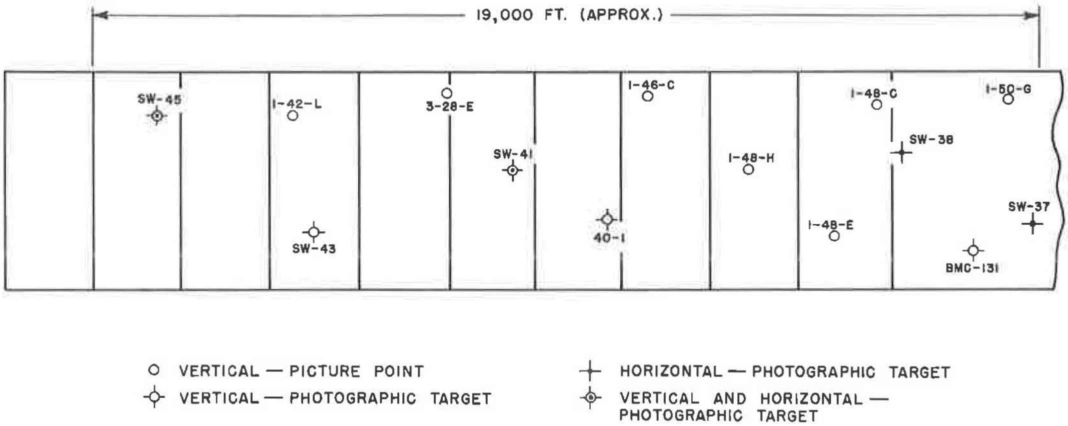

Figure 5. Distribution of ground control used in adjustment No. 1 and 1A.

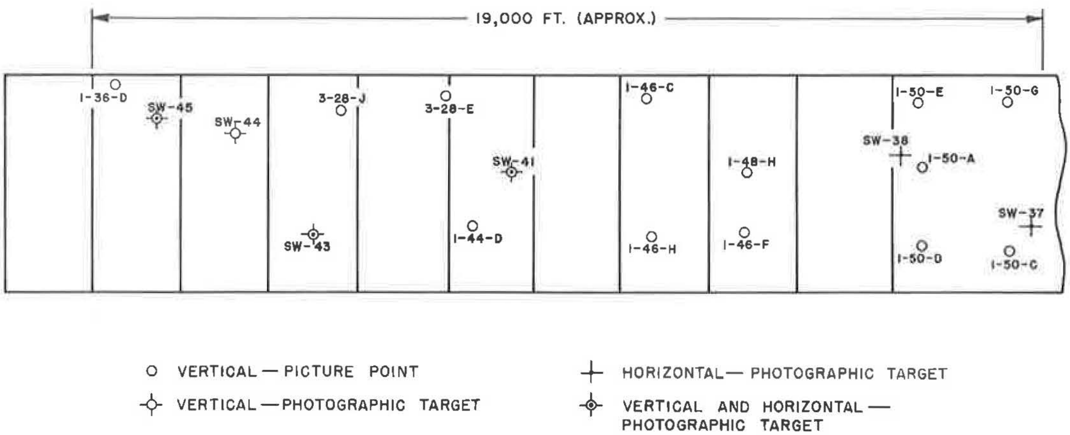

Figure 6. Distribution of ground control used in adjustment No. 2 and 2A.

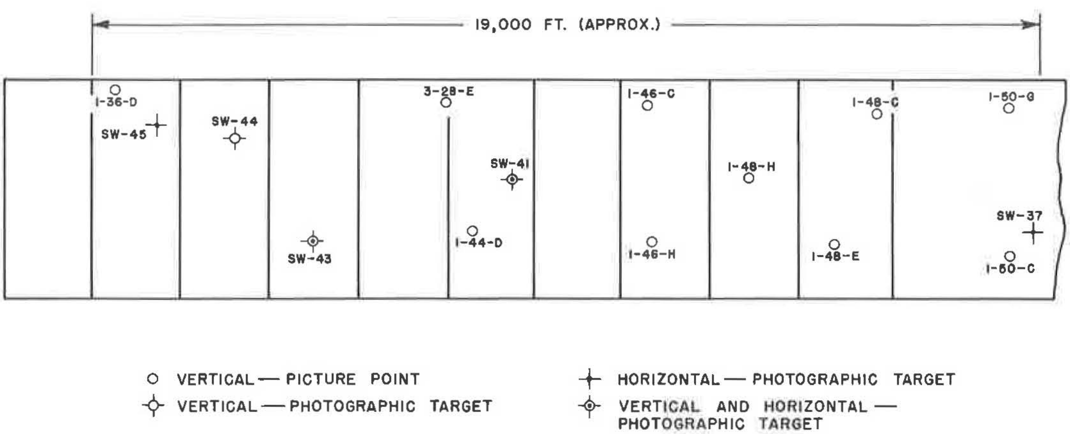

Figure 7. Distribution of ground control used in adjustment No. 3 and 3A.

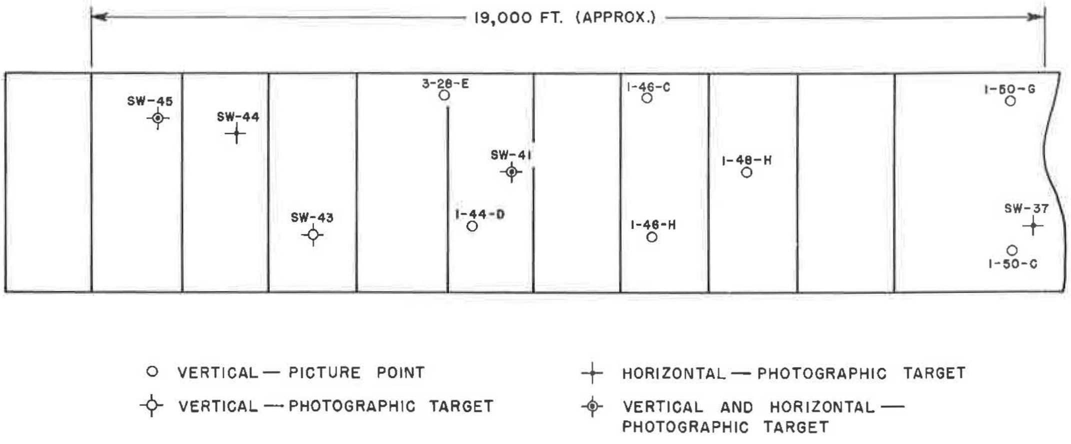

Figure 8. Distribution of ground control used in adjustment No. 4 and 4A.

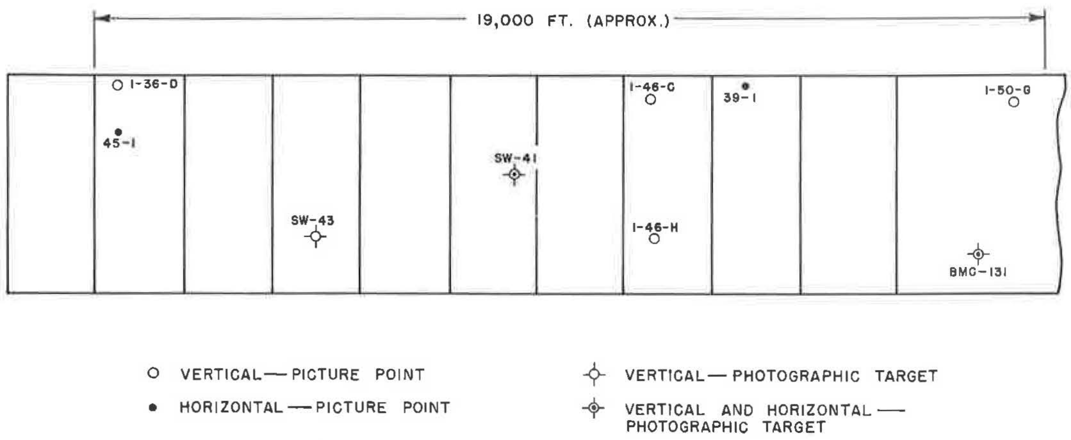

Figure 9. Distribution of ground control used in adjustment No. 5 and 5A.

Precise Photogrammetric Determination of Section Corners

G. P. KATIBAH, Photogrammetric Engineer, California Division of Highways

This investigation concerns the relative accuracy of positions of cadastral monuments determined by photogrammetric methods as compared with field methods. This is an application of numerical photogrammetry in contrast to traditional photogrammetric mapping procedures.

A second-order ground control survey, established on the State plane coordinate system, was measured through the project area. This control was sufficient for photogrammetrically positioning 72 section and quarter-section corners. The 72 corners were also tied to the control by conventional field surveying methods as a basis for comparison of the photogrammetrically determined plane coordinate positions.

Each of the corners and the selected ground control monuments were premarked with photographic targets for positive identification on the aerial photographs.

Two scales of aerial photography were employed. Each scale of photography was used in two different photogrammetric systems; thus, four sets of independent position values were developed. The systems employed were instrumental photogrammetry using a Zeiss Stereoplanigraph, and analytical photogrammetry using a Mann monocular comparator.

The results show that error propagation was not proportional to the scale of the photography. However, using the scales and procedures described, the results in all cases, are considered entirely adequate for this type of cadastral survey. Cost records show that the field method costs were about 200 percent greater than the photogrammetric method costs.

•PHOTOGRAMMETRY, as a system of measurement in which the output record is entirely in numerical terms, has been undergoing a technological evolution which promises to increase its scope of applications in scientific and engineering work. In numerical photogrammetry, the direct output of the resulting measurements is in terms of digital values. This is in contrast to the classical mapping application in which the direct output is in graphical terms, such as contour lines and planimetry.

In the spring of 1964, the California Division of Highways, in cooperation with the U. S. Bureau of Public Roads, initiated a project to investigate the application of numerical photogrammetry for determining the position of land-surveyed section corners. This is a typical right-of-way surveying problem, especially in rural areas, which is often difficult to accomplish by conventional methods on the ground with any real assurance of reliability, not to mention high costs associated with field surveying.

The project area was a 20-mile section of highway west of Needles, California, in open desert where the topography varied from flat valley floors to fairly high, steep-sided mountains (Figs. 1 and 2). The horizontal position of 72 section and quarter-section corners had to be determined on the State plane coordinate system as a basis for cadastral computations.

Paper sponsored by Committee on Photogrammetry and Aerial Surveys and presented at the 46th Annual Meeting.

Figure 1. Typical mountainous desert terrain.

There was an existing second-order traverse throughout the length of the survey project with monuments located at intervals of approximately ½ to 1 mile. This traverse, as measured by geodimeter-theodolite procedures, had been positioned on the ground for highway mapping, design, and construction purposes without any consideration for the subsequent photogrammetric surveying of section corners, which is the

Figure 2. Typical flat desert terrain showing flight line marker.

subject of this report. It did, however, provide sufficient State plane coordinate control as a framework for accomplishing the section corner survey.

This survey project was essentially an investigation to determine procedures vs accuracy of results. Therefore, it was decided to furnish two scales of aerial photography—1:12,000 (or 1000 ft/in.) and 1:18,000 (or 1500 ft/in.). The 1:12,000 scale was selected because it would comfortably cover a band more than one mile wide, and if the photographic flights were made in a north-south or east-west direction through the middle of a row of land survey sections, they would catch all section corners on the perimeter.

The 1:18,000 scale was selected because its coverage would be sufficient to photograph diagonally opposite section corners regardless of orientation of the flight line with respect to the section lines. This second scale would also make it possible to investigate the relative accuracies of positions determined photogrammetrically from aerial photography taken from the two different flight heights of 7200 ft and 9000 ft, respectively, which were required. The measurements were made by two different photogrammetric procedures. The first incorporated stereoscopic analog photogrammetry and the second was completely analytical. Both scales of photography were used in both procedures, thus permitting the development of four sets of independent data.

FIELD WORK

A search for all of the cadastral survey section corners had to be made and all found corners had to be verified. In those instances where a diligent search failed to reveal a corner, a reference mark was located in the vicinity of the lost corner for ultimately establishing a corner monument after completion of the cadastral computations. Because of the open terrain, intervisibility between a reference mark and an existing corner or quarter corner would permit the recovery of a calculated bearing for final positioning of a new corner monument from the reference mark.

When this phase of the field work was completed, the next step was to premark each section corner, quarter-section corner, and lost-corner reference mark prior to photography with a suitable target so that positive identifications on both scales of photography would be assured. The target design and dimension decided upon is shown in Figure 3.

The basic consideration for design of an aerial survey target for recovery of a definite object or position on the ground is that the target must define the object or position as an image point on the photograph. In the adopted target design, the white center square was the point to be measured photogrammetrically, To assure that this point would be definite on the photograph it was set on a black background. The two white legs simply made the target unique and distinct from any other possible white on black image that might by chance appear on the photograph, and thus aided the photogrammetrist in identification of the point.

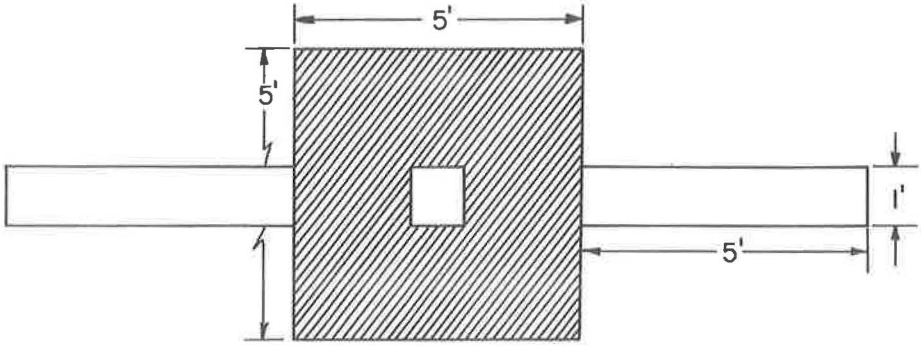

Figure 3. Target design for premarks (center square is 1 by 1 ft).

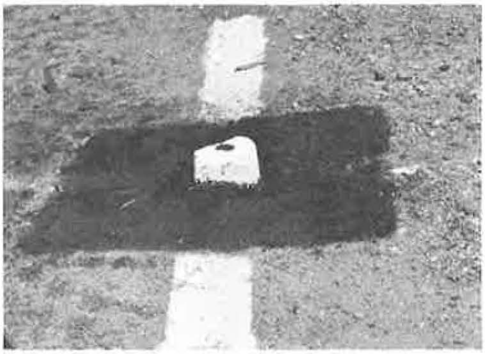

Figure 4. Premark of USC & GS monument with photographic target.

Figure 5. Close-up view of USC & GS monument.

Materials for the target had to be durable enough to withstand the elements until photography was accomplished, and also had to be as nonreflective as possible to reduce the effect of halation, or the bleeding of white images into darker surrounding areas. The materials were selected by the field crews and consisted of white casting plaster for the center square and legs, and diesel fuel for the black patch. It was discovered that 2.5 lb of lampblack mixed with 5 gal of diesel fuel made a very dense background, yet was still thin enough to be applied by spraying with a portable hand-operated pump.

This target design was used for all premarked points including section corners and geodimeter control monuments. Figure 4 shows how a USC & GS monument in the project area was premarked, which was typical of the premarking of virtually all section corners on the project. Figure 5 is a close-up view of the monument. Figure 6 shows the design applied to a geodimeter monument on the paved shoulder of the existing highway. Note that the black patch was omitted because of the black asphalt paving. Figure 7 shows how the design was varied to accommodate a rock-mound corner.

Figure 6. Premark target painted on shoulder of highway.

Figure 7. Premark of rock-mound section corner.

It is noteworthy that all targets, a total of 112, were clearly visible on the 1:12,000-scale photography, and only two were in doubt on the 1:18,000-scale photography.

Flight line markers, as shown in Figure 2, were also set by the field crews. The material for the markers was unbleached muslin 36 in. wide. The arrow was 20 ft long. These markers were of direct assistance to the aerial photographic crew for positive

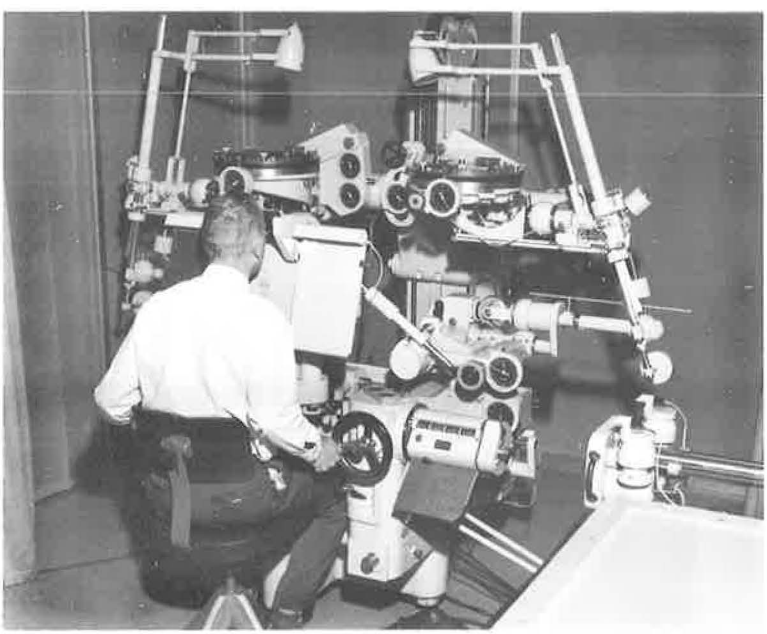

Figure 8. Zeiss Stereoplanigraph, Model C8.

identification of each flight line, especially since the terrain was somewhat devoid of features which help make a good flight map. (In other types of terrain where good contact from map to ground can be established, flight line markers may not be necessary.)

PHOTOGRAMMETRY

After all premarking was completed in the field, aerial photography was taken with a Wild RC-8 6-inch focal length aerial camera by Pictorial Crafts, Inc., of San Bernardino. High-contrast diapositive plates were made to accentuate the premarked points on both scales of photography. All targeted points were located and identified on a set of contact prints for cross reference on the diapositive plates. The project was then ready for making the photogrammetric measurements, calculations, and adjustment.

Stereoscopic Photogrammetry

This phase of the project was done by the Photogrammetry Section of the California Division of Highways. All stereoscopic measurements were made with a Zeiss Stereoplanigraph Model C8 (Fig. 8).

The 1,000-ft/in. scale photography was processed in a conventional aerial triangulation mode. An artificial image point was first marked near the principal point of each diapositive to strengthen the scaling tie between each stereoscopic model along a flight line. These points were monoscopically made with a Wild PUG instrument (Fig. 9), although a Zeiss Snap Marker could also have been used if one had been available at the time. In each model all premarked points, including control points and section corners, were measured and recorded in photogrammetric instrument coordinates from which punch cards were prepared. Conversion of photogrammetric instrument coordinates to adjusted ground coordinates on the State plane coordinate system was calculated by electronic data processing procedures using an IBM 704 computer. The programming for these calculations was based on methods explained in U. S. Coast and Geodetic Survey Technical Bulletin No. 23.

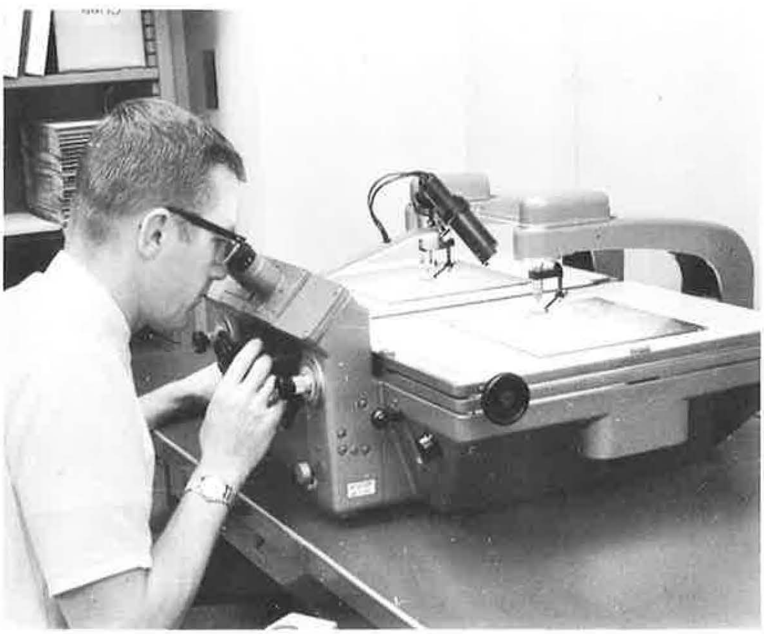

Figure 9. Wild PUG point transfer instrument.

The 1,500-ft/in. scale photography was set up in the C8 as single stereoscopic models because, at this scale, enough premarked horizontal control points were available to provide a scaling base for the individual model orientation. To utilize the adjustment program, whether for an aerial triangulation strip of photographs or a single stereoscopic model, three horizontal control points had to be measured and recorded. Since the project had been mapped previously for highway design purposes using 250-ft/in. scale photography, numerous targeted horizontal control points existed. Even though the targets on these points were old, they were still visible on the smaller scale photography and position-measurable with the C8, and served as the additional control necessary for making the adjustment calculations.

A total of 51 stereoscopic models at the contact printing scale of 1000 ft/in. covered the survey project area, whereas 28 stereoscopic models covered the area at the scale of 1500 ft/in., a reduction in the number of models by a ratio of about 1.8 to 1.

Analytical Photogrammetry

Upon completion of the stereoscopic photogrammetry work, the same diapositive plates were used for processing by analytical photogrammetric procedures. This phase was performed by Geotronics, a Division of Teledyne, Inc., of Monrovia, California.

Both sets of photographic plates, those at 1000 ft/in. and those at 1500 ft/in., had to be prepared for making monocomparator x and y coordinate measurements using a Mann Comparator (Fig. 10). The plates were set in stereoscopic pairs in a Wild PUG instrument and three artificial image points were marked along each neat line and transferred to the conjugate neat line of the adjoining photographic plates. This pattern, as diagrammed in Figure 11, was repeated on each plate of the flight strips.

Upon completion of plate preparation, each plate was placed on the stage of the comparator for monoscopic measurement of the x and y coordinates of all PUG marked points, all targeted points, and the fiducial marks. All measurements were recorded to the nearest micron. The data were then assembled for making the computations, using electronic data processing to yield adjusted ground X and Y coordinates of all section corners, quarter-section corners, and lost-corner reference points. By this

Figure 10. Mann monocomparator (courtesy of Teledyne Inc., Geotronics Div.).

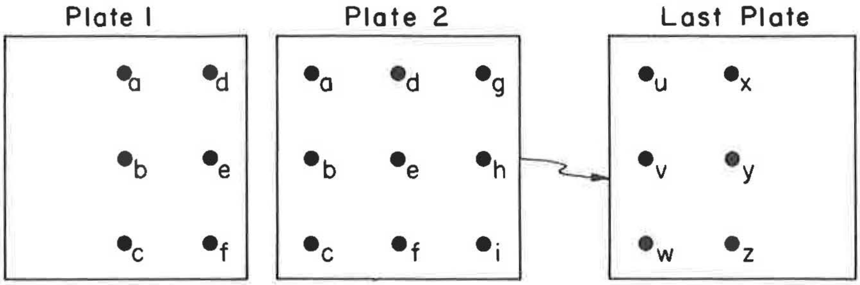

Figure 11. Pattern of artificial points.

use of the photography of both scales, the ground X and Y coordinates were derived from analytical bridging.

RESULTS

Field checks were made on 70 of the 72 section corner positions. Table 1 compares the positions derived from the four photogrammetrically made surveys with the positions determined by field procedures.

It may be difficult for those who are trained in field traverse surveying, using metes and bounds, to accept the expression of errors in terms of coordinate differences rather than a closure in terms of a proportion or fraction of a distance measured. Plane coordinate values for horizontal position, however, are the direct product of photogrammetric measuring because of the principles of simultaneous intersection, similar to the end results of a triangulation survey, from which bearings and distances are inversed from the X and Y coordinates. The opposite, of course, is the situation with a conventional bearing-distance survey made in the field from which the plane coordinates are calculated for each traverse point. Coordinate differences expressed in terms of "average" and "standard error" are more meaningful because they convey the concept of "absolute error" referred to a specific horizontal datum, which in most cases is a State plane coordinate system.

TABLE 1

PHOTOGRAMMETRICALLY DETERMINED VALUES COMPARED WITH FIELD-SURVEYED VALUES

Classification of Errors (Error values given in ft)		1000-ft/in. Scale Photography H = 6000 ft		1500 ft/in. Scale Photography H = 9000 ft	
		Stereo	Anal.	Stereo	Anal.
Average error of coordinates	ΔX	+0.17	+0.06	+0.13	+0.02
	ΔY	-0.13	-0.14	-0.18	-0.03
Standard error of coordinates	ΔX	0.50	0.04	0.61	0.44
	ΔY	0.90	0.68	0.95	0.76
Maximum error of coordinates	ΔX	+1.8	+1.4	+2.4	+1.3
	ΔY	-2.9	-2.0	-2.9	-3.1
Standard error of radial[a]		1.03	0.68	1.13	0.89
Position accuracy (standard error: H)		1:5800	1:8800	1:8000	1:10,200

[a]Vector = $\sqrt{(\Delta X)^2 + (\Delta Y)^2}$

TABLE 2

COST COMPARISONS

Photogrammetric Method	
Premarking	$1,565
Photography	825
Office work	1,030
Total	$3,520
Field Method	
Party time	$10,750

It is convenient to express position accuracy as a function of flying height above mean ground, hence it is stated as the ratio of the standard error of the radial vector to the flying height H. The results do not indicate a decrease in accuracy proportional with an increase in H, as shown in Table 1 for each of the four cases. Photogrammetric engineers would like to develop procedures to guarantee measurement of position accuracy to better than 1:10,000 of the flight height, but there continue to be some factors over which they have little control.

Factors common to both stereoscopic and analytical systems of photogrammetric determination of X and Y position, and their relative treatment, may be of interest:

1. Atmospheric refraction cannot be controlled in any way in the stereoscopic system; a mathematical equation which provides nominal correction in the analytical system is included in the routine for image coordinate refinement.
2. Deformation of the film base cannot be accounted for in the stereoscopic system; it can be corrected to some extent in the analytical system as part of the routine for image coordinate refinement.
3. Lens distortion can be compensated according to average values in the stereoscopic system, and more completely corrected in the analytical system as part of the routine for image coordinate refinement.

Because the routine of image coordinate refinement is more readily adapted to the analytical system, superior results should be expected from it compared with results from the stereoscopic system, which is analog in characteristics. This is verified in Table 1.

While there was a relative improvement in the accuracy of position measurement in both the stereoscopic and analytical systems with an increase in flight height from 6000 to 9000 ft, there was significant improvement in the stereoscopic results, amounting to 38 percent. It was earlier mentioned that the 1000-ft/in. scale photography had been processed in a conventional analog aerial triangulation mode using the Zeiss Stereoplanigraph. The 1500-ft/in. scale photography, however, had been set up as single models, which explains the relatively superior results at this scale.

With respect to costs, Table 2 gives expenditures for the photogrammetry work and expenditures for establishing field ties to the 70 section corners. In evaluating the data, it should be borne in mind that photogrammetry costs include only photography at the 1000-ft/in. scale using the Zeiss Stereoplanigraph. Also, these costs include an estimate for per diem expenses of field personnel but do not include vehicle costs and data processing costs.

CONCLUSIONS

The field method costs are about 200 percent greater than the photogrammetric method costs. Since the accuracies obtained by using photogrammetric methods are entirely adequate for this class of survey, it is concluded that the photogrammetric method offers a satisfactory solution to an otherwise costly procedure of cadastral surveying.

Cadastral Surveys by Photogrammetry

DANIEL M. McVAY, Civil Engineer, U. S. Forest Service

THE U. S. Forest Service, an Agency of the Department of Agriculture, administers some 186 million acres of National Forest and National Grasslands located in 44 states and Puerto Rico. These lands generally fall into the following three categories: (a) National Forest lands reserved from the public domain—159 million acres located mostly in the West; (b) National Forest lands acquired primarily under Weeks Law—23 million acres located mostly in the East; and (c) the National Grasslands—4 million acres located mostly in the western plains states, but also scattered elsewhere throughout the United States. These lands are called the National Forest System.

The Forest Service has more than 281,000 miles of property lines between lands in the National Forest System and lands owned or administered by others. More than 1,132,000 land survey corners are required to control these property lines.

The National Forest System lands are not grouped together in solid blocks of government ownership, but are often intermingled in a complex pattern with lands owned or administered by others. Because of the age of many of our land surveys, the complex ownership patterns, and the miles of property lines involved, the Forest Service has some complicated property line problems. The following conditions contribute to these problems:

1. Federal regulations contain no provision for the maintenance and perpetuation of the surveyed lines and corner monuments of public land surveys.
2. Even though Government survey markers are protected under Federal law, corners are frequently destroyed either intentionally or by accident. Attempts to prosecute violators are seldom successful and therefore not often undertaken.
3. Due to man's destructiveness and to time and the elements, many surveyed lines and corners have now completely disappeared.
4. In the rectangular survey system there is a common but shortsighted practice of granting title to rural parcels of land by aliquot parts of section descriptions, without requiring an official subdivision-of-section survey to mark the property lines and corners on the ground. We therefore have many miles of property lines that have never been surveyed, in addition to the miles which were once surveyed but for which the lines and corners have been obliterated and lost.
5. Cadastral surveys conducted under State authority have the same built-in decay factors and same rate of corner destruction through ignorance, carelessness, or malice.
6. For land surveys in rural areas in many States there are no regulations concerning survey accuracy or standards for property corner monumentation, no prescribed format for survey notes or plats, and no provision for filing survey records as public documents or for making the records available on request to those who need them.

Fortunately, most official U. S. Government surveys have excellent records in the form of survey notes and plats. Copies are readily available on request from the Bureau of Land Management. During ground search for survey evidence, these notes and plats enable the searcher to know what to look for, to verify evidence that is found, or to definitely establish that all evidence has disappeared.

Paper sponsored by Committee on Photogrammetry and Aerial Surveys and presented at the 46th Annual Meeting.

Notes and plats of land surveys done under State authority, however, are often difficult or impossible to find. Sometimes they are not much help if found. Corner monumentation also often leaves much to be desired. Valuable time is often spent searching in vain for the written records of survey evidence discovered on the ground, and also in searching on the ground for evidence of survey work indicated in the records to have been done.

We would certainly welcome the day when each State, in collaboration with the State Land Surveyor Societies and Boards of Professional Registration, would not only regulate the registration of licensed land surveyors, but would also:

1. Establish an Office of State Land Surveyor;
2. Prescribe land surveying accuracies (which are commensurate with specific needs);
3. Set standards for land survey corner monumentation;
4. Prescribe a format for land survey notes and plats;
5. Prescribe a format for records of corner remonumentation work;
6. Provide for a centralized official repository for the required filing of official land survey notes and plats (and corner remonumentation records) as public documents;
7. Provide for public inspection of cadastral survey records and for furnishing copies for a nominal fee on request; and
8. Provide effective protection under law for land survey monuments and accessories on the ground and for the preservation of official land survey records in a centralized office.

SURVEY AUTHORITY

National Forest System lands fall into two main categories as far as authority to do official land surveying is concerned. These are:

1. Land reserved from the public domain. These lands have never been in private ownership. Authority to conduct official land surveys on these lands is vested in the Bureau of Land Management, an Agency of the Department of the Interior.
2. Acquired lands. These lands have at one time been in private ownership. When land originally passes from U. S. Government ownership to private ownership, Federal land survey authority ends. The land becomes subject to the applicable land surveying laws of the State. Even though the U. S. Government reacquires title to these lands, the survey authority remains with the State.

Generally, either the Bureau of Land Management or the State-authorized land surveyor has authority to survey property lines between these two categories of land. The foregoing is, of course, an oversimplification of this subject. There are various exceptions, grey areas, and overlaps in these matters.

MANAGEMENT POLICY

National Forest System lands are managed under the principle of multiple-use sustained yield. This means (a) managing all the various renewable surface resources so that they are used in the combination which will best meet the needs of the people consistent with the capability of the land; (b) achieving and maintaining a continuing high-level annual or regular periodic output of these resources; and (c) accomplishing this without impairing the productivity of the land.

The manner of managing these publicly owned lands and their resources directly affects the well-being and economy of nearby communities and people. Directly or indirectly it may affect all the American people.

One of the first requirements for effective land management is to know the correct location of land ownership lines on the ground. (There are certain rather obvious advantages to managing the right areas.)

LAND LINE LOCATION PROGRAM

Public demand for increasingly intensive use of National Forest System lands and resources creates urgent needs for accurate well-marked property boundaries. Because of this need, the Forest Service in 1958 set up a special Land Line Location Branch in the Division of Engineering specifically to do cadastral work.

The purpose of the program is to locate accurately and mark adequately the property boundaries of National Forest System lands. Activities of this program fall logically into three main parts:

1. Recovering what remains of each controlling corner of each property boundary and preserving its location with an enduring corner monument;
2. Obtaining the official cadastral surveys required to reestablish the property lines and corners that are lost and to establish the needed new lines and corners; and
3. Marking these property lines so their location is apparent on the ground and setting up a continuing maintenance program to insure that the lines and corners will not be obliterated.

CADASTRAL SURVEYS BY PHOTOGRAMMETRY

Forest Service Tests

Because of its pioneering in, and successful use of, aerial photographs and photogrammetry for such important work as mapping, timber management, range inventory, fire control, pest detection and control work, road reconnaissance and location, and ground search for land survey evidence, it was probably inevitable that the Forest Service would also investigate using photogrammetry for cadastral surveying. Initial Forest Service tests, using second-order stereoscopic plotting instruments and graphic methods for distance and angle determination, did not provide the accuracy sought. Further tests, using precision optical train photogrammetric instruments, high-precision aerial cameras, special photography, pre-targeted ground points, and computational rather than graphic methods, did produce accuracy satisfactory for most Forest Service cadastral survey needs.

Of those official cadastral surveys accomplished photogrammetrically by the Forest Service, the following two projects are of special interest. One was on National Forest land reserved from the public domain. It was a dependent resurvey in T. 18 N., R. 8 E., Mount Diablo meridian, California, in the Tahoe National Forest. By this survey, the original lines and corners were reestablished. The work was done in cooperation with the Bureau of Land Management, the photogrammetric work being planned and executed by the Forest Service. This survey has been approved by the Director, Bureau of Land Management. The official plat and survey notes are on file in that Agency, from whom copies can be obtained on request. A description of this project by J. E. King (U. S. Forest Service, Retired) was published in the June 1957 issue of Photogrammetric Engineering.

The other project was on acquired National Forest land. This was a resurvey of T. 36 N., R. 9 W., fifth principal meridian, Missouri, in the Mark Twain National Forest. It was done in cooperation with Dr. Clair V. Mann, County Surveyor of Phelps County, Missouri. Photogrammetric work was planned and executed by the Forest Service. It is an official land survey under Missouri State authority. Official records of the survey are on file in the county office, and are available on request. A description of this project, prepared by Dr. Mann and titled "The Case for Adoption of Photogrammetric Methods in Land Surveying," was published in Photogrammetric Engineering, Vol. 29, No. 5, pages 556-860. A complete account of the project has also been prepared by Ray F. Fassett, Chief of the Surveys and Maps Branch of the Division of Engineering in the Forest Service Regional Office, Milwaukee, Wisconsin. Individual copies of the report may be obtained by writing to that office.

Present Application

Corner Search—Much of our present use of photogrammetry for cadastral work is for "search and rescue" operations to recover on the ground the remaining evidence of the survey corners that control our property lines so we can perpetuate them before all evidence is gone. We find that to reestablish a corner by cadastral survey after all acceptable evidence has disappeared is approximately ten times more expensive than to recover and remonument the corner before it is completely obtliterated and lost. Because of the age of a large percentage of the surveys in our areas of interest, survey evidence is rapidly disappearing. There is an urgent need to complete this corner search and rehabilitation work.

For corner search we use relatively simple but effective photogrammetric methods to plot up existing surveys on a good map base and then to transfer these plotted corner point locations to suitable aerial photography coverage of the area. Photos containing these plotted corner locations are then used to select the best route to the corner and a search is made in the immediate vicinity of the point shown on the photo as the approximate corner location. These photos, together with complete copies of the survey notes and plats, suitable maps, and a few simple tools, enable a trained and experienced corner search party to operate with minimum lost motion and to attain maximum recovery of survey evidence.

When corner search has been completed in an area we know what additional cadastral survey work must be done. We also have the information needed for selecting the most suitable method for doing the surveying. Corners with acceptable remaining evidence are remonumented and official records prepared. Corner remonumentation must be done under proper survey authority. The remonumentation party, therefore, must i include either a Bureau of Land Management surveyor or a State-licensed, registered land surveyor, depending on the status of the land involved.

When the surveyor is a Bureau of Land Management employee, that Bureau supplies its official corner monument. Also, the remonumentation notes become a part of the agency's official survey records. When the services of a registered land surveyor are used, the Forest Service furnishes a blank, brass-cap monument. The surveyor's name and registration number are stamped on the cap along with the corner designating letters and numbers at the time the monument is set. Due to the present inadequacy of State laws governing the official preparation and filing of survey records for work of this type in rural areas, the Forest Service must also provide the forms to be used to record the work and safeguard this record until arrangements can be made for its official filing. This is generally in the courthouse of the county where the land is located.

In this program, the Forest Service fully recognizes that it shares the ownership boundaries with the adjoining landowner. It respects the rights of these neighbors. It is anxious to obtain their cooperation and support in locating accurately and marking these common property lines.

Surveys—The Forest Service continues to use photogrammetric methods in other cadastral survey work when applicable. Procedures used are generally as follows:

1. Existing aerial photographs and photographic identification of the remaining corners (recovered by the procedures previously described) are used to construct an accurate large-scale plat showing previously surveyed land lines and, by protraction, the required new lines.
2. The locations of missing corners, and new corners to be established, are projected to the aerial photographs. Then the photographs are used to locate the approximate position of each corner on the ground.
3. All existing usable horizontal and vertical control is recovered and any required additional control is monumented and accurately surveyed. The surveyed position of each control point is converted to State plane coordinates.
4. A photographic target is placed on the ground at each point to be used for horizontal and vertical control and at each recovered property corner, and also at the approximate location of each property corner which has been lost and at each new corner which is to be established.

5. After the ground targets are in place, the area is photographed with a precise aerial camera.

6. The aerial photographs are used in a precision optical train photogrammetric instrument to bridge between horizontal control points and to obtain instrument coordinates for each targeted property corner. Bridge adjustments and State plane coordinates, when applicable, are then computed by electronic data processing methods.

7. Using the coordinates thus obtained for each targeted and known corner point, plus the original survey notes, the plane coordinate position is computed for each of the corners not recovered on the ground, then for each corner of the new subdivision of section corners to be established.

8. Having the photogrammetrically measured coordinate positions of the lost corner points, of the new corner points that are to be established, and of the ground targets set near them in the field, the distance and direction are computed from the target to its respective corner point location.

9. The distance and direction from the target to the corner point are measured on the ground and suitable corner monuments and accessories are set.

10. Official notes and plats are prepared.

Rights-of-Way—Photogrammetry is a useful tool in rights-of-way acquisition work. During the last five years the Forest Service has processed an average of 1500 rights-of-way cases each year. When applicable, photogrammetric methods are used in cadastral surveying needed for rights-of-way acquisition.

When photogrammetry is used in making surveys for road location and design, it is often advantageous to plan and accomplish required cadastral work along with the photogrammetric work done in connection with the road location. For discussion purpose, this cadastral work could be considered to be in two categories:

1. Unofficial measurements needed to show the right-of-way location in relation to existing land ownership lines and corners, but which do not change or add to any actual existing official land surveys on the ground, and

2. Official cadastral surveys needed to establish new lines and/or corners, or to reestablish old ones. This would involve altering or adding to existing land surveys on the ground.

In the first case, the right-of-way plat and deed description would suffice for the record. In the second case, in addition to required right-of-way plats and deed descriptions, official cadastral survey plats and notes must be prepared, certified by proper authority, and filed as public records of official land surveys.

Mineral Claims—There are many complex property lines on National Forest System land reserved from the public domain because of patented mineral claims. The method of laying out these claims results in claims crisscrossing and overlapping in some areas and leaving gaps in others. This creates many irregular property lines and isolated parcels of Government land of various shapes and sizes.

Most of these claims were surveyed many years ago. Most of the corners and lines are now difficult to find on the ground. Many others are completely lost. Our photogrammetric corner search methods are especially helpful in untangling the ownership patterns in these areas. Cadastral surveys to relocate our property lines and the corner markers which control them are frequently best accomplished by photogrammetric methods.

CONCLUSION

The Forest Service uses photogrammetric methods for cadastral surveying when this method is deemed best suited for the job at hand. Official land surveying, regardless of the system, the methods or the equipment used, must be conducted under the responsible charge of capable professional land surveyors who are duly authorized under State or Federal law to perform this service.

Sometimes the question is asked, "Is it legal to use photogrammetric methods to make cadastral surveys?" We respectfully suggest that it is the responsibility of the authorized land surveyor who is in charge of the work to design the survey and to select the methods and the equipment that will be used. His certification of the finished survey, under proper State or Federal authority, determines the legality of the survey, not the methods and procedures used to do the work.

Precision Photogrammetry and Highway Engineering

ROBERT C. RUTLAND, Engineer of Photogrammetry, Texas Highway Department

•THERE is great need on the part of highway engineers to obtain as much information as possible regarding the nature and extent of the terrain along the route of a proposed highway location. In past years this information was collected from the field in the form of rough sketches, field survey notes, traverses, field-measured cross sections, and similar data, and transported to the office for evaluation and use in design. The methods used for these on-the-ground topographic surveys were accurate and the information obtained was necessary to the designer. However, it became more and more obvious that the methods were incomplete and that the time consumed in these topographic surveys was requiring a larger and larger percentage of the overall time allotted for the completion of the highway. Therefore, the engineer had to decide whether to continue his usual method of operation under a system that offered him something less than complete information from the field or to find a new and more acceptable means which could be used as a basis for his design. Photogrammetry has supplied part of the answer to this need and is now used successfully throughout the world as an economical and rapid method of obtaining topographic information in a minimum of time.

Photogrammetry in the beginning consisted mostly of very rough and inaccurate measurements taken from the aerial photograph itself. Although the precision involved in this type of data gathering was not that required for highway design, it was at least a method of transferring information regarding the character of the terrain from the field to the office in a very rapid manner. As the science of photogrammetry became more sophisticated, and newer and more accurate instruments were available which could translate the information on the aerial photographs to maps with a consistent and accurate scale, both horizontally and vertically, a new tool came into existence which enables the engineer to spend more time in design work and less in gathering data.

This paper concerns itself with the method employed by the Texas Highway Department Division of Automation in the photogrammetric compilation of topographic and planimetric maps and measurement of cross sections, preparation of photographic mosaics, and obtainment of other engineering data. It also outlines the uses of photogrammetrically obtained data in the different phases concerned with locating and designing a highway.

GROUND CONTROL

At the beginning of the operation when the survey is in the planning stage, if time and weather permit, a high-altitude photographic flight is made of the area. Contact prints or uncontrolled photographic mosaics made from this photography are furnished to the field crews to enable them to plan the control surveying work as closely as possible before arriving at the site in the field. In many instances, the general survey can be laid out on the photographs before the field crew begins its work. It is obvious, however, that this can never be done completely and a decision must be made in the field concerning the individual placement of each survey point.

Paper sponsored by Committee on Photogrammetry and Aerial Surveys.

Since the photogrammetrically compiled maps must have complete and accurate ties to the terrain, it is of course necessary to conduct a field survey as the first step in the photogrammetric compilation of any map. The type of ground survey involved is not unique and has been used for many years and called by a variety of names, such as location survey, meander traverse, and basic control survey. As used by the Texas Highway Department, it can best be described as a survey of at least second-order accuracy, meandered through the general area of which the map is to be compiled. The survey is either closed upon itself or tied at each end to Federal Government triangulation stations or closed by celestial observations. Of necessity, this survey must be done as rapidly and as accurately as possible since its expense will exceed that of any other step involved in the photogrammetric mapping procedures. In addition, if the survey were not a rapid and efficient operation, then photogrammetry could offer no advantages over the regular ground location survey for a highway except completeness of detail. Therefore, automation in this phase is at least as important as automation of any other phase of photogrammetric methods. In the Texas Highway Department, Electrotapes are employed for all distance measuring and one-second theodolites (at times 2/10-second theodolites) are employed for measuring angles.

When need has been established for a new highway location, this Electrotape-theodolite traverse surveying usually begins on a USC&GS or a USGS triangulation monument and traverses the area to the end of the project, tying into a similar monument. The survey is conducted much as if it were a simple tie from one triangulation station to another with semipermanent points being established along the traverse at intervals of from ½ to 1 mile. Naturally it is necessary that the survey be confined to a particular corridor which the photogrammetrically compiled maps will cover so the control points will appear on the aerial photographs. Because of the very few restrictions imposed on this survey it is a rapid, therefore economical, procedure and the main concern of the crew in conducting the survey is to establish and survey control points which are intervisible, recoverable, and remain within the general area of the maps. If necessary, survey towers are used to avoid cutting lines of sight through wooded areas. Automation of this phase has progressed in many cases to the point where helicopters are used to transport the crew from point to point, and this is fully justifiable for extended surveys in rough country because of the economics involved.

To make this survey as efficient and economical as possible, the crews are equipped with 5-passenger pickups having specially constructed enclosures for the transportation of the surveying equipment. All crews are equipped with at least three portable two-way radios ("walkie-talkies"), two sets of binoculars, and two sets of more powerful two-way radios installed in the trucks for communication with the photographic aircraft, with the resident engineer, and with the walkie-talkies. Simple tape recorders have been utilized in the past to enable the instrument operators to describe the terrain, weather, instrument stations, and so forth, as quickly as possible without the necessity of manually writing this information on the data sheets. However, we discovered this type of information is rarely necessary and abandoned this means of communication completely, as it did not seem to add to the efficiency of the overall survey project.

Instruments have been tested which record the numerical information produced by the Electrotape and the theodolites directly upon magnetic tape. These units were used in the field with the hope that the ordinary data sheet could be abandoned and the process completely automated from beginning to end. However, the instruments now available on the market which are small enough and portable enough for efficient field use do not provide for checking of the information which has been entered and therefore it has been our experience that more transposition errors occurred which could not be definitely located than in the utilization of the written data sheet. Consequently, the data sheet remains in effect and has been simplified and reduced as much as possible so as not to delay the control surveying work in any manner.

We have chosen the 5-passenger pickup truck for obvious reasons. Its only competition as far as efficiency is concerned would be either the regular pickup or the station wagon. The station wagon is an excellent means of transporting personnel or equipment; however, as a transporter of both personnel and equipment, it is extremely inefficient and cumbersome. In addition, it is strictly a road car, and operations off the paved highway are severely curtailed. The normal 2-passenger pickup has the same advantage as the station wagon, inasmuch as it will transport both personnel and equipment. It is at a greater disadvantage, of course, in transporting personnel since a majority of the crew must ride in the pickup bed, which is not only unsafe on the open highway, but also extremely uncomfortable. Therefore, the 5-passenger pickup is a compromise which, in our opinion, combines the better qualities of each since it will carry 5 crew members comfortably and all the equipment necessary to maintain this crew in the field. With the installation of a steel cover on the bed of the 5-passenger pickup, a higher degree of security for the $15,000 to $20,000 worth of specialized equipment which is transported in it is obtained, particularly when compared with the station wagon. This is an important feature to consider, inasmuch as these trucks are in the field for at least a week at a time and if station wagons were used it might not be uncommon to find that the loss of equipment would be somewhat higher due to the extensive glass area in the station wagon and the rather easy entrance that can be gained into such a vehicle.

Texas is unique among the States in that its shape and area present a considerable problem in transporting the crew from one survey project to another. It was the original intention, and remains so, that the photographic aircraft be able to transport the survey crew from project to project as much as possible; however, since the Texas Highway Department began to use photogrammetry in its highway design work, it has been impossible to release the aircraft from its primary duty of securing aerial photographs and to assign it to serve as a transportation vehicle. It might be possible that the aircraft would be available to transport the crew to a project but would be unavailable for the return trip to the home office. Therefore, this step has not been satisfactorily resolved as of this time. In addition, the variance in weather conditions throughout the State presents a problem in that the departure station may be clear and suitable for aircraft flight but the landing station might be experiencing conditions which would make arrival impossible. Therefore, even though transportation is slower, it has become necessary to use the modified 5-passenger pickups for crew placement throughout the State. This slower travel can consume as much as one entire working day; however, with the advantages of the equipment and techniques which have been developed for this crew, the small percentage of the overall time can be absorbed without raising the cost of the survey by an appreciable amount.

With the utilization of these instruments and procedures, it has been possible to arrange the personnel involved in the field survey in a very efficient manner. Though this procedure must be altered for each individual project, broad outlines can be mentioned which form the framework of a normal survey project. One crew consisting of three or four men begins at the first triangulation station and selects the points to be surveyed by driving the concrete reinforcing steel rods at intervals throughout the project. Naturally, the work of this crew requires that at least one member be completely familiar with the operation and have the ability and judgment that is necessary in choosing the path for the crews that follow. The Electrotape crew follows and makes the measurements between the points that have been previously set. This crew consists of a minimum of two men but usually has an additional one or two men for transportation and carrying equipment. The theodolite crew then measures the horizontal and vertical angles from one point to another. These three basic crews can be manipulated in a variety of ways. For instance, the Electrotape crew and the monument-setting crew can be combined. This is sometimes more efficient in reducing the number of personnel required to complete a project, so long as the Electrotape is not unduly delayed because of this combination. It is also obvious that the theodolite crew could precede the Electrotape crew without a loss of time or efficiency. Therefore, some phases are completely interchangeable unless other factors intervene. If the theodolite crew precedes the Electrotape crew, then the monument crew can be combined with the theodolite crew to effect a savings in the number of personnel involved.

Another item favorable to this setup, particularly in Texas, is the fact that the entire ground control survey need not be completed in one continuous operation. In fact, it has been common in such operations to send the monumenting and Electrotape crew to one project for the initial phase of the survey and to send the theodolite crew to a different part of the State on a completely different project to finish up a survey already begun. Since this combination of crews is an Austin-based operation in the Texas Highway Department, it is essential that flexibility be maintained, particularly when the vast amount of territory that must be covered throughout the State is considered. Therefore, it is possible to have two or three surveys in different stages of completion at one time and also to have all surveys being processed to completion without delay.

Monumentation of this field control is usually done by using iron pins, usually one-inch diameter concrete reinforcing rods driven into the ground approximately 3 feet, where soil conditions will permit. Although this type of monument cannot properly be considered permanent, it seems to offer adequate service and requires much less time to install than the more familiar concrete shaft and brass cap arrangement.

When the survey has been completed, the data sheets from both the theodolite and Electrotape measurements are returned to the office where the information is automatically recorded on magnetic tape as computations on the field data sheet are being checked. This small magnetic tape is processed through a translator to produce the standard magnetic tape and then fed directly into the 1604 computer along with a traverse program to obtain the order of closure of the traverse, the State plane coordinates of each control point set and surveyed in the field, the adjusted surface coordinates of each point, the true horizontal distance between the points adjusted throughout the traverse, and the true angle at each PI station between traverse courses adjusted for closure error. The computer program used in this process is written in such a manner as to reject traverse survey data which does not meet at least second-order accuracy (1:10,000). It is interesting to note here that such a rejection by the computer is extremely rare and accuracies have been obtained of one part in 95,000 with no increase in effort beyond what has been described.

Concerning progress obtained using the procedures described, it is difficult to give an average rate per day of miles surveyed or control points set because of the extremely varying nature of the terrain and ground cover found in Texas. In areas where ground cover and trees are not a limiting factor and the terrain is easily accessible, progress has been recorded of up to 20 miles of traverse surveyed in one day. This of course requires that the vehicles be driven from point to point without excessive meandering, and that beginning and closing monuments be readily available.

TARGETING

Immediately upon completion of the ground control surveying and the verification of its closure accuracy by the computer, the survey project is targeted for the photographic flight. The Texas Highway Department has experimented with a great variety of materials and substances which could be used as photographic targets. These experiments began with a simple lime cross, progressed through a painted automobile tire, strips of aluminum foil, white plastic material and colored plastic material, centerline striping paint, regular gauze cloth, cotton muslin, strips of lumber painted white, and large aluminum or cardboard pie plates and similar materials.

For economy and ease of installation, a lime cross on the ground has no equal. However, it has many disadvantages in other regards. The main disadvantage which might be cited is that a rain shower, after completion of the targeting and prior to the photographic flight, can obliterate all targets placed along the project. In addition, in areas where cattle or other livestock are common, the targets can be scattered and identification can become extremely difficult.

The target must, of necessity, be formed by hand and therefore does not give a very clear and accurately defined image on the photography as do other materials. Pie plates and round shapes of other materials of this nature offer the danger, in certain areas, of misinterpretation of the targeted points and are not recommended. Painted strips of board are expensive and difficult to handle and transport and therefore should be used only when the supply is readily available and may be used again and again. Aluminum foil is not acceptable because of the highlights and configuration it imposes upon the photographic film. In some instances it may appear black and in some instances totally white, and in other instances any shade in between. Therefore, identification of this type of material is extremely difficult. Targets made of cloth or muslin or other woven material are very acceptable as far as registry is concerned. However, for new location work they appear to be very appetizing to local livestock and will not remain in place for any period of time. In addition they must be staked to the ground at several points to prevent the wind from destroying the configuration or completely removing the targets. In short, all types of targeting material have their shortcomings and one particular type cannot be used on all different survey projects. One project will require use of a certain material and another project something else for targeting purposes.

In the Texas Highway Department, we have narrowed the choice down to about two materials. One is the simple lime cross. Although it does have disadvantages, it still remains the simplest target to construct in the field. It is used only in those instances where targeting is completed immediately prior to the photographic flight and there is no indication that weather conditions will delay the flight for any considerable period of time. In the other instance, it has been found in large-scale mapping that white cardboard cut into strips and placed in the form of an X or a cross have served the intended purpose very well.

It is contended that almost any target can be made to work if the interval of time between the targeting and the photographic flight is kept to an absolute minimum. No great difficulty has been experienced in identifying the target on the photographs because of its configuration or the material used, or at least not nearly so much difficulty as has been experienced in targets being moved or destroyed prior to the flight. The size of the individual legs of each target are of course dependent upon the scale of the photography and therefore the flight height of the aircraft. Targets with legs 4 feet in length are adequate for photographic scales of 200 feet to one inch.

On locations along existing highways, targeting is much less of a problem. Painted targets on an existing highway can be made to remain for long periods of time—at least two or three months. In fact, this is one of the shortcomings of painted targets on highways. This type generally exists for several months after the project has been completed and is a source of extensive curiosity to the neighboring landowners or passersby. If this sort of target is used, it is recommended that a paint be employed which can be removed without much difficulty or which would wear off the pavement within a reasonable length of time.

As to the color of the target employed, white has been found to be excellent where the background is contrasting though not of an opposite color in contrast. By that it is meant that white against green or grey stands out very well and no difficulty has been experienced in regaining this point both horizontally and vertically on the stereoscopic models. However, the white target on concrete and at times even on weathered asphalt pavements appears to have a height of from 1 to 1½ feet when viewed and measured on the stereoscopic models. If the purpose of the target is strictly horizontal control this does not become a problem; however, if the target must be used as a point of vertical control, then the purpose has been defeated and the target is practically useless in accomplishing large-scale mapping by photogrammetric methods. William T. Pryor of the Bureau of Public Roads has done extensive work in the investigation of colored targeting material and it is suggested that his publications be consulted for further information concerning this subject.

In addition to the targeting of the basic control points, it is desirable to target as many as possible of the existing property corners, intersections, and so forth, with other survey lines and facilities which cross the control survey traverse. This procedure allows for the measurement of the plane coordinates of each of these points with the photogrammetric instrument and provides a tie which could become invaluable during the progress of the surveying, mapping, and engineering for the project.

AERIAL PHOTOGRAPHS

To obtain aerial photographs for photogrammetric surveying and mapping projects, the Texas Highway Department uses a Cessna 206 Skywagon aircraft equipped with a

Wild RC-8 6-inch focal length aerial camera. The Cessna 206 was originally a 6-passenger aircraft but has been modified so that it now has the pilot and co-pilot's seat and a seat for the photographer behind the camera. This leaves ample room for the storage of extra magazines and film and also provides space for to-and-fro movement by the cameraman. This type of aircraft was chosen primarily because of its ability to reach the farthest part of the State in a reasonable time and to carry sufficient fuel to enable it to remain in the air for approximately 6 hours at cruising speed, and because of its ability to fly at the rather slow speeds which are necessary in obtaining aerial photography for large-scale mapping.

The operation of the flight crew must be correlated very closely with the operation of the field survey crew. Information concerning the estimated completion date of the survey is transmitted to the Austin office for relay to the photography crew to insure that the aircraft will be available as soon after this completion as possible. Of course, weather is a contributing factor in the smooth operation of this phase.

A flight map is constructed from the photography which was obtained for the original ground survey to enable the pilot to plan the flight lines, to determine the number of photographs necessary and the amount of film required on the project, and to ascertain all other needed items. If photography was not taken for the original survey, the flight map generally consists of a United States Geological Survey topographic map or a Texas Highway Department planning and survey county map.

In the interest of economy, an effort is made to have several survey projects scattered throughout the State ready for photography flight. In this manner, the aircraft spends much less time on the ground due to poor weather conditions at any one project, since in the event one portion of the State is overcast, it may be possible to fly to and photograph a project at the opposite end of the State and utilize the aircraft to its fullest extent. In addition, an effort is always made, when a project is to be photographed, to notify the Highway Department District Offices in areas along and immediately surrounding the flight route so they may have the advantage of securing aerial photographs for purposes other than mapping at considerably less expense than would be involved in a separate aerial photography trip for each purpose.

After exposure, the film is returned to the Austin office for processing in the Reproduction Section of the Automation Division. This section can process a 200-foot roll of aerial film in approximately 2 hours and return it to the Photogrammetry Section for inspection. The aerial film negatives are studied for location of targets on markers of survey control and determination of scale and then returned to the Reproduction Section for production of blueprint positives from the roll of negatives. This continuous roll of blueprint positives is cut into individual photographs, overlapped, and stapled together to form one continuous strip. This index is inspected for excessive x-tilt and y-tilt, for excessive crab in the flight line, and to insure that the desired area had been photographed. If at the completion of this phase the flight has proven to be a successful one, contact prints are made from the roll of aerial photography negatives. The ground survey stations are identified on one set of these contact prints and this information is forwarded to either the layout section or the optical train photogrammetric instrument bridging section for further handling. Another set of contact prints is forwarded to the District Office concerned with the survey project for study and evaluation by engineers in that office. A third set of contact prints is formed into a photographic index for future reference and the recall of individual photographs.

If a photographic mosaic is to be made of the area, work is now begun in cutting and assembling the individual photographs to form a composite of the entire area of survey. An uncontrolled photographic mosaic is a valuable aid in determining a preliminary location for the highway route to avoid undesirable grades, curves, and man-made obstacles. Preparation of a controlled photographic mosaic requires at least as much field work as the photogrammetric compilation of maps and does not provide sufficient accuracy over that of an uncontrolled mosaic to justify its greater expense. A good controlled mosaic would cost nearly as much as a completed planimetric map and does not offer the same accuracy.

It has been contended that a good designer with a set of aerial photographs providing stereoscopic coverage of an area and an uncontrolled photographic mosaic of the same area can select the most feasible location for a highway route in a very short period of time and with very little difficulty. This condition does not hold true, however, if the designer intends to obtain angles and distances from the mosaic or the photographs. The camera eye sees the ground and features thereon in perspective, as if they were on a perfectly level plane. All differences in elevation are reduced to this plane along lines which radiate from the center of the camera lens. This perspective positioning of images is impossible to remove in contact printing of aerial photographs and its effect can only be reduced by projection printing techniques. When this deficiency is combined with x-tilt and y-tilt, which may occur in any photograph, the difficulties presented constitute an absolute barrier to the obtainment of accurate measurements by use of photographs without employing complicated mathematical computations.

An example of image displacements on an aerial photograph can be imagined by observing a picture taken with a 180-degree fisheye lens. The image displacement present in this type of photograph is identical to the displacement present in any aerial photograph, except in a greater degree. The science of making measurements directly from aerial photographs is an art in itself and usually requires complicated electronic computer programming and use of very precise measuring instruments.

The Texas Highway Department has recently begun to experiment with aerial color photography with fairly good results. Two types of film have been used in this experimentation. One type produces a color positive transparency and the other produces a color negative. Both have their definite places in the practice of photogrammetry. It is believed, however, that the type of film which produces a color negative has the greater promise for overall use. The color negative film is much simpler to process and it is possible to produce from the negative either a color print or a black and white print as needs may dictate. The development of this color negative film is similar to development of ordinary black and white film with the exception of closer temperature control and the number of development solutions which have to be used in the process. Color photography promises to provide extremely valuable information to the geologist, the soils engineer, the forestry engineer, the hydraulics engineer, the right-of-way engineer, and others.

EXTENSION OF CONTROL BY PHOTOGRAMMETRIC METHODS

In the compilation of large-scale maps by photogrammetric methods, such as those compiled at a scale of 40 feet to one inch, it is necessary to have a control point approximately every 300 feet along the flight strip of photographs in order that the projection-type stereoscopic plotters can be used to compile the map. There are two methods of obtaining these necessary control points. One is by ordinary field surveys and the other is by the extension of field survey control points by using photogrammetric instruments.

The methods employed in establishing the basic survey lines described previously in this paper were based on the assumption that the control would be extended using the optical train Stereoplanigraph Model C8. As has been mentioned before, the ground surveying portion of a photogrammetric surveying project must be kept to an absolute minimum if the full value of photogrammetry is to be realized. Therefore, in setting and surveying on the ground control points which are from one-half to one mile apart in the control phase it is possible to measure the position of intermediate supplemental control points, using a Stereoplanigraph or similar instrument, spaced at intervals of 300 feet along the survey path. These intermediate supplemental control points do not exist on the ground. They are, of course, selected images on the photographs of ground objects, man-made features, and photographic patterns. It is possible to measure the coordinates of these points and to use them throughout the photogrammetric process as though they had been set directly on the ground by a survey crew.

As utilized by the Texas Highway Department, the Stereoplanigraph Model C8 will measure a traverse from one-half to one mile in length or further if necessary, beginning with one field-surveyed control point and ending on another field-surveyed control point, and will furnish coordinates for any number of intermediate supplemental control points that may be desirable. The digital record of Stereoplanigraph measurements comprises the data used in a computer program to compute the X, Y, and Z coordinates of all measured points in the coordinate system used in photogrammetric compilation of the maps.

The horizontal extension of control is more than adequate for the compilation of planimetric maps at a scale as large as 40 or 20 feet to one inch, but for topographic mapping at such scales, vertical extension of control is not possible within reasonable accuracy requirements. A procedural system is now being developed in conjunction with the University of Texas Research Department which will allow both horizontal and vertical extension of control to be done on a purely mathematical basis using photogrammetrically made measurements in a computer. It is anticipated that this system, when perfected, will by no means replace the Stereoplanigraph. It will, however, supplement its abilities to a great degree. When the scale of a map to be compiled is as small as 200 feet per inch or smaller, vertical control can be bridged in the same manner as the horizontal control. When the process of establishing supplemental control points is complete, the information obtained is forwarded to the layout section.

LAYOUT

Information concerning location of the basic control points measured by the field survey crew and the supplemental control points measured by use of the Stereoplanigraph is forwarded to the layout unit to be placed on the original manuscript. This process includes utilization of an Aristo Coordinatograph with the capability of establishing and reestablishing any particular point within an accuracy of 1/1000 inch. The operator sets up and draws the coordinate grid system, identifies each grid line, and plots the control points in their proper position in relation to the grid system on the manuscript. The coordinatograph is equipped with either drawing heads or scribing heads, either of which may be utilized in this process. It is in this phase that the breakpoints of individual sheets are determined and the limits of the project are established on the manuscript.

In addition to the layout capabilities, this unit is charged with the responsibility of relating points on a previously compiled map to some coordinate system. This involves calibrating the coordinatograph to integrate its vernier system into the grid system of the existing map sheet and measuring the position of each property corner, building corner, highway intersection, or similar point in terms of plane coordinates for that particular grid system.

Plans are now being formulated to automate this instrument with the installation of magnetic tape drive. When this is complete, information obtained from the field survey crew and from the Stereoplanigraph will be processed through the computer for the production of a programmed magnetic tape which will in turn operate the coordinatograph automatically in the drawing of grid lines, setting up of coordinate systems, and plotting the position of control points. When the control data have been transferred to the manuscript, both the manuscript and the digital information are forwarded to the stereoscopic plotting unit.

STEREOSCOPIC PLOTTING AND CROSS SECTION MEASURING

The Texas Highway Department now uses nine double projection stereoscopic plotters, two of which are equipped with Auto-trol measuring and digital recording devices. Because of the work load, the plotters are operated from 6:00 a.m. to 11:00 p.m. each working day. Upon receipt of the basic control information contained on the map manuscript sheet, the stereoscopic plotter operator orients his instrument and begins to measure and delineate the actual map features. The planimetric portion of the map is completed first, then all necessary contours are measured and delineated, and spot elevations are measured and recorded. The manuscript on which this information is placed consists of a sheet of mylar plastic covered with a coating of orange-colored emulsion. The emulsion has a matte finish which will accept ink or pencil lines. The photogrammetric instrument operator traces out the information provided by the photographic images in the stereoscopic model and with a pencil delineates desired details on the manuscript, and does finishing work with a straightedge and ballpoint pen.

In the event cross section dimensions are desired on a particular project, the manuscript is properly oriented on the working surface of the stereoscopic plotter equipped with the electronic measuring and recording device. This device enables the operator, through manipulation of attachments to the tracing table of the stereoscopic plotter, to measure and have recorded automatically on IBM key-punch cards the centerline station, distance from the centerline at which the elevation is measured, either right or left, and elevation of each individual point. This phase is completely automated. The input information consists of the map manuscript directly from the layout unit, and the output information consists of computer cards which contain all necessary information for automatic plotting of cross sections or for making earthwork computations. At a map scale of 40 feet per inch, the average accuracy of a group of points on a cross section is held to plus or minus 0.25 foot. In tests conducted as a comparison between this method of measuring cross sections and the regular field method of measuring cross sections, the difference in vomume resulting from the two methods (insofar as earthwork computations are concerned) is less than 2 percent.

SCRIBING AND REPRODUCTION

When a map has been compiled using a stereoscopic plotter, it is sent to the scribing unit where the pencil lines imposed by the instrument operator are scribed. Scribing is a process whereby the emulsion coating on the plastic is scraped off by use of edged tools, each of which has a specific line width. This process turns a manuscript into a negative and prepares it for the reproduction processes which follow. The planimetric portion of the map is scribed first and a positive reproducible copy is obtained for use in preparing right-of-way sheets and for other utilizations which do not require elevation indicators. After this is complete, a scriber returns to the manuscript and scribes all contours and spot elevations which have been indicated by the stereoscopic plotter operator. Thereafter, another positive reproducible copy is made of the manuscript which is in effect a master topographic map containing all planimetric features and all elevation indicators.

The positive reproducible copies, which have been printed on translucent cronaflex material, are then sent to the Reproduction Section for a variety of processes. If the original map manuscript was compiled at a scale of 40 feet to one inch, then photographic reductions are made to produce map copies at scales of 50 or 100 feet per inch, as the design engineer may require. In addition, the scale-reduced maps are printed in a plan and profile arrangement which is complete with a profile grid on the lower half of each sheet. Such sheets are used in submission of completed construction plans. The planimetric sheets are reduced to standard plan sheet size and used by the design engineer to construct the right-of-way plans for the highway construction project.

PHOTOGRAMMETRIC COMPILATIONS AND MEASUREMENTS AND THEIR USES

The uses made by design engineers of the photogrammetrically obtained data can best be described by the step-by-step procedure of the typical work involved in designing a highway. After the need for a highway has been established, the first step is selection of a route for its location. The designers request that either a photographic mosaic or a small-scale topographic map be made of the area of survey. If it appears that several alternatives are feasible and economics will dictate selection of the best route for the location, then preparation and use of the small-scale topographic maps enable the engineers to determine the best choice. The map offers the advantages of determining and comparing various grades and alignments, and computing the approximate cost of the highway based on each alternative. If the area of survey is such that only one obvious route is feasible, then use of a photographic mosaic is the better choice. The mosaic is much cheaper to obtain than a topographic map and allows the design engineer to determine highway location control points along a route which either must be met or missed, depending on the nature of the points involved. It is obvious that the highway location can be selected in the design office, using the topographic map or the photographic mosaic and stereoscopic examination of the aerial photographs, without doing extensive field reconnaissance work.

Once the route location has been determined, the engineer is ready to begin design of the highway on the route and preparation of detailed construction plans. To facilitate this phase, topographic maps are compiled at a convenient scale for the design. The scale is usually 40 feet to one inch to gain the advantage of a one-foot contour interval on the topographic maps. The designer selects the position for PI points along the route for establishing the highway location, and identifies these points directly on the maps. From the coordinate grid lines on the maps, the plane coordinates of these PI points are measured and recorded.

These plane coordinates, together with the curve data for alignment, are furnished to the Division of Automation. Through use of the computer and automatic plotter, the exact alignment, which has yet to be staked on the ground, is plotted precisely on the maps and stationed according to whatever system the engineer desires. Once the centerline data have been secured, ground surface cross sections are measured from the topographic maps through use of the Auto-trol and its electronic read-out and recording system. The design engineer can now determine the design template of the roadway for each station, and the profile gradients for the highway from the topographic maps. This design template is furnished to the Automation Division, combined with the ground-line cross sections measured in the office, and processed through the computer. Earthwork quantities are furnished to the design engineer from this information.

The engineer may then continue with his work in establishing the necessary right-of-way limits to enclose the highway in its three dimensions. This work is usually done on the planimetric maps. If all property corners have been targeted so they can be seen on the stereoscopic models and can be plotted on the maps, it is a simple matter to determine the amount of private property that must be purchased. Property ownership boundaries and ownership data are drafted directly onto the maps, and right-of-way limits are delineated to indicate the area of taking in each individual case. If the design engineer wishes, he may furnish the Division of Automation with the plane coordinates of the corners of each parcel and he will be furnished in turn the area involved, the length and bearing of each property line, and any other information he may desire. It should be noted at this point that the designer has not as yet found it necessary to go into the field to any extensive degree in designing the highway. With information at hand, deed descriptions may be written and right-of-way deeds prepared for ultimate purchase of the essential property. Drainage areas for design of bridges and culverts can be delineated on and measured from the original small-scale topographic maps, or in some instances, from the aerial photographs.

The photographic mosaics or the topographic and planimetric maps can be used at any of the public hearings concerning the highway location and design to furnish the landowners and other interested parties all the information necessary for them to understand the intent of the Highway Department. If condemnation proceedings are necessary, large-scale aerial photography discloses exactly all improvements which have been made on a piece of property and the extent to which these improvements may be damaged or enhanced by the highway.

Although this, of course, does not constitute all the work that must be done to locate and design a highway and to prepare its detailed construction plans, it does indicate that a highway project can be surveyed photogrammetrically with a great deal of efficiency, and that the resultant engineering can be as comprehensive, detailed, and accurate as necessary.

Precision in Surveys by Use of Plane Coordinates

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•TO ACHIEVE the abundant benefits which can accrue from properly located, designed, and constructed highways, adequate surveys must be made. Not only should the surveys be sufficiently comprehensive in scope and detail, they also must be appropriately accurate and properly preserved. Unless the surveys are so made and preserved, they will fail to be of lasting benefit—like, for example, an expensive and accurate watch purchased to obtain the time of day which is then lost or destroyed instead of being kept in good condition so that it will readily furnish as needed the time every second, minute, and hour of each day for many years. Moreover, loose-ended surveys of unknown accuracy and position with respect to other surveys are like the lost or destroyed watch. They served once but where and how well will not and cannot be known, and they will not serve again regardless of their initial cost.

For more than 20 years, improvements in methods and procedures on highway construction, especially by the labor-equipment team, have kept the average cost for contract excavation near the 1925-1929 unit price index. This has occurred despite a generally uniform increase in the hourly cost of labor, and the capital and operational costs of equipment. This is a good example of savings from progress in one phase of the multiple stage work being done in providing the highways needed by present-day traffic. Similarly, progress should be made in surveying for engineering and cadastral purposes. One aspect has been the introduction and use, since 1957, of electronic distance measuring instruments. Use of such instruments has not only decreased surveying costs, it has increased the accuracy of basic control surveys and improved the ability of highway engineers to make surveys and to stake designed highway alignment and rights-of-way lines on the ground more precisely. The advantages of such procedures should not be lost for lack of an effective method of recording and using measurements made in the stages of route location, preliminary survey and design, location survey, and construction.

The location of a particular point on the surface of the earth is specific. A mathematical expression for its position, such as X and Y plane coordinates or latitude and longitude, will not alter its position on the ground. Instead, the State plane coordinates computed using the accurate measurement tie made by survey to a station marker in the national network of geodetic control surveys to a point in a survey traverse, in a triangulation network, or on a property boundary, become a precise description of the point's location. Such use of plane coordinates to express the exact position of a point, in relation to an origin which is permanent, will perpetuate knowledge of its position so that the station marker for the point, whether on or in the ground, can be replaced exactly where it was, regardless of any physical changes occurring at and surrounding the point. This is true even though the marker may have been moved or destroyed since its plane coordinates were determined.

By this precision surveying and computing procedure of determining the State plane coordinates of each point in a survey for which a marker is set in the ground, the national network of basic horizontal control is extended. Also, the accuracy of the

Paper sponsored by Committee on Photogrammetry and Aerial Surveys.

plane coordinates defining the horizontal position of each point are determinable upon completing the survey closure ties to points of known accuracy in the national network. Whenever different coordinates are determined for the same point by separate surveys, the incremental difference is an expression for the variation between the surveys. Also, this procedure precludes need for surveying a closure back to the beginning point, which is a saving in both time and money for extended linear surveys, such as for highways, pipelines, and canals. Only boundary surveys should be circuit closed, but sufficient station markers of each of such surveys should also be tied to the basic network of geodetic control. Otherwise, the continuing and increasing benefits from extensions of the basic network of horizontal control will not be realized; instead they are lost.

Thus, to take full advantage of the accuracy and efficiency of precision-made surveys tied to a network of basic control and to achieve the lasting benefits of such surveys, utilization of a system of plane coordinates and preservation of each point of significance in each survey by station markers, which are permanent in character, are the practicable and economical essentials. In making surveys for highways, there should not be any exceptions to the recognition and application of such fundamental principles. Accordingly, for surveys made to accomplish engineering and related work, the purpose of this paper is to present the principles and essentials of using the system of State plane coordinates established for each State of the United States by the U.S. Coast and Geodetic Survey. Once such use becomes usual practice by engineering stages for each highway—its location, design, right-of-way procurement, and construction—the benefits will extend much beyond those for highways. They will accrue to the makers of surveys for all other purposes. The precision watch, so to speak, will serve indefinitely without reservations. Moreover, all surveys will become interrelated and contiguously joined together to serve as an ever-increasing extension of the national network of basic geodetic control into all areas for greater usefulness.

AUTHORIZATIONS

The Federal-Aid Highway Act of 1956 initiated two significant firsts in the engineering of highways, which have been continued in all subsequent Federal-Aid Highway Acts. First, the term "construction" was amended to include "the establishment of temporary and permanent geodetic markers in accordance with specifications of the Coast and Geodetic Survey in the Department of Commerce." Since then, the costs of basic control surveying for highway surveys could be defrayed by Federal-aid funds appropriated for highways. Thus, sufficient funds were and are readily available for originating and closing each highway survey on markers of the national network of basic control. Second, the Act provided that ". . . the Secretary of Commerce may, wherever practicable, authorize the use of photogrammetric methods in mapping, and the utilization of commercial enterprise for such services." The recognition and authorization by law of two fundamental facets of efficiency and economy for accomplishing highway surveys is noteworthy and commendable. Highway engineers are thereby provided with effective and reliable methods which will not only be beneficial when used but, like the accurate and well-kept watch, will serve indefinitely. Moreover, the value will continue to increase.

FUNDAMENTALS

Having an accurate and indestructible reference for each survey point has been the fundamental concept and hope for an untold number of years of surveyors and engineers concerned with surveys and design and rights-of-way and construction. The reasons for this concept and hope are many. Among those reasons most often repeated are the basic requirements of surveys for such purposes, which include:

1. The need for distances and directions measured on the ground to be the same, insofar as practicable, as distances and directions computed between each pair of survey points using their X and Y plane coordinates. The sameness, of course, would occur only within the recognized accuracy requirements of each particular survey. Consequently, precision is governed by the purpose of the survey.

2. The fact that measurements of distance, direction, and elevation have to be made many different times. The many different measurements are made to determine position, size, and shape; to stake designed facilities (including their rights-of-way) for construction; to set alignment and grade stakes during construction; to measure volumes of excavation and embankment and other construction features before, during, and after construction; and so forth.

3. The desirability of maintaining continuity from one survey to another, regardless of the purpose for which the points within each survey were set and position measured. This means there should be a discontinuance, wherever it exists, of the practice of making open-ended surveys which serve only a limited purpose.

4. The essentiality of preserving, recovering, and using survey points many different times for numerous and often different purposes within the same area Accordingly, each significant survey point should be properly preserved by use of an adequate marker for repeated use in the future.

5. The fundamental and economic advantages of knowing the accuracy of each basic measurement of direction and distance regardless of when, where, how, and by whom made.

STATE PLANE COORDINATE SYSTEMS

Engineering and cadastral surveys are the means by which the dimensions, position, and physical relationship of features of and on the earth and the boundaries of property are determined and recorded, first on the ground, second in appropriate notes, and third as plats and maps or as digital records. Whenever the conflicts which occur between surveys made at different times by different persons and/or methods are not resolved, confusion is inevitable. The resolution of conflicts and the elimination of confusion can be accomplished in the easiest and most effective manner by using a system of plane coordinates. Thus, when and wherever there is lack of decision to use State plane coordinates in engineering and cadastral surveying, the consequence is a decision to perpetuate confusion by failing to work toward continuity and achieving the general uniformity and accuracy desirable in surveys.

Historically, the first State plane coordinate system was developed in 1933 by the Coast and Geodetic Survey in response to the request of a highway engineer in North Carolina. This initiation became the impetus for establishing a plane coordinate system for each of the 50 States and Puerto Rico. Since then, the legislatures of numerous States have adopted by law for optional use the plane coordinate system developed for each of their respective States by the Coast and Geodetic Survey.

Engineering surveys have been open-ended, and separate property descriptions have been made by metes and bounds, and worded to facilitate determining the succession of owners back to the original grantee. By adopting the system of State plane coordinates, the significant points of engineering surveys and all property corners and points on property lines can be designated precisely in each of their respective positions. Once this is done each survey point can be resurveyed and placed in its exact position whenever required. No engineering survey point or property boundary need become "lost" because of the destruction or movement of a marker or markers from their initial position. Moreover, the problems of engineers and land surveyors endeavoring to use spherical coordinates in lieu of the older, less accurate methods have been eliminated. What remains is to get the system of State plane coordinates used for all engineering and cadastral surveys, and then the utility of spherical coordinates is thereby indirectly obtained.

Details of the State Plane Coordinate Systems

Achieving the greatest number of advantages and benefits from using State plane coordinates depends largely upon the engineers who use them with experience, creative thoughts, initiative, and action. Success or failure can occur, of course, depending upon the manner in which the principles are applied, but nothing need be left to chance if the basic principles are understood and applied properly.

There are two principal systems of projecting the curved surface of the earth to a plane by mathematical procedures. The earth is spheroidal in shape, but its surface for segments, as a country or a State, may be considered spherical in shape. The radius of curvature of the sea level surface of the earth is not, of course, the same at all places. At the poles, for example, there is a general flattening of the earth's surface as compared with its curvature at the equator. Consequently, the radius of curvature at the poles is larger than at the equator. This is so, although the distance from the sea level surface to the center of the earth is smaller at the poles than at the equator. These facts are utilized in each of the systems of projecting the earth's surface to establish the system of State plane coordinates.

One system of plane coordinates is based on the transverse Mercator method of projecting the surface of the earth's spheroid to a cylinder, the straight lines of which extend east and west, and its curved lines generally north and south.

The other system is based on the Lambert conformal method of projecting the surface of the earth's spheroid to a cone, the straight lines of which extend north and south and its curved lines east and west along parallels of latitude.

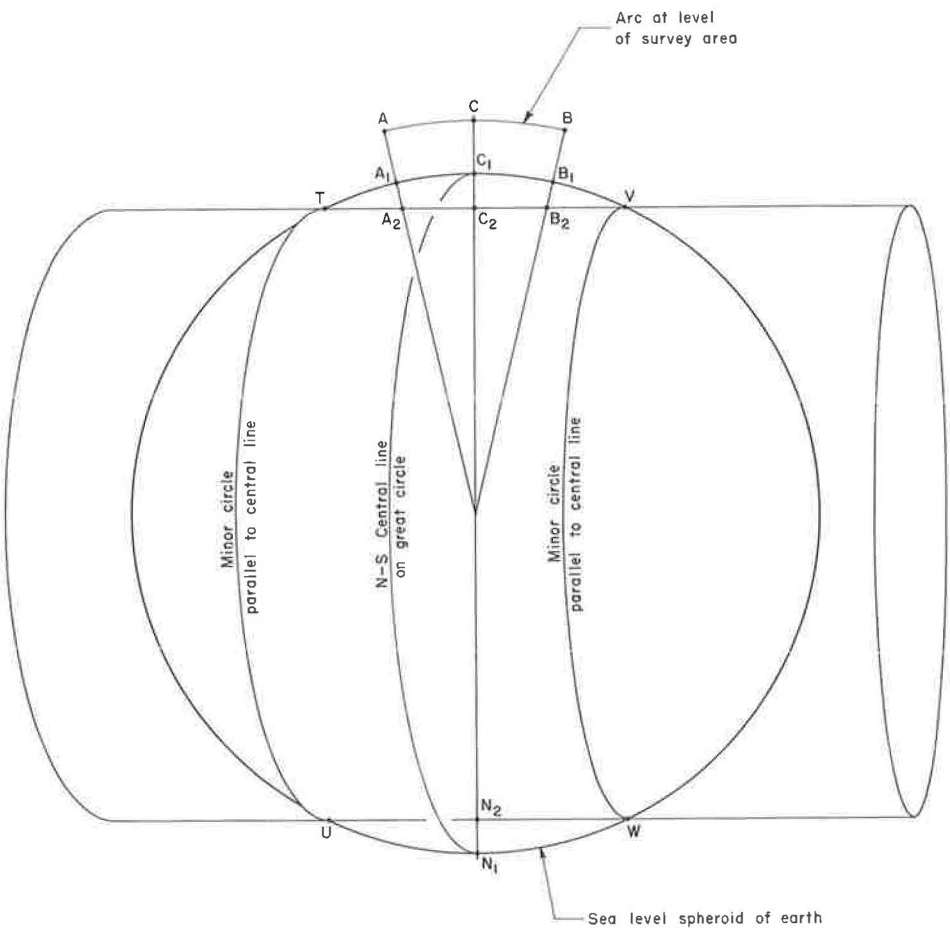

Figure 1. Transverse Mercator projection: Intersections of cylinder and spheroid, and cross section of projection.

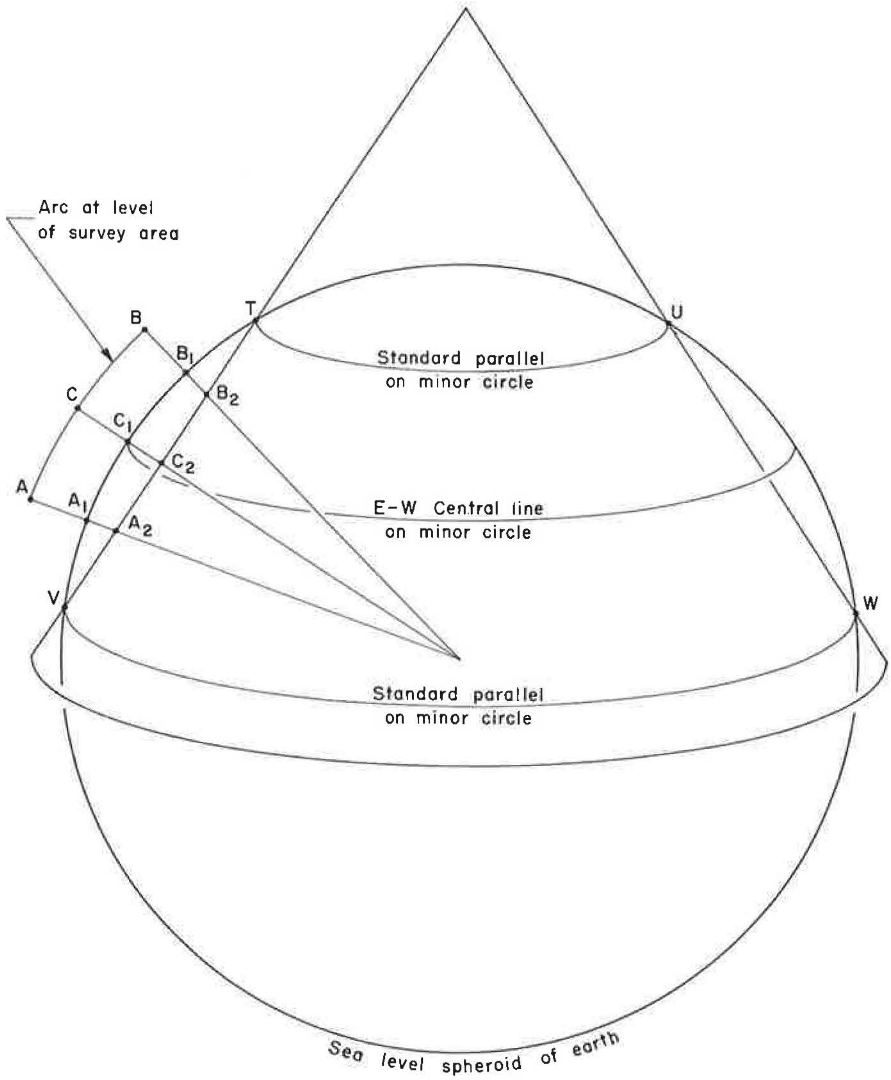

Figure 2. Lambert conformal projection: Intersections of cone and spheroid, and cross section of projection.

The intersections and representative cross sections of these separate systems of projection are illustrated in Figures 1 and 2. After projection, the cylinder and cone in effect are rolled out flat. When this is accomplished, a plane has been established onto which points from the irregular surface of the earth have been projected and plane coordinates for them have been computed (Fig. 3).

In establishing the position of control and other points on the surface of the ground in the system of State plane coordinates, whether based on the transverse Mercator or the Lambert conformal system of projection, the points on the ground are first reduced to position on the sea level arc of the earth. From that position they are further reduced to the arbitrarily established plane which serves as the datum of projection. The

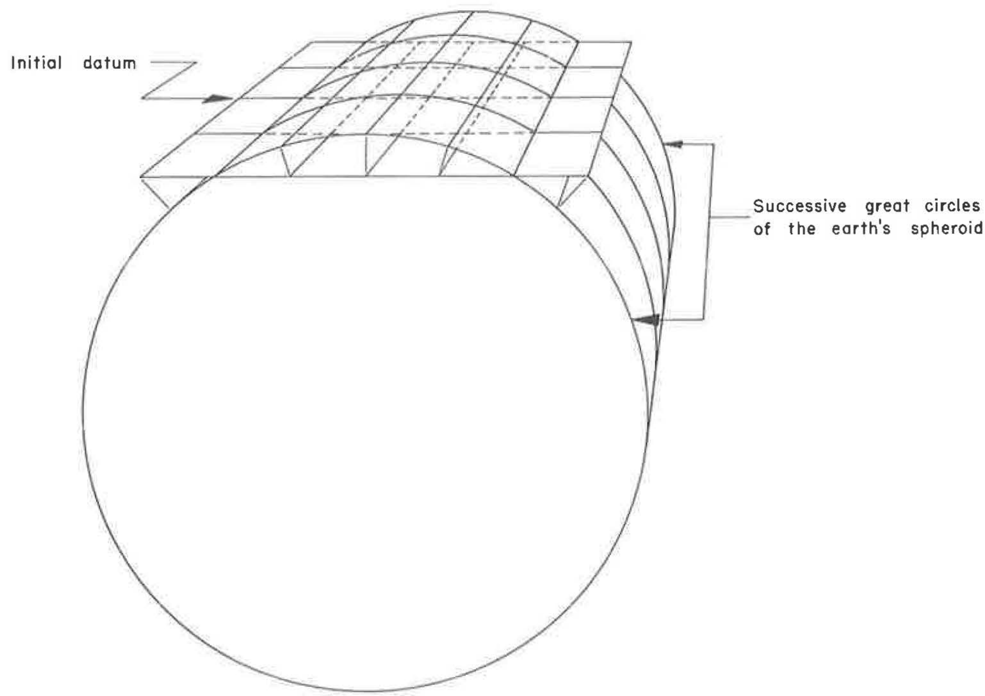

Figure 3. The initial datum formed by flattened cylinder or cone.

projection, of course, is accomplished by use of mathematical equations developed for each of the separate systems of projection.[1]

For all of each zone between the lines of intersection of the datum plane of projection with the sea level spheroid of the earth, the general effect of such projections is to cause the distance between points on the datum of projection, as computed using their X and Y plane coordinates, to be shorter than the distance on the sea level arc of the earth, also shorter than the distance measured between the points on the ground. Outside these lines of intersection, distances on the datum plane of projection are longer than distances on the sea level arc, also longer than distances measured on the ground except as altered by ground elevation in relation to the elevation of the datum plane.

Generally, the transverse Mercator system of plane coordinates is used in States long in the north-south direction, and the Lambert conformal system is used in States long in the east-west direction. The central line for each zone of a transverse Mercator system is a meridian along the selected longitude and for each zone of a Lambert conformal system is the selected central latitude. Figures 4 and 5 illustrate, respectively, the results of having, in effect, rolled the cylinder and the cone of projection out flat to form a plane containing control or other type points, A and B, for which the State plane coordinates have been computed. In each illustration, the true north-south and east-west lines are dashed. The plane coordinate grid lines of the established system are solid lines. For the transverse Mercator projection all true north-south and

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[1]Manual of Plane Coordinate Computation by Oscar S. Adams and Charles N. Claire, and The State Coordinate Systems by Hugh C. Mitchell and Lansing G. Simmons, Special Publications 193 and 235, respectively; also, the Plane Coordinate Projection Tables for each of the States and Puerto Rico, excepting Alaska, which are Special Publications 252 and 253, 255 to 259, 261 to 264, 266 to 277, 284 to 293, 302 to 306, 313 to 319, 321 to 324, 65-2, and 65-3. All of the Special Publications are by the Coast and Geodetic Surveys, U.S. Department of Commerce.

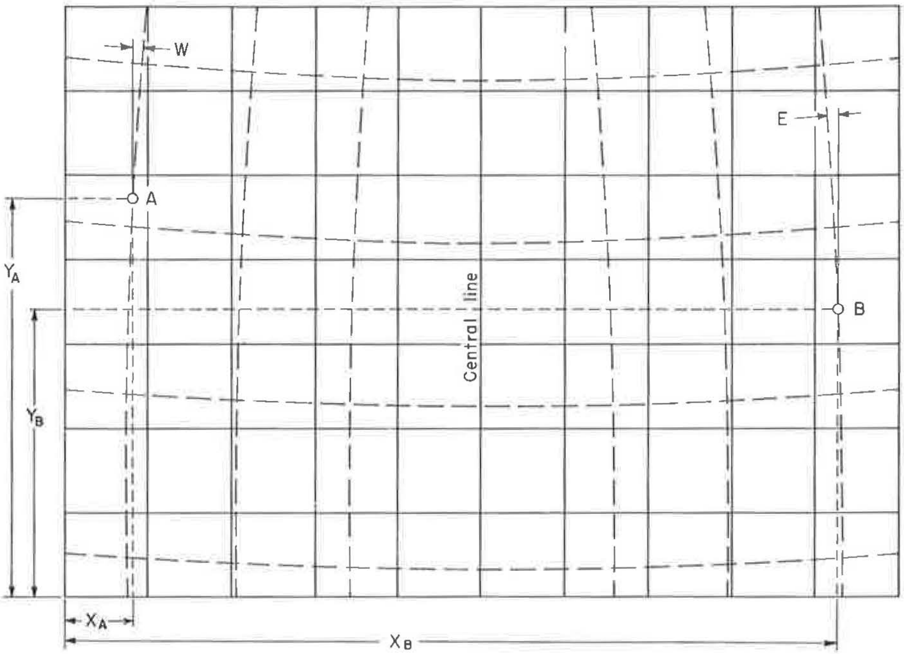

Figure 4. Plane coordinate grid on transverse Mercator projection: W and E are declinations of grid from true north at points A and B.

east-west lines are curved and for the Lambert conformal projection the north-south lines are straight and the east-west lines are curved. These characteristics result from the type of projection, after it has been flattened to form a plane. The angles W and E are declinations of the plane coordinate grid from true north at the points A and B of the respective plane coordinate systems. Only on the central longitude line of each system will true north and plane coordinate grid north be coincident. The angles W and E become progressively larger as the distance of control points is increased both west and east from the central longitude of the plane coordinate zone.

For each system of plane coordinates, the Plane Coordinate Projection Tables compiled by the Coast and Geodetic Survey contain "scale factors." The scale factors are listed in the Tables for each 5,000-foot increment of X distance east and west from the central longitude of the Mercator projection and for every minute of latitude north and south from the central latitude of the Lambert projection. Along the lines of intersection of the spheroid and plane of projection T and U, and V and W (Figs. 1 and 2) the scale factor is 1.0000000. For all segments of each plane coordinate zone between those lines the scale factor is smaller than 1.0000000. For all areas outside such intersection lines, the scale factors are larger than 1.0000000. Each scale factor expresses the ratio of distance on the datum of projection to distance on the sea level arc at the separately applicable distances from the central longitude of the Mercator and degrees and minutes of latitude from the central latitude of the Lambert projections, respectively.

To make full use of the system of State plane coordinates, not only the scale factors are significant. The effects must be considered of shortening distances from measurements made on the ground to distances on the sea level arc of the earth's spheroid. If the distance at sea level arc of the earth is unity for the scale factors, it should also be unity in determining the effects of elevation. In reality, the effects may be expressed by an equation. The distance on the ground D_g equals the distance D_s on the sea level arc of the earth multiplied by the sum of the ground elevation E at the point of

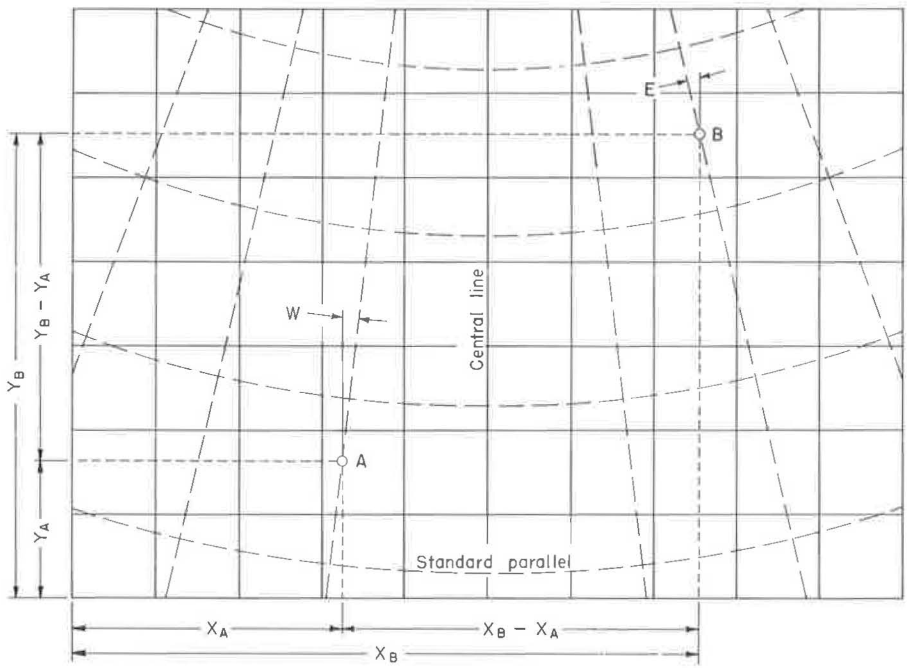

Figure 5. Plane coordinate grid on Lambert conformal projection: W and E are declinations of grid from true north at points A and B.

concern and the earth's radius of curvature R divided by that radius, thus: $D_g = D_s (E + R)/R$. Since the radius of curvature of the earth's surface is an average for the United States of 20,906,000 feet, the numerical value of the ratio of distance on the sea level arc to distance on the ground per 1,000 feet of elevation is 1.0000000 plus the result of multiplying the constant of 0.00004783315795 by the ground elevation at the point of concern in thousands of feet and fractions thereof. The consequences of such facts

TABLE 1
MAGNITUDE OF SCALE FACTORS

Number of Feet Datum Plane of Projection Is Above or Below Sea Level Spheroid of the Earth	Scale Factors, Number of Times Horizontal Distance on Datum Plane of Projection Is Larger or Smaller Than Distance on Spheroid of Earth	Difference Between Distance on Datum Plane and on Spheroid, Expressed as an Approximate Fraction of Any Total Distance
(1)	(2)	(3)
3,000	1.0001435	1:6,960
2,500	1.0001196	1:8,360
2,000	1.0000957	1:10,450
1,500	1.0000717	1:13,930
1,000	1.0000478	1:20,910
500	1.0000239	1:41,810
0	1.0000000	1:∞
- 500	0.9999761	1:41,810
-1,000	0.9999522	1:20,910
-1,500	0.9999283	1:13,930
-2,000	0.9999043	1:10,450
-2,500	0.9998804	1:8,360
-3,000	0.9998565	1:6,960

TABLE 2

MAGNITUDE OF DIFFERENCES IN DISTANCE

Elevation of Survey Point or Plane Above Sea Level Spheroid of the Earth	Number of Times Horizontal Distance on Ground at the Elevation Listed Is Larger Than Horizontal Distance on Sea Level Spheroid (a Multiplication Factor)	Difference Between Distance on Ground and on Sea Level Arc of Earth, Expressed as an Approximate Fraction of Any Total Distance
(1)	(2)	(3)
0	1.0000000	1:∞
1,000	1.0000478	1:20,910
2,000	1.0000957	1:10,450
3,000	1.0001435	1:6,960
4,000	1.0001913	1:5,220
5,000	1.0002392	1:4,180
6,000	1.0002870	1:3,480
8,000	1.0003827	1:2,610
10,000	1.0004783	1:2,090
12,000	1.0005740	1:1,740
14,000	1.0006697	1:1,490
16,000	1.0007653	1:1,300
18,000	1.0008610	1:1,160
20,000	1.0009567	1:1,045

are exemplified in Table 1, which indicates the magnitude of scale factors according to a few representative elevations of the datum of projection, and in Table 2, which contains the magnitude of differences in distance according to a few 1,000- and 2,000-foot increments of elevation of the ground. Actually, the effects of ground elevation are expressed by the multiplication factor (MF), because each distance on the sea level arc multiplied by the MF equals the applicable distance on the ground, depending on the average elevation of the points between which the horizontal distance measurements are made. Likewise, the effects of the datum elevation are expressed by the scale factor (SF), because distance on the sea level arc multiplied by the SF equals the applicable distance on the datum plane of projection.

For precision surveying, the numerals in column 2 of Tables 1 and 2 are cumulative in their effect. For example, if the datum plane of projection is 2,000 feet below and the point of concern on the ground is 4,000 feet above the sea level spheroid, the number of times the horizontal distance on the ground at the 4,000 feet of elevation is larger than the distance on the datum plane of projection would be 1.0001913 from Table 2 divided by 0.9999043 from Table 1, resulting in a combined factor of 1.0002870 from datum of projection to the ground. Another way in which the same number can be determined is to add to 1.0001913 the difference of 1.0000000 and 0.9999043. Applying the same principles, the number of times a specific distance on the ground is larger at any particular elevation than it is on the plane coordinate datum of projection can be computed easily.

As a further example, and using the principles to compute an adjustment factor for the combined effects of the elevation of the ground and of the elevation of the datum of projection, if a control point on the ground is situated where its elevation above the sea level spheroid is 6,000 feet and the elevation of the datum of projection at that point is 2,000 feet below such sea level, the actual difference would be 1.0002870 from Table 2 plus 1.0000000 minus 0.9999043 from Table 1, resulting in a combined factor of 1.0003827. Other comparisons, of course, can be made. Suffice it to say, this means there is an inherent difference in each case between ground distances and datum distances. For the preceding example, the difference amounts to 1:2,610 (column 3 of Table 2 for a total elevation of 8,000 feet from datum of projection to the ground). Similarly, where the elevation of a survey point is 2,000 feet above the sea level spheroid and the datum of projection is 1,000 feet below that spheroid, the total inherent difference between ground distances and map distances would be 1:6,960. Without making further comparisons, it is evident that measurements made on the ground cannot agree within surveying tolerances of 1:10,000 for precision engineering purposes with distances computed between points, using their coordinates on the datum of projection, unless the ground and datum of projection do not differ in elevation by more than 2,090 feet, which is the radius of curvature of the earth divided by 10,000.

One State, which uses precision electronic distance measuring instruments to survey basic control for use in the location surveying and mapping of highway routes for design and preparation of detailed construction plans, has discontinued use of State plane coordinates for its highway construction plans. This change in practice resulted from distances measured on the ground not agreeing sufficiently well with distances computed using plane coordinates of the survey control points used to compile maps on the initial datum of the State plane coordinate system. Such a change is understandable, because the map and ground distances do not agree, but it is also unfortunate indeed.

The differences can be greatly reduced by merely establishing a new datum for the plane of mapping and distance computations to make effective use of the State plane coordinates. The procedure for doing this is called datum adjustment. For several States the datum adjustment can be accomplished on a zone basis, as for Illinois and Ohio as examples, and for the more rugged States on a large area basis determined by topographic and geographic shape and elevation criteria, and bounded by county lines or other appropriate boundary lines. In extreme cases, as for surveys across a high mountain range, it may, on occasion, be necessary to make the datum adjustment on a survey project basis. This occurrence is rare, however, and largely can be avoided if the entire mountainous region within the zone of concern is thoroughly analyzed topographically as one unit and a datum adjustment factor (DAF) suitable for the unit is determined, as has been accomplished easily for New Mexico, where much surveying is done at and above elevations of 7,000 feet.

Unless the datum adjustment is made, distance differences for a substantial portion of most zones will be larger between map and ground distances than generally desirable. Thus, when a datum adjustment is not made, each distance determined from plane coordinates of points and features of maps compiled on the initial datum will have to be modified for making survey measurements on the ground for staking the designed and computed alignment and rights-of-way boundaries so each point thereof will be in its proper position on the ground. If this is not done, wherever maps are compiled on the initial datum, the ground-measured points staked using plane-coordinate-computed distances will not be where they should be on the ground. Wherever the adjustment-of-each-distance procedure is used to get points in their true position on the ground, it is not only costly in terms of time and money, it is frustrating. This is because the measured distances do not agree with the distances computed using the X and Y coordinates of the points between which the measurements are made, as between a basic control survey point and an alignment point, between a point on a tangent at the end of one curve and the beginning of another, between points on property boundaries, and so forth. Thus, to avoid such problems and the additional work they cause, adjustment of the datum of plane coordinates is not only desirable, it is easily done, and makes all surveying measurements and distance computations in agreement within acceptable surveying accuracies.

Once the adjustment has been made, the difficulties are eliminated which were initially considered ample justification for abandonment of the State plane coordinate system, such as frustrations and embarrassments experienced by not having the centerline and rights-of-way staked in their proper position on the ground unless each plane-coordinate-computed distance is changed—that is, made different from what the plane coordinates indicate it should be. The elimination results from distances on the adjusted datum, as determined from plane coordinates of points of control and map features for all maps compiled on the adjusted datum, being in agreement within the allowable surveying accuracies to the distances measured by precision methods on the ground.

To make the most effective and efficient use of the State plane coordinate system of basic control, all station markers for each highway survey should first be measured on the ground and correlated and adjusted in position on the initial datum. By this procedure, continuity is achieved throughout each zone. Then the X and Y coordinates for each basic control point so determined are multiplied by the applicable datum adjustment factor. This procedure places each basic control point on the adjusted datum. The X and Y coordinates of such points are then utilized on the adjusted datum to control all map compilation and measuring, whether done photogrammetrically or on the ground for compilation of the maps, and later to stake the designed highway alignment and rights-of-way lines on the ground for highway construction.

That which follows is a presentation of the principles and procedures applicable in making datum adjustment in order to reduce differences between distances measured on the ground and distances determined from plane coordinates of points on the maps. The principles of adjusting the datum of projection are the same for each of the two separate systems of State plane coordinates, the transverse Mercator and Lambert conformal. The datum adjustment comprises the establishment of another datum parallel with and either above or below, as necessary, the initial datum. Ordinarily, the adjustment will be a raising rather than a lowering. Wherever a plane coordinate zone is wide and the ground elevations at the edges are low, however, the initial datum may be so far above the ground in the vicinity of the edges of the zone that a lowering is necessary.

To determine the exact magnitude required in datum adjustment, the zone of consideration should be analyzed in detail. The analysis first comprises the determination

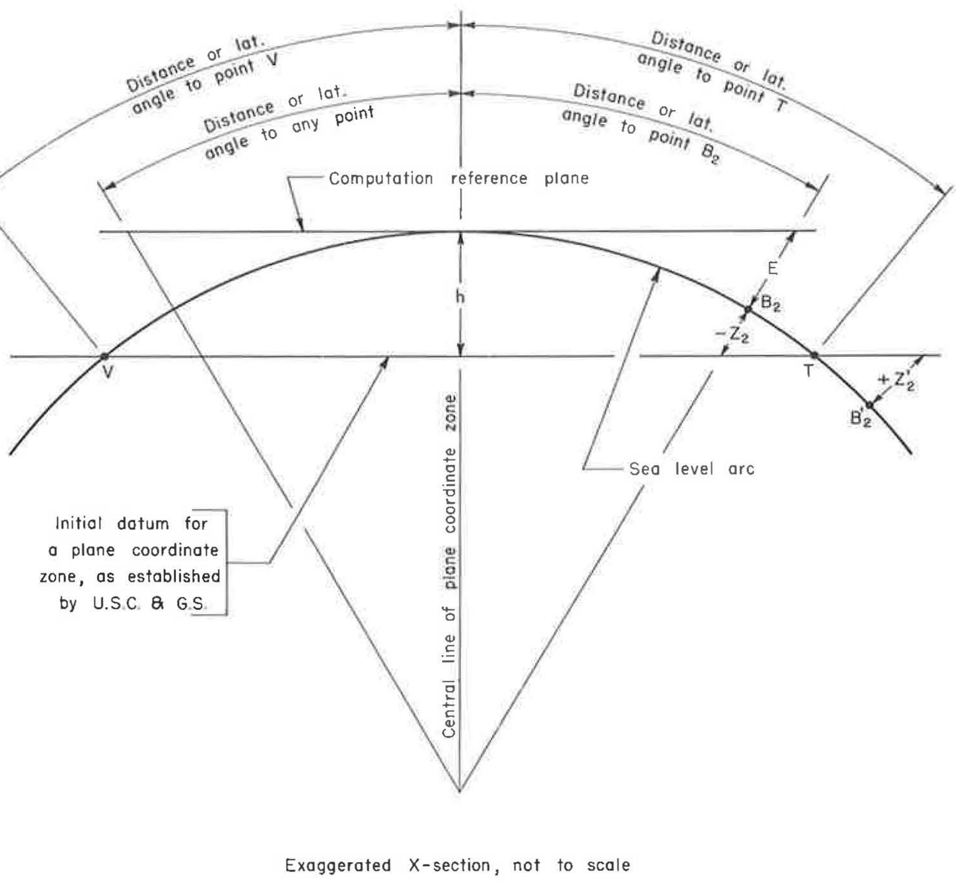

Figure 6. Use of computation reference plane.

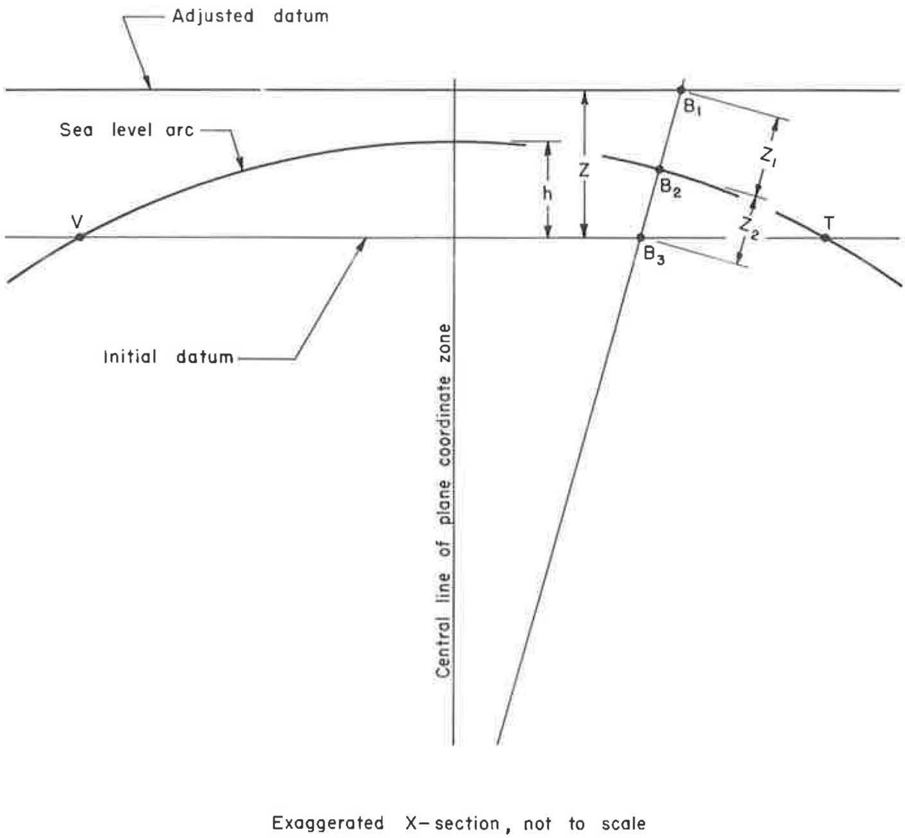

Figure 7. Elevation dimensions used in datum adjustment computations.

and recording of maximum and minimum elevations which will govern in making the datum adjustment. These elevations can best be obtained by reference to existing topographic maps, and should be recorded on a suitable analysis form.

For the transverse Mercator system of plane coordinates, the maximum and minimum ground elevations are determined and recorded for each significant increment of 5,000 feet east and west from the central line (meridian) of the zone. For a zone based on the Lambert conformal projection, the maximum and minimum elevations are determined and recorded at each significant interval of 0° 01' of latitude north and south from the central line (latitude) of the zone.

The elevation at which the initial datum was placed below the sea level arc of the earth at the central line of each zone for which the datum is to be adjusted is computed. The numerical expression of this elevation is equivalent to the scale factor at the central line minus the numeral one, multiplied by the radius of curvature of the sea level spheroid of the earth. Thus, if the scale factor is 0.9999000, the elevation of the datum at the central line is (0.9999000 minus 1.0000000) multiplied by 20,906,000, which is -2,090 feet. To compute the elevation of the initial datum at any other place within the plane coordinate zone, it is best to subtract the elevation of a computation reference plane, which is tangent to the sea level arc of the earth at the central line, from the elevation the datum is below the sea level arc at its central line for each respective distance from the central line, as 5,000 feet east and west for the transverse Mercator projection, and each 0° 01' north and south for the Lambert conformal projection, to determine the exact elevation of the initial datum with respect to the sea level arc of the

TABLE 3. Datum Adjustment Computations.
TRANSVERSE MERCATOR STATE PLANE COORDINATE SYSTEM

Area of Application ______________________________
Datum ______________ Datum Adjustment Factor ______________

Dist. E. & W. from Central Line (feet)	Elev. of Plane Tangent at Cen. Line (feet)	Datum Elevation (feet)	Actual Ground Elev. (feet)		Elevation Where 1:10,000 Difference Occurs		Difference Between Distances on the Ground and on the Datum	
			Min.	Max.	Low	High	At Min. Elev.	At Max. Elev.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	0							
5,000	1							
10,000	2							
15,000	5							
↕								
440,000	4,631							

Continuation

(1)	(2)
20,000	10
25,000	15
30,000	22
35,000	29
40,000	38
45,000	48
50,000	60
55,000	72
60,000	86
65,000	101
70,000	117
75,000	135
80,000	153
85,000	173
90,000	194
95,000	216
100,000	239
105,000	264
110,000	289
115,000	316
120,000	344
125,000	374
130,000	404
135,000	436
140,000	469
145,000	503
150,000	538
155,000	575
160,000	612
165,000	651
170,000	691
175,000	732
180,000	775
185,000	819
190,000	863
195,000	909
200,000	957
205,000	1,005
210,000	1,055
215,000	1,106
220,000	1,158
225,000	1,211
230,000	1,265
235,000	1,321
240,000	1,378
245,000	1,436
250,000	1,495
255,000	1,555
260,000	1,617
265,000	1,680
270,000	1,744
275,000	1,809
280,000	1,875
285,000	1,943
290,000	2,012
295,000	2,082
300,000	2,153
305,000	2,225
310,000	2,299
315,000	2,373
320,000	2,449
325,000	2,526
330,000	2,605
335,000	2,684
340,000	2,765
345,000	2,847
350,000	2,930
355,000	3,014
360,000	3,100
365,000	3,187
370,000	3,275
375,000	3,364
380,000	3,454
385,000	3,546
390,000	3,638
395,000	3,732
400,000	3,827
405,000	3,924
410,000	4,021
415,000	4,120
420,000	4,220
425,000	4,321
430,000	4,423
435,000	4,526
440,000	4,631

earth. Using the elevation of the initial datum of projection and the elevation of the ground at each significant increment from the central line, it is easy to determine where the new datum should be placed so it will be at the optimum elevation with respect to the ground for reducing distance differences.

The significance of utilizing the computation reference plane is illustrated in Figure 6. The relationship and elevation differences applicable in determining the adjusted datum are illustrated in Figure 7, which was prepared for illustrating use of the forms for datum adjustment computations. Tables 3 and 4 are examples, containing increments from the central line, of the separate forms used in making the datum adjustment

TABLE 4. Datum Adjustment Computations
LAMBERT CONFORMAL STATE PLANE COORDINATE SYSTEM

Area of Application ____________________
Datum ____________ Datum Adjustment Factor ____________

Angle in Min. from Cen. Line	Latitude Deg. Min. North and South from Cen. Line		Elev. of Plane Tang. at Central Line	Datum Elevation	Actual Ground Elev. (feet)		Elevation where 1:10,000 Difference Occurs		Difference between Distances on the Ground and on the Datum	
	North	South	(feet)	(feet)	Min.	Max.	Low	High	At Min. Elev.	At Max. Elev.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
00			0							
01			1							
02			4							
03			8							
04			14							
95			7,985							

Continuation

(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
05			22								
06			32	36			1,146	66			3,853
07			43	37			1,211	67			3,971
08			57	38			1,277	68			4,091
09			72	39			1,345	69			4,212
10			88	40			1,415	70			4,335
11			107	41			1,487	71			4,460
12			127	42			1,560	72			4,586
13			149	43			1,636	73			4,714
14			173	44			1,712	74			4,844
15			199	45			1,791	75			4,976
16			226	46			1,872	76			5,110
17			256	47			1,954	77			5,245
18			287	48			2,038	78			5,382
19			319	49			2,124	79			5,521
20			354	50			2,211	80			5,662
21			390	51			2,301	81			5,804
22			428	52			2,392	82			5,949
23			468	53			2,485	83			6,095
24			509	54			2,579	84			6,243
25			553	55			2,676	85			6,392
26			598	56			2,774	86			6,543
27			645	57			2,874	87			6,696
28			693	58			2,976	88			6,851
29			744	59			3,079	89			7,008
30			796	60			3,185	90			7,166
31			850	61			3,292	91			7,327
32			906	62			3,400	92			7,489
33			963	63			3,511	93			7,652
34			1,023	64			3,623	94			7,818
35			1,084	65			3,738	95			7,985

computations for the Mercator and Lambert systems of projection. Table 5 is an example of use of one of the forms in analyzing the position of the initial datum with respect to the ground for the central counties of the west zone of the New York State plane coordinate system of the Mercator projection, in preparation for making the datum adjustment. Table 6 is a follow-up use of the form in determining the datum adjustment factor, and the position of and effects of the adjusted datum on distance differences for the same area. A comparison of the difference between distances on the ground and on

TABLE 5
Datum Adjustment Computations
TRANSVERSE MERCATOR STATE PLANE COORDINATE SYSTEM

Area of Application New York - West Zone, Central Counties
Datum Initial Datum Adjustment Factor 1.0000000

Dist. E. & W. from Central Line (feet)	Elev. of Plane Tangent at Cen. Line (feet)	Datum Elevation (feet)	Actual Ground Elev. (feet)		Elevation Where 1:10,000 Difference Occurs		Difference Between Distances on the Ground and on the Datum	
			Min.	Max.	Low	High	At Min. Elev.	At Max. Elev.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	0	-1,307	246	2,400	-3,397	783	1:13,400	1:5,600
5,000	1	-1,306	246	2,407	-3,396	784	1:13,400	1:5,600
15,000	5	-1,302	246	2,220	-3,392	788	1:13,500	1:5,900
35,000	29	-1,278	246	2,294	-3,368	812	1:13,700	1:5,800
50,000	60	-1,247	246	2,230	-3,337	843	1:14,000	1:6,000
65,000	101	-1,206	246	2,410	-3,296	884	1:14,300	1:5,700
75,000	135	-1,172	246	2,392	-3,262	918	1:14,700	1:5,800
85,000	173	-1,134	246	2,274	-3,224	956	1:15,100	1:6,100
90,000	194	-1,113	246	2,145	-3,203	977	1:15,300	1:6,400
105,000	264	-1,043	246	2,230	-3,133	1,047	1:16,200	1:6,300
130,000	404	-903	246	2,400	-2,993	1,187	1:18,100	1:6,300
135,000	436	-871	246	2,450	-2,961	1,219	1:18,700	1:6,200
145,000	503	-804	246	2,210	-2,894	1,286	1:19,900	1:6,900
155,000	575	-732	246	2,548	-2,822	1,358	1:21,300	1:6,300
160,000	612	-695	610	2,125	-2,785	1,395	1:16,000	1:7,400
165,000	651	-656	620	2,020	-2,746	1,434	1:16,300	1:7,800
200,000	957	-350	610	2,210	-2,440	1,740	1:21,700	1:8,100
220,000	1,158	-149	590	2,210	-2,239	1,941	1:28,200	1:8,800
225,000	1,211	-96	590	2,230	-2,186	1,994	1:30,400.	1:8,900
235,000	1,321	+14	730	1,610	-2,076	2,104	1:29,100	1:13,000
245,000	1,436	+129	740	2,010	-1,961	2,219	1:34,200	1:11,100
270,000	1,744	+437	1,250	1,810	-1,653	2,527	1:25,700	1:15,200
275,000	1,809	+502	1,600	2,010	-1,588	2,592	1:19,000	1:13,800
290,000	2,012	+705	980	2,020	-1,385	2,795	1:76,000	1:15,800

the datum in columns 8 and 9 of Table 5 for the initial datum with those in columns 8 and 9, respectively, of Table 6 for the adjusted datum readily indicates the effectiveness of the datum adjustment—how well it reduced the differences at the places of maximum elevation within the area of datum adjustment. Between the maximum and minimum elevations, of course, the distance differences will be much less, ranging downward from differences in columns 8 and 9 of Table 6 to as small as $1:\infty$ for much of the central portion of the zone to which the datum adjustment applies. The specific manner in which the datum adjustment is accomplished is subsequently outlined.

Procedure in Determining a Datum Adjustment Factor

First the critical elevations of the ground are ascertained using the best available topographic maps. These elevations are recorded on the form for datum adjustment computations in the columns headed Actual Ground Elevation—columns 4 for minimum and 5 for maximum in Tables 3, 5, 8, and 9 when an analysis is being made of a transverse Mercator State plane coordinate system, and columns 6 for minimum and 7 for maximum in Tables 4 and 10 when a Lambert conformal State plane coordinate system is being analyzed. As mentioned previously, the elevations determined for recording are at each appropriate increment of 5,000 feet east and west from the central line of the Mercator zone, and at each appropriate increment of 0° 01' of latitude north

TABLE 6

Datum Adjustment Computations
TRANSVERSE MERCATOR STATE PLANE COORDINATE SYSTEM

Area of Application New York - West Zone, Central Counties
Datum Adjusted Datum Adjustment Factor 1.0000800
is 1,672 feet above initial datum

Dist. E. & W. from Central Line (feet)	Elev. of Plane Tangent at Cen. Line (feet)	Datum Elevation (feet)	Actual Ground Elev. (feet)		Elevation Where 1:10,000 Difference Occurs		Difference Between Distances on the Ground and on the Datum	
			Min.	Max.	Low	High	At Min. Elev.	At Max. Elev.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	0	+365	246	2,400	-1,725	2,455	1:175,600	1:10,200
5,000	1	+366	246	2,407	-1,724	2,456	1:174,200	1:10,200
15,000	5	+370	246	2,220	-1,720	2,460	1:168,500	1:11,300
35,000	29	+394	246	2,294	-1,696	2,484	1:141,200	1:11,000
50,000	60	+425	246	2,230	-1,665	2,515	1:116,700	1:11,500
65,000	101	+466	246	2,410	-1,624	2,556	1:95,000	1:10,700
75,000	135	+500	246	2,392	-1,590	2,590	1:82,300	1:11,000
85,000	173	+538	246	2,274	-1,552	2,628	1:71,500	1:12,000
90,000	194	+559	246	2,145	-1,531	2,649	1:66,700	1:13,100
105,000	264	+629	246	2,230	-1,461	2,719	1:54,500	1:13,000
130,000	404	+769	246	2,400	-1,321	2,859	1:39,900	1:12,800
135,000	436	+801	246	2,450	-1,289	2,891	1:37,600	1:12,600
145,000	503	+868	246	2,210	-1,222	2,958	1:33,600	1:15,500
155,000	575	+940	246	2,548	-1,150	3,030	1:30,100	1:13,000
160,000	612	+977	610	2,125	-1,113	3,067	1:56,900	1:18,200
165,000	651	+1,016	620	2,020	-1,074	3,106	1:52,700	1:20,800
200,000	957	+1,322	610	2,210	-768	3,412	1:29,300	1:23,500
220,000	1,158	+1,523	590	2,210	-567	3,613	1:22,400	1:30,400
225,000	1,211	+1,576	590	2,230	-514	3,666	1:21,200	1:31,900
235,000	1,321	+1,686	730	1,610	-404	3,776	1:21,800	1:275,000
245,000	1,436	+1,801	740	2,010	-289	3,891	1:19,700	1:100,000
270,000	1,744	+2,109	1,250	1,810	+19	4,199	1:24,300	1:69,900
275,000	1,809	+2,174	1,600	2,010	+84	4,264	1:36,400	1:127,400
290,000	2,012	+2,377	980	2,020	+287	4,467	1:14,900	1:58,500

and south from the central latitude of the Lambert zone. The reason for the increments is the fact that scale factors provided in the State Plane Coordinate Projection Tables are at such increments, according to the system of projection. The scale factors, of course, merely represent the ratio of distance on the initially established datum of the applicable State plane coordinate zone to distance along the sea level arc of the earth. Once all applicable elevation data are determined and recorded in the appropriate columns for the plane coordinate zone or portion thereof being analyzed, preparations have been completed for determining a datum adjustment factor which will place the adjusted datum at an appropriate height above or below the initial datum.

Before endeavoring to make the datum adjustment, however, it is best to obtain a full understanding of the actual conditions existing with respect to the initial datum. Accordingly, the first computations made in the datum adjustment procedure are for obtaining significant data regarding the initial datum, which are applicable in columns 3, 6, 7, 8, and 9 for the Mercator and in columns 2, 3, 5, 8, 9, 10, and 11 for the Lambert. Such computations can be done easily by computer—a program has been prepared for use with the IBM 1401 computer.[2] When the computations have been

[2]Program prepared by Douglas M. Reid of Region 15 and Fred W. Turner of Aerial Surveys Branch, Bureau of Public Roads.

completed, the actual elevation of the initial datum is known, as recorded in column 3 for the Mercator and column 5 for Lambert. In addition, each elevation at which the difference is as large as 1:10, 000 is known—columns 6 and 7 for Mercator and columns 8 and 9 for Lambert. Between such low and high elevations the differences between the ground measured distances and distances determined from coordinates of points on the initial datum will be less than 1:10, 000, ranging downward from each extreme to 1:∞, where the ground and datum of projection coincide. The last two columns of the datum adjustment computations express as a fraction the difference between distances on the ground for minimum and maximum elevation points selected from the maps and recorded in columns 4 anf 5 for the Mercator and columns 6 and 7 for the Lambert.

An analysis of columns 8 and 9 and 10 and 11, respectively, reveals what the actual differences are at such places of extreme elevation in the separate systems of projection. The analysis of these differences readily indicates where the adjusted datum should be placed, above or below the initial datum. Such analysis also indicates the magnitude required in the datum adjustment and establishes whether the adjustment can be made for the entire zone or will have to be made on an incremental area basis.

Except in extreme cases, a difference of 1:10, 000 between distances measured accurately on the ground and distances computed from plane coordinates of points on the mapping datum can be accepted. This being the case, as mentioned previously, the adjusted datum should be not more than 2,090 feet above or 2,090 feet below the ground at any point within the zone or other area of adjustment. Should smaller differences

TABLE 7

THE EFFECTS OF DATUM ADJUSTMENT

Datum Adjustment Factor	Resultant Change in Distance Differences Expressed as a Ratio	Elevation of Adjusted Datum With Respect to Initial Datum (feet)	Datum Adjustment Factor	Resultant Change in Distance Differences Expressed as a Ratio	Elevation of Adjusted Datum With Respect to Initial Datum (feet)
(1)	(2)	(3)	(1)	(2)	(3)
0.9999600	1:25, 000	- 836	1.0002300	1:4, 348	+ 4, 807
0.9999700	1:33, 333	- 627	1.0002400	1:4, 167	+ 5, 016
0.9999800	1:50, 000	- 418	1.0002500	1:4, 000	+ 5, 225
0.9999900	1:100, 000	- 209	1.0002600	1:3, 846	+ 5, 434
1.0000000	1:∞	0	1.0002700	1:3, 704	+ 5, 643
1.0000100	1:100, 000	+ 209	1.0002800	1:3, 571	+ 5, 852
1.0000200	1:50, 000	+ 418	1.0002900	1:3, 448	+ 6, 061
1.0000300	1:33, 333	+ 627	1.0003000	1:3, 333	+ 6, 270
1.0000400	1:25, 000	+ 836	1.0003100	1:3, 226	+ 6, 479
1.0000500	1:20, 000	+ 1, 045	1.0003200	1:3, 125	+ 6, 688
1.0000600	1:16, 667	+ 1, 254	1.0003300	1:3, 030	+ 6, 897
1.0000700	1:14, 285	+ 1, 463	1.0003400	1:2, 941	+ 7, 106
1.0000800	1:12, 500	+ 1, 672	1.0003500	1:2, 857	+ 7, 315
1.0000900	1:11, 111	+ 1, 881	1.0003600	1:2, 778	+ 7, 524
1.0001000	1:10, 000	+ 2, 090	1.0003700	1:2, 703	+ 7, 733
1.0001100	1:9, 091	+ 2, 299	1.0003800	1:2, 631	+ 7, 942
1.0001200	1:8, 333	+ 2, 508	1.0003900	1:2, 564	+ 8, 151
1.0001300	1:7, 692	+ 2, 717	1.0004000	1:2, 500	+ 8, 360
1.0001400	1:7, 143	+ 2, 926	1.0004100	1:2, 439	+ 8, 569
1.0001500	1:6, 667	+ 3, 135	1.0004200	1:2, 381	+ 8, 778
1.0001600	1:6, 250	+ 3, 344	1.0004300	1:2, 326	+ 8, 987
1.0001700	1:5, 882	+ 3, 553	1.0004400	1:2, 273	+ 9, 196
1.0001800	1:5, 556	+ 3, 762	1.0004500	1:2, 222	+ 9, 405
1.0001900	1:5, 263	+ 3, 971	1.0004600	1:2, 174	+ 9, 614
1.0002000	1:5, 000	+ 4, 180	1.0004700	1:2, 127	+ 9, 823
1.0002100	1:4, 762	+ 4, 389	1.0004800	1:2, 083	+10, 032
1.0002200	1:4, 545	+ 4, 598	1.0004900	1:2, 041	+10, 241
			1.0005000	1:2, 000	+10, 450

be desired, as, for example, 1:20,000, the adjusted datum must be not more than 1,045 feet above or below the ground at any point within the area of datum adjustment; and if a larger difference, as 1:5,000 is acceptable, the adjusted datum can be 4,180 feet above or below the ground. Visual inspection of the elevation of points on the ground and the elevation of points which are 2,090 feet below and 2,090 feet above that datum (columns 6 and 7 for Mercator, and columns 8 and 9 for Lambert) further indicates where the adjusted datum should be placed so differences between distances on the ground and distances computed using plane coordinates of points on the map do not exceed 1:10,000. Then a trial datum is selected. This is done so the elevation in columns 4 and 5 for the Mercator and columns 6 and 7 for the Lambert are not below or above the adjusted datum by a larger increment in elevation than the 2,090 feet or other permissible limitation. Actually, the elevation increment of adjustment should be 209 feet so the resultant datum adjustment factor will be an even number.

Table 7 contains representative datum adjustment factors (column 1), the resultant change in distance differences expressed as a ratio (column 2), and the elevation of the adjusted datum with respect to the initial datum (column 3). Choice of factor as a trial

TABLE 8
Datum Adjustment Computations
TRANSVERSE MERCATOR STATE PLANE COORDINATE SYSTEM

Area of Application New Mexico - West Zone, San Juan County East of R.14W. & Rio Arriba County
Datum Initial Datum Adjustment Factor 1.0000000

Dist. E. & W. from Central Line (feet)	Elev. of Plane Tangent at Cen. Line (feet)	Datum Elevation (feet)	Actual Ground Elev. (feet)		Elevation Where 1:10,000 Difference Occurs		Difference between Distances on the Ground and on the Datum	
			Min.	Max.	Low	High	At Min. Elev.	At Max. Elev.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	0	-1,741	5,500	7,475	-3,831	349	1:2,800	1:2,200
10,000	2	-1,739	5,500	7,420	-3,829	351	1:2,800	1:2,200
25,000	15	-1,726	5,450	7,050	-3,816	364	1:2,900	1:2,300
40,000	38	-1,703	5,400	7,200	-3,793	387	1:2,900	1:2,300
60,000	86	-1,655	5,380	7,500	-3,745	435	1:2,900	1:2,200
70,000	117	-1,624	5,360	7,650	-3,714	466	1:2,900	1:2,200
80,000	153	-1,588	5,300	7,200	-3,678	502	1:3,000	1:2,300
105,000	264	-1,477	5,250	6,850	-3,567	613	1:3,100	1:2,500
130,000	404	-1,337	5,200	7,200	-3,427	753	1:3,100	1:2,400
135,000	436	-1,305	6,300	7,220	-3,395	785	1:2,700	1:2,400
145,000	503	-1,238	6,590	7,405	-3,328	852	1:2,600	1:2,400
160,000	612	-1,129	6,750	7,500	-3,219	961	1:2,600	1:2,400
180,000	775	-966	6,800	7,600	-3,056	1,124	1:2,600	1:2,400
185,000	819	-922	6,600	7,500	-3,012	1,168	1:2,700	1:2,400
200,000	957	-784	6,800	7,500	-2,874	1,306	1:2,700	1:2,500
210,000	1,055	-686	6,750	7,600	-2,776	1,404	1:2,800	1:2,500
215,000	1,106	-635	6,580	8,000	-2,725	1,455	1:2,800	1:2,400
230,000	1,265	-476	6,600	8,100	-2,566	1,614	1:2,900	1:2,400
240,000	1,378	-363	6,750	8,300	-2,453	1,727	1:2,900	1:2,400
250,000	1,495	-246	6,780	9,050	-2,336	1,844	1:2,900	1:2,200
255,000	1,555	-186	6,790	9,000	-2,276	1,904	1:2,900	1:2,200
260,000	1,617	-124	6,800	8,600	-2,214	1,966	1:3,000	1:2,300
265,000	1,680	-61	6,800	8,800	-2,151	2,029	1:3,000	1:2,300
270,000	1,744	+3	6,950	8,400	-2,087	2,093	1:3,000	1:2,400
280,000	1,875	+134	7,000	7,800	-1,956	2,224	1:3,000	1:2,700

For an illustrative example pertaining to these computations, refer to Figures 3 and 6.

TABLE 9
Datum Adjustment Computations
TRANSVERSE MERCATOR STATE PLANE COORDINATE SYSTEM

Area of Application New Mexico - West Zone, San Juan County, West of R.13W.
Datum Initial Datum Adjustment Factor 1.0000000

Dist. E. & W. from Central Line (feet)	Elev. of Plane Tangent at Cen. Line (feet)	Datum Elevation (feet)	Actual Ground Elev. (feet)		Elevation Where 1:10,000 Difference Occurs		Difference between Distances on the Ground and on the Datum	
			Min.	Max.	Low	High	At Min. Elev.	At Max. Elev.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	0	-1,741						
125,000	374	-1,367	5,210	6,610	-3,457	723	1:3,100	1:2,600
135,000	436	-1,305	5,200	7,050	-3,385	785	1:3,200	1:2,500
160,000	612	-1,129	5,130	6,700	-3,219	961	1:3,300	1:2,600
170,000	691	-1,050	5,100	6,620	-3,140	1,040	1:3,300	1:2,700
190,000	863	-878	5,040	6,450	-2,968	1,212	1:3,500	1:2,800
200,000	957	-784	5,000	6,600	-2,874	1,306	1:3,600	1:2,800
220,000	1,158	-583	4,950	6,800	-2,673	1,507	1:3,700	1:2,800
240,000	1,378	-363	4,900	6,050	-2,453	1,727	1:3,900	1:3,200
255,000	1,555	-186	4,870	6,650	-2,276	1,904	1:4,100	1:3,000
265,000	1,680	-61	4,820	7,150	-2,151	2,029	1:4,200	1:2,800
280,000	1,875	+134	4,790	8,000	-1,956	2,224	1:4,400	1:2,600
290,000	2,012	+271	4,780	8,820	-1,819	2,361	1:4,600	1:2,400
305,000	2,225	+484	4,760	8,800	-1,606	2,574	1:4,800	1:2,500
325,000	2,526	+785	4,740	8,850	-1,305	2,875	1:5,200	1:2,500
330,000	2,605	+864	4,730	8,860	-1,226	2,954	1:5,400	1:2,600
345,000	2,847	+1,106	4,700	9,050	-984	3,196	1:5,800	1:2,600
350,000	2,930	+1,189	4,800	8,500	-901	3,279	1:5,700	1:2,800
360,000	3,100	+1,359	4,800	8,450	-731	3,449	1:6,000	1:2,900

will depend entirely upon results of examination of the data acquired in completing the datum adjustment computations pertaining to the initial datum, Tables 8, 9, and 10.

In the first trial, it is customary to consider the entire zone, unless the width of the zone and its ground elevations cause an adjusted datum to be too far above or below the actual ground within certain areas of the zone. Then the area of application of the trial adjustment is changed to suitable boundary limits governed by elevation, distance from the central line, and geographic shape. The change is accompanied by use of another, but appropriate, datum adjustment factor. Ordinarily, the change in area is made along governmental boundary lines, such as counties, to conform as nearly as practicable to the geographic boundaries established by ground elevation where the differences between distances on the ground and distances on the adjusted datum do not exceed the difference which is acceptable, such as 1:10,000 or 1:20,000. Examples of the completed datum adjustment computations for an adjusted datum are given in Tables 11, 12, and 13 for the areas analyzed using data in Tables 8, 9, and 10, respectively.

The principles pertaining to use of a computation reference plane in computing the elevation of the initial datum at appropriate increments from the central line for a plane coordinate zone is exemplified in Figure 6. The sea level arc of the earth is intersected by the plane of the initial datum at points V and T. The distance the datum is below the sea level arc at the central line of the plane coordinate zone is h. The central line is the central meridian of a Mercator or the central latitude of a Lambert plane coordinate zone of projection. The computation reference plane is tangent to the sea level arc of the earth at the central line of the plane coordinate zone.

The equation for computing h is $h = R(SF - 1)$, in which R is the radius of curvature of the sea level arc of the earth (an average of about 20,906,000 feet for the United

TABLE 10
Datum Adjustment Computations
LAMBERT CONFORMAL STATE PLANE COORDINATE SYSTEM

Area of Application **Arkansas - North Zone, West Central Counties**
Datum **Initial** Datum Adjustment Factor 1.0000000

Angle in Min. from Cen. Line	Latitude Deg. Min. North and South from Cen. Line		Elev. of Plane Tang. at Central Line (feet)	Datum Elevation (feet)	Actual Ground Elev. (feet)		Elevation Where 1:10,000 Difference Occurs		Difference between Distances on the Ground and on the Datum	
	North	South			Min.	Max.	Low	High	At Min. Elev.	At Max. Elev.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0	35°35'	35°35'	0	-1,340	490	2,000	-3,430	750	1:11,400	1:6,200
1	36'	34'	1	-1,339	490	2,100	-3,429	751	1:11,400	1:6,000
5	40'	30'	22	-1,318	390	2,128	-3,408	772	1:12,200	1:6,000
7	42'	28'	43	-1,297	360	2,127	-3,387	793	1:12,600	1:6,100
9	44'	26'	72	-1,268	350	2,361	-3,358	822	1:12,900	1:5,700
10	45'	25'	88	-1,252	350	2,250	-3,342	838	1:13,000	1:5,900
13	48'	22'	149	-1,191	340	2,567	-3,281	899	1:13,600	1:5,500
17	52'	18'	256	-1,084	330	2,578	-3,174	1,006	1:14,700	1:5,700
21	56'	14'	390	-950	300	2,380	-3,040	1,140	1:16,700	1:6,200
24	59'	11'	509	-831	290	2,282	-2,921	1,259	1:18,600	1:6,700
25	36°00'	10'	553	-787	290	2,285	-2,877	1,303	1:19,400	1:6,800
29	04'	06'	744	-596	490	2,385	-2,686	1,494	1:19,200	1:7,000
31	06'	04'	850	-490	990	2,370	-2,580	1,600	1:14,100	1:7,300
38	13'	34°57'	1,277	-63	1,100	1,680	-2,153	2,027	1:17,900	1:11,900
42	17'	53'	1,560	+220	1,290	1,695	-1,870	2,310	1:19,500	1:14,100
43	18'	52'	1,636	+296	1,290	1,710	-1,794	2,386	1:21,000	1:14,700

States) and SF is the scale factor at the point of concern. The scale factor is obtained from the Plane Coordinate Projection Tables for the zone of the State in which the point of concern is situated. Wherever the SF is smaller than the numeral one, h is negative, meaning the datum is below the sea level arc, and wherever the SF is larger than one the datum is above that arc.

In Figure 6, Z (in general) is the elevation of the initial datum with respect to any point of concern, as B_2 or B_2'. The h and Z are equal at the central line where E is zero. Actually, E is the elevation of the computation reference plane above any point, as B_2. Thus, in the general case, $Z = E + h$. Consequently, Z_2 is negative when h is negative and E for the point of concern is smaller in value than h, and Z_2' is positive when E is larger than the negative h. Once the datum adjustment preparations have been completed, the results are also exemplified by Figure 6, wherein all required elevations (Z_2 and Z_2') for the initial datum have been computed—column 3 of Tables 8 and 9, and column 5 of Table 10.

The next step is to determine where the adjusted datum should be placed with respect to the initial datum. An example of this placement is given in Figure 7. The upper straight line in the figure represents the cross section of the plane of adjusted datum.

For any point of concern, Z_2 represents the height of the initial datum at point B_3 below (or above) the sea level arc at point B_2, and Z_1 represents the height of the adjusted datum at point B_1 above (or below) point B_2. The scale factor (SF) applicable for point B_2 will approach unity as Z_2 approaches zero. Between the sea level arc intersections, points V and T, the SF will attain its largest deviation from unity when Z_2 is at the central line of the plane coordinate zone. For each particular point, the SF is selected from the Plane Coordinate Projection Tables compiled by the U. S. Coast and Geodetic Survey. Remember, the SF expresses the ratio of distance on the datum of

TABLE 11
Datum Adjustment Computations
TRANSVERSE MERCATOR STATE PLANE COORDINATE SYSTEM

Area of Application New Mexico - West Zone, San Juan County East of R.14W. and Rio Arriba County
Datum Adjusted is 7,942 feet above Initial Datum   Datum Adjustment Factor 1.0003800

Dist. E. & W. from Central Line (feet)	Elev. of Plane Tangent at Cen. Line (feet)	Datum Elevation (feet)	Actual Ground Elev. (feet)		Elevation Where 1:10,000 Difference Occurs		Difference between Distances on the Ground and on the Datum	
			Min.	Max.	Low	High	At Min. Elev.	At Max. Elev.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	0	+6,201	5,500	7,475	4,111	8,291	1:29,800	1:16,400
10,000	2	+6,203	5,500	7,420	4,113	8,293	1:29,700	1:17,100
25,000	15	+6,216	5,450	7,050	4,126	8,306	1:27,200	1:25,000
40,000	38	+6,239	5,400	7,200	4,149	8,329	1:24,900	1:21,700
60,000	86	+6,287	5,380	7,500	4,197	8,377	1:23,000	1:17,200
70,000	117	+6,318	5,360	7,650	4,228	8,408	1:21,800	1:15,600
80,000	153	+6,354	5,300	7,200	4,264	8,444	1:19,800	1:24,700
105,000	264	+6,465	5,250	6,850	4,375	8,555	1:17,200	1:54,300
130,000	404	+6,605	5,200	7,200	4,515	8,695	1:14,800	1:35,100
135,000	436	+6,637	6,300	7,220	4,557	8,727	1:62,000	1:35,600
145,000	503	+6,704	6,590	7,405	4,614	8,794	1:183,300	1:29,800
160,000	612	+6,813	6,750	7,500	4,723	8,903	1:331,800	1:30,400
180,000	775	+6,976	6,800	7,600	4,886	9,066	1:118,700	1:35,500
185,000	819	+7,020	6,600	7,500	4,930	9,110	1:49,700	1:43,500
200,000	957	+7,158	6,800	7,500	5,068	9,248	1:58,300	1:61,100
210,000	1,055	+7,256	6,750	7,600	5,166	9,346	1:41,300	1:60,700
215,000	1,106	+7,307	6,580	8,000	5,217	9,397	1:28,700	1:30,100
230,000	1,265	+7,466	6,600	8,100	5,376	9,556	1:24,100	1:32,900
240,000	1,378	+7,579	6,750	8,300	5,489	9,669	1:25,200	1:28,900
250,000	1,495	+7,696	6,780	9,050	5,606	9,786	1:22,800	1:15,400
255,000	1,555	+7,756	6,790	9,000	5,666	9,846	1:21,600	1:16,800
260,000	1,617	+7,818	6,800	8,600	5,728	9,908	1:20,500	1:26,700
265,000	1,680	+7,881	6,800	8,800	5,791	9,971	1:19,300	1:22,700
270,000	1,744	+7,945	6,950	8,400	5,855	10,035	1:21,000	1:45,900
280,000	1,875	+8,076	7,000	7,800	5,986	10,166	1:19,400	1:75,700

For an illustrative example applicable to results of these computations, refer to the central portion, between points N, of Figure 8.

the initially established State plane coordinate system to distance on sea level arc of the earth.

The multiplication factor (MF) expresses the ratio of distance on the adjusted datum to distance on the sea level arc of the earth, and is equal to the radius of the earth (R) plus Z_1 divided by R. For any point of concern the datum adjustment factor (DAF) is the MF for the point divided by the SF for the point. The DAF expresses the ratio of distance on the adjusted datum to distance on the initial datum. The adjusted datum is height Z above (or below) the initial datum, which is the equivalent of Z_1 plus Z_2 for any point of concern. Adjustment of the datum in this manner establishes the plane coordinate system for the area or zone requiring datum adjustment.

The results attained by placing the adjusted datum above or below the initial datum, where the adjusted datum will be not more than the predetermined permissible amount above or below any point on the ground, are subsequently illustrated in Figure 8.

TABLE 12
Datum Adjustment Computations
TRANSVERSE MERCATOR STATE PLANE COORDINATE SYSTEM

Area of Application New Mexico - West Zone, San Juan County West of R.13W.
Datum Adjusted Datum Adjustment Factor 1.0002600
is 5,434 feet above Initial Datum

Dist. E. & W. from Central Line (feet)	Elev. of Plane Tangent at Cen. Line (feet)	Datum Elevation (feet)	Actual Ground Elev. (feet)		Elevation Where 1:10,000 Difference Occurs		Difference between Distances on the Ground and on the Datum	
			Min.	Max.	Low	High	At Min. Elev.	At Max. Elev.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
0	0	+3,693						
125,000	374	+4,067	5,210	6,610	1,977	6,157	1:18,200	1:8,200
135,000	436	+4,129	5,200	7,050	2,039	6,219	1:19,500	1:7,100
160,000	612	+4,305	5,130	6,700	2,215	6,395	1:25,300	1:8,700
170,000	691	+4,384	5,100	6,620	2,294	6,474	1:29,100	1:9,300
190,000	863	+4,556	5,040	6,450	2,466	6,646	1:43,100	1:11,000
200,000	957	+4,650	5,000	6,600	2,560	6,740	1:59,700	1:10,700
220,000	1,158	+4,851	4,950	6,800	2,761	6,941	1:211,100	1:10,700
240,000	1,378	+5,071	4,900	6,050	2,981	7,161	1:122,200	1:21,300
255,000	1,555	+5,248	4,870	6,650	3,158	7,338	1:55,300	1:14,900
265,000	1,680	+5,573	4,820	7,150	3,283	7,463	1:37,800	1:11,700
280,000	1,875	+5,568	4,790	8,000	3,478	7,658	1:26,800	1:8,500
290,000	2,012	+5,705	4,780	8,820	3,615	7,795	1:22,600	1:6,700
305,000	2,225	+5,918	4,760	8,800	3,828	8,008	1:18,000	1:7,200
325,000	2,526	+6,219	4,740	8,850	4,129	8,309	1:14,100	1:7,900
330,000	2,605	+6,298	4,730	8,860	4,208	8,388	1:13,300	1:8,100
345,000	2,847	+6,450	4,700	9,050	4,450	8,630	1:11,300	1:8,300
350,000	2,930	+6,623	4,800	8,500	4,533	8,713	1:11,400	1:11,100
360,000	3,100	+6,793	4,800	8,450	4,703	8,883	1:10,400	1:12,600

Comment Regarding Table 12

The distance the western portion of large San Juan County is west of Range 13 West in New Mexico, combined with what the elevation of the adjusted datum would be for this area if the DAF of 1.0003800 were used (as for the portion of San Juan County east of this area, which is included in the example of Table 11), made use of the DAF of 1.0002600 necessary. Otherwise, optimum positioning of the adjusted datum to favor the portions of the area at elevations where most of the surveying will be done would not have been achieved. The datum established thereby is exemplified by the adjusted datum SS on the left side of Figure 8. Had the west zone of New Mexico been wider than it is, another datum adjustment for the portion farther west would be necessary. Consequently, wherever precision is necessary, it is better for engineering and cadastral surveys when the plane coordinate zone is not wide.

Figure 8 is an exaggerated diagrammatic cross section applicable to either a transverse Mercator or a Lambert conformal coordinate system.

Points V and T are intersections of sea level arc and initial datum, and V′ and T′ of the ground for one of the adjusted datums, and V″ and T″ for the other adjusted datum.

Points S and N on the adjusted datums and points G and G′ on the ground, respectively, represent the limits of applicability of the adjusted datums. The distance from A to B is a distance on the ground and distance A_1 to B_1 is a projection of that distance on the adjusted datum, which is equal to the ground distance A to B_1'. Distance A_2 to B_2 represents the projection of distance A to B on the sea level arc, and distance A_3

TABLE 13
Datum Adjustment Computations
LAMBERT CONFORMAL STATE PLANE COORDINATE SYSTEM

Area of Application **Arkansas - North Zone, West Central Counties**
Datum **Adjusted** Datum Adjustment Factor **1.0001100**
is 2,299 feet above initial datum

Angle in Min. from Cen. Line	Latitude Deg. Min. North and South from Cen. Line		Elev. of Plane Tang. at Central Line	Datum Elevation	Actual Ground Elev. (feet)		Elevation Where 1:10,000 Difference Occurs		Difference between Distances on the Ground and on the Datum	
	North	South	(feet)	(feet)	Min.	Max.	Low	High	At Min. Elev.	At Max. Elev.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0	35°35'	35°35'	0	+959	490	2,000	-1,131	3,049	1:44,500	1:20,000
1	36'	34'	1	+960	490	2,100	-1,130	3,050	1:44,400	1:18,300
5	40'	30'	22	+981	390	2,128	-1,109	3,071	1:35,300	1:18,200
7	42'	28'	43	+1,002	360	2,127	-1,088	3,092	1:32,500	1:18,500
9	44'	26'	72	+1,031	350	2,361	-1,059	3,121	1:30,600	1:15,700
10	45'	25'	88	+1,047	350	2,250	-1,043	3,137	1:29,900	1:17,300
13	48'	22'	149	+1,108	340	2,567	-982	3,198	1:27,200	1:14,300
17	52	18'	256	+1,215	330	2,578	-875	3,305	1:23,600	1:15,300
21	56'	14'	390	+1,349	300	2,380	-741	3,439	1:19,900	1:20,200
24	59'	11'	509	+1,468	290	2,282	-622	3,558	1:17,700	1:25,600
25	36°00'	10'	553	+1,512	290	2,285	-578	3,602	1:17,100	1:27,000
29	04'	06'	744	+1,703	490	2,385	-387	3,793	1:17,200	1:30,600
31	06'	04'	850	+1,809	990	2,370	-281	3,899	1:25,500	1:37,200
38	13'	34°57'	1,277	+2,236	1,100	1,680	+146	4,326	1:18,400	1:37,600
42	17'	53'	1,560	+2,519	1,290	1,695	+429	4,609	1:17,000	1:25,300
43	18'	52'	1,636	+2,595	1,290	1,710	+505	4,685	1:16,000	1:23,600

For an illustrative example applicable to results of these computations. refer to the central portion, between points N, of Figure 8.

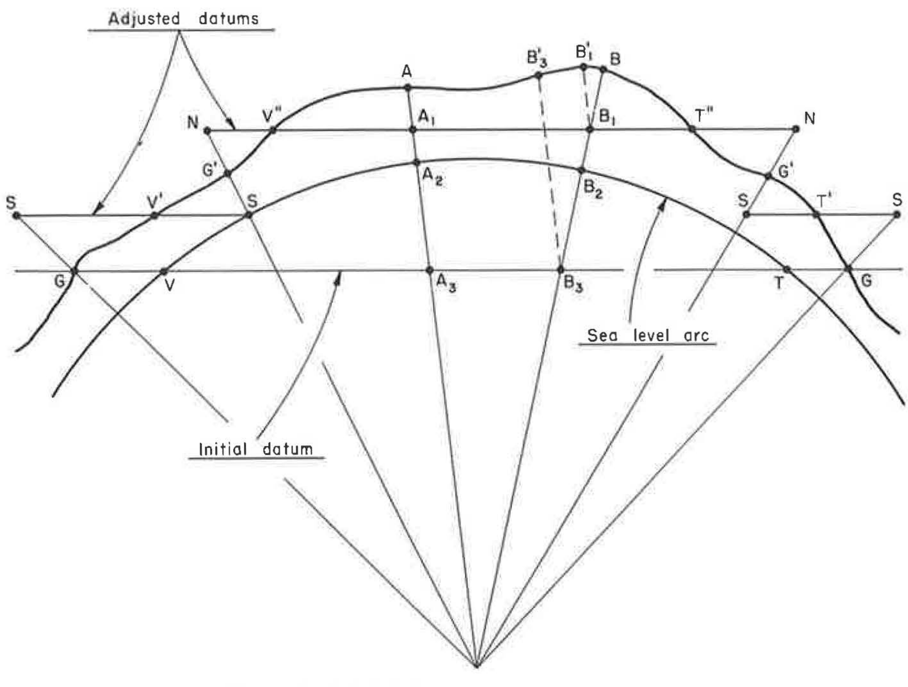

Figure 8. Initial datum and adjusted datums.

to B_3 onto the initial datum which is equal to the ground distance A to B_3'. The distance B_1' to B is the difference between the distance on the ground and distance on the adjusted datum. The distance B_3' to B is the difference between the distance on the ground and distance on the initial datum.

From the foregoing, it is obvious the adjusted datum greatly reduces the distance differences. The reduction in size of distance differences thus accomplished makes full use of the benefits from and the advantages of the State plane coordinate system of basic control.

A mathematical example of computations illustrated by Figure 7, resulting in the adjusted datum illustrated in Figure 8, is presented in the following. This example applies to both the west and central transverse Mercator zones of the State of New York. At the central line of these zones the scale factor is 0.9999375, and at 105,000 feet from the central line of these zones the scale factor is 0.9999501. Thus, in Figure 7, h at the central meridian is 1,307 feet, and Z_2 is 1,043 feet from the sea level arc of the earth down to the initial datum at the distance of 105,000 feet east and west, respectively, from the central line of each of the two zones. Actually, the radius of curvature of the earth's sea level arc of 20,906,000 feet multiplied by (0.9999375 - 1.0000000) is the 1,307 feet, and multiplied by (0.9999501 - 1.0000000) is the 1,043 feet. For this zone the trial datum adjustment factor, which proved to be the correct one for optimum solution of the adjustment problem, is 1.0000800 for the

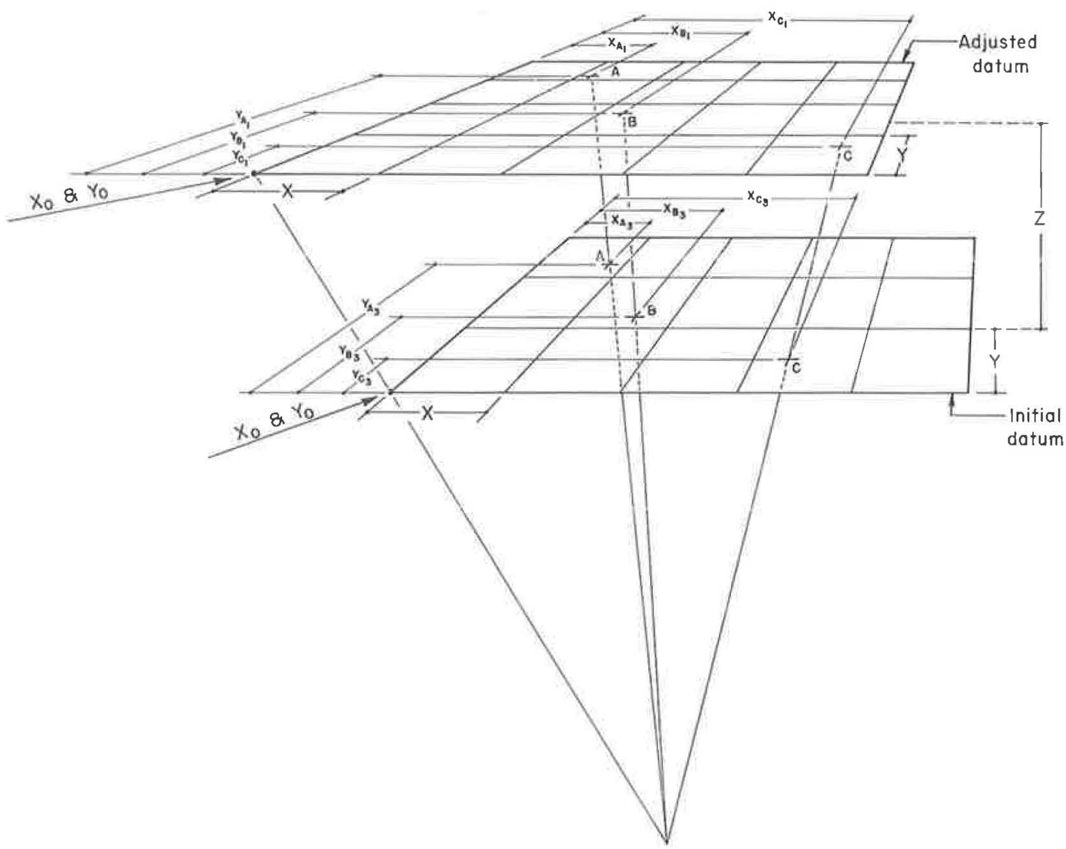

Figure 9. The datum adjustment: To reduce distance differences, datum adjustment changes the X and Y position of points with respect to X_0 and Y_0 but does not change the coordinate grid lines.

central counties of these zones. This factor places the adjusted datum 1,672 feet above the initial datum, which is actually 1.0000800 times the radius of curvature of 20,906,000 minus that radius. The same height of 1,672 feet can be computed more easily by multiplying the elevation increment of datum adjustment of 209 feet by the numeral 8. The magnitude of Z_1 for the height of the adjusted datum above the sea level arc at the distance of 105,000 feet from the central line of the zone is Z minus Z_2, or expressly, 1,672 minus 1,043, which equals 629 feet. Accordingly, the MF is 20,906,000 plus 629 divided by 20,906,000, resulting in 1.0000301. As explained previously, the datum adjustment factor is the MF divided by the SF. Thus the DAF for all points on a line 105,000 feet east and west, respectively, from the central line of the zone of consideration is 1.0000301 divided by 0.9999501, which is 1.0000800. The datum adjustment factor may be similarly computed for all points along each line at any other distance from and parallel to the central line of the zone, whether on the transverse Mercator or Lambert conformal system of projection. It should be remembered, however, that the MF and the SF will change from point to point at distances east and west from the central line of a transverse Mercator zone and at increments of latitude north and south from the central line of the Lambert conformal zone. The DAF, however, will not change.

Once the DAF has been computed, after appropriate analysis of the zone of consideration, the plane coordinates of each basic control point on the initial datum are multiplied by the DAF. The results are coordinates for the same points on the adjusted datum, which is parallel to the initial datum. The significance of this adjustment is illustrated in Figure 9. For control points A, B, and C on the initial datum the coordinates are designated as XA_3 and YA_3, XB_3 and YB_3, XC_3 and YC_3. The multiplication of such coordinates by the datum adjustment factor results in coordinates for the same points on the adjusted datum of XA_1 and YA_1, XB_1 and YB_1, and XC_1 and YC_1. Of course the diagrammatic representation in Figure 9 of the change of datum for plane coordinates of the points A, B, and C, respectively, is highly exaggerated in order to obtain separation of lines for illustrating what actually occurs. It should be noted that the X_0 and Y_0 coordinates are the same numerically for each datum.

The datum adjustment factor used to place the control points on the adjusted datum in each specific area should be appropriately recorded on each map compiled using datum adjusted control. Thereby the map users will be made fully aware of the fact that all coordinates on the map apply to the adjusted datum.

Whenever plane coordinates of points on the adjusted datum are to be reverted to plane coordinates of each point of concern on the initial datum, divide the coordinates of the points on the adjusted datum by the datum adjustment factor.

Ordinarily, all basic control surveying would be done by attaining closures on the initial datum. Then before the maps are compiled, the plane coordinates of each control point being used are separately multiplied by the DAF applicable in the mapping area to compute plane coordinates for each point which will be used to control map compilation and all subsequent surveying on the adjusted datum. Equations in the plane coordinates are required at each control point used at the border of change from one plane coordinate zone or area of datum adjustment to another.

Should complete dissimilarity between coordinates of points on the adjusted datum and of points on the initial datum be desired, this can be achieved easily by subtracting or adding, as deemed appropriate, a numerical constant of sufficient magnitude to prevent similarity. When and wherever this is done, the constant used to make the dissimilarity must be appropriately added or subtracted before dividing by the datum adjustment factor to revert the datum-adjusted plane coordinates to plane coordinates on the initial datum. Also, this constant should be recorded on each map wherever it is used to alter the plane coordinates of points before compiling each map on the adjusted datum.

All surveying for engineering and cadastral purposes which is done originating and closing on datum adjusted control points will not require further adjustment of distances unless errors are made in the measurements. This is because difference limits set by the adjustment and distances measured accurately on the ground and distances computed by use of plane coordinates of control points and of map features will be in

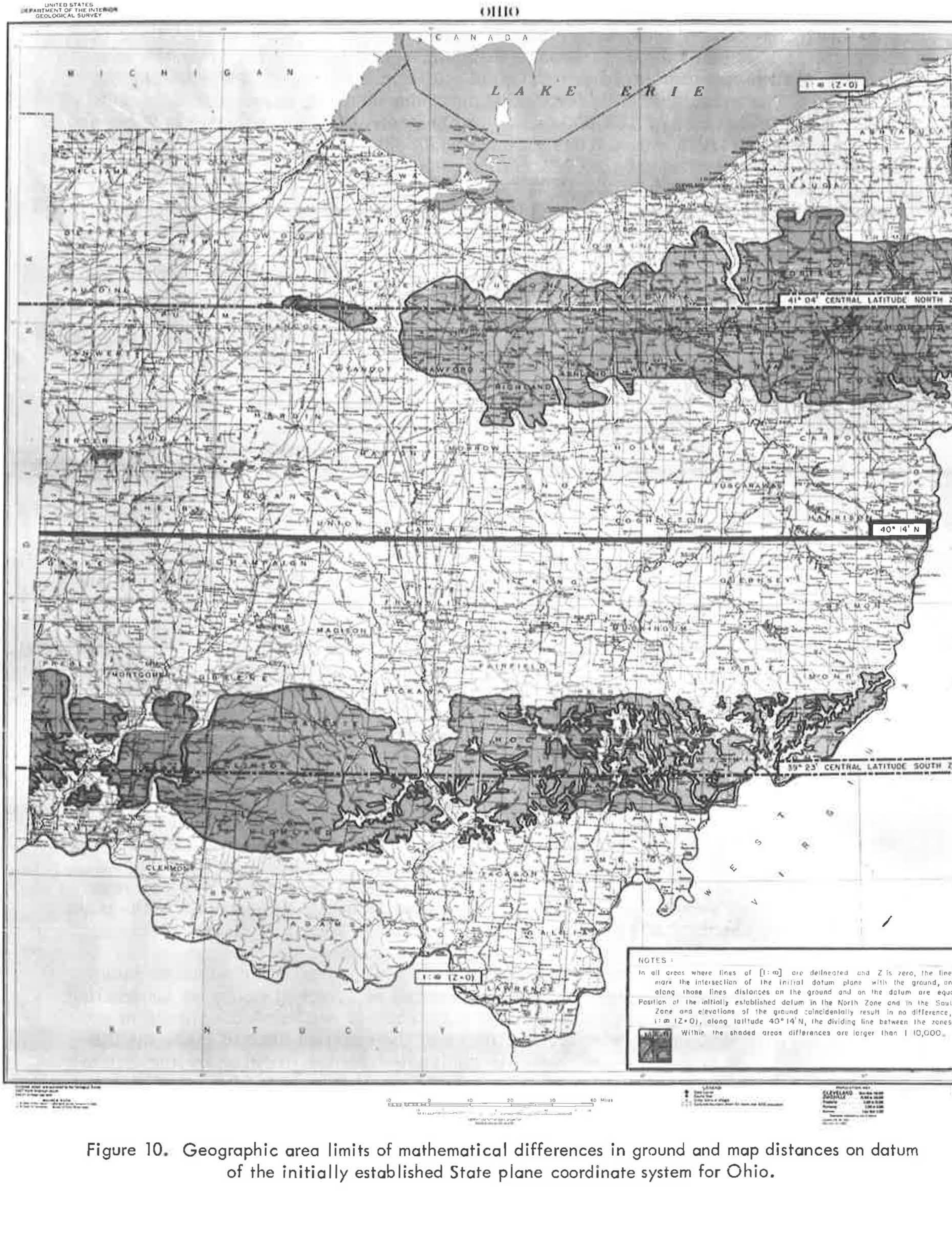

Figure 10. Geographic area limits of mathematical differences in ground and map distances on datum of the initially established State plane coordinate system for Ohio.

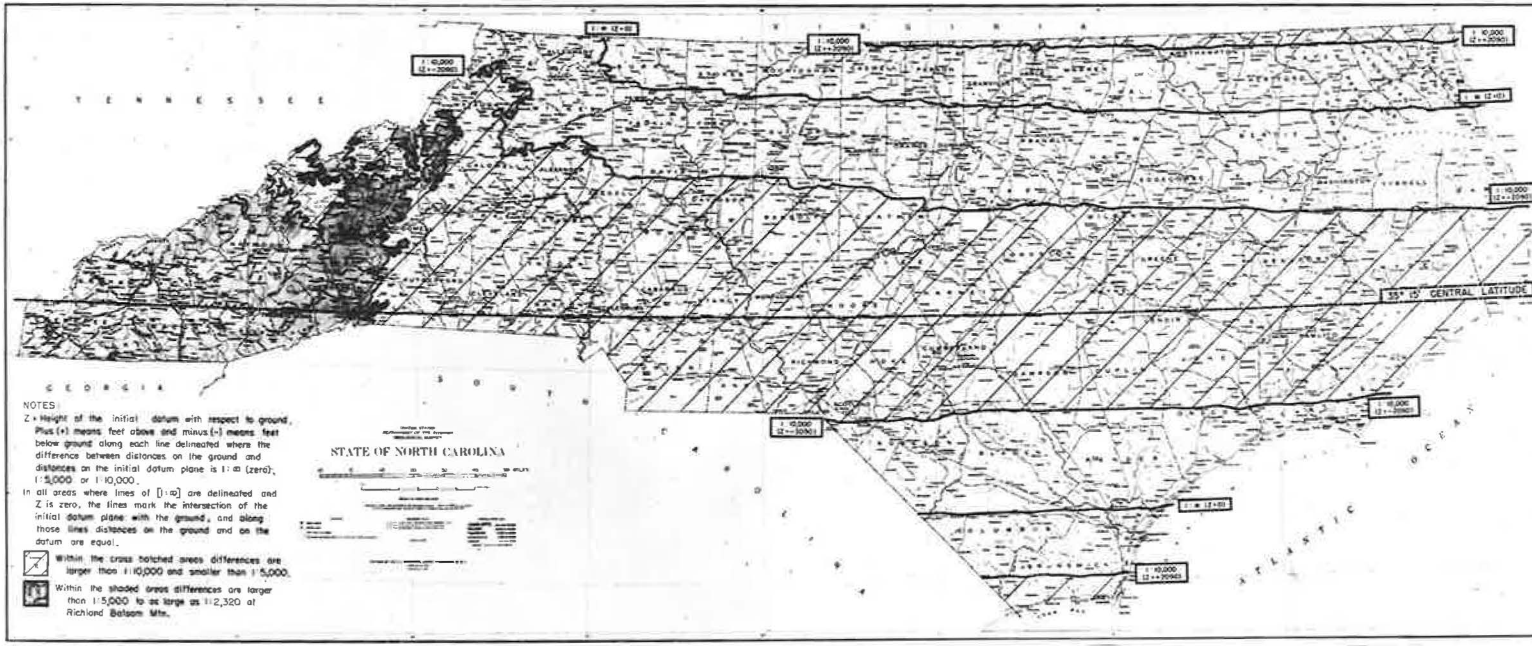

Figure 11. Geographic area limits of mathematical differences in ground and map distances on datum of the initially established State plane coordinate system for North Carolina.

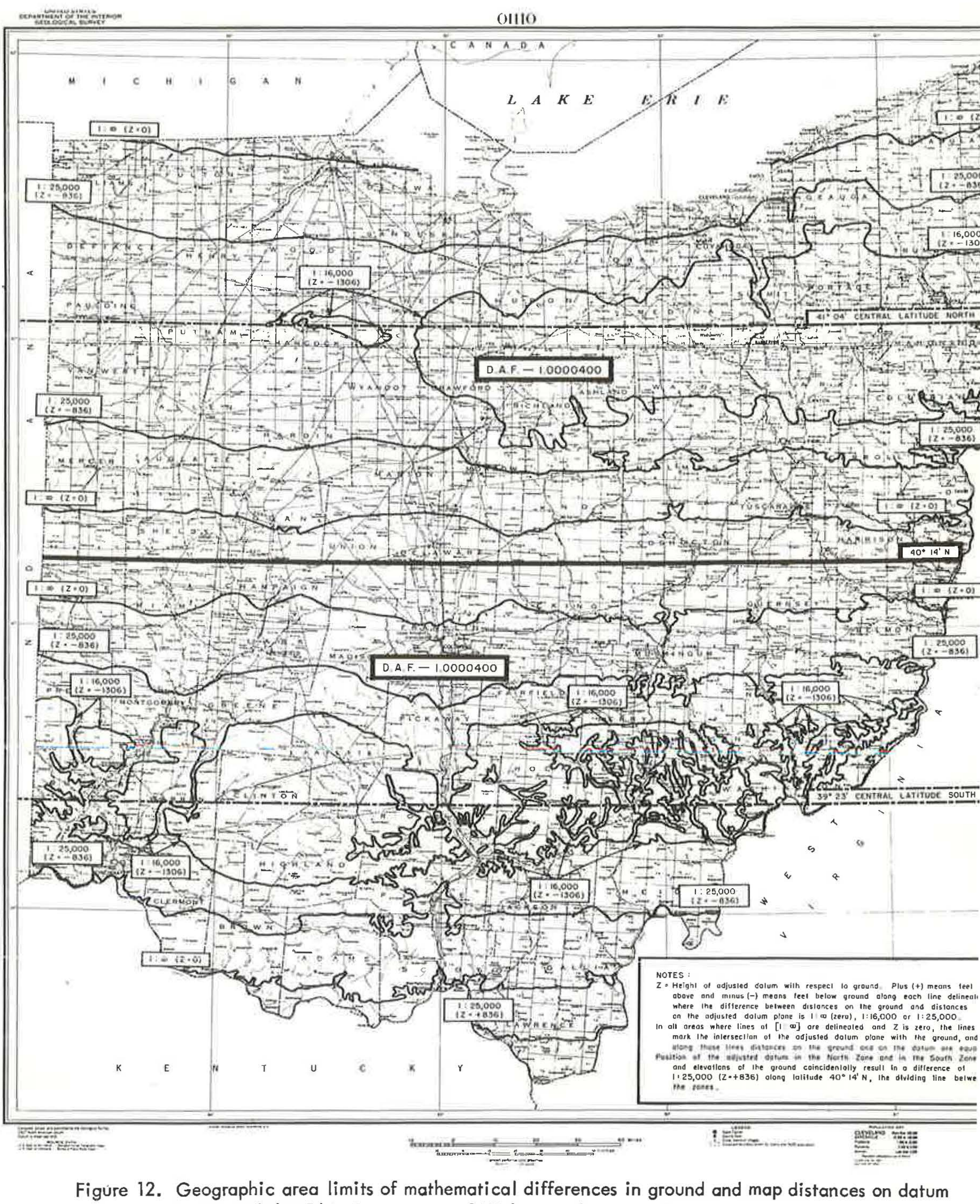

Figure 12. Geographic area limits of mathematical differences in ground and map distances on datum of the Ohio Department of Highways plane coordinate system.

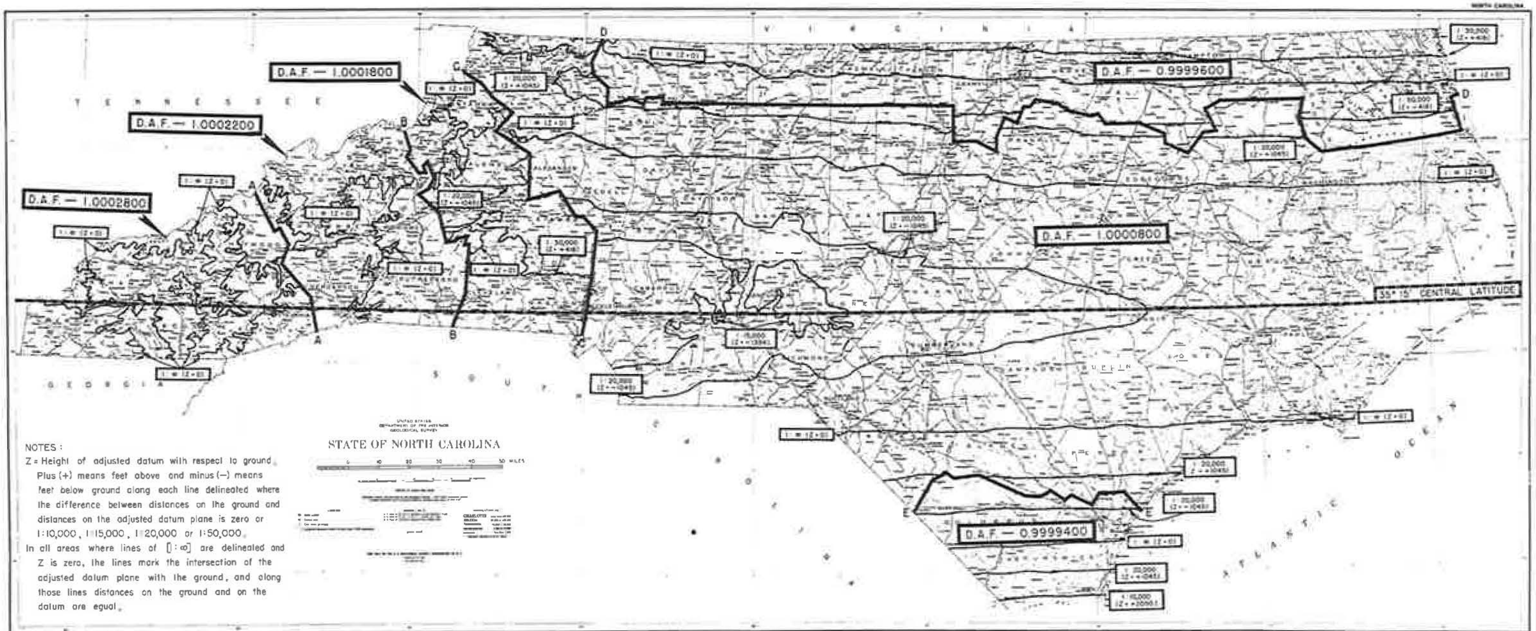

Figure 13. Geographic area limits of mathematical differences in ground and map distances on datum of the North Carolina State Highway Commission plane coordinate system.

agreement within acceptable tolerances. All intermediate points set on the ground and measured therefrom will also be on the adjusted datum. Remember, the bearing of a line between two points on the initial datum is not changed by use of the DAF which places the same points on the adjusted datum. Examples of conditions existing before datum adjustment are given in Figures 10 and 11 for the States of Ohio and North Carolina. For the same States, examples of results from adjusting the datum on a zone basis appear in Figure 12 for Ohio and on an area basis in Figure 13 for North Carolina.

Figure 10 is a planimetric map of Ohio. On this map are designated the central latitude of 41° 04'N. for the north and of 37° 23'N. for the south plane coordinate zones of the State, and the latitude of 40° 14'N. marking the general boundary between the two zones. The shaded area in the central portion of each zone indicates where the elevation of the ground is so far above the elevation of the initial datum that the difference between distances on the ground and distances on the datum are larger than 1:10,000. The lines comprising the boundary of each shaded area demarcate where the differences are 1:10,000 and the initial datum is 2,090 feet below the ground. The lines where the ground and the datum intersect are also shown. These lines have the identification insert of 1:∞ and indicate where Z is 0.

Figure 11 is a planimetric map of North Carolina. On this map the central latitude of the one plane coordinate zone of the State is designated at 35° 15'N. The central portion and portions along the north and south borders of the State are crosshatched to indicate where the elevation of the initial datum is so far below the ground within the central portion and so far above the ground near the edges of the zone that the difference between distances on the ground and on the datum are larger than 1:10,000. The lines marking the boundaries of these crosshatched areas of course indicate exactly where the 1:10,000 difference occurs. The shaded area in the western portion of the State, where the highest mountains exist, indicates where the initial datum is so far below the ground that the distance differences are larger than 1:5,000, and actually range to as large as 1:2,320. The lines where the ground and the initial datum intersect are indicated where Z is 0 and the differences are 1:∞.

From an examination of both Figures 10 and 11, it is obvious, as it was when the completed adjustment computations for the initial datum were reviewed, that a datum adjustment was necessary for each State and could be made on a zone basis for Ohio and on a geographic area basis for North Carolina with county lines marking the boundary of each area.

The maps comprising Figures 12 and 13 for Ohio and North Carolina, respectively, contain the delineations which portray results achieved by the respective datum adjustments. For Ohio, one DAF of 1.0000400 is adequate for both the north and south plane coordinate zones of the State. This is a fortunate situation.

On the map of Figure 12 various lines have been drawn. Each line designates topographically where a specific difference in distance between ground and map occurs. Along the lines where the adjusted datum is the Z distance of 1,306 feet below the ground, the difference is 1:16,000. Within the area bounded by these lines the differences are larger than 1:16,000, but none are as large as 1:10,000. The next designated line within each zone is where the adjusted datum is the Z distance of 836 feet below the ground and the differences are 1:25,000. Thus, between the lines where the datum is 1,306 feet and 836 feet below the ground, the differences range from 1:16,000 to 1:25,000. The next significant lines are where Z is 0, the datum and ground are coincident, and the differences are 1:∞, meaning nil. Thus, from an examination of this map, it is evident the datum adjustment made for each zone achieved an optimum balance between distance differences on the ground and on the datum of projection.

On Figure 13 for North Carolina the consequences of datum adjustment are also exemplified. The size of the zone comprising this State and its general shape and extremes in ground elevation precluded making a datum adjustment on a zone basis. Instead, as mentioned previously, the datum adjustment had to be made on a geographic area basis bounded by county lines. For the largest portion, comprising the central eastern part of the State, a DAF of 1.0000800 kept most of the distance differences to less than 1:20,000, a significant number to no larger than 1:15,000, and none larger than 1:10,000, as indicated by the respective lines indicating where differences of 1:15,000, 1:20,000, and 1:∞ occur. Within the extreme north and south portions of the State where ground elevations are generally low, the initial datum was too high above the ground as indicated in Figure 11 and a lowering of the datum was essential. Thus, for the northern portion a DAF of 0.9999600 was necessary, and for the southern portion a DAF of 0.9999400 was required. Such adjustments kept differences to well within the acceptable limits, not exceeding 1:10,000. In the western mountainous portion of the State, three separate datum adjustment factors were required. As can be seen by review of the lines where difference designations occur, an optimum condition was achieved. Only at the highest peaks, which were too small to delineate on this map, where little, if any, highway surveying will be done, were the differences as large as 1:10,000, and at one mountain peak the maximum difference is no larger than 1:7,700. Similar examples could be presented for other States, but Ohio, on a zone basis, and North Carolina, on a geographic area basis, are representative.

CONCLUSIONS

Use of the system of State plane coordinates within each State, by making an appropriate adjustment of the datum on a zone or area basis, is vital to accomplishing precision mapping, highway design, and location and cadastral surveying, and to achieving similarity between distances determined from coordinates of basic control points on the adjusted datum and distances measured on the ground between such points and between them and all highway and cadastral survey points. Moreover, adoption and use of State plane coordinates on an adjusted datum determined by a thorough analysis of the topography of each State on an area basis—the boundaries of each area being agreed to by all concerned after the analysis is completed—not only accomplishes the foregoing, but makes it possible to take advantage of the numerous benefits provided by the State plane coordinate system, which include:

1. Achieving the correlation and continuity desirable between independent surveys;
2. Making easy the precise computation and accurate staking of specific points of engineering facilities and property boundaries;
3. Obviating the need for remeasuring or loop closing traverses and triangulation nets to determine their accuracy (such is automatically accomplished by originating and closing each survey on station markers in the national network of basic control surveys on which the State plane coordinates in the area of the surveys are based);
4. Expressing positions, distances, and bearings in easily understood and used forms;
5. Increasing the density of control points and decreasing the distance between them whenever a survey is preserved by placement of adequately durable markers at each significant station of a traverse or triangulation survey or combination thereof, and thereby increasing the usefulness and benefits from the national network of basic control surveys; and
6. Knowing the X and Y coordinates of numerous points set and surveyed for engineering and cadastral purposes, and having them readily available as reliable and precise origin and closure station markers for use in resetting each destroyed station marker and in measuring the position of each new station marker set when making subsequent surveys.

With the foregoing in mind, all who make engineering and cadastral surveys are urged to use fully the State plane coordinate systems. By so doing, the systems are not affected adversely because each map compiled on an adjusted datum will contain the datum adjustment factor. Thus, where and when necessary, the X and Y coordinates of basic control and other points can be reverted to the initial datum from the coordinates on the adjusted datum used for measuring and mapping purposes. Of equal or greater importance, distances computed using the X and Y coordinates of points on the adjusted datum and distances surveyed on the ground will be in agreement within usable and acceptable limits.